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PREFACE

In this era of cut throat competition among **JEE** aspirants, only few are lucky enough to make it through. During the preparation, a student needs the most relevant study material to cover all the topics with proper planning to crack **JEE**.

We know there is abundance of study materials floating in the market but what We felt missing, is a self-driven and well-planned Study material. We have tried to bring clarity and Quality in the students by introducing this **Aspire for JEE**

The book covers all the important concepts and theories with a rich amount of practice questions as well. This book is designed for the perfect guidance you need.

The study material is consistent with the syllabus offered in NCERT Books as well and has been curated by industry experts under the careful observation of **Physics Wallah Team**.

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1 Units and Measurements

PHYSICAL QUANTITIES

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationships.

MEASUREMENT

Measurement is the comparison of a quantity with a standard of the same physical quantity.

Different countries followed different standards.

UNITS

All physical quantities are measured w.r.t. standard magnitude of the **same** physical quantity and these standards are called UNITS. eg. second, meter, kilogram, etc.

So the four basic properties of units are:

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

SET OF FUNDAMENTAL QUANTITIES

A set of physical quantities which are completely independent of each other and all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.

DERIVED PHYSICAL QUANTITIES

The physical quantities that can be expressed in terms of fundamental physical quantities are called derived physical quantities.eg. speed = distance/time.

SYSTEM OF UNITS

1. **FPS or British Engineering system :** In this system length, mass and time are taken as fundamential quantities and their base units are foot (ft), pound (lb) and second (s) respectively.
2. **CGS or Gaussian system :** In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).
3. **MKS system :** In this system also the fundamental quantities are length, mass and time but their fundamental units are mete (m), kilogram (kg) and second (s) respectively.

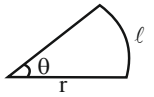
Units of some physical quantities in different systems

Type of physical Quantity	Physical Quantity	System		
		CGS	MKS	FPS
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s

4. **International system (SI) of units :** This system is modification over the MKS system. Besides the three base units of MKS system four fundamental and two supplementary units are also included in this system.

Table : SI base quantities and their units

S. No.	Physical quantity	unit	Symbol
1	Length	metre	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	kg
5	Electric current	ampere	A
6	Luminous Intensity	candela	cd
7	Amount of substance	mole	mol

Physical Quantity (SI Unit)	**Definition**
Length (m)	The distance travelled by light in vacuum in $\frac{1}{299,792,458}$ second is called **1 metre.**
Mass (kg)	The mass of a cylinder made of platinum-iridium alloy kept at International Bureau of Weights and Measures is defined as **1 kilogram**.
Time (s)	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
Electric Current (A)	If equal currents are maintained in the two parallel infinitely long wires of negligible cross-section, so that the force between them is 2×10^{-7} newton per metre of the wires, the current in any of the wires is called **1 Ampere**.
Thermodynamic Temperature (K)	The fraction $\frac{1}{273.16}$ of the thermodynamic temperature of triple point of water is called **1 Kelvin**
Luminous Intensity (cd)	**1 candela** is the luminous intensity of a blackbody of surface area $\frac{1}{600,000}m^2$ placed at the temperature of freezing platinum and at a pressure of 101,325 N/m^2, in the direction perpendicular to its surface.
Amount of substance (mole)	**The mole** is the amount of a substance that contains as many elementary entities as there are number of atoms in 0.012 kg of carbon-12.
There are two supplementary units too: **1. Plane angle (radian)**	angle = arc / radius $\theta = \ell / r$ 
2. Solid Angle (steradian)	Solid angle = Area/(radius^2)

DIMENSIONS AND DIMENSIONAL FORMULA

All the physical quantities of interest can be derived from the base quantities. "The power (exponent) of base quantity that enters into the expression of a physical quantity, is called the dimension of the quantity in that base. To make it clear, consider the physical quantity force.

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$= \text{mass} \times \frac{\text{length / time}}{\text{time}}$$

$$= \text{mass} \times \text{length} \times (\text{time})^{-2}$$

So the dimensions of force are 1 in mass, 1 in length and –2 in time. Thus

$$[\text{Force}] = MLT^{-2}$$

Similarly energy has dimensional formula given by

$$[\text{Energy}] = ML^2T^{-2}$$

i.e. energy has dimensions, 1 in mass, 2 in length and -2 in time.

Such an expression for a physical quantity in terms of base quantities is called dimensional formula.

Physical quantity can be further of four types :

(1) dimension less constant i.e. 1,2,3,π etc.
(2) Dimension less variable i.e. angle θ etc.
(3) dimensional constant i.e. G, h etc.
(4) Dimensional variable i.e. F, v, etc.

DIMENSIONAL EQUATION

Whenever the dimension of a physical quantity is equated with its dimensional formula, we get a dimensional equation.

PRINCIPLE OF HOMOGENEITY

The magnitude of a physical quantity may be added or subtracted from each other only if they have the same dimension, also the dimension on both sides of an equation must be same. This is called as principle of homogenity.

TRAIN YOUR BRAIN

Example 1: The distance covered by a particle in time t is going by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d.

Sol. The equation contains five terms. All of them should have the same dimensions. Since [x] = length, each of the remaining four must have the dimension of length.

Thus, $[a] = \text{length} = L$

$[bt] = L$, or $[b] = LT^{-1}$

$[ct^2] = L$, or $[c] = LT^{-2}$

and $[dt^3] = L$ or $[d] = LT^{-3}$

Example 2: Calculate the dimensional formula of energy from the equation $E = \frac{1}{2}mv^2$.

Sol. Dimensionally, $E = \text{mass} \times (\text{velocity})^2$, since $\frac{1}{2}$ is a number and has no dimension.

$$\text{or, } [E] = M \times \left(\frac{L}{T}\right)^2 = ML^2T^{-2}.$$

Example 3: Kinetic energy of a particle moving along elliptical trajectory is given by $K = \alpha s^2$ where s is the distance travelled by the particle. Determine dimensions of α.

Sol. $K = \alpha s^2$

$$[\alpha] = \frac{(M L^2 T^{-2})}{(L^2)}$$

$$[\alpha] = M^1 L^0 T^{-2}$$

$$[\alpha] = M T^{-2}$$

Example 4: The position of a particle at time t, is given by the equation, $x(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$.

The dimensions of v_0 & α are respectively.

(*a*) $M^0 L^1 T^0$ & T^{-1}
(*b*) $M^0 L^1 T^{-1}$ & T
(*c*) $M^0 L^1 T^{-1}$ & T^{-1}
(*d*) $M^1 L^1 T^{-1}$ & LT^{-2}

Ans. **(c)**

Sol. $[v_0] = [x][\alpha]$ & $[\alpha][t] = M^0L^0T^0$

$= M^0L^1T^{-1}$ $[\alpha] = M^0L^0T^{-1}$

USES OF DIMENSIONAL ANALYSIS

(I) To check the dimensional correctness of a given physical relation :

It is based on principle of homogeneity, which states that a given physical relation is dimensionally correct if the dimensions of the various terms on either side of the relation are the same.

Remark :

- Powers are dimensionless
- $\sin\theta$, e^θ, $\cos\theta$, $\log\theta$ gives dimensionless value and in above expression θ is dimensionless
- we can add or subtract quantity having same dimensions.

TRAIN YOUR BRAIN

Example 5: Check the accuracy of the relation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

for a simple pendulum using dimensional analysis.

Sol. The dimenstions of LHS = the dimension of

$T = [M^0L^0T^1]$

The dimensions of $RHS = \left(\frac{\text{dim.of length}}{\text{dim.of acc}^n}\right)^{1/2}$

($\because$ 2π is a dimensionless const.)

$$= \left(\frac{L}{LT^{-2}}\right) = (T^2)^{1/2} = (T) = [M^0L^0T^1]$$

(II) To establish a relation between different physical quantities:

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors by using principle of homogeneity.

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Example 6: Let us find an expression for the time period t of a simple pendulum. The time period t may possible depend upon (i) mass m of the bob of the pendulum, (ii) length ℓ of pendulum, (iii) acceleration due to gravity g at the place where the pendulum is suspended.

Let (i) $t \propto m^a$ (ii) $t \propto \ell^b$ (iii) $t \propto g^c$

Combining all the three factors, we get

$t \propto m^a\ell^bg^c$ or $t = Km^a\ell^bg^c\theta^d$

where K is a dimensionless constant of proportionality.

Writing down the dimensions on either side of equation (i), we get

$$[T] = [M^a][L^b][LT^{-2}]^c = [M^aL^{b+c}T^{-2c}]$$

Comparing dimensions, $a = 0, b + c = 0, -2c = 1$

$\therefore$ $a = 0, c = -1/2, b = 1/2$

From equation (i) $t = Km^0\ell^{1/2}g^{-1/2}$

or $$t = K\left(\frac{\ell}{g}\right)^{1/2} = K\sqrt{\frac{\ell}{g}}$$

The value of K, as found by experiment or mathematical investigation, comes out to be 2π.

$\therefore$ $$t = 2\pi\sqrt{\frac{\ell}{g}}$$

Example 7: When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Sol. Suppose the formula is $F = k\,\eta^a\,r^b\,v^c$

Then, $MLT^{-2} = [ML^{-1}T^{-1}]^aL^b\left(\frac{L}{T}\right)^c$

$$= M^a\,L^{-a+b+c}\,T^{-a-c}$$

Equating the exponents of M, L and T from both sides,

$a = 1$

$-a + b + c = 1$

$-a - c = -2$

Solving these, $a = 1$, $b = 1$ and $c = 1$

Thus, the formula for F is $F = k\eta rv$.

Example 8: If P is the pressure of a gas and ρ is its density, then find the dimension of velocity

(*a*) $P^{1/2}\rho^{-1/2}$ (*b*) $P^{1/2}\rho^{1/2}$

(*c*) $P^{-1/2}\rho^{1/2}$ (*d*) $P^{-1/2}\rho^{-1/2}$

Sol. Method - I

$[P] = [ML^{-1}T^{-2}]$...(1)

$[\rho] = [ML^{-3}]$ ...(2)

Dividing eq. (1) by (2)

$$[P\rho^{-1}] = [L^2T^{-2}]$$

$\Rightarrow$ $[LT^{-1}] = [P^{1/2}\rho^{-1/2}]$

$\Rightarrow$ $[V] = [P^{1/2}\rho^{-1/2}]$

Method - II

$v \propto P^a\,\rho^b$

$v = kP^a\,\rho^b$

$[LT^{-1}] = [ML^{-1}T^{-2}]^a\,[ML^{-3}]^b$

$a = \frac{1}{2}$, $b = -\frac{1}{2}$

$\Rightarrow$ $[V] = [P^{1/2}\rho^{-1/2}]$

Example 9: Find relationship between speed of sound in a medium (v), the elastic constant (E) and the density of the medium (ρ).

Sol. Let the speed depends upon elastic constant & density according to the relation

$v \propto E^a \rho^b$ or $v = KA^a\rho^b$ ...(1)

Where K is a dimensionless constant of proportionality

Considering dimensions of the quantities

$[v] = M^0 L T^{-1}$

$$[E] = \frac{[\text{stress}]}{[\text{strain}]} = \frac{[\text{force}]/[\text{area}]}{[\Delta\ell]/[\ell]} = \frac{[M^1L^1T^{-2}]/[L^2]}{[L^1]/[L^1]} = [M^1L^{-1}T^{-2}]$$

$\therefore$ $[E^a] = [M^a L^{-a} T^{-2a}]$

$[\rho] = [\text{mass}]/[\text{volume}] = [M]/[L^3] = [M^1L^{-3}T^0]$

$\therefore$ $[\rho^b] = [M^b L^{-3b} T^0]$

Equating the dimensions of the LHS and RHS quantities of equation (1), we get

$[M^0 L^1 T^{-1}] = [M^a L^{-a} T^{-2a}] = [M^b L^{-3b} T^0]$ or $[M^0 L^1 T^{-1}] = [M^{a+b} L^{-a-3b} T^{-2a}]$

Comparing the individual dimensions of M, L & T

$a + b = 0$ ...(2)

$-a - 3b = 1$ ...(3)

$-2a = -1$ ...(4)

Solving we get $a = \frac{1}{2}, b = -\frac{1}{2}$

Therefore the required relation is $v = K\sqrt{\frac{E}{\rho}}$

(III) To convert units of a physical quantity from one system of units to another :

It is based on the fact that,

Numerical value × unit = constant

So on changing unit, numerical value will also gets changed. If n_1 and n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$n_1u_1 = n_2u_2$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

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Example 10: Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Sol. The dimensional equation of force is

$[F] = [M^1 L^1 T^{-2}]$

Therefore if n_1, u_1 and n_2, u_2 corresponds to SI & CGS unit respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{\text{kg}}{\text{g}}\right]\left[\frac{\text{m}}{\text{cm}}\right]\left[\frac{\text{s}}{\text{s}}\right]^{-2} = 1\times1000\times100\times1 = 10^5$$

Example 11: A calorie is a unit of heat or energy and it equals about 4.2 J, where 1 J = 1 kg m^2/s^2. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β metre, the unit of time is γ second. Show that a calorie has a magnitude 4.2 $\alpha^{-1}\beta^{-2}\gamma^2$ in terms of the new units.

Sol. 1 cal = 4.2 kg m^2s^{-2}

SI	**New system**
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = \alpha$ kg
$L_1 = 1$ m	$L_2 = \beta$ metre
$T_1 = 1$ s	$T_2 = \gamma$ second

Dimensional formula of energy is $[ML^2T^{-2}]$

Comparing with $[M^aL^bT^c]$,

We find that a = 1, b = 2, c = –2

Now, $$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ s}}{\gamma\text{ s}}\right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$$

Example 12: Young's modulus of steel is 19×10^{10} N/m^2. Express it in dyne/cm^2. Here dyne is the CGS unit of force.

Sol. The unit of Young's modulus is N/m^2.

This suggest that it has dimensions of $\frac{\text{Force}}{(\text{Distance})^2}$.

Thus, $[Y] = \frac{[F]}{L^2} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

N/m^2 is in SI units,

So, $1\ N/m^2 = (1\ kg)(1m)^{-1}\ (1s)^{-2}$

and $1\ dyne/cm^2 = (1g)(1cm)^{-1}\ (1s)^{-2}$

so, $\frac{1\ N/m^2}{1 dyne/cm^2}$

$$= \left(\frac{1kg}{1g}\right)\left(\frac{1m}{1cm}\right)^{-1}\left(\frac{1s}{1s}\right)^{-2} = 1000 \times \frac{1}{100} \times 1 = 10$$

or, $1\ N/m^2 = 10\ dyne/cm^2$

or, $19 \times 10^{10}\ N/m^2 = 19 \times 10^{11}\ dyne/m^2$.

Example 13: The dimensional formula for viscosity of fluids is,

$\eta = M^1L^{-1}T^{-1}$

Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $\eta = M^1\ L^{-1}\ T^{-1}$

1 CGS units = $g\ cm^{-1}\ s^{-1}$

1 SI units = $kg\ m^{-1}\ s^{-1}$

$= 1000\ g\ (100\ cm)^{-1}\ s^{-1}$

$= 10\ g\ cm^{-1}\ s^{-1}$

Thus, 1 Poiseuilli = 10 poise

Units and dimensions of some physical quantities

Quantity	SI Unit	Dimension
Density	kg/m^3	M/L^3
Force	newton (N)	ML/T^2
Work	joule (J)(=N-m)	ML^2/T^2
Energy	joule(J)	ML^2/T^2
Power	Watt (W) (=J/s)	ML^2/T^3
Momentum	kg-m/s	ML/T
Gravitational constant	$N\text{-}m^2/kg^2$	L^3/MT^2
Angular velocity	radian/s	T^{-1}
Angular acceleration	$radian/s^2$	T^{-2}
Angular momentum	$kg\text{-}m^2/s$	ML^2/T
Moment of inertia	$kg\text{-}m^2$	ML^2
Torque	N-m	ML^2/T^2
Angular frequency	radian/s	T^{-1}
Frequency	hertz (Hz)	T^{-1}
Period	s	T
Surface Tension	N/m	M/T^2
Coefficient of viscosity	$N\text{-}s/m^2$	M/LT
Wavelength	m	L
Intensity of wave	W/m^2	M/T^3
Temperature	Kelvin (K)	K
Specific heat capacity	J/kg-K	L^2/T^2K
Stefan's constant	$W/m^2\text{–}K^4$	M/T^3K^4
Heat	J	ML^2/T^2
Thermal conductivity	W/m-K	ML/T^3K

Current density	A/m^2	I/L^2
Electrical conductivity	$1/\Omega$-m(= mho/m)	I^2T^3/ML^3
Electric dipole moment	C-m	LIT
Electric field	V/m (=N/C)	ML/IT^3
Potential (voltage)	Volt (V) (=J/C)	ML^2/IT^3
Electric flux	V-m	ML^3/IT^3
Capacitance	Farad (F)	I^2T^4/ML^2
Electromotive force	Volt (V)	ML^2/IT^3
Resistance	ohm (Ω)	ML^2/I^2T^3
Permittivity of space	$C^2/N\text{-}m^2$ (=F/m)	I^2T^4/ML^3
Permeability of space	N/A^2	ML/I^2T^2
Magnetic field	Tesla (T) (= Wb/m^2)	M/IT^2
Magnetic flux	Weber (Wb)	ML^2/IT^2
Magnetic dipole moment	N-m/T	IL^2
Inductance	Henry (H)	ML^2/I^2T^2

LIMITATIONS OF DIMENSIONAL ANALYSIS

(i) It supplies no information about dimensionless constants and the nature (vector and scalar) of physical quantities.

(ii) This method fails to derive the exact form of a physical relation, if a physical quantity depends upon more than three other mechanical physical quantities.

(iii) This method is applicable only if relation is of product type. It fails in the case of exponential and trigonometric relations.

(iv) It doesnot predict numerical correctness of formula.

SI Prefixes : The magnitudes of physical quantites vary over a wide range. The mass of an electron is 9.1×10^{-31} kg and that of our earth is about 6×10^{24} kg. Standard prefixes for certain power of 10. Table shows these prefixes

Power of 10	Prefix	Symbol
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deka	da
–1	deci	d
–2	centi	c
–3	milli	m
–6	micro	μ
–9	nano	n
–12	pico	p
–15	femto	f

MEASUREMENT OF LENGTH

You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for lengths from 10^{-3} m to 10^{2} m. A vernier callipers is used for lengths to an accuracy of 10^{-4} m. A screw gauge and a spherometer can be used to measure lengths as less as to 10^{-5} m. To measure lengths beyond these ranges, we make use of some special indirect methods.

Range of Lengths

The sizes of the objects we come across in the universe vary over a very wide range. These may vary from the size of the order of 10^{-4} m of the tiny nucleus of an atom to the size of the order of 10^{26} m of the extent of the observable universe.

We also use certain special length units for short and large lengths. These are

1 fermi = 1 f = 10^{-15} m

1 angstrom = 1 Å = 10^{-10} m (It is used mainly in measuring wavelength of light)

1 astronomical unit = 1 AU (average distance of the Sun from the Earth) = 1.496×10^{11} m

1 light year = 1 ly = 9.46×10^{15} m (distance that light travels with velocity of 3×10^{8} m s^{-1} in 1 year)

1 parsec = 3.08×10^{16} m (Parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)

MEASUREMENT OF LARGE DISTANCES

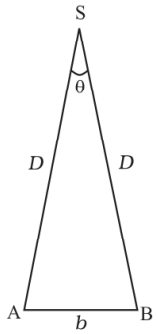
Parallax method:

Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the parallax method. When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye A (closing the right eye) and then look at the pencil through your right eye B (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called **parallax**.

The distance between the two points of observation is called the basis. In this example, the basis is the distance between the eyes. To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance AB = b at the same time as shown in Figure.We measure the angle between the two directions along which the planet is viewed at these two points. The ΔASB in Figure represented by symbol θ is called the parallax angle or parallactic angle.

As the planet is very far away, $\frac{b}{D} << 1$ and therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS = BS so that AB = b = D θ where θ is in radians.

$$D = \frac{b}{\theta} \qquad \text{....(i)}$$



Parallax method

Having determined D, we can employ a similar method to determine the size or angular diameter of the planet. If d is the diameter of the planet and α the angular size of the planet (the angle subtended by d at the earth), we have

$$\alpha = d/D \qquad \text{....(ii)}$$

The angle α can be measured from the same location on the earth. It is the angle between the two directions when two diametrically opposite points of the planet are viewed through the telescope. Since D is known, the diameter d of the planet can be determined using equation (ii).

TRAIN YOUR BRAIN

Example 14: The Sun's angular diameter is measured to be 1920". The distance D of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun ?

Sol. Sun's angular diameter a

$= 1920"$

$= 1920 \times 4.85 \times 10^{-6}$ rad

$= 9.31 \times 10^{-3}$ rad

Sun's diameter $d = \alpha D$

$= (9.31 \times 10^{-3}) \times (1.496 \times 10^{11})$ m

$= 1.39 \times 10^{9}$ m

ESTIMATION OF VERY SMALL DISTANCES

Size of a Molecule

To measure a very small size, like that of a molecule (10^{-8} m to 10^{-10} m), we have to adopt special methods. We cannot use a screw gauge or similar instruments. Even a microscope has

certain limitations. An optical microscope uses visible light to 'look' at the system under investigation. As light has wave like features, the resolution to which an optical microscope can be used is the wavelength of light.

For visible light the range of wavelengths is from about 4000 Å to 7000 Å (1 angstrom = 1 Å = 10^{-10} m). Hence an optical microscope cannot resolve particles with sizes smaller than this. Instead of visible light, we can use an electron beam. Electron beams can be focussed by properly designed electric and magnetic fields. The resolution of such an electron microscope is limited finally by the fact that electrons can also behave as waves.

The wavelength of an electron can be as small as a fraction of an angstrom. Such electron microscopes with a resolution of 0.6 Å have been built. They can almost resolve atoms and molecules in a material. In recent times, tunnelling microscopy has been developed in which again the limit of resolution is better than an angstrom. It is possible to estimate the sizes of molecules.

A simple method for estimating the molecular size of oleic acid is given below. Oleic acid is a soapy liquid with large molecular size of the order of 10^{-9} m. The idea is to first form mono-molecular layer of oleic acid on water surface. We dissolve 1 cm^3 of oleic acid in alcohol to make a solution of 20 cm^3. Then we take 1 cm^3 of this solution and dilute it to 20 cm^3 , using alcohol. So, the concentration of the solution is equal to $\left(\frac{1}{20\times 20}\right)$ cm^3 of oleic acid/cm^3 of solution. Next we lightly sprinkle some lycopodium powder on the surface of water in a large trough and we put one drop of this solution in the water.

The oleic acid drop spreads into a thin, large and roughly circular film of molecular thickness on water surface. Then, we quickly measure the diameter of the thin film to get its area A. Suppose we have dropped n drops in the water. Initially, we determine the approximate volume of each drop (V cm^3).

Volume of n drops of solution

$$= nV \text{ cm}^3$$

Amount of oleic acid in this solution

$$= nV\left(\frac{1}{20\times 20}\right)\text{cm}^3$$

This solution of oleic acid spreads very fast on the surface of water and forms a very thin layer of thickness t. If this spreads to form a film of area A cm^2 , then the thickness of the film

$$t = \frac{\text{volume of the film}}{\text{Area of the film}}$$

or, $$t = \frac{nV}{20\times 20A}\text{cm}$$

If we assume that the film has mono-molecular thickness, then this becomes the size or diameter of a molecule of oleic acid. The value of this thickness comes out to be of the order of 10^{-9} m.

TRAIN YOUR BRAIN

Example 15: If the size of a nucleus (inthe range of 10^{-15} to 10^{-11}m) is scaled up to the tip of a sharp pin, what roughly is the size of an atom ? Assume tip of the pin to be in the range 10^{-5} m to 10^{-4} m.)

Sol. The size of a nucleus is in the range of 10^{-15} m and 10^{-14} m. The tip of a sharp pin is taken to be in the range of 10^{-5} m and 10^{-4} m. Thus we are scaling up by a factor of 1m. An atom roughly of size 10^{-10} m will be scaled up to a size of 1 m. Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long.

ORDER-OF MAGNITUDE CALCULATIONS

$P = A \times 10^x$

$1/2 \le A < 5$

x is called order of magnitude

ERROR ANALYSIS IN EXPERIMENTS

Significant Figures or Digits

The *significant figures* (SF) in a measurement are the figures or digits that are known with certainity plus one that is uncertain.

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

1. **Rules to find out the number of significant figures :**

I Rule : All the non-zero digits are significant e.g. 1984 has 4 SF.

II Rule : All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF.

III Rule : All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

IV Rule : If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

V Rule : The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. e.g. 01.080 has 4 SF.

VI Rule : The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeros become significant. e.g. m = 100 kg has 3 SF.

VII Rule : When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in x = 12.3 = $1.23 \times 10^1 = .123 \times 10^2$ = $0.0123 \times 10^3 = 123 \times 10^{-1}$ each term has 3 SF only.

2. **Rules for arithmetical operations with significant figures :**

I Rule : In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. e.g. 12.587 – 12.5 = 0.087 = 0.1 ($\because$ second term contain lesser i.e. one decimal place)

II Rule : In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. e.g. $5.0 \times 0.125 = 0.625 = 0.62$

To avoid the confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in *scientific notation* (in the power of 10). In this notation every number is expressed in the form $a \times 10^b$, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number (a) is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).

The *change in the unit of measurement of a quantity does not affect the number of SF*. For example in 2.308 cm = 23.08 mm = 0.02308 m = 23080 μm each term has 4 SF.

TRAIN YOUR BRAIN

Example 16: Write down the number of significant figures in the following.

(a) 165 3SF (following rule I)
(b) 2.05 3 SF (following rules I & II)
(c) 34.000 m 5 SF (following rules I & V)
(d) 0.005 1 SF (following rules I & IV)
(e) 0.02340 N m^{-1} 4 SF (following rules I, IV & V)
(f) 26900 3 SF (see rule VI)
(g) 26900 kg 5 SF (see rule VI)

Example 17: The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Sol. length (ℓ) = 4.234 m breadth (b) = 1.005 m
thickness (t) = 2.01 cm = 2.01×10^{-2} m
Therefore area of the sheet
= $2 (\ell \times b + b \times t + t \times \ell)$
= 2 ($4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234$) m^2
= 2 (4.3604739) m^2 = 8.720978 m^2
Since area can contain a maximum of 3 SF (Rule II of article 4.2) therefore, rounding off, we get
Area = 8.72 m^2
Like wise volume = $\ell \times b \times t = 4.234 \times 1.005 \times 0.0201$ $m^3 = 0.0855289$ m^3
Since volume can contain 3 SF, therefore, rounding off, we get
Volume = 0.0855 m^3

ROUNDING OFF

To represent the result of any computation containing more than one uncertain digit, it is *rounded off* to appropriate number of significant figures.

Rules for rounding off the numbers :

I Rule : If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. $6.87 \approx 6.9$

II Rule : If the digit to be rounded off is less than 5, than the preceding digit is unaffected and is left unchanged. e.g. $3.94 \approx 3.9$

III Rule : If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. $14.35 \approx 14.4$ and $14.45 \approx 14.4$

TRAIN YOUR BRAIN

Example 18: The following values can be rounded off to four significant figures as follows :

(a) $36.879 \approx 36.88$ ($\because 9 > 5 \therefore 7$ is increased by one i.e.I Rule)

(b) $1.0084 \approx 1.008$ ($\because 4 < 5 \therefore 8$ is left unchanged i.e. II Rule)

(c) $11.115 \approx 11.12$ ($\because$ last 1 is odd it is increased by one i.e.III Rule)

(d) $11.1250 \approx 11.12$ ($\because$ 2 is even it is left unchanged i.e. III Rule)

(e) $11.1251 \approx 11.13$ ($\because 51 > 50 \therefore 2$ is incresed by one i.e. I Rule)

ERRORS IN MEASUREMENT

Definition

The difference between the true value and the measured value of a quantity is known as the error of measurement.

Classification of errors

Errors may arise from different sources and are usually classified as follows :-

Systematic or Controllable Errors : Systematic errors are the errors whose causes are known. They can be either positive or negative. Due to the known causes these errors can be minimised. Systematic errors can further be classified into three categories:

(i) Instrumental errors :- These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument. These can be reduced by using more accurate instruments.

(ii) Environmental errors :- These errors are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust, vibrations or magnetic and electrostatic fields.

(iii) Observational errors :- These errors arise due to improper setting of the apparatus or carelessness in taking observations.

Random Errors: These errors are due to unknown causes. Therefore they occur irregularly and are variable in magnitude and sign. Since the causes of these errors are not known precisely they can not be eliminated completely. For example, when the same person repeats the same observation in the same conditions, he may get different readings different times.

Random erros can be reduced by repeating the observation a large number of times and taking the arithmetic mean of all the obervations. This mean value would be very close to the most accurate reading.

Note :- If the number of observations is made n times then the random error reduces to $\left(\frac{1}{n}\right)$ times.

Example :- If the random error in the arithmetic mean of 100 observations is 'x' then the random error in the arithmetic mean of 500 observations will be $\frac{x}{5}$

Gross Errors : Gross errors arise due to human carelessness and mistakes in reading the instruments or calculating and recording the measurement results.

For example :-

(i) Reading instrument without proper initial settings.

(ii) Taking the observations wrongly without taking necessary precautions.

(iii) Exhibiting mistakes in recording the observations.

(iv) Putting improper values of the observations in calculations.

These errors can be minimised by increasing the sincerity and alertness of the observer.

REPRESENTATION OF ERRORS

Errors can be expressed in the following ways:-

1. **Mean Absolute Error :-** It is given by

$$a_m = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$$

$a_m = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$ = is taken as the true value of a quantity, if the same is not known.

$\Delta a_1 = a_m - a_1$

$\Delta a_2 = a_m - a_2$

.....

$\Delta a_n = a_m - a_n$

Final result of measurement may be written as :

$a = a_m \pm \Delta\bar{a}$

2. **Relative Error or Fractional Error :** It is given by

$$\frac{\overline{\Delta a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. **Percentage Error** $= \frac{\overline{\Delta a}}{a_m} \times 100\%$

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Example 19: The period of oscillation of a simple pendulum in an experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. find (i) mean time period (ii) absolute error in each observation and percentage error.

Sol. (i) Mean time period is given by

$$\bar{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= \frac{13.12}{5} = 2.62\text{s}$$

(ii) The absolute error in each observation is
$2.62 - 2.63 = -0.01, 2.62 - 2.56 = 0.06, 2.62 - 2.42 = 0.20,$
$2.62 - 2.71 = -0.09, 2.62 - 2.80 = -0.18$

Mean absolute error, $\overline{\Delta T} = \frac{\Sigma |\Delta T|}{5}$

$$= \frac{0.01 + 0.06 + 0.2 + 0.09 + 0.18}{5} = 0.11\text{sec}$$

$\therefore$ Percentage error

$$= \frac{\overline{\Delta T}}{\bar{T}} \times 100 = \frac{0.11}{2.62} \times 100 = 4.2\%$$

COMBINATION OF ERRORS

(i) In Sum : If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$, maximum fractional error in this case $\frac{\Delta Z}{Z} = \frac{\Delta A}{A+B} + \frac{\Delta B}{A+B}$

i.e. when two physical quantities are added then the maximum absolute error in the result is the sum of the absolute errors of the individual quantities.

(ii) In Difference : If $Z = A - B$, then maximum absolute error is $\Delta Z = \Delta A + \Delta B$ and maximum fractional error in this case $\frac{\Delta Z}{Z} = \frac{\Delta A}{A-B} + \frac{\Delta B}{A-B}$

(iii) In Product : If $Z = AB$, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

where $\Delta Z/Z$ is known as fractional error.

(iv) In Division : If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(v) In Power : If $Z = A^n$ then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

In more general form if $Z = \frac{A^x B^y}{C^q}$

then the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = x\frac{\Delta A}{A} + y\frac{\Delta B}{B} + q\frac{\Delta C}{C}$$

Applications:

1. For a simple pendulum, $T \propto l^{1/2}$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

2. For a sphere

$$A = 4\pi r^2,\ V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{\Delta A}{A} = 2.\frac{\Delta r}{r} \text{ and } \frac{\Delta V}{V} = 3.\frac{\Delta r}{r}$$

3. When two resistors R_1 and R_2 are connected

(a) In series

$$R_s = R_1 + R_2$$

$$\Rightarrow \Delta R_s = \Delta R_1 + \Delta R_2$$

$$\frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

(b) In parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

TRAIN YOUR BRAIN

Example 20: In an experiment of simple pendulum, the errors in the measurement of length of the pendulum (L) and time period (T) are 3% and 2% respectively. The maximum percentage error in the value of $\frac{L}{T^2}$ is

(*a*) 5% (*b*) 7%
(*c*) 8% (*d*) 1%

Sol. (c) Maximum percentage in the value of $\frac{L}{T^2}$

$$= \frac{\Delta L}{L} \times 100\% + 2\frac{\Delta T}{T} \times 100\%$$

$$= 3 + 2 \times 2$$

$$= 7\%$$

Example 21: If $X = \frac{A^2\sqrt{B}}{C}$, then

(*a*) $\Delta X = \Delta A + \Delta B + \Delta C$

(*b*) $\frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

(*c*) $\frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$

(*d*) $\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

Ans. (c)

Sol. $\because X = A^2 B^{1/2} C \ \therefore \ \frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$

Example 22: A body travels uniformly a distance (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is the percentage error in velocity ?

Sol. Given distance, $s = (13.8 \pm 0.2)$ m

and time $t = (4.0 \pm 0.3)$ s

$$\text{Velocity } v = \frac{s}{t} = \frac{13.8}{4.0} = 3.45 \text{m s}^{-1} = 3.5 \text{m s}^{-1}$$

$$\frac{\Delta v}{v} = \pm\left(\frac{\Delta s}{s} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right)$$

$$= \pm\left(\frac{0.8 + 4.14}{13.8 \times 40.}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right)$$

$\therefore \quad \Delta v = \pm 0.0895 \times V = \pm 0.0895 \times 3.45 = \pm 0.3087 = \pm 0.31$

Hence $v = (3.5 \pm 0.31)$ m s^{-1}

Percentage error in velocity

$$= \frac{\Delta v}{v} \times 100 = \pm 0.0895 \times 100 = \pm 8.95\% = \pm 9\%$$

Example 23: The heat generated in a circuit is given by $Q = I^2$ Rt, where I is current, R is resistance and t is time. If the percentage errors in measuring I, R and t are 2%, 1% and 1% respectively, then the maximum error in measuring heat will be–

(*a*) 2 % (*b*) 3 %

(*c*) 4 % (*d*) 6 %

Ans. (d)

Sol. $Q = I^2 Rt$ error, given I = 2%

R = 1%

t = 1.1%

Maximum possible % error

$= 2 \times 2\% + 1 \times 1\% + 1 \times 1\% = 6\%$

Example 24: Given : Resistance, $R_1 = (8 \pm 0.4)\ \Omega$ and Resistance, $R_2 = (8 \pm 0.6)\ \Omega$. What is the net resistance when R_1 and R_2 are connected in series?

(*a*) $(16 \pm 0.4)\ \Omega$

(*b*) $(16 \pm 0.6)\ \Omega$

(*c*) $(16 \pm 1.0)\ \Omega$

(*d*) $(16 \pm 0.2)\ \Omega$

Ans. (c)

Sol. $R_1 = (8 \pm 0.4)\Omega$

$R_2 = (8 \pm 0.6)\ \Omega$

$R_s = R_1 + R_2 = (16 \pm 1.0)\Omega$

Example 25: The following observations were taken for determining surface tension of water by capillary tube method: Diameter of capillary, $D = 1.25 \times 10^{-2}$ m and rise of water in capillary, $h = 1.45 \times 10^{-2}$ m.

Taking $g = 9.80$ ms^{-2} and using the relation

$T = (rgh/2) \times 10^3$ Nm^{-1}, what is the possible error in surface tension T ?

(*a*) 2.4 % (*b*) 15 %

(*c*) 1.6 % (d) 0.15 %

Ans. (c)

Sol. Given $T = (rgh/2) \times 10^3$ Nm^{-1},

$D = 1.25 \times 10^{-2}$ m, $h = 1.45 \times 10^{-2}$ m,

$g = 9.80$ ms^{-2}

$$\frac{\delta T}{T} = \frac{\delta r}{r} + \frac{\delta h}{h} + \frac{\delta g}{g}$$

after apply the above values in this relation we get $\delta T\% = 1.6\%$

MEASURING INSTRUMENT

Measurement is an important aspect of physics. Whenever we want to know about a physical quantity, we take its measurement first of all.

Instruments used in measurement are called **measuring instruments.**

Least Count: The least value of a quantity, which the instrument can measure accurately, is called the least count of the instrument.

Error: The measured value of the physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error. Every calculated quantity, which is based on measured values, has an error.

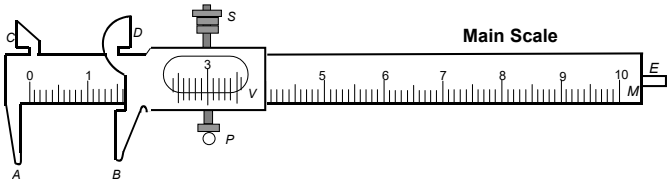
Accuracy and Precision: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.

VERNIER CALLIPERS

It is a device used to measure accurately upto 0.1 mm. There are two scales in the vernier callipers, vernier scale and main scale. The main scale is fixed whereas the vernier scale is movable along the main scale.

Its main parts are as follows:

Main scale: It consists of a steel metallic strip *M*, graduated in cm and *mm* at one edge and in inches and tenth of an inch at the other edge on same side. It carries fixed jaws *A* and *C* projected at right angle to the scale as shown in figure.



Vernier Scale: A vernier *V* slides on the strip *M*. It can be fixed in any position by screw *S*. It is graduated on both sides. The side of the vernier scale which slides over the *mm* side has ten divisions over a length of 9 mm, i.e., over 9 main scale divisions and the side of the vernier scale which slides over the inches side has 10 divisions over a length of 0.9 inch, i.e., over 9 main scale divisions.

Movable Jaws: The vernier scale carries jaws *B* and *D* projecting at right angle to the main scale. These are called movable jaws. When vernier scale is pushed towards *A* and *C*, then as *B* touches *A*, straight side of *D* will touch straight side of *C*. In this position, in case of an instrument free from errors, zeros of vernier scale will coincide with zeros of main scales, on both the cm and inch scales.

(The object whose length or external diameter is to be measured is held between the jaws *A* and *B*, while the straight edges of *C* and *D* are used for measuring the internal diameter of a hollow object).

Metallic Strip: There is a thin metallic strip *E* attached to the back side of *M* and connected with vernier scale. When the jaws *A* and *B* touch each other, the edge of strip *E* touches the edge of *M*. When the jaws *A* and *B* are separated, *E* moves outwards. The strip E is used for measuring the depth of a vessel.

Determination of least count (Vernier Constant)

Note the value of the main scale division and count the number *n* of vernier scale divisions. Slide the movable jaw till the zero of vernier scale coincides with any of the mark of the main scale and find the number of divisions $(n-1)$ on the main scale coinciding with *n* divisions of vernier scale. Then

$$nV.S.D. = (n-1)\,M.S.D. \text{ or } 1\,V.S.D. = \left(\frac{n-1}{n}\right) M.S.D.$$

$$\text{or} \quad V.C. = 1\,M.S.D. - 1\,V.S.D. = \left(1-\frac{n-1}{n}\right) \text{M.S.D.}$$

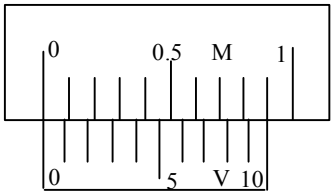
$$= \frac{1}{n} \text{ M.S.D.}$$

Determination of zero error and zero correction

For this purpose, movable jaw *B* is brought in contact with fixed jaw *A*.

One of the following situations will arise.

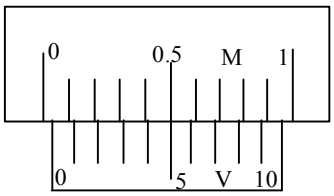
(i) Zero of Vernier scale coincides with zero of main scale (see figure)



In this case, zero error and zero correction, both are nil.

Actual length = observed (measured) length.

(ii) Zero of vernier scale lies on the right of zero of main scale (see figure)



Here 5th vernier scale division is coinciding with any main sale division.

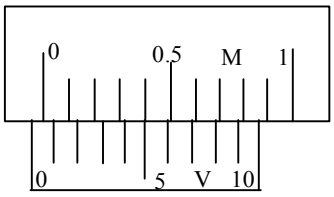
Hence, $N = 0, n = 5$, L.C. = 0.01 cm.

Zero error $N = n \times (\text{L.C.}) = 0 + 5 \times 0.01 = +0.05$ cm

Zero correction = – 0.05 cm.

Actual length will be 0.05 cm less than the observed (measured) length.

(iii) zero of the vernier scale lies left of the main scale.



Here, 5th vernier scale division is coinciding with any main scale division.

In this case, zero of vernier scale lies on the right of –0.1 cm reading on main scale.

Hence, $N = -0.1$ cm, $n = 5$, L.C. = 0.01 cm

Zero error = $N + n \times$ (L.C.) = $-0.1 + 5 \times 0.01 = -0.05$ cm.

Zero correction = +0.05 cm.

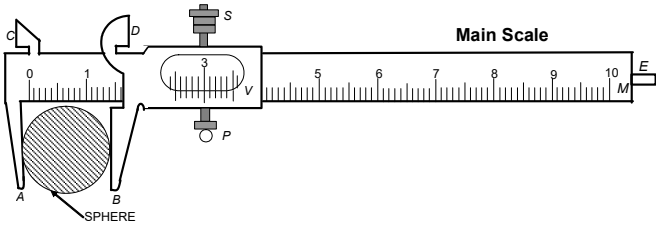
Actual length will be 0.05 cm more than the observed (measured) length.

Experiment

Aim: To measure the diameter of a small spherical/cylindrical body, using a vernier callipers.

Apparatus: Vernier callipers, a spherical (pendulum bob) or a cylinder.

Diagram:



Theory: If with the body between the jaws, the zero of vernier scale lies ahead of Nth division of main scale, then main scale reading (M.S.R.) = N.

If n^{th} division of vernier scale coincides with any division of main scale, then vernier scale reading (V.S.R.)

= $n \times$ (L.C.) (L.C. is least count of vernier callipers)

= $n \times$ (V.C.) (V.C. is vernier constant of vernier callipers)

Total reading, $T.R. = M.S.R. + V.S.R. = N + n \times (V.C.)$

Precautions (to be taken)

1. Motion of vernier scale on main scale should be made smooth (by oiling if necessary).
2. Vernier constant and zero error should be carefully found and properly recored.
3. The body should be gripped between the jaws firmly but gently (without undue pressure on it from the jaws).
4. Observations should be taken at right angles at one place and taken at least at three different places.

Sources of Error

1. The vernier scale may be loose on main scale.
2. The jaws may not be at right angles to the main scale.
3. The graduations on scale may not be correct and clear.
4. Parallax may be there in taking observations.

TRAIN YOUR BRAIN

Example 26: The least count of vernier callipers is 0.1 mm. The main scale reading before the zero of the vernier scale is 10 and the zeroth division of the vernier scale coincides with the main scale division. Given that each main scale division is 1 mm, what is the measured value?

Sol. Length measured with vernier callipers

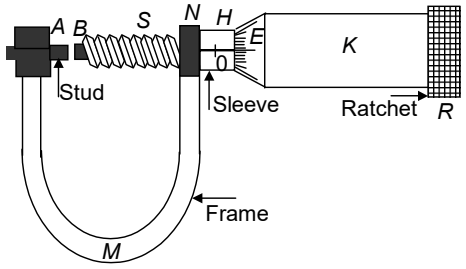
= reading before the zero of vernier scale + number of vernier divisions coinciding

with any main scale division × least count

= 10 mm + 0 × 0.1 mm = 10 mm = **1.00 cm**

SCREW GAUGE

This instrument (shown in figure) works on the principle of micrometer screw. It consists of a U-shaped frame *M*. At one end of it is fixed a small metal piece *A* of gun metal. It is called stud and it has a plane face. The other end *N* of *M* carries a cylindrical hub *H*. The hub extends few millimetre beyond the end of the frame. On the tubular hub along its axis, a line is drawn known as reference line. On the reference line graduations are in millimetre and half millimeter depending upon the pitch of the screw. This scale is called linear scale or pitch scale. A nut is threaded through the hub and the frame *N*. Through the nut moves a screw *S* made of gun metal. The front face *B* of the screw, facing the plane face *A*, is also plane. A hollow cylindrical cap *K*, is capable of rotating over the hub when screw is rotated. It is attached to the right hand end of the screw. As the cap is rotated the screw either moves in or out. The bevelled surface *E* of the cap *K* is divided into 50 or 100 equal parts. It is called the circular scale or head scale. Right hand end *R* of *K* is milled for proper grip.



In most of the instrument the milled head *R* is not fixed to the screw head but turns it by a spring and ratchet arrangement such that when the body is just held between faces *A* and *B*, the spring yields and milled head *R* turns without moving in the screw.

In an accurately adjusted instrument when the faces *A* and *B* are just touching each other, the zero marks of circular scale and pitch scale exactly coincide.

Determination of least count of screw gauge

Note the value of linear (pitch) scale division. Rotate screw to bring zero mark on circular (head) scale on reference line. Note linear scale reading i.e. number of divisions of linear scale uncovered by the cap.

Now give the screw a few known number of rotations. (one rotation completed when zero of circular scale again arrives on the reference line). Again note the linear scale reading. Find difference of two readings on linear scale to find distance moved by the screw.

Then, pitch of the screw

$$= \frac{\text{Distance moved by in n rotation}}{\text{No. of full rotation (n)}}$$

Now count the total number of divisions on circular (head) scale.

Then, least count

$$= \frac{\text{Pitch}}{\text{Total number of divisions on the circular scale}}$$

The least count is generally 0.001 cm.

Determination of zero error and zero correction

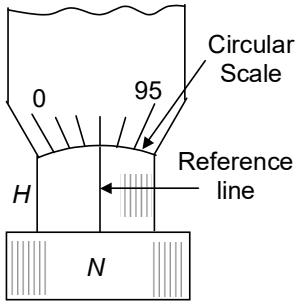
For this purpose, the screw is rotated forward till plane face *B* of the screw just touches the fixed plane face *A* of the stud and edge of cap comes on zero mark of linear scale. Screw gauge is held keeping the linear scale vertical with its zero downwards.

One of the following three situations will arise.

(i) **Zero mark of circular scale comes on the reference line** (see figure)

In this case, zero error and zero correction, both are nil

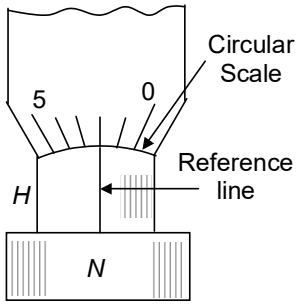
Actual thickness = Observed (measured) thickness.



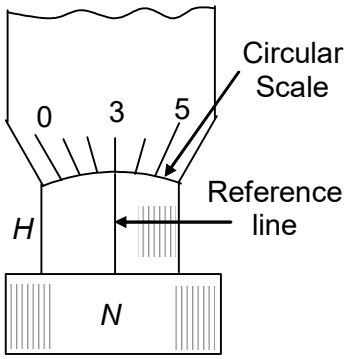
(ii) **Zero mark of circular scale remains on right of reference line and does not cross** it (see figure).

Here 2nd division on circular scale comes on reference line. Zero reading is already 0.02 mm. It makes zero error + 0.02 mm and zero correction – 0.02 mm.

Actual thickness will be 0.02 mm less than the observed (measured) thickness.



(iii) **Zero mark of circular scale goes to left on reference line after crossing it (see figure).** Here zero of circular scale has advanced from reference line by 3 divisions on circular scale. A backward rotation by 0.03 mm will make reading zero. It makes zero error – 0.03 mm & zero correction + 0.03 mm.



Actual thickness will be 0.03 mm more than the observed (measured) thickness.

Experiment

Aim: To measure diameter of a given wire using a screw gauge and find its volume.

Apparatus: Screw gauge, wire, half metre rod (scale).

Theory:

(1) Determine of least count of screw gauge

(2) If with the wire between plane faces *A* and *B*, the edge of the cap lies ahead of Nth division of linear scale.

Then, linear scale reading (L.S.R.) = N

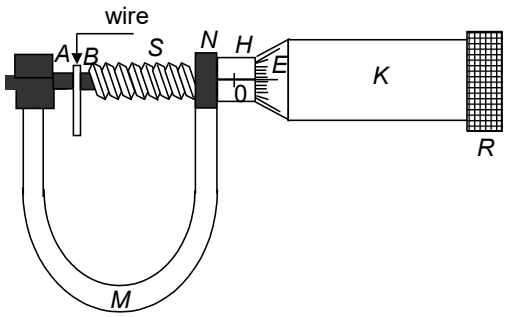
If nth division of circular scale lies over reference line.

Then, circular scale reading (C.S.R.) = $n \times$ (L.C.)

(L.C. is least count of screw gauge)

Total reading (T.R.) = L.S.R. + C.S.R. = $N + n \times$ (L.C.)

(3) If *D* be the mean diameter and *l* be the mean length of the wire. Then volume of the wire, $V = \pi \left(\frac{D}{2}\right)^2 l$



Calculation

Mean diameter of the wire,

$$D = \frac{D_1(a) + D_1(b) + ... + D_5(a) + D_5(b)}{10} = ...mm$$

$$= \ldots \text{ cm}$$

Mean length of the wire,

$$l = \frac{l_1 + l_2 + l_3}{3} = ...cm$$

Volume of the wire

$$V = \pi\left(\frac{D}{2}\right)^2 l = \ldots \text{ cm}^3$$

Result The volume of the given wire is = … cm^3

Precaution (to be taken)

1. While taking an observation, the screw must always be turned only in one direction so as to avoid the backlash error.
2. At each place, take readings in pairs i.e. in two directions at right angles to each other.
3. The wire must be straight and free from kinks.
4. Always rotate the screw by the ratchet and stop as soon as it gives one tick sound only.
5. While taking a reading, rotate the screw in only one direction so as to avoid the backlash error.

Sources of Error

(i) The screw may have friction.
(ii) The screw gauge may have back-lash error.
(iii) Circular scale divisions may not be of equal size.
(vi) The wire may not be uniform.

TRAIN YOUR BRAIN

Example 27: A vernier callipers has its main scale of 10 cm equally divided into 200 equal parts. Its vernier scale of 25 divisions coincides with 12 mm on the main scale. The least count of the instrument is–

(*a*) 0.020 cm
(*b*) 0.002 cm
(*c*) 0.010 cm
(*d*) 0.001 cm

Ans. **(b)**

Sol. In vernier calliper main scale 10 cm.

10 cm divided in 200 divisions. 1 div. $= \frac{10}{200}$

$= 0.05$ cm.

$25\,V = 24S.$

$V = \frac{24}{25}S$

$S - V = S - \frac{24}{25}S = \frac{1}{25}S$

$\because 1S = 0.05$ cm

or vernier constant $= \frac{0.05}{25} = 0.002$ cm

Least count $= 0.002$ cm

Example 28: One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 10 divisions of vernier scale coincide with 8 small divisions of the main scale, the least count of the callipers is –

(*a*) 0.005 cm (*b*) 0.05 cm
(*c*) 0.02 cm (*d*) 0.01 cm

Ans. **(c)**

Sol. 1 main scale div = 0.1 cm

$10V = 8S$

$V = \frac{8}{10}$ s.

$S - V = S - \frac{8}{10}S = \frac{2}{10}S. = \frac{1}{5}S$

But $1S = 0.1$ cm

$= \frac{0.1}{5} = 0.02$ cm

Least count = 0.02 cm

Example 29: In four complete revolutions of the cap, the distance traveled on the pitch scale is 2mm. If there are fifty divisions on the circular scale, then

(i) Calculate the pitch of the screw gauge
(ii) Calculate the least count of the screw gauge

Ans. Pitch = 0.5 mm, L.C. = 0.001 cm

Sol. Pitch of screw = Linear distance traveled in one Revolution

$P = \frac{2mm}{4} = 0.5\text{ mm} = 0.05\text{ cm}$

Least count

$$= \frac{\text{Pitch}}{\text{no. of divisions in circular scale}} = \frac{0.05}{50} = 0.001 \text{ cm}$$

Example 30: The pitch of a screw gauge 0.5 mm and there are 50 divisions on the circular scale. In measuring the thickness of a metal plate, there are five divisions on the pitch scale (or main scale) and thirty fourth division coincides with the reference line. Calculate the thickness of the metal plate.

Ans. Thickness of sheet = 2.84 mm.

Sol. Pitch of screw = 0.5 mm.

L.C. $= \frac{0.5}{50} = 0.01$ mm.

Thickness $= (5 \times 0.5 + 34 \times 0.01)$ mm

$= (2.5 + 0.34) = 2.84$ mm

Topicwise Questions

UNIT AND DIMENSIONS

Units, System of Units

1. A unit less quantity
(*a*) never has a nonzero dimension
(*b*) always has a nonzero dimension
(*c*) may have a nonzero dimension
(*d*) does not exit

2. Which of the following is not the name of a physical quantity?
(*a*) kilogram (*b*) impulse
(*c*) energy (*d*) density

3. PARSEC is a unit of
(*a*) Time (*b*) Angle
(*c*) Distance (*d*) Velocity

4. Which of the following system of units is NOT based on the unit of mass, length and time alone
(*a*) FPS (*b*) SI (*c*) CGS (*d*) MKS

5. In the S.I. system the unit of energy is-
(*a*) erg (*b*) calorie
(*c*) joule (*d*) electron volt

6. The SI unit of the universal gravitational constant G is
(*a*) $Nm\,kg^{-2}$ (*b*) Nm^2kg^{-2} (*c*) $Nm^2\,kg^{-1}$ (*d*) $Nmkg^{-1}$

7. Surface tension has unit of-
(*a*) $Joule.m^2$ (*b*) $Joule.m^{-2}$ (*c*) $Joule.m^{-1}$ (*d*) $Joule.m^3$

8. The specific resistance has the unit of-
(*a*) ohm/m (*b*) ohm/m^2 (*c*) $ohm.m^2$ (*d*) ohm.m

9. The unit of magnetic moment is-
(*a*) $amp\,m^2$ (*b*) $amp\,m^{-2}$ (*c*) amp m (*d*) $amp\,m^{-1}$

10. The SI unit of the universal gas constant R is:
(*a*) $erg\,K^{-1}\,mol^{-1}$ (*b*) $watt\,K^{-1}\,mol^{-1}$
(*c*) $newton\,K^{-1}\,mol^{-1}$ (*d*) $joule\,K^{-1}\,mol^{-1}$

11. The SI unit of Stefan's constant is:
(*a*) $Ws^{-1}\,m^{-2}\,K^{-4}$ (*b*) $J\,s\,m^{-1}\,K^{-1}$
(*c*) $J\,s^{-1}\,m^{-2}\,K^{-1}$ (*d*) $W\,m^{-2}\,K^{-4}$

Dimension, Finding Dimensional Formula

12. In SI unit the angular acceleration has unit of-
(*a*) $Nmkg^{-1}$ (*b*) ms^{-2}
(*c*) $rad.s^{-2}$ (*d*) Nkg^{-1}

13. The angular frequency is measured in $rad\,s^{-1}$. Its exponent in length are:
(*a*) -2 (*b*) -1
(*c*) 0 (*d*) 2

14. $[M\,L\,T^{-1}]$ are the dimensions of-
(*a*) power (*b*) momentum
(*c*) force (*d*) couple

15. What are the dimensions of Boltzmann's constant?
(*a*) $MLT^{-2}K^{-1}$ (*b*) $ML^2T^{-2}K^{-1}$
(*c*) M^0LT^{-2} (*d*) $M^0L^2T^{-2}K^{-1}$

16. A pair of physical quantities having the same dimensional formula is:
(*a*) angular momentum and torque
(*b*) torque and energy
(*c*) force and power
(*d*) power and angular momentum

17. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?
(*a*) torque (*b*) surface tension
(*c*) viscosity (*d*) stress

Principle of Homogeneity of Dimension

18. Force F is given in terms of time t and distance x by $F = A \sin C\,t + B \cos D\,x$ Then the dimensions of $\frac{A}{B}$ and $\frac{C}{D}$ are given by
(*a*) MLT^{-2}, $M^0L^0T^{-1}$
(*b*) MLT^{-2}, $M^0L^{-1}T^0$
(*c*) $M^0L^0T^0$, $M^0L^1T^{-1}$
(*d*) $M^0L^1T^{-1}$, $M^0L^0T^0$

19. $\int \frac{xdx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right]$. The value of n is:
You may use dimensional analysis to solve the problem.
(*a*) 0
(*b*) -1
(*c*) 1
(*d*) None of these

20. The equation for the velocity of sound in a gas states that $v = \sqrt{\gamma k_b \frac{T}{m}}$. Velocity v is measured in m/s. γ is a dimensionless constant, T is temperature in kelvin (K), and m is mass in kg. What are the units for the Boltzmann constant, k_b ?

(*a*) $kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}$ (*b*) $kg \cdot m^2 \cdot s^2 \cdot K$

(*c*) $kg \cdot m/s \cdot K^{-2}$ (*d*) $kg \cdot m^2 \cdot s^{-2} \cdot K$

21. A wave is represented by
$y = a \sin (At - Bx + C)$
where A, B, C are constants and t is in seconds & x is in metre. The Dimensions of A, B, C are-

(*a*) $T^{-1}, L, M^0L^0T^0$ (*b*) $T^{-1}, L^{-1}, M^0L^0T^0$

(*c*) T, L, M (*d*) T^{-1}, L^{-1}, M^{-1}

22. If $v = \sqrt{\frac{\gamma p}{\rho}}$, then the dimensions of γ are (p is pressure, ρ is density and v is speed of sound has their usual dimension) -

(*a*) $M^0L^0T^0$ (*b*) $M^0L^0T^{-1}$

(*c*) $M^1L^0T^0$ (*d*) $M^0L^1T^0$

23. Consider the equation $\frac{d}{dt}\left[\int \vec{F} \cdot d\vec{s}\right] = A\left[\vec{F} \cdot \vec{P}\right]$. Then dimension of A will be (where $\vec{F} \equiv$ force, $d\vec{s} \equiv$ small displacement, $t \equiv$ time and $\vec{P} \equiv$ linear momentum).

(*a*) $M^oL^oT^o$ (*b*) $M^1L^oT^o$

(*c*) $M^{-1}L^oT^o$ (*d*) $M^oL^oT^{-1}$

Application of Dimensional Analysis: Deriving New Relation

24. The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities aswhere k is a dimensionless constant

(*a*) $v^2 = k\lambda^{-1} g^{-1} \rho^{-1}$ (*b*) $v^2 = k g \lambda$

(*c*) $v^2 = k g \lambda \rho$ (*d*) $v^2 = k \lambda^3 g^{-1} \rho^{-1}$

25. Force applied by water stream depends on density of water (ρ), velocity of the stream (v) and cross–sectional area of the stream (*a*). The expression of the force should be

(*a*) ρAv (*b*) ρAv^2 (*c*) $\rho^2 Av$ (*d*) $\rho A^2 v$

Application of Dimensional Analysis: To Convert from one System of Unit

26. One watt-hour is equivalent to

(*a*) 6.3×10^3 Joule (*b*) 6.3×10^{-7} Joule

(*c*) 3.6×10^3 Joule (*d*) 3.6×10^{-3} Joule

27. The pressure of 10^6 dyne/cm^2 is equivalent to

(*a*) 10^5 N/m^2 (*b*) 10^6 N/m^2 (*c*) 10^7 N/m^2 (*d*) 10^8 N/m^2

28. $\rho = 2$ g/cm^3 convert it into MKS system-

(*a*) $2 \times 10^{-3} \frac{kg}{m^3}$ (*b*) $2 \times 10^3 \frac{kg}{m^3}$

(*c*) $4 \times 10^3 \frac{kg}{m^3}$ (*d*) $2 \times 10^6 \frac{kg}{m^3}$

29. The density of mercury is 13600 kg m^{-3}. Its value of CGS system will be:

(*a*) 13.6 g cm^{-3} (*b*) 1360 g cm^{-3}

(*c*) 136 g cm^{-3} (*d*) 1.36 g cm^{-3}

ERRORS IN MEASUREMENT

30. Which of the following measurements is most accurate ?

(*a*) 9×10^{-2} m (*b*) 90×10^{-3} m

(*c*) 900×10^{-4} m (*d*) 0.090 m

31. A system takes 70.40 second to complete 20 oscillations. The time period of the system is–

(*a*) 3.52 s (*b*) 35.2×10 s

(*c*) 3.520 s (*d*) 3.5200 s

32. The percentage error in the measurement of mass and speed are 1% and 2% respectively. What is the percentage error in kinetic energy–

(*a*) 5% (*b*) 2.5% (*c*) 3% (*d*) 1.5%

33. Number 15462 when rounded off to numbers to three significant digits will be-

(*a*) 15500 (*b*) 155

(*c*) 1546 (*d*) 150

34. Value of expression $\frac{25.2 \times 1374}{33.3}$ will be.

(All the digits in this expression are significant.)

(*a*) 1040 (*b*) 1039 (*c*) 1038 (*d*) 1041

35. Value of 24.36 + 0.0623 + 256.2 will be-

(*a*) 280.6 (*b*) 280.8 (*c*) 280.7 (*d*) 280.6224

36. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimation of the kinetic energy obtained by measuring mass and speed

(*a*) 11% (*b*) 8% (*c*) 5% (*d*) 1%

37. The random error in the arithmetic mean of 100 observations is x; then random error in the arithmetic mean of 400 observations would be

(a) $4x$ (b) $\frac{1}{4}x$

(c) $2x$ (d) $\frac{1}{2}x$

38. What is the number of significant figures in 0.310×10^3
(a) 2 (b) 3 (c) 4 (d) 6

39. Error in the measurement of radius of a sphere is 1%. The error in the calculated value of its volume is
(a) 1% (b) 3% (c) 5% (d) 7%

40. The mean time period of *second*'s pendulum is 2.00s and mean absolute error in the time period is 0.05s. To express maximum estimate of error, the time period should be written as
(a) $(2.00\pm0.01)s$ (b) $(2.00+0.025)s$
(c) $(2.00\pm0.05)s$ (d) $(2.00\pm0.10)s$

41. The unit of percentage error is
(a) Same as that of physical quantity
(b) Different from that of physical quantity
(c) Percentage error is unit less
(d) Errors have got their own units which are different from that of physical quantity measured

42. The decimal equivalent of 1/20 upto three significant figures is
(a) 0.0500 (b) 0.05000 (c) 0.0050 (d) 5.0×10^{-2}

43. Accuracy of measurement is determined by
(a) Absolute error (b) Percentage error
(c) Both (a) and (b) (d) None of these

44. A thin copper wire of length l *metre* increases in length by 2% when heated through 10ºC. What is the percentage increase in area when a square copper sheet of length l *metre* is heated through 10ºC
(a) 4% (b) 8% (c) 16% (d) 32%

45. In a vernier callipers, ten smallest divisions of the vernier scale are equal to nine smallest division on the main scale. If the smallest division on the main scale is half millimeter, then the vernier constant is–
(a) 0.5 mm (b) 0.1 mm (c) 0.05 mm (d) 0.005 mm

46. A vernier calliper has 20 divisions on the vernier scale, which coincide with 19 on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of–
(a) 0.5 mm (b) 1 mm
(c) 2 mm (d) 1/4 mm

47. A vernier callipers having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then
(a) n = 10, m = 0.5 cm (b) n = 9, m = 0.4 cm
(c) n = 10, m = 0.8 cm (d) n = 10, m = 0.2 cm

Learning Plus

UNIT AND DIMENSIONS

1. Which of the following is not the unit of time
(a) solar day (b) parallactic second
(c) leap year (d) lunar month

2. The unit of impulse is the same as that of:
(a) moment force
(b) linear momentum
(c) rate of change of linear momentum
(d) force

3. Which of the following is not the unit of energy?
(a) watt-hour
(b) electron-volt
(c) N × m
(d) kg × m/sec^2

4. A dimensionless quantity:
(a) never has a unit
(b) always has a unit
(c) may have a unit
(d) does not exit

5. If a and b are two physical quantities having different dimensions then which of the following can denote a new physical quantity
(a) a + b
(b) a – b
(c) a/b
(d) $e^{a/b}$

6. The time dependence of a physical quantity ?
$P = P_0\exp(-\alpha t^2)$ where α is a constant and t is timeThe constant α
(*a*) will be dimensionless
(*b*) will have dimensions of T^{-2}
(*c*) will have dimensions as that of P
(*d*) will have dimensions equal to the dimension of P multiplied by T^{-2}

7. Which pair of following quantities has dimensions different from each other.
(*a*) Impulse and linear momentum
(*b*) Plank's constant and angular momentum
(*c*) Moment of inertia and moment of force
(*d*) Young's modulus and pressure

8. The product of energy and time is called action. The dimensional formula for action is same as that for
(*a*) power (*b*) angular energy
(*c*) force × velocity (*d*) impulse × distance

9. What is the physical quantity whose dimensions are $[M L^2 T^{-2}]$?
(*a*) kinetic energy (*b*) pressure
(*c*) momentum (*d*) power

10. If E, M, J and G denote energy, mass, angular momentum and gravitational constant respectively, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of
(*a*) length (*b*) angle (*c*) mass (*d*) time

11. The position of a particle at time 't' is given by the relation $x(t) = \frac{V_0}{\alpha}[1 - e^{-\alpha t}]$ where V_0 is a constant and $\alpha > 0$. The dimensions of V_0 and α are respectively.
(*a*) $M^0L^1T^0$ and T^{-1} (*b*) $M^0L^1T^0$ and T^{-2}
(*c*) $M^0L^1T^{-1}$ and T^{-1} (*d*) $M^0L^1T^{-1}$ and T^{-2}

12. If force (F) is given by $F = Pt^{-1} + \alpha t$, where t is time. The unit of P is same as that of
(*a*) velocity (*b*) displacement
(*c*) acceleration (*d*) momentum

13. When a wave traverses a medium, the displacement of a particle located at x at time t is given by $y = a \sin (bt - cx)$ where a, b and c are constants of the wave. The dimensions of b are the same as those of
(*a*) wave velocity (*b*) amplitude
(*c*) wavelength (*d*) wave frequency

14. In a book, the answer for a particular question is expressed as $b = \frac{ma}{k}\left[\sqrt{1+\frac{2kl}{ma}}\right]$ here m represents mass, a represents accelerations, *l* represents length. The unit of b should be
(*a*) m/s (*b*) m/s^2
(*c*) meter (*d*) /sec

15. $\alpha = \frac{F}{V^2}\sin(\beta t)$ (here V = velocity, F = force, t = time) : Find the dimension of α and β -
(*a*) $\alpha = [M^1L^1T^0]$, $\beta = [T^{-1}]$
(*b*) $\alpha = [M^1L^1T^{-1}]$, $\beta = [T^1]$
(*c*) $\alpha = [M^1L^1T^{-1}]$, $\beta = [T^{-1}]$
(*d*) $\alpha = [M^1L^{-1}T^0]$, $\beta = [T^{-1}]$

16. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be:
(*a*) FT^2 (*b*) $F^{-1} A^2 T^{-1}$ (*c*) FA^2T (*d*) AT^2

17. If area (*A*) velocity (v) and density (ρ) are base units, then the dimensional formula of force can be represented as
(*a*) $Av\rho$ (*b*) $Av^2\rho$
(*c*) $Av\rho^2$ (*d*) $A^2v\rho$

18. The velocity of a freely falling body changes as $g^p h^q$ where g is acceleration due to gravity and h is the height. The values of p and q are:
(*a*) $1, \frac{1}{2}$ (*b*) $\frac{1}{2}, \frac{1}{2}$
(*c*) $\frac{1}{2}, 1$ (*d*) $1, 1$

19. If the unit of length is micrometer and the unit of time is microsecond, the unit of velcoity will be:
(*a*) 100 m/s (*b*) 10 m/s
(*c*) micrometers (*d*) m/s

20. In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and unit of length is 10m. In this system, one unit of power will correspond to
(*a*) 16 watts (*b*) 1/16 watts
(*c*) 25 watts (*d*) none of these

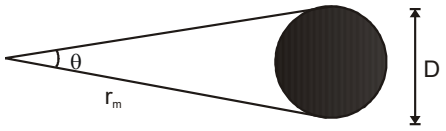
21. If the unit of force is 1 kilonewton, the length is 1 km and time is 100 second, what will be the unit of mass:
(*a*) 1000 kg (*b*) 10 kg
(*c*) 10000 kg (*d*) 100 kg

22. The units of length, velocity and force are doubled. Which of the following is the correct change in the other units?
(*a*) unit of time is doubled
(*b*) unit of mass is doubled
(*c*) unit of momentum is doubled
(*d*) unit of energy is doubled

23. If the units of force and that of length are doubled, the unit of energy will be:
(*a*) 1/4 times (*b*) 1/2 times
(*c*) 2 times (*d*) 4 times

24. If the units of M, L are doubled then the unit of kinetic energy will become
(*a*) 2 times (*b*) 4 times
(*c*) 8 times (*d*) 16 times

25. The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the act that the moon is about 384000 km away to find the approximate diameter of the moon.



(*a*) 192000 km (*b*) 3350 km
(*c*) 1600 km (*d*) 1920 km

ERRORS IN MEASUREMENT

26. The length of a rectangular plate is measured by a meter scale and is found to be 10.0 cm. Its width is measured by vernier callipers as 1.00 cm. The least count of the meter scale and vernier callipers are 0.1 cm and 0.01 cm respectively (Obviously). Maximum permissible error in area measurement is -

(*a*) ± 0.2 cm^2 (*b*) ± 0.1 cm^2
(*c*) ± 0.3 cm^2 (*d*) Zero

27. For a cubical block, error in measurement of sides is $\pm$ 1% and error in measurement of mass is $\pm$ 2%, then maximum possible error in density is -

(*a*) 1% (*b*) 5%
(*c*) 3% (*d*) 7%

28. To estimate 'g' (from $g = 4\pi^2 \frac{L}{T^2}$), error in measurement of L is $\pm$ 2% and error in measurement of T is $\pm$ 3%. The error in estimated 'g' will be -

(*a*) $\pm 8\%$ (*b*) $\pm 6\%$
(*c*) $\pm 3\%$ (*d*) $\pm 5\%$

29. The least count of a stop watch is 0.2 second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The percentage error in the time period is

(*a*) 16% (*b*) 0.8 %
(*c*) 1.8 % (*d*) 8 %

30. The dimensions of a rectangular block measured with a vernier callipers having least count of 0.1 mm is 5 mm × 10 mm × 5 mm. The maximum percentage error in measurement of volume of the block is

(*a*) 5 % (*b*) 10 %
(*c*) 15 % (*d*) 20 %

31. An experiment measures quantities x, y, z and then t is calculated from the data as $t = \frac{xy^2}{z^3}$. If percentage errors in x, y and z are respectively 1%, 3%, 2%, then percentage error in t is :

(*a*) 10 % (*b*) 4 %
(*c*) 7 % (*d*) 13 %

32. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is

(*a*) (0.34 ± 0.02) cm
(*b*) (0.17 ± 0.02) cm
(*c*) (0.17 ± 0.01) cm
(*d*) (0.34 ± 0.01) cm

33. The mass of a ball is 1.76 kg. The mass of 25 such balls is

(*a*) 0.44×10^3 kg (*b*) 44.0 kg
(*c*) 44 kg (*d*) 44.00 kg

34. Two resistors R_1 $(24 \pm 0.5)\ \Omega$ and R_2 $(8 \pm 0.3)\ \Omega$ are joined in series. The equivalent resistance is

(*a*) $32 \pm 0.33\ \Omega$ (*b*) $32 \pm 0.8\ \Omega$
(*c*) $32 \pm 0.2\ \Omega$ (*d*) $32 \pm 0.5\ \Omega$

35. The pitch of a screw gauge is 0.5 mm and there are 100 divisions on its circular scale. The instrument reads +2 divisions when nothing is put in-between its jaws. In measuring the diameter of a wire, there are 8 divisions on the main scale and 83rd division coincides with the reference line. Then the diameter of the wire is

(*a*) 4.05 mm (*b*) 4.405 mm
(*c*) 3.05 mm (*d*) 1.25 mm

36. The pitch of a screw gauge having 50 divisions on its circular scale is 1 mm. When the two jaws of the screw gauge are in contact with each other, the zero of the circular scale lies 6 division below the line of graduation. When a wire is placed between the jaws, 3 linear scale divisions are clearly visible while 31st division on the circular scale coincide with the reference line. The diameter of the wire is:

(*a*) 3.62 mm (*b*) 3.50 mm
(*c*) 3.5 mm (*d*) 3.74 mm

37. The smallest division on the main scale of a vernier callipers is 1 mm, and 10 vernier divisions coincide with 9 main scale divisions. While measuring the diameter of a sphere, the zero mark of the vernier scale lies between 2.0 and 2.1 cm and the fifth division of the vernier scale coincide with a scale division. Then diameter of the sphere is

(*a*) 2.05 cm (*b*) 3.05 cm
(*c*) 2.50 cm (*d*) None of these

Advanced Level Multiconcept Questions

JEE-ADVANCED
MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. Choose the correct statement(s):
(*a*) All quantities may be represented dimensionally in terms of the base quantities.
(*b*) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.
(*c*) The dimension of a base quantity in other base quantities is always zero.
(*d*) The dimension of a derived quantity is never zero in any base quantity.

2. Choose the correct statement(s):
(*a*) A dimensionally correct equation may be correct.
(*b*) A dimensionally correct equation may be incorrect.
(*c*) A dimensionally incorrect equation may be correct.
(*d*) A dimensionally incorrect equation must be incorrect.

3. The dimensions $ML^{-1}T^{-2}$ may correspond to
(*a*) work done by a force
(*b*) linear momentum
(*c*) pressure
(*d*) energy per unit volume

Comprehension Type Questions – 1 (No. 4 to 6)
Let us consider a particle P where is moving straight on the X-axis. We also know that the rate of change of its position is given by $\frac{dx}{dt}$; where x is its separation from the origin and t is time. This term $\frac{dx}{dt}$ is called the velocity of particle (v). Further the second derivation of x, w.r.t. time is called acceleration (*a*) or rate of change of velocity and represented by $\frac{d^2x}{dt^2}$ or $\frac{dv}{dt}$. If the acceleration of this particle is found to depend upon time as follows

$a = At + Bt^2 + \frac{Ct}{D + t^2}$ then-

4. The dimensions of A are -
(*a*) LT^{-2} (*b*) LT^{-3}
(*c*) LT^3 (*d*) L^2T^3

5. The dimensions of B are -
(*a*) LT^{-4} (*b*) L^2T^{-3}
(*c*) LT^4 (*d*) LT^{-2}

6. The dimensions of C are -
(*a*) L^2T^{-2} (*b*) LT^{-2}
(*c*) LT^{-1} (*d*) T^2

Comprehension Type Questions – 2 (No. 7 to 9)
According to coulombs law of electrostatics there is a force between two charged particles q_1 & q_2 separated by a distance r such that $F \propto q_1$, $F \propto q_2$ & $F \propto \frac{1}{r^2}$; combining all three we get $F \propto \frac{q_1q_2}{r^2}$ or $F = \frac{kq_1q_2}{r^2}$, where k is a constant which depends on the medium and is given by $1/4\pi\varepsilon_0\varepsilon_r$ where ε_0 is absolute permittivity & ε_r is relative permittivity.

But in case of protons of a nucleus there exists another force called nuclear force; which is much higher in magnitude in comparison to electrostatic force and is given by $F = \frac{Ce^{-kr}}{r^2}$.

7. What are the dimensions of C -
(*a*) $M^2L^3T^{-1}$ (*b*) ML^3T^{-3}
(*c*) ML^3T^{-2} (*d*) ML^2T^{-3}

8. What are the dimensions of k –
(*a*) L (*b*) L^2
(*c*) L^{-3} (*d*) L^{-1}

9. What are the SI units of C -
(*a*) Nm^{-2} (*b*) Nm^2
(*c*) Nm^{-3} (*d*) Nm

10. **Match the following columns**

Physical quantity	**Dimension**	**Unit**
(*a*) Gravitational constant 'G'	(P) $M^1L^1T^{-1}$	(*a*) N.m
(*b*) Torque	(Q) $M^{-1}L^3T^{-2}$	(*b*) N.s
(*c*) Momentum	(R) $M^1L^{-1}T^{-2}$	(*c*) Nm^2/kg^2
(*d*) Pressure	(S) $M^1L^2T^{-2}$	(*d*) pascal

11. Match the following:

Physical quantity	Dimension	Unit
(i) Stefan's constant 'σ'	(P) $M^1L^1T^{-2}A^{-2}$	(*a*) W/m^2
(ii) Wien's constant 'b'	(Q) $M^1L^oT^{-3}K^{-4}$	(*b*) K.m.
(iii) Coefficient of viscosity 'η'	(R) $M^1L^oT^{-3}$	(*c*) tesla .m/A
(iv) Emissive power of radiation (Intensity emitted)	(S) $M^oL^1T^oK^1$	(*d*) $W/m^2.K^4$
(v) Mutual inductance 'M'	(T) $M^1L^2T^{-2}A^{-2}$	(e) poise
(vi) Magnetic permeability 'μ_0'	(U) $M^1L^{-1}T^{-1}$	(f) henry

NUMERICAL VALUE BASED

12. Number of significant figures in $0.007\ m^2$.

13. Number of significant figures in 2.64×10^{24} kg

14. Number of significant figures in $6.032\ N\ m^{-2}$

15. The velocity of sound in a gas depends on its pressure and density. The relation between velocity, pressure and density is given by $V = Kp^a D^b$ then (a + b) is

16. A gas bubble, from an explosion under water, oscillates with a period proportional to $P^a d^b E^c$. Where P is the static pressure, d is the density and E is the total energy of the explosion. Find the values of a + b + c

17. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47th division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in two number of significant figures.

18. The density of a cube is measured by measuring its mass and the length of its sides. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is.

19. The length of the string of a simple pendulum is measured with a metre scale to be 90.0 cm. The radius of the bob plus the length of the hook is calculated to be 2.13 cm using measurements with a slide callipers. What is the effective length of the pendulum? (This effective length is defined as the distance between the point of suspension and the center of the bob).

JEE Mains & Advanced Past Years Questions

JEE-MAIN

1. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s,. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:

[*JEE Main-2016*]

(*a*) 92 ± 1.5 s (*b*) 92 ± 5.0 s
(*c*) 92 ± 1.8 s (*d*) 92 ± 3 s

2. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line? **[*JEE Main-2016*]**

(*a*) 0.75 mm (*b*) 0.80 mm
(*c*) 0.70 mm (*d*) 0.50 mm

3. The following observations were taken for determining surface tension T of water by capillary method :
Diameter of capilary, $D = 1.25 \times 10^{-2}$ m
rise of water, $h = 1.45 \times 10^{-2}$ m
Using $g = 9.80\ m/s^2$ and the simplified relation

$T = \frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to : **[*JEE Main-2017*]**

(*a*) 2.4% (*b*) 10% (*c*) 0.15% (*d*) 1.5%

4. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is:

[*JEE Main-2018*]

(*a*) 3.5% (*b*) 4.5% (*c*) 6% (*d*) 2.5%

5. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be:

[*JEE Main-2019* (*January*)]

(a) $V^{-2}A^2F^{-2}$ (b) $V^{-2}A^2F^2$

(c) $V^{-4}A^{-2}F$ (d) $V^{-4}A^2F$

6. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass 50 g, the numerical value of density of the material is: [*JEE Main-2019* (*January*)]

(a) 40 (b) 16 (c) 640 (d) 410

7. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to:

[*JEE Main-2019* (*January*)]

(a) $\sqrt{\frac{hc^5}{G}}$ (b) $\sqrt{\frac{c^3}{Gh}}$ (c) $\sqrt{\frac{Gh}{c^5}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

8. Let L, R, C and V represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{L}{RCV}$ in SI units will be:

[*JEE Main-2019* (*January*)]

(a) $[LA^{-2}]$ (b) $[A^{-1}]$ (c) [LTA] (d) $[LT^2]$

9. In SI units, the dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is

[*JEE Main-2019* (*April*)]

(a) $A^{-1}TML^3$ (b) $A^2T^3M^{-1}L^{-2}$

(c) $AT^2M^{-1}L^{-1}$ (d) $AT^{-3}ML^{3/2}$

10. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units?

[*JEE Main-2019* (*April*)]

(a) $[M^{-2}L^{-2}T^6A^3]$ (b) $[M^{-1}L^{-2}T^4A^2]$

(c) $[M^{-3}L^{-2}T^8A^4]$ (d) $[M^{-2}L^0T^{-4}A^{-2}]$

11. If surface tension (S), Moment of inertia (I) and Planck's constnat (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be:

[*JEE Main-2019* (*April*)]

(a) $S^{3/2}I^{1/2}h^0$ (b) $S^{1./2}I^{1/2}h^0$

(c) $S^{1/2}I^{1/2}h^{-1}$ (d) $S^{1/2}I^{3/2}h^{-1}$

12. Which of the following combination has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum) ?

[*JEE Main-2019* (*April*)]

(a) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (b) $\frac{\mu_0}{\varepsilon_0}$ (c) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (d) $\frac{\varepsilon_0}{\mu_0}$

13. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line. The readings of the main scale and the circular scale for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is

[*JEE Main-2019* (*January*)]

(a) 5.755 mm (b) 5.950 mm

(c) 5.725 mm (d) 5.740 mm

14. The diameter and height of a cylinder are measured by a meter scale to be 12.6±0.1 cm and 34.2 ± 0.1 cm respectively. What will be the value of its volume in appropriate significant figures?

[*JEE Main-2019* (*January*)]

(a) $4264 \pm 81\ cm^3$ (b) $4264 \pm 81.0\ cm^3$

(c) $4260 \pm 80\ cm^3$ (d) $4300 \pm 80\ cm^3$

15. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is:

[*JEE Main-2019* (*January*)]

(a) 50 (b) 200 (c) 100 (d) 500

16. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is

[*JEE Main-2019* (*April*)]

(a) 37 cm^2 (b) 37.0 cm^2

(c) 37.03 cm^2 (d) 37.030 cm^2

17. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is:

[*JEE Main-2019* (*April*)]

(a) 0.10 kg/m^3 (b) 0.31 kg/m^3

(c) 0.07 kg/m^3 (d) 0.01 kg/m^3

18. The dimensions of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is

(a) MLT^{-2} (b) $ML^{-1}T^{-2}$

(c) ML^2T^{-1} (d) ML^2T^{-2}

19. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of :

[*JEE Main-2020* (*January*)]

(a) volume (b) energy

(c) momentum (d) area

20. If momentum (P), area (*A*) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is **[*JEE Main-2020* (*September*)]**

(*a*) $\left[P^{\frac{1}{2}}AT^{-1}\right]$ (*b*) $[P^2AT^{-2}]$

(*c*) $\left[PA^{\frac{1}{2}}T^{-1}\right]$ (*b*) $[PA^{-1}T^{-2}]$

21. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be **[*JEE Main-2020* (*September*)]**

(*a*) $FA^{-1}V^0$ (*c*) FA^2V^{-1}

(*b*) FA^2V^{-2} (*d*) FA^2V^{-3}

22. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is **[*JEE Main-2020* (*September*)]**

(*a*) ML^2T^{-2} (*b*) MLT^{-2}

(*c*) $M^2L^0T^{-1}$ (*d*) ML^0T^{-3}

23. A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and length L. The dimensional formula for x is same as that of **[*JEE Main-2020* (*September*)]**

(*a*) Coefficient of viscosity

(*b*) Force constant

(*c*) Energy density

(*d*) Planck's constant

24. Dimensional formula for thermal conductivity is (here K deontes the temperature) **[*JEE Main-2020* (*September*)]**

(*a*) $MLT^{-2}K^{-2}$ (*b*) $MLT^{-3}K^{-1}$

(*c*) $MLT^{-3}K$ (*d*) $MLT^{-2}K$

25. The quantities $x = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $y = \frac{E}{B}$ and $z = \frac{I}{CR}$ are defined where C-capacitance, R-Resistance, l-length, E-Electric field, B-magnetic field and ε_0, μ_0,- free space permittivity and permeability respectively. Then **[*JEE Main-2020* (*September*)]**

(*a*) Only x and y have the same dimension

(*b*) Only x and z have the same dimension

(*c*) x, y and z have the same dimension

(*d*) Only y and z have the same dimension

26. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is : **[*JEE Main-2020* (*January*)]**

(*a*) 4.40% (*b*) 3.40%

(*c*) 2.40% (*d*) 5.40%

27. If the screw on a screw–gauge is given six rotations, it moves by 3mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is: **[*JEE Main-2020* (*January*)]**

(*a*) 0.01 cm (*b*) 0.02 mm

(*c*) 0.001 mm (*d*) 0.001 cm

28. When the temperature of a metal wire is increased from 0°C to 10°C, its length increases by 0.02%. The percentage change in its mass density will be closest to **[*JEE Main-2020* (*September*)]**

(*a*) 2.3 (*b*) 0.06

(*c*) 0.8 (*d*) 0.008

29. The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4 VSD coincides with a main scale division. The length of the cylinder is (VSD is vernier scale division) **[*JEE Main-2020* (*September*)]**

(*a*) 3.21 cm (*b*) 2.99 cm

(*c*) 3.07 cm (*d*) 3.2 cm

30. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as **[*JEE Main-2020* (*September*)]**

(*a*) 2.124 cm (*b*) 2.123 cm

(*c*) 2.125 cm (*d*) 2.121 cm

31. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is **[*JEE Main-2020* (*September*)]**

(*a*) 50 Ω (*b*) 200 Ω

(*c*) 300 Ω (*d*) 100 Ω

32. A physical quantity z depends on four observables a, b, c and d, as $\frac{a^a b^{\frac{2}{3}}}{\sqrt{c}\, d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is

[*JEE Main-2020* (*September*)]

(*a*) 13.5% (*b*) 14.5%

(*c*) 16.5% (*d*) 12.25%

33. A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm; 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as **[*JEE Main-2020* (*September*)]**

(*a*) (5.5375 ± 0.0739) mm (*b*) (5.54 ± 0.07) mm

(*c*) (5.538 ± 0.074) mm (*d*) (5.5375 ± 0.0740) mm

34. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is _____.

[*JEE Main-2020* (*September*)]

35. The vernier scale used for measurement has a positive zero error of $0.2\,mm$. If while taking a measurement it was noted that ' 0 ' on the vernier scale lies between $8.5\,cm$ and $8.6\,cm$, vernier coincidence is 6, then the correct value of measurement is cm. (least count $= 0.01\,cm$) **[*JEE Main-2021* (*March*)]**

(*a*) $8.58\,cm$ (*b*) $8.54\,cm$

(*c*) $8.56\,cm$ (*d*) $8.36\,cm$

36. In order to determine the Young's Modulus of a wire of radius $0.2\,cm$ (measured using a scale of least count $= 0.001\,cm$) and length $1\,m$ (measured using a scale of least count $= 1\,mm$), a weight of mass $1\,kg$ (measured using a scale of least count $= 1\,g$) was hanged to get the elongation of $0.5\,cm$ (measured using a scale of least count $0.001\,cm$). What will be the fractional error in the value of Young's Modulus determined by this experiment?

[*JEE Main-2021* (*March*)]

(*a*) 0.14% (*b*) 9%

(*c*) 1.4% (*d*) 0.9%

37. One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the callipers in mm is: **[*JEE Main-2021* (*March*)]**

(*a*) $\frac{10a}{R}$ (*b*) $\frac{10ra}{(r-1)}$

(*c*) $\left(\frac{R-1}{10R}\right)a$ (*d*) $\frac{10a}{(r-1)}$

JEE-ADVANCED

1. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension $\ell = 0.25$ mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement **[*IIT JEE-2012*]**

(*a*) due to the errors in the measurements of d and ℓ are the same.

(*b*) due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.

(*c*) due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.

(*d*) due to the error in the measurement of d is four time that due to the error in the measurement of ℓ.

2. Match List-I with List-II and select the correct answer using the codes given below the lists :

[*JEE Advanced-2013*]

List-I	**List-II**
P. Boltzmann constant	1. $[ML^2T^{-1}]$
Q. Coefficient of viscosity	2. $[ML^{-1}T^{-1}]$
R. Planck constant	3. $[MLT^{-3}K^{-1}]$
S. Thermal conductivity	4. $[ML^2T^{-2}K^{-1}]$

Codes :

	P	**Q**	**R**	**S**
(*a*)	3	1	2	4
(*b*)	3	2	1	4
(*c*)	4	2	1	3
(*d*)	4	1	2	3

3. The diameter of a c ylinder is measured using a vernier callipers with no zero error. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 division equivalent to 2.45 cm. The 24th division of the vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is :

[*JEE Advanced-2013*]

(*a*) 5.112 cm (*b*) 5.124 cm
(*c*) 5.136 cm (*d*) 5.148 cm

4. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and R are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56s, 0.57s, 0.54s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is (are) true? **[*JEE Advanced-2013*]**

(*a*) The error in the measurement of *r* is 10%
(*b*) The error in the measurement of *T* is 3.57%
(*c*) The error in the measurement of *T* is 2%
(*d*) The error in the determined value of g is 11%

5. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90º. The wavelength λ is exactly knowns and the error in θ is constant for all values of As θ increases from 0º : **[*JEE Advanced-2013*]**

(*a*) the absolute error in d remains constant.
(*b*) the absolute error in d increases.
(*c*) the fractional error in d remains constant.
(*d*) the fractional error in d decreases.

6. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer find that d is proportional to $S^{1/n}$. The value of n is:

[*JEE Advanced-2014*]

7. A length –scale (*l*) depends on the permittivity (ε) of a dielectric material, Boltzmann constant((k_B), the absolute temperature (*T*), the number per unit volume (*n*) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for *l* is (are) dimensionally correct?

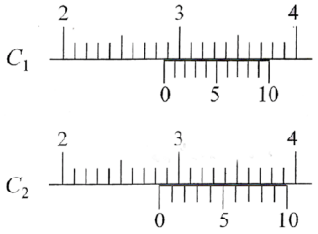
[*JEE Advanced-2016*]

(*a*) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (*b*) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(*c*) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (*d*) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

8. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale division. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are

[*JEE Advanced-2016*]



(*a*) 2.87 and 2.86 (*b*) 2.85 and 2.82
(*c*) 2.87 and 2.87 (*d*) 2.87 and 2.83

Comprehension – 1 (No. 9 and 10)

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while [$\in_0$] and [μ] stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

[*JEE Advanced - 2018*]

9. The relation between [E] and [B] is -

(*a*) [E] = [B] [L] [T] (*b*) $[E] = [B][L]^{-1}[T]$
(*c*) $[E] = [B][L][T]^{-1}$ (*d*) $[E] = [B][L]^{-1}[T]^{-1}$

10. The relation between [$\in_0$] and [μ_0] is -

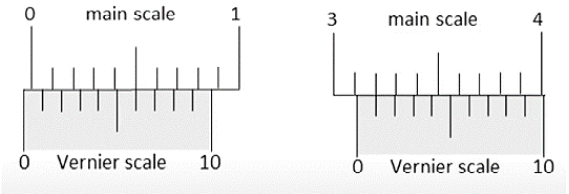
(*a*) $[\mu_0] = [\in_0][L]^2[T]^{-2}$
(*b*) $[\mu_0] = [\in_0][L]^{-2}[T]^2$
(*c*) $[\mu_0] = [\in_0]^{-1}[L]^2[T]^{-2}$
(*d*) $[\mu_0] = [\in_0]^{-1}[L]^{-2}[T]^2$

11. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement (s) is/are correct?

[*JEE Advanced - 2019*]

(*a*) The dimension of force is L^{-3}
(*b*) The dimension of energy is L^{-2}
(*c*) The dimension of power is L^{-5}
(*d*) The dimension of linear momentum is L^{-1}

12. The smallest division on the main scale of a Venire callipers is $0.1\,\text{cm}$. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calliper with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is **[*JEE Advanced-2021*]**



(*a*) $3.07\,\text{cm}$ (*b*) $3.11\,\text{cm}$

(*c*) $3.15\,\text{cm}$ (*d*) $3.17\,\text{cm}$

13. A physical quantity $\vec{S}$ is defined as $\vec{S} = (\vec{E} \times \vec{B}) / \mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity (ies) ? **[*JEE Advanced-2021*]**

(*a*) $\dfrac{\text{Energy}}{\text{charge} \times \text{current}}$

(*b*) $\dfrac{\text{Force}}{\text{Length} \times \text{Time}}$

(*c*) $\dfrac{\text{Energy}}{\text{Volume}}$

(*d*) $\dfrac{\text{Power}}{\text{Area}}$

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

ANSWER KEY

Topicwise Questions

1. (*a*) **2.** (*a*) **3.** (*c*) **4.** (*b*) **5.** (*c*) **6.** (*b*) **7.** (*b*) **8.** (*d*) **9.** (*a*) **10.** (*d*)
11. (*d*) **12.** (*c*) **13.** (*c*) **14.** (*b*) **15.** (*b*) **16.** (*b*) **17.** (*d*) **18.** (*c*) **19.** (*c*) **20.** (*a*)
21. (*b*) **22.** (*a*) **23.** (*c*) **24.** (*b*) **25.** (*b*) **26.** (*c*) **27.** (*a*) **28.** (*b*) **29.** (*a*) **30.** (*c*)
31. (*c*) **32.** (*a*) **33.** (*a*) **34.** (*a*) **35.** (*c*) **36.** (*b*) **37.** (*b*) **38.** (*b*) **39.** (*b*) **40.** (*c*)
41. (*c*) **42.** (*a*) **43.** (*b*) **44.** (*a*) **45.** (*c*) **46.** (*c*) **47.** (*c*)

Learning Plus

1. (*b*) **2.** (*b*) **3.** (*d*) **4.** (*c*) **5.** (*c*) **6.** (*b*) **7.** (*c*) **8.** (*d*) **9.** (*a*) **10.** (*b*)
11. (*c*) **12.** (*d*) **13.** (*d*) **14.** (*c*) **15.** (*d*) **16.** (*d*) **17.** (*b*) **18.** (*b*) **19.** (*d*) **20.** (*a*)
21. (*c*) **22.** (*c*) **23.** (*d*) **24.** (*c*) **25.** (*b*) **26.** (*a*) **27.** (*b*) **28.** (*a*) **29.** (*b*) **30.** (*a*)
31. (*d*) **32.** (*c*) **33.** (*b*) **34.** (*b*) **35.** (*b*) **36.** (*d*) **37.** (*a*)

Advanced Level Multiconcept Questions

1. (*a, b, c*) **2.** (*a, b, d*) **3.** (*c, d*) **4.** (*b*) **5.** (*a*) **6.** (*c*) **7.** (*c*) **8.** (*d*) **9.** (*b*)
10. $(a) \rightarrow (Q) \rightarrow (c), (b) \rightarrow (S) \rightarrow (a), (c) \rightarrow (P) \rightarrow (b), (d) \rightarrow (R) \rightarrow (d)$
11. $(i) \rightarrow (Q) \rightarrow (d), (ii) \rightarrow (S) \rightarrow (b), (iii) \rightarrow (U) \rightarrow (e), (iv) \rightarrow (R) \rightarrow (a), (v) \rightarrow (T) \rightarrow (f), (vi) \rightarrow (P) \rightarrow (c)$
12. [1] **13.** [3] **14.** [4] **15.** [1] **16.** [0] **17.** [2.6] **18.** [9] **19.** [92.1]

JEE Mains & Advanced Past Years Questions

JEE-MAIN

1. (*a*) **2.** (*b*) **3.** (*d*) **4.** (*b*) **5.** (*d*) **6.** (*a*) **7.** (*c*) **8.** (*b*) **9.** (*b*) **10.** (*c*)
11. (*b*) **12.** (*c*) **13.** (*c*) **14.** (*c*) **15.** (*b*) **16.** (*c*) **17.** (Bonus) **18.** (*b*) **19.** (*b*) **20.** (*c*)
21. (*a*) **22.** (*d*) **23.** (*c*) **24.** (*b*) **25.** (*c*) **26.** (*d*) **27.** (*d*) **28.** (*b*) **29.** (*c*) **30.** (*a*)
31. (*d*) **32.** (*b*) **33.** (*b*) **34.** [1050.00] **35.** (*b*) **36.** (*c*) **37.** (*a*)

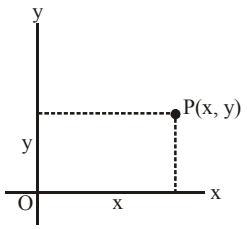
JEE-ADVANCED

1. (*a*) **2.** (*c*) **3.** (*b*) **4.** (*a, b, d*) **5.** (*d*) **6.** (*c*) **7.** (*b, d*) **8.** (*d*) **9.** (*c*) **10.** (*d*)
11. (*a, b, d*) **12.** (*c*) **13.** (*b, d*)

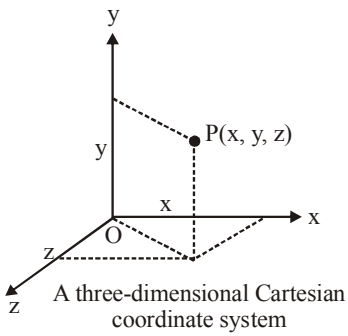
2 Vector and Calculus

LOCATION OF A POINT IN SPACE

Positions in space are designated relative to coordinate systems. The **Cartesian coordinate** system is a particularly convenient coordinate system in which positions are designated by distances (x, y, z) along three perpendicular axes that intersect at a point called the origin.



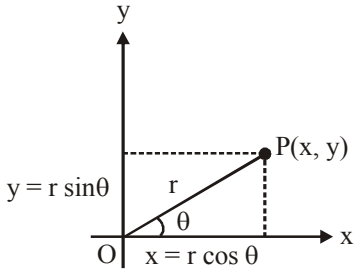
A two-dimensional Cartesian coordinate system



A three-dimensional Cartesian coordinate system

In a **polar coordinate** system positions in a plane are designated by a length r from the origin, and an angle θ usually measured from the positive x - axis. From simple trigonometry we see that the relationships between the *polar coordinates* and the *Cartesian coordinates* are

$$x = r\cos\theta$$
$$y = r\sin\theta$$



Relationship between polar and Cartesian coordinates

FRAME OF REFERENCE

A **frame of reference** is another name for the particular coordinate system with respect to which we are making observations of physical phenomena.

SCALAR & VECTOR

(i) Scalar: A **scalar** quantity requires only a number for its complete description. Mass, volume, density, pressure and temperature are all examples of scalar quantities. The mathematics of scalar quantities is the ordinary algebra of numbers.

(ii) Vector: Vector quantities require both *magnitude* and *direction* for its complete description. Velocity, acceleration, force and momentum are examples of vector quantities.

Types of vector.

(i) **Equal vectors :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction.

(ii) **Parallel vector :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when
(a) Both have same direction.
(b) One vector is scalar (positive) non-zero multiple of another vector.

(iii) **Anti-parallel vectors :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when
(a) Both have opposite direction.
(b) One vector is scalar non-zero negative multiple of another vector.

(iv) **Collinear vectors :** When the vectors under consideration can share the same support or have a common support then the considered vectors are collinear.

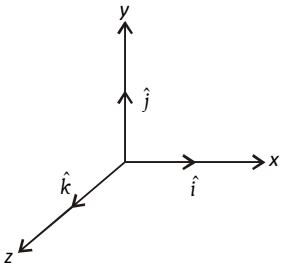
(v) **Zero vector** $\left(\vec{0}\right)$**:** A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.

(vi) **Unit vector :** A vector divided by its magnitude is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap / A hat).

Since, $\hat{A} = \dfrac{\vec{A}}{A} \Rightarrow \vec{A} = A\hat{A}$.

Thus, we can say that unit vector gives us the direction.

(vii) **Orthogonal unit vectors :** $\hat{i}, \hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a Right Handed Triad (It is a coordinate system such that when we Curl the fingers of right hand from x to y then we must get the direction of z along thumb).

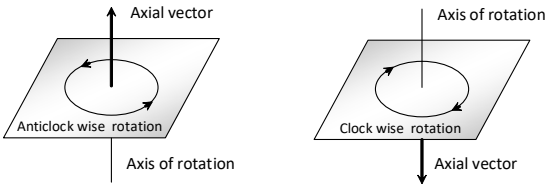


Then

$$\hat{i} = \frac{\vec{x}}{x}, \hat{j} = \frac{\vec{y}}{y}, \hat{k} = \frac{\vec{z}}{z}$$

$$\therefore \vec{x} = x\hat{i}, \vec{y} = y\hat{j}, \vec{z} = z\hat{k}$$

(viii) **Axial Vectors :** These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are example of physical quantities of this type.

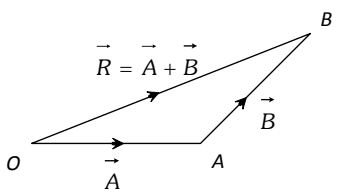


(ix) **Coplanar vector :** Three (or more) vectors are called coplanar vector if they lie in the same plane. Two (free) vectors are always coplanar.

TRIANGLE LAW OF VECTOR ADDITION OF TWO VECTORS

If two non zero vectors are represented by the two sides of a triangle taken in same order then the resultant is given by the closing side of triangle in opposite order. i.e. $\vec{R} = \vec{A} + \vec{B}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$



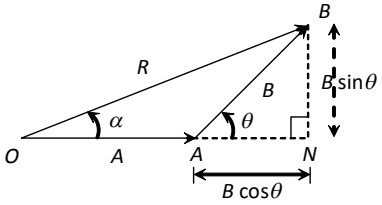
(i) Magnitude of resultant vector

In ΔABN, $\cos\theta = \frac{AN}{B}$ $\therefore$ $AN = B\cos\theta$

$\sin\theta = \frac{BN}{B}$ $\therefore$ $BN = B\sin\theta$

In ΔOBN, we have $OB^2 = ON^2 + BN^2$

$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$



$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

(ii) Direction of resultant vectors : If θ is angle between $\vec{A}$ and $\vec{B}$ then

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

PARALLELOGRAM LAW OF VECTOR ADDITION OF TWO VECTORS

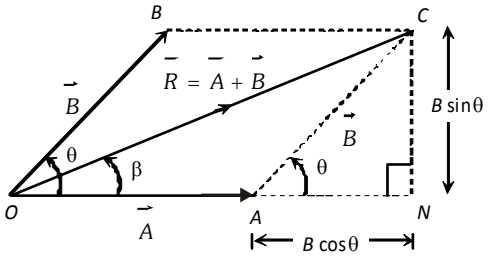
If two non zero vector are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the **two vectors**.

(i) Magnitude

Since, $R^2 = ON^2 + CN^2$

$\Rightarrow R^2 = (OA + AN)^2 + CN^2$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$



$\therefore\ R = |\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Special cases : $R = A + B$ when $\theta = 0°$

$R = A - B$ when $\theta = 180°$

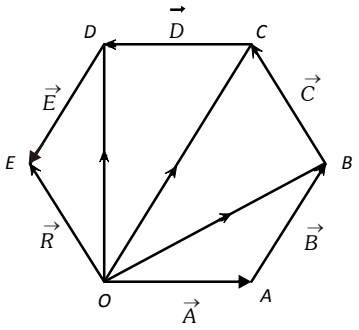
$R = \sqrt{A^2 + B^2}$ when $\theta = 90°$

(ii) Direction

$$\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$$

POLYGON LAW OF VECTOR ADDITION

If a number of non zero vectors are represented by the $(n - 1)$ sides of an n-sided polygon then the resultant is given by the closing side or the nth side of the polygon taken in opposite order. So,



$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$

$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{OE}$

NOTE: Resultant of two unequal vectors can not be zero. Resultant of three co-planar vectors may or may not be zero Resultant of three non co- planar vectors can not be zero.

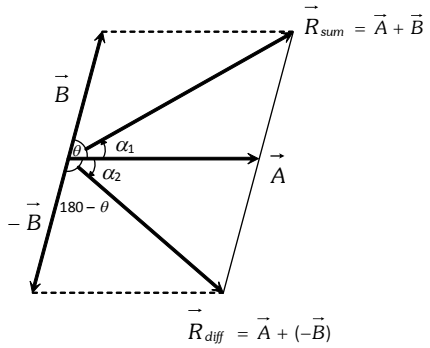
SUBTRACTION OF VECTORS

Since $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

and $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180^o - \theta)}$

Since, $\cos(180° - \theta) = -\cos\theta$



$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$$\tan\alpha_1 = \frac{B\sin\theta}{A + B\cos\theta}$$

and $\tan\alpha_2 = \dfrac{B\sin(180 - \theta)}{A + B\cos(180 - \theta)}$

But $\sin(180° - \theta) = \sin\theta$

and $\cos(180° - \theta) = -\cos\theta$

$\Rightarrow \tan\alpha_2 = \dfrac{B\sin\theta}{A - B\cos\theta}$

TRAIN YOUR BRAIN

Example 1: There are two force vectors, one of 5 N and other of 12 N at what angle the two vectors be added to get resultant vector of 17 N, 7 N and 13 N respectively

(*a*) 0°, 180° and 90° (*b*) 0°, 90° and 180°

(*c*) 0°, 90° and 90° (*d*) 180°, 0° and 90°

Ans. (a)

Sol. For 17 N both the vector should be parallel i.e. angle between them should be zero for 7 N both the vectors should be antiparallel i.e. angle between them should be 180° for 13 N both the vectors should be perpendicular to each other i.e. angle between them should be 90°.

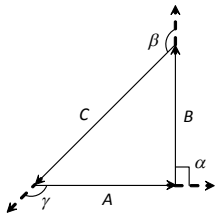
Example 2: Given that $\vec{A} + \vec{B} + \vec{C} = 0$ out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then the angles between vectors are given by

(*a*) 30°, 60°, 90° (*b*) 45°, 45°, 90°

(*c*) 45°, 60°, 90° (*d*) 90°, 135°, 135°

Ans. (d)

Sol. From polygon law, three vectors having summation zero should form a closed polygon. (Triangle) since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. i.e. the triangle should be right angled triangle



Angle between A and B, $\alpha = 90°$

Angle between B and C, $\beta = 135°$

Angle between A and C, $\gamma = 135°$

Example 3: If the sum of two unit vectors is a unit vector, then magnitude of difference is

(*a*) $\sqrt{2}$ (*b*) $\sqrt{3}$

(*c*) $1/\sqrt{2}$ (*d*) $\sqrt{5}$

Ans. (b)

Sol. Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is

$\vec{n}_s = \hat{n}_1 + \hat{n}_2$ or $n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta = 1+1+2\cos\theta$

Since it is given that n_s is also a unit vector, therefore $1 = 1+1+2\cos\theta$

or $\cos\theta = -\frac{1}{2}$ or $\theta = 120°$

Now the difference vector is $\vec{n}_d = \hat{n}_1 - \hat{n}_2$ or

$n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1+1-2\cos(120°)$

$\therefore\ n_d^2 = 2-2(-1/2) = 2+1 = 3 \Rightarrow n_d = \sqrt{3}$

Example 4: The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the, magnitudes of forces

(*a*) 12, 5 (*b*) 14, 4

(*c*) 5, 13 (*d*) 10, 8

Ans. (3)

Sol. Let P be the smaller force and Q be the greater force then according to problem -

$P+Q=18$ .....(i)

$R = \sqrt{P^2+Q^2+2PQ\cos\theta} = 12$ .....(ii)

$\tan\phi = \frac{Q\sin\theta}{P+Q\cos\theta} = \tan 90° = \infty$

$\therefore\ P + Q\cos\theta = 0$ .....(iii)

By solving (i), (ii) and (iii)

we will get P = 5 and Q = 13.

Example 5: Two vectors of 10 units & 5 units make an angle of 120° with each other. Find the magnitude & angle of resultant with vector of 10 unit magnitude.

Sol. $|\vec{a}+\vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta} = \sqrt{100+25+2\times10\times5(-1/2)}$

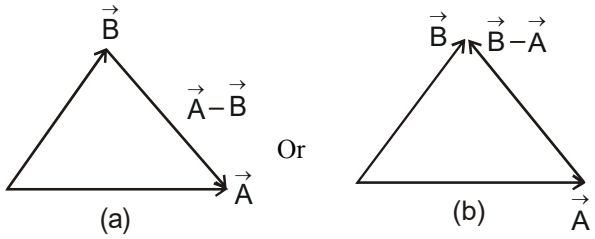
$= 5\sqrt{3}$

$\tan\alpha = \frac{5\sin120°}{10+5\cos120°} = \frac{5\sqrt{3}}{20-5} = \frac{5\sqrt{3}}{5\times3} = \frac{1}{\sqrt{3}}$

$\Rightarrow \alpha = 30°$

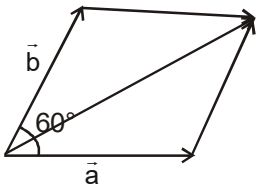
[Here shows what is angle between both vectors = 120° and not 60°]

NOTE: $\vec{A}-\vec{B}$ or $\vec{B}-\vec{A}$ can also be found by making triangles as shown in figure. (a) and (b)

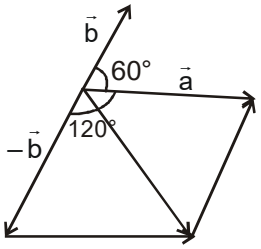


Example 6: Two vectors of equal magnitude 2 are at an angle of 60° to each other find magnitude of their sum & difference.

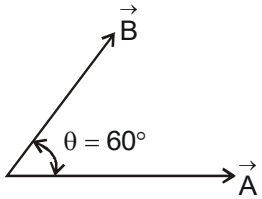
Sol. $|\vec{a}+\vec{b}| = \sqrt{2^2+2^2+2\times2\times2\cos60°} = \sqrt{4+4+4} = 2\sqrt{3}$



$|\vec{a}-\vec{b}| = \sqrt{2^2+2^2+2\times2\times2\cos120°} = \sqrt{4+4-4} = 2$



Example 7: Find $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ in the diagram shown in figure. Given A = 4 units and B = 3 units.



Sol. Addition :

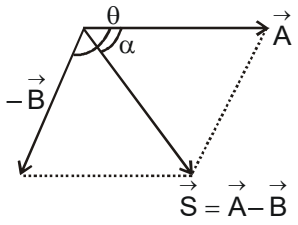
$R = \sqrt{A^2+B^2+2AB\cos\theta}$

$= \sqrt{16+9+2\times4\times3\cos60°} = \sqrt{37}$ units

$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta} = \frac{3\sin60°}{4+3\cos60°} = 0.472$

$\therefore\quad \alpha = \tan^{-1}(0.472) = 25.3°$

Thus, resultant of $\vec{A}$ and $\vec{B}$ is $\sqrt{37}$ units at angle 25.3° from $\vec{A}$ in the direction shown in figure.



Subtraction : $S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{16 + 9 - 2 \times 4 \times 3\cos 60°} = \sqrt{13}$ units

and $\tan\theta = \dfrac{B\sin\theta}{A - B\cos\theta}$

$= \dfrac{3\sin 60°}{4 - 3\cos 60°} = 1.04$

$\therefore \alpha = \tan^{-1}(1.04) = 46.1°$

Thus, $\vec{A} - \vec{B}$ is $\sqrt{13}$ units at 46.1° from $\vec{A}$ in the direction shown in figure.

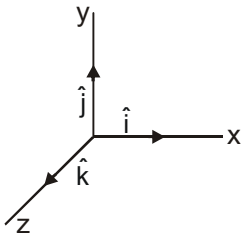
UNIT VECTOR AND ZERO VECTOR

Unit vector is a vector which has a unit magnitude and points in a particular direction. Any vector $(\vec{A})$ can be written as the product of unit vector $(\hat{A})$ in that direction and magnitude of the given vector.

$$\vec{A} = A\,\hat{A}$$

$$\text{or } \hat{A} = \frac{\vec{A}}{A}$$

A unit vector has no dimensions and unit. Unit vectors along the positive x-, y-and z-axes of a rectangular coordinate system are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively such that $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.



A vector of zero magnitude is called a **zero or a null vector**. Its direction is arbitrary.

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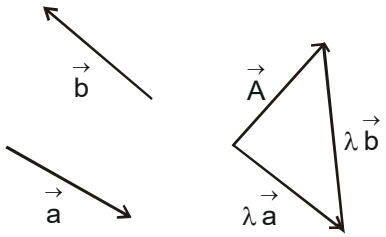
Example 8: A unit vector along East is defined as $\hat{i}$. A force of 10^5 dynes acts west wards. Represent the force in terms of $\hat{i}$.

Sol. $\vec{F} = -10^5\,\hat{i}$ dynes

RESOLUTION OF VECTORS

If $\vec{a}$ and $\vec{b}$ be any two non-zero vectors in a plane with different directions and $\vec{A}$ be another vector in the same plane. $\vec{A}$ can be expressed as a sum of two vectors-one obtained by multiplying $\vec{a}$ by a real number and the other obtained by multiplying $\vec{b}$ by another real number.



$\vec{A} = \lambda\vec{a} + \mu\vec{b}$ (where λ and μ are real numbers)

We say that $\vec{A}$ has been resolved into two component vectors namely

$\vec{A} = \lambda\vec{a} + \mu\vec{b}$ (where λ and μ are real number)

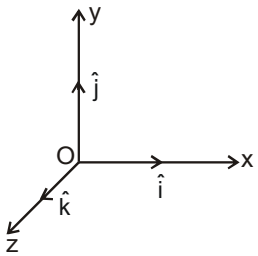
We say that $\vec{A}$ has been resolved into two component vectors namely

$\lambda\vec{a}$ and $\mu\vec{b}$

$\lambda\vec{a}$ and $\mu\vec{b}$ along $\vec{a}$ and $\vec{b}$ respectively. Hence one can resolve a given vector into two component vectors along a set of two vectors – all the three lie in the same plane.

Resolution along rectangular component:

It is convenient to resolve a general vector along axes of a rectangular coordinate system using vectors of unit magnitude, which we call as unit vectors. $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit along x, y and z-axis as shown in figure below:



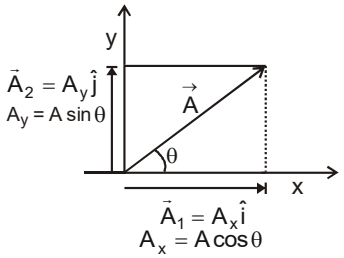
Resolution in two Dimension

Consider a vector $\vec{A}$ that lies in xy plane as shown in figure,

$\vec{A} = \vec{A}_1 + \vec{A}_2$

$\vec{A}_1 = A_x\hat{i},\ \vec{A}_2 = A_y\hat{j} \Rightarrow \vec{A} = A_x\hat{i} + A_y\hat{j}$

The quantities A_x and A_y are called x-and y-components of the vector $\vec{A}$.



A_x is itself not a vector but $A_x\hat{i}$ is a vector and so it $A_y\hat{j}$.

$A_x = A\cos\theta$ and $A_y = A\sin\theta$

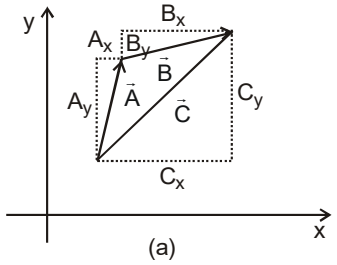
It's clear from above equation that a component of a vector can be positive, negative or zero depending on the value of θ. A vector $\vec{A}$ can be specified in a plane by two ways :

(a) its magnitude A and the direction θ it makes with the x-axis; or

(b) its components A_x and A_y

$$A = \sqrt{A_x^2 + A_y^2}, \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

NOTE: If $A = A_x \Rightarrow A_y = 0$ and if $A = A_y \Rightarrow A_x = 0$ i.e., components of a vector perpendicular to itself is always zero. The rectangular components of each vector and those of the sum $\vec{C} = \vec{A} + \vec{B}$ are shown in figure.



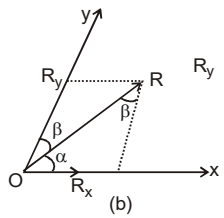
We saw that

$\vec{C} = \vec{A} + \vec{B}$ is equivalent to both

$C_x = A_x + B_x$

and $C_y = A_y + B_y$

Refer figure (b)



Vector $\vec{R}$ has been resolved in two axes x and y not perpendicular to each other. Applying sine law in the triangle shown, we have

$$\frac{R}{\sin[180° - (\alpha + \beta)]} = \frac{R_x}{\sin\beta} = \frac{R_y}{\sin\alpha}$$

or $R_x = \dfrac{R\sin\beta}{\sin(\alpha + \beta)}$ and $R_y = \dfrac{R\sin\alpha}{\sin(\alpha + \beta)}$

If $\alpha + \beta = 90°$, $R_x = R\sin\beta$ and $R_y = R\sin\alpha$

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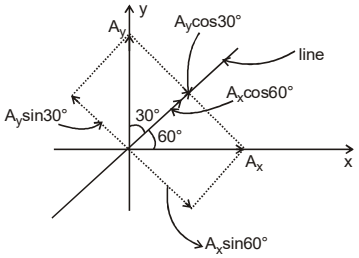
Example 9: Resolve the vector $A = A_x\hat{i} + A_y\hat{j}$ along an perpendicular to the line which make angle 60° with x-axis.

Sol.



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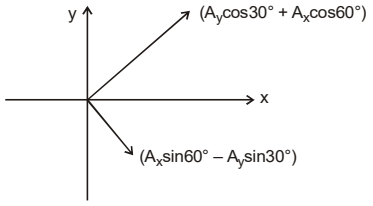
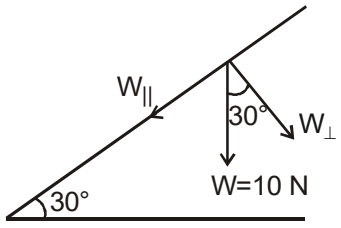


so the component along line = $|A_y\cos30° + A_x\cos60°|$

and perpendicular to line = $|A_x\sin60° - A_y\sin30°|$

Example 10: Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.

Sol. Component perpendicular to the plane



$W_{\perp} = W\cos 30°$

$= (10)\dfrac{\sqrt{3}}{2} = 5\sqrt{3}$ N **Ans.**

and component parallel to the plane

$W_{\parallel} = W\sin 30° = (10)\left(\dfrac{1}{2}\right) = 5$ N

Example 11: Resolve horizontally and vertically a force F = 8 N which makes an angle of 45° with the horizontal.

Sol. Horizontal component of $\vec{F}$ is

$F_H = F\cos 45° = (8)\left(\dfrac{1}{\sqrt{2}}\right) = 4\sqrt{2}$ N

and vertical component of $\vec{F}$ is

$F_v = F\sin 45° = (8)\left(\dfrac{1}{\sqrt{2}}\right) = 4\sqrt{2}$ N **Ans.**

PROCEDURE TO SOLVE THE VECTOR EQUATION

$$\vec{A} = \vec{B} + \vec{C} \qquad ...(1)$$

(a) There are 6 variables in this equation which are following:

(1) Magnitude of $\vec{A}$ and its direction

(2) Magnitude of $\vec{B}$ and its direction

(3) Magnitude of $\vec{C}$ and its direction.

(b) We can solve this equation if we know the value of 4 variables

[Note : two of them must be directions]

(c) If we know the two direction of any two vectors then we will put them on the same side and other on the different side.

For example

If we know the directions of $\vec{A}$ and $\vec{B}$ and $\vec{C}$'s direction is unknown then we make equation as follows:- $\vec{C} = \vec{A} - \vec{B}$

(d) Then we make vector diagram according to the equation and resolve the vectors to know the unknown values.

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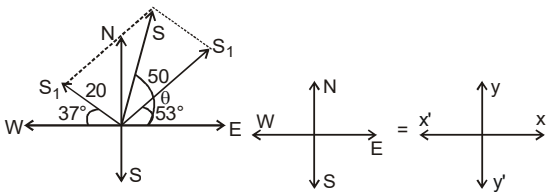
Example 12: Find the net displacement of a particle from its starting point if it undergoes two sucessive displacement given by $\vec{S}_1 = 20$m , 37° North of West, $\vec{S}_2 = 50$m , 53° North of East

Sol.



$\vec{S} = \vec{S}_1 + \vec{S}_2$

$S_x = S_{1x} + S_{2x}$

$S_y = S_{1y} + S_{2y}$

$= -20\cos 37° + 50\cos 53°$

$= 20\sin 37° + 50\sin 53°$

$= 14$

$= 52$

$S = \sqrt{S_x^2 + S_y^2} = \sqrt{(14)^2 + (52)^2} = 53.85$

Angle from west - east axis (x- axis)

$\tan\theta = \dfrac{S_y}{S_x} = \dfrac{52}{14} = \dfrac{26}{7}$

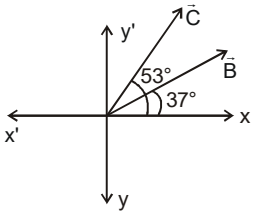
$\theta = \tan^{-1}\left(\dfrac{26}{7}\right)$

Example 13: Find magnitude of $\vec{B}$ and direction of $\vec{A}$. If $\vec{B}$ makes angle 37° and $\vec{C}$ makes 53° with x axis and $\vec{A}$ has magnitude equal to 10 and $\vec{C}$ has 5. (given $\vec{A} + \vec{B} + \vec{C} = 0$)

Sol. $-\vec{A} = \vec{C} + \vec{B}$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

$\Rightarrow -\vec{A} = -A_x\hat{i} + -A_y\hat{j}$



$A_x = -(B\cos 37° + C\cos 53°)$

$A_y = -(B\sin 37° + C\cos 53°)$

$|\vec{A}|^2 = A_x^2 + A_y^2$

$A^2 = \left(B\times\frac{4}{5} + C\times\frac{3}{5}\right)^2 + \left(B\times\frac{3}{5} + C\times\frac{4}{5}\right)^2$

$10^2 = \left(\frac{4B}{5}+3\right)^2 + \left(\frac{3B}{5}+4\right)^2$

$\Rightarrow 100 = \frac{16}{25}B^2 + \frac{9}{25}B^2 + 25 + 2\left(\frac{3\times4}{5} + \frac{4\times3}{5}\right)B$

$\Rightarrow B^2 + \frac{48}{5}B - 75 = 0$

B = 5 (magnitude can not be negative)

& Angle made by A

$\Rightarrow A_x = -(\frac{20}{5}+3) = -12$

$A_y = -(\frac{15}{5}+4) = -7$

$\tan\theta = \frac{A_y}{A_x} = \frac{-7}{-12}$

$\theta = 180° + 25° = 205°$

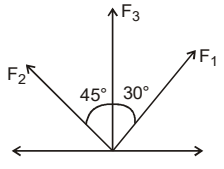
Example 14: Find the magnitude of F_1 and F_2. If F_1, F_2 make angle 30° and 45° with F_3 and magnitude of F_3 is 10 N. (given $\vec{F}_1 + \vec{F}_2 = \vec{F}_3$)

Sol. $|\vec{F}_3| = F_1\cos 30° + F_2\cos 45°$

& $F_2\sin 45°$

$= F_1\sin 30°$

$\Rightarrow 10 = \frac{\sqrt{3}F_1}{2} + \frac{F_2}{\sqrt{2}}, \frac{F_2}{\sqrt{2}} = \frac{F_1}{2}$



$\Rightarrow F_1 = \frac{20}{\sqrt{3}+1}$ & $F_2 = \frac{20\sqrt{2}}{\sqrt{3}+1}$

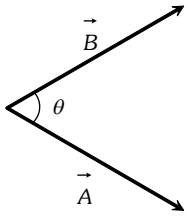
SCALAR PRODUCT OF TWO VECTORS

Definition : The scalar product (or dot product) of two vectors is defined as the product of the magnitude of two vectors with cosine of angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them, then their scalar product written as $\vec{A}.\vec{B}$ is defined as $\vec{A}.\vec{B} = AB\cos\theta$

Properties:

(i) It is always a scalar which is positive if angle between the vectors is acute (i.e., < 90°) and negative if angle between them is obtuse (i.e. 90°< θ < 180°).



(ii) It is commutative, i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(iii) It is distributive, i.e. $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(iv) As by definition $\vec{A}.\vec{B} = AB\cos\theta$

The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}.\vec{B}}{AB}\right]$

(v) Scalar product of two vectors will be maximum when $\cos\theta = \max = 1$, i.e. $\theta = 0^o$, i.e., vectors are parallel

$(\vec{A}.\vec{B})_{max} = AB$

(vi) Scalar product of two vectors will be minimum when $|\cos\theta| = \min = 0$, i.e. $\theta = 90^o$

$(\vec{A}.\vec{B})_{min} = 0$

i.e., if the scalar product of two nonzero vectors vanishes if the vectors are orthogonal.

(vii) The scalar product of a vector by itself is termed as self dot product and is given by

$(\vec{A})^2 = \vec{A}.\vec{A} = AA\cos\theta = A^2$

i.e., $A = \sqrt{\vec{A}.\vec{A}}$

(viii) In case of unit vector $\hat{n}$

$\hat{n}.\hat{n} = 1\times1\times\cos 0 = 1$

so $\hat{n}.\hat{n} = \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ix) In case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 1\times1\cos 90 = 0$

(x) In terms of components

$\vec{A}.\vec{B} = (\hat{i}A_x + \hat{j}A_y + \vec{k}A_z).(\hat{i}B_x + \hat{j}B_y + \vec{k}B_z)$

$= [A_xB_x + A_yB_y + A_ZB_z]$

Application:

(i) **Work W :** In physics for constant force work is defined as,

$W = \vec{F}.\vec{s} = Fs\cos\theta$

i.e. work is the scalar product of force with displacement.

(ii) **Power P :**

As $W = \vec{F}.\vec{s}$ or $\frac{dW}{dt} = \vec{F}.\frac{d\vec{s}}{dt}$

[As $\vec{F}$ is constant]

or $\left[\text{As}\frac{dW}{dt} = P \text{ and } \frac{d\vec{s}}{dt} = \vec{v}\right]$ i.e., power is the scalar product of force with velocity. $\left[\text{As}\frac{dW}{dt} = P \text{ and } \frac{d\vec{s}}{dt} = \vec{v}\right]$

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Example 15: $\vec{A} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ are two vectors. The angle between them will be

(*a*) 0° (*b*) 45°

(*c*) 60° (*d*) 90°

Ans. (d)

Sol. $\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{|\vec{A}|.|\vec{B}|}$

$= \frac{2\times4 + 4\times2 - 4\times4}{|\vec{A}|.|\vec{B}|} = 0$

$\therefore \quad \theta = \cos^{-1}(0°) \Rightarrow \theta = 90°$

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Example 16: In above example if vectors are perpendicular to each other then value of λ be

(*a*) 25 (*b*) 26

(*c*) -26 (*d*) -25

Ans. (b)

Sol. If $\vec{A}$ and $\vec{B}$ are perpendicular to each other then $\vec{A}.\vec{B} = 0 \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$

So, $2(-4) + 3(-6) + (-1)(-\lambda) = 0 \Rightarrow \lambda = +26$

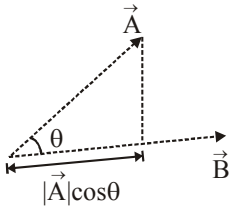
Example 17: If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$ then projection of $\vec{A}$ on $\vec{B}$ will be

(*a*) $\frac{3}{\sqrt{13}}$ (*b*) $\frac{3}{\sqrt{26}}$ (*c*) $\sqrt{\frac{3}{26}}$ (*d*) $\sqrt{\frac{3}{13}}$

Ans. (*b*)

Sol. $|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4+9+1} = \sqrt{14}$

$|\vec{B}| = \sqrt{(-1)^2 + 3^2 + 4^2} = \sqrt{1+9+16} = \sqrt{26}$



$\vec{A}.\vec{B} = 2(-1) + 3\times3 + (-1)(4) = 3$

The projection of $\vec{A}$ *on* $\vec{B} = \frac{\vec{A}.\vec{B}}{|\vec{B}|} = \frac{3}{\sqrt{26}}$

Example 18: A body, acted upon by a force of 50 N is displaced through a distance 10 meter in a direction making an angle of 60° with the force. The work done by the force be

(*a*) 200 J (*b*) 100 J

(*c*) 300 (*d*) 250 J

Ans. (d)

Sol. $W = \vec{F}.\vec{S} = FS\cos\theta$

$= 50\times10\times\cos 60° = 50\times10\times\frac{1}{2} = 250\,\text{J}.$

Example 19: A particle moves from position $3\hat{i} + 2\hat{j} - 6\hat{k}$ to $14\hat{i} + 13\hat{j} + 9\hat{k}$ due to a uniform force of $4\hat{i} + \hat{j} + 3\hat{k}$ N. If the displacement in meters then work done will be

(*a*) 100 J (*b*) 200 J

(*c*) 300 J (*d*) 250 J

Ans. (a)

Sol. $S = \vec{r_2} - \vec{r_1}$

$W = \vec{F}.\vec{S} = (4\hat{i} + \hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} + 15\hat{k})$

$= (4\times11 + 1\times11 + 3\times15) = 100\,\text{J}.$

VECTOR PRODUCT OF TWO VECTOR

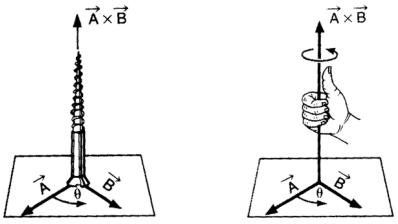
Definition:

The vector product or cross product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them, and direction perpendicular to the plane containing the two vectors in accordance with right hand screw rule.

$$\vec{C} = \vec{A} \times \vec{B}$$

Thus, if $\vec{A}$ & $\vec{B}$ are two vectors, then their vector product written as $\vec{A} \times \vec{B}$ is a vector $\vec{C}$ defined by

$$\vec{C} = \vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$



The direction of $\vec{A} \times \vec{B}$ i.e. $\vec{C}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ and in the sense of advance of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right handed screw whose axis is perpendicular to the plane framed by $\vec{A}$ and $\vec{B}$ is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A} \times \vec{B}$ i.e. $\vec{C}$.

Properties:

(i) Vector product of any two vectors is always a vector perpendicular to the plane containing these two vectors, i.e., orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

(ii) Vector product of two vectors is not commutative, i.e., $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ [but $= -\vec{B} \times \vec{A}$]

Here it is worthy to note that

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB \sin\theta$$

i.e., in case of vector $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ magnitudes are equal but directions are opposite.

(iii) The vector product is distributive when the order of the vectors is strictly maintained, i.e.

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(iv) As by definition of vector product of two vectors $\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

So $|\vec{A} \times \vec{B}| = AB \sin\theta$ i.e. $\theta = \sin^{-1}\left[\dfrac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}\right]$

(v) The vector product of two vectors will be maximum when $\sin\theta = \max = 1$, i.e., $\theta = 90^o$

$[\vec{A} \times \vec{B}]_{max} = AB \, \hat{n}$

i.e., vector product is maximum if the vectors are orthogonal.

(vi) The vector product of two non- zero vectors will be minimum when $|\sin\theta| =$ minimum $= 0$, i.e., $\theta = 0^o$ or 180°

$[\vec{A} \times \vec{B}]_{min} = 0$

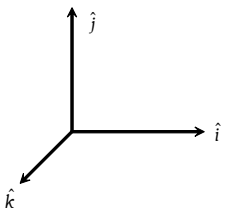
i.e. if the vector product of two non-zero vectors vanishes, the vectors are collinear.

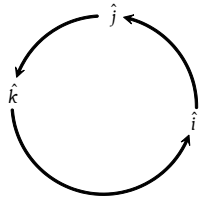
(vii) The self cross product, i.e., product of a vector by itself vanishes, i.e., null vector $\vec{A} \times \vec{A} = AA \sin 0^o \, \hat{n} = \vec{0}$

(viii) In case of unit vector $\hat{n} \times \hat{n} = \vec{0}$

so that $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$

(ix) In case of orthogonal unit vectors, $\hat{i}, \hat{j}, \hat{k}$ in accordance with right hand screw rule :





$\hat{i} \times \hat{j} = \hat{k}$,

$\hat{j} \times \hat{k} = \hat{i}$

and $\hat{k} \times \hat{i} = \hat{j}$

And as cross product is not commutative,

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

and $\hat{i} \times \hat{k} = -\hat{j}$

(x) In terms of components

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

Application:

Since vector product of two vectors is a vector, some of the vector physical quantities (particularly representing rotational effects) like torque, angular momentum, velocity and force on a moving charge in a magnetic field can be expressed as the vector product of two vectors. It is well - established in physics that:

(i) Torque $\vec{\tau} = \vec{r} \times \vec{F}$

(ii) Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

(iii) Velocity $\vec{v} = \vec{\omega} \times \vec{r}$

(iv) Force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v} \times \vec{B})$

(v) Torque on a dipole in a field $\overrightarrow{\tau_E} = \vec{p} \times \vec{E}$ and $\overrightarrow{\tau_B} = \overrightarrow{M} \times \vec{B}$

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Example 20: If two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $-4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are parallel to each other then value of λ be

(*a*) 0 (*b*) 2

(*c*) 3 (*d*) 4

Ans. (b)

Sol. Let $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} + \lambda\hat{k}$

$\vec{A}$ and $\vec{B}$ are parallel to each other $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ i.e.

$$\frac{2}{-4} = \frac{3}{-6} = \frac{-1}{\lambda} \Rightarrow \lambda = 2$$

Example 21: If $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ then value of $|\vec{A} \times \vec{B}|$ will be

(*a*) $8\sqrt{2}$ (*b*) $8\sqrt{3}$ (*c*) $8\sqrt{5}$ (*d*) $5\sqrt{8}$

Ans. (b)

Sol. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix} =$

$(1\times4 - 2\times-2)\hat{i} + (2\times2 - 4\times3)\hat{j} + (3\times-2 - 1\times2)\hat{k}$

$= 8\hat{i} - 8\hat{j} - 8\hat{k}$

$\therefore$ Magnitude of

$\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| = \sqrt{(8)^2 + (-8)^2 + (-8)^2}$

$= 8\sqrt{3}$

Example 22: In above example a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ will be

(*a*) $+\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (*b*) $-\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

(*c*) Both (1) and (2) (*d*) None of these

Sol. $\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{8\hat{i} - 8\hat{j} - 8\hat{k}}{8\sqrt{3}} = \frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

There are two unit vectors perpendicular to both $\vec{A}$ and $\vec{B}$ they are $\hat{n} = \pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

Example 23: The vectors from origin to the points A and B are $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} - 2\hat{k}$ respectively. The area of the triangle OAB be

(*a*) $\frac{5}{2}\sqrt{17}$ sq.unit (*b*) $\frac{2}{5}\sqrt{17}$ sq.unit

(*c*) $\frac{3}{5}\sqrt{17}$ sq.unit (*d*) $\frac{5}{3}\sqrt{17}$ sq.unit

Ans. (a)

Sol. Given $\overrightarrow{OA} = \vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

and $\overrightarrow{OB} = \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$$\therefore (\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

$= (12 - 2)\hat{i} + (4 + 6)\hat{j} + (3 + 12)\hat{k}$

$= 10\hat{i} + 10\hat{j} + 15\hat{k}$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{10^2 + 10^2 + 15^2}$

$= \sqrt{425} = 5\sqrt{17}$

Area of

$$\Delta OAB = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{5\sqrt{17}}{2} \text{ sq.unit.}$$

Example 24: The angle between the vectors $\vec{A}$ and $\vec{B}$ is θ. The value of the triple product $\vec{A}.(\vec{B} \times \vec{A})$ is

(*a*) A^2B (*b*) Zero

(*c*) $A^2B\sin\theta$ (*d*) $A^2B\cos\theta$

Ans. (b)

Sol. Let $\vec{A}.(\vec{B} \times \vec{A}) = \vec{A}.\vec{C}$

Here $\vec{C} = \vec{B} \times \vec{A}$ Which is perpendicular to both vector $\vec{A}$ and $\vec{B}$ $\therefore$ $\vec{A}.\vec{C} = 0$

Example 25: The torque of the force $\vec{F} = (2\hat{i} - 3\hat{j} + 4\hat{k})$N acting at the point $\vec{r} = (3\hat{i} + 2\hat{j} + 3\hat{k})$ m about the origin be

(*a*) $6\hat{i} - 6\hat{j} + 12\hat{k}$ (*b*) $17\hat{i} - 6\hat{j} - 13\hat{k}$

(*c*) $-6\hat{i} + 6\hat{j} - 12\hat{k}$ (*d*) $-17\hat{i} + 6\hat{j} + 13\hat{k}$

Ans. (b)

Sol. $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$= [(2\times4) - (3\times-3)]\hat{i} + [(2\times3) - (3\times4)]\hat{j} + [(3\times-3) - (2\times2)]\hat{k}$

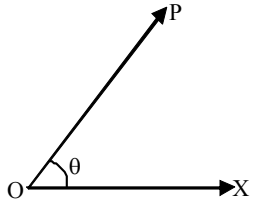
$= 17\hat{i} - 6\hat{j} - 13\hat{k}$

BASIC MATHEMATICS

Trigonometry

(I) Angle : Consider a revolving line OP. Suppose that it revolves in anticlockwise direction starting from its initial position OX.

The angle is defined as the amount of revolution that the revolving line makes with its initial position. From fig. the angle covered by the revolving line OP is given by $\theta = \angle POX$ .The angle is positive, if it is traced by the revolving line in anticlockwise direction and is **negative,** if it is covered in clockwise direction.



1 right angle = 90º (degrees)

1º = 60' (minutes)

1' = 60" (seconds)

In circular system

1 right angle = $\frac{\pi}{2}$ rad (radian)

One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle.

1 rad = $180º/\pi \approx 57º\,17'\,45'' \approx 57.3º$

(II) Trigonometrical Ratios (or T-ratios)

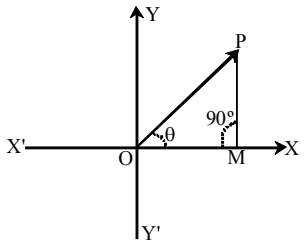
Consider the two fixed lines XOX' and YOY' intersecting at right angles to each other at point O as shown in Fig. Then.

(i) Point O is called origin.

(ii) XOX' and YOY' are known as X-axis and Y-axis respectively.

(iii) Portions XOY, YOX', X'OY' and Y'OX are called I, II, III, and IV quadrant respectively. Consider that the revolving line OP has traced out angle θ (in I quadrant) in anticlockwise direction.

From P, drop PM perpendicular to OX. Then, side OP (in front of right angle) is called hypotenuse, side MP (in front of angle θ) is called **opposite side or perpendicular** and side OM (making angle θ with hypotenuse) is called **adjacent side or base**.



The three sides of a right angled triangle are connected to each other through six different ratios, called trigonometric rations or simply T-ratios.

These are defined as below:

1. $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP}$ (read as sine of angle θ)

2. $\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP}$ (read as cosine of angle θ)

3. $\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM}$ (read as tangent of angle θ)

4. $\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP}$ (read as cotangent of angle θ)

5. $\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM}$ (read as secant of angle θ)

6. $\text{cosec}\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$ (read as cosecant of angle θ)

It can be easily proved that:

1. (a) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ (b) $\sec\theta = \frac{1}{\cos\theta}$

(c) $\cot\theta = \frac{1}{\tan\theta}$

2. (a) $\sin^2\theta + \cos^2\theta = 1$

(b) $1 + \tan^2\theta = \sec^2\theta$

(c) $1 + \cot^2\theta = \text{cosec}^2\theta$

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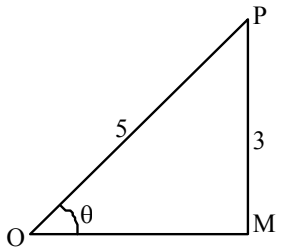
Example 26: Given $\sin\theta = \frac{3}{5}$. Find all the other T-ratios, if θ lies in the first quadrant.

Sol. In Δ OMP, $\sin\theta = \frac{3}{5}$

$\therefore$ MP = 3 and OP = 5



Hence, $OM = \sqrt{(5)^2 - (3)^2} = \sqrt{25-9} = \sqrt{16} = 4$

Now, $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$, $\tan\theta = \frac{MP}{OM} = \frac{3}{4}$, $\cot\theta = \frac{OM}{MP} = \frac{4}{3}$,

$\sec\theta = \frac{OP}{OM} = \frac{5}{4}$, $\text{cosec}\theta = \frac{OP}{MP} = \frac{5}{3}$

Example 27: If $\sec\theta = \frac{5}{3}$, find all the other T-ratios.

Ans. $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$,

$\tan\theta = \frac{4}{3}$, $\cot\theta = \frac{3}{4}$,

$\text{cosec}\theta = \frac{5}{4}$

(III) T-RATIOS OF A FEW STANDARD ANGLES:

The T-ratios of a few standard angles ranging from 0º to 180º are given in the following table

Angle(θ)	**0º**	**30º**	**45º**	**60º**	**90º**	**120º**	**135º**	**150º**	**180º**
sin θ	**0**	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	**0**
cos θ	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	**0**	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	**–1**
tan θ	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\sqrt{3}$	**∞**	$-\sqrt{3}$	**–1**	$-\frac{1}{\sqrt{3}}$	**0**

(IV) T-ratios of angles associated with multiples of 90º:

The angles such as (90º – θ), (90º + θ), (180 – θ), (180º + θ), (270º – θ), (270º + θ), etc. are called angles allied to angle θ.

The T-ratios of the following allied angle as are commonly used:

1.	(a) $\sin(-\theta) = -\sin\theta$	(b) $\cos(-\theta) = \cos\theta$	(c) $\tan(-\theta) = -\tan\theta$
2.	(a) $\sin(90^\circ - \theta) = \cos\theta$	(b) $\cos(90^\circ - \theta) = \sin\theta$	(c) $\tan(90^\circ - \theta) = \cot\theta$
3.	(a) $\sin(90^\circ + \theta) = \cos\theta$	(b) $\cos(90^\circ + \theta) = -\sin\theta$	(c) $\tan(90^\circ + \theta) = -\cot\theta$
4.	(a) $\sin(180^\circ - \theta) = \sin\theta$	(b) $\cos(180^\circ - \theta) = -\cos\theta$	(c) $\tan(180^\circ - \theta) = -\tan\theta$
5.	(a) $\sin(180^\circ + \theta) = -\sin\theta$	(b) $\cos(180^\circ + \theta) = -\cos\theta$	(c) $\tan(180^\circ + \theta) = \tan\theta$
6.	(a) $\sin(270^\circ - \theta) = -\cos\theta$	(b) $\cos(270^\circ - \theta) = -\sin\theta$	(c) $\tan(270^\circ - \theta) = \cot\theta$
7.	(a) $\tan(270^\circ + \theta) = -\cos\theta$	(b) $\cos(270^\circ + \theta) = \sin\theta$	(c) $\tan(270^\circ + \theta) = -\cot\theta$

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Example 28: Find the value of

(i) $\cos(-60^\circ)$ (ii) $\tan 210^\circ$ (iii) $\sin 300^\circ$ (iv) $\cos 120^\circ$

Sol. (i) $\cos(-60^\circ) = \cos 60^\circ = \frac{1}{2}$

(ii) $\tan 210^\circ = \tan(180^\circ + 30^\circ) = \tan 30^\circ = \frac{1}{\sqrt{3}}$

(iii) $\sin 300^\circ = \sin(270^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

(iv) $\cos 120^\circ = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$

Example 29: Find the values of the following:

1. $\tan(-30^\circ)$
2. $\sin 120^\circ$
3. $\sin 330^\circ$
4. $\cos 150^\circ$
5. $\sin 270^\circ$
6. $\cos 270^\circ$

Ans. 1. $-\frac{1}{\sqrt{3}}$ 2. $\frac{\sqrt{3}}{2}$

3. $-\frac{1}{2}$ 4. $-\frac{\sqrt{3}}{2}$

5. -1 6. 0

(V) A few important trigonometric formulae

1. Addition Formulae:

(a) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(b) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(c) $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

2. Subtraction Formulae:

(a) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(b) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(c) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

3. Multiplication Formulae:

(a) $\sin 2A = 2 \sin A \cos A$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

(c) $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

DIFFERENTIATION

(i) Finite difference

The finite difference between two values of a physical quantity is represented by Δ notation.

For example:

Difference in two values of y is written as Δy as given in the table below.

y_2	100	100	100
y_1	50	99	99.5
$\Delta y = y_2 - y_1$	50	1	0.5

(ii) Infinitely small difference:

The infinitely small difference means very-very small difference. And this difference is represented by 'd' notation instead of 'Δ'.

For example infinitely small difference in the values of y is written as 'dy'

if $y_2 = 100$ and $y_1 = 99.99999999.....$

then $dy = 0.000000.....00001$

DEFINITION OF DIFFERENTIATION

Another name for differentiation is derivative. Suppose y is a function of x or $y = f(x)$

Differentiation of y with respect to x is denoted by symbol $f'(x)$ where $f'(x) = \frac{dy}{dx}$

dx is very small change in x and dy is corresponding very small change in y.

NOTATION : There are many ways to denote the derivative of a function $y = f(x)$. Besides $f'(x)$, the most common notations are these:

y'	**"y prime" or "y dash"**	**Nice and brief but does not name the independent variable.**
$\frac{dy}{dx}$	"dy by dx"	Names the variables and uses d for derivative.
$\frac{df}{dx}$	"df by dx"	Emphasizes the function's name.
$\frac{d}{dx}f(x)$	"d by dx of f"	Emphasizes the idea that differentiation is an operation performed on f.
$?_x f$	"dx of f"	A common operator notation.
$\dot{y}$	"y dot"	One of Newton's notations, now common for time derivatives i.e. $\frac{dy}{dt}$.

SLOPE OF A LINE

It is the tan of angle made by a line with the positive direction of x-axis, measured in anticlockwise direction.

Slope = $\tan\theta$

(In 1st quadrant $\tan\theta$ is +ve & 2nd quadrant $\tan\theta$ is –ve)

In Figure - 1 slope is positive

In Figure - 2 slope is negative

$\theta < 90^\circ$ (1st quadrant)

$\theta > 90^\circ$ (2nd quadrant)

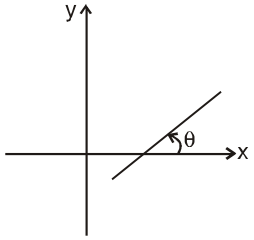


Figure - 1

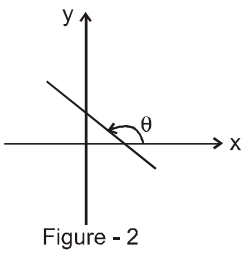
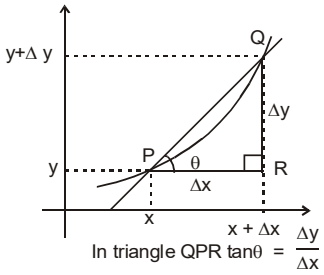


Figure - 2

AVERAGE RATES OF CHANGE

Given an arbitrary function y = f(x) we calculate the average rate of change of y with respect to x over the interval (x , x + Δx) by dividing the change in value of y, i.e. Δy = f(x + Δx) – f(x), by length of interval Δx over which the change occurred.



The average rate of change of y with respect to x over the interval $[x, x+\Delta x] = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$

Geometrically, $\frac{\Delta y}{\Delta x} = \frac{QR}{PR} = \tan\theta$ = Slope of the line PQ

therefore we can say that average rate of change of y with respect to x is equal to slope of the line joining P & Q.

THE DERIVATIVE OF A FUNCTION

We know that, average rate of change of y w.r.t. x is $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$.

If the limit of this ratio exists as $\Delta x \to 0$, then it is called the derivative of given function f(x) and is denoted as

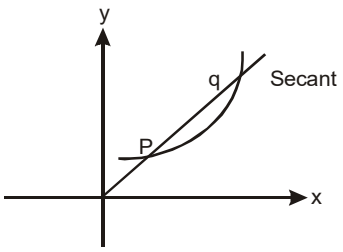
$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

GEOMETRICAL MEANING OF DIFFERENTIATION

The geometrical meaning of differentiation is very much useful in the analysis of graphs in physics. To understand the geometrical meaning of derivatives we should have knowledge of secant and tangent to a curve

Secant and tangent to a curve

Secant : A secant to a curve is a straight line, which intersects the curve at any two points.



TANGENT

A tangent is a straight line, which touches the curve at a particular point. Tangent is a limiting case of secant which intersects the curve at two overlapping points.

In the figure-1 shown, if value of Δx is gradually reduced then the point Q will move nearer to the point P. If the process is continuously repeated (Figure - 2) value of Δx will be infinitely small and secant PQ to the given curve will become a tangent at point P .

Therefore $\lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \frac{dy}{dx} = \tan\theta$

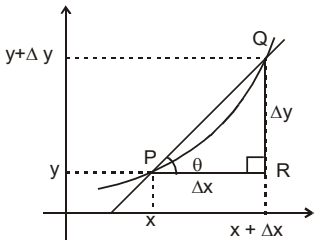


Figure - 1

we can say that differentiation of y with respect to x,i.e. $\left(\frac{dy}{dx}\right)$ is equal to slope of the tangent at point P (x, y) or $\tan\theta = \frac{dy}{dx}$

(From fig. 1, the average rate of change of y from x to x + Δx is identical with the slope of secant PQ.)

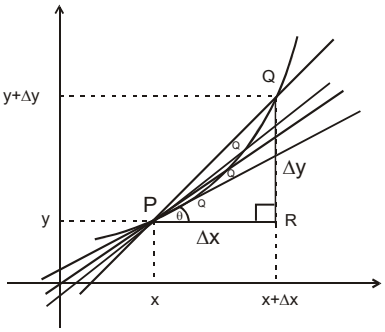


Figure - 2

Theorems of Differentiation

1. If c is constant, then
$$\frac{d}{dx}(c) = 0$$
2. If y = cu, where c is a constant and u is a function of x, then
$$\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}$$
3. If y = u ± v ± w, where, u, v and w are functions of x, then
$$\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$$

4. If $y = u\,v$ where u and v are function of x, then

$$\frac{dy}{dx} = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

5. If $y = \frac{u}{v}$, where u and v are functions of x, then

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

6. If $y = x^n$ where n is a real number, then

$$\frac{dy}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$$

Formulae for differential coefficients of trigonometric, logarithmic and exponential functions

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$
5. $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$
7. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$
8. $\frac{d}{dx}(e^x) = e^x$

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Example 30: Find $\frac{dy}{dx}$, when

(i) $y = \sqrt{x}$ (ii) $y = x^5 + x^4 + 7$

(iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Sol. (i) Here, $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

(ii) Here, $y = x^5 + x^4 + 7$

$$\frac{dy}{dx} = \frac{d}{dx}(x^5 + x^4 + 7)$$

$$= \frac{d}{dx}(x^5) + \frac{d}{dx}(x^4) + \frac{d}{dx}(7)$$

$$= 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3$$

(iii) Here, $y = x^2 + 4x^{-1/2} - 3x^{-2}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$$

$$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2})$$

$$= 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)x^{-3}$$

$$= 2x - 2x^{-3/2} + 6x^{-3}$$

Example 31: Find the derivatives of the following:

(i) $(x^3 - 3x^2 + 4)(4x^5 + x^2 - 1)$ (ii) $\frac{9x^5}{x-3}$

Sol. (i) Let $y = (x^3 - 3x^2 + 4)(4x^5 + x^2 - 1)$

$$\frac{dy}{dx} = (x^3 - 3x^2 + 4)\frac{d}{dx}(4x^5 + x^2 - 1) + (4x^5 + x^2 - 1)\frac{d}{dx}(x^3 - 3x^2 + 4)$$

$$= (x^3 - 3x^2 + 4)\left[\frac{d}{dx}(4x^5) + \frac{d}{dx}(x^2) - \frac{d}{dx}(1)\right] + (4x^5 + x^2 - 1)$$

$$\left[\frac{d}{dx}(x^3) - \frac{d}{dx}(3x^2) + \frac{d}{dx}(4)\right]$$

$$= (x^3 - 3x^2 + 4)\left[4\frac{d}{dx}(x^5) + \frac{d}{dx}(x^2) - \frac{d}{dx}(1)\right] + (4x^5 + x^2 - 1)$$

$$\left[\frac{d}{dx}(x^3) - 3\frac{d}{dx}(x^2) + \frac{d}{dx}(4)\right]$$

$$= (x^3 - 3x^2 + 4)[4 \times 5x^4 + 2x - 0] + (4x^5 + x^2 - 1)[3x^2 - 3 \times 2x + 0]$$

$$= (x^3 - 3x^2 + 4)(20x^4 + 2x) + (4x^5 + x^2 - 1)(3x^2 - 6x)$$

$$= 2x(10x^3 + 1)(x^3 - 3x^2 + 4) + 3x(x-2)(4x^5 + x^2 - 1)$$

(ii) Let $y = \frac{9x^5}{x-3}$

$$\frac{dy}{dx} = \frac{(x-3)\frac{d}{dx}(9x^5) - 9x^5\frac{d}{dx}(x-3)}{(x-3)^2}$$

$$= \frac{(x-3) \times 9\frac{d}{dx}(x^5) - 9x^5\left[\frac{d}{dx}(x) - \frac{d}{dx}(3)\right]}{(x-3)}$$

$$= \frac{(x-3)\times 9\times 5x^4 - 9x^5(1-0)}{(x-3)^2}$$

$$= \frac{45x^5 - 135x^4 - 9x^5}{(x-3)^2} = \frac{36x^5 - 135x^4}{(x-3)^2} = \frac{9x^4(4x-15)}{(x-3)^2}$$

SELF PRACTICE PROBLEMS

Find $\frac{dy}{dx}$ for the following:

1. $y = x^{7/2}$
2. $y = x^{-3}$
3. $y = x$
4. $y = x^5 + x^3 + 4x^{1/2} + 7$
5. $y = 5x^4 + 6x^{3/2} + 9x$
6. $y = ax^2 + bx + c$
7. $y = 3x^5 - 3x - \frac{1}{x}$
8. Given $S = t^2 + 5t + 3$, find $\frac{dS}{dt}$
9. Given $S = ut + \frac{1}{2}at^2$, where u and a are constants. Obtain the value of $\frac{dS}{dt}$.
10. The area of a blot of in k is growing such that after t seconds, its area is given by $A = (3t^2 + 7)$ cm^2. Calculate the rate of increase of area at t = 5 seconds.
11. The area of a circle is given by $A = \pi r^2$, where r is the radius. Calculate the rate of increase of area w.r.t. radius.

Obtain the differential coefficient (differentiation) of the following:

12. $(x-1)(2x+5)$
13. $(9x^3 - 8x + 7)(3x^5 + 5)$
14. $\frac{1}{2x+1}$
15. $\frac{3x+4}{4x+5}$
16. $\frac{x^2}{x^3+1}$

Answers

1. $\frac{7}{2}x^{5/2}$
2. $-3x^{-4}$
3. 1
4. $5x^4 + 3x^2 + 2x^{-1/2}$
5. $20x^3 + 9x^{1/2} + 9$
6. $2ax + b$
7. $15x^4 - 3 + \frac{1}{x^2}$
8. $2t + 5$
9. $u + at$
10. 30 cm^2 s^{-1}
11. $2\pi r$
12. $4x + 3$
13. $216x^7 - 144x^5 + 105x^4 + 135x^2 - 40$
14. $-\frac{2}{(2x+1)^2}$
15. $-\frac{1}{(4x+5)^2}$
16. $\frac{2x - x^4}{(x^3+1)^2}$

INTEGRATION

(I) Integration as the inverse process of differentiation

Integration is the process of finding the function, whose derivative is given. For this reason, the process of integration is the inverse process of differentiation.

Consider a function f(x), whose derivative w.r.t. x is another function f '(x) i.e.

$$\frac{d}{dx}(f(x)) = f'(x)$$

If differentiation of f(x) w.r.t.x is equal to f '(x), then f(x) + c is called the integration of f '(x), where c is called the constant of integration.

Symbolically, it is written as

$$\int f'(x)dx = f(x) + c$$

Here, f '(x) dx is called element of integration and $\int$ is called indefinite integral. Let us proceed to obtain intergral of x^n w.r.t. x.

$$\frac{d}{dx}(x^{n+1}) = (n+1)x^n$$

Since the process of integration is the inverse process of differentiation,

$$\int (n+1)x^n dx = x^{n+1} \text{ or } (n+1)\int x^n dx = x^{n+1}$$

or $$\int x^n dx = \frac{x^{n+1}}{n+1}$$

The above formula holds for all values of n, except n = – 1. It is because, for n = – 1,

$$\int x^n dx = \int x^{-1} dx = \int \frac{1}{x} dx$$

Since $\frac{1}{x}$ is differential coefficient of $\log_e x$

i.e. $$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\therefore \int \frac{1}{x} dx = \log_e x$$

Similarly, the formulae for integration of some other functions can be obtained if we know the differential coefficients of various functions.

(II) Few basic formulae of integration

Following are a few basic formulae of integration:

1. $\int x^n dx = \frac{x^{x+1}}{n+1} + c$, Provided $n \neq -1$
2. $\int \sin x dx = -\cos x + c \quad (\because \frac{d}{dx}(\cos x) = -\sin x)$

3. $\int \cos x dx = \sin x + c \quad (\because \frac{d}{dx}(\sin x) = \cos x)$

4. $\int \frac{1}{x} dx = \log_e x + c \quad (\because \frac{d}{dx}(\log_e x) = \frac{1}{x})$

5. $\int e^x dx = e^x + c \quad (\because \frac{d}{dx}(e^x) = e^x)$

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Example 32: Integrate w.r.t. x. :

(i) $x^{11/2}$ (ii) x^{-7} (iii) $x^{p/q}$

Sol. (i) $\int x^{11/2} dx = \frac{x^{\frac{11}{2}+1}}{\frac{11}{2}+1} + c = \frac{2}{13} x^{13/2} + c$

(ii) $\int x^{-7} dx = \frac{x^{-7+1}}{-7+1} + c = -\frac{1}{6} x^{-6} + c$

(iii) $\int x^{\frac{p}{q}} dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1} + c = \frac{q}{p+q} x^{(p+q)/q} + c$

Example 33: Evaluate $\int \left(x^2 - \cos x + \frac{1}{x} \right) dx$

Sol. $= \int x^2 dx - \int \cos x dx + \int \frac{1}{x} dx$

$= \frac{x^{2+1}}{2+1} - \sin x + \log_e x + c$

$= \frac{x^3}{3} - \sin x + \log_e x + c$

(III) Definite integrals

When a function is integrated between a lower limit and an upper limit, it is called a definite integral. If

$\frac{d}{dx}(f(x) = f'(x))$,

then $\int f'(x) dx$ is called indefinite integral and $\int_a^b f'(x) dx$

is called definite integral

Here, a and b are called lower and upper limits of the variable x.

After carrying out integration, the result is evaluated between upper and lower limits as explained below:

$\int_a^b f'(x) dx = |f(x)|_a^b = f(b) - f(a)$

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Example 34: Evaluate the integral : $\int_1^5 x^2 dx$

Sol. $\int_1^5 x^2 dx = \left| \frac{x^3}{3} \right|_1^5 = \frac{1}{3} |x^3|_1^5 = \frac{1}{3}((5)^3 - (1)^3)$

$= \frac{1}{3}(125 - 1) = \frac{124}{3}$

SELF PRACTICE PROBLEMS

Evaluate the following integrals:

1. $\int x^{15} dx$ **2.** $\int x^{-3/2} dx$

3. $\int (3x^{-7} + x^{-1}) dx$ **4.** $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$

5. $\int \left(x + \frac{1}{x} \right) dx$

6. $\int \left(\frac{a}{x^2} + \frac{b}{x} \right) dx$ (Where a and b are constant)

7. $\int_R^\infty \frac{GMm}{r^2} dr$ **8.** $\int_{r_1}^{r_2} -k \frac{q_1 q_2}{r^2} dr$

9. $\int_u^v M v dv$ **10.** $\int_0^\infty x^{-1/2} dx$

11. $\int_0^{\pi/2} \sin x \, dx$ **12.** $\int_0^{\pi/2} \cos x \, dx$

13. $\int_{-\pi/2}^{\pi/2} \cos x \, dx$

Answers

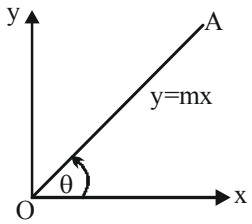
1. $\frac{x^{16}}{16}+c$ **2.** $-2x^{-1/2}+c$ **3.** $-\frac{x^{-6}}{2}+\log_e x+c$ **4.** $\frac{x^2}{2}+2x+\log_e x+c$

5. $\frac{x^2}{2}+\log_e x+c$ **6.** $-\frac{a}{x}+b\log_e x+c$ **7.** $\frac{GMm}{R}$ **8.** $kq_1q_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$

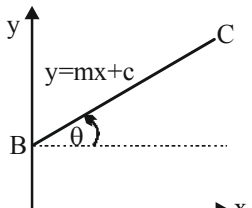
9. $\frac{1}{2}M(v^2-u^2)$ **10.** $[\infty]$ **11.** [1] **12.** [1]

13. [2]

SOME STANDARD GRAPHS AND THEIR EQUATIONS



straight line



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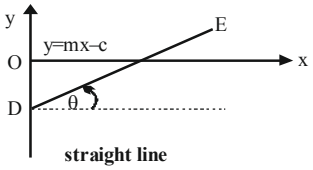

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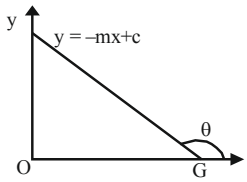

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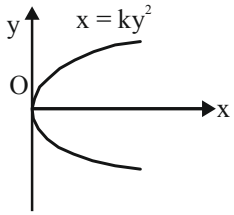
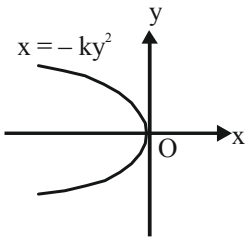


Parabola



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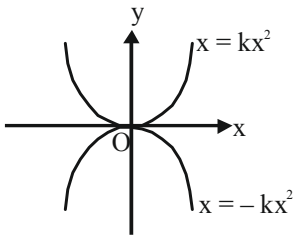
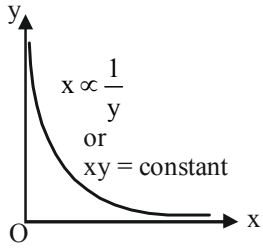
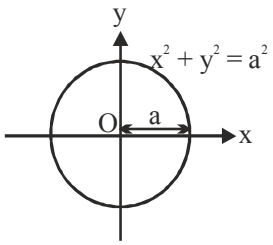


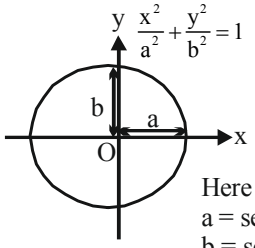
Parabola



Rectangular hyperbola

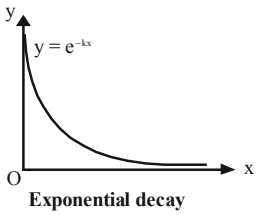


Circle



Here
a = semi major axis
b = semi minor axis
b < a

Ellipse



Exponential decay

Small angle approximation:

θ is very small and it must be in radian when you are taking approximation.

$\sin\theta \simeq \theta$, $\tan\theta \simeq \theta$

$\sin\theta \simeq \tan\theta$

$\cos\theta \simeq 1$

Example 35: Evaluate $\sin 2^o$

Sol. $2^o = 2 \times \frac{\pi}{180}\text{rad} = \frac{\pi}{90}\text{rad}$

Now

$$\sin 2^o = \sin\left(\frac{\pi}{90}\text{rad}\right) \simeq \left(\frac{\pi}{90}\right)$$

Topicwise Questions

RESULTANT OF TWO VECTOR

1. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane. The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors

(*a*) can be zero

(*b*) cannot be zero

(*c*) lies in the plane of $\vec{A}$ & $\vec{B}$

(*d*) lies in the plane of $\vec{A}$ & $\vec{A}+\vec{B}$

2. The vector sum of the forces of 10 N and 6 N can be

(*a*) 2 N (*b*) 8 N

(*c*) 18 N (*d*) 20 N

3. A set of vectors taken in a given order gives a closed polygon. Then the resultant of these vectors is a

(*a*) scalar quantity

(*b*) pseudo vector

(*c*) unit vector

(*d*) null vector.

4. A car drives 6.0 km east, then 8 km north, and then 21 km west. The magnitude of the resulting displacement from origin is

(*a*) 35 km (*b*) 23 km

(*c*) 21 km (*d*) 17 km

Diffrence of two Vector

5. Two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle θ w.r.t. each other have a resultant $\vec{c}$ which makes an angle β with $\vec{a}$. If the directions of $\vec{a}$ and $\vec{b}$ are interchanged, then the resultant will have the same

(*a*) magnitude

(*b*) direction

(*c*) magnitude as well as direction

(*d*) neither magnitude nor direction.

6. If $\vec{P}+\vec{Q}=\vec{P}-\vec{Q}$ and θ is the angle between $\vec{P}$ and $\vec{Q}$, then

(*a*) $\theta=0^\circ$ (*b*) $\theta=90^\circ$

(*c*) $P=0$ (*d*) $Q=0$

7. A particle is moving eastward with a speed of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is

(*a*) zero.

(*b*) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-west.

(*c*) 1/2 m/s^2 towards north-west.

(*d*) 1/2 m/s^2 towards north.

8. The magnitudes of 2 forces $\vec{F}_1$ & $\vec{F}_2$ and 10 N and 8 N. The angle between them is 120°. The magnitude of their difference is:

(*a*) $\sqrt{244}$ N (*b*) $\sqrt{84}$ N

(*c*) $\sqrt{164}$ N (*d*) None of these

Vectors in Unit Vector Form

9. The vector joining the points A (1, 1, –1) and B (2, –3, 4) & pointing from A to B is -

(*a*) $-\hat{i}+4\hat{j}-5\hat{k}$ (*b*) $\hat{i}+4\hat{j}+5\hat{k}$

(*c*) $\hat{i}-4\hat{j}+5\hat{k}$ (*d*) $-\hat{i}-4\hat{j}-5\hat{k}$.

10. The x and y components of a force are 2N and –3N. The force is

(*a*) $2\hat{i}-3\hat{j}$ (*b*) $2\hat{i}+3\hat{j}$

(*c*) $-2\hat{i}-3\hat{j}$ (*d*) $3\hat{i}+2\hat{j}$

11. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. The values of x are

(*a*) $-\frac{2}{3}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{2}{3}$ (*d*) 2

Dot Product

12. Three non zero vectors $\vec{A}, \vec{B}$ & $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}=0$ & $\vec{A}.\vec{C}=0$. Then $\vec{A}$ can be parallel to:

(*a*) $\vec{B}$ (*b*) $\vec{C}$

(*c*) $\vec{B}.\vec{C}$ (*d*) $\vec{B}\times\vec{C}$

13. The value of scalar product of two vectors is 8 and that of vector product is $8\sqrt{3}$. The angle between them is:

(*a*) 30º (*b*) 60º

(*c*) 120º (*d*) 150º

14. A particle is moving under the influence of force $\vec{F} = [\hat{i} - 2\hat{j}]$N, is now moved from the point (x, y, z) = (2, 1, 3) m to the point (x, y, z) = (3, 2, 4) m. How much work is done by the force $\vec{F}$ during this time period? ($W = \vec{F}.\vec{s}$)

(*a*) 0 J (*b*) –1 J

(*c*) 1 J (*d*) None of these

15. The angle between $(\hat{i}+\hat{j}+\hat{k})$ & $(2\hat{i}+2\hat{j}-2\hat{k})$ is

(*a*) $\text{Cos}^{-1}\frac{1}{3}$ (*b*) $\text{Cos}^{-1}\frac{1}{\sqrt{3}}$

(*c*) $\text{Sin}^{-1}\frac{1}{3}$ (*d*) None of these

16. If $\vec{A}.\vec{B} = \vec{C}.\vec{B}$, which of the following can not be a possible case:

(*a*) $\vec{A} = \vec{C}$

(*b*) angle between $\vec{A}$ and $\vec{B}$ = 30° and angle between $\vec{B}$ and $\vec{C}$ = 150° all three vectors are of equal mag nitude.

(*c*) $\vec{B}$ is a null vector

(*d*) $\vec{A}$, $\vec{B}$ and $\vec{C}$ are along x, y and z-axis respectively

17. The vector $5\hat{i}+2\hat{j}-\ell\hat{k}$ is perpendicular to the vector $3\hat{i}+\hat{j}+2\hat{k}$ for ℓ =

(*a*) 1 (*b*) 4.7

(*c*) 6.3 (*d*) 8.5

Cross Product

18. What is the torque of the force $\vec{F} = \left(2\vec{i}-3\vec{j}+4\vec{k}\right)$N acting at the point $\vec{r} = \left(3\vec{i}+2\vec{j}+3\vec{k}\right)$m about the origin:

(*a*) $6\vec{i}-6\vec{j}+12\vec{k}$ (*b*) $17\vec{i}-6\vec{j}-13\vec{k}$

(*c*) $-6\vec{i}+6\vec{j}-12\vec{k}$ (*d*) $-17\vec{i}+6\vec{j}+13\vec{k}$

19. If $\vec{A}\times\vec{B} = \vec{C}$, then which of the following statements is wrong

(*a*) $\vec{C}\perp\vec{A}$ (*b*) $\vec{C}\perp\vec{B}$

(*c*) $\vec{C}\perp(\vec{A}+\vec{B})$ (*d*) $\vec{C}\perp(\vec{A}\times\vec{B})$

20. Two vectors $\vec{P}$ and $\vec{Q}$ are inclined at an angle θ to each other. Which of the following is the unit vector perpendicular to $\vec{P}$ and $\vec{Q}$?

(*a*) $\frac{\vec{P}\times\vec{Q}}{PQ}$ (*b*) $\frac{\hat{P}\times\hat{Q}}{\sin\theta}$

(*c*) $\frac{\hat{P}\times\hat{Q}}{PQ\sin\theta}$ (*d*) $\frac{\vec{P}\times\hat{Q}}{PQ\sin\theta}$

Function

21. f(x) = cos x + sin x then f(π/2) will be

(*a*) 2 (*b*) 1

(*c*) 3 (*d*) 0

Direction (No. 22 to 24): Derivative of given function w.r.t. corresponding independent variable is.

22. $y = x^2 + x + 8$

(*a*) $\frac{dy}{dx} = 2x - 1$ (*b*) $\frac{dy}{dx} = -x + 1$

(*c*) $\frac{dy}{dx} = 2x + 1$ (*d*) $\frac{dy}{dx} = x - 1$

23. $s = 5t^3 - 3t^5$

(*a*) $\frac{ds}{dt} = 15t^2 + 15t^4$ (*b*) $\frac{ds}{dt} = 15t^4 + 15t^2$

(*c*) $\frac{ds}{dt} = 15t^4 - 15t^2$ (*d*) $\frac{ds}{dt} = 15t^2 - 15t^4$

24. $y = 5\sin x$

(*a*) $\frac{dy}{dx} = 3\cos x$ (*b*) $\frac{dy}{dx} = 5\cos x$

(*c*) $\frac{dy}{dx} = 5\sin x$ (*d*) $\frac{dy}{dx} = 3\sin x$

Direction (No. 25 to 28): First derivative & second derivative of given functions w.r.t. corresponding independent variable is.

25. $y = 6x^2 - 10x - 5x^{-2}$

(*a*) $12x - 10 + 10x^{-3}, 12 - 30x^{-4}$

(*b*) $10x - 12 + 20x^{-3}, 15 - 30x^{-4}$

(*c*) $12x - 10 + 15x^{-3}, 12 - 30x^{-4}$

(*d*) $10x - 15 + 12x^{-3}, 12 - 30x^{-4}$

26. $r = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$

(*a*) $12\theta^{-2} - 12\theta^{-4} + 4\theta^{-5}, 24\theta^{-3} + 48\theta^{-5} + 20\theta^{-6}$

(*b*) $-12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5}, 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$

(*c*) $-6\theta^{-2} + 12\theta^{-4} - 8\theta^{-5}, 12\theta^{-3} - 24\theta^{-5} + 10\theta^{-6}$

(*d*) $-8\theta^{-2} + 12\theta^{-4} - 6\theta^{-5}, 24\theta^{-3} - 24\theta^{-5} + 10\theta^{-6}$

27. $\omega = 3z^7 - 7z^3 + 21z^2$

(*a*) $21z^6 + 21z^2 - 42z,\ 126z^5 + 42z - 42.$

(*b*) $14z^6 - 28z^2 + 22z,\ 120z^5 - 21z + 42.$

(*c*) $28z^6 - 14z^2 + 42z,\ 122z^5 - 42z + 21$

(*d*) $21z^6 - 21z^2 + 42z,\ 126z^5 - 42z + 42$

28. $y = \sin x + \cos x$

(*a*) cosx – cosx , – sinx – sinx

(*b*) sinx – sinx , – sinx – cosx

(*c*) cosx – sinx , – sinx – cosx

(*d*) sinx + cosx , – cosx – cosx

Direction (No. 29 to 31): Derivative of given functions w.r.t. the independent variable x is.

29. $x \sin x$

(*a*) $\sin x + x \cos x$ (*b*) $\sin x - x \cos x$

(*c*) $\cos^2 x - x \sin^2 x$ (*d*) $\sin^2 x - x \cos^2 x$

30. $y = e^x \ell nx$

(*a*) $e^x \ell nx - \frac{e^x}{x}$ (*b*) $e^x \ell nx - \frac{e^x}{x^2}$

(*c*) $e^x \ell nx + \frac{e^x}{x^2}$ (*d*) $e^x \ell nx + \frac{e^x}{x}$

31. $y = (x-1)(x^2 + x + 1)$

(*a*) $\frac{dy}{dx} = 3x$ (*b*) $\frac{dy}{dx} = 3x^2$

(*c*) $\frac{dy}{dx} = 2x^2$ (*d*) $\frac{dy}{dx} = 2x$

Direction (No. 32 to 34): Derivative of given function w.r.t. the independent variable is

32. $y = \frac{\sin x}{\cos x}$

(*a*) $\sec^2 x$ (*b*) $\sec x$

(*c*) $\sec^2 2x$ (*d*) $\sec^3 2x$

33. $y = \frac{2x+5}{3x-2}$

(*a*) $y' = \frac{-19}{(3x-2)^2}$ (*b*) $y' = \frac{19}{(3x-2)^2}$

(*c*) $y' = \frac{19}{(3x-2)}$ (*d*) $y' = \frac{-19}{(3x+2)^2}$

34. $z = \frac{2x+1}{x^2-1}$

(*a*) $\frac{-2x^2 - 2x + 2}{(x^2+1)^2}$ (*b*) $\frac{-2x^2 - 2x - 2}{(x^2-1)^2}$

(*c*) $\frac{-2x^2 + 2x + 2}{(x+1)^2}$ (*d*) $\frac{-2x^2 - 2x - 2}{(x^2-1)}$

Direction (No. 35 to 39): $\frac{dy}{dx}$ for following functions is.

35. $y = (2x+1)^5$

(*a*) $10(2x+1)^3$ (*b*) $10(2x+1)^4$

(*c*) $10(2x-1)^3$ (*d*) $10(2x-1)^4$

36. $y = (4-3x)^9$

(*a*) $-8(4-3x)^8$ (*b*) $-27(4-3x)^9$

(*c*) $-27(4+3x)^9$ (*d*) $-27(4-3x)^8$

37. $y = \left(1 - \frac{x}{7}\right)^{-7}$

(*a*) $\left(1 - \frac{x}{7}\right)^8$ (*b*) $\left(1 - \frac{x}{7}\right)^{-8}$

(*c*) $\left(1 - \frac{x}{7}\right)^{-5}$ (*d*) $\left(1 - \frac{x}{7}\right)^{-4}$

38. $y = \sin(x) + \ln(x^2) + e^{2x}$

(*a*) $\cos(x) + \frac{2}{x} - 2e^{2x}$ (*b*) $\cos(x) + \frac{2}{x} + 2e^{2x}$

(*c*) $\sin(x) + \frac{2}{x} + 2e^{2x}$ (*d*) $\sin(x) + \frac{2}{x} - 2e^{2x}$

39. $y = 2\sin(\omega x + \phi)$ where ω and ϕ constants

(*a*) $2\omega\cos(\omega x + \phi)$ (*b*) $2\omega\cos(\omega x - \phi)$

(*c*) $\omega\cos(\omega x + \phi)$ (*d*) $2\omega\text{cosec}(\omega x + \phi)$

DIFFERENTIATION AS A RATE MEASUREMENT

40. Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of t. equation that relates dA / dt to dr / dt is.

(*a*) $\frac{dA}{dt} = \pi r \frac{dr}{dt}$ (*b*) $\frac{dA}{dt} = \pi r^2 \frac{dr}{dt}$

(*c*) $\frac{dA}{dt} = 2\pi r^2 \frac{dr}{dt}$ (*d*) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

41. Maximum and minimum values of function $2x^3 - 15x^2 + 36x + 11$ is

(*a*) 39, 38 (*b*) 93, 83

(*c*) 45, 42. (*d*) 59, 58

42. $y = 2u^3,\ u = 8x - 1$ Find $\frac{dy}{dx}$

(*a*) $48(8x-1)^2$ (*b*) $48(8x+1)^2$

(*c*) $48(8x-1)$ (*d*) $48(8x+1)$

43. $y = \sin u,\ u = 3x + 1$ Find $\frac{dy}{dx}$

(*a*) $3\cos(3x-1)$ (*b*) $3\cos(3x+1)$

(*c*) $3\sin(3x-1)$ (*d*) $3\sin(3x+1)$

INTEGRATION

44. $\int (2x)\,dx$ Will be

(*a*) $x^2 + C$ (*b*) $2x + C$

(*c*) $2x^2 + C$ (*d*) $-x^2 + C$

45. $\int (x^2)dx$ Will be

(*a*) $x + C$ (*b*) $2x + C$

(*c*) $\frac{x^3}{3} + C$ (*d*) $\frac{x^2}{2} + C$

46. $\int (x^2 - 2x + 1)\,dx$ Will be

(*a*) $\frac{x^3}{3} - x^2 - x + C$ (*b*) $\frac{x^3}{3} - x^2 + x + C$

(*c*) $\frac{x^3}{3} + x^2 - x + C$ (*d*) $\frac{x^3}{3} + x^2 + x + C$

47. $\int (-3x^{-4})dx$ Will be

(*a*) $x^{-3} + C$ (*b*) $x^3 + C$

(*c*) $-3x^{-3} + C$ (*d*) $3x^{-3} + C$

48. $\int (x^{-4})dx$ Will be

(*a*) $\frac{1}{3}x^{-3} + C$ (*b*) $\frac{1}{3}x^{-5} + C$

(*c*) $\frac{1}{2}x^{-2} + C$ (*d*) $-\frac{1}{3}x^{-3} + C$

49. $\int \left(\frac{5}{x^2}\right) dx$ Will be

(*a*) $-\frac{5}{x} + C$ (*b*) $\frac{5}{x} + C$

(*c*) $\frac{x}{5} + C$ (*d*) $-\frac{x}{5} + C$

50. $\int \left(\frac{3}{2}\sqrt{x}\right) dx$ Will be

(*a*) $\sqrt{x^2} + C$ (*b*) $3\sqrt{x^4} + C$

(*c*) $3\sqrt{x^2} + C$ (*d*) $\sqrt{x^3} + C$

51. $\int \left(\frac{3}{2\sqrt{x}}\right) dx$ Will be

(*a*) $2\sqrt{x^3} + C$ (*b*) $3\sqrt{x} + C$

(*c*) $\sqrt{x^3} + C$ (*d*) $\sqrt{x^4} + C$

52. $\int \left(\frac{4}{3}\sqrt[3]{x}\right) dx$ Will be

(*a*) $x^{4/3} + C$ (*b*) $x^{3/4} + C$

(*c*) $x^{2/3} + C$ (*d*) $x^{1/3} + C$

53. $\int \left(\frac{1}{3\sqrt[3]{x}}\right) dx$ Will be

(*a*) $\frac{x^{\frac{3}{4}}}{2} + C$ (*b*) $\frac{x^{\frac{2}{3}}}{3} + C$

(*c*) $x^{\frac{2}{3}} + C$ (*d*) $\frac{x^{\frac{2}{3}}}{2} + C$

54. $\int \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\right) dx$ Will be

(*a*) $\frac{3x^{\frac{4}{2}}}{4} + \frac{3x^{\frac{2}{4}}}{2} + C$ (*b*) $\frac{3x^{\frac{4}{2}}}{4} + \frac{3x^{\frac{2}{1}}}{2} + C$

(*c*) $\frac{3x^{\frac{3}{4}}}{4} + \frac{3x^{\frac{3}{2}}}{2} + C$ (*d*) $\frac{3x^{\frac{4}{3}}}{4} + \frac{3x^{\frac{2}{3}}}{2} + C$

55. $\int \left(\frac{1}{2}x^{-1/2}\right) dx$ Will be

(*a*) $x^{2/3} + C$ (*b*) $x^{1/2} + C$

(*c*) $x^{3/1} + C$ (*d*) $x^{2/1} + C$

56. $\int \left(-\frac{1}{2}x^{-3/2}\right) dx$ Will be

(*a*) $x^{-1/2} + C$ (*b*) $x^{+1/2} + C$

(*c*) $x^{-1/-2} + C$ (*d*) $x^{-2/1} + C$

57. $\int (3\sin x)\,dx$ Will be

(*a*) $+3\cos x + C$ (*b*) $+4\cos x + C$

(*c*) $-3\cos x + C$ (*d*) $3\cos x + C$

58. $\int \left(\frac{1}{3x}\right)$ **dx** Will be

(*a*) $\frac{1}{3}\ell nx + C$ (*b*) $\frac{3}{1}\ell nx + C$

(*c*) $\frac{2}{3}\ell nx + C$ (*d*) $\frac{1}{2}\ell nx + C$

59. $\int \sin 3x\,dx$,Will be(use, u = 3x)

(*a*) $-\cos 3x + C$ (*b*) $\frac{1}{3}\cos 3x - C$

(*c*) $-\frac{1}{2}\cos 3x - C$ (*d*) $-\frac{1}{3}\cos 3x + C$

60. $\int_{-2}^{1} 5\,dx$ Will be

(*a*) 15 (*b*) 16

(*c*) 17 (*d*) 18

61. $\int_{-4}^{-1} \frac{\pi}{2} d\theta$ Will be

(*a*) $\frac{3\pi}{2}$ (*b*) $\frac{2\pi}{2}$

(*c*) $\frac{3\pi}{1}$ (*d*) $\frac{2\pi}{1}$

62. Use a definite integral to find the area of the region between the given curve and the x-axis on the interval [0, b] y = 2x

(*a*) b^2 (*b*) b^3

(*c*) $2b^2$ (*d*) $\frac{b^3}{3}$

Learning Plus

1. If the angle between two forces increases, the magnitude of their resultant

(*a*) decreases

(*b*) increases

(*c*) remains unchanged

(*d*) first decreases and then increases

2. Which of the following sets of displacements might be capable of bringing a car to its returning point?

(*a*) 5, 10, 30 and 50 unit

(*b*) 5, 9, 9 and 16 unit

(*c*) 40, 40, 90 and 200 unit

(*d*) 10, 20, 40 and 90 unit

3. When two vector $\vec{a}$ and $\vec{b}$ are added, the magnitude of the resultant vector is always

(*a*) greater than (a + b)

(*b*) less than or equal to (a + b)

(*c*) less than (a + b)

(*d*) equal to (a + b)

4. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is

(*a*) 0º (*b*) 60º

(*c*) 90º (*d*) 120º

5. Vector $\vec{A}$ is of length 2 cm and is 60º above the x-axis in the first quadrant. Vector $\vec{B}$ is of length 2 cm and 60º below the x-axis in the fourth quadrant. The sum $\vec{A} + \vec{B}$ is a vector of magnitude -

(*a*) 2 along + y-axis (*b*) 2 along + x-axis

(*c*) 1 along – x axis (*d*) 2 along – x axis

6. If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60º is $\sqrt{7}Q$, then P/Q is

(*a*) 1 (*b*) 3/2

(*c*) 2 (*d*) 4

7. A man moves towards 3m north then 4m towards east and finally 5m towards 37º south of west. His displacement from origin is

(*a*) $5\sqrt{2}$ m (*b*) 0m

(*c*) 1m (*d*) 12m

8. Three forces P, Q & R are acting at a point in the plane. The angle between P & Q and Q & R are 150º & 120º respectively, then for equilibrium, forces P, Q & R are in the ratio

(*a*) 1 : 2 : 3 (*b*) 1 : 2 : $\sqrt{3}$

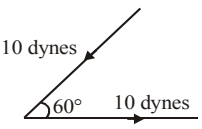
(*c*) 3 : 2 : 1 (*d*) $\sqrt{3}$: 2 : 1

9. The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between the two forces is

(*a*) 150º (*b*) 90º

(*c*) 60º (*d*) 120º

10. Two forces each numerically equal to 10 dynes are acting as shown in the following figure, then their resultant is -



(*a*) 10 dynes (*b*) 20 dynes

(*c*) $10\sqrt{3}$ dynes (*d*) 5 dynes

11. The vector $\vec{P}$ makes 120° with the x-axis and vector $\vec{Q}$ makes 30° with the y-axis. What is their resultant? (taking anticlockwise as positive)

(*a*) $P+Q$ (*b*) $P-Q$

(*c*) $\sqrt{P^2+Q^2}$ (*d*) $\sqrt{P^2-Q^2}$

12. A vector $\vec{A}$ is directed along 30^0 west of north direction and another vector $\vec{B}$ along 15^0 south of east. Their resultant cannot be in _____ direction.

(*a*) North (*b*) East

(*c*) North-East (*d*) South

13. The greatest and the least resultant of two forces acting at a point are 12N and 8N respectively. Find out the forces.

(*a*) 12 N & 8 N (*b*) 4N & 8 N

(*c*) 3 N & 9N (*d*) 10 N & 2N

14. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}-\vec{c}=\vec{0}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(*a*) 30° (*b*) 60°

(*c*) 90° (*d*) 120°

15. A car is moving on a straight road due north with a uniform speed of 50 km h^{-1} when it turns left through 90º. If the speed remains unchanged after turning, the change in the velocity of the car in the turning process is

(*a*) zero

(*b*) $50\sqrt{2}$ km h^{-1} S-W direction

(*c*) $50\sqrt{2}$ km h^{-1} N-W direction

(*d*) 50 km h^{-1} due west.

16. If the angle between the unit vectors $\hat{a}$ and $\hat{b}$ is 60º, then $|\hat{a}-\hat{b}|$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 4

17. A hall has the dimensions 10 m × 12 m × 14 m. A fly starting at one corner ends up at a diametrically opposite corner. The magnitude of its displacement is nearly

(*a*) 16m (*b*) 17m

(*c*) 18m (*d*) 21m.

18. Given : $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=5\hat{i}-6\hat{j}$. The magnitude of $\vec{A}+\vec{B}$ is

(*a*) 4 units (*b*) 10 units

(*c*) $\sqrt{58}$ units (*d*) $\sqrt{61}$ units.

19. Given : $\vec{A}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{B}=-\hat{i}-\hat{j}+\hat{k}$. The unit vector of $\vec{A}-\vec{B}$ is

(*a*) $\frac{3\hat{i}+\hat{k}}{\sqrt{10}}$ (*b*) $\frac{3\hat{i}}{\sqrt{10}}$

(*c*) $\frac{\hat{k}}{\sqrt{10}}$ (*d*) $\frac{-3\hat{i}-\hat{k}}{\sqrt{10}}$

20. The angle that the vector $\vec{A}=2\hat{i}+3\hat{j}$ makes with y-axis is-

(*a*) $\tan^{-1}(3/2)$ (*b*) $\tan^{-1}(2/3)$

(*c*) $\sin^{-1}(2/3)$ (*d*) $\cos^{-1}(3/2)$

21. If $3\hat{i}+2\hat{j}+8\hat{k}$ and $2\hat{i}+x\hat{j}+\hat{k}$ are at right angles then x=

(*a*) 7 (*b*) –7

(*c*) 5 (*d*) –4

22. If $(\vec{a}+\vec{b})$ is perpendicular to $\vec{b}$ and $\vec{a}+2\vec{b}$ is perpendicular to $\vec{a}$, $|\vec{a}|=a$, $|\vec{b}|=b$, then

(*a*) $a=b$ (*b*) $a=2b$

(*c*) $b=2a$ (*d*) $a=b\sqrt{2}$

23. If $\vec{a}_1$ and $\vec{a}_2$ are two non collinear unit vectors and if $|\vec{a}_1+\vec{a}_2|=\sqrt{3}$, then the value of $(\vec{a}_1-\vec{a}_2).(2\vec{a}_1+a_2)$ is

(*a*) 2 (*b*) $\frac{3}{2}$

(*c*) $\frac{1}{2}$ (*d*) 1

24. Two forces $\hat{i}+\hat{j}+\hat{k}$ N and $\hat{i}+2\hat{j}+3\hat{k}$ N act on a particle and displace it from (2, 3, 4) to point (5, 4, 3). Displacement is in m. Work done is:

(*a*) 5 J (*b*) 4 J

(*c*) 3 J (*d*) None of these

25. If a, b, c are three unit vectors such that a + b + c = 0, then a.b + b.c + c.a is equal to

(*a*) –1 (*b*) 3

(*c*) 0 (*d*) $-\frac{3}{2}$

26. A vector is not changed if

(*a*) it is displaced parallel to itself

(*b*) it is rotated through an arbitrary angle

(*c*) it is cross-multiplied by a unit vector

(*d*) it is multiplied by an arbitrary scalar.

27. A vector $\vec{A}$ points vertically downward & $\vec{B}$ points towards east, then the vector product $\vec{A}\times\vec{B}$ is

(*a*) along west (*b*) along east

(*c*) zero (*d*) along south

28. Find torque of a force $2\hat{i}-\hat{j}+\hat{k}$ acting at point (1, 3, 5) about origin.

(*a*) $-8i-9j+7k$ (*b*) $-8i+9j+7k$

(*c*) $8i+9j-7k$ (*d*) $8i-9j-7k$

29. The sum, difference and cross product of two vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular only if:

(*a*) $\vec{A}$ and $\vec{B}$ are perpendicular to each other and modulus of $\vec{A}$ is equal to modulus of $\vec{B}$.

(*b*) $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

(*c*) $\vec{A}$ and $\vec{B}$ are perpendicular but magnitudes are orbitrary.

(*d*) modulus of $\vec{A}$ = modulus of $\vec{B}$ and their directions are arbitrary.

Direction (No.30): The first derivative and second derivative of given functions w.r.t. the independent variable x will be

30. $y = \ell n x^2 + \sin x$

(*a*) $\frac{d^2y}{dx^2} = \frac{-2}{x^2} - \sin x$, $\frac{d^2y}{dx^2} = \frac{-2}{x^2} - \cos x$

(*b*) $\frac{dy}{dx} = \frac{2}{x} + \sin x$, $\frac{d^2y}{dx^2} = \frac{-2}{x^2} + \sin x$

(*c*) $\frac{dy}{dx} = \frac{2}{x} + \sin x$, $\frac{d^2y}{dx^2} = \frac{-2}{x^2} - \sin x$

(*d*) $\frac{dy}{dx} = \frac{2}{x} + \cos x$, $\frac{d^2y}{dx^2} = \frac{-2}{x^2} - \sin x$

Direction (No. 31 to 35) :

The derivative of given functions w.r.t. the corresponding independent variable will be

31. $y = e^x \tan x$

(*a*) $e^x(\tan x + \sec^2 x)$

(*b*) $e^x(\sec x + \tan^2 x)$

(*c*) $e^x(\tan x - \sec^2 x)$

(*d*) $e^x(\sec x - \tan^2 x)$

32. $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

(*a*) $\frac{dy}{dx} = 1 + 2x + \frac{2}{x^3} - \frac{1}{x^2}$

(*b*) $\frac{dy}{dx} = 1 + 2x + \frac{2}{x^3} + \frac{1}{x^2}$

(*c*) $\frac{dy}{dx} = 1 - 2x + \frac{2}{x^3} - \frac{1}{x^2}$

(*d*) $\frac{dy}{dx} = 1 + 2x - \frac{2}{x^3} + \frac{1}{x^2}$

33. $y = x^2 \sin x + 2x\cos x - 2\sin x$

(*a*) $x^2 \operatorname{cosec} x$ (*b*) $x^2 \sec x$

(*c*) $x^2 \cos x$ (*d*) $x^2 \sin x$

34. $y = x^2 \cos x - 2x \sin x - 2\cos x$

(*a*) $\frac{dy}{dx} = +x^2 \cos x$ (*b*) $\frac{dy}{dx} = -x^2 \cos x$

(*c*) $\frac{dy}{dx} = +x^2 \sin x$ (*d*) $\frac{dy}{dx} = -x^2 \sin x$

35. $r = (1 + \sec\theta)\sin\theta$

(*a*) $\frac{dr}{d\theta} = \cos\theta + \sec^2\theta$

(*b*) $\frac{dr}{d\theta} = \cos\theta - \sec^2\theta$

(*c*) $\frac{dr}{d\theta} = \sin\theta - \cos^2\theta$

(*d*) $\frac{dr}{d\theta} = \cos\theta - \sin^2\theta$

Direction (No. 36 to 37) :

The derivative of given functions w.r.t. the respective independent variable will be

36. $y = \frac{\sin x + \cos x}{\cos x}$

(*a*) $\frac{dy}{dx} = \cos^2 x$ (*b*) $\frac{dy}{dx} = \operatorname{cosec}^2 x$

(*c*) $\frac{dy}{dx} = \sec^2 x$ (*d*) $\frac{dy}{dx} = \sin^2 x$

37. $p = \frac{\tan q}{1 + \tan q}$

(*a*) $\frac{\sec^2 q}{(1 - \tan q)^2}$ (*b*) $\frac{\sec^2 q}{(1 + \tan q)^2}$

(*c*) $\frac{\sec^2 q}{(1 - \sec q)^2}$ (*d*) $\frac{\sec^2 q}{(1 - \sin q)^2}$

Direction (No. 38 to 40):

$\frac{dy}{dx}$ for following functions will be

38. $y = \sin^3 x + \sin^3 x$

(*a*) $3\sin^2 x \cos x + 3\cos^3 x$

(*b*) $4\sin^2 x \cos x + 4\cos^3 x$

(*c*) $2\sin^2 x \cos x + 2\cos^3 x$

(*d*) $3\sin^2 x \cos x + 4\cos^3 x$

39. $\sin^2(x^2+1)$

(*a*) $4x \sin(x^2-1)\cos(x^2+1)$

(*b*) $4x \sin(x^2+1)\cos(x^2-1)$

(*c*) $4x \sin(x^2-1)\cos(x^2-1)$

(*d*) $4x \sin(x^2+1)\cos(x^2+1)$

40. $q=\sqrt{2r-r^2}$, $\frac{dq}{dr}$ will be

(*a*) $\frac{1+r}{\sqrt{2r-r^2}}$ (*b*) $\frac{1-r}{\sqrt{2r-r^2}}$

(*c*) $\frac{1-r}{\sqrt{2r+r^2}}$ (*d*) $\frac{1+r}{\sqrt{2r+r^2}}$

41. two positive numbers x & y such that x + y = 60 and xy is maximum.

(*a*) $x=20$ & $y=10$ (*b*) $x=50$ & $y=20$

(*c*) $x=30$ & $y=30$ (*d*) $x=20$ & $y=50$

42. $\int(2x^3-5x+7)\,dx$ will be

(*a*) $\frac{x^4}{2}-\frac{5x^2}{2}+7x+C$

(*b*) $\frac{x^4}{2}-\frac{5x^2}{2}-7x+C$

(*c*) $\frac{x^4}{2}-\frac{5x^2}{2}-7x-C$

(*d*) $\frac{x^4}{2}-\frac{5x^2}{3}+7x+C$

43. $\int\left(\sqrt{x}+\sqrt[3]{x}\right)dx$ will be

(*a*) $\frac{4}{3}x^{3/2}+\frac{2}{3}x^{4/3}+C$ (*b*) $\frac{2}{3}x^{3/2}+\frac{3}{4}x^{4/3}+C$

(*c*) $\frac{3}{5}x^{3/2}+\frac{4}{2}x^{4/3}+C$ (*d*) $\frac{4}{2}x^{3/2}+\frac{2}{4}x^{4/3}+C$

44. $\int\frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt$ will be

(*a*) $2\sqrt{t}-\frac{4}{\sqrt{t}}-C$ (*b*) $2\sqrt{t}-\frac{3}{\sqrt{t}}-C$

(*c*) $2\sqrt{t}-\frac{2}{\sqrt{t}}+C$ (*d*) $2\sqrt{t}-\frac{1}{\sqrt{t}}-C$

45. $\int\frac{4+\sqrt{t}}{t^3}dt$ will be

(*a*) $-2t^{-2}-\frac{2}{3}t^{-3/2}+C$ (*b*) $-2t^{-2}+\frac{2}{3}t^{-3/2}-C$

(*c*) $2t^{-2}+\frac{2}{3}t^{-3/2}-C$ (*d*) $-2t^{-2}+\frac{2}{3}t^{-3/2}-C$

46. $\int\cos\theta(\tan\theta+\sec\theta)\,d\theta$ will be

(*a*) $-\text{cosec}\theta+\theta+C$ (*b*) $-\sec\theta+\theta+C$

(*c*) $-\sin\theta+\theta+C$ (*d*) $-\cos\theta+\theta+C$

47. $\int_{\pi}^{2\pi}\theta\,d\theta$ will be

(*a*) $\frac{3\pi^2}{2}$ (*b*) $\frac{2\pi^2}{3}$

(*c*) $\frac{4\pi^2}{3}$ (*d*) $\frac{3\pi^2}{4}$

48. $\int_0^{\pi}\cos x\,dx$ will be

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

49. $\int_0^1\frac{dx}{3x+2}$ will be

(*a*) $\frac{2}{4}\ell n\frac{2}{3}$ (*b*) $\frac{3}{1}\ell n\frac{5}{2}$

(*c*) $\frac{1}{3}\ell n\frac{2}{5}$ (*d*) $\frac{1}{3}\ell n\frac{5}{2}$

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. Which of the following is a true statement?

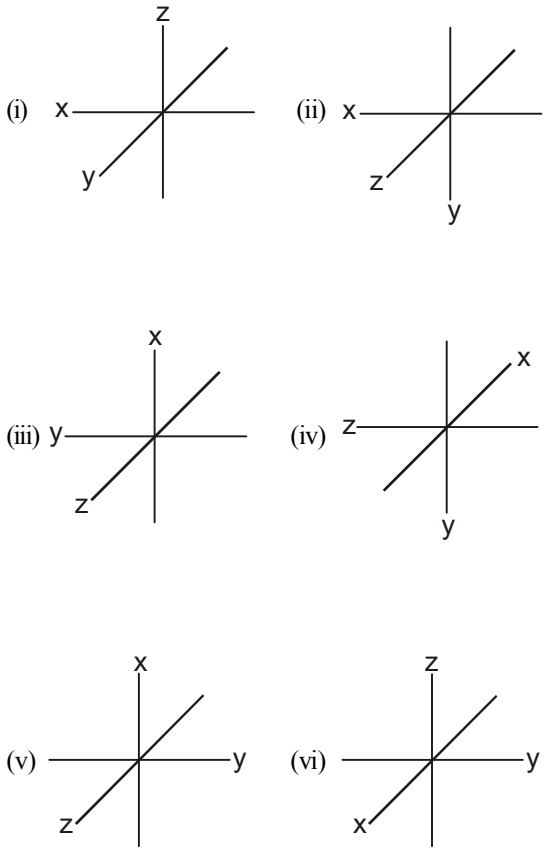
(*a*) A vector cannot be divided by another vector

(*b*) Angular displacement can either be a scalar or a vector

(*c*) Since addition of vectors is commutative therefore vector subtraction is also commutative

(*d*) The resultant of two equal forces of magnitude F acting at a point is F if the angle between the two forces is 120°.

2. Which of the arrangement of axes in fig. can be labelled "right handed coordinate system"? As usual, each axis lable indicates the positive side of the axis.



(*a*) (i), (ii) (*b*) (iii) (iv)

(*c*) (vi) (*d*) (v)

Comprehension Type Questions – 1 (No. 3 to 5)

A particle is moving along positive x-axis. Its position varies as $x = t^3 - 3t^2 + 12t + 20$, where x is in meters and t is in seconds.

3. Initial velocity of the particle is.

(*a*) 1 m/s (*b*) 3 m/s

(*c*) 12 m/s (*d*) 20 m/s

4. Initial acceleration of the particle is

(*a*) Zero (*b*) $1\ m/s^2$

(*c*) $-3 m/s^2$ (*d*) $-6\ m/s^2$

5. Velocity of the particle when its acceleration zero is

(*a*) 1 m/s (*b*) 3 m/s

(*c*) 6 m/s (*d*) 9 m/s

Comprehension Type Questions – 2 (No. 6 to 8)

Two forces $\vec{F}_1 = 2\hat{i} + 2\hat{j}$ N and $\vec{F}_2 = 3\hat{j} + 4\hat{k}$ N are acting on a particle.

6. The resultant force acting on particle is:

(*a*) $2\hat{i} + 5\hat{j} + 4\hat{k}$ (*b*) $2\hat{i} - 5\hat{j} - 4\hat{k}$

(*c*) $\hat{i} - 3\hat{j} - 2\hat{k}$ (*d*) $\hat{i} - \hat{j} - \hat{k}$

7. The angle between $\vec{F}_1$ & $\vec{F}_2$ is:

(*a*) $\theta = \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$

(*b*) $\theta = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$

(*c*) $\theta = \cos^{-1}\left(\frac{2}{3\sqrt{5}}\right)$

(*d*) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{5}\right)$

8. The component of force $\vec{F}_1$ along force $\vec{F}_2$ is:

(*a*) $\frac{5}{6}$ (*b*) $\frac{5}{3}$

(*c*) $\frac{6}{5}$ (*d*) $\frac{5}{2}$

9. Match the integrals (given in column - II) with the given functions (in column - I)

Column - I	Column - II
(*a*) $\int \sec x \tan x\, dx$	(p) $-\frac{\cos ec\, Kx}{K} + C$
(*b*) $\int \cos ec\, Kx \cot Kx\, dx$	(q) $-\frac{\cot\, Kx}{K} + C$
(*c*) $\int \cos ec^2\, Kx\, dx$	(r) $\sec x + C$
(*d*) $\int \cos\, Kx\, dx$	(s) $\frac{\sin\, Kx}{K} + C$

10. Match the statements given in column-I with statements given in column - II

Column - I	Column - II
(*a*) if $\|\vec{A}\| = \|\vec{B}\|$ and $\|\vec{A}+\vec{B}\| = \|\vec{A}\|$ then angle between $\vec{A}$ and $\vec{B}$ is	(p) 90°
(*b*) Magnitude of resultant of two forces $\|\vec{F}_1\| = 8N$ and $\|\vec{F}_2\| = 4\,N$ may be	(q) 120°
(*c*) Angle between $\vec{A} = 2\,\hat{i} + 2\,\hat{j}$ & $\vec{B} = 3\,\hat{k}$ is	(r) 12 N
(*d*) Magnitude of resultant of vectors $\vec{A} = 2\,\hat{i} + \hat{j}$ & $\vec{B} = 3\,\hat{k}$ is	(s) $\sqrt{14}$

NUMERICAL BASED QUESTIONS

11. A boy A is standing 3 m west and 4 m north to a boy B. A starts moving along a vector $\vec{a} = 1.5\hat{i} + 2\hat{j}$ with a constant speed of 2 m/s for 5 s and stops. Its new position vector with respect to the boy B is $x\hat{i} + y\hat{j}$. Find xy.

12. If the vector's $\vec{P} = a\hat{i} + aj + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - k$ are perpendicular to each other then find the positive value of 'a'.

13. Magnitude of resultant of two vector $\vec{A}$ and $\vec{B}$ is equal to 2. Angle between two vectors is 180°. If $|\vec{A}| = 3$ then find $|\vec{B}|$ (must be less then 2)

14. Two particles are moving with velocity $\vec{v}_1 = \hat{i} - 2t\,\hat{j}\,m/s$ and $\vec{v}_2 = 4\hat{i} + \hat{j}\ m/s$ respectively Time at which they are moving perpendicular to each other is.

15. Magnitude of subtraction of two vector $\vec{A}$ and $\vec{B}$ is equal to 5. Angle between both is 180°. Find the magnitude of resultant of these two vectors.

If $|\vec{A}| = 2$.

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal is **[*JEE Main-2019 (January)*]**

(*a*) 1.6×10^3 km (*b*) 3.2×10^3 km

(*c*) 6.4×10^3 km (*d*) 1.28×10^4 km

2. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A}+\vec{B})$ is 'n' times the magnitude of $(\vec{A}-\vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is:

[*JEE Main-2019 (January)*]

(*a*) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

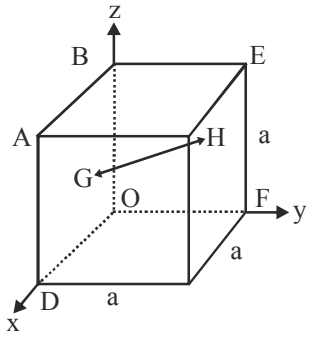
(*b*) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$

(*c*) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

(*d*) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

3. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be:

[*JEE Main-2019* (*January*)]



(*a*) $\frac{1}{2}a\left(\hat{k}-\hat{i}\right)$ (*b*) $\frac{1}{2}a\left(\hat{i}-\hat{k}\right)$

(*c*) $\frac{1}{2}a\left(\hat{j}-\hat{i}\right)$ (*d*) $\frac{1}{2}a\left(\hat{j}-\hat{k}\right)$

4. Two forces P and Q, of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. then, the angle θ is:

[*JEE Main-2019* (*January*)]

(*a*) 120° (*b*) 60°

(*c*) 90° (*d*) 30°

5. Let $\left|\vec{A}_1\right|=3,\left|\vec{A}_2\right|=5$ and $\left|\vec{A}_1+\vec{A}_2\right|=5$. The value of $\left(2\vec{A}_1+3\vec{A}_2\right).\left(3\vec{A}_1-2\vec{A}_2\right)$ is :-

[*JEE Main-2019* (*April*)]

(*a*) -112.5 (*b*) -106.5

(*c*) -118.5 (*d*) -99.5

6. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}|=|\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is,_____.

[*JEE Main-2020* (*January*)]

7. Statement I:Two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ where $\vec{P}\perp\vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3\left(P^2+Q^2\right)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2\left(P^2+Q^2\right)}$. This is possible only when $\theta_1<\theta_2$.

Statement II :In the situation given above.

$\theta_1=60^\circ$ and $\theta_2=90^\circ$ **[*JEE Main-2021* (*August*)]**

In the light of the above statements, choose the most appropriate answer from the options given below:-

(*a*) Statement I is false but Statement II is true

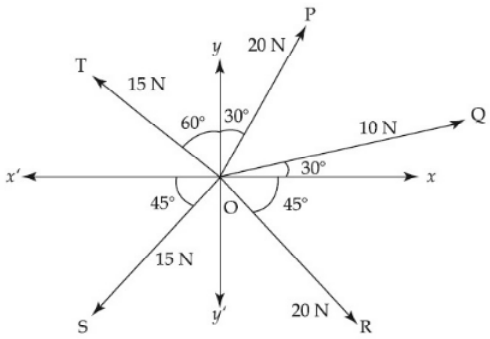
(*b*) Both Statement I and Statement II are true

(*c*) Statement I is true but Statement II is false

(*d*) Both Statement I and Statement II are false.

8. The resultant of these forces $\overrightarrow{OP},\overrightarrow{OQ},\overrightarrow{OR},\overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately N.

[Take $\sqrt{3}=1.7,\sqrt{2}=1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis] **[*JEE Main-2021* (*August*)]**

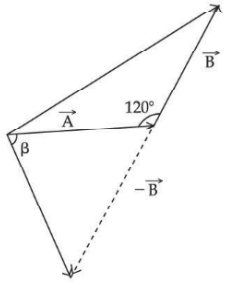


(*a*) $9.25\hat{i}+5\hat{j}$ (*b*) $3\hat{i}+15\hat{j}$

(*c*) $2.5\hat{i}-14.5\hat{j}$ (*d*) $-1.5\hat{i}-15.5\hat{j}$

9. The angle between vector $(\vec{A})$ and $(\vec{A}-\vec{B})$ is :

[*JEE Main-2021* (*August*)]



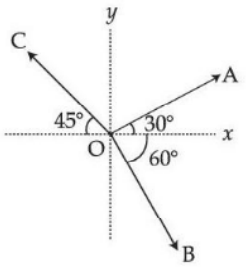
(*a*) $\tan^{-1}\left(\frac{-\frac{B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$

(*b*) $\tan^{-1}\left(\frac{A}{0.7B}\right)$

(*c*) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$

(*d*) $\tan^{-1}\left(\frac{B\cos\theta}{A-B\sin\theta}\right)$

10. The magnitude of vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$ with x-axis will be : **[*JEE Main-2021 (August)*]**



(*a*) $\tan^{-1}\dfrac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

(*b*) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$

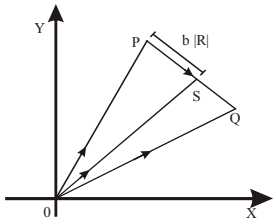
(*c*) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1-\sqrt{3}+\sqrt{2})}$

(*d*) $\tan^{-1}\dfrac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$

JEE-ADVANCED PREVIOUS YEAR'S

1. Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$ .The distance bvetween the point P and S is b$|\vec{R}|$. The general relation among vectors $\vec{P}$, $\vec{Q}$ and $\vec{S}$ is:

[*JEE Advanced-2017*]



(*a*) $\vec{S}=(b-1)\vec{P}+b\vec{Q}$ (*b*) $\vec{S}=(1-b^2)\vec{P}+b\vec{Q}$

(*c*) $\vec{S}=(1-b)\vec{P}+b^2\vec{Q}$ (*d*) $\vec{S}=(1-b)\vec{P}+b\vec{Q}$

2. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A}=a\hat{i}$ and $\vec{B}=a(\cos\omega t\,\hat{i}+\sin\omega t\hat{j})$, where a is a constant and $\omega=\pi/6 \text{rad s}^{-1}$. If $|\vec{A}+\vec{B}|=\sqrt{3}\,|\vec{A}-\vec{B}|$ at time $t=\tau$ for the first time, the value of τ, in seconds, is____

[*JEE Advanced-2018*]

ANSWER KEY

Topicwise Questions

1. (*b*) **2.** (*b*) **3.** (*d*) **4.** (*d*) **5.** (*a*) **6.** (*d*) **7.** (*b*) **8.** (*a*) **9.** (*c*) **10.** (*a*)
11. (*a*) **12.** (*d*) **13.** (*b*) **14.** (*b*) **15.** (*a*) **16.** (*b, d*) **17.** (*d*) **18.** (*b*) **19.** (*d*) **20.** (*b*)
21. (*a*) **22.** (*c*) **23.** (*d*) **24.** (*b*) **25.** (*a*) **26.** (*b*) **27.** (*d*) **28.** (*c*) **29.** (*a*) **30.** (*d*)
31. (*b*) **32.** (*a*) **33.** (*a*) **34.** (*b*) **35.** (*b*) **36.** (*d*) **37.** (*b*) **38.** (*b*) **39.** (*a*) **40.** (*d*)
41. (*a*) **42.** (*a*) **43.** (*b*) **44.** (*a*) **45.** (*c*) **46.** (*b*) **47.** (*a*) **48.** (*d*) **49.** (*a*) **50.** (*d*)
51. (*b*) **52.** (*a*) **53.** (*d*) **54.** (*d*) **55.** (*b*) **56.** (*a*) **57.** (*c*) **58.** (*a*) **59.** (*d*) **60.** (*a*)
61. (*a*) **62.** (*a*)

Learning Plus

1. (*a*) **2.** (*b*) **3.** (*b*) **4.** (*d*) **5.** (*b*) **6.** (*c*) **7.** (*b*) **8.** (*d*) **9.** (*d*) **10.** (*a*)
11. (*a*) **12.** (*d*) **13.** (*d*) **14.** (*d*) **15.** (*b*) **16.** (*b*) **17.** (*d*) **18.** (*c*) **19.** (*a*) **20.** (*b*)
21. (*b*) **22.** (*d*) **23.** (*c*) **24.** (*a*) **25.** (*d*) **26.** (*a*) **27.** (*d*) **28.** (*c*) **29.** (*d*) **30.** (*d*)
31. (*a*) **32.** (*a*) **33.** (*c*) **34.** (*d*) **35.** (*a*) **36.** (*c*) **37.** (*b*) **38.** (*a*) **39.** (*d*) **40.** (*b*)
41. (*c*) **42.** (*a*) **43.** (*b*) **44.** (*c*) **45.** (*a*) **46.** (*d*) **47.** (*a*) **48.** (*a*) **49.** (*d*)

Advanced Level Multiconcept Questions

1. (*a, b, d*) **2.** (*a, b, c*) **3.** (*c*) **4.** (*d*) **5.** (*d*) **6.** (*a*) **7.** (*b*) **8.** (*c*)
9. (*a*) $\rightarrow$ r, (*b*) $\rightarrow$ p, (*c*) $\rightarrow$ q, (*d*) $\rightarrow$ s **10.** (*a*) $\rightarrow$ q, (*b*) $\rightarrow$ r, (*c*) $\rightarrow$ p, (*d*) $\rightarrow$ s **11.** [0036] **12.** [0003] **13.** [1]
14. [2] **15.** [1]

JEE Mains & Advanced Past Years Questions

JEE-MAIN

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*a*) **5.** (*c*) **6.** [90°] **7.** (*b*) **8.** (*a*) **9.** (*c*) **10.** (*a*)

JEE-ADVANCED

1. (*d*) **2.** [2.00]

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

3 Motion in a Straight Line

INTRODUCTION

Kinematics is that branch of mechanics, which describes the motion of bodies without reference to the forces that either cause the motion or are generated as a result of the motion. Kinematics is often referred to as the 'geometry of motion'. We start our study of kinematics by first discussing in this chapter the motion of a particle. A **particle** is a physical analogous of a point. A particle is a body whose physical dimensions are so small compared with the radius of curvature of its path that we can treat the motion of a particle as that of a point.

MOTION

Change in position with respect to time is called motion.

(i) Absolute motion : Motion with respect to a body which is at complete rest is called **absolute motion** which is impossible.

(ii) Relative Motion : Motion with respect to a body which is at relative rest is called **relative motion**.

Object position may change with time due to change in only one or two or all the three co-ordinates. Accordingly, there are three types of motion.

DISTANCE AND DISPLACEMENT

Distance : The length of the actual path between initial and final positions of a particle in a given interval of time is called distance covered by the particle. Distance is the actual length of the path.

Characteristics of Distance:

- It is a scalar quantity
- It depends on the path
- It never reduces with time.
- Distance covered by a particle is always positive or zero and can never be negative.
- Dimension : $[M^0 L^1 T^0]$
- Unit : In C. G. S. centimeter (cm), In S.I. system meter (m).

Displacement: The shortest distance from the initial position to the final position of the particle is called displacement. The displacement of a particle is measured as the change in the position of the particle in a particular direction over a given time interval. It depends only on final and initial positions.

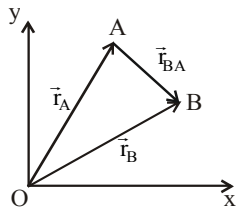
Displacement of a particle is a position vector of its final position w.r.t, initial position.

Position vector of A w.r.t. O = $\vec{OA}$

$\Rightarrow \quad \vec{r}_A = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$

Position Vector of B w.r.t. O = $\vec{OB}$

$\Rightarrow \quad \vec{r}_B = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$



Displacement $= \vec{AB} = \vec{r}_{BA}$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Characteristics of Displacement:

- It is a vector quantity.
- The displacement of a particle between any two points is equal to the shortest distance between them.
- The displacement of an object in a given time interval may be +ive, –ive or zero.
- The actual distance travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, it is less than the magnitude of the displacement, i.e. Distance ≥ | Displacement |
- Dimension : $[M^0 L^1 T^0]$
- Unit : In C. G. S. centimeter (cm), In S.I. system meter (m).

AVERAGE SPEED AND INSTANTANEOUS SPEED

Average speed is the ratio of total distance covered by a particle in a given time interval divided by the time interval.

$$v_{av.} = \frac{\Delta s}{\Delta t} \qquad (\text{where } \Delta t = t_2 - t_1)$$

Instantaneous speed at any instant t is

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- The slope of the distance-time graph provides the value of instantaneous speed.
- The average speed is defined for a time interval while the instantaneous speed is defined at an instant. The word speed normally implies instantaneous speed.

- Average speed and instantaneous speed both are scalar quantities.
- For any moving object, the average speed can never be zero or negative, i.e., $V_{av} > 0$.
- If a particle travels distances s_1, s_2, s_3 , ..., etc., at different speeds v_1, v_2, v_3, ... etc., respectively

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{\Sigma s_i}{\Sigma (s_i / v_i)} \text{ If } s_1 = s_2 = ... = s_n = s, \quad \text{then}$$

$$\frac{1}{v_{av.}} = \frac{1}{n}\left[\frac{1}{v_1} + \frac{1}{v_2} + ...\right] = \frac{1}{n}\Sigma\frac{1}{v_i}$$

Special case : If a particle moves a distance at speed v_1 and comes back to initial position with speed v_2, then

$$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

- If a particle travels at speeds v_1, v_2, ..., etc., for time intervals t_1, t_2, ...

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{v_1t_1 + v_2t_2 + ... + v_nt_n}{t_1 + t_2 + ...t_n} = \frac{\Sigma v_i t_i}{\Sigma t_i}$$

- If a particle moves for two equal intervals of time at different speeds, then $v_{av} = \frac{v_1 + v_2}{2}$

AVERAGE VELOCITY

The average velocity is the ratio of displacement from time t_1 to t_2 and the time interval $t_2 - t_1$.

$$\vec{v}_{av.} = \frac{\vec{r}_2 - \vec{r}_1}{t_1 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$$

INSTANTANEOUS VELOCITY

The instantaneous velocity at any instant t is

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

The magnitude of instantaneous velocity is always equal to the instantaneous speed.

Instantaneous speed

$$= |\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Instantaneous velocity is called simply velocity.

If an object moves along a straight line without changing its direction, then the magnitude of the average velocity is equal to the average speed, otherwise $|\vec{v}_{av.}| < v_{av.}$

- Velocity can be +ve or –ve as it is a vector but speed can never be negative as it is the magnitude of velocity.
- If a body is moving with a constant velocity, then the average velocity and instantaneous velocity are equal.
- The velocity of a body is uniform, if both magnitude and direction do not change.
- If a body moves with non-uniform velocity, then only magnitude of velocity may change or direction of velocity may change or both.
- A body can have non-zero speed and zero average velocity when a body completes one revolution around a circle, the average velocity is zero since the displacement is zero. But the average speed is not zero since the distance travelled $\neq 0$.
- If a body is moving with constant speed then its velocity may be constant or may not be constant. In case of uniform circular motion though speed remains constant but velocity changes instant to instant because of change in direction.

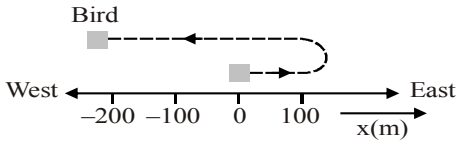
TRAIN YOUR BRAIN

Example 1: A bird flies towards east at 10 m/s for 100 m. It then turns around and flies at 20 m/s for 15 s. Find

(*a*) Its average speed

(*b*) Its average velocity

Sol. Let us take the x axis to point eastwards A sketch of



the path is shown in the figure.To find the required quantities, we need the total time interval. The first of the journey took

$$\Delta t_1 = (100 \text{ m}) / (10 \text{ m/s}) = 10\text{s},$$

and we are given t_2 = 15 s for the second part. Hence the total time interval is

$$\Delta t = \Delta t_1 + \Delta t_2 = 25 \text{ s}$$

The bird flies 100 m east and then (20 m/s)× (15s)
= 300 m west

(a) Average speed = $\frac{\text{Distance}}{\Delta t}$

$$= \frac{100\text{m} + 300\text{m}}{25\text{s}} = 16\,\text{m/s}$$

(b) The net displacement is

$\Delta x = -200$ m

So that

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{-200\text{m}}{25\text{s}} = -8 \text{ m/s}$$

The negative sign means that v_{av} is directed toward the west.

Example 2: A particle moves with speed v_1 along a particular direction. After some time it turns back and reaches the starting point again travelling with speed v_2. Find (for the whole journey)

(*a*) Average velocity (*b*) Average speed

Sol. (a) Since the particle reaches the starting point again, its displacement is zero

$$\therefore \text{ Average velocity} = \frac{\text{Net displacement}}{\text{total time}} = 0$$

(b) Let it travelled distance x while moving away as well as while moving towards the starter point.

Time taken to go away is $t_1 = \frac{x}{v_1}$

Time taken while return journey $t_2 = \frac{x}{v_2}$

$$\therefore \text{ Average speed} = \frac{x + x}{t_1 + t_2}$$

$$= \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$$

$$= \frac{2v_1v_2}{v_1 + v_2} \text{ i.e. harmonic mean of individual speeds}$$

ACCELERATION

The rate of change of velocity of an object with time is called acceleration of the object.

Let v and v′ be the velocity of the object at time t and t′ respectively, then acceleration of the body is given by

$$\vec{a} = \frac{d\vec{v}}{dt}$$

- Acceleration is a vector quantity.
- It is positive if the velocity is increasing and is negative if the velocity is decreasing.
- The negative acceleration is also called retardation or deceleration.
- **Unit:** In S.I. system m/s^2
  In C.G.S. system cm/s^2
- **Dimension :** $[M^0L^1T^{-2}]$

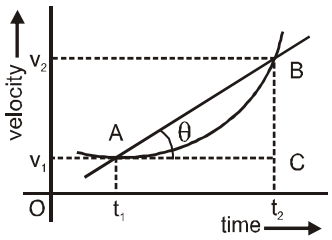
TYPES OF ACCELERATION

- **Average Acceleration**

  When an object is moving with a variable acceleration, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken i.e.,

$$\text{Average Acceleration} = \frac{\text{total change in velocity}}{\text{total time taken}}$$

Suppose the velocity of a particle is $\vec{v}_1$ at time t_1 and $\vec{v}_2$ at time t_2.



Then, Change in velocity = $\vec{v}_2 - \vec{v}_1 = \Delta\vec{v}$

Elapsed time in changing the velocity = $t_2 - t_1 = \Delta t$

Thus, $\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$

$\Rightarrow \vec{a}_{av} = \frac{BC}{AC} = \tan\theta$ = the slope of chord of v – t graph is average acceleration.

Note: If any body is accelerated with acceleration $\vec{a}_1$ till time t_1 and acceleration $\vec{a}_2$ up to time t_2 then average acceleration will

$$\vec{a}_{av} = \frac{\vec{a}_1t_1 + \vec{a}_2t_2}{t_1 + t_2}$$

- **Instantaneous Acceleration**

  The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration.

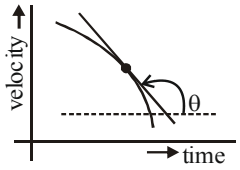
  Suppose the velocity of a particle at time $t_1 = t$ is $\vec{v}_1 = \vec{v}$ and becomes $\vec{v} = \vec{v} + \Delta\vec{v}$ at time $t_2 = t + \Delta t$,

  Then, $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$

  If Δt approaches to zero, then the rate of change of velocity will be instantaneous acceleration.

  Instantaneous acceleration $\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

  Instantaneous acceleration at a point is equal to slope of tangent at that point on velocity time graph in the graph shown.



velcity–time graph

As $\vec{v} = \frac{d\vec{x}}{dt}$ therefore $\vec{a} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}$

Thus, instantaneous acceleration of an object is equal to the second derivative of the position w.r.t. time of the object at the given instant.

Example 3: The displacement of a particle, moving in a straight line, is given by $s = 2t^2 + 2t + 4$ where s is in metres and t in seconds. The acceleration of the particle is

(*a*) 2 m/s^2 (*b*) 4 m/s^2

(*c*) 6 m/s^2 (*d*) 8 m/s^2

Sol. (b) Given $s = 2t^2 + 2t + 4$

$$\therefore \text{ velocity } (v) = \frac{ds}{dt} = 4t + 2$$

$$\text{acceleration} = \frac{dv}{dt} = 4(1) + 0 = 4 m/s^2$$

Note:

(i) It is not essential that when velocity is zero acceleration must be zero.
e.g. In vertical motion under gravity at the top point $v = 0$ but $a \neq 0$

(ii) Velocity may vary but $\frac{d\vec{v}}{dt}$ may be constant.

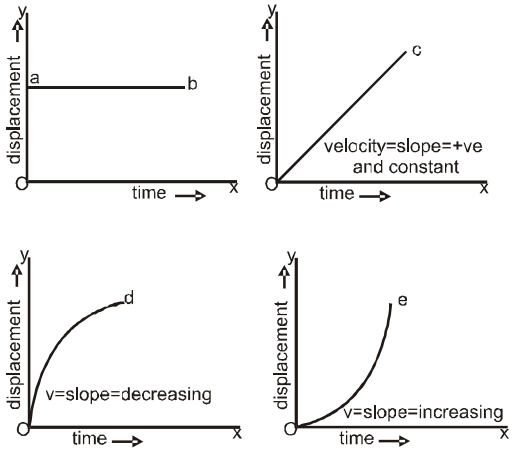
(iii) The acceleration may vary but speed may be constant e.g. In uniform circular motion.

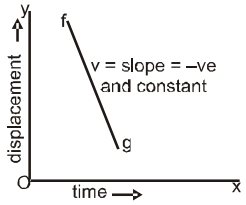
(iv) If velocity decreases w.r.t. time then acceleration is called retardation.

DISPLACEMENT-TIME GRAPHS AND THEIR CHARACTERISTICS

If the graph is :

- A straight line parallel to time-axis, shown by line ab, it means that the body is at rest, i.e., v = 0.
- A straight line inclined to x-axis (such as Oc and fg) shows that body is moving with a constant velocity.
- A straight line inclined to x-axis by an angle > 90° (line fg) represent negative velocity.
- the curve is of the type Od whose slope decreases with time, the velocity goes on decreasing, i.e., motion is retarded.
- The curve is of the type **Oe** whose slope increases with time, the velocity goes on increasing, i.e., motion is accelerated.



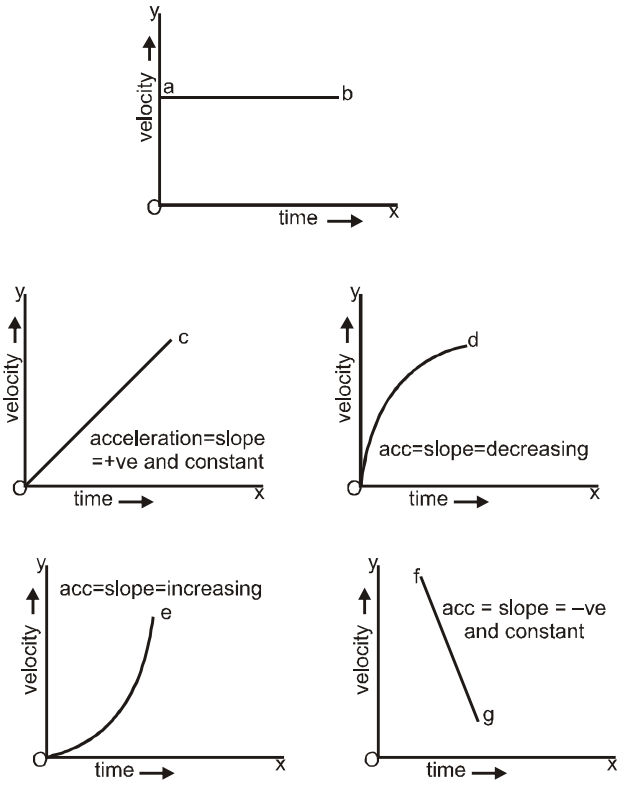


No line can ever be perpendicular to the time axis because it implies infinite velocity.

VELOCITY-TIME GRAPHS AND THEIR CHARACTERISTICS

If the graph is :

- A straight line parallel to time axis shown by line ab, it means that the body is moving with a constant velocity or acceleration (a) is zero.
- A straight line inclined to the x-axis with +ve slope (line **Oc**) it means that the body is moving with constant acceleration.
- A straight line inclined to x-axis with negative slope it means that the body is under retardation.
- A curve like **Od** whose slope decreases with time, the acceleration goes on decreasing.
- A curve like **Oe** whose slope increases with time, the acceleration goes on increasing.



No velocity-time graph can ever be perpendicular to the time-axis because it implies infinite acceleration.

The area of velocity-time graph with time axis represents the displacement of that body. For constant acceleration,

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2} at^2$ (iii) $v^2 = u^2 + 2as$

(iv) The distance covered in n^{th} second

$S_n = u + \frac{a}{2}(2n - 1)$

u = initial velocity
v = final velocity
a = acceleration
t = time interval
s = distance travelled
s_n = distance travelled in n^{th} second

Note: (i) These equations can be applied only and only when acceleration is constant. In case of circular motion or simple harmonic motion as acceleration is not constant (due to change in direction or magnitude) so these equation can not be applied.

(ii) $\vec{v} = \vec{u} + \vec{a}t$ and $\vec{s} = \vec{u}t + (1/2)\,\vec{a}\,t^2$ are vector equations, while

$\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s} \Rightarrow v^2 = u^2 + 2\vec{a}.\vec{s}$

is a scalar equation

(iii) If the velocity and acceleration are collinear, we conventionally take the direction of motion to be positive, so equation of motions becomes
$v = u + at$, $s = ut + (1/2)\,at^2$, $v^2 = u^2 + 2as$
If the velocity and acceleration are anti-parallel then,
$v = u - at$
$s = ut - (1/2)\,at^2$
$v^2 = u^2 - 2as$

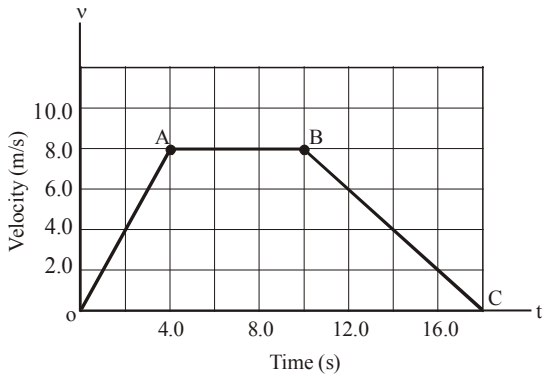
- If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then average velocity $= \frac{v+u}{2}$.
- If a body moving with uniform acceleration has velocities u and v at two points in its path, then the velocity at the midpoint of its path $= \sqrt{\frac{u^2+v^2}{2}}$.
- **For a body starting from rest and moving with uniform acceleration,**

(a) The ratio of distances covered in first one sec, two sec, three see, ... is :
$1^2 : 2^2 : 3^2 : ...$, i.e., 1 : 4 : 9 : ...
Ratio of distances covered in
1st, 2nd, 3rd sec, ... is 1 : 3 : 5 : ...

(b) The ratio of velocities after
1 sec, 2 see, 3 sec, ... is 1 : 2 : 3 : ...

TRAIN YOUR BRAIN

Example 4: What is the average acceleration for each graph segment in figure? Describe the motion of the object over the total time interval. Also calculate displacement.



Sol. Segment OA; $a = \frac{8-0}{4-0} = 2\ m/s^2$

Segment AB; graph horizontal i.e., slope zero i.e., a = 0

Segment BC; $a = \frac{0-8}{18-10} = -1 m/s^2$

The graph is trapezium. Its area between t = 0 to t = 18s is displacement

Area = displacement = $\frac{1}{2}(18 + 6) \times 8 = 96 m$

Particle accelerates uniformly for first 4 sec., then moves with uniform velocity for 6 sec. and then retards uniformly to come to rest in next 8 sec.

VERTICAL MOTION UNDER GRAVITY (FREE FALL)

Motion that occurs solely under the influence of gravity is called free fall. Thus a body projected upward or downward or released from rest are all under free fall.

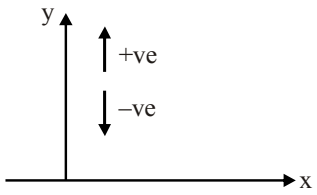
In the absence of air resistance all falling bodies have the same acceleration due to gravity, regardless of their sizes or shapes.

The value of the acceleration due to gravity depends on both latitude and altitude. It is approximately 9.8 m/s^2 near the surface of the earth. For simplicity a value of 10 m/s^2 is being used in this package. To do calculations regarding motion under gravity, we follow a proper sign convention. In this package we are taking upward direction as positive and downward as negative, Thus acceleration is taken a = – g = –10 m/s^2 no matter whether body is moving upwards or downwards, since **g** always acts downward.

Thus the equation of kinematics may be modified as

$v = u - gt$ ..... (i)

$\Delta y = y - y_0 = ut - \frac{1}{2} gt^2$ ..... (ii)

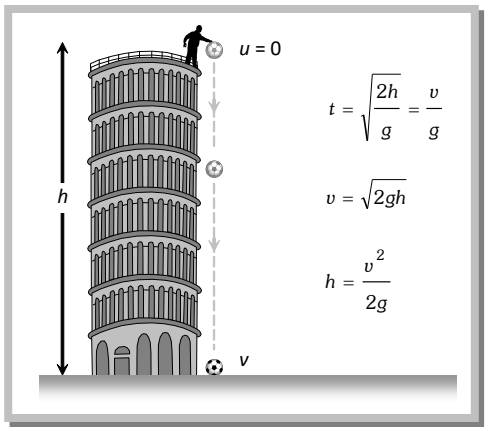


$v^2 = u^2 - 2g\,(y - y_0)$ ..... (iii)

These y_0 = position of particle at time t = 0
y = position of particle at time t.
u = velocity of particle at time t = 0
v = velocity of particle at time t.

(i) A body dropped from some height (initial velocity zero)

- Equation of motion : Taking initial position as origin and downward direction as negetive. Here we have,
  u = 0 [As body starts from rest]
  a = – g as acceleration is in the downward direction



$v = -gt$ …(i)

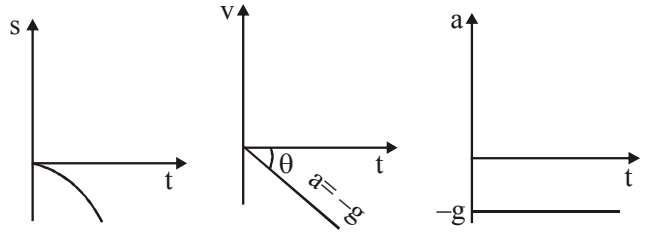
$\Delta y = -h = -\frac{1}{2}gt^2$ …(ii)

$v^2 = 2gh = 2(-g)(-h)$ …(iii)

$h_n = \frac{g}{2}(2n-1)$ ...(iv)

height covered in nth second.

- Graph of displacement velocity and acceleration with respect to time :



- As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

- The distance covered in the nth sec, $h_n = \frac{1}{2}g(2n-1)$

  So distance covered in I, II, III sec, etc., will be in the ratio of 1 : 3 : 5. This is called 'Galileo's Law of odd numbers'.

(ii) A body projected vertically downward with some initial velocity : The initial velocity is downward and will be negative

Equation of motion : $v = -u - gt$

$\Delta y = -h = -ut - \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$

$h_n = u + \frac{g}{2}(2n-1)$

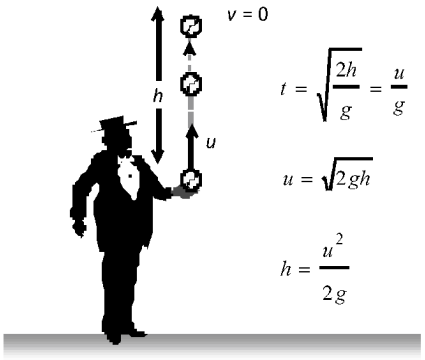
(iii) A body is projected vertically upward :

Equation of motion : Taking initial position as origin and vertically up as positive
a = - g [As acceleration is downwards]
So, if the body is projected with velocity u and after time t it reaches up to height h then

$v = u - gt;\ h = ut - \frac{1}{2}gt^2\ ;\ v^2 = u^2 - 2gh;$

$h_n = u - \frac{g}{2}(2n-1)$

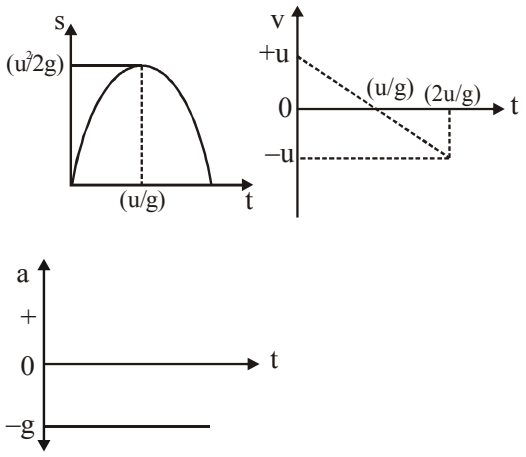
For maximum height v = 0



So from above equation
u = gt,

$h = \frac{1}{2}gt^2$ and $u^2 = 2gh$

- Graph of displacement, velocity and acceleration with respect to time (for maximum height) :



Important Points:

(i) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remains constant while speed, velocity, momentum, kinetic energy and potential energy changes.

(ii) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$

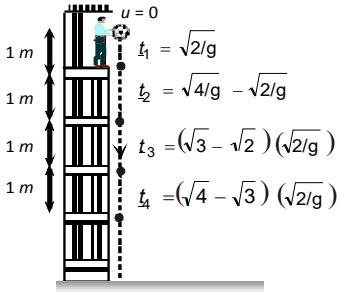
(iii) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_1) = time of ascent (t_2) = u/g

Total time of flight $T = t_1 + t_2 = \frac{2u}{g}$

(iv) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
As well as the magnitude of acceleration at any point on the path is same whether the body is moving in upwards or downward direction.

(v) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1m each will then be in the ratio of the difference in the square roots of the integers i.e.

$\sqrt{1},(\sqrt{2}-\sqrt{1}),(\sqrt{3}-\sqrt{2}),(\sqrt{4}-\sqrt{3}),.....$



TRAIN YOUR BRAIN

Example 5: A man is standing on the top of a building, throws a ball with speed 5m/s in upward direction from 30 m height above the ground level. How much time it takes to reach the ground.

Sol. u = 5m/s

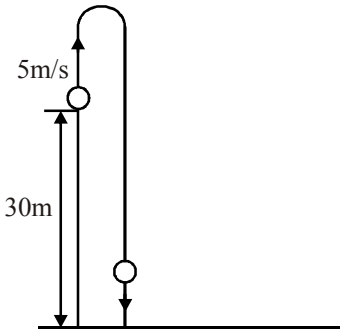
when it reaches the ground, $\Delta y = -30$ m

$\therefore$ from above equation (ii)

$$-30 = 5t - \frac{1}{2}(10)\,t^2$$

$$\Rightarrow t^2 - t - 6 = 0$$

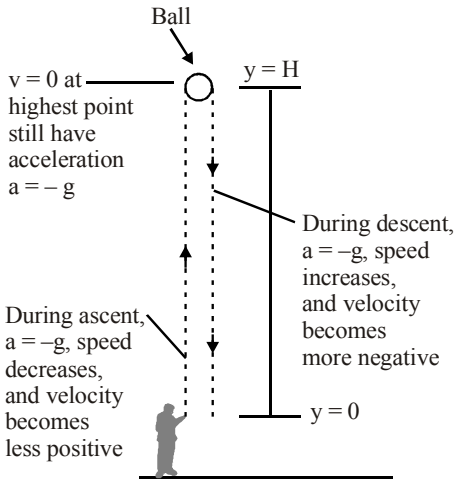
On solving, we get t = 3 & – 2



Rejecting t = – 2 sec,we get t = 3 sec

Example 6: A kid throws a ball up, with some initial speed. Comment on magnitudes and signs of acceleration and velocity of the ball.

Sol.



Important Points:

(i) During ascent, a = –g, velocity becomes less positive i.e., speed decreases

(ii) During descent, a = –g, but now it is in the direction of velocity so it is not retardation. It makes velocity becomes more negative i.e. increases v in negative direction.

Example 7: If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is (g = 10 m/s2)

(*a*) 11.25 m (*b*) 16.2 m

(*c*) 24.5 m (*d*) 7.62 m

Sol. (a) $H_{max} = \frac{u^2}{2g} = \frac{(15)^2}{2\times 10} = 11.25\text{m}$

Example 8: A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is ($g = 10\ m/s^2$)

(*a*) 25 m (*b*) 45 m

(*c*) 90 m (*d*) 125 m

Sol. (b) $h_n = \frac{g}{2}(2n-1) \Rightarrow h_{5th} = \frac{10}{2}(2\times 5-1)$

$= 45\text{m}.$

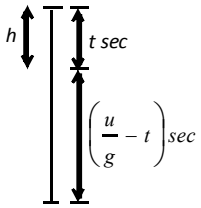
Example 9: If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(*a*) $\frac{1}{2}gt^2$ (*b*) $ut - \frac{1}{2}gt^2$

(*c*) $(u-gt)t$ (*d*) ut

Sol. (a) If ball is thrown with velocity u, then time of flight

$= \frac{u}{g}$



velocity after $\left(\frac{u}{g}-t\right)$sec: $v = u - g\left(\frac{u}{g}-t\right) = gt.$

So, distance in last 't' sec : $0^2 = (gt)^2 - 2(g)h.$

$h = \frac{1}{2}gt^2.$

Example 10: A man drops a ball downside from the roof of a tower of height 400 m. At the same time another ball is thrown upside with a velocity 50 m/s from the foot of tower. What is the height from the foot of the tower where the two balls would meet?

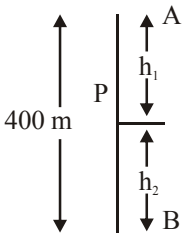
(*a*) 100 meters (*b*) 320 meters

(*c*) 80 meters (*d*) 240 meters

Sol. (c) Let both balls meet at point P after time t.

The distance travelled by ball A

$h_1 = \frac{1}{2}gt^2$ .....(i)



The distance travelled by ball B

$h_2 = ut - \frac{1}{2}gt^2$ .....(ii)

By adding (i) and (ii) $h_1 + h_2 = ut = 400$

(Given $h = h_1 + h_2 = 400$)

$\therefore$ $t = 400/50 = 8$ s and $h_1 = 320$m, $h_2 = 80$ m

Example 11: Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant

(*a*) 2.50 m (*b*) 3.75 m

(*c*) 4.00 m (*d*) 1.25 m

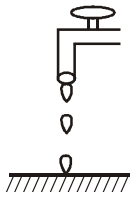
Sol. (b) Let the interval be t then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ ...(i)

For second drop $x = \frac{1}{2}gt^2$ .....(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and

hence required height $h = 5 - \frac{5}{4} = 3.75\text{m}.$



Example 12: A balloon is at a height of 81 m and is ascending upwards with a velocity of 12 m/sec. A body of 2 kg weight is dropped from it. If $g = 10\ m/s^2$ the body will reach the surface of the earth in

(*a*) 1.5 s (*b*) 4.025 s

(*c*) 5.4 s (*d*) 6.75 s

Sol. (c) As the balloon is going up we will take initial velocity of falling body = + 12 m/s,

$\Delta y = -81\text{m}$; $a = -g = -10\ m/s2$

By applying $h = ut + \frac{1}{2}gt^2$; $81 = -12t + \frac{1}{2}(10)t^2$

$\Rightarrow 5t^2 - 12t - 81 = 0$

$\Rightarrow t = \frac{12 \pm \sqrt{144+1620}}{10} = \frac{12 \pm \sqrt{1764}}{10}$

$= 5.4$ s

Example 13: A particle is dropped under gravity from rest from a height h($g = 9.8\ m/s^2$) and it travels a distance $\frac{9h}{25}$ in the last second, the height h is

(*a*) 100 m (*b*) 122.5 m

(*c*) 145 m (*d*) 167.5 m

Sol. (b) Distance travelled in n sec $= \frac{1}{2}gn^2 = h$ .....(i)

Distance travelled in n^{th} sec

$$= \frac{g}{2}(2n-1) = \frac{9h}{25} \quad \text{.....(ii)}$$

Solving (i) and (ii)

we get. h = 122.5 m .

Example 14: A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity 3u. The height of the tower is

(*a*) $3u^2/g$ (*b*) $4u^2/g$

(*c*) $6u^2/g$ (*d*) $9u^2/g$

Sol. (b) For vertical downward motion we will consider initial velocity = –u.

By applying $v^2 = u^2 + 2gh$,

$$(3u)^2 = (-u)^2 + 2gh$$

$$\Rightarrow h = \frac{4u^2}{g}$$

Example 15: A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

(*a*) 4.9 m (*b*) 9.8 m

(*c*) 19.6 m (*d*) 24.5 m

Sol. (d) The separation between two bodies, two second after the release of second body is given by :

$$s = \frac{1}{2}g(t_1^2 - t_2^2)$$

$$= \frac{1}{2} \times 9.8 \times (3^2 - 2^2) = 24.5\ m$$

Topicwise Questions

POSITION, DISTANCE AND DISPLACEMENT

1. A Body moves 6 m north. 8 m east and 10m vertically upwards, what is its resultant displacement from initial position

(*a*) $10\sqrt{2}$ (*b*) 10m

(*c*) $\frac{10}{\sqrt{2}}$m (*d*) 10×2m

2. A man goes 10m towards North, then 20m towards east then displacement is

(*a*) 22.5m (*b*) 25m
(*c*) 25.5m (*d*) 30m

SPEED AND VELOCITY

3. A person travels along a straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2 The average velocity is given by

(*a*) v_1v_2 (*b*) $\frac{v_2^2}{v_1^2}$

(*c*) $\frac{v_1+v_2}{2}$ (*d*) $\frac{2v_1v_2}{v_1+v_2}$

4. A car travels the first half of a distance between two places at a speed of 30 km/hr and the second half of the distance at 50 km/hr. The average speed of the car for the whole journey is

(*a*) 42.5 km/hr (*b*) 40.0 km/hr
(*c*) 37.5 km/hr (*d*) 35.0 km/hr

5. A person travels along a straight road for the first half time with a velocity v_1 and the next half time with a velocity v_2. The mean velocity V of the man is

(*a*) $\frac{2}{V}=\frac{1}{v_1}+\frac{1}{v_2}$ (*b*) $V=\frac{v_1+v_2}{2}$

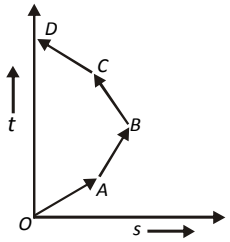
(*c*) $V=\sqrt{v_1v_2}$ (*d*) $V=\sqrt{\frac{v_1}{v_2}}$

6. If a car covers $2/5^{th}$ of the total distance with v_1 speed and $3/5^{th}$ distance with v_2 then average speed is

(*a*) $\frac{1}{2}\sqrt{v_1v_2}$ (*b*) $\frac{v_1+v_2}{2}$

(*c*) $\frac{2v_1v_2}{v_1+v_2}$ (*d*) $\frac{5v_1v_2}{3v_1+2v_2}$

7. Which of the following options is correct for the object having a straight line motion represented by the following graph



(*a*) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity.
(*b*) Velocity of the object increases uniformly
(*c*) Average velocity is zero
(*d*) The graph shown is impossible

ACCELERATION, EQUATION OF KINEMATICS: CONSANT ACCELERATION

8. A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then

(*a*) $S_1=S_2$ (*b*) $S_1=S_2/3$
(*c*) $S_1=S_2/2$ (*d*) $S_1=S_2/4$

9. A body is moving from rest under constant acceleration and let S_1 be the displacement in the first $(p-1)$ sec and S_2 be the displacement in the first P sec. The displacement in $(p^2-p+1)^{th}$ sec. will be

(*a*) S_1+S_2 (*b*) S_1S_2
(*c*) S_1-S_2 (*d*) S_1/S_2

10. A body starts from the origin and moves along the X-axis such that the velocity at any instant is given by $(4t^3-2t)$, where t is in sec and velocity is in m/s. What is the acceleration of the particle, when it is 2 m from the origin?

(*a*) 28 m/s^2 (*b*) 22 m/s^2
(*c*) 12 m/s^2 (*d*) 10 m/s^2

11. A point moves with uniform acceleration and v_1,v_2 and v_3 denote the average velocities in the three successive intervals of time t_1, t_2 and t_3. Which of the following relations is correct

(*a*) $(v_1-v_2):(v_2-v_3)=(t_1-t_2):(t_2+t_3)$

(*b*) $(v_1-v_2):(v_2-v_3)=(t_1+t_2):(t_2+t_3)$

(*c*) $(v_1-v_2):(v_2-v_3)=(t_1-t_2):(t_1-t_3)$

(*d*) $(v_1-v_2):(v_2-v_3)=(t_1-t_2):(t_2+t_3)$

12. A motor car moving with a uniform speed of 20 m/sec comes to stop on the application of brakes after travelling a distance of 10m Its acceleration is

(*a*) 20 m/sec^2 (*b*) –20 m/sec^2

(*c*) –40 m/sec^2 (*d*) +2m/sec^2

13. Which of the following four statements is false

(*a*) A body can have zero velocity and still be accelerated

(*b*) A body can have a constant velocity and still have a varying speed

(*c*) A body can have a constant speed and still have a varying velocity

(*d*) The direction of the velocity of a body can change when its acceleration is constant

14. A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in

(*a*) $\frac{20}{3}$m (*b*) 20m

(*c*) 60m (*d*) 180m

15. The position of a particle moving along the x-axis at certain times is given below:

t(s)	0	1	2	3
x(m)	–2	0	6	16

Which of the following describes the motion correctly

(*a*) Uniform, accelerated

(*b*) Uniform, decelerated

(*c*) Non-uniform, accelerated

(*d*) There is not enough data for generalization

16. A car starts from rest and moves with uniform acceleration a on a straight road from time t = 0 to t = T. After that, constant deceleration brings it to rest. In this process the average speed of the car is

(*a*) $\frac{aT}{4}$ (*b*) $\frac{3aT}{2}$

(*c*) $\frac{aT}{2}$ (*d*) aT

17. If the velocity of a particle is given by $v = (180 - 16x)^{1/2}$ m/s, then its acceleration will be

(*a*) Zero (*b*) 8 m/s^2

(*c*) – 8 m/s^2 (*d*) 4 m/s^2

18. The displacement x of a particle varies with time $t, x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(*a*) Go on decreasing with time

(*b*) Will be independent of α and β

(*c*) Drop to zero when $\alpha = \beta$

(*d*) Go on increasing with time

19. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then

(*a*) $S = \frac{1}{2}ft^2$ (*b*) $S = \frac{1}{4}ft^2$

(*c*) $S = \frac{1}{72}ft^2$ (*d*) $S = \frac{1}{6}ft^2$

20. A body is starts from rest with acceleration 2 m/s^2 till it attains the maximum velocity then retards to rest with 3 m/s^2. If total time taken is 10 second then maximum speed attained is

(*a*) 12 m/s (*b*) 8 m/s

(*c*) 6 m/s (*d*) 4 m/s

MOTION UNDER GRAVITY

21. A stone falls from a balloon that is descending at a uniform rate of 12 m/s. The displacement of the stone from the point of release after 10 sec is

(*a*) 490 m (*b*) 510 m

(*c*) 610 m (*d*) 725 m

22. Two bodies of different masses m_a and m_b are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distances are

(*a*) a : b (*b*) b : a

(*c*) $\sqrt{a} : \sqrt{b}$ (*d*) a^2 : b^2

23. A ball P is dropped vertically and another ball Q is thrown horizontally with the same velocities from the same height and at the same time. If air resistance is neglected, then

(*a*) Ball P reaches the ground first

(*b*) Ball Q reaches the ground first

(*c*) Both reach the ground at the same time

(*d*) The respective masses of the two balls will decide the time

24. A stone dropped from the top of the tower touches the ground in 4 sec. The height of the tower is about

(*a*) 80 m (*b*) 40m

(*c*) 20m (*d*) 160m

25. A body is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time t/2 sec

(*a*) At h/2from the ground

(*b*) At h/4 from the ground

(*c*) Depends upon mass and volume of the body

(*d*) At 3h/4 from the ground

26. A body is slipping from an inclined plane of height h and length l. If the angle of inclination is θ, the time taken by the body to come from the top to the bottom of this inclined plane is

(*a*) $\sqrt{\frac{2h}{g}}$ (*b*) $\sqrt{\frac{2l}{g}}$

(*c*) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ (*d*) $\sin\theta\sqrt{\frac{2h}{g}}$

27. A man in a balloon rising vertically with an acceleration of 4.9 m/sec^2 releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reached by the ball is (g = 9.8m/sec^2)

(*a*) 14.7 m (*b*) 19.6 m
(*c*) 9.8 m (*d*) 24.5 m

28. A rocket is fired upward from the earth's surface such that it creates an acceleration of 19.6 m/sec^2. If after 5 sec its engine is switched off, the maximum height of the rocket from earth's surface would be

(*a*) 245 m (*b*) 490 m
(*c*) 980 m (*d*) 735 m

29. A ball of mass m_1 and another ball of mass m_2 are dropped from equal height. If time taken by the balls are t_1 and t_2 respectively, then

(*a*) $t_1 = \frac{t_2}{2}$ (*b*) $t_1 = t_2$

(*c*) $t_1 = 4t_2$ (*d*) $t_1 = \frac{t_2}{4}$

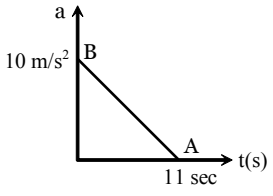
30. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from rest and from the same point 'O' along three different frictionless paths. The speeds of the three objects, on reaching the ground, will be in the ratio of

(*a*) $m_1 : m_2 : m_3$ (*b*) $m_1 : 2m_2 : 3m_3$

(*c*) 1 : 1 : 1 (*d*) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

VARIABLE ACCELERATION

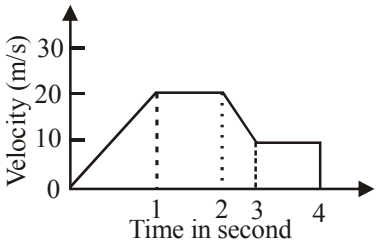
31. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be



(*a*) 110 m/s (*b*) 55 m/s
(*c*) 550 m/s (*d*) 660 m/s

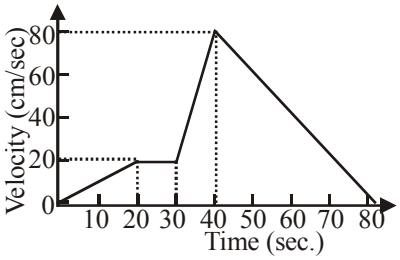
GRAPHS

32. The variation of velocity of a particle with time moving along a straight line is illustrated in the following figure. The distance travelled by the particle in four seconds is



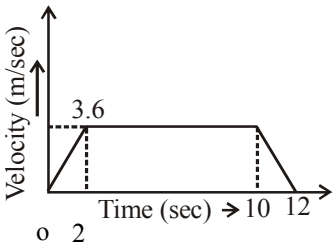
(*a*) 60 m (*b*) 55 m
(*c*) 25 m (*d*) 30 m

33. The v – t graph of a moving object is given in figure. The maximum acceleration is



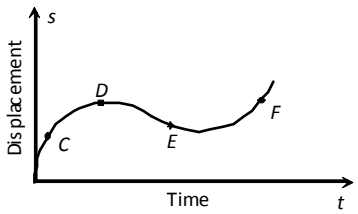
(*a*) 1cm/sec^2 (*b*) 2 cm/sec^2
(*c*) 3 cm/sec^2 (*d*) 6 cm/sec^2

34. A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers?



(*a*) 3.6 m
(*b*) 28.8 m
(*c*) 36.0 m
(*d*) Cannot be calculated from the above graph

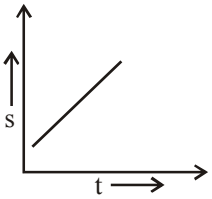
35. The displacement-time graph of moving particle is shown below

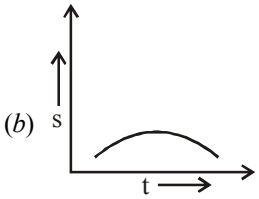


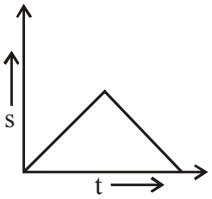
The instantaneous velocity of the particle is negative at the point

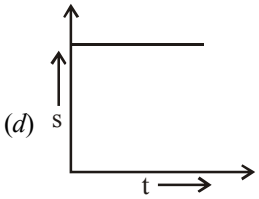
(*a*) D (*b*) F
(*c*) C (*d*) E

36. Which of the following graph represents uniform motion

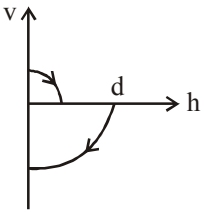
(*a*) 

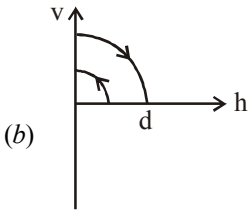
(*b*) 

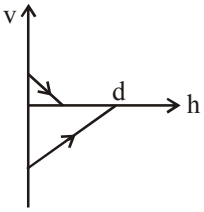
(*c*) 

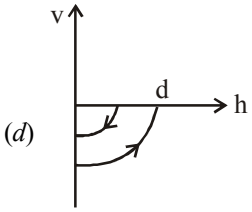
(*d*) 

37. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground is

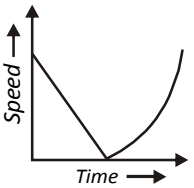
(*a*) 

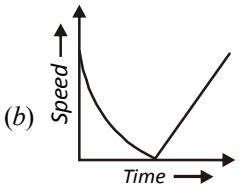
(*b*) 

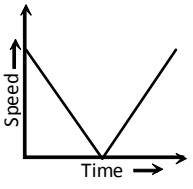
(*c*) 

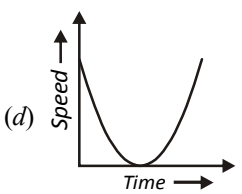
(*d*) 

38. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its height if the air resistance (constent) is not ignored

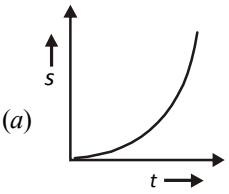
(*a*) 

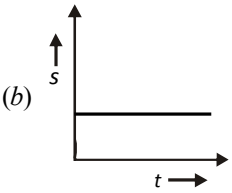
(*b*) 

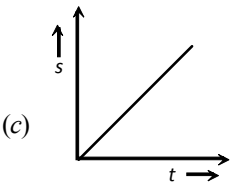
(*c*) 

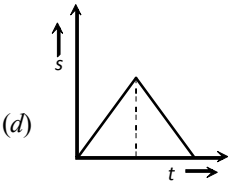
(*d*) 

39. Which graph represents the uniform acceleration

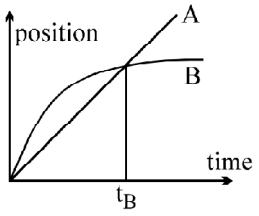
(*a*) 

(*b*) 

(*c*) 

(*d*) 

40. The graph shows position as a function of time for two trains running on parallel tracks. Which one of the following statements is true?



(*a*) At time t_B, both trains have the same velocity.
(*b*) Both trains have the same velocity at some time after t_B
(*c*) Both trains have the same velocity at some time before t_B.
(*d*) Somewhere on the graph, both trains have the same acceleration.

Learning Plus

1. A particle covers half of the circle of radius r. Then the displacement and distance of the particle are respectively-

(*a*) $2\pi r, 0$ (*b*) $2r, \pi r$

(*c*) $\frac{\pi r}{2}, 2r$ (*d*) $\pi r, r$

2. A car travels from A to B at a speed of 20 km h^{-1} and returns at a speed of 30 km h^{-1}. The average speed of the car for the whole journey is

(*a*) 5 km h^{-1} (*b*) 24 km h^{-1}

(*c*) 25 km h^{-1} (*d*) 50 km h^{-1}

3. A car runs at constant speed on a circular track of radius 100 m taking 62.8 s on each lap. What is the average speed and average velocity on each complete lap ?

(*a*) velocity 10 m/s speed 10 m/s

(*b*) velocity zero, speed 10 m/s

(*c*) velocity zero, speed zero

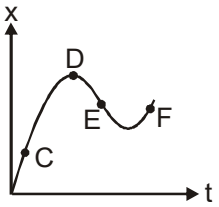
(*d*) velocity 10 m/s, speed zero

4. The displacement of a body is given by $2s = gt^2$ where g is a constant. The velocity of the body at any time t is

(*a*) gt (*b*) gt/2

(*c*) $gt^2/2$ (*d*) $gt^3/3$

5. The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is zero at the point



(*a*) C (*b*) D

(*c*) E (*d*) F

6. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as

(*a*) 1 : 2 : 4 (*b*) 1 : 2 : 5

(*c*) 1 : 3 : 5 (*d*) 1 : 3 : 9

7. A body starts from rest with constant acceleration, the ratio of distances travelled by the body during 4th and 5th seconds is:

(*a*) 7/5 (*b*) 7/9

(*c*) 7/3 (*d*) 3/7

8. A particle, after starting from rest , experiences, constant acceleration for 20 seconds. If it covers a distance of S_1, in first 10 seconds and distance S_2 in next 10 sec, then

(*a*) $S_2 = S_1/2$ (*b*) $S_2 = S_1$

(*c*) $S_2 = 2S_1$ (*d*) $S_2 = 3S_1$

9. A body sliding on a smooth inclined plane requires 4sec to reach the bottom after starting from rest at the top. How much time does it take to cover one fourth the distance starting from the top

(*a*) 1sec (*b*) 2 sec

(*c*) 0.4sec (*d*) 1.6 sec

10. A body is dropped from a height h under acceleration due to gravity g. If t_1 and t_2 are time intervals for its fall for first half and the second half distance, the relation between them is

(*a*) $t_1 = t_2$ (*b*) $t_1 = 2t_2$

(*c*) $t_1 = 2.414\, t_2$ (*d*) $t_1 = 4t_2$

11. A body is thrown upward and reaches its maximum height. At that position-

(*a*) its velocity is zero and its acceleration is also zero

(*b*) its velocity is zero but its acceleration is maximum

(*c*) its acceleration is minimum

(*d*) its velocity is zero and its acceleration is the acceleration due to gravity

12. A particle is moving so that its displacement s is given as $s = t^3 - 6t^2 + 3t + 4$ meter. Its velocity at the instant when its acceleration is zero will be -

(*a*) 3 m/s (*b*) –12 m/s

(*c*) 42 m/s (*d*) –9 m/s

13. The motion of a body is given by the equation $\frac{dv(t)}{dt} = 6.0 - 3v(t)$, where v(t) is speed in m/s and t in sec. If body was at rest at t = 0 find the wrong option.

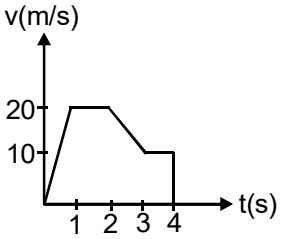
(*a*) The terminal speed is 2.0 m/s

(*b*) The speed varies with the time as $v(t) = 2(1 - e^{-3t})$ m/s

(*c*) The speed is 0.1 m/s when the acceleration is half the initial value

(*d*) The magnitude of the initial acceleration is 6.0 m/s^2

14. The variation of velocity of a particle moving along straight line is shown in the figure. The distance travelled by the particle in 4 s is

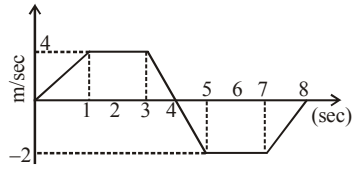


(*a*) 25m (*b*) 30m
(*c*) 55m (*d*) 60m

15. The displacement time graphs of two particles A and B are straight lines making angles of respectively 30º and 60º with the time axis. If the velocity of A is v_A and that of B is v_B then the value of $\frac{v_A}{v_B}$ is

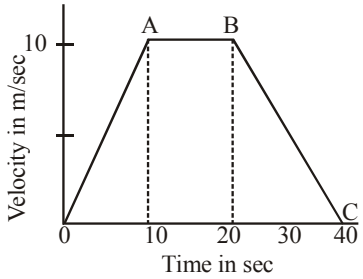
(*a*) 1/2 (*b*) $1/\sqrt{3}$
(*c*) $\sqrt{3}$ (*d*) 1/3

16. The v-t graph of a linear motion is shown in adjoining figure. The distance from origin after 8 seconds is-



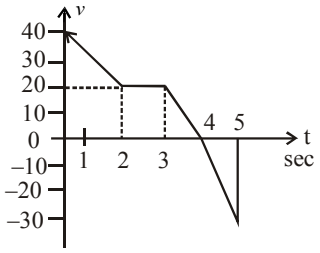
(*a*) 18 meters (*b*) 16 meters
(*c*) 8 meters (*d*) 6 meters

17. The adjoining curve represents the velocity-time graph of a particle, its acceleration values along OA, AB and BC in metre/sec^2 are respectively-



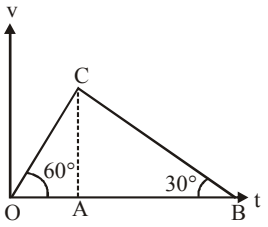
(*a*) 1, 0, -0.5 (*b*) 1, 0, 0.5
(*c*) 1, 1, 0.5 (*d*) 1, 0.5, 0

18. In the following velocity-time graph of a body, the distance and displacement travelled by the body in 5 second in meters will be -



(*a*) 75,115 (*b*) 105, 75
(*c*) 45, 75 (*d*) 95, 55

19. The velocity-time graph of body is shown in figure. The ratio of the ..... during the intervals OA andAB is ..... which of the following statement is wrong



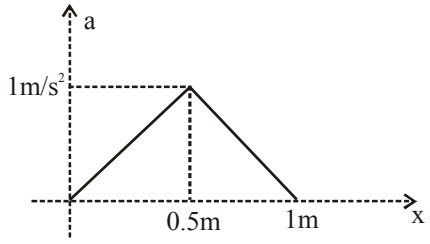
(*a*) magnitude of average velocities, 1

(*b*) $\frac{OA}{AB}, \frac{1}{4}$

(*c*) magnitude of average accelerations, inverse of ratio of distances covered

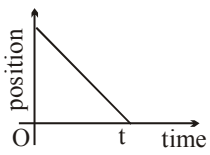
(*d*) distance covered, 1 : 3

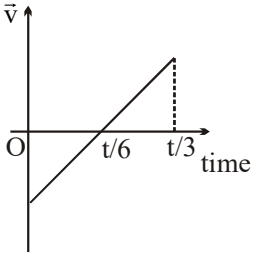
20. A body initially at rest, starts moving along x-axis in such a way so that its acceleration vs displacement plot is as shown in figure. The maximum velocity of particle is

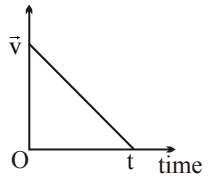


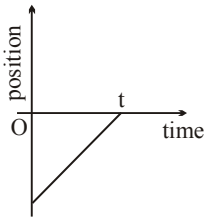
(*a*) 1 m/s (*b*) 6 m/s
(*c*) 2 m/s (*d*) none

21. For which of the following graphs the average velocity of a particle moving along a straight line for time interval (0, t) must be negative.

(*a*) 

(*b*) 

(*c*) 

(*d*) 

22. Each of four particles move along x-axis. Their coordinates (in meters) as functions of time (in seconds) are given by

particle 1 : $x(t) = 3.5 - 2.7t^3$

particle 2 : $x(t) = 3.5 + 2.7t^3$

particle 3 : $x(t) = 3.5 + 2.7t^2$

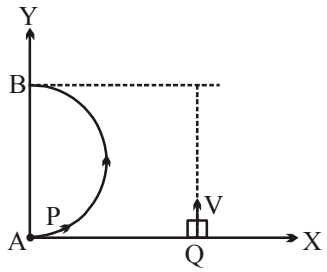
particle 4 : $x(t) = 3.5 - 3.4t - 2.7t^2$

Which of these particles have constant acceleration ?

(*a*) All four (*b*) Only 1 and 2

(*c*) Only 2 and 3 (*d*) Only 3 and 4

23. A particle P starts from origin as shown and moves along a circular path. Another particle Q crosses x-axis at the instant particle P leaves origin. Q moves with constant speed v parallel to y-axis and is all the time having y-coordinate same as that of P. When P reaches diametrically opposite at point B, its average speed is



(*a*) πV (*b*) $\frac{\pi V}{2}$

(*c*) $\frac{V}{2}$ (*d*) None of these

24. A particle is projected up from ground with initial speed v_0. Starting from time $t = 0$ to $t = t_1$,

(*a*) Distance travelled and magnitude of displacement are not equal if $t_1 < \frac{v_0}{g}$

(*b*) Distance travelled and magnitude of displacement are equal if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

(*c*) Distance travelled and magnitude of displacement may not be equal if $0 < t_1 < \frac{2v_0}{g}$

(*d*) The magnitude of displacement is greater than the distance travelled if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

25. Two bodies P and Q have to move equal distances starting from rest. P is accelerated with 2a for first half distance then its acceleration becomes a for last half, where as Q has acceleration a for firsthalf and acceleration 2a for last half, then for whole journey.

(*a*) Average speed of P is more than that of

(*b*) Average speed of both will be same.

(*c*) Maximum speed during the journey is more for P.

(*d*) Maximum speed during the journey is more for

26. A particle moves along the x axis from x_i to x_f. Of the following values of the initial and final coordinates, which results in a negative displacement?

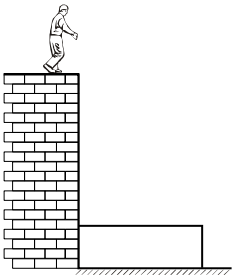
(*a*) $x_i = 4m, x_f = 6m$

(*b*) $x_i = -4m, x_f = -8m$

(*c*) $x_i = -4m, x_f = 2m$

(*d*) $x_i = -4m, x_f = -2m$

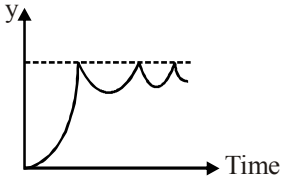
27. Suppose that a man jumps off a building 202 m high onto cushions having a total thickness of 2m. If the cushions are crushed to a thickness of 0.5 m, what is the man's acceleration as he slows down?



(*a*) $10\ m/s^2$ (*b*) $\frac{4000}{3}\ m/s^2$

(*c*) $50\ m/s^2$ (*d*) $20\ m/s^2$

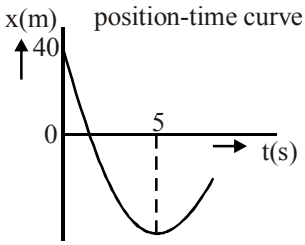
28. The graph below describes the motion of a ball rebounding from a horizontal surface being released from a point above the surface. The quantity represented on the y-axis is the ball's



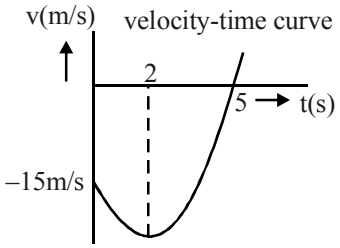
(*a*) displacement (*b*) velocity
(*c*) acceleration (*d*) momentum

29. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in meters and t in seconds. Which of the graph does not represent the motion of the particle?

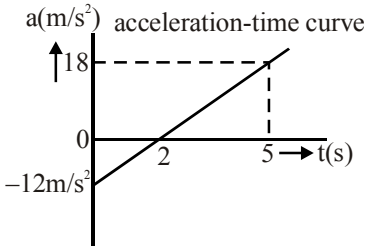
(*a*)



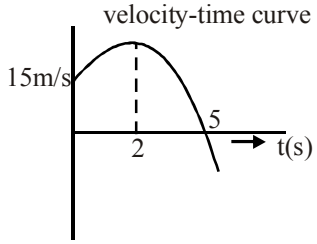
(*b*)



(*c*)



(*d*)



30. A Trolley is moving away from a stop with an acceleration $a = 0.2\ m/s^2$. After reaching the velocity u = 36 km / h, it moves with a constant velocity for the time of 2 min. Then, it uniformly slows down, and stops after further travelling a distance of 100 m. Find the average speed all the way between stops.

(*a*) $\frac{76}{17}$ m/s (*b*) $\frac{208}{21}$ m/s

(*c*) $\frac{85}{12}$ m/s (*d*) $\frac{155}{19}$ m/s

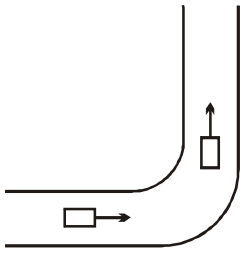
31. Two cars start to move simultaneously with the same speed from point A to point B. The first moves in a straight line connecting A and B, uniformly, and the second - on the bypass road, made as a half-circle connecting the same points. The speed of the second uniformly increases so that at the end of the path its speed is doubled. Which car will arrive earlier at point B?

(*a*) 1st car
(*b*) 2nd car
(*c*) both will reach simultaneously
(*d*) depends on values of R and v

32. A particle is dropped from rest. The particle first covers a distance x_1 in time t_1 and then a distance x_2 in further time t_2. If ratio of time $\frac{t_1}{t_2} = \frac{1}{\left(\sqrt{2}-1\right)}$, find the correct option:

(*a*) $x_1 = x_2$ (*b*) $x_1 > x_2$
(*c*) $x_1 < x_2$ (*d*) Unpredictable

33. A car is moving with 20 ms^{-1} from west to east and takes left turn in 5 sec without changing its speed. Find average acceleration of the car during this period of 5 sec.



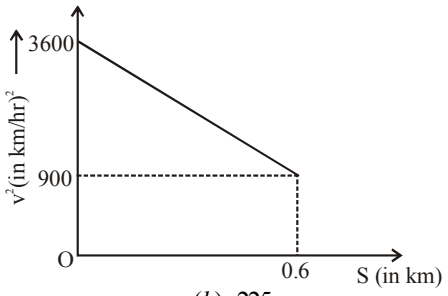
(*a*) $4\sqrt{2}\ ms^{-2}$ N–E

(*b*) $4\sqrt{2}\ ms^{-2}$ N–W

(*c*) $4\ ms^{-2}$ N – E

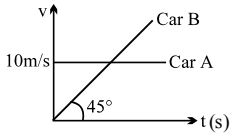
(*d*) Zero

34. A graph between the square of the velocity of a particle and the distance 'S' moved by the particle is shown in the figure. The acceleration of the particle in kilometer per hour square is



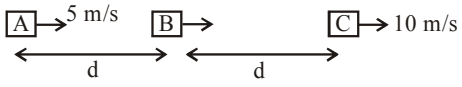
(*a*) 2250 (*b*) 225
(*c*) –2250 (*d*) –225

35. Initially car A is 10.5 m ahead of car B. Both start moving at time t = 0 in the same direction along a straight line. The velocity time graph of two cars is shown in figure. The time when the car B will catch the car A, will be



(*a*) t = 21 sec (*b*) t = $2\sqrt{5}$ sec
(*c*) 20 sec (*d*) none

36. Three persons A, B, C are moving along a straight line as shown with constant and different speeds. When B catches C, the seperation between A & C becomes 4d, then the speed of B is



(*a*) 15 m/s
(*b*) 10 m/s
(*c*) 12.5 m/s
(*d*) not possible

37. Two balls are projected simultaneously with the same speed from the top of a tower, one vertically upwards and the other vertically downwards. If the first ball strikes the ground with speed 20 m/s then speed of second ball when it strikes the ground is.
(*a*) 10 m/s
(*b*) 20 m/s
(*c*) 40 m/s
(*d*) Data insufficient

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

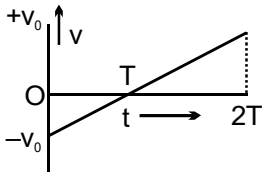
1. Mark the correct statements for a particle going on a straight line
(*a*) if the velocity is zero at any instant, the acceleration should also be zero at that instant
(*b* if the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval
(*c*) if the velocity and acceleration have opposite sign, the object is slowing down
(*d*) if the position and velocity have opposite sign, the particle is moving towards the origin.

2. Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional motion
(*a*) $|\vec{v}|$ must decrease when $\vec{a} < 0$
(*b*) Speed must increase when $\vec{a} > 0$
(*c*) Speed will increase when both $\vec{v}$ and $\vec{a}$ are < 0
(*d*) Speed will decrease when $\vec{v} < 0$ and $\vec{a} > 0$

3. A particle has intial velocity 10 m/s. It moves due to constant retarding force along the line of velocity which produces a retardation of 5 m/s^2. Then -
(*a*) the maximum displacement in the direction of initial velocity is 10 m
(*b*) the distance travelled in first 3 seconds is 7.5 m
(*c*) the distance travelled in first 3 seconds is 12.5 m
(*d*) the distance travelled in first 3 seconds is 17.5 m

4. The displacement x of a particle depend on time t as $x = \alpha t^2 - \beta t^3$
(*a*) particle will return to its starting point after time α/β.
(*b*) the particle will come to rest after time $\frac{2\alpha}{3\beta}$
(*c*) the initial velocity of the particle was zero but its initial acceleration was not zero.
(*d*) no net force act on the particle at time $\frac{\alpha}{3\beta}$

5. The figure shows the velocity (v) of a particle plotted against time (t)



(*a*) The particle changes its direction of motion at some point
(*b*) The acceleration of the particle remains constant
(*c*) The displacement of the particle is zero
(*d*) The initial and final speeds of the particle are the same

6. A particle moves with constant speed v along a regular hexagon ABCDEF in the same order. Then the magnitude of the average velocity for its motion from A to -
(*a*) F is v/5 (*b*) D is v/3
(*c*) C is v $\sqrt{3}/2$ (*d*) B is v

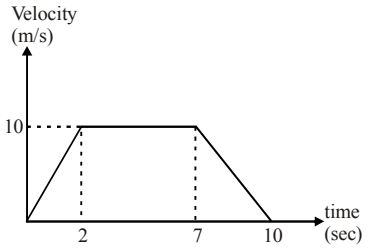
Comprehension Type Questions – 1 (No. 7 to 9)
A boy is standing on a open truck. Truck is moving with an acceleration 2 m/s^2 on horizontal road. When speed of truck is 10 m/s and reaches to a electric pole, boy projected a ball with a velocity 10 m/s in vertical upward direction relative to himself (take g = 10 m/s^2). Neglect the height of boy and truck.

7. The distance of ball from pole where ball land is
(*a*) 20m (*b*) 10m
(*c*) 30m (*d*) 40m

8. Maximum height of ball from ground is -
(*a*) 5m (*b*) 7.5m
(*c*) 2.5m (*d*) 10m

9. Speed of truck at the instant when boy see that ball is moving backward horizontally is -
(*a*) 14 m/s (*b*) 10 m/s
(*c*) 12 m/s (*d*) Data is insufficient

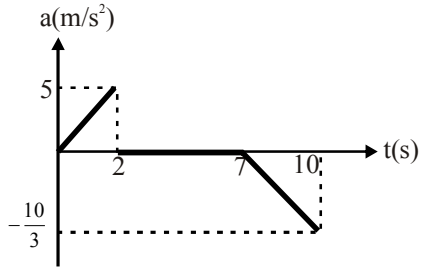
Comprehension Type Questions – 2(No. 10 to 12)
The velocity-time graph of a car moving on a straight track is given below. The car weighs 1000 kg.
(Use F = ma)

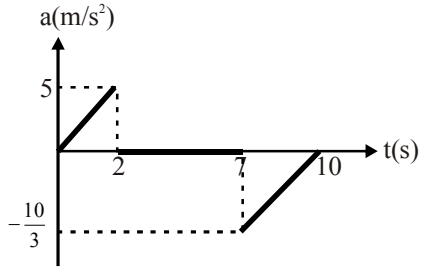


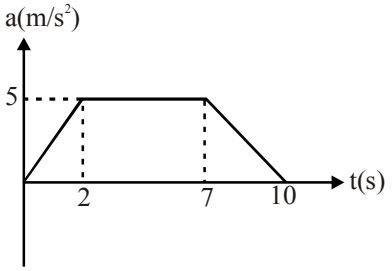
10. The distance travelled by the car during the whole motion is:
(*a*) 50m (*b*) 75m
(*c*) 100m (*d*) 150m

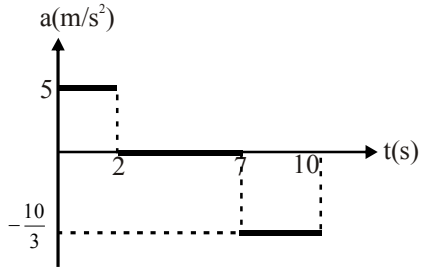
11. The braking force required to bring the car to a stop with in one second from the maximum speed is:
(*a*) $\frac{10000}{3}$N
(*b*) 5000 N
(*c*) 10000 N
(*d*) $\frac{5000}{3}$N

12. Correct acceleration-time graph representing the motion of car is:

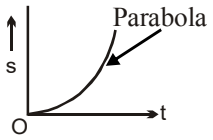
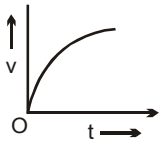
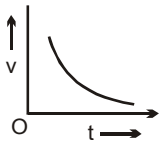
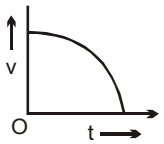
(*a*)


(*b*)


(*c*)


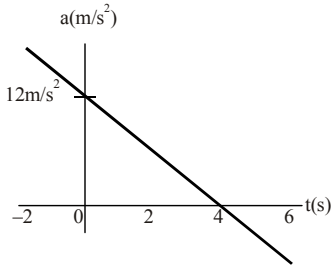
(*d*)


13. Match list **I** with List **II** and select the correct answer using the codes given below the lists.

Column-I		**Column-II**
(*a*) Dacceleration decreasing with time	(P)	
(*b*) Velocity increasing with time	(Q)	
(*c*) Magnitude of acceleration increasing with time	(R)	
(*d*) Body going farther away from the starting point with time	(S)	

NUMERICAL VALUE BASED

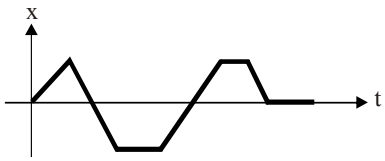
14. Figure gives the acceleration a versus time t for a particle moving along an x-axis. At t = –2.0s, the particle's velocity is 7.0 m/s. What is its velocity (in m/s) at t = 6.0 s?



15. A particle moves in xy-plane according to the equation x = 3t, y = 25 – 4t. What is the minimum distance of the particle (in m) from the origin ? Both x & y are in m.

16. A rocket rises vertically up from the surface of earth so that it's distance from the earth's surface is $l = ct^2$ where c is a constant. After 10 sec. the rocket has travelled 2 km. Determine it's speed (in m/s) at that moment.

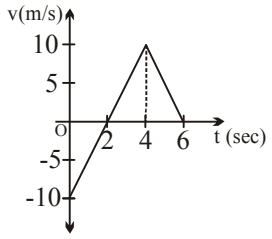
17. Consider a particle moving on a straight line with varying velocity. Its position time graph is as shown.



Find the number of times its velocity changes during motion.

18. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s^2). It is at instantaneous rest at $x = 0$. At what position x (in metre) will the particle again come to instantaneous rest?

19. The figure shows the graph of velocity-time for a particle moving in a straight line. If the average speed for 6 sec is 'b' and the average acceleration from 0 sec to 4 sec is 'c' find magnitude of bc (in m^2/s^3).

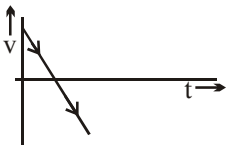


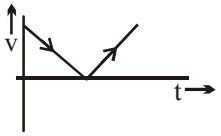
JEE Mains & Advanced Past Years Questions

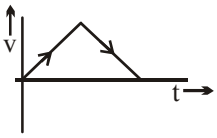
JEE-MAIN
PREVIOUS YEAR'S

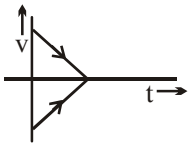
1. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

[*JEE Main-2017*]

(*a*) 

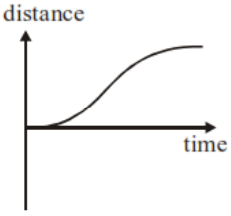
(*b*) 

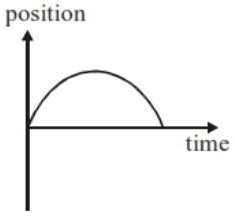
(*c*) 

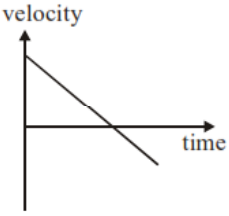
(*d*) 

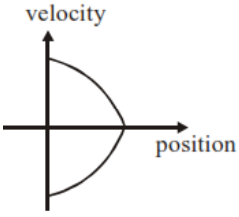
2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

[*JEE Main-2018*]

(*a*) 

(*b*) 

(*c*) 

(*d*) 

3. A particle moves from the point $(2.0\hat{i}+4.0\hat{j})$m , at t = 0 with an initial velocity $(5.0\hat{i}+4.0\hat{j})\,ms^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i}+4.0\hat{j})\,ms^{-2}$. What is the distance of the particle from the origin at time 2s ?

[*JEE Main-2019* (*January*)]

(*a*) 15m (*b*) $20\sqrt{2}$ m

(*c*) 5m (*d*) $10\sqrt{2}$ m

4. The position co-ordinates of a particle moving in a 3-D coordinates system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is:

[*JEE Main - 2019* (*January*)]

(*a*) $\sqrt{2}\,a\omega$ (*b*) $a\omega$

(*c*) $\sqrt{3}\,a\omega$ (*d*) $2a\omega$

5. In a car race on straight road, car A takes a time 't' less than car B at the finish and passes finishing point with a speed 'V' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to

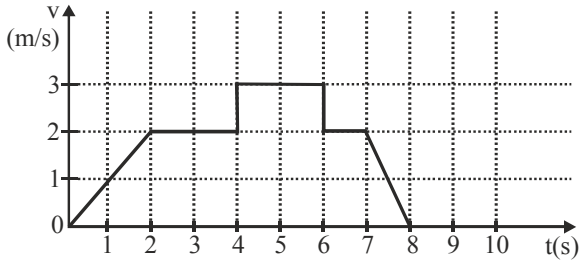
[*JEE Main-2019* (*January*)]

(*a*) $\frac{2a_1a_1}{a_1+a_2}t$ (*b*) $\sqrt{2a_1a_2}\,t$

(*c*) $\sqrt{a_1a_2}\,t$ (*d*) $\frac{a_1+a_2}{2}t$

6. A particle starts from the origin at time t = 0 and moves along the positive x-axis. the graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s?

[*JEE Main-2019* (*January*)]



(*a*) 10m (*b*) 6m

(*c*) 3m (*d*) 9m

7. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be:

[*JEE Main-2019* (*April*)]

(*a*) $a+\frac{b^2}{4c}$ (*b*) $a+\frac{b^2}{c}$

(*c*) $a+\frac{b^2}{2c}$ (*d*) $a+\frac{b^2}{3c}$

8. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + \left(4 - 20t^2\right)\hat{j}$. What is the magnitude of the acceleration at t = 1 ?

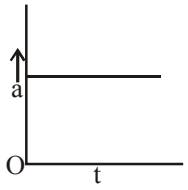
[*JEE Main-2019* (*April*)]

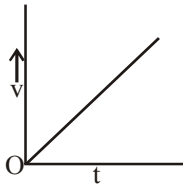
(*a*) 40 (*b*) 100
(*c*) 25 (*d*) 50

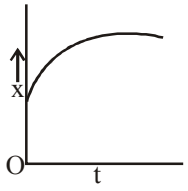
9. A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. if the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to: **[*JEE Main-2019* (*April*)]**

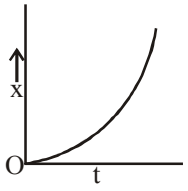
(*a*) 0.4 ms^{-1} (*b*) 0.1 ms^{-1}
(*c*) 0.3 ms^{-1} (*d*) 0.7 ms^{-1}

10. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify the figure that is not correctly representing the motion qualitatively. **[*JEE Main-2019* (*April*)]**
(a = acceleration, v = velocity, x = displacement, t = time)

(*a*) 

(*b*) 

(*c*) 

(*d*) 

(*a*) (*a*), (*b*), (*c*) (*b*) (*a*)
(*c*) (*c*) (*d*) (*b*), (*c*)

11. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at t = 0)

[*JEE Main-2019* (*April*)]

(*a*) $\frac{b^2\tau}{4}$ (*b*) $\frac{b^2\tau}{2}$

(*c*) $b^2\tau$ (*d*) $\frac{b^2\tau}{\sqrt{2}}$

12. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is .....

[*JEE Main-2020* (*January*)]

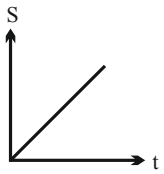
13. A particle starts from the origin at t = 0 with an initial velocity of 3.0 $\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\ \hat{i} + 4.0\hat{j})$ m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32m is D meters. The value of D is :

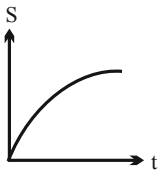
[*JEE Main-2020* (*January*)]

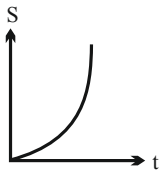
(*a*) 50 (*b*) 40
(*c*) 32 (*d*) 60

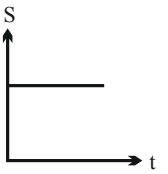
14. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale)

[*JEE Main-2020* (*September*)]

(*a*) 

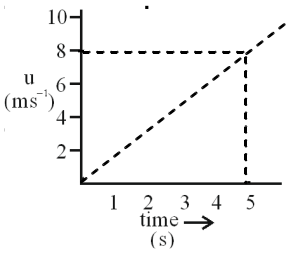
(*b*) 

(*c*) 

(*d*) 

15. The speed versus time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5 s will be _____.

[*JEE Main-2020* (*September*)]



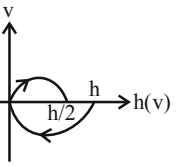
16. Starting from the origin at time t = 0, with initial velocity $5\hat{j}\,ms^{-1}$, a particle moves in the x-y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})ms^{-2}$. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 are, respectively

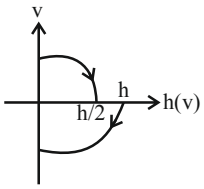
[*JEE Main-2020* (*September*)]

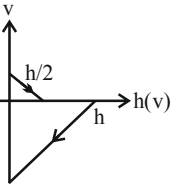
(*a*) 4 s and 52 m (*b*) 2 s and 24 m
(*c*) 5 s and 25 m (*d*) 2 s and 18 m

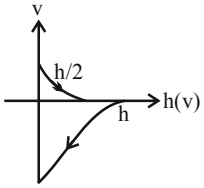
17. A tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by (graph are drawn schematically and on not to scale).

[*JEE Main-2020* (September**)]**

(*a*) 

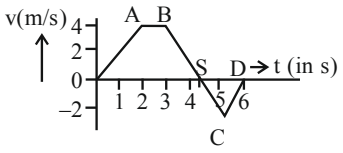
(*b*) 

(*c*) 

(*d*) 

18. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is

[*JEE Main-2020* (*September*)]



(*a*) $\frac{37}{3}$m

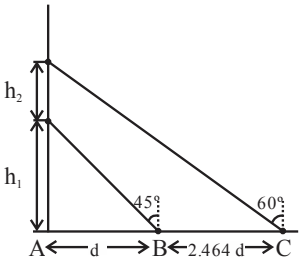
(*b*) 11 m

(*c*) 12 m

(*d*) $\frac{49}{4}$m

19. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan30° = 0.5774)

[*JEE Main-2020* (*September*)]



(*a*) d

(*b*) 0.732 d

(*c*) 1.464 d

(*d*) 0.464 d

20. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity]

[*JEE Main-2020* (*September*)]

(*a*) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$

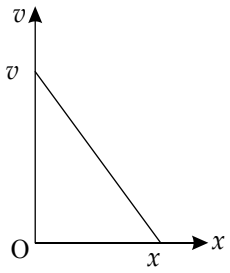
(*b*) $t = 1.8\sqrt{\frac{h}{g}}$

(*c*) $t = \sqrt{\frac{2h}{3g}}$

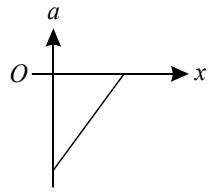
(*d*) $\frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

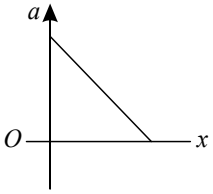
21. The velocity - displacement graph of a particle is shown in the figure.

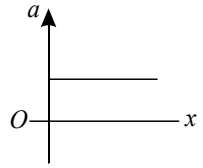
[*JEE Main-2021* (*March*)]

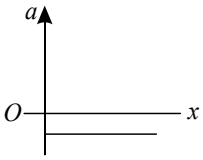


The acceleration - displacement graph of the same particle is represented by:

(*a*) 

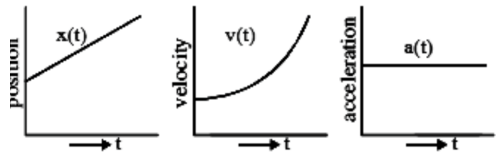
(*b*) 

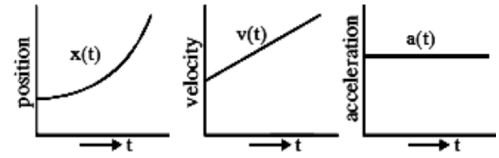
(*c*) 

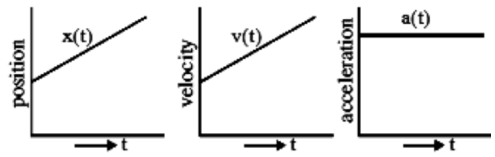
(*d*) 

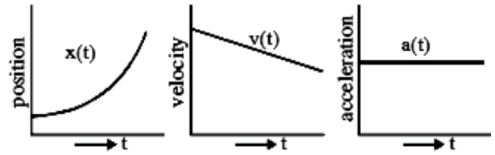
22. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by:

[*JEE Main-2021* (*March*)]

(*a*) 

(*b*) 

(*c*) 

(*d*) 

23. A rubber ball is released from a height of 5m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take $g = 10\,ms^{-2}$) **[*JEE Main-2021 (March)*]**

(*a*) $2.50\,ms^{-1}$ (*b*) $3.0\,ms^{-1}$

(*c*) $2.0\,ms^{-1}$ (*d*) $3.50\,ms^{-1}$

24. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time $(t = 1)$ is: **[*JEE Main-2021 (March)*]**

(*a*) $v_0 + g + f$ (*b*) $v_0 + \frac{g}{2} + \frac{F}{3}$

(*c*) $v_0 + 2g + 3F$ (*d*) $v_0 + \frac{g}{2} + F$

JEE-ADVANCED PREVIOUS YEAR'S

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to **[*JEE Advanced-2017*]**

(*a*) R^3 (*b*) R

(*c*) $R^{2/3}$ (*d*) $\frac{1}{R}$

2. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \hat{i}$ where $E_0 = 1.0\,N\,C^{-1}$ and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is ..... **[*JEE Advanced-2018*]**

ANSWER KEY

Topicwise Questions

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*b*) **6.** (*d*) **7.** (*c*) **8.** (*b*) **9.** (*a*) **10.** (*b*)
11. (*b*) **12.** (*b*) **13.** (*b*) **14.** (*d*) **15.** (*c*) **16.** (*c*) **17.** (*c*) **18.** (*d*) **19.** (*c*) **20.** (*a*)
21. (*c*) **22.** (*c*) **23.** (*c*) **24.** (*a*) **25.** (*d*) **26.** (*c*) **27.** (*a*) **28.** (*d*) **29.** (*b*) **30.** (*c*)
31. (*b*) **32.** (*b*) **33.** (*d*) **34.** (*c*) **35.** (*d*) **36.** (*a*) **37.** (*a*) **38.** (*c*) **39.** (*a*) **40.** (*c*)

Learning Plus

1. (*b*) **2.** (*b*) **3.** (*b*) **4.** (*a*) **5.** (*b*) **6.** (*c*) **7.** (*b*) **8.** (*d*) **9.** (*b*) **10.** (*c*)
11. (*d*) **12.** (*d*) **13.** (*c*) **14.** (*c*) **15.** (*d*) **16.** (*a*) **17.** (*a*) **18.** (*b*) **19.** (*b*) **20.** (*a*)
21. (*a*) **22.** (*d*) **23.** (*b*) **24.** (*c*) **25.** (*a*) **26.** (*b*) **27.** (*b*) **28.** (*a*) **29.** (*d*) **30.** (*d*)
31. (*a*) **32.** (*a*) **33.** (*b*) **34.** (*c*) **35.** (*a*) **36.** (*c*) **37.** (*b*)

Advanced Level Multiconcept Questions

1. (*b*, *c*, *d*) **2.** (*c*, *d*) **3.** (*a*, *c*) **4.** (*a*, *b*, *c*, *d*) **5.** (*a*, *b*, *c*, *d*) **6.** (*a*, *c*, *d*) **7.** (*a*) **8.** (*a*) **9.** (*c*)
10. (*b*) **11.** (*c*) **12.** (*d*) **13.** (*a*)–R; (*b*)–P, Q; (*c*)–S; (*d*)–P, Q, R, S **14.** [0055] **15.** [0015]
16. [0400] **17.** [0006] **18.** [0004] **19.** [0025]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*a*) **5.** (*c*) **6.** (*d*) **7.** (*d*) **8.** (*d*) **9.** (*d*) **10.** (*c*)
11. (*b*) **12.** (*c*) **13.** (*d*) **14.** (*c*) **15.** [20] **16.** (*d*) **17.** (*b*) **18.** (*a*) **19.** (*a*) **20.** (*a*)
21. (*a*) **22.** (*b*) **23.** (*a*) **24.** (*b*)

JEE-ADVANCED
PREVIOUS YEAR'S

1. (*b*) **2.** (2.00)

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

4 Motion in Plane

PROJECTILE MOTION IN TWO AND THREE DIMENSIONS

When a particle is moving in space then its motion can be broken up in three coordinate axis (x, y and z). The motion in these three directions is governed only by velocity and acceleration in that particular direction and is totally independent of the velocities and acceleration in other directions.

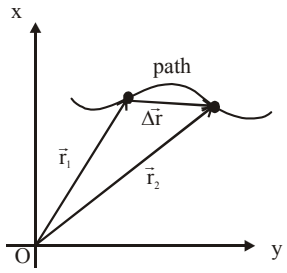
Lets say a particle is moving in space.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad .....(1)$$

Gives position of particle in space.

Velocity

Differentiating $\vec{r}$ w.r.t. time gives us velocity vector of particle at that time.



$$\vec{v} = \frac{d\vec{r}}{dt} \quad .....(2)$$

To write equation in a unit vector form, we substitute $\vec{r}$ from equation (1)

$$\vec{v} = \frac{d\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

This equation can be simplified some what by writing it is

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \quad .....(3)$$

Where the scalar components of $\vec{v}$ are

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt} \quad .....(4)$$

ACCELERATION

Similarly, if we differentiate $\vec{v}$ w.r.t. time we get acceleration of particle

$$\vec{a} = \frac{d\vec{v}}{dt} \quad .....(5)$$

If the velocity changes in either magnitude or direction (or both), the particle must have an acceleration.

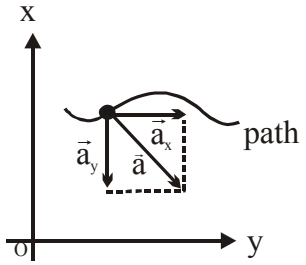
We can write equation (5) in a vector form by substituting $\vec{v}$ from equation (3) to obtain

$$\vec{a} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

We can rewrite this as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \quad .....(6)$$

where the scalar components of $\vec{a}$ are



$$a_x = \frac{dv_x}{dt}; a_y = \frac{dv_y}{dt}; a_z = \frac{dv_z}{dt} \quad .....(7)$$

Thus, we can find the scalar components of $\vec{a}$ by differentiating the scalar components of $\vec{v}$.

Figure shows an acceleration vector $\vec{a}$ and its scalar components for a particle moving in two dimensions.

Now, collecting equations of motion related to x and y axes separately

x-axis	**y-axis**
$v_x = \frac{dx}{dt}$	$v_y = \frac{dy}{dt}$
$a_x = \frac{dv_x}{dt}$	$a_y = \frac{dv_y}{dt}$

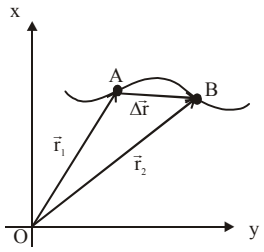
Thus we can see that motion in plane is composed of two straight line motions. These motions are completely independent of each other. Only thing connecting them is fact that they are occurring simultaneously.

Velocity is Along Tangent of Path

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position

$$v = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$



$\Delta \vec{r}$ will be along tangent at point A. when $\Delta \vec{r}$ tends to 0. The result is the same in three dimensions.

Problem solving strategy

for solving any problem of two dimensional motion, following is standard algorithm.

1. Break the motion in two separate one dimensional motions
2. Solve both motions separately.
3. Connect the final equations thoroughly using 'time' (because it is the only common thing).

PROJECTILE MOTION

It consists of two independent motions, a horizontal motion at constant speed and a vertical motion under acceleration due to gravity.

In order to deal with problems in projectile motion, one has to choose a coordinate system. Let's take horizontal as x-axis and vertical upward direction as y-axis, then

$\mathbf{a_x = 0}$ and $\mathbf{a_y = -g}$; since there is only one force "mg" downward (ignore air resistance)

Equation along x-axis **Equations along y-axis**

$v_x = u_x$ (constant) $\qquad v_y = u_y - gt$

$\Delta x = u_x t \qquad \Delta y = u_y t - \frac{1}{2} gt^2$

$v_y^2 = u_y^2 - 2g(\Delta y)$

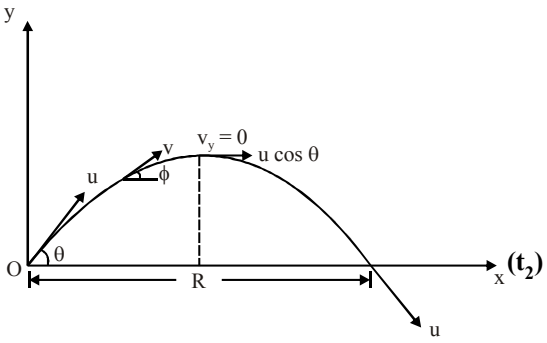
If an object is dropped from rest or projected up or down, it follows straight line path. If its initial velocity is not along the line of force it follows parabolic path which is proved mathematically in this topic later on.

Projectile Motion

Projectile motion is the motion of an object through into the air at some angle under the action of gravity. The path followed by the particle is a curved path.

The particle which is projectted is known as projectile. The path followed by trajectory.

PROJECTILE THROWN FROM THE GROUND LEVEL



A particle is projected from ground level at an angle θ from horizontal with speed u.

$\therefore u_x = u \cos\theta$ and $u_y = u \sin\theta$

At any instant, $v_x = u_x = u \cos\theta$ & $v_y = u_y - gt = u \sin\theta - gt$

$\Delta x = (u \cos\theta)\, t$ & $\Delta y = (u \sin\theta)\, t - \frac{1}{2} gt^2$

Time of flight (T) :

Let it strikes the ground again at time T.

i.e. for $t = T$, $\Delta y = 0 = u_y T - \frac{1}{2} gT^2$

$$\therefore \quad \boxed{T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}}$$

Horizontal Range (R)

It is horizontal displacement till time t = T

i.e. $R = u_x T$

$$\boxed{R = \frac{2u_x u_y}{g}}$$

$$\text{i.e. } R = \frac{2(u\cos\theta)(u\sin\theta)}{g} = \frac{u^2 \sin(2\theta)}{g}$$

Maximum height (H)

$H = \Delta y$ when $v_y = 0$

$\therefore \quad v_y^2 = u_y^2 - 2g(H) = 0$

$$\therefore \quad \boxed{H = \frac{u_y^2}{2g} = \frac{u^2 \sin^2\theta}{2g}}$$

From above formulae, we can observe

(i) $T \propto u_y$ i.e. depends only on vertical component of initial velocity

(ii) $H \propto u_y^2$ i.e. depends only on vertical component of initial velocity

(iii) $R \propto u_x u_y$ i.e. depends both on horizontal and vertical components of initial velocity.

Velocity at any general point

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

$$= \sqrt{u^2 + g^2t^2 - 2(u\sin\theta)gt}$$

If angle which direction of motion makes at an instant is ϕ, then

$$\tan\phi = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

$\tan\phi$ is positive during its upward motion i.e. before reaching highest point and after that $\tan\phi$ is negative.

TRAIN YOUR BRAIN

Example 1: A particle is projected with 20 m/s at an angle 60° with the horizontal. At what time it is moving at an angle 45° with the horizontal while moving downwards.

Sol. $u_x = 20\cos 60° = 10$ m/s

& $u_y = 20\sin 60° = 10\sqrt{3}$ m/s

At required instant, $\tan\phi = -1$

i.e. $\frac{u_y - gt}{u_x} = -1$

i.e. $\frac{10\sqrt{3} - 10t}{10} = -1$

on solving, we get $t = (\sqrt{3}+1)$ sec

Example 2: A particle is projected in the X-Y plane with y-axis along vertical. At 2 sec after projection the velocity of the particle makes an angle 45° with the X - axis and 4 sec after projection, it moves horizontally. Find the velocity of projection.

Sol. At t = 2 sec, $\tan\phi = \frac{u_y - 10(2)}{u_x} = 1$ ($\therefore \phi = 45°$)

$\Rightarrow$ $u_y - 20 = u_x$ ..... (i)

Also $\frac{1}{2}$ (time of fhight) = 4 sec

$\Rightarrow \frac{1}{2}\left(\frac{2u_y}{g}\right) = 4$

$\Rightarrow u_y = 40$ m/s

$\therefore$ From equation (i), $u_x = 20$ m/s

$\therefore u = \sqrt{u_x^2 + u_y^2} = 20\sqrt{5}$ m/s

Equation of Trajectory:

$y = u\sin\theta\, t - (1/2)\, gt^2$

and $x = (u\cos\theta)\, t \Rightarrow t = \frac{x}{u\cos\theta}$

From these equations, (eliminating t)

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$

The above relation between x and y is equation of parabola, which proves that the trajectory i.e. path of projectile is parabolic.

Note:

- The direction of acceleration of the projectile is different from its velocity direction.
- The acceleration of the projectile remains constant in magnitude and direction that is why the path of its motion is parabola, As from equation it is clear, it is equation of parabola.

TRAIN YOUR BRAIN

Example 3: A projectile is projected with kinetic energy K. If it has the maximum possible horizontal range, then its kinetic energy at the highest point will be:

(*a*) 0.25 K (*b*) 0.5 K (*c*) 0.75 K (*d*) 0.0 K

Sol. When range is maximum $\theta = 45°$

Now $u_x = u\cos 45° = u/\sqrt{2}$

At the highest point, the net velocity of projectile is u_x

$\therefore$ K.E. $= (1/2)\, m\, (u/\sqrt{2})^2$

$= (1/2)\, (1/2\, mu^2) = (1/2)\, K = 0.5$ K

Hence correct answer is (b)

Example 4: When the angle of elevation of a gun are 60° and 30° respectively. The height it shoots are h_1 and h_2 respectively, h_1/h_2 equals to-

(*a*) 3/1 (*b*) 1/3 (*c*) 1/2 (*d*) 2/1

Sol. For angle of elevation of 60°, we have maximum height

$$h_1 = \frac{u^2\sin^2 60°}{2g} = \frac{3u^2}{8g}$$

For angle of elevation of 30°, we have maximum height

$$h_2 = \frac{u^2\sin^2 30°}{2g} = \frac{u^2}{8g} \Rightarrow \frac{h_1}{h_2} = \frac{3}{1}$$

Hence correct answer is (*a*)

Example 5: A particle is projected from ground at angle 45° with initial velocity $20\sqrt{2}$ m/s. Find:

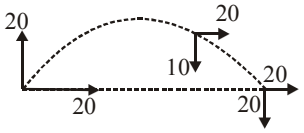
(*a*) change in velocity during in its whole journey

(*b*) magnitude of average velocity in a time interval from t = 0 to t = 3 s.

Sol. (*a*) $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$= 20\hat{i} - 20\hat{j} - (20\hat{i} + 20\hat{j})$

$= 40\hat{j}$ m/s



(*b*) $\vec{v}_i = 20\hat{i} + 20\hat{j}$

$\vec{v}_f = 20\hat{i} - 10\hat{j}$

$\Delta v = v_f - v_i = -30\hat{j}$

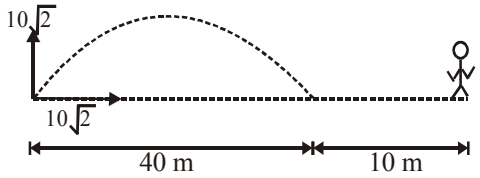
$\Delta\vec{r} = (60\hat{i}) + (20\times 3 - \frac{1}{2}\times 10\times 9)\hat{j} = 60\hat{i} + 15\hat{j}$

$v_{avg} = \frac{\Delta r}{\Delta t}$

$\vec{v}_{av} = 20\hat{i} + 5\hat{j}$

Example 6: The coach throws a baseball to a player with an initial speed of 20 m/s at an angle of 45° with the horizontal. At the moment the ball is thrown, the player is 50 m from the coach. At what speed and in what direction must the player run to catch the ball at the same height at which it was released?

Sol.



Time of flight

$s_y = 10\sqrt{2}t - \frac{1}{2}\times 10t^2$

$t = \frac{10\sqrt{2}}{5} = 2\sqrt{2}$ sec

Range $= 10\sqrt{2}\times 2\sqrt{2} = 40$ m

$v = \frac{10m}{2\sqrt{2}} = \frac{5}{\sqrt{2}} ms^{-1}$

Example 7: If T be the total time of flight of a projectile and H be the maximum height attained by it from the point of projection, then H/T will be- (u = projection velocity and θ = projection angle)

(*a*) (1/2) u sinθ (*b*) (1/4) u sinθ

(*c*) u sinθ (*d*) 2u sinθ

Sol. (*b*) Total time of flight = T = $\frac{2u\sin\theta}{g}$, Maximum height attained H = $\frac{u^2\sin^2\theta}{2g}$

Now $\frac{H}{T} = \frac{u\sin\theta}{4}$

Example 8: If a baseball player can throw a ball at maximum distance = d over a ground, the maximum vertical height to which he can throw it, will be- (Ball have same initial speed in each case)

(*a*) d/2 (*b*) d

(*c*) 2d (*d*) d/4

Sol. The range $R = \frac{u^2\sin 2\theta}{g}$

$\therefore$ Maximum range $R_{max} = d = \frac{u^2}{g}$ ...(1)

Height $H = \frac{u^2\sin^2\theta}{2g}$

$\therefore$ Maximum height

$H_{max} = \frac{u^2}{2g}$ ....(2)

From (1) & (2), $H_{max} = d/2$

Hence correct answer is (a)

Example 9: A projectile can have the same range R for two angles of projections. If t_1 and t_2 be the times of flight in two cases, then the product of times of flight will be-

(*a*) $t_1t_2 \propto R$ (*b*) $t_1t_2 \propto R^2$

(*c*) $t_1t_2 \propto 1/R$ (*d*) $t_1t_2 \propto 1/R^2$

Sol. $t_1 = \frac{2u\sin\theta}{g}$

$t_2 = \frac{2u\sin(90^\circ-\theta)}{g}$

$= \frac{2u\cos\theta}{g}$

$t_1t_2 = \frac{2}{g}\cdot\frac{u^2\sin 2\theta}{g} = \frac{2}{g}\cdot R$

where R is the range, Hence $t_1t_2 \propto R$

Hence correct answer is (a)

Example 10: A ball projected with speed 'u' at angle of projection 15º has range R. The other angle of projection at which the range with same initial speed 'u' is R–

(*a*) 45º (*b*) 35º

(*c*) 75º (*d*) 90º

Sol. Range of projectile, $R = \frac{u^2\sin 2\theta}{g}$

The range is same for two angles θ_1 and θ_2 provided $\theta_2 = 90^\circ - \theta_1$

At an angle θ_1, range $R_1 = \frac{u^2\sin 2\theta_1}{g}$

At an angle of projection θ_2,

$$\text{range } R_2 = \frac{u^2 \sin 2\theta_2}{g} = \frac{u^2 \sin 2(90°-\theta_1)}{g}$$

$$= \frac{u^2 \sin 2\theta_1}{g}$$

$\Rightarrow R_1 = R_2$

$\therefore$ other angle $= 90° - \theta_1 = 90° - 15° = 75°$

Hence correct answer is (c)

HORIZONTAL PROJECTION

In Horizontal Direction

(i) Initial velocity $u_x = u$

(ii) Acceleration $= 0$

(iii) Horizontal velocity of particle remains same after time t horizontal velocity $= v_x = u$

(iv) Range $x = ut$

In Vertical Direction

(i) Initial velocity $u_y = 0$

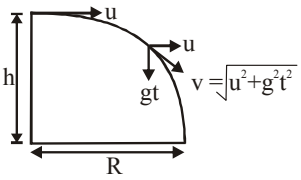
(ii) Acceleration = 'g' downward

(iii) Velocity of particle after time $t = -gt \Rightarrow gt$ (downward)

(iv) Displacement $y = (1/2)\, gt^2$ (downward)

Velocity at a general point P(x, y):

$$v = \sqrt{v_x^2 + v_y^2} \qquad \tan \phi = \frac{v_y}{v_x}$$



ϕ is angle made by v with horizontal in clockwise direction

Time of flight:

$$-h = u_y t - (1/2)\, gt^2 = 0 - \frac{1}{2}\, gt^2$$

$$t = \pm \sqrt{\frac{2h}{g}}$$

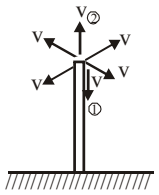
Rejecting negative time $t = \sqrt{\frac{2h}{g}}$

Range:

$$R = u_x\, t = u \sqrt{\frac{2h}{g}}$$

Note: If a projectile is projected with initial in horizontal velocity u and another particle is dropped from same height at the same time, both the projectile would strike the ground at the same instant both will have same vertical components of velocity but their net velocities would be different.

Relative motion of one projectile w.r.t. motion of particle dropped from same height at the same time would be in straight line joining them.



All the particles thrown with same initial velocity would strike the ground with same speed at different times irrespective of their initial direction of velocities.

(a) Time would be least for the particle thrown with velocity v downward i.e. particle (1)

(b) Time would be maximum for the particle thrown with velocity v vertically upwards i.e. particle (2)

TRAIN YOUR BRAIN

Example 11: An aeroplane is in a level flight of 144 km/hr at an altitude of 1000 m. How far horizontally from a given target should a bomb be released from it to hit the target?

(*a*) 566 m (*b*) 671.43 m

(*c*) 471.34 m (*d*) 371.34 m

Sol. $y = (1/2)\, gt^2$ (downward)

$\Rightarrow \quad 1000 = (1/2) \times 10 \times t^2$

$\Rightarrow \quad t = 14.15$ sec

$$x = ut = \left(\frac{144 \times 5}{18}\right) \times 14.15 = 566 \text{ m}$$

Hence correct answer is (a)

Example 12: An aeroplane is flying in horizontal direction with a velocity 600 km/hr and at a height 1960 m. When it is vertically above the point A, on the ground, a body is dropped from it. The body strikes the ground at point B. The distance AB equal to

(*a*) 3.333 km (*b*) 33.33 km

(*c*) 333.3 km (*d*) 33.33 m

Sol. The velocity of plane in horizontal direction $= (500/3)$ m/s

Initial velocity of body in vertical direction, $v_y = 0$

If t is time taken by the body to reach the earth, then from relation

$s = ut + (1/2)\, at^2$

we have

$h = (1/2)\, gt^2$

$$\Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20 \text{ sec}$$

$\therefore$ Distance traversed by body in horizontal direction $AB = v_x\, t$

$\Rightarrow (500/3) \times 20 = 3.333$ km

Hence correct answer is (a)

PROJECTED FROM SOME HEIGHT AT SOME ANGLE

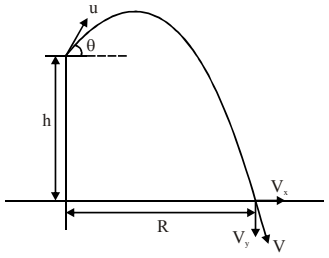
Case I: When projected at some angle θ with the horizontal towards upward direction.

Let it takes time $\mathbf{t_1}$ (time of flight) to strike the ground

$\Delta y = -h$ When $t = \mathbf{t_1}$

$$\therefore \; -h = (u\sin\theta)\, t_1 - \frac{1}{2} g t_1^2$$

$$\Rightarrow \; t_1^2 - \left(\frac{2u\sin\theta}{g}\right) t_1 - \frac{2h}{g} = 0$$



$$\therefore \; \mathbf{t_1} = \frac{T + \sqrt{T^2 + 8h/g}}{2}$$

$$\left(\text{where } T = \frac{2u\sin\theta}{g}\right)$$

Also $\mathbf{R} = \Delta x = u_x t_1$ (putting values of t_1 & u_x we can find **R** whenever required)

When it reaches ground $v_x = u\cos\theta$

& $v_y^2 = u_y^2 - 2g(\Delta y)$

$$\Rightarrow v_y = \sqrt{u^2\sin^2\theta - 2g(-h)} = \sqrt{u^2\sin^2\theta + 2gh}$$

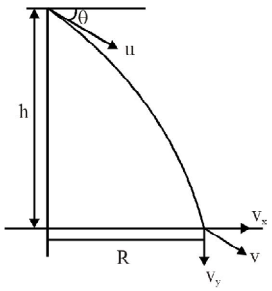
$$\therefore \; v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$$

Case II: When projected at angle θ with horizontal towards downward direction

Here $u_y = -u\sin\theta$

Thus, if it takes time

$\mathbf{t_2}$ to strike the ground then



$$-h = -(u\sin\theta)\, t_2 - \frac{1}{2} g t_2^2$$

$$\Rightarrow \; t_2^2 + \left(\frac{2u\sin\theta}{g}\right) t_2 - h = 0$$

$$\therefore \; t_2 = \frac{\sqrt{T^2 + 8h/g} - T}{2}$$

Also $R = u_x t_2$

Here on reaching ground, $v_x = u\cos\theta$

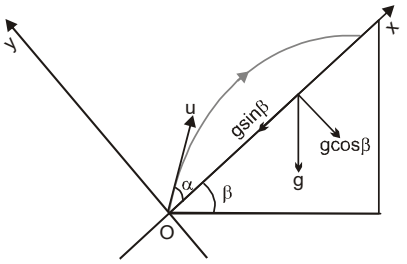
and $v_y = \sqrt{(u\sin\theta)^2 - 2g(-h)}$

$$\therefore \; v = \sqrt{v_y^2 + v_x^2} = \sqrt{u^2 + 2gh}$$

PROJECTION ON AN INCLINED PLANE

Case (i): Particle is projected up the incline

Here α is angle of projection w.r.t. the inclined plane.



x and y axis are taken along and perpendicular to the incline as shown in the diagram.

In this case: $a_x = -g\sin\beta$

$u_x = u\cos\alpha$

$a_y = -g\cos\beta$

$u_y = u\sin\alpha$

Time of flight (T): When the particle strikes the inclined plane y becomes zero

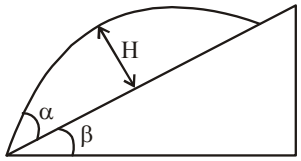
$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \; 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \; T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Where $u_\perp$ and $g_\perp$ are component of u and g perpendicular to the incline.

Maximum height from inclined plane (H):



When half of the time is elapsed y coordinate is equal to maximum distance from the inclined plane of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_\perp^2}{2g_\perp}$$

Range along the inclined plane (R):

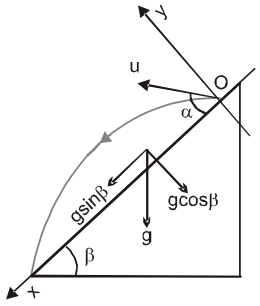
When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$\Rightarrow \quad R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$$

Case (ii) : Particle is projected down the incline



In this case:

$a_x = g\sin\beta$ $u_x = u\cos\alpha$

$a_y = -g\cos\beta$ $u_y = u\sin\alpha$

Time of flight (T):

When the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow 0 = u\sin\alpha T - \frac{1}{2}g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (H):

When half of the time is elapsed y coordinate is equal to maximum height of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_\perp^2}{2g_\perp}$$

Range along the inclined plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$\Rightarrow \quad R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$$

Table: Standard results for projectile motion on an inclined plane

	Up the Incline	Down the Incline
Range	$\frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$	$\frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\frac{2u\sin\alpha}{g\cos\beta}$	$\frac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\frac{\pi}{4}-\frac{\beta}{2}$	$\frac{\pi}{4}+\frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

Here α is the angle of projection with the incline and β is the angle of incline.

RELATIVE MOTION

Introduction:

A person travelling by train may ask his co-passenger (Has delhi come?). This statement seems to claim that Delhi is moving towards our passenger. We may say that his understanding of word is wrong as all of us know that 'actually' it is the train which is moving not the huge city Delhi.

If we observe from passenger's point of view we will realise relative to him all parts of train are at rest, which means relative to him train is at rest. Still when passenger gets down from train he is in a city, thus it must be the cities which are moving.

The point is that when a person standing on ground says that 'actually' it is the train (hence passengers sitting in train) which is moving and reaching to Delhi (which of course is stationary), he is also right. The difference here is of reference frame.

Reference Frame

Reference frame is an axis system from which motion is observed. A clock is attached to measure time. Reference frame can be stationary or moving. There are two types of reference frame.

If the velocities of two bodies are known with respect to a common frame of reference, velocity of one body relative to other can be determined. Thus, if velocities of two bodies P and Q with respect to earth are $\vec{v}_P$, and $\vec{v}_Q$ respectively, then the velocity of Q relative to P is expressed as follows: $\vec{v}_{QP} = \vec{v}_Q - \vec{v}_P$

- If two bodies P and Q are moving along the same line in the same direction with velocities v_P and v_Q relative to ground, the velocity of Q relative to P is: $v_{QP} = v_Q - v_P$.

(a) If it is positive, the direction of v_{QP} is same as that of Q and

(b) If it is negative, the direction of v_{QP} is opposite to that of Q.

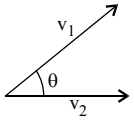
- It is worth noting here that in dealing with the motion of two bodies relative to each other, $\vec{v}_{rel.}$ is the difference of velocities of two bodies if they are moving in the same direction and is the sum of two velocities if they are moving in opposite directions.

If two bodies have velocities $\vec{v}_1$ and $\vec{v}_2$

then the relative velocity of first body with respect to second body

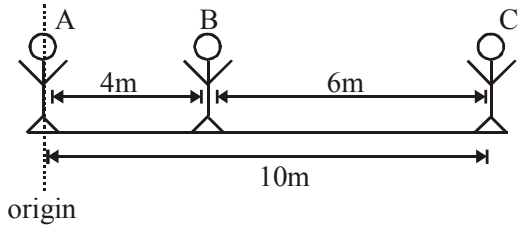
$$\vec{v}_{rel.} = \vec{v}_1 - \vec{v}_2 = \vec{v}_1 + (-\vec{v}_2)$$

$$v_{relative} = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$$



TRAIN YOUR BRAIN

Example 13: The position of three man A, B and C as shown in figure. Then find position of one man with respect to other (take +ve direction towards right and –ve towards left)



Here,

Position of B w.r.t. A is 4 m towards right. ($x_{BA} = +4m$)

Position of C w.r.t. A is 10 m towards right . ($x_{CA} = +10m$)

Position of C w.r.t. B is 6 m towards right ($x_{CB} = +6m$)

Position of A w.r.t. B is 4 m towards left. ($x_{AB} = -4$ m)

Position of A w.r.t. C is 10 m towards left. ($x_{AC} = -10m$)

Example 14: Two objects A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively as shown.



(i) Find the velocity of A with respect to B.

(ii) Find the velocity of B with respect to A

Sol. $v_A = +10$, $v_B = -12$

(i) $v_{AB} = v_A - v_B = (10) - (-12) = 22$ m/s.

(ii) $v_{BA} = v_B - v_A = (-12) - (10) = -22$ m/s.

BOAT-RIVER PROBLEM

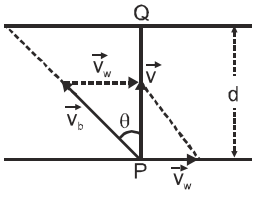
Let velocity of boat in still water = $\vec{v}_b$ and velocity of flow of water in river = $\vec{v}_w$

width of river = d

To cross the river in the shortest path:

It is required that the boat starting from P must reach the opposite point Q along the shortest path PQ.

For the shortest path, the boat should be rowed upstream making an angle θ with PQ such that PQ gives the direction of resultant velocity $\sin\theta = \frac{v_w}{v_b}$



and $v^2 = v_b^2 - v_w^2$. $t = \frac{s}{v} = \frac{s}{\sqrt{v_b^2 - v_w^2}}$

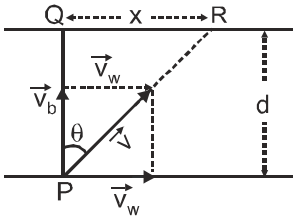
To cross the river in the shortest time

For the boat to cross the river in shortest time, the boat should be directed along PQ.

Let the resultant velocity making an angle θ with PQ = v

$$\tan\theta = \frac{v_w}{v_b} \text{ and } v^2 = v_b^2 + v_w^2$$

$\therefore$ Time for crossing the river $t = \frac{d}{v_b}$



Now the boat reaches the point R rather than reaching point Q. If QR = x, then

$$\tan\theta = \frac{v_w}{v_b} = \frac{x}{d} \qquad \text{or } x = d \times \frac{v_w}{v_b}$$

If a man swims downstream in a river,
then the time taken by the man to cover a distance d is

$$t_1 = \frac{d}{v_b + v_w}$$

If a man swims upstream in a river, then the time taken by him to cover a distance d is

$$t_2 = \frac{d}{v_b - v_w} \qquad \text{So, } \frac{t_1}{t_2} = \frac{v_b - v_w}{v_b + v_w}$$

TRAIN YOUR BRAIN

Example 15: A 400 m wide river is flowing at a rate of 2.0 m/s. A boat is sailing with a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.

[a] find the time taken by the boat to reach the opposite bank.

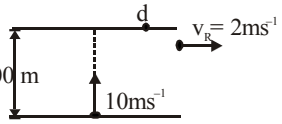
[b] How far from the point directly opposite to the starting point does the boat reach the opposite bank.

[c] In what direction does the boat actually move w.r.t. ground w.r.t. river.

[d] If he can walk at 5 ms^{-1} find the time he will take coming directly to other end.

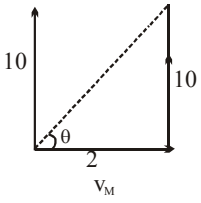
Ans. [a] 40 s [b] 80 m [c] $\tan^{-1} 5$ (downstream) with the river flow

Sol.



v_{MR} is velocity of man w.r.t. river i.e. in still river so in ground reference frame.

[a] time $= \frac{400}{10} = 40$ sec



[b] drift $= 2 \times 40 = 80$ m

[c] $\tan\theta = \frac{10}{2} = 5$

$\theta = \tan^{-1}(5)$ w.r.t. river flow.

[d] Time taken to come to directly other end.

$$= 40 \text{ sec} + \frac{80}{5} = 40 + 16 = 56 \text{ (sec)}$$

Example 16: A man can swim at the rate of 5 km/ h in still water. A 1 km wide river flows at the rate of 3 km/h. The man wishes to swim across the river directly opposite to the starting point.

(*a*) Along what direction must the man swim?

(*b*) What should be his resultant velocity (w.r.t. earth)?

(*c*) How much time will he take to cross the river ?

Ans. (a) 127° w.r.t the river flow or 37° w.r.t. perpendicular in backward direction.

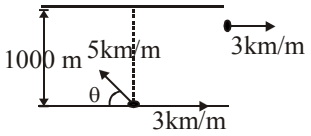
(b) 4 km/hr.

(c) 15 min.

Sol. $5\cos\theta = 3$

$\cos\theta = 3/5$

$\theta = 53°$



(a) so $180° - 53° = 127°$ w.r.t. river flow

(b) 4 km/m

(c) time $= 100/4 = 1/4 \times 60 = 15$ min

RAIN MAN PROBLEMS

In these type of problems we again come across three terms $\vec{v}_r$, $\vec{v}_m$ and $\vec{v}_{rm}$, Here,

$\vec{v}_r$ = velocity of rain

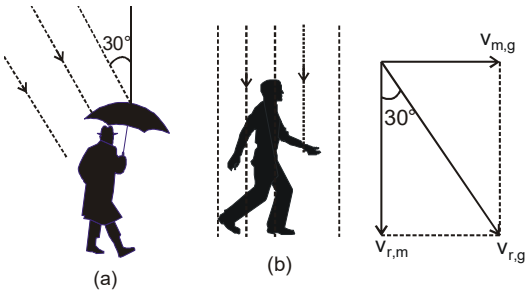
$\vec{v}_m$ = velocity of man (it may be velocity of cyclist or velocity of motorist also)

and $\vec{v}_{rm}$ = velocity of rain with respect to man.

Here, $\vec{v}_{rm}$ is the velocity of rain which appears to the man. Now, let us take one example of this.

Example 17: A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (a) the road, (b) the moving man.

Sol. When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground. The situation when the man runs is shown in the figure



Here $\vec{v}_{r,g}$ = velocity of the rain with respect to the ground

$\vec{v}_{m,g}$ = velocity of the man with respect to the ground and

$\vec{v}_{r,m}$ = velocity of the rain with respect to the man.

We have, $\vec{v}_{r,g} = \vec{v}_{r,m} + \vec{v}_{m,g}$ ...(i)

Taking horizontal components, equation (i) gives

$v_{r,g} \sin 30° = V_{m,g} = 10$ km/h

or, $V_{r,g} = \dfrac{10 \text{ km/h}}{\sin 30°} = 20$km/h

Taking vertical components, equation (i) gives

$v_{r,g} \cos 30° = v_{r,m}$ or, $v_{r,m} = (20 \text{ km/h}) \dfrac{\sqrt{3}}{2} = 10\sqrt{3}$ km/h

Example 18: To a man walking at the rate of 3 km/h the rain appears to fall vertically. When he increases his speed to 6 km/h it appears to meet him at an angle of 45° with vertical. Find the speed of rain.

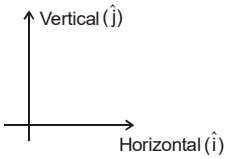
Sol. Let $\hat{i}$ and $\hat{j}$ be the unit vectors in horizontal and vertical directions respectively.

Let velocity of rain

$\vec{v}_r = a\hat{i} + b\hat{j}$ ...(i)

Then speed of rain will be

$|\vec{v}_r| = \sqrt{a^2 + b^2}$



In the first case $\vec{v}_m$ = velocity of man = $3\hat{i}$

$\therefore \quad \vec{v}_{rm} = \vec{v}_r - \vec{v}_m = (a-3)\hat{i} + b\hat{j}$

It seems to be in vertical direction. Hence,

$a - 3 = 0$ or $a = 3$

In the second case $\vec{v}_m = 6\hat{i}$

$\therefore \quad \vec{v}_{rm} = (a-6)\hat{i} + b\hat{j} = -3\hat{i} + b\hat{j}$

This seems to be at 45° with vertical.

Hence, $|b| = 3$

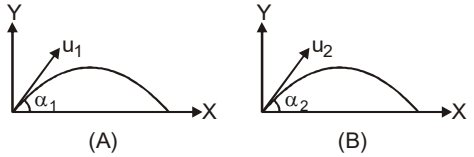
Therefore, from Eq. (ii) speed of rain is

$|\vec{v}_r| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}$ km/h

RELATIVE MOTION BETWEEN TWO PROJECTILES

Let us now discuss the relative motion between two projectiles or the path observed by one projectile of the other. Suppose that two particles are projected from the ground with speeds u_1 and u_2 at angles α_1 and α_2 as shown in Fig.A and B. Acceleration of both the particles is g downwards. So, relative acceleration between them is zero because

$a_{12} = a_1 - a_2 = g - g = 0$



i.e., the relative motion between the two particles is uniform.

Now

$u_{1x} = u_1 \cos \alpha_1$, $u_{2x} = u_2 \cos \alpha_2$

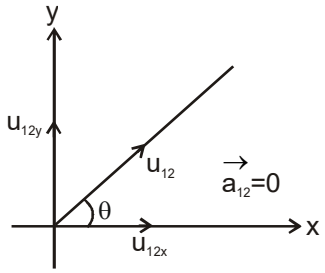
$u_{1y} = u_1 \sin \alpha_1$ and $u_{2y} = u_2 \sin \alpha_2$

Therefore, $u_{12x} = u_{1x} - u_{2x} = u_1 \cos \alpha_1 - u_2 \cos \alpha_2$

and $u_{12y} = u_{1y} - u_{2y} = u_1 \sin \alpha_1 - u_2 \sin \alpha_2$

u_{12x} and u_{12y} are the x and y components of relative velocities of 1 with respect to 2.

Hence, relative motion of 1 with respect to 2 is a straight



line at an angle $\theta = \tan^{-1}\left(\dfrac{u_{12y}}{u_{12x}}\right)$ with positive x-axis.

Now, if $u_{12x} = 0$ or $u_1 \cos \alpha_1 = u_2 \cos \alpha_2$, the relative motion is along y-axis or in vertical direction (as $\theta = 90°$). Similarly, if $u_{12y} = 0$ or $u_1 \sin \alpha_1 = u_2 \sin \alpha_2$, the relative motion is along x-axis or in horizontal direction (as $\theta = 0°$).

Note: Relative acceleration between two projectiles is zero. Relative motion between them is uniform. Therefore, condition of collision of two particles in air is that relative velocity of one with respect to the other should be along line joining them, i.e., if two projecticles A and B collide in mid air, then $\vec{V}_{AB}$ should be along AB or $\vec{V}_{BA}$ along BA.

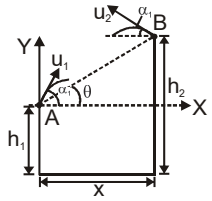
1. Condition for collision of two projectiles:

Consider the situation shown in the figure. For projectiles to collide, direction of velocity of A with respect to B has to be along line AB.

Here, $v_{ABx} = u_1 \cos \alpha_1 + u_2 \cos \alpha_2$

$v_{ABy} = u_1 \sin \alpha_1 - u_2 \sin \alpha_2$

Let, direction of velocity vector of A(wrt B) is making an



angle β with +ve X-axis, which is given by

$$\tan \beta = \frac{v_{ABy}}{v_{ABx}} = \frac{u_1 \sin \alpha_1 - u_2 \sin \alpha_2}{u_1 \cos \alpha_1 + u_2 \cos \alpha_2}$$

For collision to take place, $\tan \beta = \tan \theta = \dfrac{h_2 - h_1}{x}$

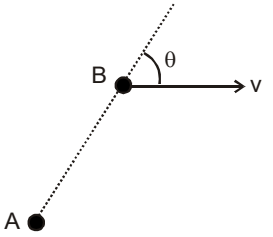
VELOCITY OF APPROACH / SEPARATION IN TWO DIMENSION

It is the component of relative velocity of one particle w.r.t. another, along the line joining them.

If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

TRAIN YOUR BRAIN

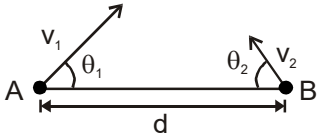
Example 19: Particle A is at rest and particle B is moving with constant velocity v as shown in the diagram at t = 0. Find their velocity of separation



Sol. $v_{BA} = v_B - v_A = v$

v_{sep} = component of v_{BA} along line AB = $v\cos\theta$

Example 20: Two particles A and B are moving with constant velocities v_1 and v_2 . At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find their velocity of approach.



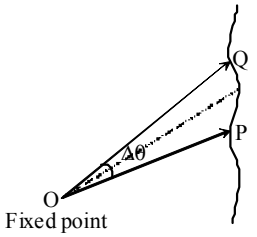
Sol. Velocity of approach is relative velocity along line AB

$v_{APP} = v_1\cos\theta_1 + v_2\cos\theta_2$

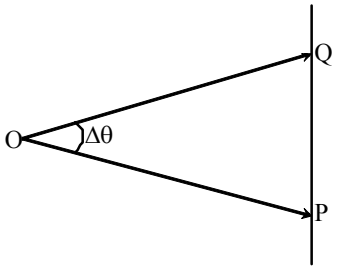
CIRCULAR MOTION

Angular Displacement

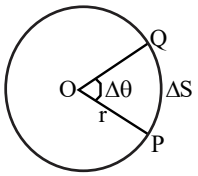
Introduction: Angle subtended by position vector of a particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement.



(a) Particle moving in an arbitrary path



(b) Particle moving in straight line



(c) Particle moving in circular path

(i) Infinitesimaly small Angular displacement is a vector quantity.

(ii) Its direction is perpendicular to plane of rotation and given by right hand screw rule.

Note: Clockwise angular displacement is taken as negative and anticlockwise displacement as positive.

(iii) For circular motion $\Delta S = r \times \Delta\theta$

(iv) Its unit is radian (in M.K.S)

Note: Always change degree into radian, if it occurs in numerical problems.

Note: 1 radian = $\frac{360^o}{2\pi} \Rightarrow \pi$ radian = 180º

(v) It is a dimensionless quantity
i.e. dimension $[M^0L^0T^0]$

Angular Velocity

It is defined as the rate of change of angular displacement of a body or particle moving in circular path.

(i) It is a vector quantity.

(ii) Its direction is same as that of angular displacement i.e. perpendicular to plane of rotation.

Note: If the particle is revolving in the clockwise direction then the direction of angular velocity is perpendicular to the plane inwards. Whereas in case of anticlockwise direction the direction will be outwards.

(iii) Its unit is Radian/sec

(iv) Its dimension is $[M^0L^0T^{-1}]$

TYPES OF ANGULAR VELOCITY:

- **Average Angular Velocity:**

$$\omega_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}} = \frac{\Delta\theta}{\Delta t}$$

It is a scalar quantity because finite angular displacement is a scalar quantity.

- **Instantaneous Angular velocity:**

The instantaneous angular velocity is defined as the angular velocity at some particular instant.

Instantaneous angular velocity

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \vec{\omega} = \frac{d\vec{\theta}}{dt}$$

Note: Instantaneous angular velocity can also be called as simply angular velocity.

RELATION BETWEEN LINEAR VELOCITY AND ANGULAR VELOCITY

We have $\omega = \frac{d\theta}{dt} = \frac{d\theta}{ds} \cdot \frac{ds}{dt} = \frac{1}{r} \cdot v$

$[\because d\theta = \frac{ds}{dr}, \text{angle} = \frac{\text{arc}}{\text{radius}}$ and

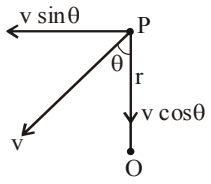
$v = \frac{ds}{dt}$ = linear velocity]

$\Rightarrow v = r\omega$

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

Note: (i) When a particle moves along a curved path, its linear velocity at a point is along the tangent drawn at that point

(ii) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v\sin\theta}{r}$



ANGULAR ACCELERATION

The rate of change of angular velocity is defined as angular acceleration.

If $\Delta\omega$ be change in angular velocity in time Δt, then angular acceleration

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

[If $\vec{\alpha} \parallel \vec{\omega}$ ie $\theta = 0$, particle is accelerating]

[If $\theta = 180^\circ$ then particle is decelerating], where θ is the angle between $\vec{\alpha}$ & $\vec{\omega}$.

(i) It is a vector quantity

(ii) Its direction is that of change in angular velocity

(iii) Unit : rad/sec^2

(iv) Dimension : $M^0L^0T^{-2}$

RELATION BETWEEN ANGULAR ACCELERATION AND LINEAR ACCELERATION

Linear acceleration = Rate of change of linear velocity

$\Rightarrow a = \frac{dv}{dt}$ ....(i)

Angular acceleration = Rate of change of angular velocity

$\Rightarrow \alpha = \frac{d\omega}{dt}$ ....(ii)

From (i) & (ii)

$\frac{a}{\alpha} = \frac{dv}{d\omega} = \frac{d(r\omega)}{d\omega} = r\frac{d\omega}{d\omega}$ [r is constant] $= r$

$\Rightarrow a = \alpha r$

In vector form $\vec{a} = \vec{\alpha} \times \vec{r}$

TRAIN YOUR BRAIN

Example 21: If θ depends on time t in following way $\theta = 2t^2 + 3$ then

(a) Find out average ω upto 3 sec. (b) ω at 3 sec

Sol. $\omega_{avg} = \frac{\text{Total angular displacement}}{\text{total time}} = \frac{\theta_f - \theta_i}{t_2 - t_1}$

$\theta_f = 2(3)^2 + 3 = 21$ rad

$\theta_i = 2(0) + 3 = 3$ rad.

So, $\omega_{avg} = \frac{21-3}{3} = 6$ rad/sec

$\omega_{instantaneous} = \frac{d\theta}{dt} = 4t$

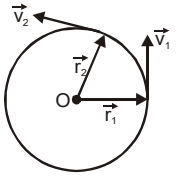
$\omega_{at\ t=3\ sec} = 4 \times 3 = 12$ rad/sec

CIRCULAR MOTION

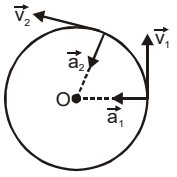
Uniform Circular Motion:

Motion of a particle along the circumference of a circle with a constant speed is called uniform circular motion.

Uniform circular motion is an accelerated motion because in uniform circular motion the position of particle goes on changing and the position vector keeps on changing. As a result the velocity of the particle goes on changing it's direction.



(a)



(b)

Speed of the particle remains constant.

The position vector and velocity vector goes on changing, though their magnitudes remain constant.

The acceleration of the particle is always towards center.

In case of uniform circular motion:

- Position vector is always perpendicular to the velocity vector, i.e. $\vec{r} \cdot \vec{v} = 0$.
- Velocity vector is always perpendicular to the acceleration vector, i.e., $\vec{v} \cdot \vec{a} = 0$.
- The centripetal acceleration vector is directed opposite to the radius vector.
- A force towards centre (i.e., centripetal force) must act so that direction of $\vec{v}$ keeps on changing which forces the particle to describe the circular path.
- The work done by centripetal force is zero because $\vec{F} = m\vec{a}$ is perpendicular to $\vec{v}$ or displacement vector $\Delta\vec{r}$.
- The force only changes the direction of motion, i.e., velocity but not the magnitude of velocity, so the kinetic energy of the particle remains constant or speed of the particle does not change although a force is acting on the particle.

Non-uniform circular motion:

If the speed of the particle moving in a circle is not constant, the acceleration has both the radial and the tangential components.

Examples of non uniform circular motion:

A merry-go-round spinning up from rest to full speed, or a ball whirling around in a verticle circle.

The net acceleration is neither parallel nor perpendicular to the velocity.

We can resolve the acceleration vector into two components:

(a) Radial Acceleration:

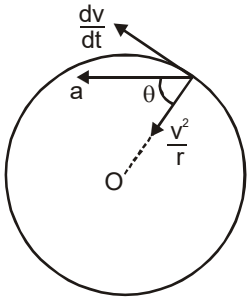
The component of the acceleration towards the centre is

$\omega^2 r = \frac{v^2}{r}$

a_r perpendicular to the velocity

$\Rightarrow$ changes only the direction of velocity.

Acts just like the acceleration in uniform circular motion.



$a_c = \frac{v^2}{r} = \omega^2 r \quad [a_c = a_r]$

Centripetal force $F_c = \frac{mv^2}{r} = m\omega^2 r$

(b) Tangential Acceleration:

The component along the tangent (along the direction of motion) is $a_t = \frac{dv}{dt}$ parallel to the velocity(since it is tangent to the path)

$\Rightarrow$ changes magnitude of the velocity and acts just like one-dimensional acceleration

$\Rightarrow$ $a_t = \frac{dv}{dt}$ where $v = \frac{ds}{dt}$ and s = length of arc

Tangential force $F_t = ma_t$

Net acceleration vector is obtained by vector addition of these two components.

The magnitude of the net acceleration

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

In non-uniform circular motion:

Speed $|\vec{v}| \neq$ constant

angular velocity $\omega \neq$ constant

If at any instant

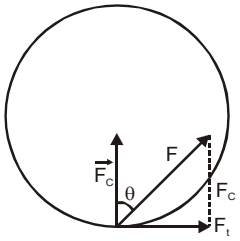
$\Rightarrow$ v = magnitude of velocity of particle

$\Rightarrow$ r = radius of circular path

$\Rightarrow$ ω = angular velocity of a particle,

then, at that instant $v = r\omega$

Net force on the particle in non uniform circular motion

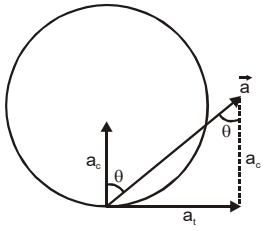


$\vec{F} = \vec{F}_c + \vec{F}_t \Rightarrow \quad F = \sqrt{F_c^2 + F_t^2}$

If θ is the angle made by F with F_c

then $\tan\theta = \frac{F_t}{F_c} \Rightarrow \quad \theta = \tan^{-1}\left[\frac{F_t}{F_c}\right]$ [angle between F_c and F_t is 90º]

Angle between F and F_t is $(90^\circ - \theta)$



Net acceleration, $a = \sqrt{a_c^2 + a_t^2} = \frac{F_{net}}{m}$

The angle made by 'a' with a_c, $\tan\theta = \frac{a_t}{a_c} = \frac{F_t}{F_c}$

The direction of this resultant acceleration makes an angle θ with the radius.

where $\tan\theta = \frac{\frac{dv}{dt}}{\frac{v^2}{r}}$

TRAIN YOUR BRAIN

Example 22: A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changes from 5.0 m/s to 6.0 m/s in 2.0s, find the angular acceleration.

Sol. The tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

($\because$ Here speed increases uniformly $a_t = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$)

$$= \frac{6.0 - 5.0}{2.0}\ m/s^2 = 0.5\ m/s^2$$

The angular acceleration is $\alpha = a_t/r$

$$= \frac{0.5\,m/s^2}{20\,cm} = 2.5\ rad/s^2$$

Example 23: A particle moves in a circle of radius 20 cm. Its linear speed at any time is given by v = 2t where v is in m/s and t is in seconds. Find the radial and tangential acceleration at t = 3 seconds and hence calculate the total acceleration at this time.

Sol. The linear speed at 3 seconds is

$v = 2 \times 3 = 6$ m/s

The radial acceleration at 3 seconds

$$= \frac{v^2}{r} = \frac{6\times 6}{0.2} = 180\ m/s^2$$

The tangential acceleration is given by

$\frac{dv}{dt} = 2$, because v = 2t.

$\therefore$ tangential acceleration is 2 m/s^2.

Net Acceleration = $\sqrt{a_r^2 + a_t^2} = \sqrt{(180)^2 + (2)^2}$

$= 180.01$ m/s^2

Is it possible for a car to move in a circular path in such a way that it has a tangential acceleration but no centripetal acceleration?

Example 24: A particle moves in a circle of radius 2.0 cm at a speed given by v = 4t, where v is in cm/s and t in seconds.

(a) Find the tangential acceleration at t = 1 s.

(b) Find total accleration at t = 1 s.

Sol. (a) Tangential acceleration

$$a_t = \frac{dv}{dt} \quad \text{or} \quad a_t = \frac{d}{dt}(4t) = 4\ cm/s^2$$

$$a_c = \frac{v^2}{R} = \frac{(4)^2}{2} = 8$$

$$\Rightarrow a = \sqrt{a_t^2 + a_c^2} = \sqrt{(4)^2 + (8)^2} = 4\sqrt{5}\ m/s^2$$

SPECIAL NOTE

- Since in both uniform and non-uniform circular motion F_c is perpendicular to velocity, so work done by centripetal force will be zero in both the cases.
- In uniform circular motion $F_t = 0$, as $a_t = 0$, so work done will be zero by tangential force.
- In non-uniform circular motion $F_t \neq 0$, so work done by tangential force is non zero.
- Rate of work done by net force in non-uniform circular motion = rate of work done by tangential force

$$\Rightarrow P = \frac{dW}{dt} = \vec{F}_t . \vec{v} = \vec{F}_t . \frac{d\vec{x}}{dt}$$

- In a circle as tangent and radius are always normal to each other, so $\vec{a}_t \perp \vec{a}_r$.

  Net acceleration in case of circular motion

  $a = \sqrt{a_r^2 + a_t^2}$

  Here it must be noted that a_t governs the magnitude of $\vec{v}$ while a_r its direction of motion, so that

$\Rightarrow$ if $a_r = 0$ and $a_t = 0$, $a \to 0$ motion is uniform translatory

$\Rightarrow$ if $a_r = 0$ but $a_t \neq 0$, $a \to a_t$ motion is accelerated translatory

$\Rightarrow$ if $a_r \neq 0$ but $a_t = 0$, $a \to a_r$ motion is uniform circular

$\Rightarrow$ if $a_r \neq 0$ and $a_t \neq 0$, $a \to \sqrt{a_r^2 + a_t^2}$ motion is nonuniform circular

The centripetal force can be expressed as :

$$\vec{F} = m\vec{a} = -m\omega^2\vec{r} = -m\omega^2 r\hat{r} = -\left(\frac{mv^2}{r}\right)\hat{r}$$

(a) For earth's motion round the sun, the centripetal force is provided by the gravitational force.

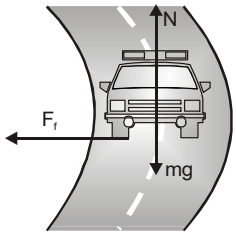
(b) For electron's motion around the nucleus in hydrogen atom, centripetal force is provided by the electrostatic force.

- If the body on a circular path comes to rest, i.e., $\vec{v} \to 0$, the body will move along the radius towards the centre and if radial acceleration a_r vanishes, the body will fly off along the tangent. So a tangential velocity and a radial acceleration are must for uniform circular motion.

- As $F = \frac{mv^2}{r} \neq 0$, so the body is not in equilibrium and linear momentum of the particle moving on the circle is not conserved. However, as the force is central, i.e., $\tau = 0$, so angular momentum is conserved.

A CAR TAKING A TURN ON A LEVEL ROAD

When a car takes a turn on a level road, the portion of the turn can be approximated by an



arc of a circle of radius r. If the car makes the turn at a constant speed v, then there must be some centripetal force acting on the car. This force is generated by the friction between the tyres and the road.

The maximum frictional force is $F_f = \mu_s N, \mu_s$ = the coefficient of static friction.

Then the maximum safe velocity v is given by

$$\frac{mv^2}{r} = \mu_s N \Rightarrow \quad v = \sqrt{\mu_s rg} \quad \text{or} \quad \mu_s = \frac{v^2}{rg}$$

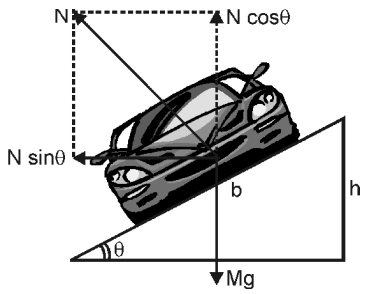
It is important to note that safe velocity is independent of the mass of the car.

BANKING OF TRACKS

Roads on large highways are generally banked so that a vehicle may make a safe and easier turn without depending on friction, road bend at the curved path is raised a little on the side away from the centre of the curved path.

Due to banking of the road, a component of the normal force points towards the centre of curvature of the road.

Necessary centripetal force required for circular motion is supplied by this component. The vertical component of the normal force is balanced by the weight of the vehicle.



$$N\cos\theta = mg \text{ and } N\sin\theta = \frac{mv^2}{r}$$

$$\therefore \tan\theta = \frac{v^2}{rg} \qquad \sin\theta = \frac{h}{b} \Rightarrow v = \sqrt{rg\tan\theta}$$

For a road with angle of banking θ, the speed v at which minimum wear and tear of tyre takes place is $v = \sqrt{rg\tan\theta}$

TRAIN YOUR BRAIN

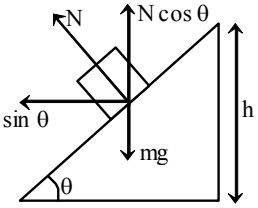
Example 25: A railway track is banked for a speed v, by making the height of the outer rail (h) higher than that of the inner rail. The distance between the rails is d. The radius of curvature of the track is r-

(*a*) $\frac{h}{d} = \frac{v^2}{rg}$ (*b*) $\tan\left(\sin^{-1}\frac{h}{d}\right) = \frac{v^2}{rg}$

(*c*) $\tan^{-1}\left(\frac{h}{d}\right) = \frac{v^2}{rg}$ (*d*) $\frac{h}{r} = \frac{v^2}{dg}$

Ans. (b)

Sol.



$$\sin\theta = \frac{h}{d} \qquad N\cos\theta = mg \qquad \ldots.(1)$$

$$N\sin\theta = \frac{mv^2}{r} \qquad \ldots.(2)$$

$$\therefore \frac{(2)}{(1)} \Rightarrow \tan\theta = \frac{v^2}{rg}$$

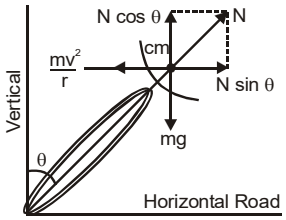
$$\Rightarrow \tan\left[\sin^{-1}\frac{h}{d}\right] = \frac{v^2}{rg}$$

TURNING OF A CYCLIST AROUND A CORNER ON THE ROAD

When a cyclist turns around a corner on the road, he needs a centripetal force $\frac{mv^2}{r}$. The forces acting on the cyclist are (a) weight (mg) and (b) the normal force (N). In order to generate the necessary centripetal force, the cyclist bends inwards by an angle θ w.r.t. vertical.

$N\cos\theta = mg$

and $N\sin\theta = \frac{mv^2}{r}$



So, $\tan\theta = \frac{v^2}{rg}$, i.e.,

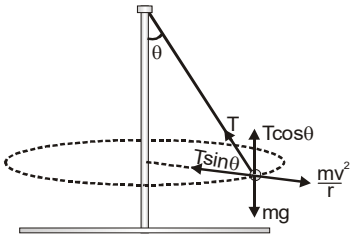
Angle θ should be : $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

CONICAL PENDULUM

(i) If a small bob of mass m tied to a string is whirled in a horizontal circle, the string will not remain horizontal but the string becomes inclined to the vertical and sweeps a cone while the body moves on a horizontal circle with uniform speed. This arrangement is called **conical pendulum.**

(ii) In case of conical pendulum, the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force, i.e.,

$T\cos\theta = mg$ and $T\sin\theta = \frac{mv^2}{r}$ or $\tan\theta = \frac{v^2}{rg}$



Also $T = m\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$

Hence $v = \sqrt{rg\tan\theta}$

i.e., $\omega = \sqrt{\frac{g\tan\theta}{r}}$

Hence, time period

$t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

(iii) Time period t is independent of the mass of the body and depends on L cos θ (= h)
i.e., distance between point of suspension and centre of circle.

(iv) If $\theta = 90°$, the pendulum becomes horizontal and it follows that $v = \infty$, $T = \infty$ and $t = 0$
It is practically impossible.

Topicwise Questions

RELATIVE MOTION

One Dimensional Relative Motion

1. Two trains, each 50 m long are travelling in opposite direction with velocity 10 m/s and 15 m/s. The time of crossing is

(*a*) 2s (*b*) 4s

(*c*) $2\sqrt{3}$ s (*d*) $4\sqrt{3}$ s

2. A 210 meter long train is moving due North at a of 25m/s. A small bird is flying due South a little above the train with speed 5m/s. The time taken by the bird to cross the train is

(*a*) 6 s (*b*) 7 s

(*c*) 9 s (*d*) 10 s

3. A stone is dropped from a building, and 2 seconds later another stone is dropped. (Both a are dropped from rest.) How far apart are the two stones by the time the first one has reached a speed of 30 m/s ?

(*a*) 80 m (*b*) 100 m

(*c*) 60 m (*d*) 40 m

Two Dimensional Relative Motion

4. A bird is flying towards south with a velocity 40km/hr and a train is moving with a velocity 40 km/hr towards east. What is the velocity of the bird w.r.t. an observer in train :–

(*a*) $40\sqrt{2}$ km/hr. North – East

(*b*) $40\sqrt{2}$ km/hr. South – East

(*c*) $40\sqrt{2}$ km/hr. South – West

(*d*) $40\sqrt{2}$ km/hr. North – West

5. A train is moving towards east and a car is along north, both with same speed. The observed direction of car to the passenger in the train is

(*a*) East-north direction (*b*) West-north direction

(*c*) South-east direction (*d*) None of these

Rain Man Problem

6. A man is walking on a road with a velocity 3 km/hr. Suddenly rain starts falling. The velocity of rain is 10 km/hr in vertically downward direction. the relative velocity of the rain with respect to man is :–

(*a*) $\sqrt{13}$ km/hr (*b*) $\sqrt{7}$ km/hr

(*c*) $\sqrt{109}$ km/hr (*d*) 13 km/hr

7. A boy is running on the plane road with velocity (v) with a long hollow tube in his hand. The water is falling vertically downwards with velocity (u). At what angle to the vertical, he must incline the tube so that the water drops enters in it without touching its side :–

(*a*) $\tan^{-1}\left(\frac{v}{u}\right)$ (*b*) $\sin^{-1}\left(\frac{v}{u}\right)$

(*c*) $\tan^{-1}\left(\frac{u}{v}\right)$ (*d*) $\cos^{-1}\left(\frac{v}{u}\right)$

8. A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/hr then he finds that rain drops are hitting his head vertically, then speed of rain drops with respect to moving man :–

(*a*) 20 km/hr. (*b*) $10\sqrt{3}$ km/hr.

(*c*) $\frac{10}{\sqrt{3}}$ km/hr. (*d*) 10 km/hr.

River Man Problem

9. A boat is sent across a river with a velocity of 8 km/hr. If the resultant velocity of boat is 10 km/hr, then velocity of the river is:

(*a*) 10 km/hr (*b*) 8 km/hr

(*c*) 6 km/hr (*d*) 4 km/hr

10. A boat is moving with velocity of $3\hat{i}+4\hat{j}$ in river and water is moving with a velocity of $-3\hat{i}-4\hat{j}$ with respect to ground. Relative velocity of boat with respect to water is:

(*a*) $-6\hat{i}-8\hat{j}$ (*b*) $6\hat{i}+8\hat{j}$

(*c*) $8\hat{i}$ (*d*) $6\hat{i}$

11. A river is flowing from W to E with a speed of 5 m/min. A man can swim in still water with a velocity 10 m/min. In which direction should the man swim so as to take the shortest possible path to go to the south.

(*a*) 30° with downstream (*b*) 60° with downstream

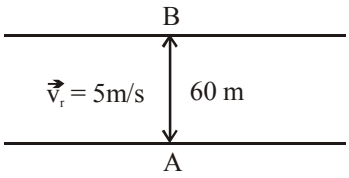
(*c*) 120° with downstream (*d*) South

12. A river is flowing from west to east at a speed of 5 meters per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction

(*a*) due north (*b*) 30° east of north

(*c*) 30° north of west (*d*) 60° east of north

13. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at a distance of 60 meter in 5 sec. His velocity in still water should be



(*a*) 12 m/s (*b*) 13 m/s

(*c*) 5 m/s (*d*) 10 m/s

Distance of Nearest Approach

14. A body is projected vertically up at t = 0 with a velocity of 98 m/s. Another body is projected from the same point with same velocity after 4 seconds. Both bodies will meet after :–

(*a*) 6 s (*b*) 8 s

(*c*) 10 s (*d*) 12 s

PROJECTILE

Two Dimensional Motion: General Study

15. The point from where a ball is projected is taken as the origin of the coordinate axes. The x and y components of its displacement are given by $x = 6t$ and $y = 8t - 5t^2$. What is the velocity of projection?]

(*a*) 6 m s^{-1} (*b*) 8m s^{-1}

(*c*) 10 m s^{-1} (*d*) 14 m s^{-1}

PROJECTILE MOTION: PROJECTILE HITTING GROUND AT SAME HORIZONTAL LEVEL

16. A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be

(*a*) 2R (*b*) R/2

(*c*) R (*d*) 4 R

17. If the initial velocity of a projectile be doubled, keeping the angle of projection same, the maximum height reached by it will

(*a*) Remain the same

(*b*) Be doubled

(*c*) Be quadrupled

(*d*) Be halved

18. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are

(*a*) Perpendicular to each other

(*b*) Parallel to each other

(*c*) Inclined to each other at an angle of 45°

(*d*) Antiparallel to each other

19. The range of a particle when launched at an angle of 15º with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45° to the horizontal

(*a*) 1.5 km (*b*) 3.0 km

(*c*) 6.0 km (*d*) 0.75 km

20. A stone is projected from the ground with velocity 25 m/s. Two seconds later, it just clears a wall 5 m high. The angle of projection of the stone is ($g = 10 m/sec^2$)

(*a*) 30º (*b*) 45º

(*c*) 50.2º (*d*) 60º

21. A projectile thrown with a speed ν at an angle θ has a range R on the surface of earth. For same ν and θ, its range on the surface of moon will be

(*a*) R/6 (*b*) 6R

(*c*) R/36 (*d*) 36 R

22. A particle reaches its highest point when it has covered exactly one half of its horizontal range. The corresponding point on the displacement time graph is characterised by

(*a*) Negative slope and zero curvature

(*b*) Zero slope and negative curvature

(*c*) Zero slope and positive curvature

(*d*) Positive slope and zero curvature

23. If the range of a gun which fires a shell with muzzle speed V is R, then the angle of elevation of the gun is

(*a*) $\cos^{-1}\left(\frac{V^2}{Rg}\right)$ (*b*) $\cos^{-1}\left(\frac{gR}{V^2}\right)$

(*c*) $\frac{1}{2}\left(\frac{V^2}{Rg}\right)$ (*d*) $\frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$

PROJECTILE THROWN FROM SOME HEIGHT ABOVE GROUND

24. A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following

(*a*) Straight path (*b*) Circular path

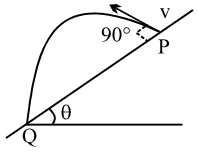
(*c*) Parabolic path (*d*) Hyperbolic path

25. An aeroplane moving horizontally with a speed of 720 km/h drops a food pocket, while flying at a height of 396.9 m. the time taken by a food pocket to reach the ground and its horizontal range is
(**Take** g = 9.8 m/sec^2)
(*a*) 3 sec and 2000 m (*b*) 5 sec and 500 m
(*c*) 8 sec and 1500 m (*d*) 9 sec and 1800 m

Projection on Inclined Plane, Two Dimensional

26. Find time of flight of projectile thrown horizontally with speed 10 ms^{-1} from a long inclined plane which makes an angle of $\theta = 45°$ with the horizontal.
(*a*) $\sqrt{2}$ sec (*b*) $2\sqrt{2}$ sec
(*c*) 2 sec (*d*) none

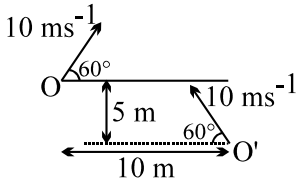
27. If time taken by the projectile to reach Q is T, then PQ =



(*a*) Tv sin θ (*b*) Tv cos θ
(*c*) Tv sec θ (*d*) Tv tan θ

Relative Motion Between Two Projectile

28. Two particles are projected simultaneously from two points O and O' such that 10 m is the horizontal and 5 m is the vertical distance between them as shown in the figure. They are projected at the same inclination 60° to the horizontal with the same velocity 10 ms^{-1}. The time after which their separation becomes minimum is



(*a*) 2.5 sec (*b*) 1 sec
(*c*) 5 sec (*d*) 10 sec

CIRCULAR MOTION

Kinematics of Circular Motion

29. If a particle moves in a circle describing equal angles in equal times, its velocity vector
(*a*) Remains constant
(*b*) Changes in magnituden
(*c*) Changes in direction
(*d*) Changes both in magnitude and direction

30. A particle moves with constant angular velocity in a circle. During the motion its
(*a*) Energy is conserved
(*b*) Momentum is conserved
(*c*) Energy and momentum both are conserved
(*d*) None of the above is conserved

31. The ratio of angular speeds of minute hand and hour hand of a watch is
(*a*) 1 : 12 (*b*) 6 : 1
(*c*) 12 : 1 (*d*) 1 : 6

32. The length of second's hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is
(*a*) Zero (*b*) $\frac{\pi}{30\sqrt{2}}$ cm / sec
(*c*) $\frac{\pi}{30}$ cm / sec (*d*) $\frac{\pi\sqrt{2}}{30}$ cm / sec

33. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$
(*a*) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (*b*) $-18\hat{i} - 13\hat{j} + 2\hat{k}$
(*c*) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (*d*) $6\hat{i} - 2\hat{j} + 8\hat{k}$

34. A wheel is at rest. Its angular velocity increases uniformly and becomes 80 radian per second after 5 second. The total angular displacement is:
(*a*) 800 rad (*b*) 400 rad
(*c*) 200 rad (*d*) 100 rad

35. A wheel is of diameter 1m. If it makes 30 revolutions/sec., then the linear speed of a point on its circumference will be.
(*a*) 30π m/s (*b*) π m/s
(*c*) 60π m/s (*d*) π/2 m/s

36. In uniform circular motion (angular momentum $L = m\vec{r} \times \vec{v}$)
(*a*) Both the angular velocity and the angular momentum vary
(*b*) The angular velocity varies but the angular momentum remains constant.
(*c*) Both the angular velocity and the angular momentum stay constant
(*d*) The angular momentum varies but the angular velocity remains constant.

Centripetal/Tangential/Net Acceleration

37. A body is moving in a circular path with a constant speed. It has
(*a*) A constant velocity
(*b*) A constant acceleration
(*c*) An acceleration of constant magnitude
(*d*) An acceleration which varies with time

38. Two bodies of mass 10 kg and 5 kg moving in concentric orbits of radii R and r such that their periods are the same. Then the ratio between their centripetal acceleration is

(*a*) R/r (*b*) r/R

(*c*) R^2 / r^2 (*d*) r^2 / R^2

39. A car travels north with a uniform velocity. It goes over a piece of mud which sticks to the tyre. The particles of the mud, as it leaves the ground are thrown

(*a*) Vertically upwards (*b*) Vertically inwards

(*c*) Towards north (*d*) Towards south

40. A stone is tied to one end of a string 50 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 10 revolutions in 20 s, what is the magnitude of acceleration of the stone

(*a*) 493 cm/s^2 (*b*) 720 cm/s^2

(*c*) 860 cm/s^2 (*d*) 990 cm/s^2

41. Two particles P and Q are located at distances r_P and r_Q respectively from the axis of a rotating disc such that $r_P > r_Q$:

(*a*) Both P and Q have the same acceleration

(*b*) Both P and Q do not have any acceleration

(*c*) P has greater acceleration than Q

(*d*) Q has greater acceleration than P

42. A string breaks if its tension exceeds 10 newtons. A stone of mass 250 gm tied to this string of length 10 cm is rotated in a horizontal circle. The maximum angular velocity of rotation can be.

(*a*) 20 rad/s (*b*) 40 rad/s

(*c*) 100 rad/s (*d*) 200 rad/s

43. A particle moving along a circular path due to a centripetal force having constant magnitude is an example of motion with:

(*a*) constant speed and velocity

(*b*) variable speed and velocity

(*c*) variable speed and constant velocity

(*d*) constant speed and variable velocity.

44. The formula for centripetal acceleration in a circular motion is.

(*a*) $\vec{\alpha} \times \vec{r}$ (*b*) $\vec{\omega} \times \vec{v}$

(*c*) $\vec{\alpha} \times \vec{v}$ (*d*) $\vec{\omega} \times \vec{r}$

45. A particle moves in a circular orbit under the action of a central attractive force inversely proportional to the distance 'r'. The speed of the particle is.

(*a*) Proportional to r^2

(*b*) Independent of r

(*c*) Proportional to r

(*d*) Proportional to 1/r

46. A particle of mass m is moving with constant velocity $\vec{v}$ on smooth horizontal surface. A constant force $\vec{F}$ starts acting on particle perpendicular to velocity v. Radius of curvature after force F start acting is :

(*a*) $\frac{mv^2}{F}$ (*b*) $\frac{mv^2}{F\cos\theta}$

(*c*) $\frac{mv^2}{F\sin\theta}$ (*d*) none of these

47. If the radii of circular paths of two particles of same masses are in the ratio of 1 : 2, then in order to have same centripetal force, their speeds should be in the ratio of :

(*a*) 1 : 4 (*b*) 4 : 1

(*c*) $1:\sqrt{2}$ (*d*) $\sqrt{2}$: 1

48. A particle is moving in a horizontal circle with constant speed. It has constant

(*a*) Velocity (*b*) Acceleration

(*c*) Kinetic energy (*d*) Displacement

49. What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and halve the angular velocity ω ?

(*a*) the centripetal acceleration remains unchanged

(*b*) the centripetal acceleration is halved

(*c*) the centripetal acceleration is doubled

(*d*) the centripetal acceleration is quadrupled

50. A particle is moving along a circular path. the angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega}, \vec{v}, \vec{\alpha}$ and $\vec{a}_c$. Which of the following relations is not correct?

(*a*) $\vec{\omega} \perp \vec{v}$ (*b*) $\vec{\omega} \perp \vec{\alpha}$

(*c*) $\vec{\omega} \perp \vec{a}_c$ (*d*) $\vec{v} \perp \vec{a}_c$

51. A particle is moving in circular path with constant tangential acceleration, time t after the begining of motion the direction of net acceleration is at 45° to radius vector at the instant. The angular acceleration of the particle at time 't' is proportional to:

(*a*) $\frac{1}{t}$ (*b*) $\frac{1}{t^2}$

(*c*) $\frac{3}{t}$ (*d*) t^0

52. A car is travelling with linear velocity v on a circular road of radius r. If it is increasing it speed at the rate of 'a' metre/sec^2, then the resultant acceleration will be-

(*a*) $\sqrt{\left(\frac{v^2}{r^2} - a^2\right)}$ (*b*) $\sqrt{\left(\frac{v^4}{r^2} + a^2\right)}$

(*c*) $\sqrt{\left(\frac{v^4}{r^2} - a^2\right)}$ (*d*) $\sqrt{\left(\frac{v^2}{r^2} + a^2\right)}$

Mechanics of Circular Motion

53. If the overbridge is concave instead of being convex, the thrust on the road at the lowest position will be

(*a*) $mg+\frac{mv^2}{r}$ (*b*) $mg-\frac{mv^2}{r}$

(*c*) $\frac{m^2v^2g}{r}$ (*d*) $\frac{v^2g}{r}$

54. A motor cyclist moving with a velocity of 72 km/hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 meters. The acceleration due to gravity is 10 m/sec^2. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than

(*a*) $\theta=\tan^{-1}6$ (*b*) $\theta=\tan^{-1}2$

(*c*) $\theta=\tan^{-1}25.92$ (*d*) $\theta=\tan^{-1}4$

55. The force required to keep a body in uniform circular motion is

(*a*) Centripetal force (*b*) Centrifugal force

(*c*) Resistance (*d*) None of the above

56. The magnitude of the centripetal force acting on a body of mass m executing uniform motion in a circle of radius r with speed v is

(*a*) mvr (*b*) mv^2/r

(*c*) v/r^2m (*d*) v/rm

57. A ball of mass 0.25 kg attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved

(*a*) 14 m/s (*b*) 3 m/s

(*c*) 3.92 m/s (*d*) 5 m/s

58. If a particle of mass m is moving in a horizontal circle of radius r with a centripetal force ($-k/r^2$), the total energy is

(*a*) $-\frac{k}{2r}$ (*b*) $-\frac{k}{r}$

(*c*) $-\frac{2k}{r}$ (*d*) $-\frac{4k}{r}$

59. A car when passes through a convex bridge exerts a force on it which is equal to

(*a*) $Mg+\frac{Mv^2}{r}$ (*b*) $\frac{Mv^2}{r}$

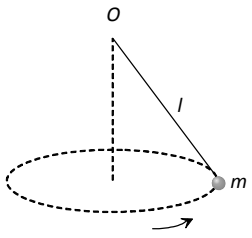
(*c*) Mg (*d*) $Mg-\frac{Mv^2}{r}$

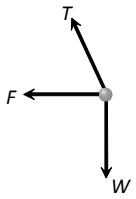
60. An unbanked curve has a radius of 60 m. The maximum speed at which a car can make a turn if the coefficient of static friction is 0.75, is

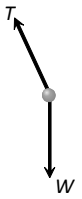
(*a*) 2.1 m/s (*b*) 14 m/s

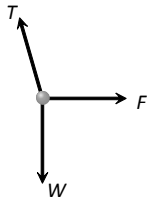
(*c*) 21 m/s (*d*) 7 m/s

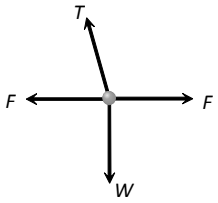
61. A point mass m is suspended from a light thread of length *l*, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are



(*a*)


(*b*)


(*c*)


(*d*)


62. A stone of mass 0.5 kg tied with a string of length 1 metre is moving in a circular path with a speed of 4 m/sec. The tension acting on the string in newton is -

(*a*) 2 (*b*) 8

(*c*) 0.2 (*d*) 0.8

63. A particle of mass m is executing a uniform motion along a circular path of radius r. If the magnitude of its linear momentum is p, the radial force acting on the particle will be.

(*a*) pmr (*b*) rm/p

(*c*) mp^2/r (*d*) p^2/mr

64. When the road is dry and the coefficient of friction is μ, the maximum speed of a car in a circular path is 10 m/s, if the road becomes wet and $\mu' = \mu/2$. What is the maximum speed permitted ?

(*a*) 5 m/s (*b*) 10 m/s

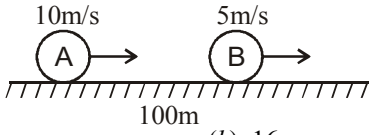
(*c*) $10\sqrt{2}$ m/s (*d*) $5\sqrt{2}$ m/s

Learning Plus

RELATIVE MOTION

1. Two trains each of length 50 m are approaching each other on parallel rails. Their velocities are 10 m/sec and 15 m/sec. They will cross each other in -
(*a*) 2 sec (*b*) 4 sec
(*c*) 10 sec (*d*) 6 sec

2. An object A is moving with 10 m/s and B is moving with 5 m/s in the same direction of positive x-axis. A is 100 m behind B as shown. Find time taken by A to Meet B



(*a*) 18 sec. (*b*) 16 sec.
(*c*) 20 sec. (*d*) 17 sec.

3. A thief is running away on a straight road with a speed of 9 m s^{-1}. A police man chases him on a jeep moving at a speed of 10 m s^{-1}. If the instantaneous separation of the jeep from the motorcycle is 100m, how long will it take for the police man to catch the thief?
(*a*) 1s (*b*) 19s
(*c*) 90s (*d*) 100s

4. A bus is moving with a velocity 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100s. If, the bus is at a distance of 1 km from the scooterist, with what velocity should the scooterist chase the bus?
(*a*) 50 ms^{-1} (*b*) 40 ms^{-1}
(*c*) 30 ms^{-1} (*d*) 20 ms^{-1}

5. A body is thrown up in a lift with a velocity u relative to the lift and the time of flight is found to be 't'. The acceleration with which the lift is moving up is:
(*a*) $\frac{u - gt}{t}$ (*b*) $\frac{2u - gt}{t}$
(*c*) $\frac{u + gt}{t}$ (*d*) $\frac{2u + gt}{t}$

6. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car (as, seen by thief), according to thief in the car?
(a) 105 m/s (*b*) 100 m/s
(*c*) 110 m/s (*d*) 90 m/s

7. Two cars get closer by 9 m every second while travelling in the opposite directions. They get closer by 1 m every second while travelling in the same directions. What are the speeds of the cars?
(*a*) 5 ms^{-1} and 4 ms^{-1} (*b*) 4 ms^{-1} and 3 ms^{-1}
(*c*) 6 ms^{-1} and 3 ms^{-1} (*d*) 6 ms^{-1} and 5 ms^{-1}

8. A jet airplane travelling from east to west at a speed of 500 km h^{-1} ejects out gases of combustion at a speed of 1500 km h^{-1} with respect to the jet plane. What is the velocity of the gases with respect to an observer on the ground ?
(*a*) 1000 km h^{-1} in the direction west to east
(*b*) 1000 km h^{-1} in the direction east to west
(*c*) 2000 km h^{-1} in the direction west to east
(*d*) 2000 km h^{-1} in the direction east to west

9. A helicopter is flying south with a speed of 50 kmh^{-1}. A train is moving with the same speed towards east. The relative velocity of the helicopter as seen by the passengers in the train will be towards.
(*a*) north east (*b*) south east
(*c*) north west (*d*) south west

10. Two particles are moving with velocities v_1 and v_2. Their relative velocity is the maximum, when the angle between their velocities are:
(*a*) zero (*b*) $\pi/4$
(*c*) $\pi/2$ (*d*) π

11. Two billiard balls are rolling on a flat table. One has velocity components v_x = 1m/s, $v_y = \sqrt{3}$ m/s and the other has components v_x = 2m/s and v_y = 2 m/s. If both the balls start moving from the same point, the angle between their path is-
(*a*) 60° (*b*) 45°
(*c*) 22.5° (*d*) 15°

12. A man walks in rain with a velocity of 5 kmh^{-1}. The rain drops strike at him at an angle of 45° with the horizontal. Velocity of rain if it is falling vertically downward -
(*a*) 5 kmh^{-1} (*b*) 4 kmh^{-1}
(*c*) 3 kmh^{-1} (*d*) 1 kmh^{-1}

13. To man running at a speed of 25 m/sec,the rain drops appear to be falling at an angle of 45° from the vertical. If the rain drops are actually falling vertically downwards, then velocity in m/sec is
(*a*) 25 (*b*) $25\sqrt{3}$
(*c*) $25\sqrt{2}$ (*d*) 4

14. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is -

(*a*) 1 (*b*) 3

(*c*) 4 (*d*) $\sqrt{41}$

15. A boat can go along a lake and return in time T_0 at a speed V. On a rough day there is uniform current at speed u to help the onward journey and impede the return journey. If the time taken to go along and return on the rough day be T, then $T/T_0 =$

(*a*) $1-\frac{u^2}{V^2}$ (*b*) $\frac{1}{1-\frac{u^2}{V^2}}$

(*c*) $1+\frac{u^2}{V^2}$ (*d*) $\frac{1}{1+\frac{u^2}{V^2}}$

16. Two boats were going down stream with different velocities. When one overtook the other a plastic ball was dropped from one of the boats into water. Some time later both boats turned back simultaneously & went at the same speeds as before (relative to the water) towards the spot where the ball had been dropped. Which boat will reach the ball first?

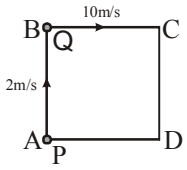
(*a*) the boat which has greater velocity (relative to water)

(*b*) the boat which has lesser velocity (relative to water)

(*c*) both will reach the ball simultaneously

(*d*) cannot be decided unless we know the actual values of the velocities and the time after which they turned around

17. Two men P & Q are standing at corners A & B of square ABCD of side 8 m. They start moving along the track with constant speed 2 m/s and 10 m/s respectively. Find the time when they will meet for the first time.



(*a*) 2 sec (*b*) 3 sec

(*c*) 1 sec (*d*) 6 sec

PROJECTILE

18. At an instant t , the co-ordinates of a particle are $x = at^2$, $y = bt^2$ and $z = 0$, then its speed at the instant t will be

(*a*) $t\sqrt{a^2+b^2}$ (*b*) $2t\sqrt{a^2+b^2}$

(*c*) $\sqrt{a^2+b^2}$ (*d*) $2t^2\sqrt{a^2+b^2}$

19. A bullet is fired horizontally from a rifle at a distant target. Ignoring the effect of air resistance, which of the following is correct? Horizontal Acceleration and Vertical Acceleration are given by:

(*a*) $10\ ms^{-2}$, $10\ ms^{-2}$ (*b*) $10\ ms^{-2}$, $0\ ms^{-2}$

(*c*) $0\ ms^{-2}$, $10\ ms^{-2}$ (*d*) $0\ ms^{-2}$, $0\ ms^{-2}$

20. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ rad should strike a given target in the same horizontal plane. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation should the gun be fired to hit the target.

(*a*) $\frac{5\pi}{36}$ rad (*b*) $\frac{11\pi}{36}$ rad

(*c*) $\frac{7\pi}{36}$ rad (*d*) $\frac{13\pi}{36}$ rad.

21. A projectile is thrown with a speed v at an angle θ with the vertical. Its average velocity between the instants it crosses half the maximum height is

(*a*) v sin θ, horizontal and in the plane of projection

(*b*) v cos θ, horizontal and in the plane of projection

(*c*) 2v sin θ, horizontal and perpendicular to the plane of projection

(*d*) 2v cos θ, vertical and in the plane of projection.

22. During projectile motion, acceleration of a particle at the highest point of its trajectory is

(*a*) g

(*b*) zero

(*c*) less than g

(*d*) dependent upon projection velocity

23. The maximum range of a projectile is 22 m. When it is thrown at an angle of 15° with the horizontal, its range will be-

(*a*) 22 m (*b*) 6m

(*c*) 15 m (*d*) 11 m

24. The equation of projectile is $y = 16x - \frac{5x^2}{4}$. The horizontal range is-

(*a*) 16 m (*b*) 8m

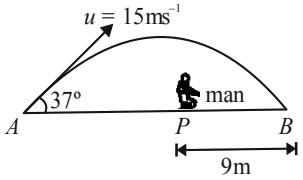
(*c*) 3.2 m (*d*) 12.8 m

25. If four balls A, B, C, D are projected with same speed at angles of 15°, 30°, 45° and 60° with the horizontal respectively, the two balls which will fall at the same place will be-

(*a*) *a* and *b* (*b*) *a* and *d*

(*c*) *b* and *d* (*d*) *a* and *c*

26. A ball is hit by a batsman at an angle of 37º as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.



(*a*) 3 ms^{-1} (*b*) 5 ms^{-1}
(*c*) 9 ms^{-1} (*c*) 12 ms^{-1}

27. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?
(*a*) The one with the farthest range.
(*b*) The one which reaches maximum height
(*c*) The one with the greatest initial velocity
(*d*) The one leaving the bat at 45° with respect to the ground.

28. A particle is projected from the ground with velocity u at angle θ with horizontal. The horizontal range, maximum height and time of flight are R, H and T respectively. They are given by,

$$R = \frac{u^2 \sin 2\theta}{g},\ H = \frac{u^2 \sin^2 \theta}{2g} \text{ and } T = \frac{2u \sin \theta}{g}$$

Now keeping u as fixed, θ is varied from 30° to 60°. Then,

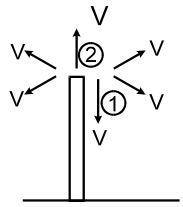
(*a*) R will first increase then decrease, H will increase and T will decrease
(*b*) R will first increase then decrease while H and T both will increase
(*c*) R will decrease while H and T will increase
(*d*) R will increase while H and T will increase

29. A point mass is projected, making an acute angle with the horizontal. If angle between velocity $\vec{v}$ and acceleration $\vec{g}$ is θ at any time t during the motion, then θ is given by

(*a*) $0^\circ < \theta < 90^\circ$ (*b*) $\theta = 90^\circ$
(*c*) $\theta < 90^\circ$ (*d*) $0^\circ < \theta < 180^\circ$

30. One stone is projected horizontally from a 20 m high cliff with an initial speed of 10 ms^{-1}. A second stone is simultaneously dropped from that cliff. Which of the following is true?
(*a*) Both strike the ground with the same speed.
(*b*) The ball with initial speed 10 ms^{-1} reaches the ground first.
(*c*) Both the balls hit the ground at the different time.
(*d*) Both strike the ground with different speed

31. Particles are projected from the top of a tower with same speed at different angles as shown. Which of the following are True ?



(*a*) All the particles would strike the ground with (same) speed.
(*b*) All the particles would strike the ground with (same) speed simultaneously.
(*c*) Particle 1 will be the first to strike the ground.
(*d*) (*a*) & (*c*) both

32. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at angle 45º. What is the vertical component of velocity when it strikes the ground?

(*a*) $18\sqrt{2}$ ms^{-1} (*b*) 18 ms^{-1}
(*c*) $9\sqrt{2}$ ms^{-1} (*d*) 9 ms^{-1}

33. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y varies with time t in second as:

$x = 10\sqrt{3}\,t$ and $y = 10t - t^2$

The maximum height attained by the ball is
(*a*) 100 m (*b*) 75 m
(*c*) 50 m (*d*) 25 m

34. The angle of projection of a body is 15º . The other angle for which the range is the same as the first one is equal to-
(*a*) 30º (*b*) 45º
(*c*) 60º (*d*) 75º

35. An aeroplane is flying at a height of 1960 m in horizontal direction with a velocity of 360 km/hr. When it is vertically above the point. A on the ground, it drops a bomb. The bomb strikes a point B on the ground, then the time taken by the bomb to reach the ground is-

(*a*) $20\sqrt{2}$ sec (*b*) 20 sec
(*c*) $10\sqrt{2}$ sec (*d*) 10 sec

36. A ball is projected from top of a tower with a velocity of 5 m/s at an angle of 53º to horizontal. Its speed when it is at a height of 0.45 m from the point of projection is
(*a*) 2 m/s
(*b*) 3 m/s
(*c*) 4 m/s
(*d*) data insufficient

37. An aeroplane flying at a constant velocity releases a bomb. As the bomb drops down from the aeroplane.
(*a*) it will always be vertically below the aeroplane
(*b*) it will always be vertically below the aeroplane only if the aeroplane is flying horizontally
(*c*) it will always be vertically below the aeroplane only if the aeroplane is flying at an angle of 45° to the horizontal.
(*d*) it will gradually fall behind the aeroplane if the aeroplane is flying horizontally

38. A particle is projected at angle 37° with the incline plane in upward direction with speed 10 m/s. The angle of incline plane is given 53°. Then the maximum height above the incline plane attained by the particle will be
(*a*) 3m (*b*) 4m
(*c*) 5m (*d*) zero

39. The velocity at the maximum height of a projectile is half of its initial velocity u. Its range on the horizontal plane is:

(*a*) $\frac{2u^2}{3g}$ (*b*) $\frac{\sqrt{3}\,u^2}{2g}$

(*c*) $\frac{u^2}{3g}$ (*d*) $\frac{u^2}{2g}$

CIRCULAR MOTION

40. Two racing cars of masses m_1 and m_2 are moving in circles of radii r and 2r respectively ; their angular speeds are equal. The ratio of the time taken by cars to complete one revolution is :
(*a*) $m_1 : m_2$ (*b*) 1 : 2
(*c*) 1 : 1 (*d*) $m_1 : 2m_2$

41. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be :
(*a*) 2π & 0 mm/s (*b*) $2\sqrt{2}\ \pi$ & 4.44 mm/s
(*c*) $2\sqrt{2}\ \pi$ & 2π mm/s (*d*) 2π & $2\sqrt{2}\ \pi$ mm/s

42. A wheel is subjected to uniform angular a cceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1 ; in the next 2 sec, it rotates through an additional angle θ_2. The ratio of θ_2 / θ_1 is-
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 5

43. The ratio of angular speed of hours hand and seconds hand of a clock is-
(*a*) 1 : 1 (*b*) 1 : 60
(*c*) 1 : 720 (*d*) 3600 : 1

44. The linear and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively it will be at a distance from the axis of rotation -
(*a*) 50m (*b*) 1/2 m
(*c*) 1m (*d*) 2m

45. The earth, radius 6400 km, makes one revolution about its own axis in 24 hours. The centripetal acceleration of a point on its equator is nearly -

(*a*) $340\frac{cm}{sec^2}$ (*b*) $3.4\frac{cm}{sec^2}$

(*c*) $34\frac{cm}{sec^2}$ (*d*) $0.34\frac{cm}{sec^2}$

46. A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of particle in m/s^2 is -
(*a*) π^2 (*b*) $8\pi^2$
(*c*) $4\pi^2$ (*d*) $2\pi^2$

47. If angular velocity of a disc depends an angle rotated θ as $\omega = \theta^2 + 2\theta$, then its angular acceleration α at $\theta = 1$ rad is
(*a*) 8 rad/sec^2 (*b*) 10 rad/sec^2
(*c*) 12 rad/sec^2 (*d*) None

48. A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal smooth surface. The maximum tension the string can withstand is 16 N. The maximum speed of revolution of the stone without breaking it, will be :
(*a*) 20 ms^{-1} (*b*) 16 ms^{-1}
(*c*) 14 ms^{-1} (*d*) 12 ms^{-1}

49. On horizontal smooth surface a mass of 2 kg is whirled in a horizontal circle by means of a string at an initial angular speed of 5 revolutions per minute. Keeping the radius constant the tension in the string is doubled. The new angular speed is nearly :
(*a*) 14 rpm (*b*) 10 rpm
(*c*) 2.25 rpm (*d*) 7 rpm

50. A coin placed on a rotating turntable just slips if it is placed at a distance of 16 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of
(*a*) 1 cm (*b*) 2 cm
(*c*) 4 cm (*d*) 8 cm

51. A stone is projected with speed u and angle of projection is θ. Find radius of curvature at t = 0.

(*a*) $\frac{u^2\cos^2\theta}{g}$ (*b*) $\frac{u^2}{g\sin\theta}$

(*c*) $\frac{u^2}{g\cos\theta}$ (*d*) $\frac{u^2\sin^2\theta}{g}$

52. In a circus, stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be :

(*a*) $\sqrt{2gR}$ (*b*) $2gR$

(*c*) $\sqrt{3gR}$ (*d*) $\sqrt{gR}$

53. A stone tied to a string is rotated in a vertical plane. If mass of the stone is m, the length of the string is r and the linear speed of the stone is v when the stone is at its lowest point, then the tension in the string at the lowest point will be :

(*a*) $\frac{mv^2}{r}+mg$ (*b*) $\frac{mv^2}{r}-mg$

(*c*) $\frac{mv^2}{r}$ (*d*) mg

54. A curved section of a road is banked for a speed v. If there is no friction between road and tyres of the car, then :

(*a*) car is more likely to slip at speeds higher than v than speeds lower than v

(*b*) car can remain in static equilibrium on the curved section

(*c*) car will not slip when moving with speed v

(*d*) none of the above

55. A man is standing on a rough ($\mu = 0.5$) horizontal disc rotating with constant angular velocity of 5 rad/sec. At what distance from centre should he stand so that he does not slip on the disc?

(*a*) $R \le 0.2\,m$ (*b*) $R > 0.2\,m$

(*c*) $R > 0.5\,m$ (*d*) $R > 0.3\,m$

56. A road is banked at an angle of 30° to the horizontal for negotiating a curve of radius $10\sqrt{3}$ m. At what velocity will a car experience no friction while negotiating the curve?

(*a*) 54 km/hr (*b*) 72 km/hr

(*c*) 36 km/hr (*d*) 18 km/hr

57. A car moving on a horizontal road may be thrown out of the road in taking a turn :

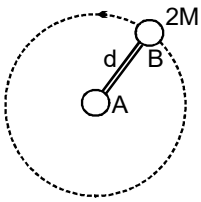
(*a*) By the gravitational force

(*b*) Due to lack of sufficient centripetal force

(*c*) Due to friction between road and the tyre

(*d*) Due to reaction of earth

58. The dumbell is placed on a frictionless horizontal table. Sphere A is attached to a frictionless pivot so that B can be made to rotate about A with constant angular velocity. If B makes one revolution in period P, the tension in the rod is



(*a*) $\frac{4\pi^2 Md}{P^2}$ (*b*) $\frac{8\pi^2 Md}{P^2}$

(*c*) $\frac{4\pi^2 Md}{P}$ (*d*) $\frac{2Md}{P}$

59. A string can bear a maximum tension of 100 Newton without breaking. A body of mass 1 kg is attached to one end of 1 m length of thin string and it is revolved in a horizontal plane. The maximum linear velocity which can be imparted to the body without breaking the string, will be -

(*a*) 10 m/s (*b*) 1 m/s

(*c*) 100 m/s (*d*) 1000 m/s

60. A cyclist is moving on a circular track of radius 80 m with a velocity of 72 km/hr. He has to lean from the vertical approximately through an angle

(*a*) $\tan^{-1}(1/4)$ (*b*) $\tan^{-1}(1)$

(*c*) $\tan^{-1}(1/2)$ (*d*) $\tan^{-1}(2)$

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/ NUMERICAL RELATIVE MOTION

1. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself

(*a*) The ball will always return to him

(*b*) The ball will never return to him

(*c*) The ball will return to him if the cart moves with constant velocity

(*d*) The ball will fall behind him if the cart moves with some positive acceleration

2. A train is running with uniform velocity in east direction. A car is running on a road parallel to the track with uniform speed. After some time, the road becomes perpendicular to railway track. The car driver notices that initial speed of train with respect to itself was 7 m/s and later on it became 13 m/s. What can be the true speed of the driver ?

(*a*) 12 m/s (*b*) 5 m/s

(*c*) 13 m/s (*d*) 7 m/s

3. State which of the following statement(s) is/are false.
   (*a*) If two particles are neither approaching nor separating from each other, then their relative velocity is zero.
   (*b*) If relative velocity of particle B with respect to A is $\vec{v}_1$, relative velocity of particle C with respect to B is $\vec{v}_2$ and particle A moves with velocity $\vec{v}_0$ with respect to ground, then the velocity of C with respect to ground cannot be zero. (assuming $\vec{v}_1$, $\vec{v}_2$ & $\vec{v}_0$ to be non zero)
   (*c*) Four dogs are running along a line in the same direction, such that each is running relative to the dog in front of him with equal speed. Then the rate of separation between the third and the first dog is same as that of the fourth & the second dog. (where first, second, third and fourth are taken in order)
   (*d*) At some instant of time at a place, two particles are observed and it is found that their relative velocity is zero. Then they will remain stationary with respect to each other.

4. A block is thrown with a velocity of 2 ms^{-1} (relative to ground) on a belt, which is moving with velocity 4 ms^{-1} in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 sec of the throwing then choose the correct statements (s)
   (*a*) Displacement with respect to ground is zero after 2.66s and magnitude of displacement with respect to ground is 12 m after 4 sec.
   (*b*) Magnitude of displacement with respect to ground in 4 sec is 4 m.
   (*c*) Magnitude of displacement with respect to belt in 4 sec is 12 m.
   (*d*) Displacement with respect to ground is zero in 8/3 sec.

5. A river is flowing with a velocity of 2 m/s. A boat is moving downstream along the river. Velocity of the boat in still water is 3 m/s. A person standing on the boat throws a ball (w.r.t. himself) in a plane -perpendicular to the direction of motion of the boat with 10 m/s at 60° with the horizontal. When the ball reaches highest point of its path.
   (*a*) The speed of ball w.r.t. man standing on boat is 5 m/s
   (*b*) The speed of ball w.r.t. river is 3 m/s
   (*c*) The speed of ball w.r.t. river is 0 m/s
   (*d*) The speed of ball w.r.t. ground is $5\sqrt{2}$ m/s

6. Two particles are projected from the same point with the same speed in the same vertical plane at different angles with the horizontal. A frame of reference is fixed to one particle. The position vector of the other particle as observed from the frame is $\vec{r}$. Which of the following statements is/are **incorrect**?
   (*a*) $\vec{r}$ is a constant vector
   (*b*) $\vec{r}$ changes in magnitude and direction with time
   (*c*) the magnitude of $\vec{r}$ increases linearly with time, its direction does not change
   (*d*) the direction of $\vec{r}$ changes with time, its magnitude may or may not change depending on the angles of projection.

7. Consider a shell that has a muzzle velocity of 45 ms^{-1} fired from the tail gun of an airplane moving horizontally with a velocity of 215 ms^{-1}. The tail gun can be directed at any angle with the vertical in the plane of motion of the airplane. The shell is fired when the plane is above point A on ground, and the plane is above point B on ground when the shell hits the ground. (Assume for simplicity that the Earth is flat)
   (*a*) Shell may hit the ground at point A.
   (*b*) Shell may hit the ground at point B.
   (*c*) Shell may hit a point on earth which is behind point A.
   (*d*) Shell may hit a point on earth which is ahead of point B.

PROJECTILE

8. A projectile is projected at an angle α (> 45°) with an initial velocity u. The time t at which its horizontal component will equal the vertical component in magnitude:
   (*a*) $t = \frac{u}{g}(\cos\alpha - \sin\alpha)$
   (*b*) $t = \frac{u}{g}(\cos\alpha + \sin\alpha)$
   (*c*) $t = \frac{u}{g}(\sin\alpha - \cos\alpha)$
   (*d*) $t = \frac{u}{g}(\sin^2\alpha - \cos^2\alpha)$

9. At what angle should a body be projected with a velocity 24 ms^{-1} just to pass over the obstacle 14 m high at a distance of 24 m. [Take g = 10 ms^{-2}]
   (*a*) $\tan\theta = 19/5$ (*b*) $\tan\theta = 1$
   (*c*) $\tan\theta = 3$ (*d*) $\tan\theta = 2$

10. Choose the correct alternative (s)

(*a*) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone is 2h.

(*b*) The angle of projection for a projectile motion whose range R is n times the maximum height is $\tan^{-1}(4/n)$

(*c*) The time of flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R\tan\theta$ where θ is the angle of projection.

(*d*) A ball is thrown vertically up. Another ball is thrown at an angle θ with the vertical. Both of them remain in air for the same period of time. Then the ratio of heights attained by the two ball 1 : 1.

11. A ball is rolled off along the edge of a horizontal table with velocity 4 m/s. It hits the ground after time 0.4s. Which of the following are correct?

(*a*) The height of the table is 0.8 m

(*b*) It hits the ground at an angle of 60° with the vertical

(*c*) It covers a horizontal distance 1.6 m from the table

(*d*) It hits the ground with vertical velocity 4 m/s

CIRCULAR MOTION

12. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen (in the plane of motion) from an inertial frame of reference.

(*a*) This is not possible.

(*b*) There are other forces on the particle.

(*c*) The resultant of the other forces is $\frac{mv^2}{r}$ towards the centre.

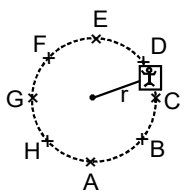
(*d*) The resultant of the other forces varies in magnitude as well as in direction.

13. Which of the following quantities may remain constant during the motion of an object along a curved path:

(*a*) speed (*b*) velocity

(*c*) acceleration (*d*) magnitude of acceleration

14. A machine, in an amusement park, consists of a cage at the end of one arm, hinged at O. The cage revolves along a vertical circle of radius r (ABCDEFGH) about its hinge O, at constant linear speed $v = \sqrt{gr}$. The cage is so attached that the man of weight 'w' standing on a weighing machine, inside the cage, is always vertical. Then which of the following is correct



(*a*) the reading of his weight on the machine is the same at all positions

(*b*) the weight reading at A is greater than the weight reading at E by 2 w.

(*c*) the weight reading at G = w

(*d*) the ratio of the weight reading at E to that at A=0

(*e*) the ratio of the weight reading at A to that at C=2

15. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ to ϕ. For an angular displacement θ, $[|\theta| < \phi]$ the tension in the string and velocity of the bob are T and v respectively. The following relations hold good under the above conditions :

(*a*) $T\cos\theta = Mg$

(*b*) $T - Mg\cos\theta = \frac{Mv^2}{L}$

(*c*) Tangential acc. = $g\sin\theta$

(*d*) $T = Mg\cos\theta$

16. A car of mass m attempts to go on the circular road of radius r, which banked for a speed of 36 km/hr. The friction coefficient between the tyre and the road is negligible.

(*a*) The car cannot make a turn without skidding.

(*b*) If the car turns at a speed less than 36 km/hr, it will slip down

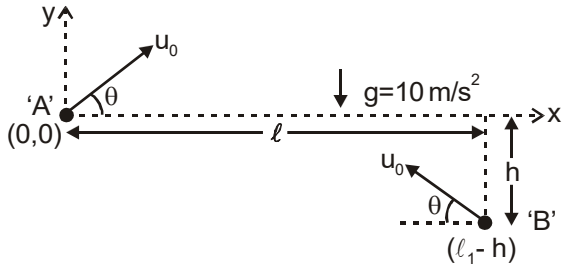
(*c*) If the car turns at the constant speed of 36 km/hr, the force by the road on the car is equal to $\frac{mv^2}{r}$

(*d*) If the car turns at the correct speed of 36 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$

RELATIVE MOTION

Comprehension – 1 (Q.17 to Q.19)

Two particles 'A' and 'B' are projected in the vertical plane with same initial speed u_0 from part (0, 0) and (ℓ , – h) towards each other as shown in figure at t = 0.



17. The path of particle 'A' with respect to particle 'B' will be
(*a*) parabola
(*b*) straight line parallel to x-axis.
(*c*) straight line parallel to y-axis
(*d*) none of these.

18. Minimum distance between particle A and B during motion will be :
(*a*) ℓ (*b*) h
(*c*) $\sqrt{\ell^2 + b^2}$ (*d*) $\ell + h$

19. The time when separation between A and B is minimum is:

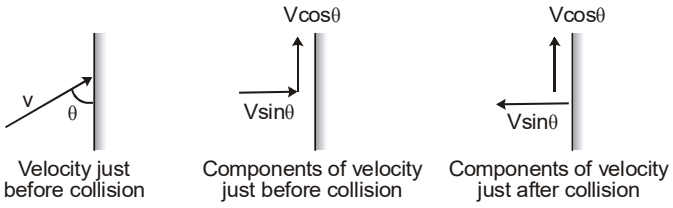
(*a*) $\frac{x}{u_0 \cos\theta}$ (*b*) $\sqrt{\frac{2h}{g}}$

(*c*) $\frac{\ell}{2u_0 \cos\theta}$ (*d*) $\frac{2\ell}{u_0 \cos\theta}$

PROJECTILE

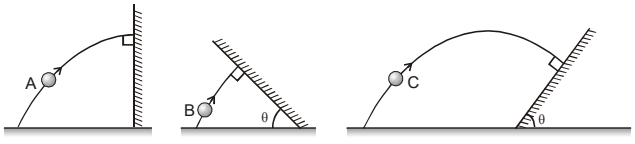
Comprehension-2 (Q. No 20 to 22)

We know how by neglecting the air resistance, the problems of projectile motion can be easily solved and analysed. Now we consider the case of the collision of a ball with a wall. In this case the problem of collision can be simplified by considering the case of elastic collision only. When a ball collides with a wall we can divide its velocity into two components, one perpendicular to the wall and other parallel to the wall. If the collision is elastic then the perpendicular component of velocity of the ball gets reversed with the same magnitude.



The other parallel component of velocity will remain constant if given wall is smooth.

Now let us take a problem. Three balls 'A' and 'B' & 'C' are projected from ground with same speed at same angle with the horizontal. The balls A,B and C collide with the wall during their flight in air and all three collide perpendicularly with the wall as shown in figure.



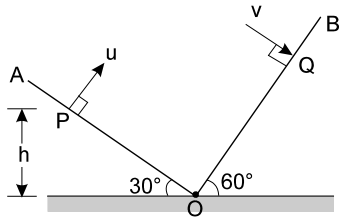
20. Which of the following relation about the maximum height H of the three balls from the ground during their motion in air is correct :
(*a*) $H_A = H_C > H_B$ (*b*) $H_A > H_B = H_C$
(*c*) $H_A > H_C > H_B$ (*d*) $H_A = H_B = H_C$

21. If the time taken by the ball A to fall back on ground is 4 seconds and that by ball B is 2 seconds. Then the time taken by the ball C to reach the inclined plane after projection will be :
(*a*) 6 sec. (*b*) 4 sec.
(*c*) 3 sec. (*d*) 5 sec.

22. The maximum height attained by ball 'A' from the ground is:
(*a*) 10m
(*b*) 15m
(*c*) 20m
(*d*) Insufficient information

Comprehension-3 (Q. No 23 to 26)

Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}\ \frac{m}{s}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q
(Take g = 10 m/s^2). Then



23. The time of flight is from P to Q is :-
(*a*) 5 Sec.
(*b*) 2 sec.
(*c*) 1 sec.
(*d*) None of these

24. The speed with which the particle strikes the plane OB is:-
(*a*) 10 m/s (*b*) 20 m/s
(*c*) 30 m/s (*d*) 40 m/s

25. The height h of point P from the ground is :-
(*a*) $10\sqrt{3}$ m (*b*) 10m
(*c*) 5m (*d*) 20m

26. The distance PQ is :-
(*a*) 20m (*b*) $10\sqrt{3}$ m
(*c*) 10m (*d*) 5m

CIRCULAR MOTION

Comprehension – 4 (Q. No. 27 to 29)

A particle undergoes uniform circular motion. The velocity and angular velocity of the particle at an instant of time is $\vec{v} = 3\hat{i} + 4\hat{j}$ m/s and $\vec{\omega} = x\hat{i} + 6\hat{j}$ rad/sec.

27. The value of x in rad/s is

(*a*) 8 (*b*) -8

(*c*) 6 (*d*) can't be calculated

28. The radius of circle in metres is

(*a*) 1/2 m (*b*) 1m

(*c*) 2m (*d*) can't be calculated

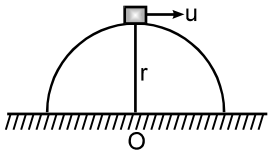
29. The acceleration of particle at the given instant is

(*a*) $-50\hat{k}$ (*b*) $-42\hat{k}$

(*c*) $2\hat{i} + 3\hat{j}$ (*d*) can't be calculated

Comprehension – 5 (Q. No. 30 to 33)

A small block of mass m is projected horizontally from the top of the smooth and fixed hemisphere of radius r with speed u as shown. For values of $u \geq u_0$, ($u_0 = \sqrt{gr}$) it does not slide on the hemisphere. [i.e. leaves the surface at the top itself]



30. For $u = 2u_0$, it lands at point P on ground. Find OP.

(*a*) $\sqrt{2}$ r (*b*) 2r

(*c*) 4r (*d*) $2\sqrt{2}$ r

31. For $u = u_0/3$, find the height from the ground at which it leaves the hemisphere.

(*a*) $\frac{19r}{9}$ (*b*) $\frac{19r}{27}$

(*c*) $\frac{10r}{9}$ (*d*) $\frac{10r}{27}$

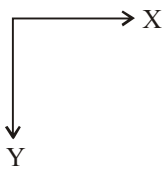
32. Find its net acceleration at the instant it leaves the hemisphere.

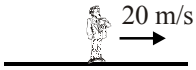
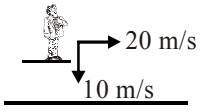
(*a*) g/4 (*b*) g/2

(*c*) g (*d*) g/3

RELATIVE MOTION

33. Rain is falling at velocity $10\hat{i} + 10\hat{j}$ m/s. Then match the direction of velocity of rain w.r.t. man in column-II from column-I.



	Column-I	Column-II
(*a*)		(P) →
(*b*)		(Q) ↓
(*c*)		(R) ↘
(*d*)		(S) ↙
		(T) ←

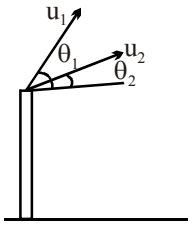
34.

	Column-I	Column-II
(*a*)	If swimmer can swim at 5m/sec in still water and if velocity of water flow is 4m/sec then angle between direction of swimming and direction of river flow to minimize drift.	(P) 53°
(*b*)	If swimmer can swim at 5m/sec in still water and velocity of flow is 3m/sec then angle between direction of velocity of swimmer with respect to river and the direction of river flow if swimmer crosses the river in minimum time .	(Q) 127°
(*c*)	If swimmer can swim at 4 m/sec and velocity of flow is 3m/sec then angle of resultant velocity (w.r.t. ground) with the direction of river flow if swimmer swims perpendicular to flow of river.	(R) 143°
(*d*)	Angle between direction of fluttering of flag and north if wind blows towards south west direction with a velocity $3\sqrt{2}$ m/sec. Man moves with a velocity 7m/sec along west, holding flag in his hand.	(S) 90°

35. Column-I shows certain situations with certain conditions and column-II shows the parameters in which situations of Column-I match. Which can be possible combination.

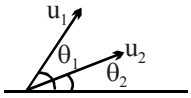
Column-I **Column-II**

(*a*) $u_1 = u_2$; $\theta_1 = \theta_2$ (P)



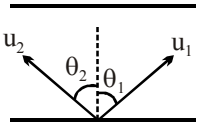
Two projectiles are projected from a height such that they strike ground at the same time.

(*b*) $u_1 > u_2$; $\theta_1 > \theta_2$ (Q)



Two projectiles under standard ground to ground projection such that horizontal range is same.

(*c*) $u_1 < u_2$; $\theta_2 > \theta_1$ (R)



Two swimmer starting from same point on a river bank such that time of crossing is same. u_1 & u_2 are velocities relative to river.

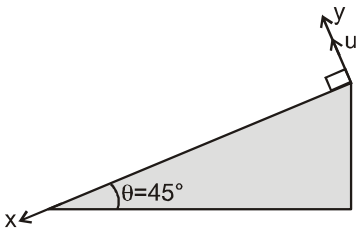
36. Match the following :

A ball is thrown vertically upward in the air by a passenger (relative to himself) from a train that is moving as given in column I ($v_{ball} << v_{escape}$). Correctly match the situation as described in the column I, with the paths given in column II.

Column I	**Column II**
(*a*) Train moving with constant acceleration on a slope then path of the ball as seen by the passenger.	(p) Straight line
(*b*) Train moving with constant acceleration on a slope then path of the ball as seen by a stationary observer outside.	(q) Parabolic
(*c*) Train moving with constant acceleration on horizontal ground then path of the ball as seen by the passenger.	(r) Elliptical
(*d*) Train moving with constant acceleration on horizontal ground then path of the ball as seen by a stationary observer outside.	(s) Hyperbolic
	(t) Circular

PROJECTILE

37. An inclined plane makes an angle $\theta = 45°$ with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at t = 0 sec. x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in column I with the results in column II. (g in column II is acceleration due to gravity.)



Column I	**Column II**
(*a*) The instant of time at which velocity of stone is parallel to x-axis	(p) $\frac{2\sqrt{2}\,u}{g}$
(*b*) The instant of time at which velocity of stone makes an angle $\theta = 45°$ with positive x-axis.	(q) $\frac{2u}{g}$
(*c*) The instant of time till which (starting from t = 0) component of displacement along x-axis become half the range on inclined plane is	(r) $\frac{\sqrt{2}\,u}{g}$
(*d*) Time of flight on inclined plane is	(s) $\frac{u}{\sqrt{2}\,g}$

38. A particle is projected from level ground. Assuming projection point as origin, x-axis along horizontal and y-axis along vertically upwards. If particle moves in x-y plane and its path is given by $y = ax - bx^2$ where a, b are positive constants. Then match the physical quantities given in column-I with the values given in column-II. (g in column II is acceleration due to gravity.)

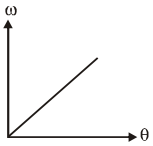
Column I	**Column II**
(*a*) Horizontal component of velocity	(p) $\frac{a}{b}$
(*b*) Time of flight	(q) $\frac{a^2}{4b}$
(*c*) Maximum height	(r) $\sqrt{\frac{g}{2b}}$
(*d*) Horizontal range	(s) $\sqrt{\frac{2a^2}{bg}}$

CIRCULAR MOTION

39. Each situation in column I gives graph of a particle moving in circular path. The variables ω,θ and t represent angular speed (at any time t) , angular displacement (in time t) and time respectively. Column II gives certain resulting interpretation. Match the graphs in column I with statements in column II.

Column-I **Column-II**

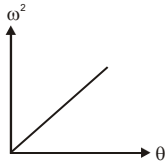
(*a*)



ω - θ graph

(p) Angular acceleration of particle is uniform

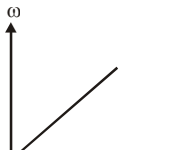
(*b*)



ω² - θ graph

(q) Angular acceleration of particle is non-uniform

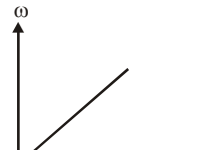
(*c*)



ω - t graph

(r) Angular acceleration of particle is directly proportional to t.

(*d*)



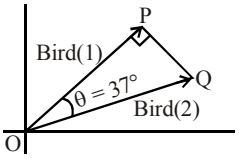
ω - t² graph

(s) Angular acceleration of particle is directly proportional to θ.

NUMERICAL BASED QUESTIONS

Relative Motion

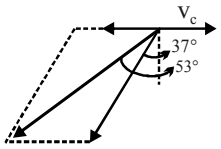
40. Two birds are at origin. They both start to fly in straight lines in different directions making an angle $\theta = 37°$ with each other at the same time as shown.



Bird(1) on reaching a point P takes a right angle turn to reach point Q simultaneously with Bird (2). If the bird(1) was flying with a constant speed of 7 km/hr then what must be the constant speed of bird (2) (in km/hr)?

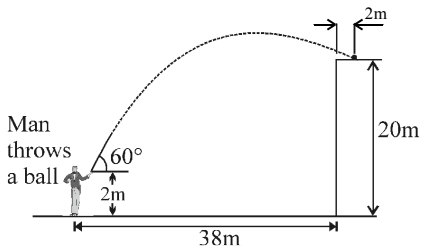
41. Shore based radar indicates that a ferry boat is moving on a river with a speed of 10 m/s at an angle of 30° North of east. The instruments on the ferry boat indicate that it moves with a speed of 10 m/s at an angle of 30° North of west relative to the river. What is the true speed (in m/s) at which river flows? Round off to nearest integer.

42. A car is travelling in steady rain with constant acceleration in a straight line. When it begins to move the driver sees that the raindrops make track at an angle of 37° with the vertical on the side window. After 20 sec., the raindrops make track at an angle of 53° with vertical in same direction. Find the acceleration of the car in cm/s^2. Rain is falling at 3 m/s.



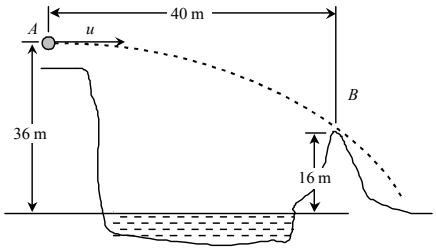
PROJECTILE

43. A soft ball is thrown at an angle of $\alpha = 60°$ above the horizontal. It lands a distance d = 2m from the edge of a flat roof, whose height is h = 20 m ; the edge of the roof is l = 38 m from the thrower (see fig.). At what speed (in m/s) was the softball thrown?



44. When a particle is projected at an angle of 15° from horizontal, it's range is found to be 3.5 m. What would be it's range (in m) if it is projected at an angle of 45° from horizontal with same speed?

45. With what minimum horizontal velocity 'u', (in m/s) can a boy throw a rock at A so that it just clear the obstruction at B

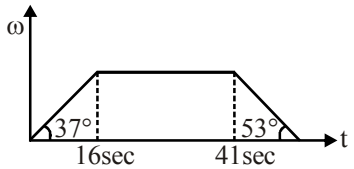


CIRCULAR MOTION

46. An electric drill starts from rest and rotates with constant angular acceleration. After it has rotated through an angle θ, the magnitude of centripetal acceleration of a point on the drill is twice the magnitude of tangential acceleration. What is the angle (in rad.)

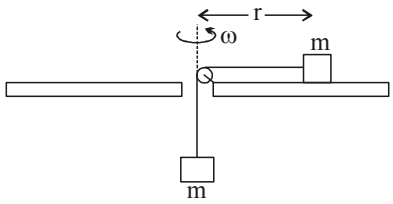
47. Find the radius (in cm) of a rotating disc if the velocity of a point on rim is 5 times the velocity of a point located 5 m closer to the centre.

48. The angular velocity of a body moving in a circular path is shown in graph below. What is the average angular velocity (in rad/s) for the entire motion ? Approximate the answer to nearest integer.

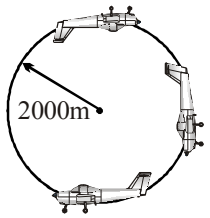


49. Two equal masses m are attached by a string. One mass lies at radial distance r from the centre of a horizontal turntable which rotates with constant angular velocity ω = 2 rad s^{-1}, while the second hangs from the string inside the turntable's hollow spindle (see fig.). The

coefficient of static friction between the turntable and the mass lying on it $\mu_s = 0.5$. The maximum and minimum values such that the mass lying on the turntable does not slide are r_{max}, r_{min}. Then $(r_{max} + r_{min})$ (in meter).



50. An air craft loops the vertical loop as shown with constant speed. At the top most point, the normal force exerted on the pilot by his seat is $\frac{1}{3}$ times the force exerted by the seat at the lowest point. What is the speed (in m/s) of the plane?



JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

RELATIVE MOTION

1. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when: (i) they are moving in the same direction, and (ii) in the aopposite direction is :

[*JEE Main-2019 (January)*]

(*a*) $\frac{11}{5}$ (*b*) $\frac{5}{2}$

(*c*) $\frac{3}{2}$ (*d*) $\frac{25}{11}$

2. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in : **[*JEE Main - 2019 (April)*]**

(*a*) 4.2 hrs. (*b*) 2.2 hrs.

(*c*) 3.2 hrs. (*d*) 2.6 hrs.

3. The stream of a river is flowing with a speed of 2km/h. A swimmer can swim at a speed of 4km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight ?

[*JEE Main-2019 (April)*]

(*a*) 60° (*b*) 150°

(*c*) 90° (*d*) 120°

4. A particle is moving along the x-axis with its coordinate with time t given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is

[*JEE Main-2020 (January)*]

(*a*) 580 (*b*) 500

(*c*) 540 (*d*) 560

5. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60º from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45º. The value of β is close to

[*JEE Main-2020 (September)*]

(*a*) 0.37 (*b*) 0.41

(*c*) 0.73 (*d*) 0.50

PROJECTILE

6. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[*JEE Main - 2019 (January)*]**

(*a*) $y = x^2 +$ constant (*b*) $y^2 = x +$ constant

(*c*) $y^2 = x^2 +$ constant (*d*) $xy =$ constant

7. Two guns A and B can fire bullets at speed 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the buyllets fired by the two guns, on the ground is:

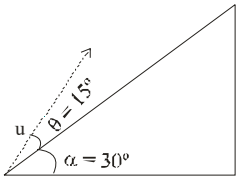
[*JEE Main-2019 (January)*]

(*a*) 1 : 16 (*b*) 1 : 2

(*c*) 1 : 4 (*d*) 1 : 8

8. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$ from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to: **[*JEE Main-2019 (April)*]**

(Take $g = 10\ ms^{-2}$)



(*a*) 14 cm (*b*) 20 cm

(*c*) 18 cm (*d*) 26cm

9. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ? **[*JEE Main-2019 (April)*]**

(*a*) $R^2 = 2\,h_1\,h_2$ (*b*) $R^2 = 16\,h_1\,h_2$

(*c*) $R^2 = 4\,h_1\,h_2$ (*d*) $R^2 = h_1\,h_2$

10. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then $(g = 10\ ms^{-2})$

[*JEE Main-2019 (April)*]

(*a*) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

(*b*) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

(*c*) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$

(*d*) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$

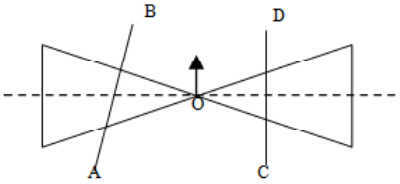
11. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is: **[*JEE Main-2019 (April)*]**

(*a*) R/g (*b*) R/4g

(*c*) 2R/g (*d*) R/2g

CIRCULAR MOTION

12. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to: **[*JEE Mains-2016*]**



(*a*) turn left

(*b*) turn right.

(*c*) go straight.

(*d*) turn left and right alternately.

13. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then, **[*JEE Main-2018*]**

(*a*) $T \propto R^{\frac{n}{2}+1}$ (*b*) $T \propto R^{(n+1)/2}$

(*c*) $T \propto R^{n/2}$ (*d*) $T \propto R^{3/2}$ for any n

14. A body is projected at t = 0 with a velocity $10\ ms^{-1}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1 s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\ ms^{-2}$ the radius of R is : **[*JEE Main-2019 (January)*]**

(*a*) 10.3 m (*b*) 2.8 m

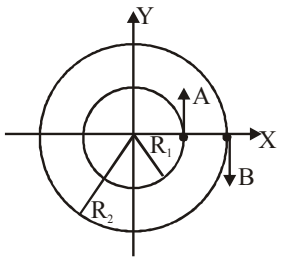
(*c*) 2.5 m (*d*) 5.1 m

15. A particle is moving along a circular path with a constant speed of $10\ ms^{-1}$. What is the magnitude of the change in velocity of the particle, when it moves through an angle of $60°^{+}$ around the centre of the circle ? **[*JEE Main-2019 (January)*]**

(*a*) $10\sqrt{3}$ m/s (*b*) 0

(*c*) $10\sqrt{2}$ m/s (*d*) 10 m/s

16. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure: **[*JEE Main - 2019 (January)*]**



The relative velocity $\overrightarrow{v_A} - \overrightarrow{v_B}$ at $t = \frac{\pi}{2\omega}$ is given by :

(*a*) $\omega(R_1 + R_2)\hat{i}$ (*b*) $-\omega(R_1 + R_2)\hat{i}$

(*c*) $\omega(R_2 - R_1)\hat{i}$ (*d*) $\omega(R_1 - R_2)\hat{i}$

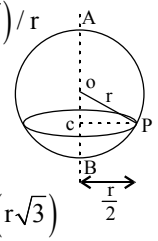
17. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to: **[*JEE Main-2019* (*April*)]**

(*a*) $\left(g\sqrt{3}\right)/r$

(*b*) $\frac{\sqrt{3}g}{2r}$

(*c*) $2g/r$

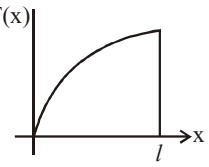
(*d*) $2g/\left(r\sqrt{3}\right)$



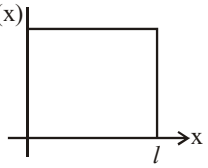
18. A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

[*JEE Main-2019* (*April*)]

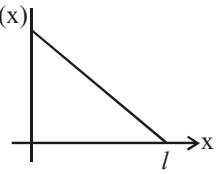
(*a*)



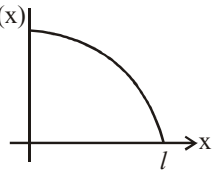
(*b*)



(*c*)



(*d*)



19. A particle of mass m is fixed to one end of light spring having force constant k and unstretched length ℓ. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is : **[*JEE Main-2020* (*January*)]**

(*a*) $\frac{m\ell\omega^2}{k-m\omega^2}$ (*b*) $\frac{m\ell\omega^2}{k-\omega m}$

(*c*) $\frac{m\ell\omega^2}{k+m\omega^2}$ (*d*) $\frac{m\ell\omega^2}{k+m\omega}$

20. A particle moves such that its position vector $\vec{r}(t)=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$ where ω is a constant and t is time, then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle :

[*JEE Main-2020* (*January*)]

(*a*) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(*b*) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$

(*c*) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

(*d*) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin.

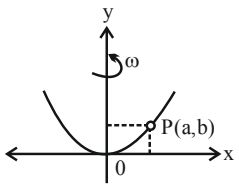
21. A spring mass system (mass m, spring constant k and natural length ℓ) rests in equilibrium on a horizontal disc. The free and of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option **[*JEE Main-2020* (*January*)]**

(*a*) $\sqrt{\frac{2}{3}\left(\frac{m\omega^2}{k}\right)}$ (*b*) $\frac{m\omega^2}{3k}$

(*c*) $\frac{2m\omega^2}{k}$ (*d*) $\frac{m\omega^2}{k}$

22. A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction)

[*JEE Main-2020* (*September*)]



(*a*) $2\sqrt{2gC}$ (*b*) $2\sqrt{gC}$

(*c*) $\sqrt{\frac{2g}{C}}$ (*d*) $\sqrt{\frac{2gC}{ab}}$

23. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of

[*JEE Main-2020* (*September*)]

(*a*) 10^{-1} (*b*) 10^{-2}

(*c*) 10^{-4} (*d*) 10^{-3}

24. A mosquito is moving with a velocity $\vec{v} = 0.5t\hat{i}\hat{j} + 3t\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after $2s$? **[*JEE Main-2021* (*March*)]**

(*a*) $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from *y*-axis

(*b*) $\tan^{-1}\left(\frac{5}{2}\right)$ from *y*-axis

(*c*) $\tan^{-1}\left(\frac{2}{3}\right)$ from *x*-axis

(*d*) $\tan^{-1}\left(\frac{5}{2}\right)$ from *x*-axis

25. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection 8 and the maximum height attained H are respectively given by: **[*JEE Main-2021* (*March*)]**

(*a*) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$ (*b*) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$

(*c*) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$ (*d*) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

26. A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a: **[*JEE Main-2021* (*March*)]**

(*a*) hyperbola

(*b*) parabola in the direction of motion of plane

(*c*) straight line vertically down the plane

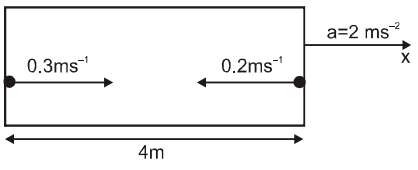
(*d*) parabola in a direction opposite to the motion of plane

JEE-ADVANCED PREVIOUS YEAR'S

RELATIVE MOTION

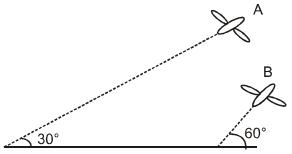
1. A rocket is moving in a gravity free space with a constant acceleration of $2ms^{-2}$ along $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of $0.3ms^{-1}$ relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of $0.2ms^{-1}$ from its right end relative to the rocket. The time in seconds when the two balls hit each other is:

[*JEE Advanced-2014*]



PROJECTILE

2. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure . The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0s, an observer in A finds B at a distance of 500m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is: **[*JEE Advanced-2014*]**



3. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is .....

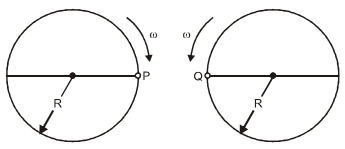
[*JEE Advanced-2018*]

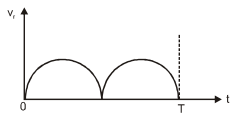
4. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1

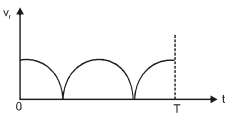
After hitting the ground, ball rebounds at the same angle q but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8\ V_1$, the value of α is_____ **[*JEE Advanced-2019*]**

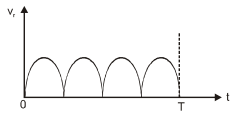
CIRCULAR MOTION

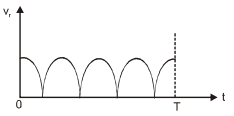
5. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane.At time t = 0 , the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. as function of times best represented by **[*IIT JEE-2012*]**



(*a*) 

(*b*) 

(*c*) 

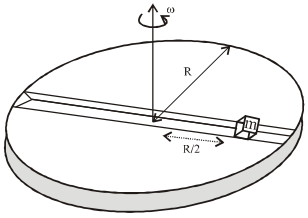
(*d*) 

Comprehension– 01 (Q.6 and 7)

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass *m* moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in inertial frame of reference is $\vec{F}_{rot} = \vec{F}_{in} + 3\text{m}(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$,

Where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter –clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega \hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r}$ = (R/2) $\hat{i}$ at *t* = 0 and is constrained to move only along the slot. **[*JEE Advanced-2016*]**



6. The distance *r* of the block at time *t* is

(*a*) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (*b*) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

(*c*) $\frac{R}{2}\cos 2\,\omega t$ (*d*) $\frac{R}{2}\cos\,\omega t$

7. The net reaction of the disc on the block is

(*a*) $-m\omega^2 R\cos\omega t\,\hat{j} - mg\hat{k}$

(*b*) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\,\hat{j} + mg\hat{k}$

(*c*) $m\omega^2 R\sin\omega t\,\hat{j} - mg\hat{k}$

(*d*) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\,\hat{j} + mg\hat{k}$

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ANSWER KEY

Topicwise Questions

1. (*b*) **2.** (*b*) **3.** (*d*) **4.** (*c*) **5.** (*b*) **6.** (*c*) **7.** (*a*) **8.** (*b*) **9.** (*c*) **10.** (*b*)
11. (*c*) **12.** (*a*) **13.** (*b*) **14.** (*d*) **15.** (*c*) **16.** (*d*) **17.** (*c*) **18.** (*a*) **19.** (*b*) **20.** (*a*)
21. (*b*) **22.** (*b*) **23.** (*d*) **24.** (*c*) **25.** (*d*) **26.** (*c*) **27.** (*d*) **28.** (*b*) **29.** (*c*) **30.** (*a*)
31. (*c*) **32.** (*d*) **33.** (*b*) **34.** (*c*) **35.** (*a*) **36.** (*c*) **37.** (*c*) **38.** (*a*) **39.** (*d*) **40.** (*a*)
41. (*c*) **42.** (*a*) **43.** (*d*) **44.** (*b*) **45.** (*b*) **46.** (*a*) **47.** (*c*) **48.** (*c*) **49.** (*a*) **50.** (*b*)
51. (*b*) **52.** (*b*) **53.** (*a*) **54.** (*b*) **55.** (*a*) **56.** (*b*) **57.** (*a*) **58.** (*a*) **59.** (*d*) **60.** (*c*)
61. (*c*) **62.** (*b*) **63.** (*d*) **64.** (*d*)

Learning Plus

1. (*b*) **2.** (*c*) **3.** (*d*) **4.** (*d*) **5.** (*b*) **6.** (*a*) **7.** (*a*) **8.** (*a*) **9.** (*d*) **10.** (*d*)
11. (*d*) **12.** (*a*) **13.** (*a*) **14.** (*b*) **15.** (*b*) **16.** (*c*) **17.** (*b*) **18.** (*b*) **19.** (*c*) **20.** (*d*)
21. (*a*) **22.** (*a*) **23.** (*d*) **24.** (*d*) **25.** (*c*) **26.** (*b*) **27.** (*b*) **28.** (*b*) **29.** (*d*) **30.** (*d*)
31. (*d*) **32.** (*b*) **33.** (*d*) **34.** (*d*) **35.** (*b*) **36.** (*c*) **37.** (*a*) **38.** (*a*) **39.** (*b*) **40.** (*c*)
41. (*d*) **42.** (*c*) **43.** (*c*) **44.** (*d*) **45.** (*b*) **46.** (*c*) **47.** (*c*) **48.** (*d*) **49.** (*d*) **50.** (*c*)
51. (*c*) **52.** (*d*) **53.** (*a*) **54.** (*c*) **55.** (*a*) **56.** (*c*) **57.** (*b*) **58.** (*b*) **59.** (*a*) **60.** (*c*)

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. (*c*, *d*) **2.** (*a*, *b*) **3.** (*a*,*b*,*d*) **4.** (*b*, *c*, *d*) **5.** (*a*, *d*) **6.** (*a*,*b*,*d*) **7.** (*b*, *d*) **8.** (*b*, *c*) **9.** (*a*, *b*)
10. (*a*, *b*, *c*, *d*) **11.** (*a*, *c*, *d*) **12.** (*b*, *d*) **13.** (*a*, *c*, *d*) **14.** (*b*, *c*, *d*, *e*) **15.** (*b*, *c*) **16.** (*b*, *d*) **17.** (*b*) **18.** (*b*)
19. (*c*) **20.** (*a*) **21.** (*c*) **22.** (*c*) **23.** (*b*) **24.** (*a*) **25.** (*c*) **26.** (*a*) **27.** (*b*) **28.** (*a*)
29. (*a*) **30.** (*d*) **31.** (*b*) **32.** (*c*) **33.** (*a*) Q (*b*) R (*c*) S (*d*) T **34.** (*a*) R (*b*) S (*c*) P (*d*) Q
35. (*a*) P, Q, R (*b*) Q, R, S (*c*) Q, R, S **36.** (*a*) Q (*b*) Q (*c*) Q (*d*) Q **37.** (*a*) R (*b*) S (*c*) Q (*d*) P
38. (*a*) R (*b*) S (*c*) Q (*d*) P **39.** (*a*) Q, S (*b*) P (*c*) P (*d*) Q, R **40.** [0005] **41.** [0017] **42.** [0007] **43.** [0025]
44. [0007] **45.** [0020] **46.** [0001] **47.** [0625] **48.** [0009] **49.** [0005] **50.** [0200]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*a*) **2.** (*d*) **3.** (*d*) **4.** [580] **5.** (*c*) **6.** (*c*) **7.** (*a*) **8.** (*b*) **9.** (*b*) **10.** (*a*)
11. (*c*) **12.** (*a*) **13.** (*b*) **14.** (*b*) **15.** (*d*) **16.** (*c*) **17.** (*d*) **18.** (*d*) **19.** (*a*) **20.** (*d*)
21. (*d*) **22.** (*a*) **23.** (*d*) **24.** (*a*) **25.** (*a*) **26.** (*c*)

JEE-ADVANCED
PREVIOUS YEAR'S

1. [*b*] **2.** [*e*] **3.** [30.00] **4.** [4.00] **5.** (*a*) **6.** (*b*) **7.** (*d*)



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In this era of cut throat competition among **JEE** aspirants, only few are lucky enough to make it through. During the preparation, a student needs the most relevant study material to cover all the topics with proper planning to crack **JEE**.

We know there is abundance of study materials floating in the market but what We felt missing, is a self-driven and well-planned Study material. We have tried to bring clarity and Quality in the students by introducing this **Aspire for JEE**

The book covers all the important concepts and theories with a rich amount of practice questions as well. This book is designed for the perfect guidance you need.

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2. Vector & Calculus **17-25**

3. Motion in a Straight Line **26-37**

4. Motion in a Plane **38-57**

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1 Units and Measurements

Topicwise Questions

1. (*a*) It is obvious.

2. (*a*) Kilogram is not a physical quantity, its a unit.

3. (*c*) PARSEC is a unit of distance.
It is used in astronomical science.

4. (*b*) System is NOT based on unit of mass, length and time alone,
This system is based on all 7 Fundamental physical quantities and 2 supplementary physical quantities.

5. (*c*) S.I. unit of energy is Joule.

6. (*b*) SI unit of universal gravitational constant G is -

We know $F = \frac{GM_1M_2}{R^2}$

Here M_1 and M_2 are mass

R = Distance between them M_1 and M_2

F = Force

$$G = \frac{FR^2}{M_1M_2} = \frac{N - m^2}{kg^2}$$

So, Unit of G = $N–m^2 kg^{-2}$

7. (*b*) Surface Tension (T) :-

$$T = \frac{J}{A} = \frac{J}{m^2}$$

So S.I. unit of surface tension is $joule/m^{+2}$

8. (*d*) Here ρ is specific resistance.

$$R = \frac{\rho l}{A} \Rightarrow ohm = \frac{\rho m}{m^2} \qquad \Rightarrow \rho = ohm \times m$$

9. (*a*) Here i = current
A = crossectional Area
M = iA
= Amp. m^2

10. (*d*) Unit of universal gas constant (R)
PV = nRT P → Pressure
V → Volume

$$R = \frac{PV}{nT}$$ T → Temperature

$$= \frac{N/m^2 \times m^3}{mol. \times K}$$ R → Univ. Gas. Const.

n → No. of male

$$= \frac{N - m}{mol. \times K} = Joule\, K^{-1} mol^{-1}$$

{n – m = joule}

11. (*d*) Stefan-Constant(σ)
Unit → $w/m^2\text{-}k^4 = wm^{-2}k^{-4}$

12. (*c*) S.I. unit of the angular acceleration is rad/s^2.
α = angular velocity/time = $rad/s^2 = rad.s^{-2}$

13. (*c*) Angular Frequency (f) $= \frac{1}{T} = M^oL^oT^{-1}$

So, here dimension in length is zero

14. (*b*) P = mvm → mass
v → velocity
Dimesion of $[P] = [MLT^{-1}]$

15. (*b*) Boltz mann's const. (k) → J → Joule
K → Kelvin
Unit → J/k

$$\text{Dimension} = \frac{M^1L^2T^{-2}}{K^1} = M^1L^2T^{-2}K^{-1}$$

16. (*b*) Find out Dimension of each physical quantity in all options.
$AM = Mvr^2 = ML^2T^{-1}$

Dimension of Torque (τ)= $\vec{r} \times \vec{F}$ →Force = MLT^{-2}
$(\tau) = L^1 \times M^1L^1T^{-2}$
It is also a dimension of Energy. = ML^2T^{-2}
Power = $W/t = ML^2T^{-2}/T = ML^2T^{-3}$

17. (*d*) Find dimension in all options.

Here stress = Force/Area

$$= \frac{M^1L^1T^{-2}}{L^2}$$

stress = $[M^1L^{-1}T^{-2}]$

18. (*c*) All the terms in the equation must have the dimension of force

$\therefore$ $[A \sin C\, t] = MLT^{-2}$

$\Rightarrow$ $[A][M^0L^0T^0] = MLT^{-2}$

$\Rightarrow$ $[A] = MLT^{-2}$

Similarly, $[B] = MLT^{-2}$

$$\therefore \quad \frac{[A]}{[B]} = M^oL^oT^o$$

Again $[Ct] = M^oL^oT^o \Rightarrow [C] = T^{-1}$

$[Dx] = MLT^o \Rightarrow [D] = L^{-1}$

$$\Rightarrow \quad \frac{[C]}{[D]} = M^oL^1T^{-1}.$$

19. (*c*) $$\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right].$$

L.H.S. is the dimensionless as

denominator $2ax - x^2$ must have the dimension of $[x]^2$ ($\because$ we can add or substract only if quantities have same dimension)

$$\therefore \quad \left[\sqrt{2ax - x^2}\right] = [x]$$

Also, dx has the dimension of [x]

$$\therefore \quad \frac{x\,dx}{\sqrt{2ax - x^2}} \text{ is having dimension L}$$

Equating the dimension of L.H.S. & R.H.S. we have

$[a^n] = M^0L^1T^0$ { $\therefore \sin^{-1}\left(\frac{x}{a} - 1\right)$ must be dimensionless

$\therefore$ $n = 1$

20. (*a*)

21. (*b*) $y = a \sin(At - Bx + C)$

Angle has no dimensions so

Dimensions of $At = M^0L^0T^0$

$\Rightarrow$ $A = T^{-1}$

Dimensions of $Bx = M^0L^0T^0$

$\Rightarrow$ $B = L^{-1}$

Dimensions of $C = M^0L^0T^0$

22. (*a*) Dimensions of $\sqrt{\frac{\gamma p}{\rho}}$ = Dimensions of v

$\gamma L^2T^{-2} = L^2T^{-2}$

$\gamma = M^0L^0T^0$

23. (*c*) $$A = \frac{d\ work}{dt}\ \frac{1}{F}P$$

$$[A] = \frac{[work]}{[time]\frac{[work]}{[displacement]} \times [P]}$$

$$= \frac{L}{T}MLT^{-1} = \frac{1}{M}$$

24. (*b*) $[v] = [k][\lambda^a \rho^b g^c] \Rightarrow LT^{-1} = L^a M^b L^{-3b} L^c T^{-2c}$

$\Rightarrow$ $LT^{-1} = M^b L^{a-3b+c} T^{-2c}$

$\Rightarrow$ $a = ½, b = 0, c = ½$

so, $v^2 = kg\lambda$

25. (*b*) It is obvious

26. (*c*) $$P = \frac{W}{t}$$

Watt = Joule/sec.

Joule = Watt-sec.

One watt-hour = 1 watt×60×60 sec

1 Hour=60×60sec. = 3600 watt-sec

= 3600 Joule

= 3.6×10^3 Joule

27. (*a*) Given

$P = 10^6$ dyne/cm^2

$n_1u_1 = n_2u_2$

$$n_1\left[M_1^1L_1^{-1}T_1^{-2}\right] = 10^6\left[M_2^1L_2^{-1}T_2^{-2}\right]$$

$$n_1 = 10^6\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^{-1}\left[\frac{T_2}{T_1}\right]^{-2}$$

$$= 10^6\left[\frac{1}{1000}\right]^1\left[\frac{1}{100}\right]^{-1}$$

$$\Rightarrow 10^6 \times \frac{10^2}{10^3} = 10^5 \text{ N/m}^2$$

28. (*b*) $\rho = 2 g/cm^3$

$n_1u_1 = n_2u_2$

$$n_1\left[M_1^1L_1^{-3}\right] = 2\left[M_2^1L_2^{-3}\right]$$

$$n_1 = 2\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^{-3}$$

$$= 2\left[\frac{10^{-3}}{1}\right]^1\left[\frac{10^{-2}}{1}\right]^{-3}$$

$= 2 \times 10^{-3} \times 10^6$

$= 2 \times 10^3$ Kg/m^3

29. (*a*) $n_2 = 13600\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-3}$

$= 13600\left[\frac{1000}{1}\right]^1\left[\frac{100}{1}\right]^{-3}$

$n_2 = 13.6\ gcm^{-3}$

30. (*c*) Measurement 900×10^{-4} m is most accurate as significant figure is 3,

31. (*c*) 70.40s four significant figures.
Time period = 3.520 sec. (4 significant figure)

32. (*a*) $KE = \frac{1}{2}mv^2$

$\frac{\Delta k}{k} \times 100 = 1\% + 2 \times 2\% = 5\%$

33. (*a*) The third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore , be increased by 1. The digits to be dropped should be replaced by zeros because they appear to the left of the decimal. Thus, 15462 becomes 15500 on rounding to three significant digits.

34. (*a*) We have $\frac{25.2 \times 1374}{33.3} = 1039.7838.....$

Out of the three numbers given in the expression 25.2 and 33.3 have 3 significant digits and 1374 has four. The answer should have three significant digits. Rounded 1039.7838 .... to three significant digits, it becomes 1040.
Thus , we write.

$\frac{25.2 \times 1374}{33.3} = 1040.$

35. (*c*) 24.36
0.0623
256.2
Now the first column where a doubtful digit occurs is the one just next to the decimal point (256.2). All digits right to this column must be dropped after proper rounding. The table is rewritten and added below
24.4
0.1
256.2
———
280.7 The sum is 280.7

36. (*b*) $\therefore E = \frac{1}{2}mv^2$

$\therefore$ % Error in K.E.
= % error in mass + 2 × % error in velocity
$= 2 + 2 \times 3 = 8\%$

37. (*b*)

38. (*b*) Number of significant figures are 3, because 10^3 is decimal multiplier.

39. (*b*)

$\because V = \frac{4}{3}\pi r^3$

$\therefore$ % error is volume $= 3 \times$ % error in radius $= 3 \times 1 = 3\%$

40. (*c*) Mean time period $T = 2.00\ sec$
& Mean absolute error = $\Delta T = 0.05\ sec.$
To express maximum estimate of error, the time period should be written as $(2.00 \pm 0.05)\ sec$

41. (*c*)

42. (*a*) $\frac{1}{20} = 0.05$

$\therefore$ Decimal equivalent upto 3 significant figures is 0.0500

43. (*b*)

44. (*a*) Since percentage increase in length = 2 %
Hence, percentage increase in area of square sheet
$= 2 \times 2\% = 4\%$

45. (*c*) 1 main scale div = 0.5 mm
$10V = 9S$

$V = \frac{9}{10}S$

$S - V = S - \frac{9}{10}S = \frac{1}{10}S.$

$\therefore$ Vernier constant $= \frac{0.5\ mm}{10} = 0.05\ mm$

46. (c) $20V = 19s., V = \frac{19}{20}S$

$S - V = S - \frac{19}{20}S \Rightarrow$ vernier $= \frac{S}{20}$

$0.1\ mm = \frac{S}{20}$

$\therefore$ $1s = 20 \times 0.1\ mm = 2\ mm.$

47. (*c*) 1 VC = 1 MSD – VSD

$1VC = 0.1\ cm - \frac{n}{m}$

$0.02\ cm = \frac{1}{10} - \frac{n}{m}$

$\frac{n}{m} = \frac{1}{10} - \frac{2}{100}$

$n = 10, m = 0.8\ cm$

Learning Plus

1. (*b*) Solar day → Time far Earth to wake a complete rotation on its axis

Parallactic second [1 Parsec] → It is a distance corresponding to a parallex of one second of arc.

Leap year → A leap year is year (time) Containing one extra day.

Lunar Month → A lunar month is the time between two identical view moons of full moons.

1 Lunar month = 29.53059 days.

2. (*b*) Unit of impulse = ⇒ Impulse = Force × time

$$= \text{kg}\ \frac{\text{m}}{\text{sec}^2}\text{sec} = \text{kg}\frac{\text{m}}{\text{sec}} = \text{mv}$$

The unit is same as the unit of linear momentum.

3. (*d*) Energy W = f × d = Nm

W = eV = electron-volt

W = p × t = Watt hour

So, kg × m/sec^2 is not the unit of energy.

4. (*c*) Dimensionless quantity may have a unit

Ex. Angle Unit → Radian

Dimension → $M^oL^oT^o$

5. (*c*) Only same physical quantities can be added or substracted,

It's only multiply and divided only.

So, a/b denote a new physical quantity.

6. (*c*) $P = P_o\,\text{Exp}(-\alpha t^2)$

Here Exp $(-\alpha t^2)$ is a dimensionless

So, dimension of $[\alpha t^2] = M^oL^oT^o$

$$\text{So, } [\alpha] = \frac{M^oL^oT^o}{T^2}$$

$$[\alpha] = M^oL^oT^{-2}$$

7. (*c*) By Checking the dimension in all options

(*c*) Moment of Inertia = Mr^2

$= M^1L^2T^o$

Moment of force = r × F

$= L^1 \times M^1L^1T^{-2}$

$= M^1L^2T^{-2}$

8. (*d*) Action = Energy × Time = $M^1L^2T^{-2} \times T^1$

$= M^1L^2T^{-1}$

It is same as dimension of Impulse × distance

$= MLT^{-1} \times L^1 = M^1L^2T^{-1}$

9. (*a*) $M^1L^2T^{-2}$ is a dimension of kinetic energy.

10. (*b*) $\frac{EJ^2}{M^5G^2}$ J=mvr, $J = [ML^2T^{-1}]$

$= [M^0L^0T^0]$

Dimension of Angle = $[M^0L^0T^0]$

11. (*c*) $x(t) = \frac{v_o}{\alpha}[1 - e^{-\alpha t}]$

Dimension of v_o and α

Here $e^{-\alpha t}$ is dimensionless so,

$[\alpha]\,[t] = M^oL^oT^o$

$$[\alpha] = \frac{M^oL^oT^o}{T^1} = T^{-1}$$

$[\alpha] = M^oL^oT^{-1}$

Here $1-e^{-\alpha t}$ is a number

$$[x(t)] = \frac{V_o}{\alpha}$$

$[V_o] = [L^1]\,[T^{-1}]$

$[V_o] = M^oL^1T^{-1}$

12. (*d*) $F = Pt^{-1} + \alpha t$

Here F and Pt^{-1} is a same

Physical quantity

$[F] = [Pt^{-1}]$

$$[P] = \frac{[F]}{[t^-]} = [F \times t] = ML\,T^{-2} \times T = MLT^{-1}$$

We find it is same as dimension of momentum $= MLT^{-1}$

13. (*d*) Y = a sin (bt – cx)

Dimension of b

Here bt is dimensionless

$[bt] = M^oL^oT^o$

$$[b] = \frac{M^oL^oT^o}{[T^1]} = M^oL^oT^{-1}$$

It is a dimension of wave frequency.

14. (*c*) Here $\sqrt{1 + \frac{2k\ell}{ma}}$ is a number.

It's a dimensionless quantity.

$$\left[\frac{2k\ell}{ma}\right] = [M^oL^oT^o]$$

$$[K] = \frac{[m][a]}{[\ell]}$$

$$= \frac{M^1L^1T^{-2}}{L^1} = M^1L^oT^{-2}$$

So dimession of [b] is

$$[b] = \left[\frac{ma}{K}\right] = \left[\frac{MLT^{-2}}{MT^{-2}}\right]$$

$[b] = L$

unit of b is metre

15. (*d*) $\alpha = \frac{F}{V^2}\sin(\beta t)$

Here sin (βt) is dimensionless.

$[\beta t] = M^oL^oT^o$

$$\beta = \frac{M^oL^oT^o}{T^1} = \left[T^{-1}\right]$$

$$[\alpha] = \left[\frac{F}{V^2}\right]$$

$$= \frac{M^1L^1T^{-2}}{[L^1T^{-1}]^2} = \frac{M^1L^1T^{-2}}{L^2T^{-2}}$$

$[\alpha] = [M^1L^{-1}T^o]$

16. (*d*) $L \alpha FAT$

$L = K F^aA^bT^c$ .... (*a*)

$M^oL^1T^o = K[M^1L^1T^{-2}] [L^1T^{-2}]^b [T]^c$

$M^oL^1T^o = K[M^a] [L^{a+b}] [T^{-2a-2b+c}]$

By comparesion and solving we find

$[a = 0] [b = 1] [c = 2]$

Put these value in Equa. (*a*)

$[L = F^oA^1T^2]$

17. (*b*) $F \alpha Av\rho$

$F = KA^a v^b \rho^c$

$= K[L^2]^a [L^1T^{-1}]^b [M^1L^{-3}]^c$

$F = K[M^cL^{2a+b-3c} T^{-b}]$

$M^1L^1T^{-2} = K[M^c L^{2a+b-3c} T^{-b}]$

$c = 1$

$-2 = -b \Rightarrow b = 2$

and

$2a + b - 3c = 1$

$2a + 2 - 3 = 1 \Rightarrow a = 1$

So $F = A^1 v^2 q^1$

$\therefore F = Av^2\rho$

18. (*b*) $V = g^p h^q$

$V = Kg^p h^q$

$[L^1T^{-1}] = [L^1T^{-2}]^p [L^1]^q$

$L^1T^{-1} = L^{p+q} T^{-2p}$

By comparing both sides

$p + q = 1, -2p = -1$

$p = 1/2, q = 1/2$

19. (*d*) Unit of length is micrometer

Unit of time is microsecond

$$\because \text{ Velocity } = \frac{\text{Displacement}}{\text{Time taken}}$$

$$= \frac{10^{-6}\,m}{10^{-6}\,sec} = m/sec$$

20. (*a*) $n_1u_1 = n_1u_1$

$$n_1\left[M_1^1L_1^2T_1^{-3}\right] = 1\left[M_2^1L_2^2T_2^{-3}\right]$$

$$n_1 = \left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^2\left[\frac{T_2}{T_1}\right]^{-3}$$

$$= \left[\frac{20}{1}\right]^1\left[\frac{10}{1}\right]^2\left[\frac{5}{1}\right]^{-3}$$

$$= \frac{20\times100}{5\times5\times5} = 16$$

$n_1 = 16$

Unit of power in new system = 16 Watt.

21. (*c*) $10^3(N) = M^1L^1T^{-2}$

$10^3 = [M]^1 [10^3]^1 [100]^{-2}$

$$M = \frac{10^3}{10^3 \times (100)^{-2}} = 10000\ kg$$

22. (*c*) In new system

Length $\rightarrow$ m 2m

Velocity $\rightarrow$ m/sec. 2m/sec

Force $\rightarrow$ kgm/sec^2 2kgm/sec^2

$\therefore$ Momentum (P) = mv = kg m/sec.

$$P = kg\frac{m}{sec}\times\frac{m}{m}\times\frac{sec}{sec}$$

$$P = kg\frac{m}{sec^2}\times\frac{m}{(m/sec)}$$

In new system

$$P^1 = \left(2kg\frac{m}{sec^2}\right)\times\frac{(2m)}{(2m/sec)}$$

$$P^1 = (2kg\,m/sec) = 2P$$

So, Here unit of momentum is doubled.

23. (*d*) Unit of Energy $= \text{kg}\frac{\text{m}^2}{\text{sec}^2}$

$= \left(\text{kg}\frac{\text{m}}{\text{sec}^2}\right)\times(\text{m})$

Now unit of force and length are doubled.

$= \left(2\text{kg}\frac{\text{m}}{\text{sec}^2}\right)(2\text{m}) = 4\text{kg}\frac{\text{m}^2}{\text{sec}^2}$

So, Unit of Energy is 4 times.

24. (*c*) $\text{K.E.} = \frac{1}{2}\text{mv}^2$

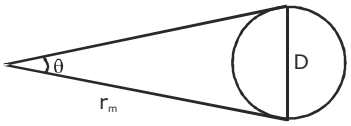
Dimension $= M^1L^2T^{-2}$

Now M.L are doubled

$= (2M)^1 (2L)^2 (T^{-2}) = 8\, M^1L^2T^{-2}$

So, K.E. will become 8 times.

25. (*b*) Take small angle approximation



$\text{Sin}\,\theta = \frac{\text{D}}{\text{r}_\text{m}}$

$\text{Sin}\,0.50° = \frac{\text{D}}{\text{r}_\text{m}}$

$0.50\times\frac{\pi}{180} = \frac{\text{D}}{384000}$

$\text{D} = 0.50\times\frac{\pi}{180}\times 384000$

$\text{D} = 3349.33 \Rightarrow \text{D} \approx 3350\,\text{km}.$

26. (*a*) $\text{A} = \ell\text{b} = 10.0\times 1.00 = 10.00$

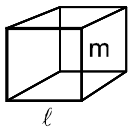
$\frac{\Delta\text{A}}{\text{A}} = \frac{\Delta\ell}{\ell} + \frac{\Delta\text{b}}{\text{b}}$

$\frac{\Delta\text{A}}{10.00} = \frac{0.1}{10.0} + \frac{0.01}{1.00}$

$\Rightarrow \Delta\text{A} = 10.00\left(\frac{1}{100}+\frac{1}{100}\right)$

$= 10.00\left(\frac{2}{100}\right) = \pm 0.2\,\text{cm}^2.$

27. (*b*)



$\rho = \frac{\text{m}}{\text{V}} = \frac{\text{m}}{\ell^3}$

Given : $\frac{\Delta\text{m}}{\text{m}} = \pm\ 2\% = \pm 2\times 10^{-2}\frac{\Delta\ell}{\ell}$

$= \pm\ 1\% = \pm\ 1\times 10^{-2}$

$\frac{\Delta\rho}{\rho} = \frac{\Delta\text{m}}{\text{m}} + 3\frac{\Delta\ell}{\ell}$

$= 2\times 10^{-2} + 3\times 10^{-2}$

$= 5\times 10^{-2} = 5\%$

28. (*a*) $\text{g} = 4\pi^2\frac{\ell}{\text{T}^2}$

$\frac{\Delta\ell}{\ell} = 2\% = \pm 2\times 10^{-2}$

$\frac{\Delta\text{T}}{\text{T}} = \pm\ 3\% = \pm\ 3\times 10^{-2}$

$\Rightarrow \frac{\Delta\text{g}}{\text{g}} = \frac{\Delta\ell}{\ell} + \frac{2\Delta\text{T}}{\text{T}}$

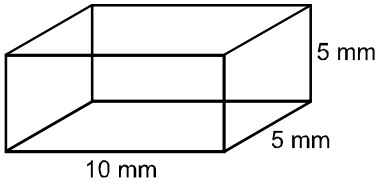
$= 2\times 10^{-2} + 2\times 3\times 10^{-2} = 8\times 10^{-2} = \pm\ 8\,\%$

29. (*b*) $\Delta\text{t} = 0.2$ s.

$\text{t} = 25$ s

$\text{T} = \frac{\text{t}}{\text{N}} \Rightarrow \frac{\Delta\text{T}}{\text{T}} = \frac{\Delta\text{t}}{\text{t}} = \frac{0.2}{25} = 0.8\,\%$

30. (*a*)



$\text{v} = \ell\text{bh}$

$\frac{\Delta\text{v}}{\text{v}} = \frac{\Delta\ell}{\ell} + \frac{\Delta\text{b}}{\text{b}} + \frac{\Delta\text{h}}{\text{h}}$

$= \frac{0.1}{10} + \frac{0.1}{5} + \frac{0.1}{5} = \frac{0.5}{10} = \pm 5\%$

31. (*d*) $\frac{\Delta\text{x}}{\text{x}} = 1\% = 10^{-2}$

$\frac{\Delta\text{y}}{\text{y}} = 3\% = 3\times 10^{-2}$

$\frac{\Delta\text{z}}{\text{z}} = 2\% = 2\times 10^{-2}$

$\text{t} = \frac{\text{xy}^2}{\text{z}^3}$

$$\frac{\Delta t}{t}=\frac{\Delta x}{x}+\frac{2\Delta y}{y}+\frac{3\Delta z}{z}$$

$$=10^{-2}+2\times3\times10^{-2}+3\times2\times10^{-2}$$

$$=13\times10^{-2}\ \therefore\%\text{ error in t}=\frac{\Delta t}{t}\times100=13\%$$

32. (*c*) $D=(4.23\pm0.01)$ cm
$d=(3.89\pm0.01)$ cm
$\Delta t=(D-d)/2$

$$=\frac{(4.23\pm0.01)-(3.89\pm0.01)}{2}$$

$$=\frac{(4.23-3.89)\pm(0.01+0.01)}{2}$$

$=(0.34\pm0.02)/2$ cm
$=(0.17\pm0.01)$cm

33. (*b*) $m=1.76$ kg
$M=25\,m$
$=25\times1.76$
$=44.0$ kg
Note: Mass of one unit has three significant figures and it is just multiplied by a pure number (magnified). So result should also have three significant figures.

34. (*b*) $R_1=(24\pm0.5)\,\Omega$
$R_2=(8\pm0.3)\,\Omega$
$R_S=R_1+R_2$
$=(32\pm0.8)\,\Omega$

35. (*b*) $\Delta\ell=0.5$ mm
N = 100 divisions
zero correction = 2 divisions
Reading = Measured value + zero correction

$$=(8\times0.5)\text{ mm}+(83-2)\times\frac{0.5}{100}.$$

$$=4\text{ mm}+81\times\frac{0.5}{100}\text{ mm}$$

$=4.405$ mm

36. (*d*) $\Delta\ell=1$ mm
N = 50 division
zero error = –6 Divisions
$=-0.12$ mm
Diameter = Measured value + zero correction

$$=3\times1+(6+31)\times\frac{1}{50}$$

$=3+0.74=3.74$ mm

37. (*a*) $D=2\times1+5\times\frac{10-9}{100}=2.05$ cm

Advanced Level Multiconcept Questions

1. (*a*), (*b*), (*c*), (*d*)
All A,B & C are obvious.
(d) Derived quantity may have zero dimension in certain base quantity.
For example acceleration which is derived quantity, has zero dimensions in mass which is base quantity.

2. (*a*), (*c*), (*d*)
It is obvious

3. (*c*), (*d*)
By checking dimension in each option

$$\text{Pressure}=\frac{F}{A}=\frac{[M^1L^1T^{-2}]}{[L^2]}=[M^1L^{-1}T^{-2}]$$

$$\text{Energy per unit volume}=\frac{\text{Energy}}{\text{Volume}}$$

$$=\frac{[M^1L^2T^{-2}]}{[L^3]}=[M^1L^{-1}T^{-2}]$$

4. (*b*) $A=\frac{a}{t}=LT^{-3}$

5. (*a*) LT^{-4}

6. (*c*) $c=at=[LT^{-1}]$

7. (*c*) $F=\frac{[c]}{r^2}\Rightarrow[c]=Fr^2=[MLT^{-2}]L^2$

8. (*d*) Kr = constant

$$K=\frac{\text{constant}}{r}=\frac{1}{L}=L^{-1}$$

9. (*b*) $F=\frac{C}{r^2}\Rightarrow C=Fr^2=Nm^2$

10. (*a*) $\rightarrow$ (Q) $\rightarrow$ (*c*) ,(*b*) $\rightarrow$ (S) $\rightarrow$ (*a*), (*c*) $\rightarrow$ (P) $\rightarrow$ (*b*), (*d*) $\rightarrow$ (R) $\rightarrow$ (*d*)

$$F=G\frac{m_1m_2}{r^2}\Rightarrow[G]=\frac{[F][r^2]}{[m_1m_2]}=\frac{MLT^{-2}L^2}{M^2}=M^{-1}L^3T^{-2}$$

$[\text{Torque}]=[f]\,[d]=MLT^{-2}L=ML^2T^{-2}$
$[\text{Momentum}]=[m]\,[v]=MLT^{-1}$

$$[p]=\frac{[F]}{[A]}=\frac{MLT^{-2}}{L^2}=ML^{-1}T^{-2}.$$

11. (i) → (Q) → (*d*), (ii) → (S) → (*b*), (iii) → (U) → (e), (iv) → (R) → (*a*), (v) → (T) → (f)
(vi) → (P) → (*c*)

(i) $U = \sigma AT^4 \Rightarrow [\sigma] = \frac{[U]}{[A][T^4]} = \frac{ML^2T^{-3}}{L^2K^4} = MT^{-3}K^{-4}$

(ii) $\lambda T = b \Rightarrow [b] = [\lambda][T] = LK$

(iii) $F = 6\pi\eta rv \Rightarrow [\eta] = \frac{[F]}{[r][v]} = \frac{MLT^{-2}}{L.\,LT^{-1}} = ML^{-1}T^{-1}$

(iv) $I = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = ML^oT^{-3}$

(v) Energy $= \frac{1}{2}Mi^2 \Rightarrow [M] = \frac{[E]}{[i^2]} = ML^2T^{-2}A^{-2}$

(vi) $\frac{[U]}{[V]} = \frac{[B^2]}{[2\mu_0]}$

$= [\mu_0] = \frac{[B^2][V]}{[U]}$

Also , $F = qVB \Rightarrow B = \frac{F}{qv}$

$[\mu_0] = \frac{(F)^2[V]}{[q^2v^2][U]} = MLT^{-2}A^{-2}$

12. [1] **13.** [3] **14.** [4] **15.** [1] **16.** [0]
17. 2.6 cm^2 (in two significant figures)
18. [9%]
19. [92.1 cm]
Least count of slide calliper is 1 mm. Hence effective length of Pendulum = 90 + 2.1 = 92.1 cm

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*a*) $\overline{x} = \frac{90+91+95+92}{4} = 92$

$\Delta x_1 = 90 - 92 = -2$

$\Delta x_2 = 91 - 92 = -1$

$\Delta x_3 = 95 - 92 = 3$

$\Delta x_4 = 92 - 92 = 0$

$|\Delta x| = \frac{2+1+3+0}{4} = 1.5$

2. (*b*) $LC = \frac{0.5}{50} = 0.01mm$

Zero error = –(50 – 45) × 0.01 *mm* = –0.05
Thickness = (0.5 + 25× 0.01) – (–0.05) = 0.80 *mm*

3. (*d*) $T = \frac{rhg}{2} \times 10^3$ N/m

$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$

$100 \times \frac{\Delta T}{T} = \left(\frac{10^{-2} \times 0.01}{1.25 \times 10^{-2}} + \frac{10^{-2} \times 0.01}{1.45 \times 10^{-2}}\right)100$

$= (0.8 + 0.689)$

$= (1.489)$

$= 100 \times \frac{\Delta T}{T} = 1.489\%$

$\simeq 1.5\%$

4. (*b*) Density $= \frac{\text{Mass}}{\text{Volume}}$

$\frac{I\Delta d}{d} = \frac{1\Delta M}{M} + \frac{3\Delta L}{L}$

$= 1.5 + 3(a)$

$= 4.5\%$

5. (*d*) $\frac{F}{A} = y.\frac{\Delta\ell}{\ell}$; $[Y] = \frac{F}{A}$

Now from dimension

$F = \frac{ML}{T^2}$; $L = \frac{F}{M}.T^2$

$L^2 = \frac{F^2}{M^2}\left(\frac{V}{A}\right)^4 \quad \because T = \frac{V}{A}$

$L^2 = \frac{F^2}{M^2A^2}\frac{v^4}{A^2} \quad F = MA$

$L^2 = \frac{V^4}{A^2}$

$[Y] = \frac{[F]}{[A]} = F^1V^{-4}A^2$

6. (*a*) $\frac{128kg}{m^3} = \frac{125(50g)(20)}{(25m)^4(4)^3}$

$= \frac{128}{64}(20) = 40$

7. (*c*) $t = G^a h^b c^c$

$\Rightarrow M^\circ L^\circ T^1 = (M^{-1} L^3 R^{-2})^a (ML^2T^{-1})^b (LT^{-1})^c$

$\Rightarrow -a + b = \Rightarrow a = b$

$\Rightarrow 3a + 2b + c = 0$

$\Rightarrow c = -5a$

$\Rightarrow -2a - b - c = 1$

$\Rightarrow a = \frac{1}{2}; b = \frac{1}{2}; c = \frac{5}{2}$

8. (*b*) $\frac{L}{RCV} = \left[A^{-1}\right]$

9. (*b*) dimension of of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2][\mu_0] = [MLT^{-2}A^{-2}]$

dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}L^{-3}T^4A^2}{MLT^{-2}A^{-2}}\right]^{\frac{1}{2}}$

$= [M^{-2}L^{-4}T^6A^4]^{1/2} = [M^{-1}L^{-2}T^3A^2]$

10. (*c*) $X = 5YZ^2$

$Y = \frac{X}{5Z^2}$

$[Y] = \frac{[X]}{[Z^2]}$

$= \frac{A^2.M^{-1}L^{-2}.T^4}{(MA^{-1}T^{-2})^2}$

$= M^{-3}.L^{-2}.T^8.A^4$

11. (*b*) $p = k\, S^a I^b h^c$

where k is dimensionless constant

$MLT^{-1} = (MT^{-2})^a (ML^2)^b (ML^2T^{-1})^c$

$a + b + c = 1$

$2b + 2c = 1$

$-2a - c = -1$

$a = \frac{1}{2} \qquad b = \frac{1}{2} \quad c = 0$

$S^{1/2}I^{1/2}h^0$

12. (*c*) $[\varepsilon_0] = M^{-1}L^{-3}\ T^4\ A^2$

$[\mu_0] = M\ L\ T^{-2}\ A^{-2}$

$[R] = M\ L^2\ T^{-3}\ A^{-2}$

$[R] = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$

13. (*c*) Zero error $= 0 + 3 \times \frac{0.5\text{mm}}{100} = 0.015$ mm

$\text{MSR} = 5.5 + 48 \times \frac{0.5}{100}$

$= 5.74$ mm.

$\therefore$ Thickness $= 5.74 - 0.015 = 5.725$ mm

14. (*c*) $v = \pi R^2 h = \frac{\pi}{4} D^2 h$

$= 4260\ \text{cm}^2$

$\because \quad \frac{\Delta v}{v} = 2\frac{\Delta D}{D} + \frac{\Delta h}{h}$

$= \left(2 \times \frac{0.1}{12.6} + \frac{0.1}{34.2}\right) v$

$= \frac{2x426}{12.6} + \frac{426}{34.2}$

$= 67.61 + 12.459 = 80.075$

$\therefore \quad v = 4260 \pm 80\ \text{cm}^3$

15. (*b*) Least count $= \frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$\Rightarrow 5 \times 10^{-6} = \frac{10^{-3}}{N} \qquad \Rightarrow N = 200$

16. (*c*) Total Area $= A_1 + A_2 + \ldots\ldots A_7$

$= A + A + \ldots\ldots 7$ times

$= 37.03\ \text{m}^2$

Addition of 7 terms all having 2 terms beyond decimal, so final answer must have 2 terms beyond decimal (as per rules of significant digits.)

17. **(Bonus)** $\rho = \frac{m}{v}$

maximum % error is S will be given by

$$\frac{\Delta\rho}{\rho} \times 100\% = \left(\frac{\Delta m}{m}\right) \times 100\% + 3\left(\frac{\Delta L}{L}\right) \times 100\% \quad \ldots\ldots\text{(i)}$$

which is only possible when error is small which is not the case in this question.

Yet if we apply equation (i), we get

$\Delta\rho = 3100\ \text{kg/m}^3$

Now, we will calculate error, without using approximation.

$$\rho_{min} = \frac{m_{min}}{v_{max}} = \frac{9.9}{(0.11)^3} = 7438\ \text{kg/m}^3$$

$$\& \ \rho_{max} = \frac{m_{max}}{v_{min}} = \frac{10.1}{(0.09)^3} = 13854.6\ \text{kg/m}^3$$

$\Delta\rho = 67416.6\ \text{kg/m}^3$

No option is matching.

Therefore this question should be awarded bonus

18. (*b*) Energy density in magnetic field $= \frac{B^2}{2\mu_0}$

$= \frac{\text{Force} \times \text{displacement}}{(\text{displacement})^3} = \frac{MLT^{-2}L}{L^3} = ML^{-1}T^{-2}$

19. (*b*) $[ML^2T^{-2}]$

$[hc] = [ML^3T^{-2}]$

$[c] = [LT^{-1}]$

$[G] = [M^{-1}L^3T^{-2}]$

20. (*c*) Energy = Force × Distance

$\Rightarrow$ $[\text{Energy}] = \frac{P}{T} \times \sqrt{A}$

$= \left[PA^{\frac{1}{2}}T^{-1} \right]$

21. (*a*) $\because$ $[\text{Young's modulus}] = \left[\frac{\text{Force}}{\text{Area}}\right]$

$\Rightarrow$ $[\text{Young's modulus}] = FA^{-1}$

$\Rightarrow$ $[\text{Young's modulus}] = FA^{-1}V^0$

22. (*d*) Solar constant $= \frac{E}{AT}$

$\frac{M^1L^2T^{-2}}{L^2T} = M^1T^{-3}$

23. (*c*) $= \frac{IFv^2}{WL^4}$

$\therefore$ $[x] = \frac{(ML^2) \times (MLT^{-2}) \times (LT^{-1})^2}{(ML^2T^{-2}) \times L^4}$

$= ML^{-1}T^{-2}$

= [Energy density]

24. (*b*) $\frac{dQ}{dt} = \frac{KA(\Delta T)}{x}$

$\Rightarrow$ $[K] = \frac{ML^2T^{-2} \times L}{T \times L^2 \times K}$

$= MLT^{-3}K^{-1}$

25. (*c*) $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = X$

$C = \frac{E}{B} = Y$

$\tau = RC = t$

$\Rightarrow$ $[X] = [Y] = [Z]$

26. (*d*) (official)

(*a*) (Reso)

$\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta g}{g} + \frac{\Delta L}{L}\right)$

$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta L}{L}$; $= 2\left(\frac{1}{50}\right) + \frac{0.1}{25.0}$

$= 4.4\,\%$

27. (*d*) Pitch $= \frac{3}{6} = 0.5\text{mm}$

L.C. $= \frac{0.5\text{mm}}{50} = \frac{1}{500}\text{mm} = 0.01\text{mm}$

$= 0.001$ cm

28. (*b*) $\rho = \frac{M}{V} \Rightarrow \left|\frac{\Delta\rho}{\rho} \times 100\right| = \left|\frac{\Delta V}{V} \times 100\right|$

$\left|\frac{\Delta\rho}{\rho} \times 100\right| = 3\alpha\Delta T \times 100$ .....(i)

Given $\frac{\Delta\ell}{\ell} = 2 \times 10^{-4}$ $\Rightarrow \alpha\Delta T = 2 \times 10^{-4}$

$\Rightarrow$ $\alpha = 2 \times 10^{-5}$

From (i), $\frac{\Delta\rho}{\rho} \times 100 = 6 \times 10^{-5} \times 10 \times 100 = 0.06$

29. (*c*) Least count of V.C $= \left(1 - \frac{9}{10}\right) \times 1\text{mm}$

$= 0.1$ mm

$\therefore$ Zero error $= 7 \times 0.1 = 0.7$ mm

positive error

Measured value $= (31 + 4 \times 0.1)$ mm

$= 31.4$ mm

$\therefore$ Length of cylinder $= 31.4 - 0.7$

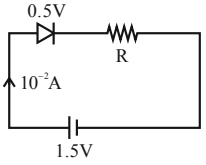
$= 30.7$ mm

$= 3.07$ cm

30. (*a*) Least count $= \frac{0.1}{50}$ cm $= 0.002$ cm

Thickness of object = Main scale Reading + Circular scale reading × least count

31. (*d*)



$R = \frac{V}{I}$

$\frac{1}{10^{-2}} = 100\Omega$

32. (*b*) $\frac{\Delta z}{z} = 2\frac{\Delta A}{A} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + 3\frac{\Delta d}{d}$

$= 2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5 = 14.5\%$

33. (*b*) $d_{av} = 5.5375\,mm$

$\Delta d = 0.07395\,mm$

$\because$ Measured data are up to two digits after decimal

$\therefore$ $d = (5.54 \pm 0.07)\,mm$

34. [1050.00]

$$\rho = \frac{m}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \qquad \therefore \quad \% \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3.\left(\frac{\Delta d}{d}\right)$$

$$= 6 + 3 \times 1.5 = 10.5\% = \left(\frac{1050}{100}\right)\%$$

35. (*b*) Reading $= MSR + VSD \times LC -$ zero error

$$\text{Reading} = 8.5 + \frac{(0.1)\times 6}{10} - \frac{0.2}{10} = 8.54\,cm$$

36. (*c*) $$\frac{\Delta Y}{Y} = \left(\frac{\Delta m}{m}\right) + \left(\frac{\Delta g}{g}\right) + \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta l}{l}\right) + \left(\frac{\Delta L}{L}\right)$$

$$= \left(\frac{1g}{1kg}\right) + 0 + 2\left(\frac{\Delta r}{r}\right) + \left(\frac{\Delta l}{l}\right) + \left(\frac{\Delta L}{L}\right)$$

$$= \left(\frac{1g}{1kg}\right) + 2\left(\frac{0.001cm}{0.2\,cm}\right) + \left(\frac{0.001cm}{0.5\,cm}\right) + \left(\frac{0.001m}{1m}\right)$$

$$= \left(\frac{1}{1000}\right) + 2\left(\frac{1\times 10}{2\times 10^3}\right) + \left(\frac{1}{5}\times\frac{10^2}{10^3}\right) + \left(\frac{1}{10^3}\right)$$

$$= \frac{1}{1000} + \frac{1}{100} + \frac{2}{10^3} + \frac{1}{10^3}$$

$$= \frac{1+10+2+1}{1000} = \frac{14}{1000}\times 100\%$$

$$= 1.4\%\,.$$

37. (*a*) $VSD = (n-1)\,MSD$

$$\Rightarrow VSD = \left(\frac{R-1}{R}\right) MSD$$

$$\Rightarrow \text{L.C.} = 1\,MSD - 1\,VSD$$

$$= 1\,MSD - \left(\frac{R-1}{R}\right) MSD$$

$$= 1\,MSD - 1\,MSD + \frac{MSSD}{R} = \frac{MSQD}{R} = \frac{a}{R}\,cm$$

$$= \frac{10R}{R}\,mm$$

JEE-ADVANCED PREVIOUS YEAR'S

1. (*a*) $d = \Delta\ell = \frac{0.5}{100}\,mm$

$$y = \frac{4MLg}{\pi\ell d^2}$$

$$\left(\frac{\Delta y}{y}\right)_{max} = \frac{\Delta\ell}{\ell} + 2\frac{\Delta d}{d}$$

error due to ℓ measurement $\frac{\Delta\ell}{\ell} = \frac{0.5/100\,mm}{0.25\ mm}$

error due to d measurement

$$2\frac{\Delta d}{d} = \frac{2\times\frac{0.5}{100}}{0.5\ mm} = \frac{0.5/100}{0.25}$$

So error in y due to ℓ measurement = error in y due to d measurement

2. (*c*) (p) $U = \frac{1}{2}kT \Rightarrow ML^2T^{-2} = [k]\,K \Rightarrow [K] = ML^2T^{-2}K^{-1}$

(q) $F = \eta A\frac{dv}{dx} \Rightarrow [\eta] = \frac{MLT^{-2}}{L^2LT^{-1}L^{-1}} = ML^{-1}\,T^{-1}$

(r) $E = h\nu \Rightarrow ML^2T^2 = [h]\,T^{-1} \Rightarrow [h] = ML^2\,T^{-1}$

(s) $\frac{dQ}{dt} = \frac{kA\Delta\theta}{\ell} \Rightarrow [k] = \frac{ML^2T^{-3}L}{L^2K} = MLT^{-3}\,K^{-1}$

3. (*b*) $50\,VSD = 2.45\,cm$

$1\,VSD = \frac{2.45}{50}\,cm = 0.049\,cm$

Least count of vernier = 1MSD – 1 VSD

$= 0.05\,cm - 0.049\,cm$

$= 0.001\,cm$

Thickness of the object = Main scale reading + vernier scale reading × least count

$= 5.10 + (24)(0.001)$

$= 5.124\,cm.$

4. (*a, b, d*) $T = 2\pi\sqrt{\frac{7}{5}\frac{(R-r)}{g}}$

$$\ln T = \ln 2\pi\sqrt{\frac{7}{5}} + \frac{1}{2}\ln(R-r) + \frac{1}{2}\ln g$$

$$\frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta R + \Delta r}{R-r}$$

$$\frac{\Delta g}{g}\times 100 = \left(2\times\frac{0.02}{0.556} + \frac{1+1}{60-10}\right)\times 100$$

$= 11.2\% \simeq 11\%$

$$\frac{\Delta r}{r}\times 100 = \frac{1}{10}\times 100 = 10\%$$

$$\frac{\Delta T}{T}\times 100 = \frac{0.02}{0.556}\times 100 = 3.57\%$$

Hence, (a, b, d)

5. (*d*) $2d\sin\theta = \lambda$

$d = \frac{\lambda}{2\sin\theta}$

differntiate

$\partial(d) = \frac{\lambda}{2}\ \partial(\text{cosec}\theta)$

$$\partial(d) = \frac{\lambda}{2}(-\cos ec\theta \cot\theta)\partial\theta$$

$$\partial(d) = \frac{-\lambda\cos\theta}{2\sin^2\theta}\partial\theta$$

as θ = increases , $\frac{\lambda\cos\theta}{2\sin^2\theta}$ decreases

2nd Method

$$d = \frac{\lambda}{2\sin\theta}$$

$$\ell n\ d = \ell n\ \lambda - \ell n\, 2 - \ell n\ \sin\theta$$

$$\frac{\Delta(d)}{d} = 0 - 0 - \frac{1}{\sin\theta}\times\cos\theta(\Delta\theta)$$

Fractional error $|+(d)| = |\cot\theta\ \Delta\theta|$

Absoulute error $\Delta d = (d\cot\theta)\ \Delta\theta$

$$\frac{d}{2\sin\theta}\times\frac{\cos\theta}{\sin\theta}$$

$$\Delta d = \frac{\cos\theta}{\sin^2\theta}$$

6. [3]

$$d = k\,(\rho)^a\,(S)^b\,(f)^c$$

$$\left[\frac{M}{L^3}\right]^a\left[\frac{M^1L^2T^{-2}}{L^2\,T}\right]^b\left[\frac{1}{T}\right]^c$$

$$0 = a + b$$

$$1 = -3a \Rightarrow a = -\frac{1}{3}$$

So $b = \frac{1}{3}$

So $n = 3$

7. (*b,d*)

$$\left[k_B T\right] = \left[Fl\right]$$

$$\&\ \left[\frac{q^2}{\in}\right] = \left[Fl^2\right]$$

8. (*d*) $R_1 = 2.8 + 0.01\times 7 = 2.87$

$$R_2 = 2.8 + (8\text{MSD} - 7\text{VSD}) = 2.8 + (8\times 0.1 - 7\times\frac{11}{10})$$

$= 2.83$

Hence, (*d*)

9. (*c*) We have $\frac{E}{C} = B$

$$\therefore\ [B] = \frac{[E]}{[C]} = [E]L^{-1}T^1$$

$$\Rightarrow\ [E] = [B]\,[L]\,[T]^{-1}$$

10. (*d*) We have

$$C = \frac{1}{\sqrt{\mu_0\in_0}}$$

$$\therefore\ [C^2] = \left[\frac{1}{\mu_0\in_0}\right]$$

$$\Rightarrow\ L^2T^{-2} = \frac{1}{[\mu_0][\in_0]}$$

$$\Rightarrow\ [\mu_0] = [\mu_0] = [\in_0]^{-1}\,[L]^{-2}\,[T]^2$$

11. (*a,b,d*)

Mass = $M^0L^0T^0$

MVr = $M^0L^0T^0$

$$M^0\frac{L^1}{T^1}.\ L^1 = M^0L^0T^0$$

$L^2 = T^1$ .....(*a*)

Force = $M^1L^1T^{-2}$ (in SI)

$= M^0L^1L^{-4}$ (In new system from equation (*a*))

$= L^{-3}$

Energy = $M^1L^2T^{-2}$ (In SI)

$= M^0L^2L^{-4}$(In new system from equation (*a*))

$= L^{-2}$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$

$= M^1L^2T^{-3}$ (in SI)

$= M^0L^2L^{-6}$(In new system from equation (*a*))

$= L^{-4}$

Linear momentum = $M^1L^1T^{-1}$ (in SI)

$= M^0L^1L^{-2}$(In new system from equation(*a*))

$= L^{-1}$

12. (c) Given 10VSD = 9MSD

$$1\text{VSD} = \frac{9}{10}\text{MSD}$$ Least count

$$= 1\text{MSD} - 1\text{VSD} = \left(1 - \frac{9}{10}\right)\text{MSD}$$

$= 0.1\text{MSD} = 0.1\times 0.1\text{cm} = 0.01\text{cm}$

As '0' of V.S. lie before ' 0 ' of M.S.

Zero error $= -[10 - 6]$ L.C.

$= -4\times 0.01\text{cm}$

$= -0.04\text{cm}$

Reading $= 3.1\text{cm} + 1\times\text{LC} = 3.4\text{cm} + 1\times 0.01\text{cm} = 3.11\text{cm}$

True diameter = Reading - Zero error

$= 3.11 - (-0.04)\text{cm} = 3.15\text{cm}$

13. (*b, d*)

$$\vec{S} = [\vec{E}\times\vec{B}]\frac{1}{\mu_0}$$

S is pointing vector denotes flow of energy per unit area per unit time

$$\vec{S} = \frac{\text{watt}}{m^2}$$

Hence *b*, *d* are correct

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

2 Vector and Calculus

Topicwise Questions

1. (*b*)

2. (*b*) $|P-Q| \le R \le |P+Q|$
If $P = 10N$ & $Q = 6N$
$4 \le R \le 16$

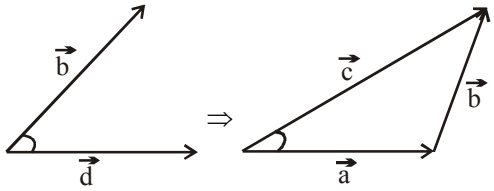
3. (*d*) Initial & final position are coincide.

4. (*d*) $\vec{S} = 6\hat{i} + 8\hat{j} - 21\hat{i}$

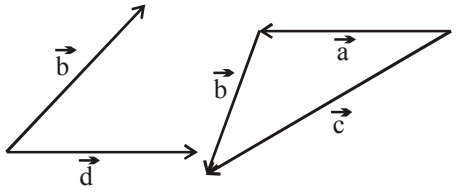
$= -15\hat{i} + 8\hat{j}$

$\therefore \quad |\vec{S}| = 17$

5. (*a*)



Now direction one interchanged



Change in direction but angle between A & B is change so no change in magnitude.

6. (*d*) $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$

$\vec{2Q} = 0$

$\vec{Q} = 0$

$|\vec{Q}| = 0$

7. (*b*) $\vec{a}_{avg} = \dfrac{\vec{v}_f - \vec{v}_i}{t} = \dfrac{5\hat{j} - 5\hat{i}}{10} = \dfrac{1}{\sqrt{2}}\left(\dfrac{\hat{j}}{\sqrt{2}} - \dfrac{\hat{i}}{\sqrt{2}}\right)$

8. (*a*) Let $R = |F_1 - F_2|$ Basically we here to find 'R'

$R^2 = |F_1 - F_2|^2$ Square

$= |F_1|^2 + |F_2|^2 - 2\vec{F}_1.\vec{F}_2$

$= 164 + 80 = 244$

9. (*c*) $\vec{AB} = \vec{OB} - \vec{OA}$

$= 2\hat{i} - 3\hat{j} + 4\hat{k} - \hat{i} - \hat{j} + \hat{k}$

$= \hat{i} - 4\hat{j} + 5\hat{k}$

10. (*a*) $\boxed{2\hat{i} - 3\hat{j}}$ $\boxed{\vec{F} = F_x\hat{i} + F_y\hat{j}}$

12. (*a*, *d*)

13. (*b*) $AB\cos\theta = 8$

$AB\sin\theta = 8\sqrt{3}$

$\tan\theta = \sqrt{3}$

$\theta = 60°$.

14. (*b*) The displacement is

$\vec{d} = ([3, 2, 4] - [2, 1, 3])\,m = [1, 1, 1]\,m.$

Then $\vec{F} \cdot \vec{d} = [(1)(1) + (-2)(1) + (0)(1)]N.m = -1\,J.]$

15. (*a*)

16. (*b*, *d*) $\vec{A} \cdot \vec{B} = AB\cos 30°$

$\vec{B} \cdot \vec{C} = BC\cos 150° = BC\cos(180° - 30°) = -BC\cos 30°$

$\Rightarrow$ If $A = B = C$

$\Rightarrow \vec{A} \cdot \vec{B} \neq \vec{B} \cdot \vec{C}$

17. (*d*) $\vec{A} \cdot \vec{B} = 15 + 2 - 2l = 0$

$2\ell = 17 \Rightarrow \ell = \dfrac{17}{2} = 8.5$

18. (*b*)

19. (*d*) From the property of vector product, we notice that $\vec{C}$ must be perpendicular to the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ and $(\vec{A} + \vec{B})$ vector also must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ must be perpendicular to $(\vec{A} + \vec{B})$ also but the cross product $(\vec{A} \times \vec{B})$ gives a vector $\vec{C}$ which can not be perpendicular to itself. Thus the last statement is wrong.

20. (*b*) $\vec{P}\times\vec{Q} = PQ\sin\theta$

required unit vector $= \dfrac{\vec{P}\times\vec{Q}}{|\vec{P}\times\vec{Q}|} = \dfrac{\vec{P}\times\vec{Q}}{PQ\sin\theta} = \dfrac{\hat{P}\times\hat{Q}}{\sin\theta}$

21. (*b*)

22. (*c*) $\dfrac{dy}{dx} = 2x+1$

23. (*d*) $\dfrac{ds}{dt} = 15t^2 - 15t^4$

24. (*b*)

25. (*a*) $\dfrac{dy}{dx} = 12x - 10 + 10x^{-3}$, $\dfrac{d^2y}{dx^2} = 12 - 30x^{-4}$

26. (*b*) $\dfrac{dr}{d\theta} = -12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5}$,

$\dfrac{d^2r}{d\theta^2} = 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$

27. (*d*) $\dfrac{d\omega}{dz} = 21z^6 - 21z^2 + 42z$,

$\dfrac{d^2\omega}{dz^2} = 126z^5 - 42z + 42$

28. (*c*) $\dfrac{dy}{dx} = \cos x - \sin x$, $\dfrac{d^2y}{dx^2} = -\sin x - \cos x$

30. (*d*) $e^x \ell nx + \dfrac{e^x}{x}$

31. (*b*) $\dfrac{dy}{dx} = 3x^2$

32. (*a*) $\sec^2 x$

33. (*a*) $y' = \dfrac{-19}{(3x-2)^2}$

34. (*b*) $\dfrac{dz}{dx} = \dfrac{-2x^2 - 2x - 2}{(x^2-1)^2}$

35. (*b*) With $u = (2x+1)$

$y = u^5 : \dfrac{dy}{dx} = \dfrac{dy}{du} = 5u^4 \,.\, 2 = 10(2x+1)^4$

36. (*d*) $\dfrac{dy}{dx} = -27(4-3x)^8$

37. (*b*) With $u = \left(1 - \left(\dfrac{x}{7}\right)\right)$ $y = u^{-7} : \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= -7u^{-8} \cdot \left(-\dfrac{1}{7}\right) = \left(1 - \dfrac{x}{7}\right)^{-8}$

38. (*b*) $\cos(x) + \dfrac{2}{x} + 2e^{2x}$

39. (*a*) $2\omega\cos(\omega x + \varphi)$

40. (*d*) $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$

41. (*a*) $y_{max} = 39$, $y_{min} = 38$

42. (*a*) $\dfrac{dy}{dx} = 48(8x-1)^2$

43. (*b*) $3\cos(3x+1)$

44. (*a*) x^2

45. (*c*) $\dfrac{x^3}{3}$

46. (*d*) $\dfrac{x^3}{3} - x^2 + x$

47. (*a*) x^{-3}

48. (*d*)

49. (*a*) $-\dfrac{5}{x}$

50. (*d*) $\sqrt{x^3}$

51. (*b*) $3\sqrt{x}$

52. (*a*) $x^{4/3}$

53. (*d*) $\dfrac{x^{\frac{2}{3}}}{2}$

54. (*d*) $\dfrac{3x^{\frac{4}{3}}}{4} + \dfrac{3x^{\frac{2}{3}}}{2}$

55. (*b*) $x^{1/2}$

56. (*a*) $x^{-1/2}$

57. (*c*)

58. (*a*) $\dfrac{1}{3}\ell nx$

59. (*d*) $-\dfrac{1}{3}\cos 3x + C$

60. (*a*) 15

61. (*a*) $\dfrac{3\pi}{2}$

62. (*a*) Using n subintervals of length $\Delta x = \dfrac{b}{n}$ and right-endpoint values : Area $= \int_0^b 2x\,dx = b^2$

Learning Plus

1. (*a*) θ will increase plan cos θ will decrease.

2. (*b*) Net displacement must be zero.

3. (*b*) $|P-Q| \le R \le (P+Q)$

4. (*d*) $|\bar{A}+\bar{B}| = \bar{A} = \bar{B}$

$A^2 + B^2 + 2AB\cos\theta = A^2$

$2A^2 + 2A^2\cos\theta^o = A^2$

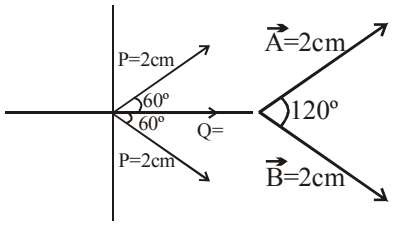
$2A^2 + 2A^2\cos\theta^o = A^2$

$2\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^o$

5. (*b*)



Resulting of $\bar{A}$ & $\bar{B}$ also have same unit 2 cm.

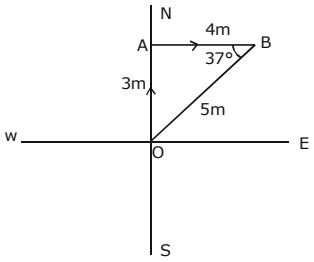
6. (*c*) $R^2 = P^2 + Q^2 + 2PQ\cos60°$

$7Q^2 = P^2 + Q^2 + PQ \Rightarrow P^2 - 6Q^2 + PQ = 0$

$$\left(\frac{P}{Q}\right)^2 + \left(\frac{Q}{Q}\right) - 6 = 0$$

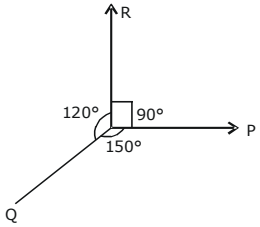
$$\Rightarrow \left(\frac{P}{Q}+3\right)\left(\frac{P}{Q}-2\right) = 0 \Rightarrow \frac{P}{Q} = 2$$

7. (*b*)



Displacement = 0

8. (*d*)

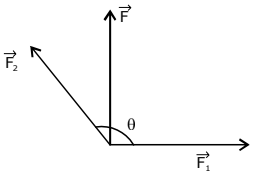


$$\frac{P}{\sin120°} = \frac{Q}{\sin90°} = \frac{R}{\sin150°}$$

$$\frac{P}{\sqrt{3/2}} = \frac{\theta}{1} = \frac{R}{\frac{1}{2}}$$

$P : Q\ R = \sqrt{3} : 2 : 1$

9. (*d*) $|\vec{F}_2| = |\vec{F}_2|$



$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} \Rightarrow \alpha = 90°$$

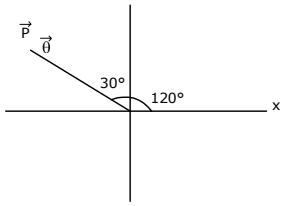
then $F_1 + F_2\cos\theta = 0 \Rightarrow \cos\theta = -\frac{1}{2}$

$\theta = 120°$

10. (*a*) $|\vec{F}_1| = |\vec{F}_2| = \text{dyne} \Rightarrow \theta = 120°$

$|\vec{F}| = 2|\vec{F}_1|\cos\theta/2 \Rightarrow 10$ dyne

11. (*a*)

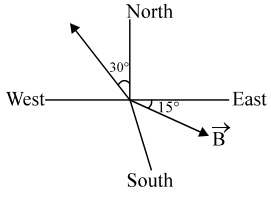


Angle b/w $\vec{P}$ & $\vec{\theta}$ is 0

so Resultant = $\sqrt{P^2 + \theta^2 + 2PQ\cos\theta}$

$|\vec{R}| = P + Q$

12. (*d*)



The direction of resultant must lie in between smaller angle between $\vec{A}$ and $\vec{B}$. There fore resultant in south direction is not possible.

13. (*d*) $(F_{net})_{max} = F_1 + F_2 = 12\ N$

$(F_{net})_{max} = F_1 - F_2 = 8N$

on solving ; $F_1 = 10\ N$ and $F_2 = 2N$

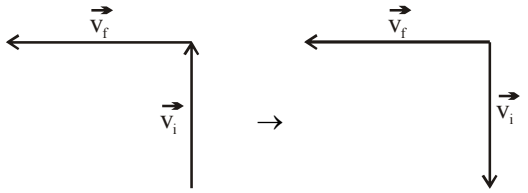
14. (*d*) $|\vec{a}+\vec{b}| = |\vec{c}|$

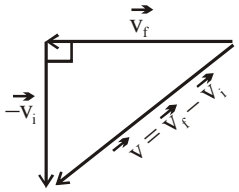
$1 + 1 + 2.1.1\cos\theta = 1$

$\cos\theta = -\frac{1}{2}h$

$\theta \to 120°$

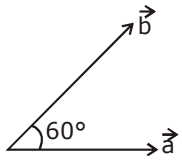
15. (*b*) $\overrightarrow{\Delta_v} = \overrightarrow{v_f} - \overrightarrow{v_i}$





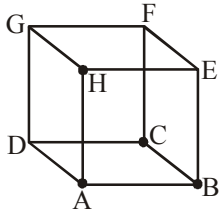
$|v_f - v_i| = 50\sqrt{2}$

16. (*b*) $|\hat{a}-\hat{b}| = \sqrt{a^2+b^2-2ab\cos\theta}$



$\theta = 60° \Rightarrow |\hat{a}-\hat{b}| = 1$

17. (*d*)



Start from A to F

$AC^2 = AB^2 + CB^2$

$AC^2 = 10^2 + 12^2$

$AC^2 = 100 + 144 = 244$

$AF^2 = AC^2 + CF^2 = 244 + 196$

$AF^2 = 440$

$AF = 20.99 \simeq 21$

18. (*c*) $\overline{A} + \overline{B} = 7\hat{i} - 3\hat{j}$

$|A+B| = \sqrt{4a+9} = \sqrt{58}$

19. (*a*) $\overline{A} - \overline{B} = 3\hat{i} - \hat{k}$

$A - B = \frac{\overline{A}-\overline{B}}{|A-B|} = \frac{3\hat{i}-\hat{k}}{\sqrt{10}}$

20. (*b*) $\vec{A} = 2\hat{i} + 3\hat{j}$

$\vec{B} = \hat{j} \Rightarrow \vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\cos\theta = \frac{3}{\sqrt{13}} \Rightarrow \tan\theta = 2/3 \Rightarrow \theta = \tan^{-1}(2/3)$

21. (*b*) $\vec{A} = 3\hat{i} + 2\hat{j} + 8\hat{k}$

$\vec{B} = 2\hat{i} + x\hat{j} + \hat{k}$

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta \quad \theta = 90°$

$\vec{A}.\vec{B} = 0$

$6 + 2x + 8 = 0,\ x = -7$

22. (*d*) $\vec{a}\cdot\vec{b} + b^2 = 0$

$a^2 + 2\vec{a}\cdot\vec{b} = 0$

$a^2 - 2b^2 = 0$

23. (*c*) $a_1 = a_2 = 1$ and $a_1^2 + a_2^2 + 2a_1a_2\cos\theta\ (\sqrt{3})^2 = 3$

Or $1 + 1 + 2\cos\theta = 3$ or $\cos\theta = \frac{1}{2}$

Now $(\vec{a}_1 - \vec{a}_2)\ (2\vec{a}_1 + \vec{a}_2) = 2a_1^2 - a_2^2 - a_1a_2\cos\theta$

$= 2 - 1 - \frac{1}{2} = \frac{1}{2}$

24. (*a*) $\vec{F} = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$ N

& $\vec{d} = (5\hat{i} + 4\hat{j} + 3\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = 3\hat{i} + \hat{j} - \hat{k}$ m

$\therefore\ W = \vec{F}\cdot\vec{d} = (2\hat{i} + 3\hat{j} + 4\hat{k})\cdot(3\hat{i} + \hat{j} - \hat{k}) = 6 + 3 - 4 = 5\ J$

25. (*d*) if $\vec{a} + \vec{b} + \vec{c} = 0$

$(\vec{a} + \vec{b} + \vec{c})\cdot(\vec{a} + \vec{b} + \vec{c}) = 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2[(\vec{a}\,\vec{b}) + (\vec{b}\,\vec{c}) + (\vec{c}\,\vec{a})] = 0$

$(\vec{a}\,\vec{b}) + (\vec{b}\,\vec{c}) + (\vec{c}\,\vec{a}) = -3/2$

26. (*a*) By the defination of equal vector.

27. (*d*) $(\overline{A}\times\overline{B}) \perp A$

$(\overline{A}\times\overline{B}) \perp B$

28. (*c*)

29. (*d*) $\left(\vec{A}+\vec{B}\right)\cdot\left(\vec{A}-\vec{B}\right)=0$[$\therefore$ two vectors are perpendicular]

$A^2-B^2=0$

$A=B$

Also $\vec{A}+\vec{B}$ & $\vec{A}-\vec{B}$ are in plane of $\vec{A}$ & $\vec{B}$ whereas $\vec{A}\times\vec{B}$ is perpendicular to both of them.

30. (*d*) $\frac{dy}{dx}=\frac{2}{x}+\cos x$, $\frac{d^2y}{dx^2}=\frac{-2}{x^2}-\sin x$

$y=\ell nx^2+\sin x$

$\Rightarrow y=\ell n\,x^2+\sin x \Rightarrow y=2\ell nx+\sin x$

$\Rightarrow \frac{dy}{dx}=\frac{2}{x}+\cos x$

also $\frac{d^2y}{dx}=\frac{-2}{x^2}-\sin x$

31. (*a*) $e^x(\tan x+\sec^2 x)$

$y=e^x\tan x \Rightarrow \frac{dy}{dx}=e^x\tan x+e^x\sec^2 x$

32. (*a*) $y=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$

$\Rightarrow \frac{dy}{dx}=\left(1-\frac{1}{x^2}\right)\left(x-\frac{1}{x}+1\right)+\left(x+\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right)$

$\Rightarrow \frac{dy}{dx}=1+2x+\frac{2}{x^3}-\frac{1}{x^2}$

33. (*c*) $x^2\cos x$

$y=x^2\sin x+2x\cos x-2\sin x$

$\Rightarrow \frac{dy}{dx}=2x\sin x+x^2\cos x+2\cos x-2x\sin x-2\cos x$

$\Rightarrow \frac{dy}{dx}=x^2\cos x$

34. (*d*) $\frac{dy}{dx}=-x^2\sin x$

$y=x^2\cos x-2x\sin x-2\cos x$

$\Rightarrow \frac{dy}{dx}=2x\cos x-x^2\sin x-2\sin x-2x\cos x+2\sin x$

$\frac{dy}{dx}=-x^2\sin x$

35. (*a*) $\frac{dr}{d\theta}=\cos\theta+\sec^2\theta$

$r=(1+\sec\theta)\sin\theta$

$\Rightarrow \frac{dx}{d\theta}=\sec\theta\tan\theta\sin\theta+(1+\sec\theta)\cos\theta$

$\Rightarrow \frac{dr}{d\theta}=\cos\theta+\sec^2\theta$

36. (*c*) $\frac{dy}{dx}=\sec^2 x$

$y=\frac{\sin x+\cos x}{\cos x}$

$\Rightarrow \frac{dy}{dx}=\frac{\cos x(\cos x-\sin x)-(\sin x+\cos x)\times-\sin x}{\cos^2 x}$

$\frac{dy}{dx}=\frac{1}{\cos^2 x}=\sec^2 x$

37. (*b*)

$p=\frac{\tan q}{1+\tan q}$

$\Rightarrow \frac{dp}{dq}=\frac{(1+\tan q)\sec^2 q-\tan q\sec^2 q}{(1+\tan q)^2}$

$\Rightarrow \frac{dp}{dq}=\frac{\sec^2 q}{(1+\tan q)^2}$

38. (*a*) $y=\sin^3x+\sin3x$

$\Rightarrow y=y_1+y_2 \Rightarrow \frac{dy}{dx}=\frac{dy_1}{dx}+\frac{dy_2}{dx}$

$\Rightarrow y_1=\sin^3x$

$\Rightarrow y_1=u^3 \qquad u=\sin x$

$\Rightarrow \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

$\Rightarrow \frac{dy_1}{du}=3u^2 \qquad \frac{du}{dx}=\cos x$

$\Rightarrow \frac{dy_1}{dx}=3\sin^2 x\cos x$

$\Rightarrow y_2=\sin3x$

$\Rightarrow y_2 = \sin v \Rightarrow \frac{dy_2}{dv} = \cos v \quad v = 3x$

$\Rightarrow \frac{dy_2}{dx} = 3\cos 3x \qquad \frac{dv}{dx} - 3$

$\Rightarrow \frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$

$\Rightarrow \frac{dy}{dx} = 3\sin^2 x \cos x + 3\cos 3x$

39. (*d*) $y = \sin^2(x^2+1)$

$\Rightarrow y = u^2 \qquad u = \sin v \quad v = x^2 + 1$

$\Rightarrow \frac{dy}{du} = 2u \qquad \frac{du}{dv} = \cos v$

$\Rightarrow \frac{dv}{dx} = 2x \quad \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = 2u\cos v 2x \Rightarrow 4x\sin(x^2+1)\cos(x^2+1)$

40. (*b*) $\frac{1-r}{\sqrt{2r-r^2}}$

$\Rightarrow q = \sqrt{2r-r^2}$

$\Rightarrow q = \sqrt{u} \qquad u = 2r - r^2$

$\Rightarrow \frac{dq}{du} = \frac{1}{2\sqrt{u}} \qquad \frac{du}{dr} = 2 - 2r$

$\Rightarrow \frac{dq}{dr} = \frac{dq}{du}.\frac{du}{dr} \Rightarrow = \frac{1}{2\sqrt{u}}.(2-2r)$

$\Rightarrow \frac{1-r}{\sqrt{2r-r^2}}$

41. (*c*) $x = 30$ & $y = 30$

$x + y = 60$

$\Rightarrow y = 60 - x \qquad \Rightarrow xt = 60x - x^2$

$\Rightarrow z = 60x - x^2 \qquad \Rightarrow \frac{dz}{dx} = 60 - 2x$

$\Rightarrow \frac{dz}{dx} = 0 \Rightarrow x = 30 \qquad \Rightarrow \frac{d^2z}{dx^2} = -\text{ve maximum}$

$\Rightarrow x = 30 \qquad y = 30$

42. (*a*) $\frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$

43. (*b*) $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

44. (*c*) $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$

45. (*a*)

46. (*d*) $-\cos\theta + \theta + C$

47. (*a*) $\frac{3\pi^2}{2}$

48. (*a*)

49. (*d*) $\frac{1}{3}\ell n\frac{5}{2}$

Advanced Level Multiconcept Questions

1. (*a*,*b*,*d*)

2. (*a*,*b*,*c*) By right handed co-ordinate system.

3. (*c*) $x = t^3 - 3t^2 + 12t + 20$

$v = \frac{dx}{dt} = 3t^2 - 6t + 12$

$t = 0 \Rightarrow v = 12$ m/s

4. (*d*) $a = \frac{dv}{dt} = 6t - 6$

$t = 0 \Rightarrow a = -6$ m/s^2

5. (*d*) $a = 0 \Rightarrow t = 1$ sec.

$v = 3t^2 - 6t + 12 = 9$ m/s

6. (*a*) $\vec{F}_R = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + 5\hat{j} + 4\hat{k}$

7. (*b*) $\cos\theta = \frac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_1||\vec{F}_2|} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$

8. (*c*) : $F_1\cos\theta = \frac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_2|} = \frac{6}{5}$

9. (*a*) $\rightarrow$ r, (*b*) $\rightarrow$ p, (*c*) $\rightarrow$ q, (*d*) $\rightarrow$ s

(A) $\int \sec x \tan x\,dx = \sec x + C$

(B) $\int \text{cosec } kx \cot kx\,dx = \frac{\cos kx}{k} + C$

(C) $\int \text{cosec}^2 kx\,dx = -\frac{\cot kx}{k} + C$

(D) $\int \cos dx = \frac{\sin kx}{k} + C$

10. (*a*) $\rightarrow$ q, (*b*) $\rightarrow$ r, (*c*) $\rightarrow$ p, (*d*) $\rightarrow$ s

(A) $|\vec{A} + \vec{B}| = A^2 + B^2 + 2AB\cos\theta$

$A^2 = A^2 + A^2 + 2A^2\cos\theta \cos\theta = -\frac{1}{2} \quad \theta = 120$

(B) $F_1 \sim F_2 \le R \le F_1 + F_2$

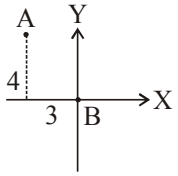
Here $F_1 \sim F_2 = 4$ and $F_1 + F_2 = 12$

(C) $\cos\theta = \dfrac{\vec{A}-\vec{B}}{|A|+|B|} = \dfrac{0}{2\sqrt{2}\times 3} = 0 \; \theta = 90^\circ$

(D) $\vec{A}+\vec{B} = 2\hat{i}+\hat{j}+3\hat{k}$

$|\vec{A}+\vec{B}| = \sqrt{2^2+1+3^2} = \sqrt{14}$

11. [0036]



$\vec{r}_f - \vec{r}_i = \vec{v}t$

$\vec{r}_f - \left[3(\hat{i})+4(\hat{j})\right] = 2\left(\dfrac{3\hat{i}+4\hat{j}}{5}\right)\times 5$

$\vec{r}_f + 3\hat{i} - 4\hat{j} = 6\hat{i}+8\hat{j}$

$\therefore \; \vec{r}_f = 3\hat{i}+12\hat{j}$

12. [0003] $a^2 - 2a - 3 = 0$

$a^2 - 3a + a - 3 = 0$

$a(a-3)+1(a-3) = 0$

$\therefore \; a = 3$

13. [1] (Given $|A| = 3$)

$|\vec{A}| - |\vec{B}| = 2$

$|\vec{B}| > |\vec{A}|$

$|\vec{B}| = 1$

14. [2] $\vec{v}_1.\vec{v}_2 = 0 \Rightarrow t = 2\,\text{sec}$

15. [1] $|\vec{A}|+|\vec{B}| = 5$

$2+|\vec{B}| = 5$

resultant of $|\vec{A}|$ and $|\vec{B}|$

$|\vec{B}| - |\vec{A}| = 3-2 = 1$

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*b*) $E_1 = -\dfrac{GMm}{R+h} - \left(-\dfrac{GMm}{R}\right)$

$E_2 = -\dfrac{1}{2}m\left(\sqrt{\dfrac{GM}{R+h}}\right)^2 = -\dfrac{GMm}{2(R+h)}$

$E_1 = E_2 \; ; h = \dfrac{R}{2}$

2. (*a*) $|\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta$
$= n^2(A^2 + B^2 - 2AB\cos\theta)$

$\Rightarrow \cos\theta(1+n^2) = \dfrac{2a^2(n^2-1)}{2a^2} \quad [A = B = a]$

$\cos\theta = \dfrac{n^2-1}{n^2+1}$

3. (*b*) $1\left(0, \dfrac{a}{2}, \dfrac{a}{2}\right)$

$2\left(\dfrac{a}{2}, \dfrac{a}{2}, 0\right)$

$\vec{r}_2 - \vec{r}_1 = \dfrac{a}{2}\hat{i} - \dfrac{a}{2}\hat{k}$

Unit vector $= \dfrac{\hat{i}-\hat{k}}{\sqrt{2}}$

4. (*a*) $2|\vec{P}+\vec{Q}| = |\vec{P}+2\vec{Q}|$

$\Rightarrow 13 + 12\cos\theta = 10 + 6\cos\theta$

$\cos = -\dfrac{1}{2}$

$\theta = 120^\circ.$

5. (*c*) $|\vec{A}_1| = 3 \qquad |\vec{A}_2| = 5 \qquad |\vec{A}_1 + \vec{A}_2| = 5$

$|\vec{A}_1 + \vec{A}_2| = \sqrt{|\vec{A}_1|^2 + |\vec{A}_2|^2 + 2|\vec{A}_1||\vec{A}_2|\cos\theta}$

$5 = \sqrt{9+25+2\times 3\times 5\cos\theta}$

$\cos\theta = -\dfrac{9}{2\times 3\times 5} = -\dfrac{3}{10}$

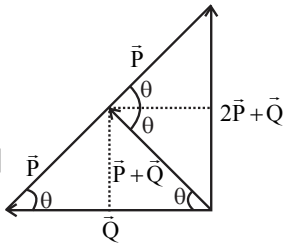
$(2\vec{A}_1 + 3\vec{A}_2).(3\vec{A}_1 - 2\vec{A}_2)$

$= 6|\vec{A}_1|^2 + 9\vec{A}_1.\vec{A}_2 - 4\vec{A}_1\vec{A}_2 - 6|\vec{A}_2|^2$

$$54 + 5 \times 3 \times 5\left(-\frac{3}{10}\right) - 6 \times 25$$

$$= 54 - 150 - \frac{45}{2} = -118.5$$

6. [90°]



So angle between $(2\vec{P}+\vec{Q})$ and $\vec{Q}$ is 90°

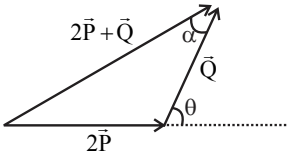
Alternate solution

$$\left|\vec{P}+\vec{Q}\right| = \left|\vec{P}\right|$$

$$P^2 + Q^2 + 2PQ\cos\theta = P^2$$

$$\Rightarrow Q + 2P\cos\theta = 0$$

$$\Rightarrow \cos\theta = -\frac{Q}{2P}$$



$$\tan\alpha = \frac{2P\sin\theta}{2P\cos\theta + Q} = \infty \because [2P\cos\theta + Q = 0]$$

$$\alpha = 90^o$$

7. (*b*) According to question the given data is two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$

$$\vec{A} = \vec{P} + \vec{Q} \Rightarrow \vec{B} = \vec{P} - \vec{Q} \Rightarrow \vec{P} \perp \vec{Q}$$

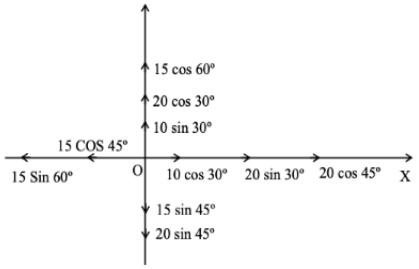
$$\Rightarrow |\vec{A}| = |\vec{B}| = \sqrt{2\left(P^2 + Q^2\right)(1+\cos\theta)}$$

$$\Rightarrow \text{For } |\vec{A}+\vec{B}| = \sqrt{3\left(P^2+Q^2\right)} \Rightarrow \theta_1 = 60^\circ$$

$$\Rightarrow \text{For } |\vec{A}+\vec{B}| = \sqrt{2\left(P^2+Q^2\right)}n \Rightarrow \theta_2 = 90^\circ n$$

According to solution Both Statement I and Statement II are true

8. (*a*)



Force in x-direction,

$$\vec{F_x} = \left(10 \times \frac{\sqrt{3}}{2} + 20\left(\frac{1}{2}\right) + 20\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{\sqrt{3}}{2}\right)\right)\hat{i}$$

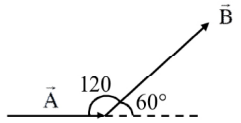
$$= 9.25\hat{i}$$

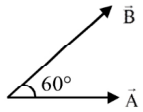
Force in y-direction,

$$\vec{F_y} = \left(15\left(\frac{1}{2}\right) + 20\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 20\left(\frac{1}{\sqrt{2}}\right)\right)\hat{j}$$

$$= 5\hat{j}$$

9. (*c*)





According to digram,

Angle between $\vec{A}$ and $\vec{B}, \theta = 60^\circ$

Angle between $\vec{A}$ and $-\vec{B}, \theta = 120^\circ$

If angle between $\vec{A}$ and $\vec{A}-\vec{B}$ is α

then $\tan\alpha = \frac{|-\vec{B}|\sin\theta}{\vec{A}+|-\vec{B}|_{\cos\theta}}$

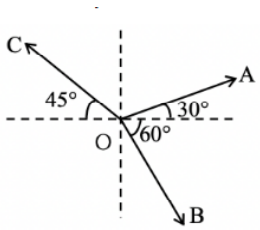
$$= \frac{B\sin 120^\circ}{A + B\cos 120^\circ}$$

$$= \frac{B\frac{\sqrt{3}}{2}}{A - \frac{B}{2}}$$

$$\Rightarrow \tan\alpha = \frac{\sqrt{3}B}{2A - B}$$

Hence, the angle between vector $(\vec{A})$ and $(\vec{A}-\vec{B})$ is $\tan\alpha = \frac{\sqrt{3}B}{2A-B}$

10. (*a*)



According to diagram, let magnitude be equal to

$$\overrightarrow{OA} = \lambda\left[\cos 30^\circ \hat{i} + \sin 30 \hat{j}\right] = \lambda\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$

$$\overrightarrow{OB} = \lambda\left[\cos 60^\circ \hat{i} - \sin 60 \hat{j}\right] = \lambda\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$$

$$\overrightarrow{OC} = \lambda\left[\cos 45^\circ (-\hat{i}) + \sin 45 \hat{j}\right] = \lambda\left[-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$$

$$\therefore \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$$

$$= \lambda\left[\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)\hat{i} + \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\hat{j}\right]$$

$$\tan^{-1}\left[\frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}}\right] = \tan^{-1}\left[\frac{\sqrt{2} - \sqrt{6} - 2}{\sqrt{6} + \sqrt{2} + 2}\right]$$

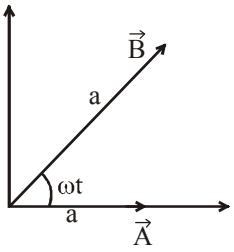
$$= \tan^{-1}\left[\frac{1 - \sqrt{3} - \sqrt{2}}{\sqrt{3} + 1 + \sqrt{2}}\right]$$

Hence option (*a*).

JEE-ADVANCED PREVIOUS YEAR'S

1. (*d*) $\vec{S} = \vec{P} + b\vec{R} = \vec{P} + b\left(\vec{Q} - \vec{P}\right) = \vec{P}(1-b) + b\vec{Q}$

2. [2.00 sec]



$$|\vec{A} + \vec{B}| = 2a\cos\frac{\omega t}{2}$$

$$|\vec{A} + \vec{B}| = 2a\sin\frac{\omega t}{2}$$

$$\text{So, } 2a\cos\frac{\omega t}{2} = \sqrt{3}\left(2a\sin\frac{\omega t}{2}\right)$$

$$\tan\frac{\omega t}{2} = \frac{\pi}{6} \Rightarrow \omega t = \frac{\pi}{3}$$

$$\frac{\pi}{6}t = \frac{\pi}{3} \qquad t = 2.00 \text{ sec}$$

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

3 Motion in a Straight Line

Topicwise Questions

1. (*a*) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad \therefore r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{6^2 + 8^2 + 10^2} = 10\sqrt{2}\text{m}$

2. (*a*) $\vec{r} = 20\hat{i} + 10\hat{j}$

$\therefore r = \sqrt{20^2 + 10^2} = 22.5\text{m}$

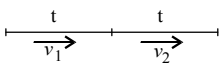
3. (*d*) As the total distance is divided into two equal parts

therefore distance average speed $= \dfrac{2v_1v_2}{v_1 + v_2}$.

4. (*c*) Distance average speed $= \dfrac{2v_1v_2}{v_1 + v_2}$

$= \dfrac{2\times 30\times 50}{30+50} = \dfrac{75}{2} = 37.5\,\text{km/hr.}$

5. (*b*)



$V_{Ar} = \dfrac{S}{T} = \dfrac{v_1 t + v_2 t}{2t} = \dfrac{v_1 + v_2}{2}$

6. (*d*) Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{x}{\frac{2x/5}{v_1} + \frac{3x/5}{v_2}} = \dfrac{5v_1v_2}{3v_1 + 2v_2}.$

7. (*c*) From given figure, it is clear that the net displacement is zero. So average velocity will be zero.

8. (*b*) As $S = ut + \dfrac{1}{2}at^2 \therefore S_1 = \dfrac{1}{2}a(10)^2 = 50a$ .....(i)

As $v = u + at$

velocity acquired by particle in 10 sec $v = a \times 10$

For next 10 sec, $S_2 = (10a)\times 10 + \dfrac{1}{2}(a)\times(10)^2$

$S_2 = 150a$ .....(ii)

From (i) and (ii) $S_1 = S_2/3$

9. (*a*) From $S = ut + \dfrac{1}{2}at^2$

$S_1 = \dfrac{1}{2}a(P-1)^2$ and $S_2 = \dfrac{1}{2}aP^2\,[\text{As } u = 0]$

From $S_n = u + \dfrac{a}{2}(2n-1)$

$S_{\left(P^2-P+1\right)^{th}} = \dfrac{a}{2}\left[2\left(P^2 - P + 1\right) - 1\right] = \dfrac{a}{2}\left[2P^2 - 2P + 1\right]$

It is clear that $S_{\left(P^2-P+1\right)^{th}} = S_1 + S_2.$

10. (*b*) $v = 4t^3 - 2t$ (given) $\therefore a = \dfrac{dv}{dt} = 12t^2 - 2$

and $x = \int_0^t v\,dt = \int_0^t \left(4t^3 - 2t\right)dt = t^4 - t^2$

When particle is at 2m from the origin $t^4 - t^2 = 2$

$\Rightarrow t^4 - t^2 - 2 = 0\,\left(t^2 - 2\right)\left(t^2 + 1\right) = 0 \Rightarrow t = \sqrt{2}\,\text{sec}$

Acceleration at $t = \sqrt{2}$ sec given by,

$a = 12t^2 - 2 = 12\times 2 - 2 = 22\text{m/s}^2.$

11. (*b*) Let u_1, u_2, u_3 and u_4 be velocities at time t = 0, t_1, $(t_1 + t_2)$ and $(t_1 + t_2 + t_3)$ respectively and acceleration is a then

$v_1 = \dfrac{u_1 + u_2}{2}, v_2 = \dfrac{u_2 + u_3}{2}$ and $v_3 = \dfrac{u_3 + u_4}{2}$

Also $u_2 = u_1 + at_1, u_3 = u_1 + a\left(t_1 + t_2\right)$

and $u_4 = u_1 + a\left(t_1 + t_2 + t_3\right)$

By solving, we get $\dfrac{v_1 - v_2}{v_2 - v_3} = \dfrac{(t_1 + t_2)}{(t_2 + t_3)}.$

12. (*b*) From $v^2 = u^2 + 2aS$

$\Rightarrow 0 = u^2 + 2aS$

$\Rightarrow a = \dfrac{-u^2}{2S} = \dfrac{-(20)^2}{2\times 10} = -20\text{m/s}^2.$

13. (*b*) Constant velocity means constant speed as well as same direction throughout.

14. (*d*) $S \propto u^2$ If u becomes 3 times then S will become 9 times i.e. $9 \times 20 = 180m$.

15. (*c*) Instantaneous velocity $v = \frac{\Delta x}{\Delta t}$

By using the data from the table

$$v_1 = \frac{0-(-2)}{1} = 2m/s, \quad v_2 = \frac{6-0}{1} = 6m/s$$

$$v_3 = \frac{16-6}{1} = 10m/s.$$

16. (*c*) For First part,
$u = 0$, $t = T$ and acceleration $= a$

$$\therefore v = 0 + aT = aT \text{ and } S_1 = 0 + \frac{1}{2}aT^2 = \frac{1}{2}aT^2$$

For Second part,
$u = aT$, retardation $= a_1$, $v = 0$ and time taken $= T_1$ (let)

$$\therefore 0 = u - a_1T_1 \Rightarrow aT = a_1T_1$$

and from $v^2 = u^2 - 2aS_2$

$$\Rightarrow S_2 = \frac{u^2}{2a_1} = \frac{1}{2}\frac{a^2T^2}{a_1} \quad (v = aT)$$

$$S_2 = \frac{1}{2}aT \times T_1 \left(\text{As } a_1 = \frac{aT}{T_1}\right)$$

$$V_{Ar} = \frac{S_1 + S_2}{T + T_1} = \frac{aT}{2}$$

17. (*c*) $v = (180 - 16x)^{1/2}$

As $a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt}$

$$\therefore a = \frac{1}{2}(180-16x)^{-1/2} \times (-16)\left(\frac{dx}{dt}\right)$$

$$= -8(180-16x)^{-1/2} \times v$$

$$= -8(180-16x)^{-1/2} \times (180-16x)^{1/2} = -8m/s^2.$$

18. (*d*) $x = ae^{-\alpha t} + be^{\beta t}$

Velocity $v = \frac{dx}{dt} = \frac{d}{dt}\left(ae^{-at} + be^{\beta t}\right)$

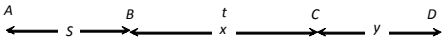
$$= a.e^{-\alpha t}\left(-\alpha + be^{\beta t}.\beta\right) = -a\,\alpha e^{-\alpha t} + b\beta e^{\beta t}$$

Acceleration $= -a\,\alpha e^{\alpha t}(\alpha) + b\beta e^{bt}.\beta$

$$= a\alpha^2\, e^{-\alpha t} + b\beta^2\, e^{\beta t}$$

Acceleration is positive so velocity goes on increasing with time.

19. (*c*) Let car starts from point A from rest and moves up to point B with acceleration f.



Velocity of car at point B, $v = \sqrt{2fS}$

[As $v^2 = u^2 + 2as$]

Car moves distance BC with this constant velocity in time t

$x = \sqrt{2fS}.t$ .....(i) [As $s = ut$]

So the velocity of car at point C also will be $\sqrt{2fs}$ and finally car stops after covering distance y.

Distance CD $\Rightarrow y = \frac{\left(\sqrt{2fS}\right)^2}{2(f/2)} = \frac{2fS}{f} = 2S$ ....(ii)

$$[\text{As } v^2 = u^2 - 2as \Rightarrow s = u^2/2a]$$

So, the total distance $AD = AB + BC + CD = 15S$ (given)

$\Rightarrow S + x + 2S = 15S \Rightarrow x = 12S$

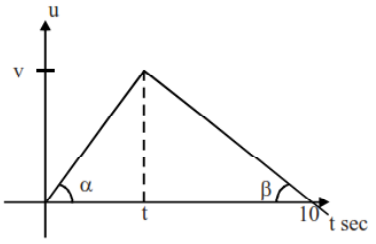
Substituting the value of x in equation (i) we get

$x = \sqrt{2fS}.t \Rightarrow 12S = \sqrt{2fS}.t$

$\Rightarrow 144S^2 = 2fS.t^2$

$\Rightarrow S = \frac{1}{72}ft^2$.

20. (*a*)



$\tan\alpha = 2$, $\tan\beta = 3$

$$2 = \frac{v}{t}, \qquad 3 = \frac{v}{10-t}$$

$v = 2t$, $v = 30 - 3t$

$30 - 3t = 2t \quad \Rightarrow t = 6$ sec

$v = 12$ m/s

21. (*c*) $u = 12$ m/s, $g = 9.8$ m/sec^2, $t = 10$ sec

Displacement $= ut + \frac{1}{2}gt^2$

$$= 12 \times 10 + \frac{1}{2} \times 9.8 \times 100 = 610m.$$

22. (*c*) $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{2h/g}$

$$t_a = \sqrt{\frac{2a}{g}} \text{ and } t_b = \sqrt{\frac{2b}{g}} \Rightarrow \frac{t_a}{t_b} = \sqrt{\frac{a}{b}}$$

23. (*c*) Vertical component of velocities of both the balls are same and equal to zero. So $t = \sqrt{\frac{2h}{g}}$.

24. (*a*) $h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times (4)^2 = 80\text{m}.$

25. (*d*) Let the body after time t/2 be at x from the top, then

$$x = \frac{1}{2}g\frac{t^2}{4} = \frac{gt^2}{8} \quad \ldots(i)$$

$$h = \frac{1}{2}gt^2 \quad \ldots(ii)$$

Eliminate t from (i) and (ii), we get $x = \frac{h}{4}$

$\therefore$ Height of the body from the ground $= h - \frac{h}{4} = \frac{3h}{4}$.

26. (*c*) Force down the plane = mg sin θ

$\therefore$ Acceleration down the plane = g sin θ

Since $l = 0 + \frac{1}{2}g\sin\theta t^2$

$$\therefore t^2 = \frac{2l}{g\sin\theta} = \frac{2h}{g\sin^2\theta} \Rightarrow t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}.$$

27. (*a*) Height travelled by ball (with balloon) in 2 sec

$$h_1 = \frac{1}{2}a\,t^2 = \frac{1}{2}\times 4.9\times 2^2 = 9.8\text{m}$$

Velocity of the balloon after 2 sec

$v = a\,t = 4.9\times 2 = 9.8\,\text{m/s}$

Now if the ball is released from the balloon then it acquire same velocity in upward direction.

Let it move up to maximum height h_2

$$v^2 = u^2 - 2gh_2 \Rightarrow 0 = (9.8)^2 - 2\times(9.8)\times h_2 \therefore h_2 = 4.9\text{m}$$

Greatest height above the ground reached by the ball $= h_1 + h_2 = 9.8 + 4.9 = 14.7$ m.

28. (*d*) Given $a = 19.6\,\text{m/s}^2 = 2g$

Resultant velocity of the rocket after 5 sec

$v = 2g\times 5 = 10g$ m/s

Height achieved after 5 sec, $h_1 = \frac{1}{2}\times 2g\times 25 = 245\text{m}$

On switching off the engine it goes up to height h_2 where its velocity becomes zero.

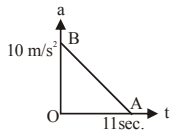
$0 = (10g)^2 - 2gh_2 \Rightarrow h_2 = 490$ m

$\therefore$ Total height of rocket = 245 + 490 = 735 m.

29. (*b*) The time of fall is independent of the mass.

30. (*c*) Speed of the object at reaching the ground $v = \sqrt{2gh}$.

31. (*b*) The area under acceleration time graph gives change in velocity. As acceleration is zero at the end of 11 sec

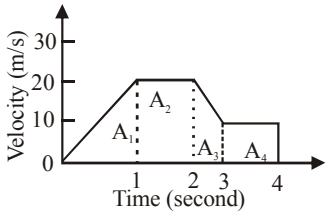


i.e. v_{max} = Area of ΔOAB

$$= \frac{1}{2}\times 11\times 10 = 55\,\text{m/s}$$

32. (*b*) Distance = Area under

v -- t graph = $A_1 + A_2 + A_3 + A_4$



$$= \frac{1}{2}\times 1\times 20 + (20\times 1) + \frac{1}{2}(20+10)\times 1 + (10\times 1)$$

$= 10 + 20 + 15 + 10 = 55$m.

33. (*d*) Maximum acceleration means maximum change in velocity in minimum time interval.

In time interval t =30 to t = 40 sec

$$a = \frac{\Delta v}{\Delta t} = \frac{80-20}{40-30} = \frac{60}{10} = 6\,\text{cm/sec}^2.$$

34. (*c*) Area of trapezium $= \frac{1}{2}\times 3.6\times(12+8) = 36.0\text{m}.$

35. (*d*) Slope of displacement time graph is negative only at point E.

36. (*a*) This graph shows uniform motion because line having a constant slope.

37. (*a*) For the given condition initial height h = d and velocity of the ball is zero. When the ball moves downward its velocity increases and it will be maximum when the ball hits the ground & just after the collision it becomes half and in opposite direction. As the ball moves upward its velocity again decreases and becomes zero at height d/2. This explanation match with graph (*a*).

38. (*d*) For upward motion

Effective acceleration = – (g + a)

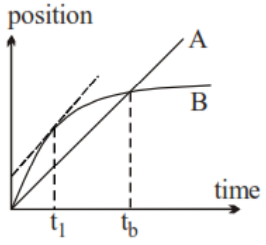
and for downward motion

Effective acceleration = (g – a)

But both are constants. So the slope of speed-time graph will be constant.

39. (*a*) Since slope of graph remains constant for velocity-time graph.

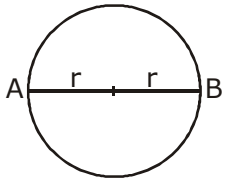
40. (*c*)



(*c*) Both have same velocity at some time before t_b at time t_1

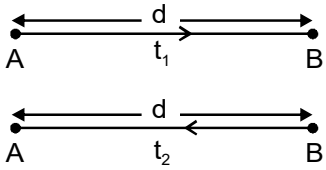
JEE-MAIN OBJECTIVE QUESTIONS

1. (*b*)



Displacement = 2r
distance = πr

2. (*b*) From A to B $t_1 = \frac{d}{20}$ hr $\Rightarrow$ From B to A $t_2 = \frac{d}{30}$ hr



$\therefore$ Average Speed $= \frac{\text{Total Distance}}{\text{Total Time}}$

$= \frac{2d}{t_1 + t_2} = \frac{2d}{\frac{d}{20} + \frac{d}{30}} \Rightarrow v = 24$ km/hr

3. (*b*) $t = 62.8$ sec

in each lap car travel a distance $= 2\pi R$

$= 2 \times 3.14 \times 100 = 628$ m

In each lap displacement of the car = 0

Average speed

$= \frac{\text{Total Distance}}{\text{Total Time}} = \frac{628}{62.8} = 10$ m/s

Average Velocity $= \frac{\text{Total Displacement}}{\text{Total Time}} = 0$

4. (*a*) $2s = gt^2 \Rightarrow s = \frac{1}{2}gt^2$

$v = \frac{ds}{dt} = gt$

5. (*c*) $V_{inst} = \frac{dx}{dt}$ (slop of x-t graph)

At C $\tan\theta$ =+ve At E $\theta > 90^\circ (-\text{ve slop})$

At D $\theta = 0^\circ$ At F $\theta < 90^\circ (+\text{ve slope})$

$\therefore$ At D, V = 0 As ten $\theta = 0$

6. (*c*) let the acceleration of the body is a and $u = 0$

then $x_1 = \frac{1}{2}at^2 = \frac{1}{2}a(10)^2$

$x_2 = \frac{1}{2}a(20)^2 - x_1 \Rightarrow x_2 = \frac{1}{2}a(20)^2 - \frac{1}{2}a(10)^2$

$= \frac{1}{2}a(10)(30) \Rightarrow x_3 = \frac{1}{2}a(30)^2 - \frac{1}{2}a(20)^2$

$= \frac{1}{2}a(10)(50) \Rightarrow \therefore x_1 : x_2 : x_3 = 1 : 3 : 5$

7. (*b*) $S_2 = \frac{1}{2} \times a \times 4$

$S_3 = \frac{1}{2} \times a \times 9$

$S_4 = \frac{1}{2} \times a \times 16$

$S_5 = \frac{1}{2} \times a \times 25$

distance travelled by body in 4th sec $= \frac{1}{2}a[7]$

distance travelled by body in 5th sec $= \frac{1}{2}a[9]$

ratio = 7 : 9

8. (*d*) Let constant acceleration = a

$S = \frac{1}{2}at^2$

$S_1 = \frac{1}{2}a \times 10^2 = 50a$

$S_2 = \frac{1}{2}a \times 20^2 - \frac{1}{2}a \times 10^2 = 150a$

$S_2 = 3S_1$

9. (*b*) In inclined initial u = 0

$S = \frac{1}{2}at^2$ and $a = g\sin\theta$

$l = \frac{1}{2}g\sin\theta \times (4)^2$ ...(i)

$\frac{\ell}{4} = \frac{1}{2}g\sin\theta\, t^2$ ...(ii)

From (i) and (ii)

t = 2 sec

10. (*c*) $\frac{h}{2} = \frac{1}{2}gt_1^2$ ....(*a*)

$h = \frac{1}{2}g(t_1 + t_2)^2$ ....(*b*)

From equation (*a*) and (*b*)

$2t_1^2 = (t_1 + t_2)^2$

$\sqrt{2}t_1 = t_1 + t_2$

$(\sqrt{2} - 1)t_1 = t_2$

$t_1 = \frac{t_2}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$t_1 = \left(\sqrt{2}+1\right) t_2$

11. (*d*) At H_{max}, $v = 0$

Acceleration constant & it is due to gravity

$|a| = g$

12. (*d*) $v = \frac{ds}{dt} = 3t^2 - 12t + 3$

$a = \frac{dv}{dt} = 6t - 12$

$a = 0 \Rightarrow t = 2\text{sec}$

$V_{2sec} = 3(2)^2 - 12(2) + 3 = +12 - 24 + 3$

$= -9$ m/s

13. (*c*) $\frac{dv}{dt} = 6 - 3v \Rightarrow \frac{dv}{6-3v} = dt$

Integrating both sides, $\int \frac{dv}{6-3v} = \int dt$

$\Rightarrow \frac{\log_e(6-3v)}{-3} = t + K_1$

$\Rightarrow \log_e(t - 3v) = -3t + K_2$ ...(i)

Att = 0, v = 0 \ $\log_e 6 = K_2$

Substituting the value of K_2 in equation (i)

$\log_e(t - 3v) = -3t + \log_e 6$

$\Rightarrow \log_e\left(\frac{6-3v}{6}\right) = -3t$ Þ $e^{-3t} = \frac{6-3v}{6}$

$\Rightarrow t - 3v = 6e^{-3t}$ Þ $3v = 6(1 - e^{-3t})$

$\Rightarrow v = 2(1 - e^{-3t})$

$\therefore v_{trminal} = 2$ m/s (When t = ¥)

Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}\left[2\left(1 - e^{-3t}\right)\right] = 6e^{-3t}$

Initial acceleration = 6 m/s^2.

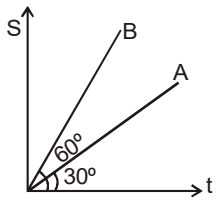
14. (*c*) From graph it is clear that velocity is always positive during its motion

so displacement = distance

displacement = Area under V-t curve

$= \frac{1}{2} \times 20 \times 1 + 20 \times 1 + \frac{1}{2} \times 1 \times 30 + 1 \times 10 \Rightarrow 55$ m

15. (*d*)



$\frac{V_A}{V_B} = \frac{\tan 30°}{\tan 60°} \Rightarrow \therefore \frac{V_A}{V_B} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

16. (*d*) Total Distance = Area under the curve (Position + Negative)

$= \frac{1}{2} \times 4 \times 1 + 4 \times 2 + 1 \times 4 \times \frac{1}{2} - \frac{1}{2} \times 2 \times 1 - 2 \times 2 - \frac{1}{2} \times 1 \times 2$

$= 2 + 8 + 2 - 1 - 4 - 1 = 6$ meter

17. (*a*) (acceleration) = Slope = $\frac{\Delta v}{\Delta t}$

$OA \rightarrow \frac{10}{10} = 1$

$AB \rightarrow 0 = 0$

$BC \rightarrow \frac{-10}{20} = -0.5$

18. (*b*) Distance = Total Area = 105 m

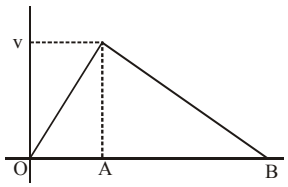
Displacement = 90 - 15 = 75m

(–ve y asxis area) – (–ve y axis area)

19. (*b*) Avg. velocity = for OA = $\frac{O + v}{2}$

For OB = $\frac{v + O}{2}$ constant acceleration

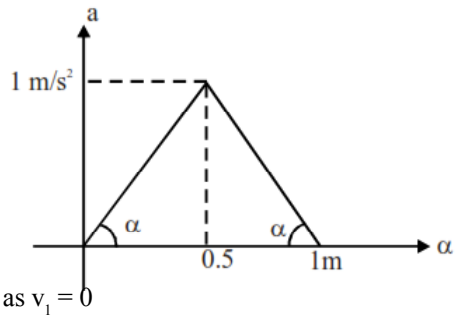
$\frac{\text{Area OAC}}{\text{Area ABC}} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3$



Distance coverd in (OA) = $\frac{1}{2}\left(\sqrt{3}\right)\left(t_{OA}\right)^2$

Distance covered in (AB) = $\frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(t_{AB}\right)^2$

20. (*a*) $\int a\,dx = \frac{v_2^2 - v_1^2}{2} = \frac{v_2^2}{2}$



as $v_1 = 0$

$\frac{1}{2} \times 1 \times 1 = \frac{v_2^2}{2}$

$\Rightarrow v_2 = 1\text{m/s}$

21. (*a*) $< \vec{v} > = \frac{\vec{s}}{t} = \frac{\Delta \vec{x}}{t}$

(*a*) +ve disp., +ve velocity
(*b*) zero disp.
(*c*) slope is +ve, so velocity is +ve.
(*d*) slope is –ve, so velocity is –ve.

22. (*d*) $a = \frac{d^2x}{dt^2}$ is constant for 3 and 4

23. (*b*) Avg. speed $= \frac{\text{dist}}{\text{time}} = \frac{\pi R}{2R/v} = \frac{nV}{2}$

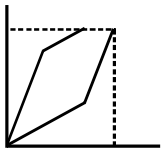
24. (*c*)



At t_1. disp $\neq$ distance

where $0 < t_1 < \frac{2r_0}{g}$

25. (*a*) for P : $v^2_{max} = 2 \times 2a \times \frac{L}{2} + 2 \times a \times \frac{L}{2}$

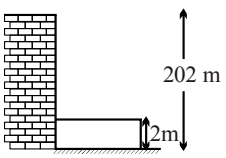


for Q : $v^2_{max} = 2 \times a \times \frac{L}{2} + 2 \times 2a \times \frac{L}{2}$

from velocity time graph time taken by P will be less. So average speed greater than

26. (*b*) Δx is negative for (2)

27. (*b*) Use basic kinematics,



$v_2 - v_0 = 2a\,(x - x_0)$

$v = v = \sqrt{2ax} = \sqrt{2(9.8)200} = 62.61$ m/s

$v^2 - v_0^2 = 2a\,(x - x_0)$

$0^2 - 62.61^2 = 2a(0.5 - 2)$

$a = 1307\ \text{m/s}^2$

$a = 133\ g$

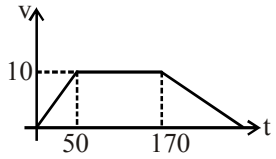
28. (*a*) Acceleration is constant except during collision. So displacement time graph is parabolic.

29. (*d*) $x = t^3 - 6t^2 - 15t + 40$

$v = t^2 - 12t - 15 = 0 \text{at}\ \ t = 5$

$\therefore$ A is wrong

30. (*d*)



$v = 36 \times \frac{5}{18} = 10$ m/s

$v = at$

$10 = 0.2 \times t$

$t = 50$

$\frac{1}{2} \times t \times 10 = 100$

$t = 20$ sec.

dist. $= \frac{1}{2} \times (190 + 120) \times 10 = 1550$ m

$v_{av} = \frac{1550}{190} = \frac{155}{19}$ m/s

31. (*a*) $t_1 = \frac{2R}{v}$

for 2nd car, $v_{av} = \frac{v + 2v}{2} = \frac{\pi R}{t_2}$

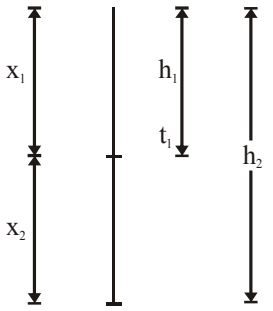
$\Rightarrow t_2 = \frac{2\pi R}{3v} > t_1$

32. (*a*) $t_1 : t_2 = 1 : \left(\sqrt{2} - 1\right)$

So, $t_1 = 1$ and $t_1 + t_2 = \sqrt{2}$

$x_1 = h_1 = \frac{1}{2} g t_1^2 = \frac{1}{2} g$

$h_2 = \frac{1}{2} g\,(t_1 + t_2)^2$



$= \frac{1}{2} g \times 2 = g$

$x_2 = (h_2 - h_1) = \frac{1}{2} g$

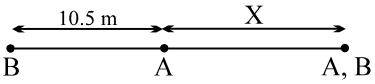
$x_1 = x_2$

33. (*b*) avg. $a = \frac{20\sqrt{2}}{5} = 4\sqrt{2}\ ms^{-2} N - W$

34. (*c*) Since, $V_f^2 - V_i^2 = 2aS$

$a = -2250\ kmh^{-2}$

35. (*a*)



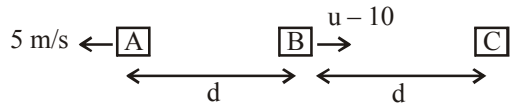
$X = 10t$

$10.5 + X = \frac{1}{2} \times 1 \times t^2$

$\Rightarrow t^2 - 20t - 21 = 0$

$\Rightarrow t = -1, 21$ sec

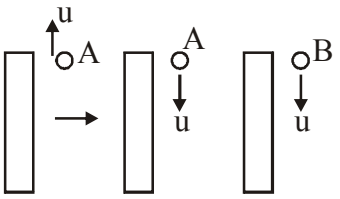
36. (*c*) with respect to C



$\frac{d}{u-10} = \frac{2d}{5}$

$\Rightarrow u = 12.5$ m/s

37. (*b*)



Ball A has the same speed u when its crosses the top end of the tower while going down which is the intital speed of B.

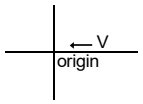
As both have same initial velocities, so their velocities are same upon reaching the ground.

Advanced Level Multiconcept Questions

1. (*b,c,d*) (*b*) $\Rightarrow a = \frac{dv}{dt}$

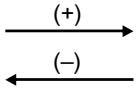
(*c*) $\frac{\vec{v}}{\vec{a}}$ Object is slowing down.

(*d*)



the particle is moving towards origin.

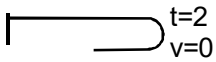
2. (*c,d*)



$\frac{\vec{v}}{\vec{a}} \cdot (v)\uparrow$ $\frac{\vec{v}}{\vec{a}} \cdot (v)\uparrow$

$\frac{\vec{v}}{\vec{a}} \cdot (v)\downarrow$ $\frac{\vec{v}}{\vec{a}} \cdot (v)\downarrow$

3. (*a,c*) $v = 10 - 5t$



When v = 0 at t = 2 sec.

Max displacement $= 10t - \frac{5t^2}{2}$

put $t = 2 \Rightarrow 20 - 10 = 10$m

Distance traveled in first 3 seconds

$= 10 + \left(0 + \frac{1}{2} \times 5 \times (1)^2\right) \Rightarrow = 12.5$ m

4. (*a,b,c,d*) $X = \alpha T^2 - \beta t^3$

(*a*) $0 = \alpha t^2 - \beta t^3 \Rightarrow t = \frac{\alpha}{\beta}$

(*b*) $v = \frac{dx}{dt} = 2\alpha t - 3\beta t^2 \Rightarrow v = 0 \Rightarrow t = \frac{2\alpha}{3\beta}$

(*c*) $a = \frac{d^2x}{dt^2} = 2\alpha - 6\beta t$

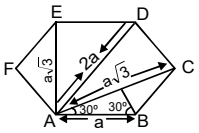
when $t = 0 \Rightarrow a = 2\alpha$; $v = 0$

(*d*) Acceleration at $t = \frac{\alpha}{3\beta}$; $a = 0$

$\therefore$ net force = 0

5. (*a,b,c,d*)
(a) At T (velocity changes its direction)
(b) slope constant
(c) Upper area = Lower area
(d) Initial speed = final speed.

6. (*a,c,d*)



(*a*) A to F

$$\text{Average velocity} = \frac{\text{Total Displacement}}{\text{Total time}}$$

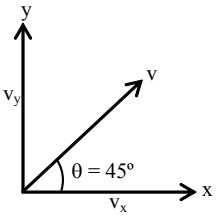
$$= \frac{a}{5a/v} = \frac{v}{5}$$

(*b*) A to D $= \dfrac{2a}{3a/v} = \dfrac{2}{3}v$

(*c*) A to C $= \dfrac{a\sqrt{3}}{2a/v} = \dfrac{v\sqrt{3}}{2}$

(*d*) A to B $= \dfrac{a}{a/v} = v$

7. (*a*)



Take x-axis along motion of truck and y-axis vertically upward

$\vec{v}_{ball/boy} = (10\ \hat{j})$ m/s

$\vec{v}_{boy/g} = (10\hat{i})$ m/s

$\vec{v}_{ball/g} = (10 + 10)$

Distance of ball from pole

$$= \frac{u^2 \sin 2\theta}{g} = \frac{(10\sqrt{2})^2 \times \sin 90}{g}$$

$= 20$ m

8. (*a*) Maximum height $= \dfrac{10^2}{2g} = 5$ m

9. (*c*) At maximum height, velocity of ball is horizontal.
Time taken by ball to reach maximum height

$$t_H = \frac{u_y}{g} = \frac{10}{10} = 1 \text{ sec}$$

Velocity of truck is
$v = u + at$
$= 10 + 2 \times 1 = 12$ m/s

10. (*b*) Distance travelled = Are $= \dfrac{1}{2} \times (10 + 5) \times 10 = 75$ m

11. (*c*) $ma = |-(10)(1000)\text{kg}|$
$= 10000$ N

12. (*d*)

13. (*a*) R ; (*b*)-P,Q ; (*c*)-S ; (*d*)-P,Q,R,S

(*a*) $v > 0$ and $\dfrac{dv}{dt} < 0$ or

$v > 0$ and $\dfrac{dv}{dt} > 0$

(*b*) $\dfrac{ds}{dt} > 0$ or

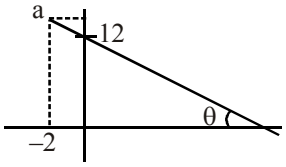
$v > 0$ and $\dfrac{dv}{dt} > 0$ or

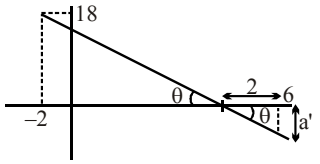
$v < 0$ and $\dfrac{dv}{dt} < 0$

(*c*) $\left|\dfrac{dv}{dt}\right|$ is increasing

(*d*) $x > 0$, and $v > 0$ or
$x < 0$ and $v < 0$

14. [0055] ΔV = area under graph





$\tan\theta = \dfrac{12}{4} = 3$

$\dfrac{a}{6} = 3 \Rightarrow a = 18$

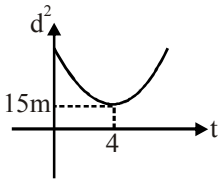
$\dfrac{a'}{2} = \tan\theta \Rightarrow a' = 6$

$\text{Area} = \dfrac{1}{2} \times 18 \times 6 - \dfrac{1}{2} \times 6 \times 2$

$V_6 - 7 = 54 - 6 = 48$

$V_6 = 55$ m/s

15. [0015] $d^2 = x^2 + y^2 = (3t)^2 + (25 - 4t)^2$



$= 625 - 200t + 25t^2$

$d^2 = 625 - 200 \times 4 + 25 \times 16 = 1025 - 800 = 225$

$d = 15$ m

16. [0400] $\ell = ct^2$

$2000 = c \times 10^2$

$c = 20$ m/s^2

$v = \dfrac{d\ell}{dt} = 2ct = 2 \times 20 \times 10 = 400$ m/s

17. [0006] g x –t graphs

Every change in shape is a change in velocity

18. [0004] $\dfrac{vdv}{dx} = 4 - 2x$

$\int_0^v vdv = \int_0^x (4 - 2x)dx$

$\Rightarrow \dfrac{v^2}{2} = 4x - x^2$

when $v = 0$, $4x - x^2 = 0$

$x = 0, 4$

$\therefore$ At $x = 4$, the particle will again come to rest.

19. [0025] Average speed $= \dfrac{\text{dis tan ce travelled}}{\text{time taken}}$

$b = \dfrac{\text{total area}}{\text{total time}} = \dfrac{10 + 20}{6} = \dfrac{30}{6} = 5$ m/s

Average acceleration $= \dfrac{\text{change in velocity}}{\text{time taken}}$

$C = \dfrac{10 - (-10)}{4} = \dfrac{20}{4} = 5$ m/s^2

$bc = (5)(5) = 25$ m^2/s^3

JEE Mains & Advanced Past Years Questions

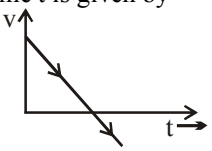
JEE-MAIN
PREVIOUS YEAR

1. (*a*) Velocity at any time t is given by

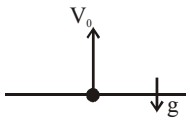
$v = u + at$

$v = v_0 + (-g)t$

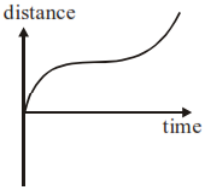
$v = v_0 - gt$



$\Rightarrow$ straight line with negative slope



2. (*a*) In this question option (*b*) and (*d*) are the corresponding position - time graph and velocity – position graph of option (*c*) and its distance – time graph is given as



Hence incorrect graph is option (*a*)

3. (*b*) $\vec{s} = \left(5\hat{i} + 4\hat{j}\right)2 + \dfrac{1}{2}\left(4\hat{j} + 4\hat{j}\right)4$

$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\vec{r}_2 - \vec{r}_1 = 18\hat{i} + 16\hat{j}$

$\vec{r}_2 = 20\hat{i} + 20\hat{j}$

$|\vec{r}_f| = 20\sqrt{2}$

4. (*a*) $v_x = \dfrac{dx}{dt} = -a\omega \sin \omega t$

$v_y = \dfrac{dy}{dt} = a\omega \cos \omega t$

$v_z = \dfrac{dz}{dt} = a\omega$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2 + v_z^2} = a\omega\sqrt{2}$

5. (*c*) $\sqrt{\dfrac{2\ell}{a_2}} - \sqrt{\dfrac{2\ell}{a_1}} = t \Rightarrow \dfrac{\sqrt{2\ell}}{t} = \dfrac{\sqrt{a_1 a_2}}{\sqrt{a_1} - \sqrt{a_2}}$

$\sqrt{2a_1\ell} - \sqrt{2a_2\ell} = v \Rightarrow \dfrac{\sqrt{2\ell}}{v} = \dfrac{1}{\sqrt{a_1} - \sqrt{a_2}}$

$\Rightarrow \dfrac{v}{t} = \sqrt{a_1 a_2} \quad \Rightarrow v = \left(\sqrt{a_1 a_2}\right)t$

6. (*d*) $r_{t=5}$ = area

$$=\left(\frac{1}{2}\times 2\times 2+2\times 2+3\times 1\right)\text{m}$$

$$=(2+4+3)\,\text{m}\quad =9\,\text{m}.$$

7. (*d*) $x = at + bt^2 - ct^3$

$$v=\frac{dx}{dt}=a+2bt-3ct^2$$

$$a=\frac{dv}{dt}=2b-6ct=0\ \Rightarrow\ t=\frac{b}{3c}$$

$$v_{\left(at\,t=\frac{b}{3c}\right)}=a+2b\left(\frac{b}{3c}\right)-3c\left(\frac{b}{3c}\right)$$

$$=a+\frac{b^2}{3c}$$

8. (*d*) $\vec{r}=15t^2\hat{i}+\left(4-20t^2\right)\hat{j}$

$$\vec{v}=\frac{d\vec{r}}{dt}=30t\hat{i}+\left(-40t\right)\hat{j}$$

$$\vec{a}=\frac{d\vec{v}}{dt}=30\hat{i}-40\hat{j}$$

$$|\vec{a}|=50\text{m}/\text{s}^2$$

9. (*d*) $m = 20\,g$, $u = 1$ m/s, $v = ?$

$S = 20\times 10^{-2}$ m

$$a=\frac{-2.5\times10^{-2}}{20\times10^{-3}}\text{m}/\text{s}^2$$

$v^2 = u^2 + 2as$

$$v^2=1-\ 2\times\frac{2.5\times10^{-2}}{20\times10^{-3}}\times\frac{20}{100}$$

$$v=\frac{1}{\sqrt{2}}\simeq 0.7\text{m/s}$$

10. (*c*) Given initial velocity u = 0 and acceleration is constant

At time t

$v = 0 + at \Rightarrow v = at$

also $x=0(t)+\frac{1}{2}at^2\ \Rightarrow\ x=\frac{1}{2}at^2$

Graph (*a*) ; (*b*) and (*d*) are correct.

11. (*b*) $v=b\sqrt{x}$

$$\frac{dv}{dt}=\frac{b}{2\sqrt{x}}\frac{dx}{dt}\qquad a=\frac{bv}{2\sqrt{x}}$$

$$a=\frac{b\left(b\sqrt{x}\right)}{2\sqrt{x}}$$

$$\frac{dv}{dt}=a=\frac{b^2}{2}\qquad v=\frac{b^2}{2}\tau$$

12. [3] $x^2 = at^2 + 2bt + c$

$2xv = 2at + 2b$

$xv = at + b$

$v^2 + Ax = a$

$$Ax=a-\left(\frac{at+b}{x}\right)^2$$

$$A=\frac{a(at^2+2bt+c)-(at+b)^2}{x^3}$$

$$A=\frac{ac-b^2}{x^3}$$

$A \propto x^{-3}$

13. (*d*) $S_y=u_yt+\frac{1}{2}a_yt^2$

$$32=0+\frac{1}{2}\times4t^2\quad\Rightarrow\ t=4\ \text{sec}$$

$$S_x=u_xt+\frac{1}{2}a_xt^2$$

$$=3\times4+\frac{1}{2}\times6\times16=60\,\text{m}.$$

14. (*c*) $\frac{1}{2}mv^2=p\times t$

$v \propto \sqrt{t}$

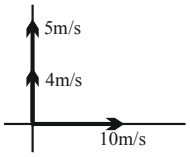
$\frac{ds}{dt}\propto\sqrt{t}$

$s \propto t^{3/2}$

15. [20] Distance travelled = Area under the u-t graph

$$\therefore\ \Delta S=\frac{1}{2}\times5\times8=20$$

16. (*d*) At t = t, (20 m, y_0)

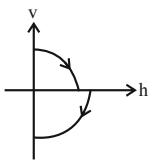


$$\therefore\ t=\sqrt{\frac{20\times2}{10}}=2\text{s}$$

$$\therefore\ y_0=5\times2+\frac{1}{2}\times4\times2^2$$

$= 18$ m

17. (*b*) At H = h, v = 0



At h = 0, $v=\sqrt{2gh}$

Also a = –g, throughout this motion

18. (*a*) Distance $= |A_1| + |A_2|$

$$= \frac{1}{2}\left(1+\frac{13}{3}\right)\times 4 + \frac{1}{2}\times\frac{5}{3}\times 2$$

$$= \frac{37}{3}\ \text{m}$$

19. (*a*) $\frac{h_1}{d} = \tan 45^\circ$

$$\frac{h_1 + h_2}{d + 2.464d} = \tan 30^\circ$$

$\Rightarrow h_2 = d$

20. (*a*) $t_0 = \sqrt{\frac{2h}{g}}$

$$\Rightarrow V_0 = \sqrt{\frac{2h}{g}}\times g = \sqrt{2gh}$$

$\therefore$ t_1 = time to reach top $= \frac{v_0}{g}$

$\Rightarrow H = h + h' = 2h$

$\therefore$ t_2 = time of fall $= \sqrt{\frac{2\times(2h)}{g}} = 2\sqrt{\frac{h}{g}}$

$\therefore$ Total time $= t_1 + t_2$

$$= (2+\sqrt{2})\sqrt{\frac{h}{g}} = 3.4\sqrt{\frac{h}{g}}$$

21. (*a*) The slope of the given v versus x graph is $m = -\frac{v_0}{x_0}$ and intercept is $c = +v_0$. Hence, varies with x as

$$v = -\left(\frac{\partial_0}{x_0}\right)x + v_0 \ldots\ldots(1)$$

where v_0 and x_0 are constants of motion. Differentiating with respect to time t_1 we have

$$\frac{dv}{dt} = -\left(\frac{2w}{x_0}\right)\frac{dx}{\dot{\alpha}t} \text{ or } a = -\left(\frac{20}{x_0}\right)y \ldots\ldots(2)$$

Using Eq. (1) in Eq.(2), weget

$$a = -\left(\frac{v_0}{x_0}\right)\left(-\frac{v_0}{x_0}x + v_0\right) \Rightarrow a = \left(\frac{\partial_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus the graph of a versus x is a straight line having a positive slope $= \left(\frac{v_0}{x_0}\right)^2$ and negative intercept $= -\frac{v_0^2}{x_0}$. Hence the correct choice is (*a*).

22. (*b*) Acceleration (a) is constant

$\therefore v \propto t$ (Straight line graph)

And $x \propto t^2$ (parabolic graph)

23. (*a*) Total distance

$$d = h + 2e^2h + 2e^4h + 2e^6h + 2e^8h + \ldots\ldots$$

$$d = h + 2e^2h\left(1 + e^2 + e^4 + e^6 + \ldots\ldots\right)$$

$$d = \frac{\left(1-e^2\right)k + 2\varepsilon^2\kappa}{1-\epsilon^2} = \frac{\kappa\left(1+e^2\right)}{1-t^2}$$

Total time $= T + 2eT + 2e^2T + 2e^3T + \ldots\ldots$

Total time $= T + 2eT\left(1 + e + e^2 + e^3 + \ldots\ldots\right)$

$$= T + 2e.T\left(\frac{1}{1-2}\right)$$

Total time $= \frac{T(1+e)}{1-e}$

Average speed of the ball

$$= \frac{5}{1}\left(\frac{1+e^2}{(1+e)(1-e)}\frac{(1-e)}{(1+e)}\right)$$

$$V_{avg} = \frac{5\left(1+e^2\right)}{(1+e)^2}$$

$\because h^1 = e^2h$

$$\frac{81}{100} = e^2$$

$$e = \frac{9}{10} = 0.9$$

$$V_{\text{Qug}} = \frac{s\left(1+\frac{s1}{10}\right)}{(1+0.9)^2}$$

$= 2.50\,\text{m/sec.}$

24. (*b*) $V = V_0 + gt + Ft^2$

$$\frac{dx}{dt} = V_0 + gt + Ft^2 \Rightarrow \int_{x=0}^{x} dx = \int_{t=0}^{t=1}\left(V_0 + gt + Ft^2\right)dt$$

$$\Rightarrow x = \left[v_0t + \frac{g^0}{2} + \frac{F^2}{3}\right]_{t=0}^{t=1}$$

$$\Rightarrow x = V_0 + \frac{g}{2} + \frac{F}{3}$$

JEE-ADVANCED PREVIOUS YEAR'S

1. (*b*) $m = \frac{4\pi R^2}{3} \times \rho$

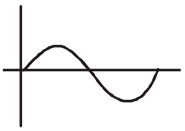
$$\ell n(m) = \ell n\left(\frac{4\pi}{3}\right) + \ell n(\rho) + 3\ell n(R)$$

$$0 = 0 + \frac{1}{\rho}\frac{d\rho}{dt} + \frac{3}{R}\frac{dR}{dt}$$

$$\Rightarrow \left(\frac{dR}{dt}\right) = v \propto -R \times \frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$$

$v \propto R$

2. [2.00] $n = 10^{-3}$ kg $q = 1C$ $t = 0$

$E = E_0 \sin \omega t =$ 

Force on particle will be

$F = qE = qE_0 \sin \omega t$

at v_{max}, a, $F = 0qE_0 \sin \omega t = 0$

$F = qE_0 \sin \omega t$

$$\frac{dv}{dt} = q\frac{E_0}{m}\sin \omega t$$

$$\int_0^v dv = \int_0^{\pi/\omega} \frac{qE_0}{m}\sin \omega t\, dt$$

$$v - 0 = \frac{qE_0}{m\omega}\left[-\cos \omega t\right]_0^{\pi/\omega}$$

$$v - 0 = \frac{qE_0}{m\omega}\left[(-\cos \pi) - (-\cos 0)\right]$$

$$v = \frac{1 \times 1}{10^{-3}10^3} \times 2 = 2\,m/s$$

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

4 Motion in Plane

Topicwise Solutions

1. (*b*) $\text{Time} = \dfrac{\text{Total length}}{\text{Relative velocity}} = \dfrac{50+50}{10+15} = \dfrac{100}{25} = 4 \text{ sec}$

2. (*b*) Relative velocity of bird w.r.t train= 25 + 5 = 30 m/s
time taken by the bird to cross the train

$$t = \frac{210}{30} = 7 \text{ sec}$$

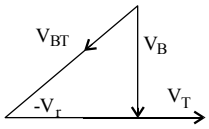
3. (*d*) At t = 3 sec, 1st stone will have speed of 30 m/s

$$h_1 = \frac{1}{2} \times 10 \times 9 = 45\text{m}$$

$$h_2 = \frac{1}{2} \times 10 \times 1^2 = 5\text{m}$$

$$h_1 - h_2 = 40\text{m}$$

4. (*c*)



$$\vec{V}_{BT} = \vec{V}_B - \vec{V}_T$$

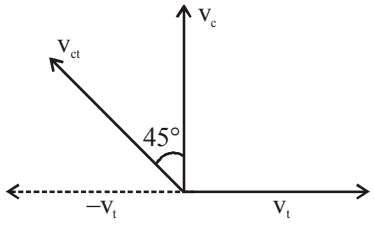
$$V_{BT} = \sqrt{40^2 + 40^2}$$

$$= 40\sqrt{2}$$

Direction $\rightarrow$ SW

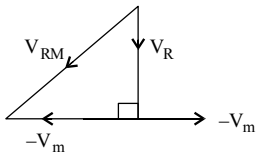
5. (*b*) $\vec{v}_{ct} = \vec{v}_c - \vec{v}_t$

$$\vec{v}_{ct} = \vec{v}_c + (-\vec{v}_t)$$



Velocity of car w.r.t. train (v_{ct}) is towards West – North

6. (*c*)

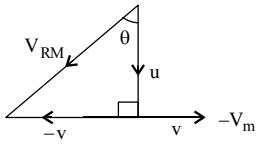


$$\vec{V}_{RM} = \vec{V}_R - \vec{V}_m$$

$$V_{RM} = \sqrt{V_R^2 + V_m^2}$$

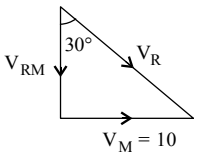
$$= \sqrt{109}$$

7. (*a*)



$$\tan\theta = \frac{v}{u}$$

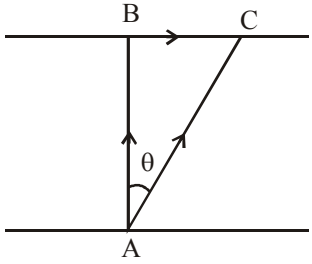
8. (*b*)



$$\tan 30° = \frac{V_m}{V_{RM}} = \frac{10}{V_{RM}}$$

$$V_{RM} = 10\sqrt{3}$$

9. (*c*) Given $\overrightarrow{AB}$ = Velocity of boat = 8 km/hr



$\overrightarrow{AC}$ = Resultant velocity of boat = 10 km/hr

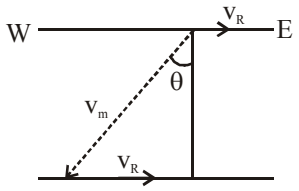
$\overrightarrow{BC}$ = Velocity of river = $\sqrt{AC^2 - AB^2}$

$= \sqrt{(10)^2 - (8)^2} = 6$ km / hr

10. (*b*) The relative velocity of boat w.r.t. water

$= v_{boat} - v_{water} = (3\hat{i} + 4\hat{j}) - (-3\hat{i} - 4\hat{j}) = (6\hat{i} + 8\hat{j})$

11. (*c*) For shortest possible path man should swim with an angle (90+θ) with downstream.

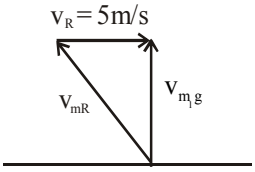


From the fig, $\sin\theta = \frac{v_r}{v_m} = \frac{5}{10} = \frac{1}{2} \Rightarrow \therefore \theta = 30°$

So angle with downstream = 90° + 30° = 120°

12. (*a*) Due north will take himacross in shortest time.

13. (*b*)

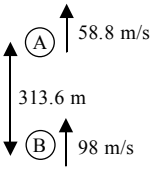


$\frac{60\text{m}}{5} = \sqrt{v_m^2 - v_2^2}$

$\Rightarrow 144 + 5^2 = v_m^2$

$\Rightarrow v_m = 13\,\text{m/s}$

14. (*d*) At t = 4s



$a_{ml} = 0$

$s_{ml} = U_{nl}t$

$t = \frac{313.6}{39.2} = 8$s.

$T = t + 4 = 12$s

15. (*c*) $v_x = \frac{dx}{dt} = 6$ and $v_y = \frac{dy}{dt} = 8 - 10t$

$= 8 - 10 \times 0 = 8$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10\ \text{ms}^{-1}$

16. (*d*) $R = \frac{u^2 \sin 20}{g} \therefore R \propto u^2$. If initial velocity be doubled then range will become four times.

17. (*c*) $H = \frac{u^2 \sin^2\theta}{2g} \therefore H \propto u^2$. If initial velocity be doubled

18. (*a*) Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means angle between $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

19. (*b*) $R_{15°} = \frac{u^2 \sin(2\times 15°)}{g} = \frac{u^2}{2g} = 1.5\ km$

$R_{45°} = \frac{u^2 \sin(2\times 45°)}{g} = \frac{u^2}{g} = 1.5\times 2 = 3\,km.$

20. (*a*) For vertical upward motion $h = ut - \frac{1}{2}gt^2$

$5 = (25\sin\theta)\times 2 - \frac{1}{2}\times 10\times(2)^2$

$\Rightarrow 25 = 50\sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°.$

21. (*b*) Range is given by $R = \frac{u^2 \sin 2\theta}{g}$

On moon $g_m = \frac{g}{6}$. Hence $R_m = 6R$.

22. (*b*) $y = ut\sin\theta - \frac{gt^2}{2}$

$V_r = 0$ (zero shape)

$a_r = -g$ (negative curvature)

23. (*d*) $R = \frac{v^2 \sin 2\theta}{g} \Rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{v^2}\right)$.

24. (*c*) Due to constant velocity along horizontal and vertical downward force of gravity stone will hit the ground following parabolic path.

25. (*d*) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 396.9}{9.8}} \simeq 9$ sec and u = 720 km/ hr

= 200 m/s

$\therefore$ R = u × t = 200 × 9 = 1800 m

26. (*c*) $T = \frac{2u\sin 45°}{g\cos 45°} = 2\,\text{sec}$

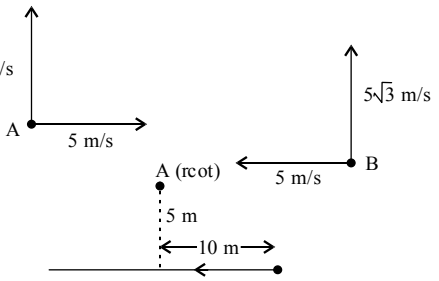
27. (*d*) $PQ = \frac{1}{2} g \sin\theta T^2$

$T = \frac{2v}{g\cos\theta}$

$PQ = \frac{1}{2} g \sin\theta \left(\frac{2v}{g\cos\theta}\right) T$

$PQ = Tv \tan\theta$

28. (*b*)



$t = \frac{10}{10} = /s$

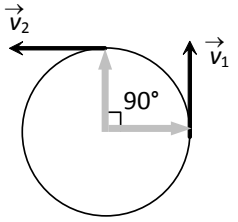
29. (*c*) It is always directed in a direction of tangent to circle.

30. (*a*) In uniform circular motion (constant angular velocity) kinetic energy remains constant but due to change in velocity of particle its momentum varies.

31. (*c*) $\omega_{min} = \frac{2\pi}{60}\frac{Rad}{min}$ and $\omega_{hr} = \frac{2\pi}{12\times 60}\frac{Rad}{min}$

$\therefore \frac{\omega_{min}}{\omega_{hr}} = \frac{2\pi/60}{2\pi/12\times 60}$

32. (*d*) In 15 second's hand rotate through 90°.



Change in velocity $\left|\overrightarrow{\Delta v}\right| = 2v\sin(\theta/2)$

$= 2(r\omega)\sin(90^\circ/2) = 2\times 1\times\frac{2\pi}{T}\times\frac{1}{\sqrt{2}}$

$= \frac{4\pi}{60\sqrt{2}} = \frac{\pi\sqrt{2}}{30}\frac{cm}{sec}$ [As T = 60 sec]

33. (*b*) $\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$.

34. (*c*) $\omega = 80$ rad/sec, $t = 5$ sec, $\omega_0 = 0$

$\theta = ?$

If α constant, then

$\theta = \left(\frac{\omega+\omega_0}{2}\right) t = \left(\frac{80+0}{2}\right) 5 = 200$ rad **Ans.**

35. (*a*) $V = \omega.r \Rightarrow V = 30\times 2\pi\times\frac{1}{2} = 30\pi$

36. (*c*) Direction & megnitude of Angular velocity & angular momentum stays unstant.

37. (*c*) Centripetal acceleration $= \frac{v^2}{r}$ = constant. Direction keeps changing.

38. (*a*) $\frac{a_R}{a_r} = \frac{\omega_R^2\times R}{\omega_r^2\times r} = \frac{T_r^2}{T_R^2}\times\frac{R}{r} = \frac{R}{r}$ [As $T_r = T_R$]

39. (*d*) The particle performing circular motion flies off tangentially.

40. (*a*) $\alpha = 4\pi^2 n^2 r = 4\pi^2\left(\frac{1}{2}\right)^2\times 50 = 493$ cm/s^2.

41. (*c*) Angular velocity of every particle of disc is same

$a_p = \omega^2 r_p, a_Q = \omega^2 r_Q$

$\because r_p > r_Q \Rightarrow a_p > a_Q$

42. (*a*)

43. (*d*) Centripetal force is constant in magnitude that means speed is constant and due to change in direction velocity is variable.

44. (*b*)

45. (*b*) $F = K\frac{1}{r}\frac{k}{r} = \frac{mv^2}{r}$ $v = \sqrt{\frac{k}{m}}$

so independent to r

46. (*a*) Force is perpendicular to $\vec{v}$

$R = \frac{v^2}{a_\perp} \Rightarrow R = \frac{mv^2}{F}$

47. (*c*) $F_{C1} = F_{C2}$

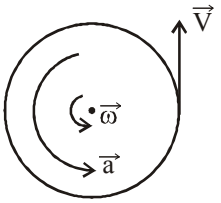
$\Rightarrow \frac{mv_1^2}{r_1} = \frac{mv_2^2}{r_2}$

$\frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{1}{\sqrt{2}}$

48. (*c*) $K = \frac{mV^2}{2}$

49. (*a*) $a_c = wv$

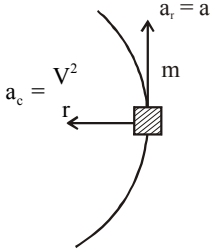
50. (*b*)



51. (*b*) $a_t = \alpha r = \omega^2 r = \alpha^2 t^2 r$

$\alpha \times \frac{1}{t^2}$

52. (*b*) $|\vec{a}| = \sqrt{a_c^2 + a_t^2} = \sqrt{\frac{v^2}{r^2} + a^2}$



53. (*a*) Thrust at the lowest point of concave bridge

$= mg + \frac{mv^2}{r}$

54. (*b*) v = 72 km / hour = 20 m/sec

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20\times 20}{20\times 10}\right) = \tan^{-1}(2)$

55. (*a*) centripetal force = $\frac{mV^2}{r}$

56. (*b*)

57. (*a*) $T = \frac{mv^2}{r} \Rightarrow 25 = \frac{0.25\times v^2}{1.96} \Rightarrow v = 14$ m/s.

58. (*a*) $\frac{mv^2}{r} = \frac{k}{r^2}$

$\Rightarrow mv^2 = \frac{k}{r}$; K.E. $= \frac{1}{2}mv^2 = \frac{k}{2r}5$

$P.E = \int F\,dr = \int \frac{k}{r^2}dr = -\frac{k}{r}$

$\therefore$ Total energy = K.E. + P.E. $= \frac{k}{2r} - \frac{k}{r} = -\frac{k}{2r}$.

59. (*d*) $F = mg - \frac{mv^2}{r}$.

60. (*c*) $v_{max} = \sqrt{\mu rg} = \sqrt{0.75\times 60\times 9.8} = 21$m/s.

61. (*c*) T = Tension, W = weight and F = centrifugal force.

62. (*b*) $T = \frac{mv^2}{r}$

$= \frac{0.5\times(4)^2}{1} = 8$N

63. (*d*) $p = mv$, & $F = mv^2/r \Rightarrow F = m\left(\frac{p}{m}\right)^2/r$

$\Rightarrow F = \frac{p^2}{mr}$

64. (*d*) $v_{max} = \sqrt{\mu rg}$

Learning Plus

1. (*b*) Total Length of 2 trains = 50 + 50 = 100

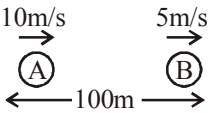
Velocity $V_1 = 10$

$V_2 = 15$

$V_1 + V_2 = 25$

time $= \frac{100}{25} = 4$ sec

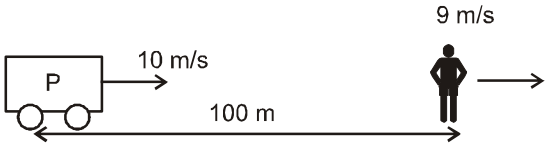
2. (*c*)



$V_{AB} = 10 - 5 = 5$m/s

$t = \frac{100}{5} = 20$ sec

3. (*d*) $\vec{V}_{PT} = \vec{V}_P - \vec{V}_T = 10 - 9 = 1$ m/s



So time taken $t = \frac{100}{\vec{V}_{PT}} = \frac{100}{1} = 100$ sec.

4. (*d*) $V_{rel} = \frac{S_{rel}}{t} = \frac{1000}{100} = 10$ m/s.

$\therefore$ $V_S - V_B = 10$

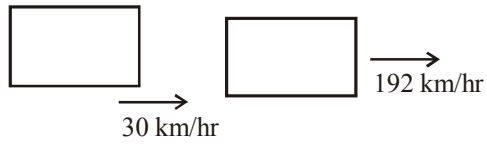
$\Rightarrow$ $V_S = 10 + V_B = 10 + 10 = 20$ m/s. **Ans.**

5. (*b*) We have, $S_{rel} = u_{rel}t + \frac{1}{2}a_{rel}\,t^2$

$\Rightarrow 0 = ut - \frac{1}{2}(a+g)\,t^2$

$\Rightarrow a = \frac{2u}{t} - g = \frac{2u - gt}{t}$

6. (*a*)



$\frac{25}{3}$ m/s = 30 km/h

Muzzle speed = velocity of bullet w.r.t. revolver
= velocity of bullet w.r.t. van

$150 = V_b - V_v$

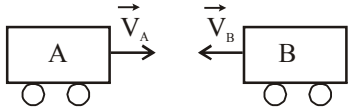
$150 = V_b - \frac{25}{3} \Rightarrow V_b = \frac{475}{3}$ m/s w.r.t. ground

Now speed with which bullet hit thief's car
= velocity of bullet w.r.t car

$= V_{bc} = V_b - V_c$

$= \frac{475}{3} - \frac{160}{3} = \frac{315}{3} = 105$ m/s

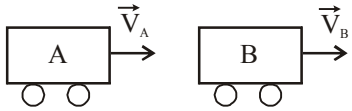
7. (*a*)



In opposite direction

$\vec{V}_{AB} = \vec{V}_A + \vec{V}_B$

$9 = \vec{V}_A + \vec{V}_B$ ..... (*a*)



In same direction -

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

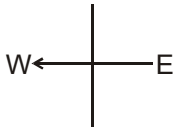
$1 = \vec{V}_A - \vec{V}_B$ ..... (*b*)

From equation (*a*) & (*b*)

$\vec{V}_A = 5$ m/s

$\vec{V}_B = 4$ m/s

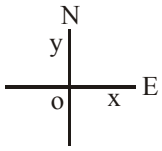
8. (*a*)



$\vec{V}_A = -500\,\hat{i} \quad \Rightarrow \vec{V}_{GA} + \vec{V}_A = V_{GAS}$

$\vec{V}_{GA} = 1500\,\hat{i} \quad \Rightarrow 1500\,\hat{i} - 500\,\hat{i} = 1000\,\hat{i}$

9. (*d*)



$\vec{V}_r = 50\,(-\hat{j}) - 50\,\hat{i} = 50\,(-\hat{i} - \hat{j})$ i.e., in south west

10. (*d*) $\vec{V}_{12} = \vec{V}_1 - \vec{V}_2$

$|\vec{V}_{12}| = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$

If $\cos\theta = -1$

$|\vec{V}_{12}|_{max} = \sqrt{V_1^2 + V_2^2 + 2V_1V_2}$

$|\vec{V}_{12}|_{max} = (V_1 + V_2)$

So $|\vec{V}_{12}|$ is maximum when $\cos\theta = -1$ and $\theta = \pi$

11. (*d*) $\vec{V}_1 = \hat{i} + \sqrt{3}\,\hat{j}$ $\tan\theta_1 = \sqrt{3}$ i.e., $\theta_1 = 60$.

$\vec{V}_2 = 2\hat{i} + 2\,\hat{j}$ $\tan\theta_2 = 1$ i.e., $\theta_2 = 45^\circ$ $\theta_1 - \theta_2 = 15^\circ$

12. (*a*) $\vec{V}_r = v_y\,j$

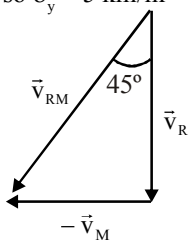
$\vec{v}_m = 5\hat{i}$

$\vec{V}_r - \vec{V}_m = (-5)\,\hat{i} + v_y\hat{j}$

$\tan\theta = 1 = \frac{v_y}{5}$

so $v_y = 5$ km/hr

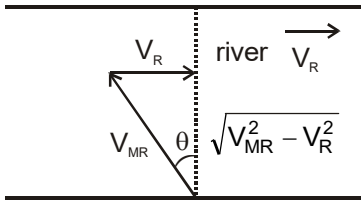
13. (*a*)



$\vec{v}_{RM} = \vec{v}_R - \vec{v}_m$

so $v_R = v_M$

14. (*b*) 15 min = 1/4 hr.



$t = \frac{d}{V_y} \Rightarrow \frac{1}{4} = \frac{1}{\sqrt{V_{MR}^2 - V_R^2}} = \frac{1}{4} = \frac{1}{\sqrt{5^2 - V_R^2}}$

$\Rightarrow V_R = 3$ km/h

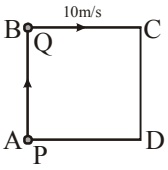
15. (*b*) $T_0 = \frac{d}{V} + \frac{d}{V} = \frac{2d}{V}$

$T = \frac{d}{V+u} + \frac{d}{V-u} = \frac{2Vd}{V^2-u^2}$

$T = \frac{V^2 T_0}{V^2-u^2} = \frac{T_0}{1-\frac{u^2}{V^2}}$

$\frac{T}{T_0} = \frac{1}{1-\frac{u^2}{V^2}}$

16. (*c*) Viewing the motion from river frame ; both boats will reach the ball simultaneously

17. (*b*) 

a = 8 m

They meet when Q displace 8 × 3 m
more than p $\Rightarrow$ relative displacement = relative velocity × time.
8 × 3 = (10 – 2) t t = 3 sec

18. (*b*) $V_x = 2at$

$V_y = 2bt$

$V = 2t\sqrt{a^2+b^2}$

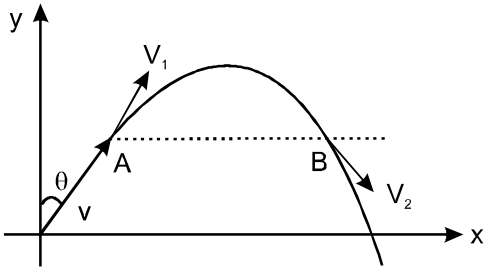
19. (*c*) In projectile motion Horizontal accelereation a_x = o & Vertical acceleration $a_y = g = 10 m/s^2$ a_x = o
$a_x = 0$
$a_y = 10$ (down)
$\Rightarrow$ only "C" is correct

20. (*d*) $\theta_1 = \frac{5\pi}{36}$

Another angle for same range is the complementary angle θ_2

Then $\theta_2 = \frac{\pi'}{2} - \left(\frac{5\pi}{36^o}\right)^c = \frac{18-5}{36}\pi \Rightarrow \theta_2 = \frac{13}{36}\pi$

"D" **Ans.**

21. (*a*) 

Avg. vel. b/w A & B $= \frac{\vec{V}_1 + \vec{V}_2}{2}$ ($\because$ Acceleration is constant = g)

Now, if $\vec{V}_1 = V_{1x}\,\hat{i} + V_{1y}\,\hat{j}$

Thanì $\vec{V}_2 = V_{1x}\,\hat{i} - V_{1y}\,\hat{j}$

($\because$ both A & B are at same lavel)

$\therefore \quad \frac{\vec{V}_1 + \vec{V}_2}{2} = V_{1x}\,\hat{i} = V\sin\theta\,\hat{i}$

($\because \theta$ is from vertical) " B "

22. (*a*) Gravitational acceleration is constant near the surface of the earth.

23. (*d*) $R = u^2 \sin 2\theta / g$
$R_{max} = u^2/g$

$22 = \frac{u^2}{g}$

for $\theta = 15°$

$R = \frac{u^2 \sin 30}{g} = 22 \times \frac{1}{2} = 11 m$

24. (*d*) $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$y = x\tan\theta - \tan\theta\frac{x^2}{R}$

Compare from eq^n.
$\tan\theta = 16$

$\frac{\tan\theta}{R} = \frac{5}{4}$

$R = \frac{64}{5} = 12.8 m$

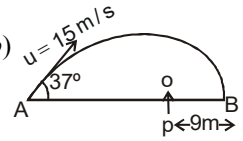
$\tan\theta = 16$

$\frac{1}{2}\frac{g}{u^2\cos^2\theta} = \frac{5}{4.2}$

$R = \frac{u^2\sin 2\theta}{g} = \frac{2g}{5} \times \frac{2\times 16}{g}$

$R = 12.8 m$

25. (*c*) Range of θ and 90-θ is same
If $\theta = 30°$
So 90 - $\theta = 60°$

26. (*b*) 

In this process both time taken is same.

$T = \frac{2u\sin\theta}{g}$

$$T = \frac{2u\sin 37^\circ}{g}$$

$$= \frac{2\times 15\times 3}{10\times 5} = 1.8\,\text{Sec}$$

$$\text{Minimum Velocity} = \frac{9}{1.8} = 5\,\text{m/s}$$

27. (*b*) $\therefore H_{max} = \frac{u_y^2}{2g}$

$\Rightarrow u_y = \sqrt{2gH}$

$\therefore T = \frac{2u_y}{g} = \frac{2\sqrt{2gH}}{g} = 2\sqrt{2}\sqrt{\frac{H}{g}}$

28. (*b*) As $\theta\uparrow$, H and T both increases

But R $\uparrow$ from 0º to 45º & at θ = 45° Max then decreases

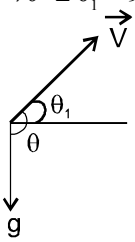
Ans (*b*) R$\uparrow$ then $\downarrow$ [θ from from 30º to 60º]

while H$\uparrow$ and T$\uparrow$.

29. (*d*) Acute Angle of Velocity with horizontal possible is – 90º to + 90º hence angle with g is 0° to 180º.

θ_1 is acute

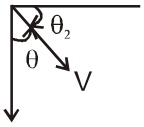
$\Rightarrow 0^\circ \le \theta_1 < 90^\circ$(during the upward journey of mass)



from fig. $\theta = 90^\circ + \theta_1$

or, $90^\circ \le \theta < 180^\circ$ .....(*a*)

During downward motion



$0^\circ < \theta_2 < 90^\circ$

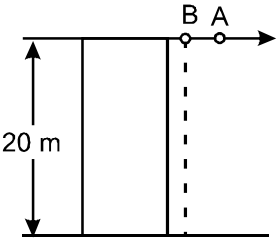
$\theta = 90^\circ - \theta_2$

$0^\circ < \theta < 90^\circ$ .....(*b*) From eq. (*a*) and (*b*)

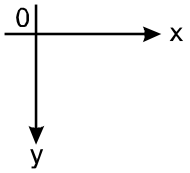
i.e.,$0 < \theta < 90^\circ \cup 90^\circ \le \theta < 180^\circ$

$\Rightarrow 0^\circ < \theta < 180^\circ$ "D"

30. (*d*)



$10\ \text{m/s} = u_A$;



$u_B = 0\ \text{m/s} u_A = 10\ \hat{i}$

On reaching the ground,

Both will have same vertical velocity

$V_y^2 = u_y^2 + 2a_y s_y$

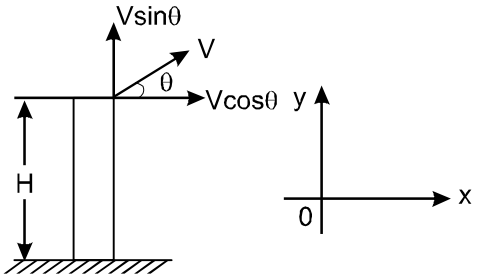
since u_y = 0 for both A & B

$a_y = g$ for both A & B

s_y = 20 m for both A & B

Thats why the time taken by both are same

31. (*d*)



Let final vel be V_2

Now v_{2x} = horizontal component of velocity

$V_{2x} = V\cos\theta$ & $V_{2y}^2 = (V\sin\theta)^2 + 2(-g)(-H)$

$\therefore V_{2y}^2 = V^2\sin^2\theta + 2gH$

$\Rightarrow V_2^2 = V_{2x}^2 + V_{2y}^2$

$= (V\cos\theta)^2 + [V^2\sin^2\theta + 2gH]$

$V_2^2 = V^2 + 2gH$

i.e., $V = \sqrt{v^2 + 2gH}$ $\left\{V_2 = \sqrt{V^2 + 2gH}\right\}$

This magnitude of final velocity is independent on θ

$\Rightarrow$ all particles strike the ground with the same speed.

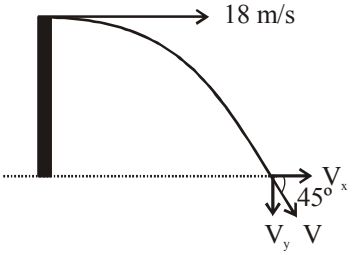
i.e., 'A' is correct.

In vertical motion

The highest velocity (initial) along the direction of displacement is possessed by particle (*a*). Hence particle (*a*) will reach the ground earliest. [since a_y and s_y are same for all]

i.e., 'C' is correct

32. (*b*) $\tan 45^\circ = \frac{v_y}{v_x}$



$\Rightarrow v_y = v_x = 18\text{m/s}$

33. (*d*) $(Y_{max}) \Rightarrow \frac{dY}{dt} = 0$

$\Rightarrow \frac{d}{dt}(10t - t^2) = 10 - 2t \Rightarrow t = 5$

$\Rightarrow Y_{max} = 10(5) - 5^2 = 25$ m **Ans** "D"

34. (*d*) (i) For θ and 90-θ
Range is same
$\theta = 15^0$
$90-\theta = 75^0$

(ii) $R = \frac{u\sin\theta . u\cos\theta}{g}$

$\therefore \sin(90 - \theta) = \cos\theta$
$\therefore \cos(90 - \theta) = \sin\theta$

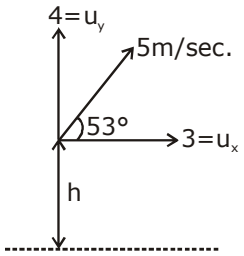
35. (*b*) Vel. of Bomb is same as the vel. of aeroplane.
$u_x = 360$ km/h & $v_y = 0$.

$S_y = u_y t + \frac{1}{2} a_y t^2$, Here $u = 0$

$1960 = \frac{1}{2} \times 9.8 t^2$

$t = 20$ sec

36. (*c*) $v_y^2 - u_y^2 = 2 a_y S$



$v_y^2 - (4)^2 = -2 \times 10 \times 0.45$

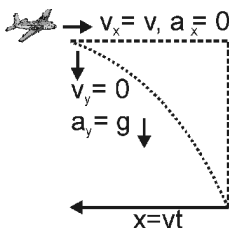
$v_y^2 = 7 m^2/s^2$

$v_x = 5\cos 53^\circ = 3$m/s(always remains same)

$\therefore V_{net} = \sqrt{(V_x)^2 + (V_y)^2} = \sqrt{9+7}$

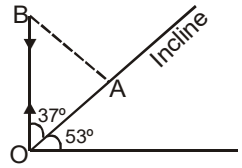
$= 4 m/s$

37. (*a*)



Because horizontal velocity of plane and bomb is always same.

38. (*a*) $OB = \frac{u^2}{2g} = 5$m



$\therefore$ AB = OB sin37° = 3m.

39. (*b*) At maximum height $v = u\cos\theta$

$\frac{u}{2} = v \Rightarrow \frac{u}{2} = u\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2} \quad \Rightarrow \theta = 60^\circ$

$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin(120^\circ)}{g}$

$= \frac{u^2 \cos 30^\circ}{g} = \frac{\sqrt{3}\,u^2}{2g}$

40. (*c*) Speed $v_1 = \frac{2\pi r}{t}$

$v_2 = \frac{2\pi r}{t}$

$\omega_1 = \frac{v_1}{r} = \frac{2\pi}{t} \Rightarrow \omega_2 = \frac{v_2}{2r} = \frac{2\pi}{t}$

$\omega_1 = \omega_2 \Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{1}$

41. (*d*) $\omega_{second} = \frac{2\pi}{T} = \frac{2\pi}{60}$ rad/sec.

$v = \omega.r = \frac{2\pi}{60} \times 0.06$ m/s $= 2\pi$ mm/s

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \sqrt{2}\,v = 2\sqrt{2}\,\pi$ mm/s

42. (*c*) Given, $\omega_0 = 0$, $t = 2$ sec.
$\theta = 0$, next 2 sec., $\theta = O_2$

$\theta_1 = \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha 2^2 = 2\alpha$

$\theta_2 = \frac{1}{2}\alpha(2+2)^2 - \frac{1}{2}\alpha 2^2 = 6\alpha$

$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = 3$

43. (*c*) $\omega_1 = \frac{2\pi}{T_1}$, $\omega_2 = \frac{2\pi}{T_2}$

$\omega_1 : \omega_2 = T_2 : T_1$
$T_1 = 12 \times 60 \times 60$ sec.

$T_2 = 60$ sec.

$\omega_1 : \omega_2 = 60 : (12 \times 60 \times 60)$

$\omega_1 : \omega_2 = 1 : 720$

44. (*d*) Given

$a = 10 m/sec^2 \Rightarrow \alpha = 5 rad/sec^2$

$a = \alpha r$

$r = \frac{10}{5} = 2$ m

45. (*b*) $a_c = \omega^2 R = \frac{4\pi^2}{T^2} R = \frac{4 \times 3.14^2 \times 6400 \times 10^5}{(24 \times 60 \times 60)^2}$

$\omega^2 R = \frac{4\pi^2}{T^2} R = \frac{4 \times 3.14^2 \times 6400 \times 10^5}{(24 \times 60 \times 60)^2} = 3.4 cm/sec^2$

46. (*c*) Given r = 25 cm, n = 2

$\omega = 2\pi \times 2$ rad / s $\Rightarrow a_c = \omega^2 r$

$= (4\pi)^2 \times 0.25 = 16\pi^2 \times 0.25 = 4\pi^2$

47. (*c*) Given

$\omega = \theta^2 + 2\theta$

$\frac{d\omega}{d\theta} = 2\theta + 2 \Rightarrow \left.\frac{d\omega}{d\theta}\right|_{t=1} = 2\theta + 2 = 4$

$\alpha = \frac{\omega d\omega}{d\theta} = (\theta^2 + 2\theta).(2\theta + 2) = 12$ rad/sec^2

48. (*d*) r = 144 m, m = 16 kg, T_{max} = 16 N

$T = \frac{mv^2}{r}$

$v = \sqrt{\frac{Tr}{M}} = \sqrt{\frac{16 \times 144}{16}} = 12$ m/s

49. (*d*) $T = m\omega^2 r$

$\Rightarrow T^1 = 2T = m\omega_1^2 r$

$\omega_1 = \sqrt{2}\ \omega = \sqrt{2} \times 5 = \sqrt{50} \sim 7$ rev/min

50. (*c*) For just slip $\Rightarrow \mu mg = m\omega^2 r$

here ω is double then radius is $1/4^{th}$

$r' = 4$ cm

51. (*c*) At t = 0,

$a_\perp = g \cos\theta$,

$R = \frac{v^2}{a_\perp} = \frac{u^2}{g\cos\theta}$

52. (*d*) For circular motion in vertical plane normal reaction is minimum at highest point and it is zero, minimum speed of motorbike is -

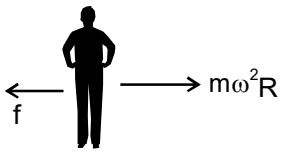
$mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{gR}$

53. (*a*) $T - mg = \frac{mv^2}{r}$ (centripetal force at lowest point)

$T = \frac{mv^2}{r} + mg$

54. (*c*) Car will not slip when moving with speed v

55. (*a*)



$\mu mg \geq \frac{mv^2}{R}$

$0.5\ mg \geq m \times (5)^2 \times R$

$\frac{0.5 \times 10}{25} \geq R$

$R \leq 0.2$ m

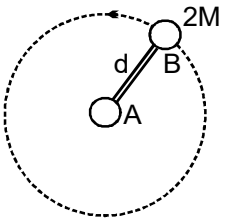
56. (*c*) $v = \sqrt{Rg\tan\theta}$

$R = 10\sqrt{3}$ m, $\theta = 30°$

$= \sqrt{10\sqrt{3} \times 10 \times \frac{1}{\sqrt{3}}} = 10$ m/sec $= 36$ km/hr

57. (*b*) Here required centripetal force is provided by friction force. Due to lack of sufficient centripetal force car thrown out of the road in taking a turn.

58. (*b*) Given



$P = \frac{2\pi}{\omega} \Rightarrow \omega^{-1} = \frac{P}{2\pi}$

$T = 2M\omega^2 d = \frac{8\pi^2 Md}{P^2}$

59. (*a*) The maximum bearable Tension

$T = \frac{mv^2}{l}$

$T_{max} = 10$ N, $m = 1$, $v = ?$, $l = 1$

$\upsilon = \sqrt{\frac{Tl}{m}} = \sqrt{\frac{100 \times 1}{1}} = 10$ m/s

60. (*c*) Given that

v = 72 km/h., r = 80 m

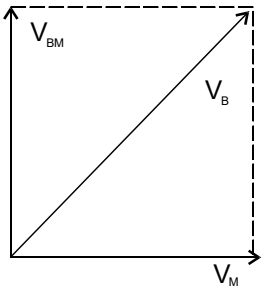
We know that

$\tan\theta = \frac{v^2}{rg} = \frac{20 \times 20}{80 \times 10} = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

Advanced Level Multiconcept Questions

1. (*c*, *d*)

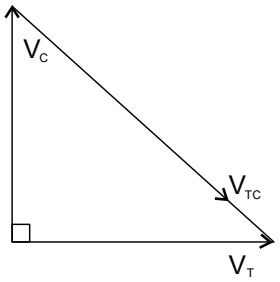


Ball will be catched by man if the cart is unaccelerated. If there is some positive accelerations, then ball will fall behind.

2. (*a*, *b*) $\xrightarrow{V_T}$

$\xrightarrow{V_c}$

$|V_T - V_C| = 7$

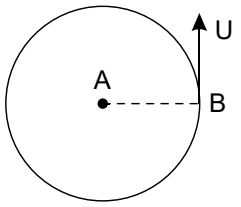


$V_T^2 + V_C = 13^2$

If $V_T = 12, V_C = 5$

If $V_T = 5, \quad V_C = 12$

3. (*a*, *b*, *d*)



Particles are neither approaching nor separating.
But, Relative velocity is V.

$\overrightarrow{V_1} = \overrightarrow{V_B} - \overrightarrow{V_A}$ ....(1)

$\overrightarrow{V_2} = \overrightarrow{V_C} - \overrightarrow{V_B}$ ....(2)

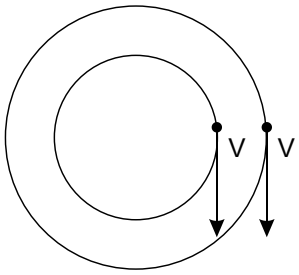
$\overrightarrow{V_0} = \overrightarrow{V_A}$ ....(3)

Adding (1) (2) & (3),

$\overrightarrow{V_c} = \overrightarrow{V_1} + \overrightarrow{V_2} + \overrightarrow{V_0}$

V_C can be zero if $\overrightarrow{V_c} = \overrightarrow{V_1} + \overrightarrow{V_2} + \overrightarrow{V_0} = 0$

Rate of seperation of dog 1 w.r.t. dog 3 = 2V
Rate of seperation of dog 2 w.r.t. dog 4 = 2V



At this instant $V_{rel} = 0$
They would continue to move w.r.t each other,

4. (*b*,*c*,*d*)

V = U + at

-4 = 2 + a + 4

$a = \frac{-3}{2}$ m/s²

$S_G = V_G t + \frac{a_G t^2}{2}$

$0 = 2t - \frac{3}{4}t^2$

$t = \frac{8}{2} = 2.66s$

Hence, option (D)
In 4S,
SG= 2x

$S_G = 2 \times 4 \frac{-3}{4} \times 4^2$

$= -4m$

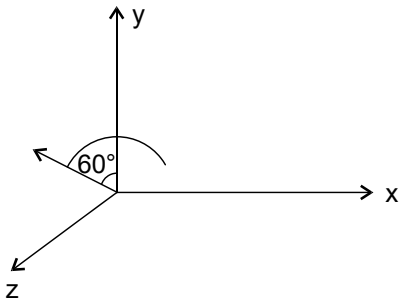
∴ optin (B)
Wrt Belt

$S_B = U_B t + \frac{1}{2} a_B t^2$

$= 6 \times 4 - \frac{3}{4} \times 4^2$

$= 12m$

5. (*a*, *d*)



$\overrightarrow{V_R} = 2t$ (River)

$\vec{V}_{Boat} = 3t + 2t = 5t$

At $t = 0, \vec{V}_{Ball} = 5t + 5j + 5\sqrt{3}\,k$

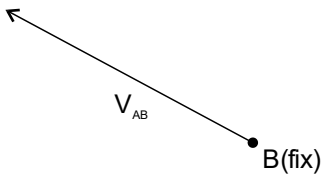
At

Highest Point, $\vec{V}_{Ball} = 5t + 5j$

$\vec{V}_{Ball,\ man} = 5t$

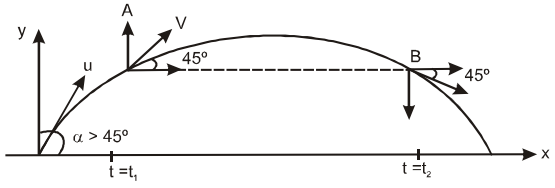
$\left|\vec{V}_{Ball,}\right| = 5\sqrt{2}$

6. (*a*,*b*)



7. (*b*,*d*)

8. (*b*,*c*)



$\Rightarrow$ Time at which $V_x = V_y$ is what we are solving $V_x = V_y$

Now, $V_x = u\cos\alpha$

$V_y = u\sin\alpha - gt$

$\Rightarrow u\cos\alpha = u\sin\alpha - gt$ $\{\because V_y = V_x\}$; at $t = t_1$ (say)

$\Rightarrow t_1 = \dfrac{u}{g}(\sin\alpha - \cos\alpha)$ **"C"**

Also when $V_y \equiv V_x$ {i.e., when we choose 'y' axis as – y'}

at $t = t_2$ (say)

$-u\cos\alpha = u\sin\alpha - gt_2$

$\Rightarrow t_2 = \dfrac{u}{g}(\sin\alpha + \cos\alpha)$ **"B"**

9. (*a*,*b*) $x = 24 = u\cos\theta.t$

$\Rightarrow t = \dfrac{24}{24\cos\theta} = \dfrac{1}{\cos\theta}$

$y = 14 = u\sin\theta t - \dfrac{1}{2}gt^2$

$\Rightarrow 14 = \dfrac{u\sin\theta}{\cos\theta} - \dfrac{5}{\cos^2\theta}$

$\Rightarrow 14 = u\tan\theta - 5\sec^2\theta$

$\Rightarrow 5\tan^2\theta - 24\tan\theta + 19 = 0$

$\Rightarrow \tan\theta = 1, 19/5.$

10. (*a*,*b*,*c*,*d*) $h = \dfrac{u^2}{2g} \Rightarrow u = \sqrt{2gh}$

(a) $R_{max} = \dfrac{u^2}{g} = 2h$

(b) $R = nH_{max}$

$\dfrac{u^2\sin 2\theta}{g} = n\dfrac{u^2\sin^2\theta}{2g}$

$4 = n\tan\theta$

$\theta = \tan^{-1}\left(\dfrac{4}{n}\right)$

(*c*) $gT^2 = g\times\dfrac{4u^2\sin^2\theta}{g^2} = \dfrac{2\times u^2\sin 20}{g}\times\tan\theta$

$gT^2 = 2R\tan\theta$

(*d*) $T = \dfrac{2u_y}{g}, H_{max} = \dfrac{u_y^{\,2}}{2g}$

$\therefore$ Ratio 1:1

11. (*a*,*c*,*d*) $T = \sqrt{\dfrac{2H}{g}} \Rightarrow 0.4 = \sqrt{\dfrac{2H}{g}}$

$\Rightarrow H = 0.8m$

$R = 0.4 \times 4 = 1.6m$

and $Uy = \sqrt{2gH} = \sqrt{2\times 10\times 0.8} = 4m/s \Rightarrow \theta = 45°$

12. (*b*,*d*) (*b*) There are other forces on the particle

(*d*) The resultant of the other forces varies in magnitude as well as in direction.

13. (*a*,*c*,*d*) In curved path, may be circular or parabolic.

In circular path speed and magnitude of acceleration are constant.

In parabolic path acceleration is constant.

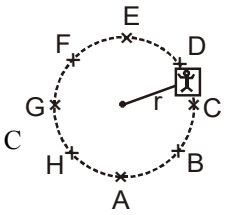
14. (*b*,*c*,*d*,*e*) $v = \sqrt{gr} \Rightarrow$ At **A**

$N = mg + \dfrac{mv^2}{r} = 2mg$ $[v = \sqrt{gr}]$

at E

$$N + \frac{mv^2}{r} = mg$$

$\Rightarrow$ N = 0 $\Rightarrow$ At G and N = mg



15. (*b,c*) $T - mg\cos\theta = \frac{mv^2}{L}$

Tangential Acceleration = g sin θ

16. (*b,d*) When speed of car is 36 km/hr, car can make a turn without skidding. If speed is less than 36 km/hr than tendency of slipping is downward so it will slip down. If speed is greater than 36 km/hr than tendency of slipping upward so it will slip up.

If the car's turn at correct speed 36 km/hr

N cos θ = mg

$$N\sin\theta = \frac{mv^2}{r}$$

$$N = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)}$$

17. (*b*) **18.** (*b*)

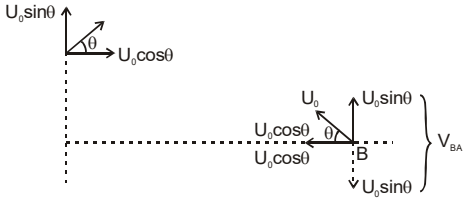
Sol. (17 & 18)

The path of a projectile as observed by other projec tile is a straight line.

$V_A = u\cos\theta\ \hat{i} + (u\sin\theta - gt)\hat{j}$. $V_{AB} = (2u\cos\theta)\hat{i}$

$V_B = -u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$.

$a_{BA} = g - g = 0$



The vertical component $u_0 \sin\theta$ will get cancelled. The relative velocity will only be horizontal which is equal to $2u_0\cos\theta$.

Hence B will travel horizontally towards left w.r.t. A with constant speed $2u_0\cos\theta$ and minimum distance will be h.

19. (*c*) Time to attain this separation will obviously be

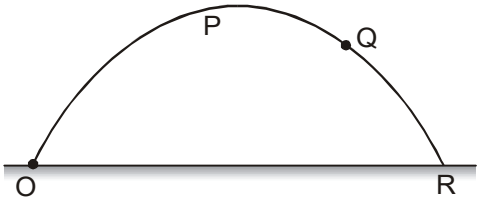
$$\frac{S_{rel}}{V_{rel}} = \frac{\ell}{2u_0\cos\theta}.$$

20. (*a*) $H_A = H_C > H_B$

Obviously A just reaches its maximum height and C has crossed its maximum height which is equal to A as u and θ are same. But B is unable to reach its max. height.

21. (*c*) Time of flight of A is 4 seconds which is same as the time of flight if wall was not there.

Time taken by C to reach the inclined roof is 1 sec.



$T_{OR} = 4$

$T_{QR} = 1$

$\therefore$ $T_{OQ} = T_{OR} - T_{QR} = 3$ seconds

22. (*c*) from above $= \frac{2u\sin\theta}{g} = 4$

$\therefore$ $u\sin\theta = 20$ m/s $\Rightarrow$ vertical component is 20 m/s.

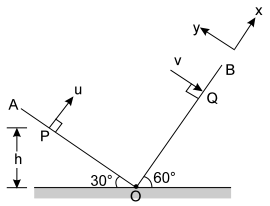
for maximum height

$v^2 = u^2 + 2as \Rightarrow 0^2 = 20^2 - 2 \times 10 \times s$

s = 20 m.

23. (*b*) Let us choose the x and y directions along OB and OA respectively. Then

$u_x = u = 10\sqrt{3}$ m/s, $u_y = 0$



$a_x = -g\sin 60° = -5\sqrt{3}$ m/s^2

and$a_y = -g\cos 60° = -5$ m/s^2

At point Q, x-component of velocity is zero. Hence, substituting in

$v_x = u_x + a_x t$

$0 = 10\sqrt{3} - 5\sqrt{3}t$

$$\text{or } t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2s$$

24. (*a*) At point Q, $v = v_y = u_y + a_y t$

$\therefore$ $v = 0 - (5)(b) = -10$ m/s

Here, negative sign implies that velocity of particle at Q is along negative y direction.

25. (*c*)

Distance PO = |displacement of particle along y-direction |

Here, $s_y = u_y t + \frac{1}{2} a_y t^2$

$= 0 - \frac{1}{2}(5)(b)^2 = -10$ m

$\therefore$ PO = 10 m

Therefore, $h = PO \sin 30° = (10)\left(\frac{1}{2}\right)$

or $h = 5$ m

26. (*a*) Distance OQ = displacement of particle along x-direction = s_x

Here $s_x = u_x t + \frac{1}{2} a_x t^2$

$= (10\sqrt{3})(b) - \frac{1}{2}(5\sqrt{3})(b)^2 = 10\sqrt{3}$ m

or $OQ = 10\sqrt{3}$ m

$\therefore \quad PQ = \sqrt{(PO)^2 + (OQ)^2}$

$= \sqrt{(10)^2 + (10\sqrt{3})^2} = \sqrt{100 + 300} = \sqrt{400}$

$\therefore \quad PQ = 20$ m

27. (*b*) **28.** (*a*) **29.** (*a*)

Sol. (30 to 32)

The angular velocity and linear velocity are mutually perpendicular

$\therefore \ \vec{v} \cdot \vec{\omega} = 3x + 24 = 0$ or $x = -8$

The radius of circle $r = \frac{v}{\omega} = \frac{5}{10} = \frac{1}{2}$ meter

The acceleration of particle undergoing uniform circular motion is

$\vec{a} = \vec{\omega} \times \vec{v} = (-8\hat{i} + 6\hat{j}) \times (3\hat{i} + 4\hat{j})$

$= -50\hat{k}$

$\therefore \ \vec{v} \cdot \vec{\omega} = 3x + 24 = 0$ or $x = -8$

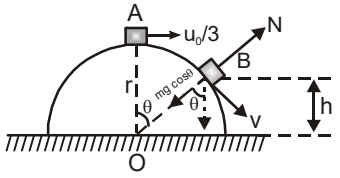
30. (*d*) $mg = \frac{mu_0^2}{r} \Rightarrow u_0 = \sqrt{gr}$

Now, along vertical

$r = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2r}{g}}$

Along horizontal; $OP = 2u_0 t = 2\sqrt{2}\, r$

31. (*b*) As at B it leaves the hemisphere,



$\therefore \quad N = 0$

$mg \cos\theta = \frac{mV^2}{r}$

$mg \frac{h}{r} = \frac{mV^2}{r}$

$mv^2 = mgh$ .....(*a*)

By energy conservation between A and B

$mgr + \frac{1}{2} m \left(\frac{u_0}{3}\right)^2 = mgh + \frac{1}{2} mv^2$

Put u_0 and $mv^2 \therefore \ h = \frac{19r}{27}$

32. (*c*) As $a_c = \frac{v^2}{r} = g \cos\theta$

$\therefore \ a_t = g \sin\theta$

$\therefore \ a_{net} = g$

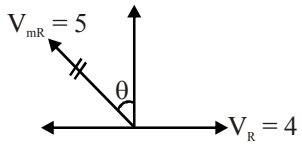
Alternate Solution:

when block leave only the force left is mg.

$\therefore \ a_{net} = g.$

33. (*a*) Q (*b*) R (*c*) S (*d*) T

34. (*a*) -R; (*b*)-S ; (*c*)-P ; (*d*)-Q

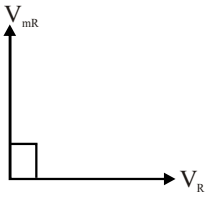


(*a*) $V_{mR} \sin\theta = V_R$

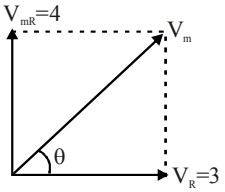
$\Rightarrow \sin\theta = \frac{4}{5} \qquad \Rightarrow \theta = 53°$

$\Rightarrow \theta + 90 = 143°$

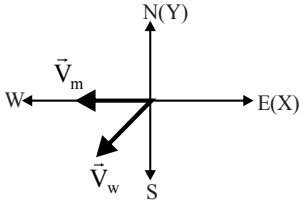
(*b*) Direction of velocity with the direction of flow = 90°

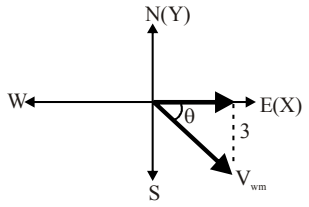


(*c*) $\tan\theta = \frac{4}{3} \quad \Rightarrow \quad \theta = 53°$



(*d*) $\vec{V}_w = -3\hat{i} - 3\hat{j} \Rightarrow \quad \vec{V}_m = -7\hat{i}$





$$\vec{V}_{wm} = \vec{V}_w - \vec{V}_m$$

$$= -3\hat{i} - 3\hat{j} + 7\hat{i} = 4\hat{i} - 3\hat{j}$$

$$\tan\theta = \frac{3}{4} \Rightarrow \theta = 37° \Rightarrow \theta + 90 = 127°$$

35. (*a*) P,Q,R (*b*) Q, R, S (*c*) Q,R,S

36. (*a*)q,(*b*)q,(*c*)q,(*d*)q

In all cases, angle between velocity and net force (in the frame of observer) is in between 0° and 180° (excluding both values, in that path is straight line).

37. (*a*)r (*b*)s (*c*)q (*d*)p Time of flight

$$T = \frac{2u}{g\cos 45°} = \frac{2u}{g\cos 45°} = \frac{2\sqrt{2}u}{g} \quad \therefore D \to p$$

Velocity of stone is parallel to x-axis at half the time of flight.

$$\therefore A \to r$$

At the instant stone make 45° angle with x-axis its velocity is horizontal.

$$\therefore \text{ The time is } = \frac{u\sin 45°}{g} = \frac{u}{\sqrt{2}\,g} \therefore B \to s$$

The time till its displacement along x-axis is half the range is

$$= \frac{1}{\sqrt{2}}T = \frac{2u}{g} \therefore C \to q$$

38. (*a*)r,(*b*)s,(*c*)q,(*d*)p Equation of path is given as $y = ax - bx^2$

Comparing it with standard equation of projectile;

$$y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$$

$$\tan\theta = a, \ \frac{g}{2u^2\cos^2\theta} = b$$

Horizontal component of velocity = $u\cos\theta = \sqrt{\frac{g}{2b}}$

$$\text{Time of flight } T = \frac{2u\sin\theta}{g} = \frac{2(u\cos\theta)\tan\theta}{g}$$

$$= \frac{2\left(\sqrt{\frac{g}{2b}}\right)a}{g} = \sqrt{\frac{2a^2}{bg}}$$

$$\text{Maximum height } H = \frac{u^2\sin^2\theta}{2g} = \frac{[u\cos\theta.\tan\theta]^2}{2g}$$

$$= \frac{\left[\sqrt{\frac{g}{2b}}.\right]^2}{2g} = \frac{a^2}{4b}$$

$$\text{Horizontal range } R = \frac{u^2\sin 2\theta}{2g} = \frac{2(u\sin\theta)(u\cos\theta)}{g}$$

$$= \frac{2\left[\sqrt{\frac{g}{2b}}.a\right]\left[\sqrt{\frac{g}{2b}}\right]}{g} = \frac{a}{b}$$

39. (*a*)q,s,(*b*)p(*c*)p,(*d*)q,r

From graph (a) $\Rightarrow \omega = k\theta$

where k is positive constant

$$\text{angular acceleration} = \omega\frac{d\omega}{d\theta} = k\theta \times k = k^2\theta$$

$\therefore$ angular acceleration is non uniform and directly proportional to θ.

$\therefore$ (*a*) q, s

From graph (b) $\Rightarrow \omega^2 = k\theta$.

Differentiating both sides with respect to θ.

$$2\omega\frac{d\omega}{d\theta} = k \quad \text{or} \quad \omega\frac{d\omega}{d\theta} = \frac{k}{2}$$

Hence angular acceleration is uniform.

$\therefore$ (*b*) p

From graph (c) $\Rightarrow \omega = kt$

$$\text{angular acceleration} = \frac{d\omega}{dt} = k$$

Hence angular acceleration is uniform

$\Rightarrow$ (*c*) p

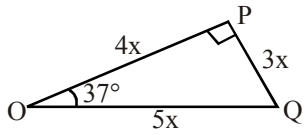
From graph (d) $\Rightarrow \omega = kt^2$

$$\text{angular acceleration} = \frac{d\omega}{dt} = 2kt$$

Hence angular acceleration is non uniform and directly proportional to t.

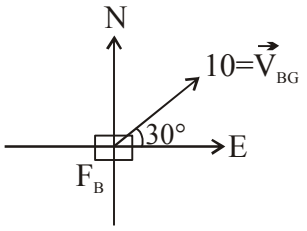
$\therefore$ (*d*) q,r**.**

40. [0005] Bird (*a*) travels 7x and Bird (*b*) travels 5x in same time,

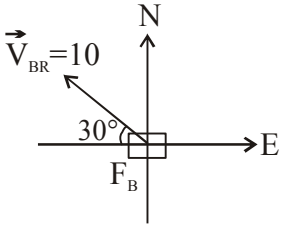


$$\text{so } \frac{v_2}{v_1} = \frac{s_2}{s_1} = \frac{5x}{7x} \text{ or } v_2 = \frac{5}{7}(v_1) = 5 \text{ km/hr}$$

41. [0017] Ground frame:



River frame :



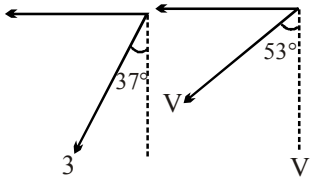
$$\vec{V}_{BG} = \vec{V}_{BR} + \vec{V}_{RG}$$

$$\Rightarrow\ 5\sqrt{3}\,\hat{i} + 5\hat{j} = -5\sqrt{3}\,\hat{i} + 5\hat{j} + \vec{V}_{RG}$$

$$\therefore\ \vec{V}_{RG} = 10\sqrt{3}\,\hat{i}$$

$\therefore$ V_{RG} = 17.3 m/s

42. [0007]



$$\frac{3V}{5} = \frac{12}{5}$$

V = 4

$$\frac{4V}{5} = \frac{9}{5} + V_C = \frac{7}{5}$$

$V_C = 0 + a_C t$

$a_C = 7$

43. [0025] $y = x\tan\theta - \frac{1}{2} g \frac{x^2}{v^2\cos^2\theta}$

x = 38 + 2 = 40

y = 18

θ = 60° $\Rightarrow$ v = 25 m/s

44. [0007] $R = \frac{u^2\sin(2\theta)}{g}$ $\therefore$ $R \propto \sin(2\theta)$ (for same speed of projection)

$$\therefore\ \frac{R'}{R} = \frac{\sin(2\times 45°)}{\sin(2\times 15°)}$$

$$\therefore\ R' = 3.5 \times \frac{1}{0.5} = 7\text{ m}$$

45. [0020] $-20 = 40\tan 0 - \frac{1}{2}\frac{g\times 40^2}{u^2\cos^2 0°}$

$u^2 = \frac{1600}{4} = 400$

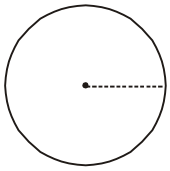
u = 20 m/s

46. [0001] $\omega^2 R = 2R\alpha$

$\omega^2 = 2\alpha = 2\alpha\theta$

θ = 1 rad.

47. [0625] $v = wR$



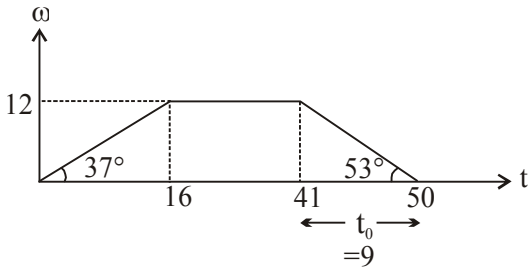
$v' = w(R-5) = \frac{\omega R}{5}$

5R – 25 = R

$R = \frac{25}{4}\text{m} = 6.25$ m

48. [0009] $< \omega > \frac{\int \omega\,dt}{\int dt} = \frac{\text{Area under graph}}{\text{time}}$

$= \frac{\frac{1}{2}\times 12[25+50]}{50} = 9$ r/s.



49. [0005] T = mg

$m r_{max}\omega^2 = T + \mu mg$

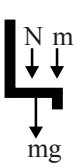
$m r_{min}\omega^2 = T - \mu mg$

$m(r_{max} + r_{min})\omega^2 = 2mg$

$\therefore$ $r_{max} + r_{min} = \frac{2g}{\omega^2} = 5$ m

50. [0200]

$N_b - mg = \frac{mv^2}{R}$





$N_b = mg + \frac{mv^2}{R}$

$$N_T + mg = \frac{mv^2}{R}$$

$$N_T = \frac{mv^2}{R} - mg = \frac{1}{3}\left(mg + \frac{mv^2}{R}\right)$$

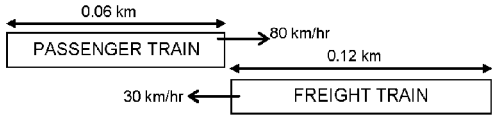
$$\frac{2}{3}\frac{mv^2}{R} = \frac{4}{3}mg$$

$$v = \sqrt{2gR} = \sqrt{20\times 2000} = 200 \text{ m/s}$$

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*a*) -



Time taken if both moving in same direction

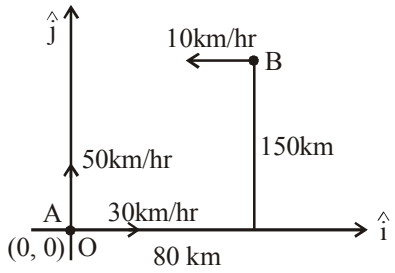
$$t_1 = \frac{\text{distance}}{\text{speed}} = \frac{0.12+0.06}{80-30} = \frac{0.18}{50}$$

Time taken if moving in opposite direction.

$$t_2 = \frac{\text{distance}}{\text{speed}} = \frac{0.12+0.06}{80+30} = \frac{0.18}{110}$$

$$\frac{t_1}{t_2} = \frac{11}{5}$$

2. (*d*) If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as :



$\vec{v}_A = (30\hat{i} + 50\hat{j})$ km/hr

$\vec{v}_B = -10\hat{i}$ km/hr

$\vec{r}_A = 0\hat{i} + 0\hat{j}$

$\vec{r}_B = (80\hat{i} + 150\hat{j})$ km

Time after which distance between them will be mini mum

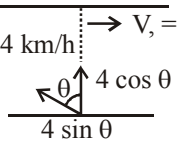
$$t = -\frac{\vec{r}_{BA}\cdot\vec{v}_{BA}}{|\vec{v}_{BA}|^2} \text{ ; where } \vec{r}_{BA} = (80\hat{i}+150\hat{j}) \text{ km}$$

$$\vec{v}_{BA} = -10\hat{i} - (30\hat{i}+50\hat{j}) = (-40\hat{i}-50\hat{j}) \text{ km/hr}$$

$$\therefore\ t = -\frac{(80\hat{i}+150\hat{j})\cdot(-40\hat{i}-50\hat{j})}{|-40\hat{i}+50\hat{j}|^2}$$

$$= \frac{3200+7500}{4100}\text{ hr} = \frac{10700}{4100}\text{ hr} = 2.6 \text{ hrs}$$

3. (*d*) $V_{mt} = 4$ km/h, $V_r = 2$ km/h



For swimmer to cross the river straight

$\Rightarrow$ 4 sin θ = 2

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

So, angle with direction of river flow = 90° + θ = 120°

Option (*d*)

4. [580] $X_1 = -3t^2 + 8t + 10$

$\vec{v}_1 = (-6t+8)\hat{i}$

$= 2\hat{i}$

$Y_2 = 5 - 8t^3$

$\sqrt{v} = |\vec{v}_2 - \vec{v}_1| = |-24\hat{j} - 2\hat{i}|$

$\sqrt{v} = \sqrt{24^2 + 2^2}$

v = 580

5. (*c*) Given

$\frac{v^1}{v} = \tan 60^\circ$ ...(i) $\longrightarrow$ v, 60°

$\frac{v^1}{v(1+\beta)} = \tan 45^\circ$ ...(ii) $\longrightarrow$ $v(1+\beta)$, 45°

From (i) and (ii)

β = 0.73

6. (*c*) $\frac{dx}{dt} = y; \frac{dy}{dt} = x$

$\frac{dx}{dy} = \frac{y}{x}$

$\Rightarrow y^2 = x^2 + c$

7. (*a*) Area $\propto \pi(\text{Range})^2 \propto V^4$

$\therefore \frac{A_1}{A_2} = \left(\frac{1}{2}\right)^4$

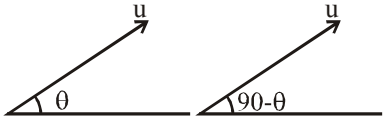
8. (*b*) $t = \frac{2 \times 2\sin 15^\circ}{g\cos 30^\circ}$

$S = 2\cos 15^\circ \times t - \frac{1}{2} g \sin 30^\circ t^2$

Put values and solve

S = 20 cm

9. (*b*)



For same range angle of projection will be θ & 90 – θ

$R = \frac{u^2 2\sin\theta\cos\theta}{g}$

$h_1 = \frac{u^2\sin^2\theta}{g}$

$h_2 = \frac{u^2\sin^2(90-\theta)}{g}$

$\frac{R^2}{h_1 h_2} = 16$

10. (*a*) Equation of trajectory is given as

$y = 2x - 9x^2$ ....(i)

Comparing with equation :

$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}.x^2$ ....(ii)

We get ;

$\tan\theta = 2$

$\therefore \cos\theta = \frac{1}{\sqrt{5}}$

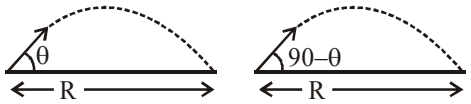
Also, $\frac{g}{2u^2\cos^2\theta} = 9$

$\Rightarrow \frac{10}{2\times 9\times\left(\frac{1}{\sqrt{5}}\right)^2}\times u^2$

$\Rightarrow u^2 = \frac{25}{9}$

$\Rightarrow u = \frac{5}{3}\text{m/s}$

11. (*c*) Range will be same for time t_1 & t_2, so angles of projection will be 'θ' & '90° – θ'



$t_1 = \frac{2u\sin\theta}{g}$ $t_2 = \frac{2u\sin(90^\circ-\theta)}{g}$

and $R = \frac{u^2\sin 2\theta}{g}$

$t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right]$

$\frac{2R}{g}$

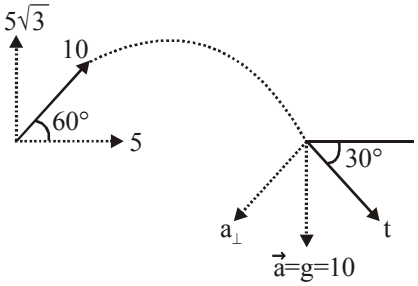
12. (*a*)

13. (*b*) $m\omega^2 R = \text{Force} \propto \frac{1}{R^n}$ $\Rightarrow \omega^2 \propto \frac{1}{R^{n+1}}$

$\Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$

time period $T = \frac{2\pi}{\omega} \propto R^{\frac{n+1}{2}}$

14. (*b*)



at t = 1

$u_x = 5$, $u_y = 5\sqrt{3}$

$v_y = 5\sqrt{3} - 10$; $v_x = 5$

$\tan\theta = -(2-\sqrt{3}) \Rightarrow \theta = -30^\circ$

$R = \frac{v^2}{a_\perp} = \frac{10^2}{(10\cos 30^\circ)}$

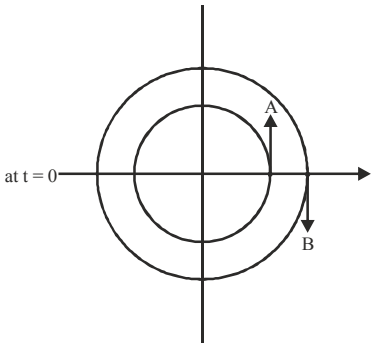
$= \frac{10}{\sqrt{3}}\times 2 = \frac{20}{\sqrt{3}}\text{m}$

$\frac{5^2 + \left(10-5\sqrt{3}\right)^2}{10\cos\theta} = \frac{200-100\sqrt{3}}{10\times 0.965} = 2.8\text{ m}$

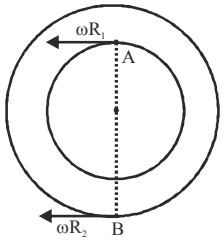
15. (*d*) $\Delta\vec{v} = 2v\sin\left(\frac{\theta}{2}\right)$

$= 2 \times 10 \times \sin(30°)$ $= 10$ m/s

16. (*c*)



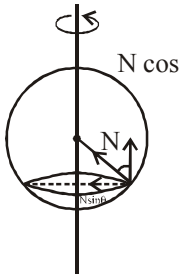
at $t = \frac{\pi}{2\omega}$



$\vec{v}_A - \vec{v}_B = -\omega R_1\hat{i} + \omega.R_2\hat{i}$

$= \omega(R_2 - R_1)\hat{i}$

17. (*d*)



$N\sin\theta = m\frac{r}{2}\omega^2$ .....(*a*)

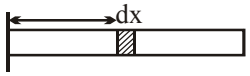
$N\cos\theta = mg$ .....(*b*)

$\tan\theta = \frac{r\omega^2}{2g}$

$\frac{r}{2\frac{\sqrt{3}r}{2}} = \frac{r\omega^2}{2g}$

$\omega^2 = \frac{2g}{\sqrt{3}r}$

18. (*d*)

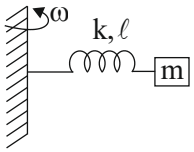


$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell} \frac{m}{\ell} dx\omega^2 x$

$= \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$

$T = \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$

19. (*a*)



$m\omega^2(\ell + x) = kx$

$\left(\frac{\ell}{x} + 1\right) = \frac{k}{m\omega^2}$

$x = \frac{\ell m\omega^2}{k - m\omega^2}$

20. (*d*) $\vec{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = \omega\left(-\sin\omega t\hat{i} + \cos\omega t\hat{j}\right)$

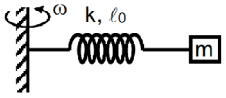
$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\hat{i} + \sin\omega t\hat{j})$

$\vec{a} = -\omega^2\vec{r}$ $\therefore$ $\vec{a}$ is antiparallel to $\vec{r}$

$\vec{v}.\vec{r} = \omega\left(-\sin\omega t \ \cos\omega t + \cos\omega t \ \sin\omega t\right) = 0$

so $\vec{v} \perp \vec{r}$

21. (*d*)



$m\omega^2(\ell_0 + x) = kx$

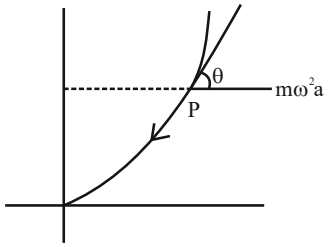
$\left(\frac{\ell_0}{x} + 1\right) = \frac{k}{m\omega^2}$

$x = \frac{\ell_0 m\omega^2}{k - m\omega^2}$

$k >> m\omega^2$

So, $\frac{x}{\ell_0}$ is equal to $\frac{m\omega^2}{k}$.

22. (*a*)



$y = 4Cx^2$

$\Rightarrow \dfrac{dy}{dx} = \tan\theta = 8Cx$

At P, $\tan\theta = 8Ca$

For steady circular motion

$mg\sin\theta = mw^2a\cos\theta$

$\Rightarrow \tan\theta = \dfrac{\omega^2 a}{g}$

$\Rightarrow 8Ca \times g = \omega^2 \times a$

$\Rightarrow \omega = \sqrt{8gC}$

$\Rightarrow \omega = 2\sqrt{2gC}$

23. (*d*) $a = \omega^2 \times \ell$

$= \left(\dfrac{2\pi}{T}\right)^2 \times \ell$

$= \left(\dfrac{2\pi}{60}\right)^2 \times 0.1$

$= 1.1 \times 10^{-3}\ m/s^2$

24. (*a*) $\vec{v} = \left(0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}\right) m/s$

At $t = 2s$

$\vec{v} = (2\hat{i} + \hat{j}\hat{j} + g\hat{k})$

Direction cosine along y-axis,

$\cos\theta = \dfrac{(v\hat{j})}{\sqrt{g^2 + \epsilon^2 + 2^2}} = \dfrac{6}{\sqrt{121}} = \dfrac{6}{11}$

$\therefore \sin\theta = \dfrac{\sqrt{85}}{11}$

and $\tan\theta = \dfrac{\sqrt{85}}{6}$

$\therefore$ Mosquito make angle $\tan^{-1}\left(\dfrac{\sqrt{85}}{6}\right)$ from y -axis.

25. (*a*) $y = \alpha x - \beta x^2$

comparing with trajectory equation

$y = x\tan\theta - \dfrac{1}{2}\dfrac{\xi y^2}{x^2\cos^2\phi}$

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}\alpha$

$\Rightarrow \beta = \dfrac{1}{2}\dfrac{\theta}{2\cos^2\theta}$

$\Rightarrow u^2 = \dfrac{g}{2\beta 00 < 9}$

Maximum height H:

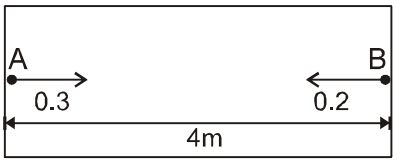
$H = \dfrac{2^2\sin^2\theta}{2\theta} = \dfrac{\theta}{2\beta\,\text{coc}\,\alpha\beta}\dfrac{\sin^2\beta}{2\theta}$

$\Rightarrow H = \dfrac{\tan^2\beta}{4\beta} = \dfrac{\alpha^2}{4\beta}$

26. (*c*) Relative velocity of bomb w.r.t. observer in plane = 0. Bomb will fall down vertically So, it will move in straight line w.r.t. Observer.

JEE-ADVANCED PREVIOUS YEAR'S

1. [2]



consider motion of two balls with respect to rocket

Maximum distance of ball A from left wall =

$\dfrac{u^2}{2a} = \dfrac{0.3\times 0.3}{2\times 2} = \dfrac{0.09}{4} \approx 0.02\ m$

so collision of two balls will take place very near to left wall

For B

$S = ut + \dfrac{1}{2}at^2$

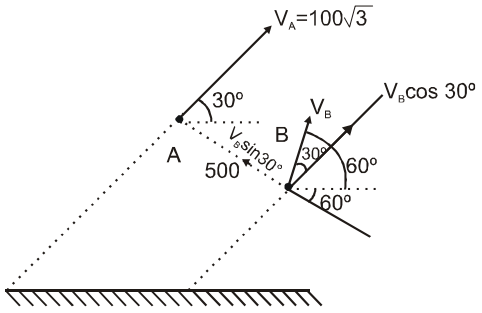
$-4 = -0.2t - \left(\dfrac{1}{2}\right)2t^2$

$\Rightarrow t^2 + 0.2t - 4 = 0$

$\Rightarrow t = \dfrac{-0.2 \pm \sqrt{0.04 + 16}}{2} = 1.9$

nearest integer = 2s

2. **[5]**



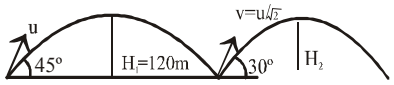
For relative motion perpendicular to line of motion of A

$V_A = 100\sqrt{3} = V_B \cos 30^\circ$

$\Rightarrow V_B = 100$ m/s

$t_0 = \dfrac{50}{V_B \sin 30^\circ} = \dfrac{500}{200 \times \frac{1}{2}} = 5\,\text{sec}$

3. [30.00]



$H_1 = \dfrac{u^2 \sin^2 45}{2g} = 120$

$\Rightarrow \dfrac{u^2}{4g} = 120$ .....(i)

when half of kinetic energy is lost $v = \dfrac{u}{\sqrt{2}}$

$H_2 = \dfrac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30}{2g} = \dfrac{u^2}{16g}$ .....(ii)

from (i) & (ii)

$H_2 = \dfrac{H_1}{4} = 30\text{m}$ on 30.00

4. [4.00] Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time}}$

Total time taken = $t_1 + t_2 + t_3 + ...$

$= t_1 + \dfrac{t_1}{\alpha} + \dfrac{t_1}{\alpha^2} + ...$

Total time = $\dfrac{t_1}{1 - \frac{1}{\alpha}}$

Total displacement = $v_1 t_1 + v_2 t_2 + ...$l

$= v_1 t_1 + \dfrac{v_1}{\alpha} \cdot \dfrac{t_1}{\alpha} + ...$

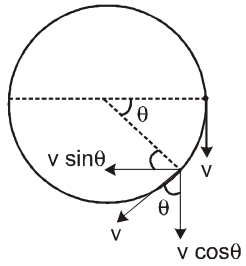
$= \dfrac{v_1 t_1}{1 - \frac{1}{\alpha^2}}$

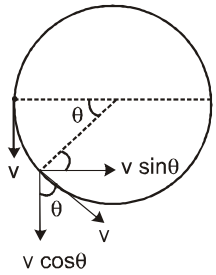
On solving

$< v > = \dfrac{v_1 \alpha}{\alpha + 1} = 0.8 v_1$

$\alpha = 4.00$

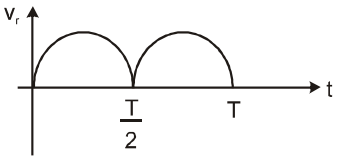
5. (*a*)





$v_r = |2\ v \sin\theta)|$

$= |2v \sin \omega t)|$



6. (*b*) $a = \omega^2 r$

$\Rightarrow \int_0^v v dv = \omega^2 \int_{R/2}^r r dr \Rightarrow v = \omega\sqrt{r^2 - \dfrac{R^2}{4}}$

$\& \int_{R/2}^r \dfrac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \omega \int_0^t dt$

$\Rightarrow r = \dfrac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$

Hence, (*b*)

7. (*d*) $\vec{F}_{rot} = -m\omega^2 r\hat{i} + 2mv_{rot}\omega(-\hat{j}) + m\omega^2 r\hat{i}$

$= 2mv_{rot}\omega(-\hat{j})$

$= 2m\dfrac{\omega R}{4}\left(e^{\omega t} - e^{-\omega t}\right)\omega(-\hat{j})$

$\vec{F}_{net} = -\vec{F}_{rot} + mg\hat{k} = \dfrac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$

Hence, (*d*)

SAHEEL

5 LAWS OF MOTION

SAHEEL

FORCE

A force changes the state of rest or motion of a body. When a force act on a body it start moving if it is at rest or if it is in motion then to stop it, or to deflect it from its initial path of motion. Force is defined as an interaction between two bodies.

BASIC FORCES IN NATURE

(i) Gravitational force
(ii) Electromagnetic force
(iii) Nuclear force
(iv) Weak force

- Other well known forces like frictional force, elastic force, viscous force, spring force, intermolecular force are manifestations of electromagnetic forces.
- Electromagnetic force is about 10^{36} times stronger than gravitational force while weak forces are about 10^{25} times as strong as gravitational force.

$F_N > F_E > F_W > F_G$

$F_G : F_W : F_E : F_N :: 1 : 10^{25} : 10^{36} : 10^{38}$

- Gravitational and electromagnetic forces are central or conservative forces and obey inverse square law.
- The gravitational and the electromagnetic forces are long range forces having infinite range while nuclear and weak forces are very short range forces.
- The nuclear force is the strongest force while the gravitational force is the weakest force of nature.
- Nuclear force is a non-central force and varies inversely with some higher power of distance.
- Beta-decay can be explained only on the basis of weak forces.

TYPES OF FORCES

1. **Contact forces :**
   Whenever two bodies come in contact they exert forces on each other, that are called **contact forces.**
   (a) Normal reaction (N or R) : It is the component of contact force normal to the surface. It measures how strongly the surfaces in contact are pressed together.
   (b) Frictional force (f) : It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two surfaces in contact.
   (c) Tension : The force exerted by the end of a taut string, rope or chain is called the tension. The direction of tension is always pulling in nature.
   (d) Spring force : Every spring resists any attempt to change its length, the more you change its length the harder it resists. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is spring constant or stiffness constant (units N/m).
2. **Non contact forces :**
   These forces do not involve physical contact between two points and are also called **field forces** e.g. Gravitational force,electric force.
   Note : Weight : Weight of a body is the force with which earth attracts it. It is also defined as force of gravity or the gravitational force.

NEWTON'S FIRST LAW OF MOTION OR LAW OF INERTIA

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Definition of force from Newton's first law of motion "Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line".

Inertia : Inertia is the property of the body due to which the body oppose the change of state by itself. Inertia of a body is measured by mass of the body.

Inertia $\propto$ mass

Heavier the body, greater is the force required to change its state and hence greater is inertia. The reverse is also true. i.e. lighter body has less inertia.

TYPES OF INERTIA

Inertia of rest : It is the inability of a body to change by itself, its state of rest.

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly the passengers sitting inside tends to fall backwards.
- The dust particles in a blanket fall off when it is beaten with a stick.

Inertia of motion : It is the inability of a body to change by itself its state of uniform motion.

- When a bus or train stop suddenly, a passenger sitting inside tends to fall forward.
- A person jumping out of a speeding train may fall forward.

Inertia of direction : It is the inability of a body to change by itself its direction of motion.

- When a car rounds a curve suddenly, the person sitting inside is thrown outwards.
- Rotating wheels of vehicle throw out mud. Mudguard over the wheels stop this mud.

NEWTON'S SECOND LAW OF MOTION

Rate of change of momentum of a body is directly proportional to the external force applied on it and the change in momentum takes place in the direction of force

$$\vec{F} \propto \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\boxed{\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}} \qquad \text{if m = constant}$$

$$\boxed{\vec{F} = \vec{v}\frac{dm}{dt}} \qquad \text{(as in case of conveyor belt)}$$

Special points:

1. This law gives magnitude, unit and dimension of force.
2. There are two types of unit of force :
   (a) Absolute unit
   (b) Gravitational or Practical unit
   In M.K.S. : Newton (N)
   In M.K.S. : kg-wt or kg-f
   In C.G.S. : Dyne (dyne)
   In C.G.S. : gm-wt or gm-f
   1 kg-wt = 9.8 N
   1 gm-wt = 980 dyne

NEWTON'S THIRD LAW OF MOTION

It states that to every action, there is an equal (in magnitude) and opposite (in direction) reaction.

If a body A exerts of force $\vec{F}$ on another body B, then B exerts a force $-\vec{F}$ on A, the two forces acting along the same line. The two forces $\vec{F}$ and $-\vec{F}$ connected by Newton's third law are called action-reaction pair. Any one may be called 'action' and the other 'reaction'.

"Action and reaction acts on different bodies hence they never cancel each other".

FREE BODY DIAGRAM

A free body diagram is a diagram showing the chosen body by itself, "free" of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces applied to the body by the various other bodies that interact with it.

Be careful to include all the forces acting on the body, but the equally careful not to include any force that the body exerts on any other body. In particular, the two forces in an action-reaction pair must never appear in the same free-body diagram because they never act on the same body.

[Forces that a body exerts on itself are never included, since these can't affect the body's motion.]

NOTE:

- The forces exerted on bodies of a given system by bodies situated outside are called external forces.
- The forces of interaction between bodies composing a system are called internal forces.
- A single isolated force is physically impossible.
- Whenever one force acts on a body it gives rise to another force called reaction.
- Total internal force in an isolated system is always zero.

PROBLEM SOLVING STRATEGY

Newton's laws refer to a particle and relate the forces acting on the particle to its mass and to its acceleration. But before writing any equation from Newton's law, you should be careful about which particle you are considering. The laws are applicable to an extended body too which is nothing but collection of a large number of particles.

Follow the steps given below in writing the equations :

Step 1 : Select the body

The first step is to decide the body on which the laws of motion are to be applied. The body may be a single particle, an extended body like a block, a combination of two blocks-one kept over another or connected by a string. The only condition is that **all the parts of the body or system must have the same acceleration.**

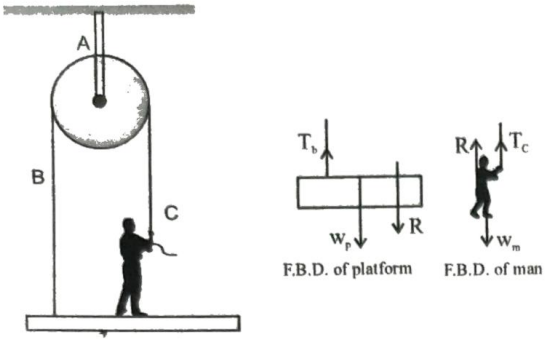
Step 2 : Identify the forces

Once the system is decided, list down all the forces acting on the system due to all the objects in the environment such as inclined planes, strings, springs etc. However, any force applied by the system shouldn't be included in the list. You should also be clear about the nature and direction of these forces.

Step 3 : Make a Free-body diagram (FBD)

Make a separate diagram representing the body by a point and draw vectors representing the forces acting on the body with this point as the common origin.

This is called a free-body diagram of the body.



Look at the adjoining free-body diagrams for the platform and the man. Note that the force applied by the man on the rope hasn't been included in the FBD.

Once you get enough practice, you'd be able to identify and draw forces in the main diagram itself instead of making a separate one

Step 4 : Select axes and Write equations

When the body is in equillibrium then choose the axis in such a manner that maximum number of force lie along the axis.

If the body is moving with some acceleration then first find out the direction of real acceleration and choose the axis. One is along the real acceleration direction and other perpendicular to it.

Write the equations according to the newton's second law ($F_{net} = ma$) in the corresponding axis.

EQUILIBRIUM OF CONCURRENT FORCE

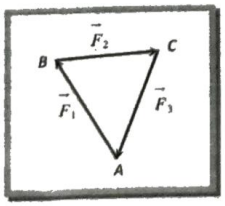
(1) If all the forces working on a body are acting on the same point, then they are said to be concurrent.

(2) A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.

(3) The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.

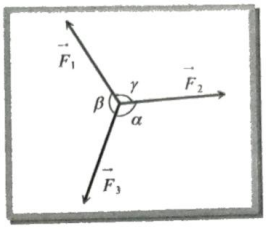
(4) Mathematically for equilibrium $\sum F_{net} = 0$ or $\sum F_x = 0$; $\sum F_y = 0$; $\sum F_z = 0$,

(5) Three concurrent forces will be in equilibrium, if they can be represented completely by three sides of a triangle taken in order.



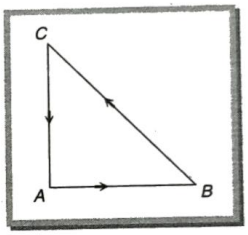
(6) **Lami's Theorem :** For concurrent forces

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



TRAIN YOUR BRAIN

Example 1: Three forces starts acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity



(*a*) $\vec{v}$ remaining unchanged

(*b*) Less than $\vec{v}$

(*c*) Greater than $\vec{v}$

(*d*) $\vec{v}$ in the direction of the largest force BC

Sol. : (a) Given three forces are in equilibrium i.e. net force will be zero. It means the particle will move with same velocity.

Example 2: Two forces are such that the sum of their magnitudes is 18 N and their resultant is perpendicular to the smaller force and magnitude of resultant is 12. Then the magnitudes of the forces are

(*a*) 12 N, 6 N (*b*) 13 N, 5N

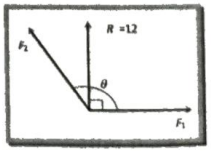
(*c*) 10 N, 8 N (*d*) 16 N, 2 N

Sol. (b) Let two forces are F_1 and F_2 ($F_1 < F_2$).

According to problem : $F_1 + F_2 = 18$ ...(i)

Angle between F_1 and resultant (R) is 90°

$$\therefore \quad \tan 90 = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} = \infty$$



$$\Rightarrow \cos\theta = -\frac{F_1}{F_2} \quad ...(ii)$$

and $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$144 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$ ...(iii)

by solving (i). (ii) and (iii)

we get $F_1 = 5N$ and $F_2 = 13N$

Example 3: The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is

(*a*) 60 ° (*b*) 120°

(*c*) 150° (*d*) 90°

Sol. Let forces are F and 2F and angle between them is θ and resultant makes an angle α with the force F.

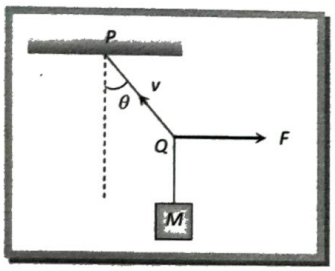
$$\tan\alpha = \frac{2F\sin\theta}{F + 2F\cos\theta} = \tan 90 = \infty$$

$\Rightarrow F + 2F\cos\theta = 0$

$\therefore \cos\theta = -1/2$

or $\theta = 120°$

Example 4: A mass M is suspended by a rope from a rigid support at P as shown in the figure. Another rope is tied at the end Q, and it is pulled horizontally with a force F. If the rope PQ makes angle with the vertical then the tension in the string PQ is



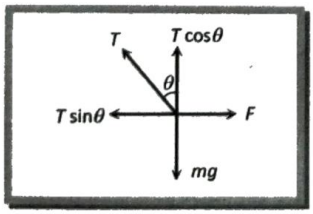
(*a*) F sin θ (*b*) F/sin θ

(*c*) F cos θ (*d*) F/cos θ

Sol. (b) From the figure

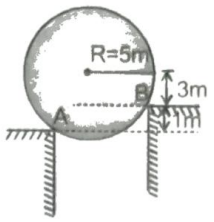
For horizontal equilibrium

T sin θ = F



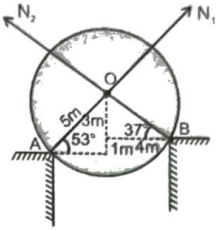
$\therefore T = \frac{F}{\sin\theta}$

Example 5

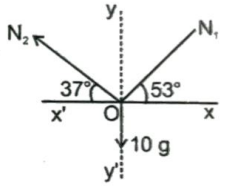


Find out the normal reaction at point A and B if the mass of sphere is 10 kg.

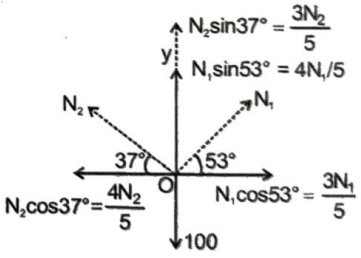
Sol.



Now F.B.D.



Now resolve the forces along x & y direction



$\because$ The body is in equilibrium so equate the force in x & y direction

In x-direction $\frac{3N_1}{5} = \frac{4N_2}{5}$ ...(1)

In y-direction $\frac{3N_2}{5} + \frac{4N_1}{5} = 100$ ...(2)

after solving above equation

$N_1 = 80$ N, $N_2 = 60$ N

MOTION OF A BLOCK ON A HORIZONTAL SMOOTH SURFACE

(a) When subjected to a horizontal pull.

(b) When subjected to a pull acting at an angle θ to the horizontal.

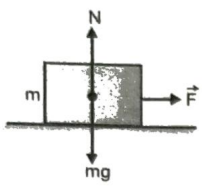


Fig-I

$N = mg$

and $a = \frac{F}{m}$

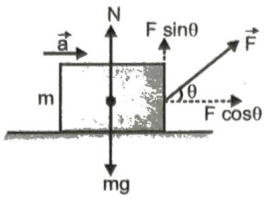
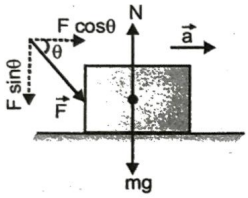


Fig-II

$N = mg - F \sin\theta$

and $a = \frac{F\cos\theta}{m}$

(c) When subjected to a push acting at an angle θ to the horizontal.



$N = mg + F \sin\theta$ and $a = \frac{F\cos\theta}{m}$

MOTION OF A BODY ON A SMOOTH INCLINED PLANE

(a) When smooth inclined plane is fixed

(b) When the smooth inclined plane is moving horizontally with an acceleration b.

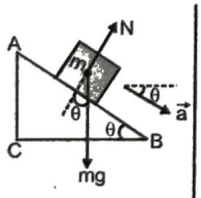


Fig-I

$N = mg\cos\theta$

and $a = g\sin\theta$

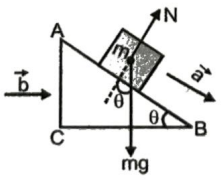


Fig-II

$N = m(g\cos\theta + b\sin\theta)$

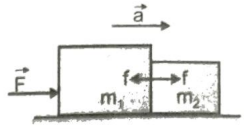
and $a = (g\sin\theta - b\cos\theta)$

MOTION OF BODIES IN CONTACT

Force of contact:

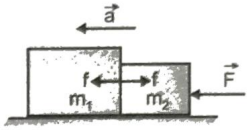
(a) Two bodies are kept in contact and force is applied on the body of mass m_1.

$$a = \frac{F}{m_1 + m_2} \quad \text{and} \quad f = \frac{m_2 F}{m_1 + m_2}$$

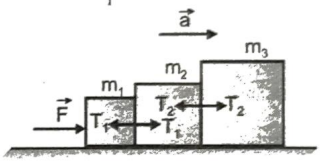


(b) Two bodies are kept in contact and force is applied on the body of mass m_2

$$a = \frac{F}{m_1 + m_2} \quad \text{and} \quad f = \frac{m_1 F}{m_1 + m_2}$$



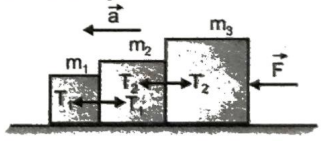
(c) Three bodies are kept in contact and force is applied on the body of mass m_1



$$a = \frac{F}{m_1 + m_2 + m_3} \quad T_1 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

$$T_2 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(d) Three bodies are kept in contact and force is applied on the body of mass m_3



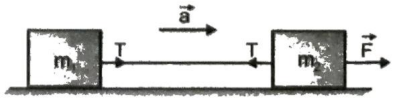
$$a = \frac{F}{m_1 + m_2 + m_3} \quad T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$

$$T_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)}$$

MOTION OF CONNECTED BODIES

(a) Two bodies are connected by a string and placed on a smooth horizontal surface

$$a = \frac{F}{m_1 + m_2} \quad T = \frac{m_1 F}{m_1 + m_2}$$

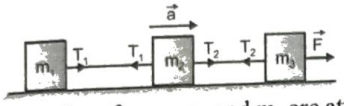


(b) When three bodies are connected through strings as shown in fig. and placed on a smooth horizontal surface.

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

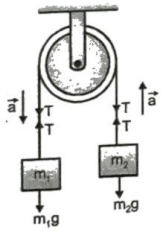
$$T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$

$$T_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)}$$



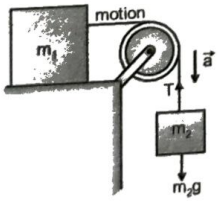
(c) When two bodies of mass m_1 and m_2 are attached at the ends of a string passing over a pulley as shown in the figure (neglecting the mass of pulley).

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} \qquad T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$$



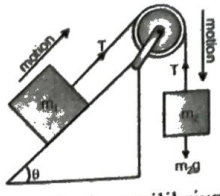
(d) When two bodies of masses m_1 and m_2 are attached at the ends of a string passing over a pulley in such a way that mass m_1 rests on a smooth horizontal table and mass m_2 is hanging vertically

$$a = \frac{m_2 g}{(m_1 + m_2)} \qquad T = \frac{m_1 m_2 g}{(m_1 + m_2)}$$



(e) If in the above case, mass m_1 is placed on a smooth inclined plane making an angle θ with horizontal as shown in figure, then

$$a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2} \qquad T = \frac{m_1 m_2 g(1 + \sin\theta)}{m_1 + m_2}$$



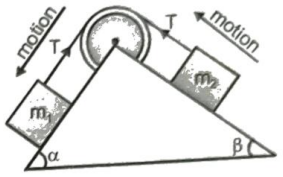
If the system remains in equilibrium, then $m_1 g \sin\theta = m_2 g$

(f) If case (d), masses m_1 and m_2 are placed on inclined planes making angles α and β with the horizontal respectively, then.

$\Rightarrow$ If $m_1 \sin\alpha > m_2 \sin\beta$

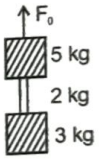
then $a = \frac{g(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)}$

$$T = \frac{m_1 m_2}{m_1 + m_2}(\sin\alpha + \sin\beta)\, g$$



TRAIN YOUR BRAIN

Example 6: A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at 2 m/s² by an external force F_0.

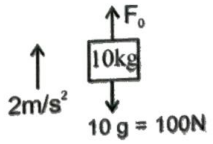


(*a*) What is F_0?
(*b*) What is the force on rope?
(*c*) What is the tension at middle point of the rope? ($g = 10\ m/s^2$)

Sol. For calculating the value of F_0, consider two blocks with the rope as a system.

F.B.D. of whole system

(a)



$F_0 - 100 = 10 \times 2$

$F = 120\ N$ ...(1)

(b) According to Newton's second law, net force on rope.

$F = ma = (2)(2) = 4\ N$ ...(2)

(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.

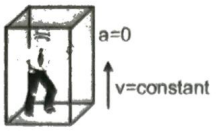
$T - 4g = 4 \times (2) = 48\ N$



APPARENT WEIGHT OF A BODY IN A LIFT

The lift is at rest or moving with uniform velocity

a = 0 :



$mg - N = 0$ or $N = mg$ or $W_{app.} = W_0$

$W_{app.} = N$ = reaction of supporting surface

and $W_0 = mg$ = true weight.

When the lift moves upwards with an acceleration a :



$N - mg = ma$ or $N = m(g + a) = mg\left(1 + \frac{a}{g}\right)$

$\therefore W_{app.} = W_0\left(1 + \frac{a}{g}\right)$

When the lift moves downwards with an acceleration a :

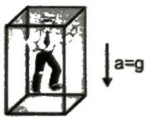


$mg - N = ma$ or $N = m(g - a) = mg\left(1 - \frac{a}{g}\right)$

$\therefore W_{app.} = W_0\left(1 - \frac{a}{g}\right)$

Here, if $a > g$, $W_{app.}$ will be negative. Negative apparent weight will mean that the body is pressed against the roof of the lift instead of floor.

When the lift falls freely, i.e., a = g :



$N = m(g - g) = 0$ or $W_{app.} = 0$

MONKEY CLIMBING A ROPE

The tension in the rope = T



$T = mg$ $T = m(g + a)$ $T = m(g - a)$

SYSTEMS OF VARIABLE MASS

According to Newton's 2nd law : $\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

If the mass of the system is variable and the velocity of escaping mass is constant $\frac{d\vec{v}}{dt} = 0$ and $\vec{F} = \vec{v}\frac{dm}{dt}$

Bullet fired from a machine gun :



If 'n' bullets each of mass m are fired per unit time from a machine gun. The force required to hold the gun = $v \times \frac{dm}{dt} = v(mn) = mnv$.

Sand is dropped on to a conveyor belt:

If sand is dropped on a conveyor belt at a constant rate,



extra force required to maintain the constant speed of the belt = $v\frac{dm}{dt}$

Jet of liquid coming out from a pipe :

If a jet of liquid coming out from a pipe of uniform area of cross-section A hits a wall normally with a velocity v.

The force applied by the jet on the wall = $Av^2\rho$

(ρ = density of liquid)

Rocket :

Let the rocket ejecting gases, with a constant velocity = $v_{rel.}$ (relative to rocket)

By Newton's 2nd law : $F_{ext.} + v_{rel.}\left(\frac{dm}{dt}\right) = ma$

thrust on the rocket = $v_{rel}\frac{dm}{dt}$ (the reaction force exerted by the leaving gases on the rocket)

In the absence of air resistance, if the rocket moves up with an acceleration = a

thrust = $v_{rel.}\frac{dm}{dt} = ma + mg$

If the rocket just moves up with constant velocity then

thrust = $v_{rel}\frac{dm}{dt} = mg$

CONSTRAINED MOTION:

String Constraint

When two objects are connected through a string and if the string have the following properties :

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them.

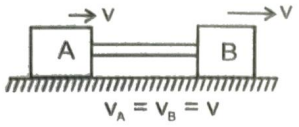
(i) 1st format : - When string is fixed



The block B moves with velocity v. i.e. each particle of block B moves with velocity v.

If string remain attached to block B it is necessary that velocity of each particle of string is same = v (v_s = v)

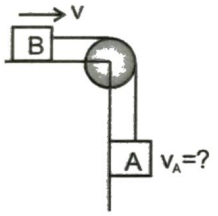
Now we can say that Block A also moves with velocity v.



If pulley is fixed then the velocity of all the particles of string is same along the string.

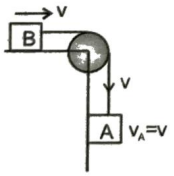
TRAIN YOUR BRAIN

Example 7:

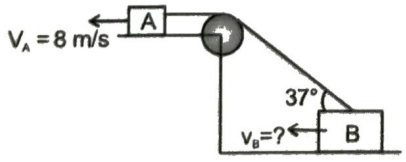


Sol. In the above situation block B is moving with velocity v. Then speed of each point of the string is v along the string.

∴ speed of the block A is also v



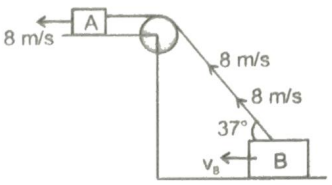
Example 8:



Sol. ∵ Block A is moving with velocity 8 ms^{-1}.

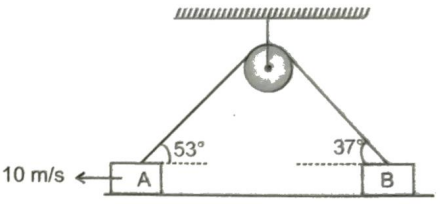
∴ velocity of every point on the string must be 8m/s along the string.

The real velocity of B is v_B. Then the string will not break only when the compoent of v_B along string is 8 m/s.

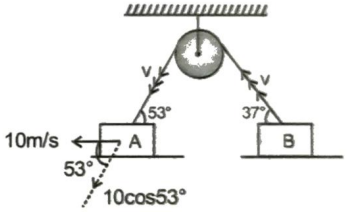


$$\Rightarrow \quad v_B \cos 37^\circ = 8 \quad \Rightarrow \quad v_B = \frac{8}{\cos 37^\circ} = 10 \text{ m/sec}$$

Example 9: Find out the velocity of block B in a pulley block system as shown in figure.



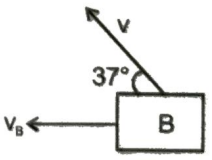
Sol. Let us assume the velocity of all the particle of string is v.



10 m/s is the real velocity of block A then its component along string is v.

$$\Rightarrow \quad 10 \cos 53^\circ = v \qquad \ldots(1)$$

If v_B is the real velocity of block B then it component



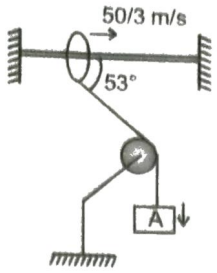
along string is v then

$v_B \cos 37^\circ = v$ ...(2)

from (1) & (2) $v_B \cos 37^\circ = 10 \cos 53^\circ$

$$\Rightarrow \quad v_B = \frac{10 \times 3/5}{4/5} = \frac{30}{4} = \frac{15}{2} \text{ m/sec}$$

Example 10:



What is the velocity of block A in the figure as shown above.

Sol. The component of velocity of ring along string = velocity of A

$$= \frac{50}{3}\cos 53° = v_A \Rightarrow v_A = 10 \text{ m/s}$$

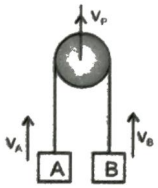
In the first format only two points of string are attached or touched to moving bodies.

(ii) 2nd format When pulley is also moving

To understand this format we consider the following example in which pulley is moving with velocity v_p and both block have velocity v_A & v_B respectively as shown in figure.

If we observe the motion of A and B with respect to pulley. Then the pulley is at rest. Then from first format.

$v_{AP} = -v_{BP}$



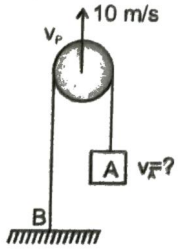
(–ve sign indicate the direction of each block is opposite with respect to Pulley)

$$v_A - v_p = -v_B + v_p \Rightarrow v_p = \frac{v_A + v_B}{2}$$

To solve the problem put the values of v_A, v_B, & v_p with sign.

TRAIN YOUR BRAIN

Example 11:



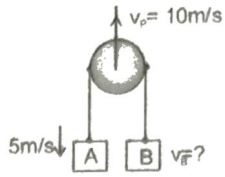
Sol. $v_p = \frac{v_A + v_B}{2}$

Putting $v_p = 10 \text{ ms}^{-1}$, $v_B = 0$,

we get

$v_A = 20 \text{ ms}^{-1}$ (upward direction)

Example 12:

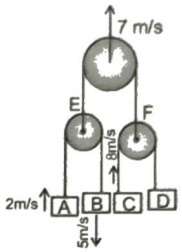


Sol. If we take upward direction as +ve then

$$10 = \frac{-5 + v_B}{2}$$

$v_B = 25$ m/sec (in upward direction)

Example 13:



Find out the velocity of Block D

Sol. From 2nd format of constrained motion

$$v_E = \frac{v_A + v_B}{2}$$

$$v_E = \frac{2-5}{2} = -3/2$$

(If upward direction is taken to be +ve)

$v_E = -3/2$ m/s

Now $\frac{v_E + v_F}{2} = 7$ m/s

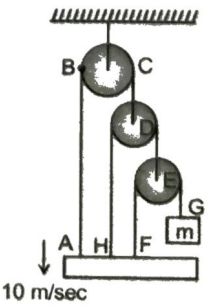
$$\Rightarrow 7 = \frac{-3/2 + v_F}{2} \Rightarrow 14 + 3/2 = v_F \Rightarrow v_F = \frac{31}{2}$$

Now $\frac{v_C + v_D}{2} = v_F \Rightarrow \frac{8 + v_D}{2} = \frac{31}{2}$

$\Rightarrow v_D = 31 - 8$

$v_D = 23$ m/s (upward direction)

Example 14:



Find the velocity of point G.

Sol. In string ABCD from first format of constraint

$V_D = 10$ m/s↑

Now $v_D = \frac{v_H + v_E}{2}$

$v_H = 10$ m/s ↓ if upward direction is taken to be positive

$+10 = \frac{-10 + v_E}{2} \Rightarrow v_E = 30$ m/s ↑

Now $\frac{v_F + v_G}{2} = v_E \Rightarrow 30 = \frac{-10 + v_G}{2}$

$60 + 10 = v_G$

$v_G = 70$ m/s↑

In IInd format three or four Points of the string is attached to the moving bodies.

(iii) 3rd format of string constraint :

SOLVING STRATEGY

1. First choose the longest string in the given problem which contains the point of which velocity/acceleration to be find out.
2. Now mark a point on the string wherever it comes in contact or leaves the contact of real bodies.
3. If due to motion of a point, length of the part of a string increases, then its speed will be taken +ve otherwise –ve.

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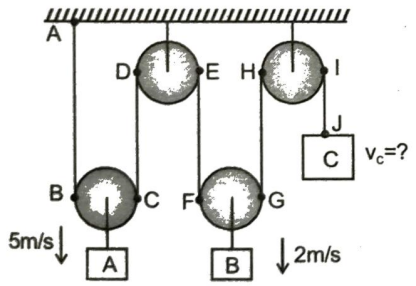
Example 15:



Sol.

Step 1. We choose a longest string ABCDEFGHIJ in which we have to find out velocity of point J (v_c)

Step 2. Mark all the point A, B .....

Step 3. Write equation

$v_A + v_B + v_C + v_D + v_E + v_F + v_{G+} v_H + v_I + v_J = 0$

$v_A = v_D = v_E = v_H = v_I = 0$

(No movement of that point because attached to fixed objects)

$\Rightarrow v_B + v_c + v_F + v_G + v_J = 0$ ...(1)

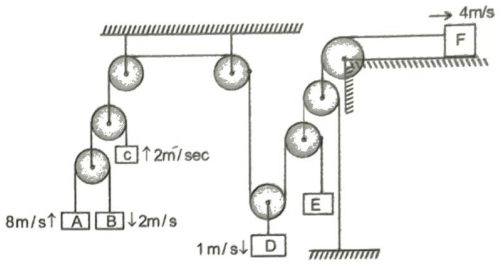
$v_B = v_C = 5$ m/s(increases the length)

$v_F = v_G = 2$m/s(It also increases the length)

Let us assume C is moving upward with velocity v_c so v_c is negative because it decreases the length

$\Rightarrow 5 + 5 + 2 + 2 - v_c = 0 \quad v_C = 14$ m/sec (upward)

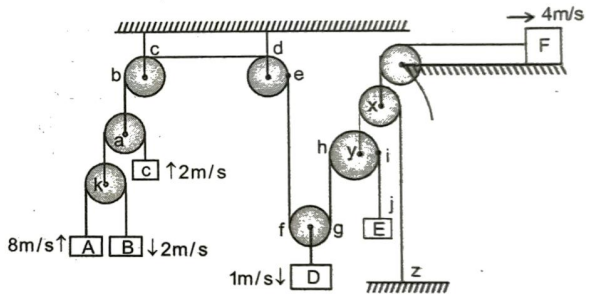
Example 16:



Find out the velocity of block E as shown in figure.

Sol.

Step-1 We first choose the longest string in which point j (block E) lie. (abcdefghij)



Step 2 : Now write equation according to the velocity of each point (either increase or decrease the length)

$v_a + v_b + v_c + v_d + v_e + v_f + v_g + v_h + v_i + v_j = 0$ ...(1)

Now find value of v_a, v_b ..... in a following way

$v_k = \frac{v_A + v_B}{2}$ (from second format)

$= \frac{8-2}{2} = 3$ m/sec. (upward)

$v_a = \frac{v_K + v_C}{2}$ (from 2nd format)

$= \frac{3+2}{2} = 5/2$ m/sec. (upward)

$v_x = 4$m/s (from first format of constrain)

from 2nd format of constrain $v_x = \frac{v_y + v_z}{2}$

$\therefore \quad v_z = 0$ (fixed)

$\Rightarrow v_y = 2\, v_x = 8$ m/s (upward)

$\Rightarrow$ Now $v_a = -5/2$ m/s (decreases the length)

$v_b = v_c = v_d = v_e = 0$ (attached to fixed object)

$v_f = v_g =$ 1m/s (increases the length)

$v_h = v_i = v_y = 8$ m/s (increase the length)

Let us assume block E move upward then $v_j = -v_E$ (decrease the length)

Puting the above values in eq. (1)

$\Rightarrow -5/2 + 1 + 1 + 8 + 8 - v_E = 0$

$v_E = 31/2$ m/s (upward)

(iv) Steps to solve more complex problem of String Constraint:

Step 1. Identify all the objects and number of strings in the problem.

Step 2. Assume variable to represent the parameters of motion such as displacement, velocity acceleration etc.

(i) Object which moves along a line can be specified by one variable.

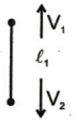
(ii) Object moving in a plane are specified by two variables.

(iii) Objects moving in 3-D requires three variables to represent the motion.

Step 3. Identify a single string and divide it into different linear sections and write in the equation format.

$\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ell_5 + \ell_6 = \ell$

Step 4. Differentiate with respect to time



$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} + \frac{d\ell_3}{dt} + ... = 0$$

$\frac{d\ell_1}{dt}$ = represents the rate of increment of the portion 1, end points are always in contact with some object so take the velocity of the object along the length of the string $\frac{d\ell_1}{dt} = V_1 + V_2$

Take positive sign if it tends to increase the length and negative sign if it tends to decrease the length. Here $+V_1$ represents that upper end is tending to increase the length at rate V_1 and lower end is tending to increase the length at rate V_2.

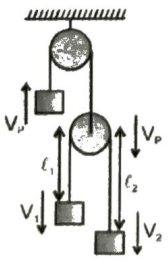
Step 5. Repeat all above steps for different-different strings.Let us consider a problem given below

Here $\ell_1 + \ell_2 =$ constant

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} = 0$$

$$(V_1 - V_P) + (V_P - V_2) = 0$$

$$V_P = \frac{V_1 + V_2}{2}$$



Similarly,

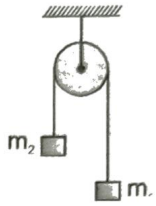
$a_P = \frac{a_1 + a_2}{2}$ Remember this result

TRAIN YOUR BRAIN

Example 17: Two blocks of masses m_1 and m_2 are attached at the ends of an inextensible string which passes over a smooth massless pulley. If $m_1 > m_2$, find :

(i) the acceleration of each block

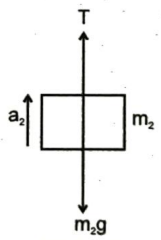
(ii) the tension in the string.



Sol. The block m_1 is assumed to be moving downward and the block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction. If the values of a_1 and a_2 come out to be positive then only the assumed directions are correct; otherwise the body moves in the opposite direction. Since the pulley is smooth and massless, therefore, the tension on each side of the pulley is same.

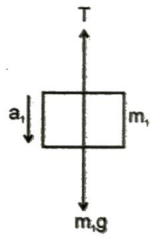
The free body diagram of each block is shown in the figure.

F.B.D. of m_2



F.B.D. of m_2

F.B.D. of m_1



F.B.D. of m_1

Applying Newton's second Law on blocks m_1 and m_2

Block m_1 $\quad m_1g - T = m_1a_1$ ...(1)

Block m_2 $\quad -m_2g + T = m_2a_2$ ...(2)

Number of unknowns : T, a_1 and a_2 (three)

Number of equations: only two

Obviously, we require one more equation to solve the problem. Note that whenever one finds the number of equations less than the number of unknowns, one must think about the constraint relation. Now we are going to explain the mathematical procedure for this.

How to determine Constraint Relation ?

(1) Assume the direction of acceleration of each block, e.g. a_1 (downward) and a_2 (upward) in this case.

(2) Locate the position of each block from a fixed point (depending on convenience), e.g. centre of the pulley in this case.

(3) Identify the constraint and write down the equation of constraint in terms of the distance assumed. For example, in the chosen problem, the length of string remains constant is the constraint or restriction.

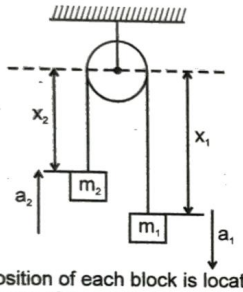
Thus, $x_1 + x_2 = \text{constant}$

Differentiating both the sides w.r.t. time we get

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

Each term on the left side represents the velocity of the blocks.

Since we have to find a relation between accelerations, therefore we differentiate it once again w.r.t. time.



Position of each block is located w.r.t. centre of the pulley

Thus $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$

Since, the block m_1 is assumed to be moving downward (x_1 is increasing with time)

$\therefore \frac{d^2x_1}{dt^2} = +a_1$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

$\therefore \frac{d^2x_2}{dt^2} = -a_2$

Thus $a_1 - a_2 = 0$

or $a_1 = a_2 = a$ (say) is the required constraint relation.

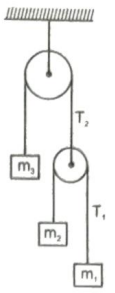
Substituting $a_1 = a_2 = a$ in equations (1) and (2) and solving them, we get

(i) $a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] g$ (ii) $T = \left[\frac{2m_1m_2}{m_1 + m_2}\right] g$

Example 18: A system of three masses m_1, m_2 and m_3 are shown in the figure. The pulleys are smooth and massless; the strings are massless and inextensible.

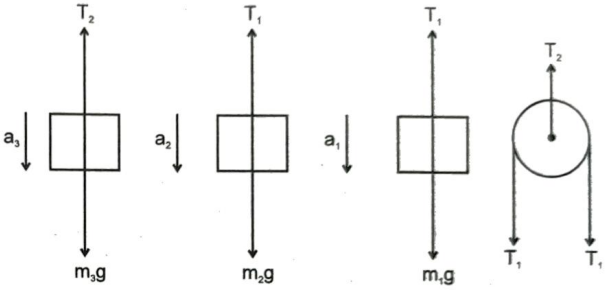
(i) Find the tensions in the strings.

(ii) Find the acceleration of each mass.



Sol. All the blocks are assumed to be moving downward and the free body diagram of each block is shown in figure.

F.B.D. m_3 **F.B.D. m_2** **F.B.D. m_1** **F.B.D. of pulley**



Applying Newton's Second Law to

Block m_1 : $m_1g - T_1 = m_1a_1$ ...(1)

Block m_2 : $m_2g - T_1 = m_2a_2$ ...(2)

Block m_3 : $m_3g - T_2 = m_3a_3$ ...(3)

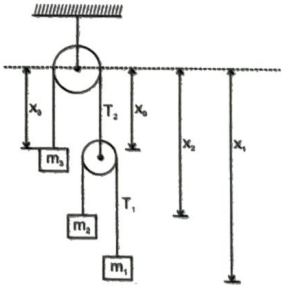
Pulley : $T_2 = 2T_1$ ...(4)

Number of unknowns a_1, a_2, a_3, T_1 and T_2 (Five)

Number of equations : Four

The constraint relation among accelerations can be obtained as follows

For upper string



$x_3 + x_0 = c_1$

For lower string

$(x_2 - x_0) + (x_1 - x_0) = c_2$

$x_2 + x_1 - 2x_0 = c_2$

Eliminating x_0 from the above two relations,

we get $x_1 + x_1 + 2x_3 = 2c_1 + c_2 = \text{constant}$.

Differentiating twice with respect to time,

we get $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} + 2\frac{d^2x_3}{dt^2} = 0$

or $a_1 + a_2 + 2a_3 = 0$ ...(5)

Solving equations (1) to (5), we get

(i) $T_1 = \left[\frac{4m_1m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g$; $T_2 = 2T_1$

(ii) $a_1 = \left[\frac{4m_1m_2 + m_1m_3 - 3m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g$;

$a_2 = \left[\frac{3m_1m_3 - m_2m_3 - 4m_1m_2}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

$a_3 = \left[\frac{4m_1m_2 - m_3(m_1+m_2)}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

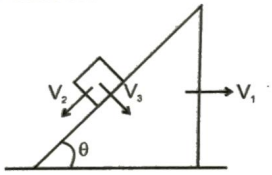
WEDGE CONSTRAINT

Conditions:

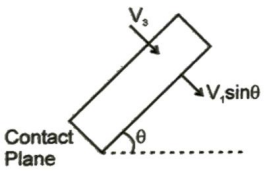
(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.



$V_3 = V_1 \sin\theta$

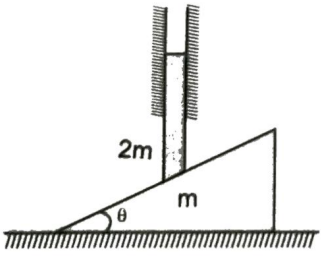


In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

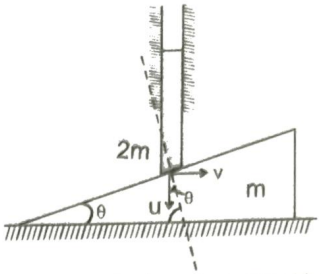
TRAIN YOUR BRAIN

Example 19: A rod of mass 2m moves vertically downward on the surface of wedge of mass as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.



Sol. Using wedge constraint.

Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.



perpendicular to contact of two blocks

$u \cos\theta = v \sin\theta$

$\frac{u}{v} = \tan\theta \qquad \Rightarrow u = v\tan\theta$

SPRING FORCE

Many spring follow Hooke's law for small extension and compression. That is, the extension or compression – the increase or decrease in length from the relaxed length – is proportional to the force applied to the ends of the spring.

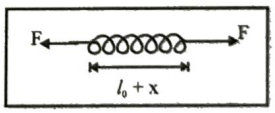
Hooke's Law for an ideal spring $F = k\Delta L$

In equation F is the magnitude of the force exerted on each end of the spring and ΔL is the modulus of change in length of the spring from its relaxed length. The constant k is called the spring constant for a particular spring. The SI units of a spring constant are N/m.

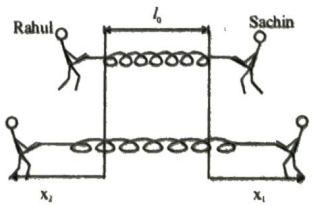
When we say an ideal spring, we mean a spring that obeys Hooke's law and is also massless. Since we have assumed spring to be massless we know forces acting on both ends have to be equal and opposite, to have net force on spring to be zero.

NOTE:

If we look at FBD of the spring we will note that force on spring must act from both ends.



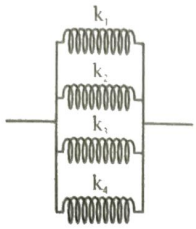
Lets say Rahul and Sachin are pulling a spring from two ends as shown. Rahul moves x_2 and Sachin moves x_1.



The force acting on Rahul and Sachin is $k(x_1 + x_2)$, Not kx_2 on Rahul and kx_1 on Sachin. Force due to spring is kx where x is defined as $|l - l_0|$, where l is present length and l_0 is natural length.

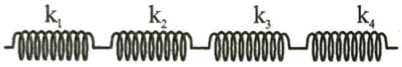
EQUIVALENT SPRING CONSTANT

(a) When springs are connected in parallel then we can replace them by single spring of spring constant k_e where $k_e = k_1 + k_2$.



For more than two spring $k = k_1 + k_2 + k_3 + ...$

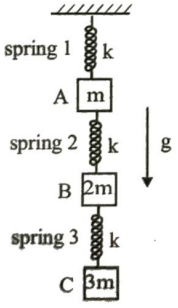
(b) When springs are connected in series then we can replaced them by single spring of spring constant k_e where $1/k_e = 1/k_1 + 1/k_2$. As spring constants are not equal so extensions will not be equal , but total extension y can be written as sum of two extensions $y = y_1 + y_2$ for more than two springs.



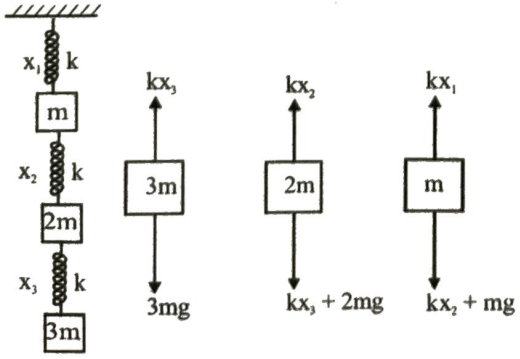
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + ...$$

TRAIN YOUR BRAIN

Example 20: The system shown in the figure is in equilibrium. Find the initial acceleration of A,B, and C just after the spring-2 is cut.



Sol.



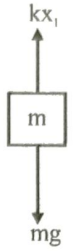
$3mg = kx_3$ ...(1)

$2mg + kx_3 = kx_2$

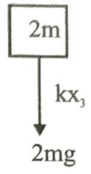
$2mg + 3mg = kx_2 \Rightarrow 5mg = kx_2$ ...(2)

$kx_1 = 6m$ ...(3)

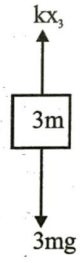
when spring 2 is cut spring force in other two strings remain unchaged.



$kx_1 - mg = ma_3 \Rightarrow a_3 = 5g\uparrow$



$kx_3 + 2mg = 2ma_2 \Rightarrow a_2 = \frac{5g}{2}\downarrow$



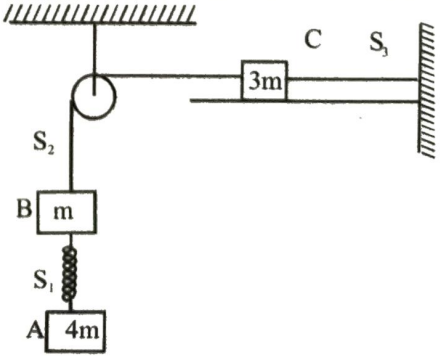
acceleration of 3m will be zero.

IMPORTANT POINT

It is important to remember that ropes can change tension instantaneously while spring need to move to change tension, so in this example tension in spring is not changing instantaneously

TRAIN YOUR BRAIN

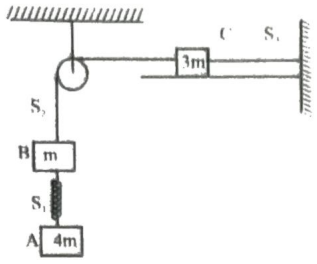
Example 21: In following setup pulley, strings and spring are light. Initially all masses are in equilibrium and at rest.

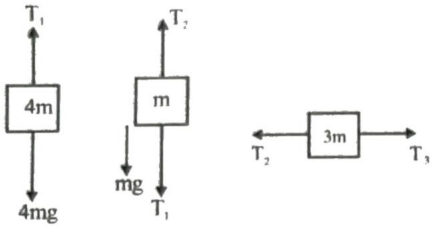


(a) find tension in spring and tension in ropes

(b) find accleration of masses immediately after the string S_3 is cut.

Sol. (a)





applying Newton's 2nd law to block A $4mg - T_1 = 0$

applying Newton's 2nd law to block B $mg - T_1 - T_2 = 0$

applying Newton's 2nd law to block C $T_3 - T_2 = 0$

Solving $T_1 = 4\ mg$

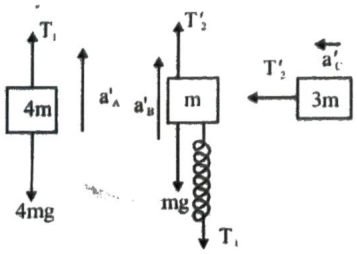
$T_2 = 5\ mg$

$T_3 = 5mg$

Here spring is behaving same as string except that it is strechedwhile string can not strech.

[b] The most important point in this problem is that any object of finite mass can not change its position instantaneously as this require ← infinite velocity.

Thus immediately after cutting the string S_3 all masses will remain at same position and force due to spring will not change. As force of spring is kx. At the same time we would like to emphasize that tension in string S_2 will change instataneously (Tension is a self adjusuting force). To maintain constraint relation blocks B and C have same magnitude of acceleration. We can identify all the forces acting on all objects. Only tension T in string S_2 is unknown force all other forces are known. considering FBD,



We know from part [a] that tension in spring is T_1 and $T_1 = 4\ mg$

Writing Newton's Second law for A

$4mg - T_1 = 4ma_A'$

Writing Newton's Second law for B

$T_1 + mg - T_2' = ma_B'$

Writing Newton's Second law for C

$T_2' = 3ma_C'$

Quantities which may have different value from part (a) are represented using symbol, eg fnsion in string s_2 is T_2' others which have same value as part (a) have been retained with same symbol.

as $a_B' = a_C'$

solving we get $a_A' = 0$ $\qquad a_B' = a_C' = \frac{5}{4}g$

PSEUDO FORCE

Motion in Accelerated Frames:

Till now we have restricted ourselves to apply Netwon's laws of motion, only to describe observation that are made in an inertial frame of reference. In this part, we learn how Netwon's laws can be applied by an observer in a noninertial reference frame. For example consider a block kept on smooth surface of a compartment of train.

If the train accelerates, the block accelerates toward the back of the train. We may conclude based on Newton's second law F = ma that a force is acting the block to cause it to accelerate, but the Newton's second law is not applicable from this non - inertial frame. So we can not relate observed acceleration with the force acting on the block.

If we still want to use Newton's second law we need to apply a pseudo force, acting in backward direction, i.e. opposite to the acceleration of noninertial reference frame. This force explains the motion of block towards the back of car. The fictitious force is equal to – ma, where a is the acceleration of the noninertial reference frame. Fictitious force appears to act on an object in the same way as a real force, but real forces are always interactions between two objects. On the other hand there is no second object for a fictitious force.

TRAIN YOUR BRAIN

Example 22: A small ball of mass m hangs by a cord from the ceiling of a compartment of a train that is accelerating to the right as shown in figure (a). Analyze the situation for two observers A and B.

Sol. The observer A is on the ground, in an inertial frame. He sees the compartment is accelerating and knows that the deviation of the cord provides the ball, required horizontal force. The noninertial observer on the compartment, can not see that car's motion so that he is not aware of its acceleration. Because he does not know of this acceleration, he will say that

Newtons' second law is not valid as the object has net horizontal force [the horizontal component of tension] but no horizontal acceleration.

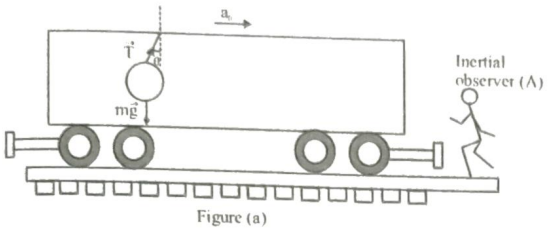


Figure (a)

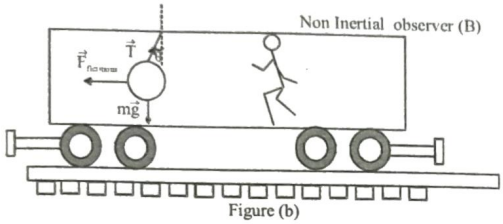


Figure (b)

For the inertial observer, ball has a net force in the horizontal direction and is in equilibrium in the vertical direction. For the non-inertial observer, we apply fictitious force towards left and consider it to be in equilibrium.

According to the inertial observer A, the ball experience two forces, T exerted by the cord and the weight.

Apply Newton's second law in horizontal and vertical directions we get.

Inertial observer: $T\sin\theta - ma = 0$

$T\cos\theta - mg = 0$

According to the noninertial observer B riding in the car [fig. b], the ball is always at rest and so its acceleration is zero. The noninertial observer applies a fictitious force in the horizontal direction of magnitude ma toward left. This fictitious force balances the horizontal component of T and thus the net force on the ball is zero.

Apply Newton's second law in horizontal and vertical direction we get.

Non-inertial observer: $T\sin\theta - F = 0$

$T\cos\theta - mg = 0$

The non-inertial observer B obtaines the same equations as the inertial observer. The physical explanation of the cord's deflection, however, differs in the two frames of reference.

TRAIN YOUR BRAIN

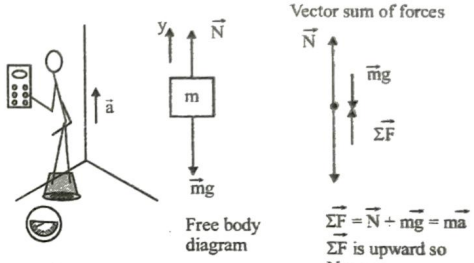
Example 23: Apparent weight in an Elevator

A passenger weighing 600 N rides in an elevator. What is the apparent weight of the passenger in each of the following situations? In each case, the magnitude of the elevator's acceleration is 0.500 m/s². [a] The passenger is on the first floor and has pushed the button for the 15th floor; the elevator is beginning to move upward. [b] the elevator is slowing down as it nears the 15th floor.

Sol. Lets understand this situation by making FBD of the person inside the elevator.

Please note that ma is not being shown in FBD. **as ma is not the force, it is effect of the net force acting on the person.** Only two actual forces act on the person in elevator, normal contact force by the scale's surface and weight. By writing Newton's 2nd law we can find the normal force from the known weight and the acceleration. W = 600 N ; magnitude of the acceleration is a = 0.500 m/s² . To find : W'

We expect the apparent weight W' = N to be greater then the true weight - the floor must push up with a force greater then W to cause an upward acceleration.



[a] [b] [c]

$N - W = ma_y$

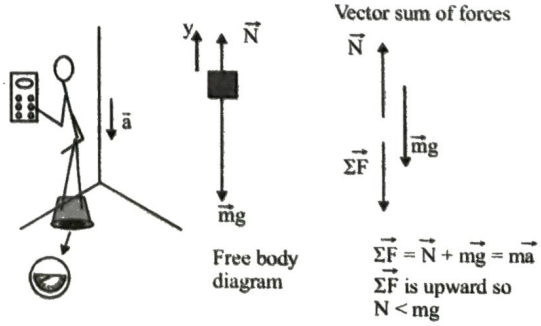
Since W = mg, we can substitute m = W/g.

$$N = W + ma = W + \frac{W}{g}a_y = W\left(1+\frac{a_y}{g}\right)$$

$$= 600\,N \times \left(1+\frac{0.500\,m/s^2}{10\,m/s^2}\right) = 630\,N$$

By Newton's third law same force will be acting on the scale. Hence scale will measure your weight larger than the actual weight.

(b) When the elevator approaches the 15th floor, it is sloving down while still moving upward; its acceleration is downward ($a_y < 0$) as in figure.



[a] [b] [c]

The normal force must be less than the weight to have an downward net force.

$$N = W\left(1-\frac{a}{g}\right)$$

$$= 600N\times\left(1-\frac{0.500m/s^2}{10m/s^2}\right) = 570\ N$$

FRICTION

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a force of friction. Forces of friction are very important in our everday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Friction force is of two types

(1) Static friction 'f_s'

(2) Kinetic friction 'f_k'

(1) Static Friction:

The static friction between two contact surfaces is given by $f_s \le \mu_s N$, where N is the normal force between the contact surfaces and μ_s is a constant which depends on the nature of the surfaces and is called **'coefficient of limiting friction'**.

Static friction acts on stationary objects. Its values satisfy the condition

$f_s \le \mu_s N$

μ_s = Co-efficient of limiting friction

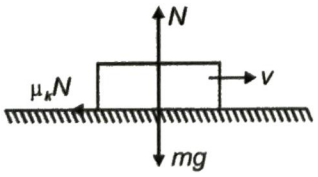
Static friction takes its peak value ($f_{s(max)} = \mu_s N$) when one surface is 'about to slide' on the other. Static friction in this case is called limiting friction.

(2) Kinetic Friction (f_k)

It acts on the two contact surfaces only when there is relative sliping or relative motion between two contact surfaces.

$$f_k = \mu_k N$$

The relative motion of a contact surface with respect to each other is opposed by a force given by $f_k = \mu_k N$, where N is the normal force between the contact surfaces and μ_k is a constant called **'coefficient of kinetic friction'**, which depends, largely, on the nature of the contact surfaces.



NOTE:

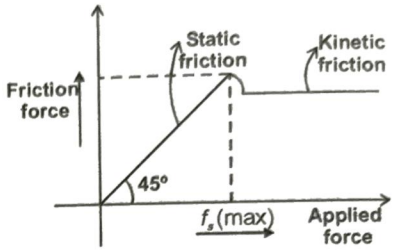
(1) Value of μ_k is always less than μ_s ($\mu_k < \mu_s$) from experimental observation.

(2) If only coefficient of friction (μ) is given by a problem then use $\mu_s = \mu_k = \mu$

(3) Value of μ_s and μ_k is independent of surface area. It depends only on surface properties of contact surface.

(4) μ_k is independent of relative speed.

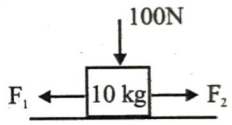
(5) μ_s & μ_k are properties of a given pair of surfaces i.e. for wood to wood combination μ_1, then for wood to iron μ_2 and so on.



Graphical representation of variation of friction force with the force applied on a body

TRAIN YOUR BRAIN

Example 24: Find frictional force acting on the block in the following cases given that $\mu = 0.1$



Sol. **Case - 1** $F_1 = 0$

$F_2 = 5$ N fr = 5 N (towards left)

$F_2 = 15$ N fr = 15 N (towards left)

$F_2 = 25$ N fr = 20 N (towards left)

Case - 2 $F_1 = 20$ N

$F_2 = 10$ N fr = 10 N (towards right)

$F_2 = 20$ N fr = 0

$F_2 = 42$ N fr = 20 N (towards left)

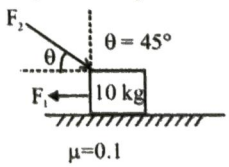
$F_2 = 50$ N fr = 20 N (towards left)

Example 25: Find friction force acting on block in the following cases.

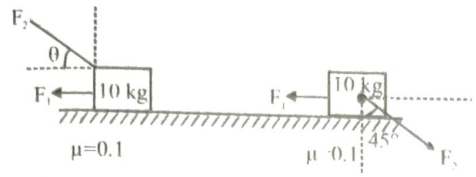
Case - I $F_1 = 0$, $F_2 = 5\sqrt{2}$

Case - II $F_1 = 0$, $F_2 = 50\sqrt{2}$

Case - III $F_1 = 5N$, $F_2 = 5\sqrt{2}$

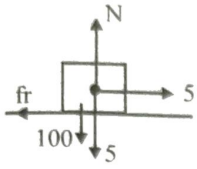


Sol.



Both of the above are same cases.

Case-I

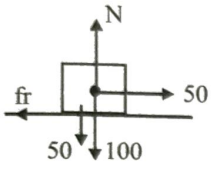


$N = 105$ N

$fr_{max} = 0.1 \times 105 = 10.5$ N

$fr = 5$N

Case-II

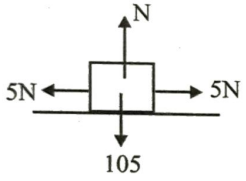


$N = 150$

$fr_{max} = 0.1 \times 150 = 15$ N

so block will skid and fr = 15 N

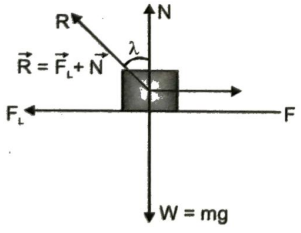
Case-III



$fr = 0$

ANGLE OF FRICTION

The angle of friction is the angle which the resultant of limiting friction F_L and



normal reaction N makes with the normal reaction. It is represented by λ,

Thus from the figure. $\tan\lambda = \frac{F_L}{N}$

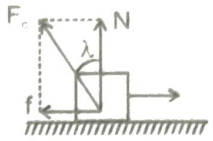
or $\tan\lambda = \mu \quad (\because F_L = \mu N)$

For smooth surfaces, λ = 0 (zero)

TRAIN YOUR BRAIN

Example 26: A body of mass 400 g slides on a rough horizontal surface. If the frictional force is 3.0 N, find (a) the angle made by the contact force on the body with the vertical and (b) the magnitude of the contact force. Take g = 10 m/s^2

Sol. Let the contact force on the block by the surface be F_c which makes an angle λ with the vertical (shown in figure)



The component of F_c perpendicular to the contact surface is the normal force N and the component of F parallel to the surface is the firction f. As the surface is horizontal, N is vertically upward. For vertical equilirbrium,

$N = Mg = (0.400\ kg)(10\ m/s^2) = 4.0$ N

The frictional force is f = 3.0 N

(a) $\tan\lambda = \frac{f}{N} = \frac{3}{4}$ or, $\lambda = \tan^{-1}(3/4) = 37°$

(b) The magnituded of the contact force is

$$F = \sqrt{N^2 + f^2}$$

$$= \sqrt{(4.0N)^2 + (3.0N)^2} = 5.0N$$

MOTION ON A ROUGH INCLINED PLANE

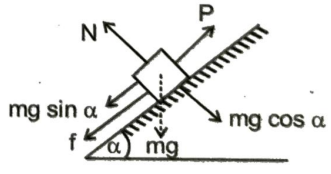
Suppose a motion up the plane takes place under the action of pull P acting parallel to the plane

$N = mg\cos\alpha$

Frictional force acting down the plane

$F = \mu N = \mu\, mg\cos\alpha$

Appling Newton's second law for motion up the plane



$P - (mg\sin\alpha + f) = ma$

$P - mg\sin\alpha - \mu\, mg\cos\alpha = ma$

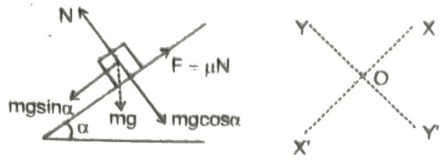
If P = 0 the block may slide downwards with an acceleration a. The frictional force would then act up the plane

$mg\sin\alpha - f = ma$

or, $mg\sin\alpha - \mu\, mg\cos\alpha = ma$

TRAIN YOUR BRAIN

Example 27: A 20 kg box is gently placed on a rough inclined plane of inclination 30° with horizontal. The coefficient of sliding friction between the box and the plane is 0.4. Find the acceleration of the box down the incline.



Sol. In solving inclined plane problems, the X and Y directions along which the forces are to be considered, may be taken as shown. The components of weight of the box are

(i) mg sin α acting down the plane and

(ii) mg cos α acting perpendicular to the plane.

$N = mg \cos \alpha$

$mg \sin \alpha - \mu N = ma \Rightarrow mg \sin \alpha - \mu mg \cos \alpha = ma$

$a = g \sin \alpha - \mu g \cos \alpha = g (\sin \alpha - \mu \cos \alpha)$

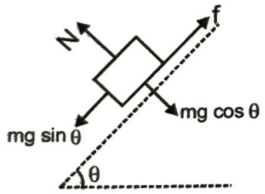
$$= 9.8\left(\frac{1}{2} - 0.4 \times \frac{\sqrt{3}}{2}\right) = 4.9 \times 0.3072 = 1.505 \text{ m/s}^2$$

The box accelerates down the plane at 1.505 m/s².

ANGLE OF REPOSE

Consider a rough inclined plane whose angle of inclination θ with ground can be changed. A block of mass m is resting on the plane. Coefficient of (static) friction between the block and plane is μ.

For a given angle θ, the FBD (Free body diagram) of the block is



Where f is force of static friction on the block. For normal direction to the plane, we have N=mg cosθ

As θ increases, the force of gravity down the plane, mg sin θ, increases. Friction force resists the slide till it attains its maximum value.

$f_{max} = \mu N = \mu mg \cos \theta$

Which decreases with θ (because cos θ decreases as θ increases)

Hence, beyond a critical value $\theta = \theta_c$, the blocks starts to slide down the plane. The critical angle is the one when mg sin θ is just equal of f_{max}, i.e., when

$mg \sin \theta_c = \mu mg \cos \theta_c$

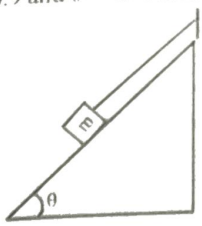
or $\tan \theta_c = \mu$

where θ_C is called angle of repose

If $\theta > \theta_c$, block will slide down. For $\theta < \theta_c$ the block stays at rest on the incline.

TRAIN YOUR BRAIN

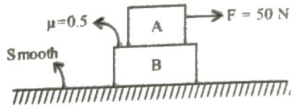
Example 28: If μ = 0.9 and θ = 45°, then find tension in string



Sol. As $\theta < \theta_c$ block has no tendency to move hence tension in the string will be zero.

TWO BLOCK PROBLEMS

Example 29: Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient is as shown in the figure? $M_A = M_B = 10$kg



Sol. Method of solving

Step 1 : Make force diagram.

Step 2 : Show static friction force by f because value of friction is not known.

Step 3 : Calculate separately for two cases.

Case 1 : Move together

Step 4 : Calculate acceleration.

Step 5 : Check value of friction for above case.

Step 6 : If required friction is less than available it means they will move together else move separately.

Step 7: (a) above acceleration will be common acceleration for both

Case 2 : Move separately

Step 7(b) If they move separately then kinetic friction is involved. whose value is μN.

Step 8 : Calculate acceleration for above case.



$f_{max} = \mu N$

$\therefore$ f ≤ 50 N (available friction)

(i) $a = \dfrac{50}{10+10} = 2.5 \text{ m/s}^2$

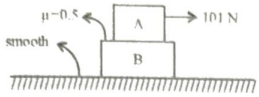
(ii) Check friction for B : $f = 10 \times 2.5 = 25$

$fr_{max} = \mu N = 0.5 \times 150 = 50$ N

25 N is required which is less than available friction hence they will move together.

and $a_A = a_B = 2.5 \text{ m/s}^2$

Example 30: Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?

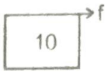


Sol. $f_{max} = 50$ N

$\therefore$ $f \leq 50$ N



(i) If they move together $a = \frac{101}{20} = 5.05$ m/s²

(ii) Check friction on B 

$f = 10 \times 5.05 = 50.5$ (required)

$50.5 > 50$ (therefore required > available)

Hence they will not move together.

(iii) Hence they move separately so kinetic friction is involved.



$\therefore$ for $a_A = \frac{101-50}{10} = 5.1$ m/s² $\Rightarrow a_B = \frac{50}{10} = 5$ m/s²

Also $a_A > a_B$ as force is applied on A.

Topicwise Questions

NEWTON'S LAWS OF MOTION

Laws of Motion : Theoritical Discription

1. A rider on horse back falls when horse starts running all of a sudden because
   (*a*) Rider is taken back
   (*b*) Rider is suddenly afraid of falling
   (*c*) Inertia of rest keeps the upper part of body at rest whereas lower part of the body moves forward with the horse
   (*d*) None of these

2. When a train stops suddenly, passengers in the running train feel an instant jerk in the forward direction because
   (*a*) The back of seat suddenly pushes the passengers forward
   (*b*) Inertia of rest stops the train and takes the body forward
   (*c*) Upper part of the body continues to be in the state of motion whereas the lower part of the body in contact with seat remains at rest
   (*d*) Nothing can be said due to insufficient data

3. A man getting down a running bus falls forward because
   (*a*) Due to inertia of rest, road is left behind and man reaches forward
   (*b*) Due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road
   (*c*) He leans forward as a matter of habit
   (*d*) Of the combined effect of all the three factors stated in (*a*), (*b*) and (*c*)

4. A boy sitting on the topmost berth in the compartment of a train which is just going to stop on a railway station, drops an apple aiming at the open hand of his brother sitting vertically below his hands at a distance of about 2 metre. The apple will fall
   (*a*) Precisely on the hand of his brother
   (*b*) Slightly away from the hand of his brother in the direction of motion of the train
   (*c*) Slightly away from the hand of his brother in the direction opposite to the direction of motion of the train
   (*d*) None of these

5. A force of 100 dynes acts on a mass of 5 gm for 10 sec. The velocity produced is
   (*a*) 2 cm/sec (*b*) 20 cm/sec
   (*c*) 200 cm/sec (*d*) 2000 cm/sec

6. A machine gun is mounted on a 2000 kg car on a horizontal frictionless surface. At some instant the gun fires bullets of mass 10 gm with a velocity of 500 m/sec with respect to the car. The number of bullets fired per second is ten. The average thrust on the system is
   (*a*) 550 N (*b*) 50 N
   (*c*) 250 N (*d*) 250 dyne

7. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin
   (*a*) 5 m/s^2 (*b*) 10 m/s^2
   (*c*) 3 m/s^2 (*d*) 15 m/s^2

8. In a rocket of mass 1000 kg fuel is consumed at a rate of 40 kg/s. The velocity of the gases ejected from the rocket is 5×10^4 m/s. The thrust on the rocket is
   (*a*) 2×10^3 N (*b*) 5×10^4 N
   (*c*) 2×10^6 N (*d*) 2×10^9 N

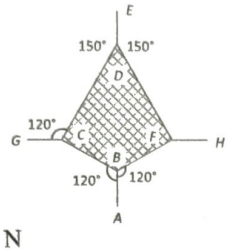
9. Two balls of masses m_1 and m_2 are separated from each other by a powder charge placed between them. The whole system is at rest on the ground. Suddenly the powder charge explodes and masses are pushed apart. The mass m_1 travels a distance s_1 and stops. If the coefficients of friction between the balls and ground are same, the mass m_2 stops after travelling the distance
   (*a*) $s_2 = \frac{m_1}{m_2} s_1$ (*b*) $s_2 = \frac{m_2}{m_1} s_1$
   (*c*) $s_2 = \frac{m_1^2}{m_2^2} s_1$ (*d*) $s_2 = \frac{m_2^2}{m_1^2} s_1$

10. A force vector applied on a mass is represented as $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ and accelerates with 1m/s^2. What will be the mass of the body
    (*a*) $10\sqrt{2}$ kg (*b*) $2\sqrt{10}$ kg
    (*c*) 10 kg (*d*) 20 kg

11. A cricket ball of mass 250 g collides with a bat with velocity 10 m/s and returns with the same velocity within 0.01 second. The force acted on bat is
    (*a*) 25 N (*b*) 50 N
    (*c*) 250 N (*d*) 500 N

12. A body of mass 2 kg is moving with a velocity 8 m/s on a smooth surface. If it is to be brought to rest in 4 seconds, then the force to be applied is
(a) 8 N (b) 4 N
(c) 2 N (d) 1 N

13. The adjacent figure is the part of a horizontally stretched net. Section AB is stretched with a force of 10 N. The tensions in the sections BC and BF are



(a) 10 N, 11 N
(b) 10 N, 6 N
(c) 10 N, 10 N
(d) Can't calculate due to insufficient data

14. When the speed of a moving body is doubled
(a) Its acceleration is doubled
(b) Its momentum is doubled
(c) Its kinetic energy is doubled
(d) Its potential energy is doubled

15. A ball of mass m moves with speed v and it strikes normally with a wall and reflected back normally, if its time of contact with wall is t then find force exerted by ball on wall
(a) $\frac{2mv}{t}$ (b) $\frac{mv}{t}$
(c) mvt (d) $\frac{mv}{2t}$

16. A particle moves in the xy-plane under the action of a force F such that the components of its linear momentum p at any time t are $p_x = 2\cos t$, $p_y = 2\sin t$. The angle between F and p at time t is
(a) 90° (b) 0°
(c) 180° (d) 30°

17. Swimming is possible on account of
(a) First law of motion
(b) Second law of motion
(c) Third law of motion
(d) Newton's law of gravitation

18. A man is at rest in the middle of a pond on perfectly smooth ice. He can get himself to the shore by making use of Newton's
(a) First law (b) Second law
(c) Third law (d) All the laws

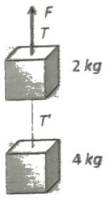
19. If a force of 250 N act on body, the momentum acquired is 125 kg-m/s. What is the period for which force acts on the body
(a) 0.5 sec (b) 0.2 sec
(c) 0.4 sec (d) 0.25 sec

20. An aircraft is moving with a velocity of 300 ms–1. If all the forces acting on it are balanced, then
(a) It still moves with the same velocity
(b) It will be just floating at the same point in space
(c) It will fall down instantaneously
(d) It will lose its velocity gradually

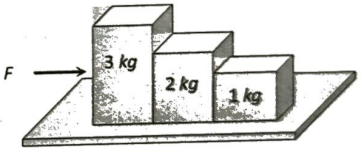
APPLICATION OF FORCE/IMPULSE, STATICS AND DYNAMICS INVOLVING SINGLE SYSTEM

21. Two forces of magnitude F have a resultant of the same magnitude F. The angle between the two forces is
(a) 45° (b) 120°
(c) 150° (d) 60°

22. Two blocks are connected by a string as shown in the diagram. The upper block is hung by another string. A force F applied on the upper string produces an acceleration of 2 m/s² in the upward direction in both the blocks. If T and T' be the tensions in the two parts of the string, then
(a) T = 70.8 N and T'= 47.2 N
(b) T = 58.8 N and T'= 47.2 N
(c) T = 70.8 N and T' = 58.8 N
(d) T = 70.8 N and T' = 0



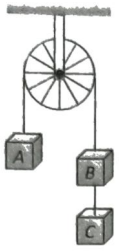
23. Consider the following statements about the blocks shown in the diagram that are being pushed by a constant force on a frictionless table



A. All blocks move with the same acceleration
B. The net force on each block is the same Which of these statements are/is correct
(a) A only
(b) B only
(c) Both A and B
(d) Neither A nor B

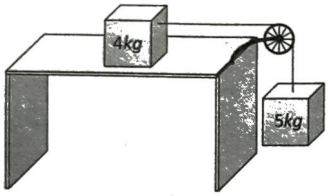
24. A rope of length L is pulled by a constant force F. What is the tension in the rope at a distance x from the end where the force is applied
(a) $\frac{FL}{x}$ (b) $\frac{F(L-x)}{L}$
(c) $\frac{FL}{L-x}$ (d) $\frac{Fx}{L-x}$

25. Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the figure The tension in the string connecting weights B and C is



(*a*) Zero (*b*) 13 N
(*c*) 3.3 N (*d*) 19.6 N

26. Two masses of 4 kg and 5 kg are connected by a string passing through a frictionless pulley and are kept on a frictionless table as shown in the figure. The acceleration of 5 kg mass is



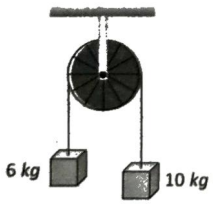
(*a*) 49 m/s² (*b*) 5. 44 m/s²
(*c*) 19.5 m/s² (*d*) 2.72 m/s²

27. Three solids of masses m_1, m_2 and m_3 are connected with weightless string in succession and are placed on a frictionless table. If the mass m_3 is dragged with a force T, the tension in the string between m_2 and m_3 is

(*a*) $\frac{m_2}{m_1+m_2+m_3}T$ (*b*) $\frac{m_3}{m_1+m_2+m_3}T$

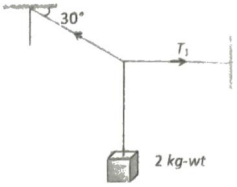
(*c*) $\frac{m_1+m_2}{m_1+m_2+m_3}T$ (*d*) $\frac{m_2+m_3}{m_1+m_2+m_3}T$

28. A light string passes over a frictionless pulley. To one of its ends a mass of 6 kg is attached. To its other end a mass of 10 kg is attached. The tension in the thread will be



(*a*) 24.5 N (*b*) 2.45 N
(*c*) 79 N (*d*) 73.5 N

29. A body of weight 2kg is suspended as shown in the figure. The tension T_1 in the horizontal string (in kg wt) is



(*a*) $2/\sqrt{3}$ (*b*) $\sqrt{3}/2$
(*c*) $2\sqrt{3}$ (*d*) 2

CONSTRAINT RELATION, DYNAMICS OF MULTI SYSTEM SPRING

30. A man is standing at a spring platform. Reading of spring balance is 60 kg wt. If man jumps outside platform, then reading of spring balance
(*a*) First increases then decreases to zero
(*b*) Decreases
(*c*) Increases
(*d*) Remains same

31. A bird is sitting in a large closed cage which is placed on a spring balance. It records a weight of 25 N. The bird (mass m = 0.5 kg) flies upward in the cage with an acceleration of 2 m/s². The spring balance will now record a weight of

(*a*) 24 N (*b*) 25 N
(*c*) 26 N (*d*) 27 N

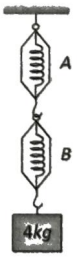
32. The tension in the spring is



(*a*) Zero (*b*) 2.5 N
(*c*) 5 N (*d*) 10 N

33. A block of mass 4 kg is suspended through two light spring balances A and B. Then A and B will read respectively

(*a*) 4 kg and zero kg
(*b*) Zero kg and 4 kg
(*c*) 4 kg and 4 kg
(*d*) 2 kg and 2 kg



Non Inertial Reference frame and Pseudo force

34. A person is standing in an elevator. In which situation he finds his weight less than actual when
(*a*) The elevator moves upward with constant acceleration
(*b*) The elevator moves downward with constant acceleration.
(*c*) The elevator moves upward with uniform velocity
(*d*) The elevator moves downward with uniform velocity

35. A body of mass 4 kg weighs 4.8 kg when suspended in a moving lift. The acceleration of the lift is
(*a*) 9.80 ms^{-2} downwards (*b*) 9.80 ms^{-2} upwards
(*c*) 1.96 ms^{-2} downwards (*d*) 1.96 ms^{-2} upwards

36. The mass of a lift is 500 kg. When it ascends with an acceleration of 2 m/s^2, the tension in the cable will be [g = 10 m/s^2]
(*a*) 6000 N (*b*) 5000 N
(*c*) 4000 N (*d*) 50 N

37. If in a stationary lift, a man is standing with a bucket full of water, having a hole at its bottom. The rate of flow of water through this hole is R0. If the lift starts to move up and down with same acceleration and then that rates of flow of water are Ru and Rd, then
(*a*) R0 > Ru > Rd (*b*) Ru > R0 > Rd
(*c*) Rd > R0 > Ru (*d*) Ru > Rd >R0

38. A plumb line is suspended from a ceiling of a car moving with horizontal acceleration of a. What will be the angle of inclination with vertical
(*a*) tan– 1(a/g) (*b*) tan–1 (g/a)
(*c*) cos–1 (a/g) (*d*) cos–1 (g/a)

FRICTION

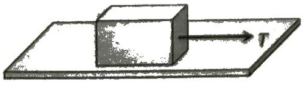
Static Friction

39. The coefficient of friction μ and the angle of friction λ are related as
(*a*) $\sin\lambda = \mu$ (*b*) $\cos\lambda = \mu$
(*c*) $\tan\lambda = \mu$ (*d*) $\tan\mu = \lambda$

40. A force of 98 N is required to just slide body of mass 100 kg over ice. The coefficient of static friction is
(*a*) 0.6 (*b*) 0.4
(*c*) 0.2 (*d*) 0.1

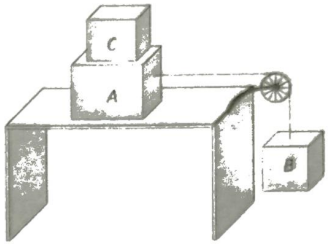
41. Maximum value of static friction is called
(*a*) Limiting friction (*b*) Rolling friction
(*c*) Normal reaction (*d*) Coefficient of friction

42. In the figure shown, a block of weight 10 N resting on a horizontal surface. The coefficient of static friction between the block and the surface is $\mu_s = 0.4$. A force of 3.5 N will keep the block in uniform motion, once it has been set in motion. A horizontal force of 3 N is applied to the block, then the block will



(*a*) Move over the surface with constant velocity
(*b*) Move having accelerated motion over the surface
(*c*) Will not move
(*d*) First it will move with a constant velocity for some time and then will have accelerated motion

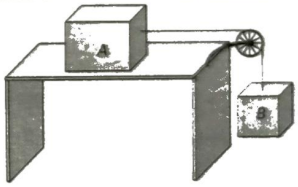
43. Two masses A and B of 10 kg and 5 kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown. The coefficient of static friction of A with table is 0.2. The minimum mass of C that may be placed on A to prevent it from moving is



(*a*) 15 kg (*b*) 10 kg
(*c*) 5 kg (*d*) 12 kg

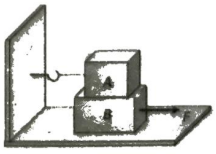
44. Which of the following statements is not true?
(*a*) The coefficient of friction between two surfaces increases as the surface in contact are made rough
(*b*) The force of friction acts in a direction opposite to the applied force
(*c*) Rolling friction is greater than sliding friction
(*d*) The coefficient of friction between wood and wood is less than 1

45. The blocks A and B are arranged as shown in the figure. The pulley is frictionless. The mass of A is 10 kg. The coefficient of friction of A with the horizontal surface is 0.20. The minimum mass of B to start the motion will be



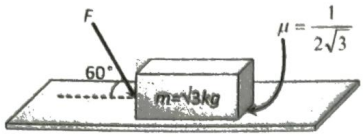
(*a*) 2 kg (*b*) 0.2 kg
(*c*) 5 kg (*d*) 10 kg

46. A block A with mass 100 kg is resting on another block B of mass 200 kg. As shown in figure a horizontal rope tied to a wall holds it. The coefficient of friction between A and B is 0.2 while coefficient of friction between B and the ground is 0.3. The minimum required force F to start moving B will be



(*a*) 900 N (*b*) 100 N
(*c*) 1100 N (*d*) 1200 N

47. What is the maximum value of the force F such that the block shown in the arrangement, does not move?



(*a*) 20 N (*b*) 10 N
(*c*) 12 N (*d*) 15 N

Kinetic Friction

48. A car is moving along a straight horizontal road with a speed v_0. If the coefficient of friction between the tyres and the road is μ, the shortest distance in which the car can be stopped is

(*a*) $\frac{v_0^2}{2\mu g}$ (*b*) $\frac{v_0}{\mu g}$

(*c*) $\left(\frac{v_0}{\mu g}\right)^2$ (*d*) $\frac{v_0}{\mu}$

49. A body B lies on a smooth horizontal table and another body A is placed on B. The coefficient of friction between A and B is μ. What acceleration given to B will cause slipping to occur between A and B

(*a*) μg (*b*) g/μ
(*c*) μ/g (*d*) $\sqrt{\mu g}$

50. A 500 kg horse pulls a cart of mass 1500 kg along a level road with an acceleration of 1 ms^{-2}. If the coefficient of sliding friction is 0.2, then the force exerted by the horse on cart in forward direction is

(*a*) 3000 N (*b*) 4000 N
(*c*) 5000 N (*d*) 4500 N

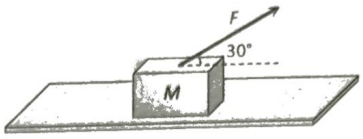
51. A horizontal force of 129.4 N is applied on a 10 kg block which rests on a horizontal surface. If the coefficient of friction is 0.3, the acceleration should be

(*a*) 9.8 m/s^2 (*b*) 10 m/s^2
(*c*) 12.6 m/s^2 (*d*) 19.6 m/s^2.

52. If μ_s, μ_k and μ_r are coefficients of static friction, sliding friction and rolling friction, then

(*a*) $\mu_s < \mu_k < \mu_r$ (*b*) $\mu_k < \mu_r < \mu_s$
(*c*) $\mu_r < \mu_k < \mu_s$ (*d*) $\mu_r = \mu_k = \mu_s$

53. A block of mass M = 5 kg is resting on a rough horizontal surface for which the coefficient of friction is 0.2. When a force F = 40 N is applied, the acceleration of the block will be ($g = 10\ m/s^2$)



(*a*) 5.73 m/sec^2
(*b*) 8.0 m/sec^2
(*c*) 3.17 m/sec^2
(*d*) 10.0 m/sec^2

54. The coefficient of friction between a body and the surface of an inclined plane at 45° is 0.5. If $g = 9.8\ m/s^2$, the acceleration of the body downwards in m/s^2 is

(*a*) $\frac{4.9}{\sqrt{2}}$ (*b*) $4.9\sqrt{2}$

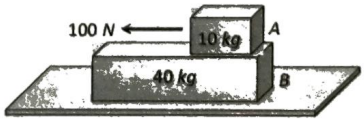
(*c*) $19.6\sqrt{2}$ (*d*) 4.9

55. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A body starting from the rest at top comes back to rest at the bottom if the coefficient of friction for the lower half is given by

(*a*) $\mu = \sin\theta$ (*b*) $\mu = \cot\theta$
(*c*) $\mu = 2\cos\theta$ (*d*) $\mu = 2\tan\theta$

Double Block Problem

56. A 40 kg slab rests on a frictionless floor as shown in the figure. A 10 kg block rests on the top of the slab. The static coefficient of friction between the block and slab is 0.60 while that of the kinetic friction is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8\ m/s^2$, the resulting acceleration of the slab will be



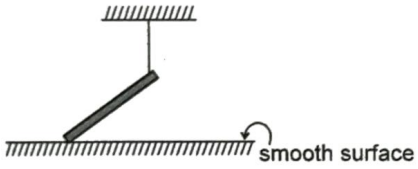
(*a*) 0.98 m/s^2 (*b*) 1.47 m/s^2
(*c*) 1.52 m/s^2 (*d*) 6.1 m/s^2

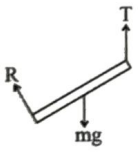
NEWTON'S LAWS OF MOTION

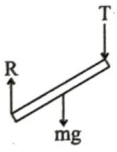
1. Let E, G and N represents the magnitude of electromagnetic, gravitational and nuclear forces between two protons at a given separation (1 fermi metre). Then
   (*a*) N < E < G (*b*) E > N > G
   (*c*) G > N > E (*d*) N > E > G

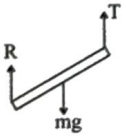
2. A man getting down a running bus, falls forward because-
   (*a*) due to inertia of rest, road is left behind and man reaches forward
   (*b*) due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road
   (*c*) he leans forward as a matter of habbit
   (*d*) of the combined effect of all the three factors stated in (*a*), (*b*) and (*c*)

3. Which figure represents the correct F.B.D. of rod of mass m as shown in figure:



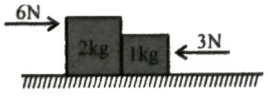
(*a*) 

(*b*) 

(*c*) 

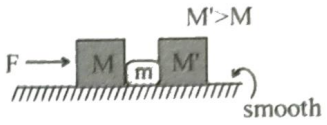
(*d*) None of these

4. Two forces of 6N and 3N are acting on the two blocks of 2kg and 1kg kept on frictionless floor. What is the force exerted on 2kg block by 1kg block ?



(*a*) 1N (*b*) 2N
(*c*) 4N (*d*) 5N

5. A constant force F is applied in horizontal direction as shown. Contact force between M and m is N and between m and M' is N' then

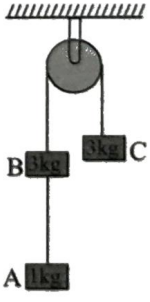


(*a*) N = N' (*b*) N > N'
(*c*) N' > N (*d*) cannot be determined

6. In which of the following cases the net force is not zero?
   (*a*) A kite skillfully held stationary in the sky
   (*b*) A ball freely falling from a height
   (*c*) An aeroplane rising upwards at an angle of 45° with the horizontal with a constant speed
   (*d*) A cork floating on the surface of water

7. Two persons are holding a rope of negligible weight tightly at its ends so that it is horizontal. A 15 kg weight is attached to the rope at the mid point which now no longer remains horizontal. The minimum tension required to completely straighten the rope is:
   (*a*) 15 kg
   (*b*) $\frac{15}{2}$ kg
   (*c*) 5 kg
   (*d*) Infinitely large (or not possible)

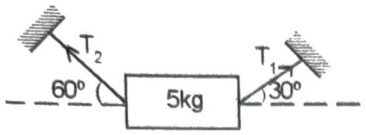
8. In the system shown in the figure, the acceleration of the 1 kg mass and the tension in the string connecting between A and B is:



(*a*) $\frac{g}{4}$ downwards , $\frac{8g}{7}$ (*b*) $\frac{g}{4}$ upwards, $\frac{g}{7}$

(*c*) $\frac{g}{7}$ downwards , $\frac{6}{7}$g (*d*) $\frac{g}{2}$ upwards , g

9. A body of mass 5 kg is suspended by the strings making angles 60° and 30° with the horizontal -



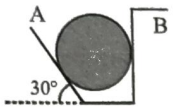
(A) $T_1 = 25$ N (B) $T_2 = 25$ N

(C) $T_1 = 25\sqrt{3}$ N (D) $T_2 = 25\sqrt{3}$ N

(*a*) A, B (*b*) A, D

(*c*) C, D (*d*) B, C

10. The 50 kg homogeneous smooth sphere rests on the 30° incline A and bears against the smooth vertical wall B. Calculate the contact forces at A and B.



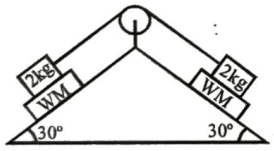
(*a*) $N_B = \frac{1000}{\sqrt{3}}$N, $N_A = \frac{500}{\sqrt{3}}$N

(*b*) $N_A = \frac{1000}{\sqrt{3}}$N, $N_B = \frac{500}{\sqrt{3}}$N

(*c*) $N_A = \frac{100}{\sqrt{3}}$N, $N_B = \frac{500}{\sqrt{3}}$N

(*d*) $N_A = \frac{1000}{\sqrt{3}}$N, $N_B = \frac{50}{\sqrt{3}}$N

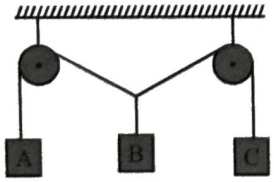
11. Find out the reading of the weighing machine in the following cases.



(*a*) $10\sqrt{3}$ (*b*) $10\sqrt{2}$

(*c*) $20\sqrt{3}$ (*d*) $30\sqrt{3}$

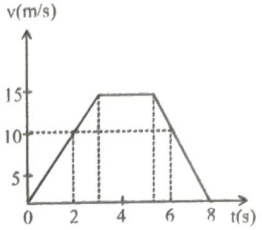
12. Three blocks A, B and C are suspended as shown in the figure. Mass of each blocks A and C is m. If system is in equilibrium and mass of B is M, then:



(*a*) M = 2m (*b*) M < 2 m

(*c*) M > 2m (*d*) M = m

13. A particle of mass 50 gram moves on a straight line. The variation of speed with time is shown in figure. find the force acting on the particle at t = 2, 4 and 6 seconds.



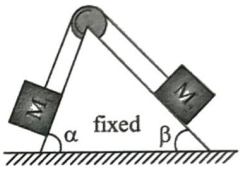
(*a*) 0.25 N along motion, zero, 0.25 opposite to motion

(*b*) 0.25 N along motion, zero, 0.25 along motion

(*c*) 0.25 N opposite to motion, zero, 0.25 along motion

(*d*) 0.25 N opposite to motion, zero, 0.25 opposite motion

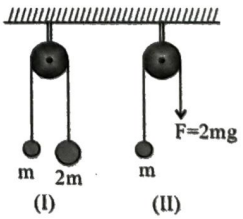
14. Two masses M_1 and M_2 are attached to the ends of a light string which passes over a massless pulley attached to the top of a double inclined smooth plane of angles of inclination α and β. If $M_2 > M_1$ and $\beta > \alpha$ then the acceleration of block M_2 down the incline will be:



(*a*) $\frac{M_2 g(\sin\beta)}{M_1 + M_2}$ (*b*) $\frac{M_1 g(\sin\alpha)}{M_1 + M_2}$

(*c*) $\left[\frac{M_2 \sin\phi - M_1 \sin\alpha}{M_1 + M_2}\right] g$ (*d*) zero

15. The pulley arrangements shown in figure are identical, the mass of the rope being negligible. In case I, the mass m is lifted by attaching a mass 2m to the other end of the rope. In case II, the mass m is lifted by pulling the other end of the rope with a constant downward force F = 2mg, where g is acceleration due to gravity. The acceleration of mass in case I is:



(*a*) zero

(*b*) more than that in case II

(*c*) less than that in case II

(*d*) equal to that in case II

16. A fireman wants to slide down a rope. The rope can bear a tension of $\frac{3}{4}$ th of the weight of the man. With what minimum acceleration should the fireman slide down:

(a) $\frac{g}{3}$ (b) $\frac{g}{6}$

(c) $\frac{g}{4}$ (d) $\frac{g}{2}$

17. A rope of mass 5 kg is moving vertically in vertical position with an upwards force of 100 N acting at the upper end and a downwards force of 70 N acting at the lower end. The tension at midpoint of the rope is

(a) 100 N (b) 85 N

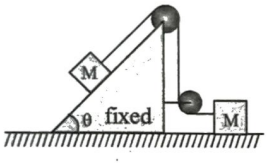
(c) 75 N (d) 105 N

18. A body is moving with a speed of 1 m/s and a force F is needed to stop it in a distance x. If the speed of the body is 3 m/s the force needed to stop it in the same distance x will be

(a) 1.5 F (b) 3F

(c) 6 F (d) 9F

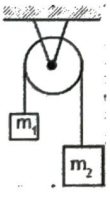
19. Two blocks, each having mass M, rest on frictionless surfaces as shown in the figure. If the pulleys are light and frictionless, and M on the incline is allowed to move down, then the tension in the string will be :



(a) $\frac{2}{3}$ Mg sin θ (b) $\frac{3}{2}$ Mg sin θ

(c) $\frac{Mg\sin\theta}{2}$ (d) 2 Mg sin θ

20. Two masses are hanging vertically over frictionless pulley. The acceleration of the two masses is-



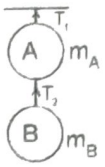
(a) $\frac{m_1}{m_2}g$ (b) $\frac{m_2}{m_1}g$

(c) $\left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$ (d) $\left(\frac{m_1 + m_2}{m_2 - m_1}\right)g$

21. Two objects A and B of masses m_A and m_B are attached by strings as shown in fig. If they are given upward acceleration, then the ratio of tension $T_1 : T_2$ is -

(a) (mA + mB)/mB

(b) (mA + mB)/mA



(c) $\frac{m_A + m_B}{m_A - m_B}$

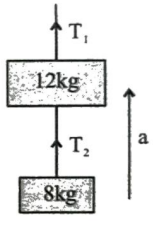
(d) $\frac{m_A - m_B}{m_A + m_B}$

22. A monkey of mass 20 kg is holding a vertical rope. The rope can break when a mass of 25 kg is suspended from it. What is the maximum acceleration with which the monkey can climb up along the rope?

(a) 7 ms^{-2} (b) 10 ms^{-2}

(c) 5 ms^{-2} (d) 2.5 ms^{-2}

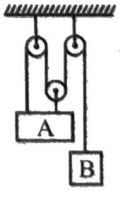
23. A body of mass 8 kg is hanging from another body of mass 12 kg. The combination is being pulled by a string with an acceleration of 2.2 m s^{-2}. The tension T_1 and T_2 will be respectively:
(use g = 9.8m/s^2)



(a) 200 N, 80 N (b) 220 N, 90 N

(c) 240 N, 96 N (d) 260 N, 96 N

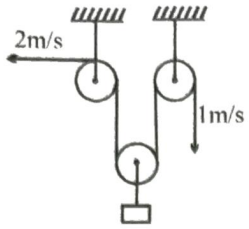
24. At a given instant, A is moving with velocity of 5 m/s upwards. What is velocity of B at the time



(a) 15 m/s↓ (b) 15 m/s↑

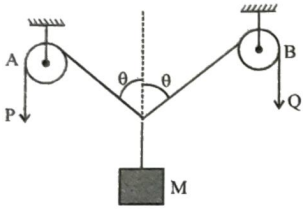
(c) 5 m/s ↓ (d) 5 m/s ↑

25. Find the velocity of the hanging block if the velocities of the free ends of the rope are as indicated in the figure.



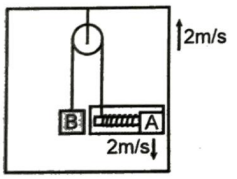
(*a*) 3/2 m/s ↑ (*b*) 3/2 m/s ↓
(*c*) 1/2 m/s ↑ (*d*) 1/2 m/s ↓

26. In the arrangement shown in fig. the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. Mass M moves upwards with a speed.



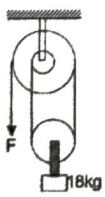
(*a*) 2 U cos θ (*b*) U cos θ
(*c*) $\frac{2U}{\cos\theta}$ (*d*) $\frac{U}{\cos\theta}$

27. In the figure shown the velocity of lift is 2 m/s while string is winding on the motor shaft with velocity 2 m/s and block A is moving downwards with a velocity of 2 m/s, then find out the velocity of block B.



(*a*) 2 m/s ↑ (*b*) 2 m/s ↓
(*c*) 4 m/s ↑ (*d*) 8 m/s ↑

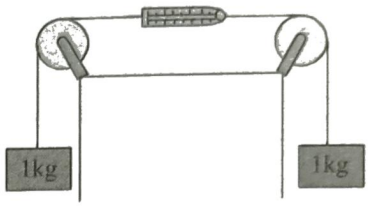
28. In the figure at the free end a force F is applied to keep the suspended mass of 18 kg at rest. The value of F is-



(*a*) 180 N (*b*) 90 N
(*c*) 60 N (*d*) 30 N

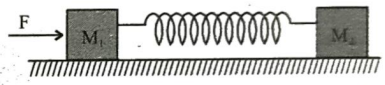
29. If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may be-
(A) going up with increasing speed
(B) going down with increasing speed
(C) going up with uniform speed
(D) going down with uniform speed
(*a*) A, D (*b*) A, B, C
(*c*) C, D (*d*) A, B

30. In the given figure, what is the reading of the spring balance ?



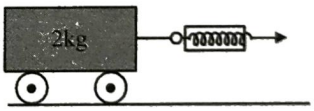
(*a*) 10 N (*b*) 20 N
(*c*) 5 N (*d*) zero

31. Two blocks of masses M_1 and M_2 are connected to each other through a light spring as shown in figure. If we push mass M_1 with force F and cause acceleration a_1 in right direction in mass M_1, what will be the magnitude of acceleration in M_2?



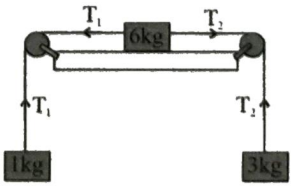
(*a*) F/M_2 (*b*) $F/(M_1 + M_2)$
(*c*) a_1 (*d*) $(F - M_1a_1)/M_2$

32. A massless spring balance is attached to 2 kg trolley and is used to pull the trolley along a flat surface as shown in the fig. The reading on the spring balance remains at 10 kg during the motion. The acceleration of the trolley is (Use $g = 9.8\ ms^{-2}$)



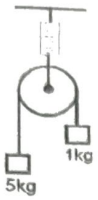
(*a*) 4.9 ms^{-2} (*b*) 9.8 ms^{-2}
(*c*) 49 ms^{-2} (*d*) 98 ms^{-2}

33. Three masses of 1 kg , 6 kg and 3 kg are connected to each other with threads and are placed on table as shown in figure. What is the acceleration with which the system is moving? Take $g = 10m\ s^{-2}$.



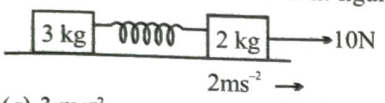
(*a*) Zero (*b*) 1 m s^{-2}
(*c*) 2 m s^{-2} (*d*) 3 m s^{-2}

34. In the figure a smooth pulley of negligible weight is suspended by a spring balance. Weights of 1kg and 5 kg are attached to the opposite ends of a string passing over the pulley and move with acceleration because of gravity. During the motion, the spring balance reads a weight of -



(*a*) 6 kg
(*b*) less than 6 kg
(*c*) more than 6 kg
(*d*) may be more or less than 6 kg

35. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 ms^{-2} as shown in figure.



(*a*) 3 ms^{-2} (*b*) 2 ms^{-2}
(*c*) 0.5 ms^{-2} (*d*) zero

36. The ratio of the weight of a man in a stationary lift & when it is moving downward with uniform acceleration 'a' is 3 : 2 . The value of 'a' is : (g = acceleration. due to gravity)

(*a*) (3/2) g (*b*) g
(*c*) (2/3) g (*d*) g/3

FRICTION

37. A block is placed on a rough floor and a horizontal force F is applied on it. The force of friction f by the floor on the block is measured for different values of F and a graph is plotted between them

(A) The graph is a straight line of slope 45°
(B) The graph is straight line parallel to the F axis
(C) The graph is a straight line of slope 45° for small F and a straight line parallel to the F-axis for large F.
(D) There is small kink on the graph

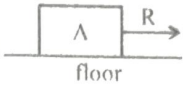
(*a*) C, D (*b*) A, D
(*c*) A, B (*d*) A, C

38. A monkey of mass m is climbing a rope hanging from the roof with acceleration a. The coefficient of static friction between the body of the monkey and the rope is μ. Find the direction and value of friction force on the monkey.

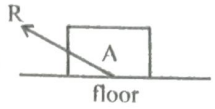
(*a*) Upward, F = m(g + a)
(*b*) downward, F = m(g + a)
(*c*) Upward, F = mg
(*d*) downward, F = mg

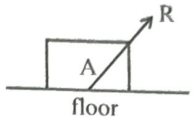


39. A box 'A' is lying on the horizontal floor of the compartment of a train running along horizontal rails from left to right. At time 't', it decelerates. Then the reaction R by the floor on the box is given best by

(*a*)


(*b*)


(*c*)


(*d*)


40. A body is placed on a rough inclined plane of inclination θ. As the angle θ is increased from 0° to 90° the contact force between the block and the plane

(*a*) remains constant
(*b*) first remains constant then decreases
(*c*) first decreases then increases
(*d*) first increases then decreases

41. A chain is lying on a rough table with a fraction 1/n of its length hanging down from the edge of the table. if it is just on the point of sliding down from the table, then the coefficient of friction between the table and the chain is-

(*a*) $\frac{1}{n}$ (*b*) $\frac{1}{(n-1)}$

(*c*) $\frac{1}{(n+1)}$ (*d*) $\frac{n-1}{(n+1)}$

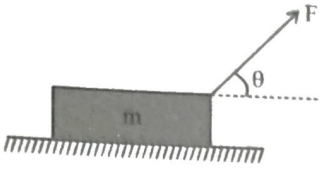
42. For the equilibrium of a body on an inclined plane of inclination 45°. The coefficient of static friction will be

(*a*) greater than one (*b*) less than one
(*c*) zero (*d*) less than zero

43. A block of mass 5 kg and surface area 2 m^2 just begins to slide down an inclined plane when the angle of inclination is 30°. Keeping mass same, the surface area of the block is doubled. The angle at which this starts sliding down is:

(*a*) 30° (*b*) 60°
(*c*) 15° (*d*) None of these

horizontally is :



(a) $\frac{F\cos\theta}{m}$ (b) $\frac{\mu F\sin\theta}{M}$

(c) $\frac{F}{m}(\cos\theta+\mu\sin\theta)-\mu g$ (d) none of these

If it slides down a rough inclined plane of some inclination, its velocity on reaching the bottom is v, where n is a number greater than 0. The coefficient of friction is given by-

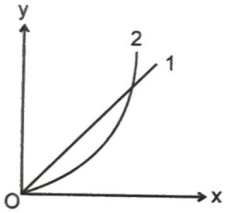
(a) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right)$ (b) $\mu = \cot\theta\left(1-\frac{1}{n^2}\right)$

(c) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right)^{1/2}$ (d) $\mu = \cot\theta\left(1-\frac{1}{n^2}\right)^{1/2}$

Advanced Level Multiconcept Questions

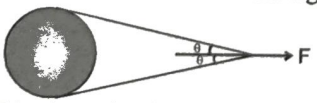
MCQ/COMPREHENSION/MATCHING/ NUMERICAL

1. A particle is resting on a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time according to law $F = \alpha . t$, where α is a constant. For the figure shown which of the following statements is/are correct ?



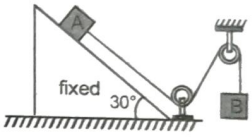
(a) Curve 1 shows acceleration against time
(b) Curve 2 shows velocity against time
(c) Curve 2 shows velocity against acceleration
(d) None of these

2. A string is wrapped round a log of wood and it is pulled with a constant force F as shown in the figure.



(a) tension T in the string increases with increase in θ
(b) tension T in the string decreases with increase in θ
(c) tension $T > F$ if $\theta > \pi/3$
(d) tension $T > F$ if $\theta > \pi/4$

3. Two blocks A and B of equal mass m are connected through a massless string and arranged as shown in figure. Friction is absent everywhere. When the system is released from rest.



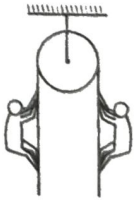
(a) tension in string is $\frac{mg}{2}$

(b) tension in string is $\frac{mg}{4}$

(c) acceleation of A is g/2

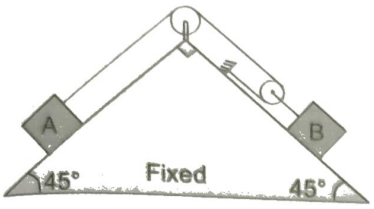
(d) acceleration of A is $\frac{3}{4}g$

4. Two men of unequal masses hold on to the two sections of a light rope passing over a smooth light pulley. Which of the following are possible?



(a) The lighter man is stationary while the heavier man slides with some acceleration
(b) The heavier man is stationary while the lighter man climbs with some acceleration
(c) The two men slide with the same acceleration in the same direction
(d) The two men move with accelerations of the same magnitude in opposite directions

5. Two blocks A and B of mass 10 kg and 40 kg are connected by an ideal string as shown in the figure. Neglect the masses of the pulleys and effect of friction. ($g = 10\ m/s^2$)



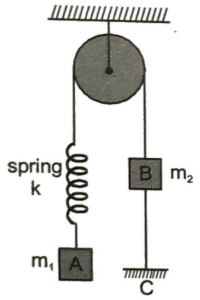
(*a*) The acceleration of block A is $\frac{5}{\sqrt{2}}\ ms^{-2}$

(*b*) The acceleration of block B is $\frac{5}{2\sqrt{2}}\ ms^{-2}$

(*c*) The tension in the string is $\frac{125}{\sqrt{2}}$ N

(*d*) The tension in the string is $\frac{150}{\sqrt{2}}$ N

6. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC . Just after the thread BC is burnt:



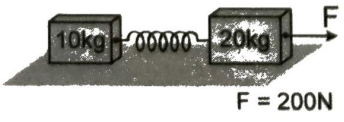
(*a*) acceleration of m_2 will be upwards

(*b*) magnitude of acceleration of both blocks will be equal to $\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

(*c*) acceleration of m_1 will be equal to zero

(*d*) magnitude of acceleration of two blocks will be non–zero and unequal.

7. Two blocks of masses 10 kg and 20 kg are connected by a light spring as shown. A force of 200 N acts on the 20 kg mass as shown. At a certain instant the acceleration of 10 kg mass is 12 ms^{-2} towards right direction.



(*a*) At that instant the 20 kg mass has an acceleration of 12 ms^{-2}.

(*b*) At that instant the 20 kg mass has an acceleration of 4 ms^{-2}.

(*c*) The stretching force in the spring is 120 N.

(*d*) The collective system moves with a common acceleration of 30 ms^{-2} when the extension in the connecting spring is the maximum.

8. A particle stays at rest as seen in a frame. We can conclude that

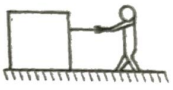
(*a*) the frame is inertial.

(*b*) resultant force on the particle is zero.

(*c*) if the frame is inertial then the resultant force on the particle is zero.

(*d*) if the frame is noninertial then there is a nonzero resultant force.

9. A man pulls a block heavier than himself with a light horizontal rope. The coefficient of friction is the same between the man and the ground, and between the block and the ground.



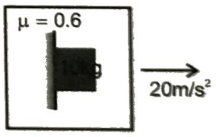
(*a*) The block will not move unless the man also moves

(*b*) The man can move even when the block is stationary

(*c*) If both move, the acceleration of the man is greater than the acceleration of the block

(*d*) None of the above assertions is correct

10. Car is accelerating with acceleration = 20 m/s^2. A box that is placed inside the car, of mass m = 10 kg is put in contact with the vertical wall as shown. The friction coefficient between the box and the wall is $\mu = 0.6$.



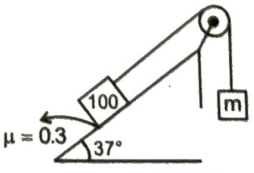
(*a*) The acceleration of the box will be 20 m/sec^2

(*b*) The friction force acting on the box will be 100 N

(*c*) The contact force between the vertical wall and the box will be $100\sqrt{5}$ N

(*d*) The net contact force between the vertical wall and the box is only of electromagnetic in nature.

11. The value(s) of mass m for which the 100 kg block remains in static equilibrium is



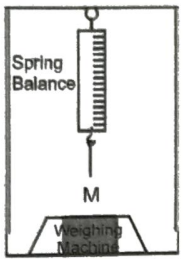
(*a*) 35 kg (*b*) 37 kg

(*c*) 83 kg (*d*) 85 kg

Comprehension – 1 (No. 12 to 14)

Figure shows a weighing machine kept in a lift. Lift is moving upwards with acceleration of 5 m/s². A block is kept on the weighing machine. Upper surface of block is attached with a spring balance. Reading shown by weighing machine and spring balance is 15 kg and 45 kg respectively.

Answer the following questions. Asume that the weighing machine can measure weight by having negligible deformation due to block, while the spring balance requires larger expansion: (take g = 10 m/s²)



12. Mass of the object in kg and the normal force acting on the block due to weighing machine are:

(*a*) 60 kg, 450 N (*b*)40 kg, 150 N

(*c*) 80 kg, 400 N (*d*) 10 kg, zero

13. If lift is stopped and equilibrium is reached. Reading of weighing machine and spring balance will be:

(*a*) 40 kg, zero (*b*) 10 kg, 20 kg

(*c*) 20 kg, 10 kg (*d*) zero, 40 kg

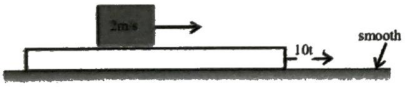
14. Find the acceleration of the lift such that the weighing machine shows its true weight.

(*a*) $\frac{45}{4}$ m/s² (*b*) $\frac{85}{4}$ m/s²

(*c*) $\frac{22}{4}$ m/s² (*d*) $\frac{60}{4}$ m/s²

Comprehension – 2 (No. 15 to 17)

A small block of mass 1 kg starts moving with constant velocity 2 m/s on a smooth long plank of mass 10 kg which is also pulled by a horizontal force F = 10 t N where t is in seconds and F is in newtons. (the initial velocity of the plank is zero).



15. Displacement of 1 kg block with respect to plank at the instant when both have same velocity is

(*a*) $4\frac{4}{3}$m (*b*) 4 m

(*c*) $\frac{8}{3}$m (*d*) 2 m

16. The time ($t \neq 0$) at which displacement of block and plank with respect to ground is same will be:

(*a*) 12 s (*b*) $2\sqrt{3}$ s

(*c*) $3\sqrt{3}$ s (*d*) $\sqrt{3}/2$ s

17. Relative velocity of plank with respect to block when acceleration of plank is 4 m/s² will be -

(*a*) Zero (*b*) 10 m/s

(*c*) 6 m/s (*d*) 8 m/s

Comprehension – 3 (No. 18 to 20)

An object of mass 2 kg is placed at rest in a frame (S_1) moving with velocity $10\hat{i}+5\hat{j}$ m/s and having acceleration $5\hat{i}+10\hat{j}$ m/s². The object is also seen by an observer standing in a frame (S_2) moving with velocity $5\hat{i}+10\hat{j}$ m/s

18 Calculate 'Pseudo force' acting on object. Which frame is responsible for this force?

(*a*) $F = -10\,\hat{i} - 20\,\hat{j}$ due to acceleration of frame S_1

(*b*) $F = -20\,\hat{i} - 20\,\hat{j}$ due to acceleration of frame S_1

(*c*) $F = -10\,\hat{i} - 30\,\hat{j}$ due to acceleration of frame S_1

(*d*) None of these

19. Calculate net force acting on object with respect to S_2 frame.

(*a*) $F = 20\hat{i}+20\hat{j}$ (*b*) $F = 10\hat{i}+20\hat{j}$

(*c*) $F = 5\hat{i}+20\hat{j}$ (*d*) $0F = 10\hat{i}+5\hat{j}$

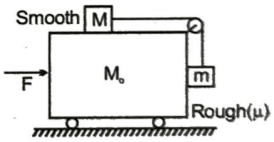
20. Calculate net force acting on object with respect of S_1 frame.

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) none of these

Comprehension – 4 (No. 21 and 22)

Imagine a situation in which the horizontal surface of block M_0 is smooth and its vertical surface is rough with a coefficient of friction μ



21. Identify the correct statement(s)

(*a*) If F=0, the blocks cannot remain stationary

(*b*) For one unique value of F, the blocks M and m remain stationary with respect to M_0

(*c*) The limiting friction between m and M_0 is independent of F

(*d*) There exist a value of F at which friction force is equal to zero.

22. In above problem, choose the correct value(s) of F which the blocks M and m remain stationary with respect to M_0

(*a*) $(M_0+M+m)\frac{g}{\mu}$ (*b*) $\frac{m(M_0+M+m)g}{M-\mu m}$

(*c*) $(M_0+M+m)\frac{mg}{M}$ (*d*) none of these

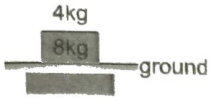
23. Column-I gives four different situation. In final statement of each situation two vector quantities are compared. The result of comparison is given in Column-II. Match the statement in Column-I with the correct comparison(s) in Column-II:

Column-I **Column-II**

(*a*) Stone is projected from ground at an angle θ with horizontal ($\theta \neq 90°$). Neglect the effect of air friction. Then between the two instants when it is at same height (above ground), its average velocity and horizontal component of velocity are — (p) same in magnitude

(*b*) For four particles A, B, C & D, the velocities of one with respect to other are given as $\vec{V}_{DC}$ is 20 m/s towards north, $\vec{V}_{BC}$ is 20 m/s towards east and $\vec{V}_{BA}$ is 20 m/s towards south. Then $\vec{V}_{BC}$ and $\vec{V}_{AD}$ are — (q) different in magnitude

(*c*) Two blocks of masses 4 and 8 kg are placed on ground as shown. Then the net force exerted by earth on block of mass 8 kg and normal reaction exerted by 8 kg block on earth are (note that earth includes ground) — (r) same in direction



(*d*) For a particle undergoing rectilinear motion with uniform acceleration, the magnitude of displacement is half the distance covered in some time interval. The magnitude of final velocity is less than magnitude of initial velocity for this time interval. Then the initial velocity and average velocity for this time interval are — (s) opposite in direction

24. Column-I gives four different situations involving two blocks of mass m_1 and m_2 placed in different ways on a smooth horizontal surface as shown. In each of the situations horizontal forces F_1 and F_2 are applied on blocks of mass m_1 and m_2 respectively and also $m_2F_1 < m_1F_2$. Match the statements in Column-I with corresponding results in Column-II.

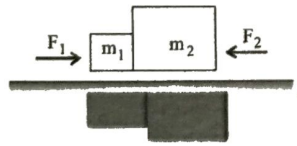
Column-I **Column-II**

(*a*) 

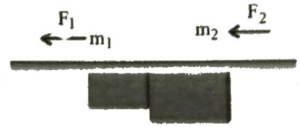
. Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is — (p) $\frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} - \frac{F_2}{m_2}\right)$

(*b*) 

. Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is — (q) $\frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right)$

(*c*) 

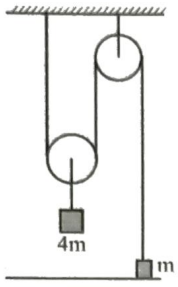
. The magnitude of normal reaction between the blocks is — (r) $\frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right)$

(*d*) 

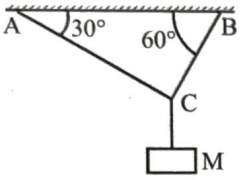
. The magnitude of normal reaction between the blocks is — (s) $m_1 m_2\left(\frac{F_1 + F_2}{m_1 + m_2}\right)$

NUMERICAL VALUE BASED

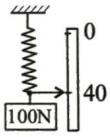
25. The mass of the body which is hanging on the rope attached to the movable pulley is four times as much as the mass of the body which is fixed to the ground. At a given instant the fixed body is released. What is its initial acceleration (in m/s^2) ? (The mass of the pulleys and the ropes is negligible.)



26. If mass M is 2 kg what is the tension in string AC? (in N)

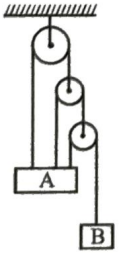


27. In ideal spring, with a pointer attached to its end, hangs next to a scale. With a 100 N weight attached and at rest, the pointer indicates '40' on the scale as shown. Using a 200 N weight instead results in 60 on the scale. Using an unknown weight 'X' instead results in '30' on the scale. Find value of 'X' (in newton).

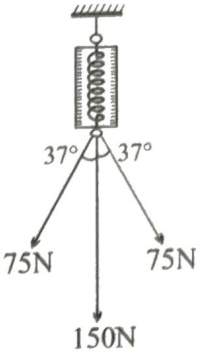


28. A block of mass 5kg is being pulled up by a constant force of 90 N starting at ground from rest. 2 sec later, a part of the block having mass 3kg falls off. Find the distance between the two parts when 3kg part hits the ground.

29. If $M_B = 10$ kg, find the value of M_A (in kg), so that block A remains in equilibrium. The pulleys are ideal.



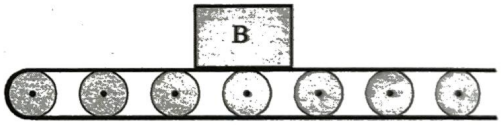
30. The scale in **Figure** is being pulled on by three ropes. What net force (in N) does the spring scale read?



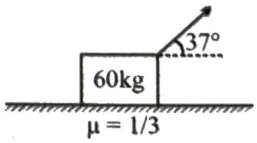
31. A rope has a length of 12 m and a mass of 16 kg. The rope hangs from a rigid support. An operator whose mass is 80 kg slides down the rope at a constant speed of 0.8 m/s. What is the tension (in N) in the rope at a point 6 m from the top when the man has slid below this point?

32. Compute the least acceleration (in m/s^2) with which a 50 kg man can slide down a rope, if the rope can withstand a tension of 300 N.

33. The conveyor belt is moving at speed 4 m/s. If the coefficient of static friction between the conveyor and the package B of mass 10 kg is 0.2, determine the shortest time (in sec) the belt can stop so that the package does not slide on the belt.



34. A force P pulls a block at a constant speed across the floor. What is the magnitude of the force (in N)?

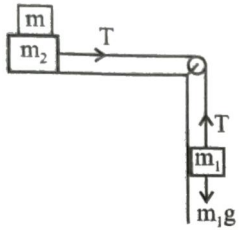


JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. Two masses $m_1 = 5kg$ and $m_2 = 10kg$, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :

[JEE Main-2018]



(*a*) 23.3 kg (*b*) 43.3 kg
(*c*) 10.3 kg (*d*) 18.3 kg

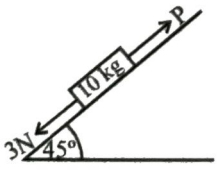
2. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$)

[JEE Main-2019 (January)]

(*a*) 200 N (*b*) 140 N
(*c*) 70 N (*d*) 100 N

3. block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block does not move downward? (take = $g=10ms^{-2}$)

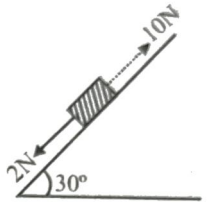
[JEE Main-2019 (January)]



(*a*) 32 N (*b*) 18 N
(*c*) 23 N (*d*) 25 N

4. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is :

[JEE Main-2019 (January)]



(*a*) $\frac{\sqrt{3}}{2}$ (*b*) $\frac{\sqrt{3}}{4}$

(*c*) $\frac{1}{2}$ (*d*) $\frac{2}{3}$

5. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is :

[JEE Main-2019 (April)]

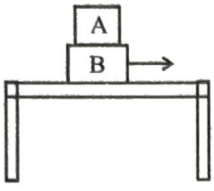
(*a*) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(*b*) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(*c*) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

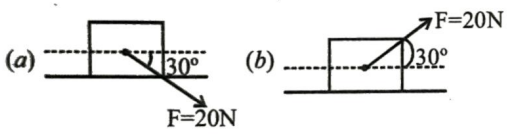
(*d*) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

6. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is:
(Take g = 10 m/s^2) [*JEE Main-2019 (April)*]



(*a*) 16 N (*b*) 40 N
(*c*) 12 N (*d*) 8 N

7. A block of mass 5 kg is (i) pushed in case (*a*) and (ii) pulled in case (*b*). by a force F = 20N. Making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (*b*) and case (*a*) will be :($g = 10\ ms^{-2}$)
[*JEE Main-2019 (April)*]



(*a*) 0 ms^{-2} (*b*) 0.8 ms^{-2}
(*c*) 0.4 ms^{-2} (*d*) 3.2 ms^{-2}

8. A mass of 10 kg is suspended by a rope of length 4m, from the ceiling. a force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take g = 10ms^{-2} and the rope to be massless)
[*JEE Main-2020 (January)*]

(*a*) 75 N (*b*) 90 N
(*c*) 100 N (*d*) 70 N

9. A small ball of mass m is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2, where v is its speed. The maximum height attained by the ball is [*JEE Main-2020 (September)*]

(*a*) $\frac{1}{2K}\tan^{-1}\frac{ku^2}{g}$ (*b*) $\frac{1}{K}\ln\left(1+\frac{ku^2}{2g}\right)$

(*c*) $\frac{1}{2K}\ln\left(1+\frac{ku^2}{g}\right)$ (*d*) $\frac{1}{K}\tan^{-1}\frac{ku^2}{2g}$

10. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v . It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{1}{1000}$. The nearest integer to I is _____. [*JEE Main-2020 (September)*]

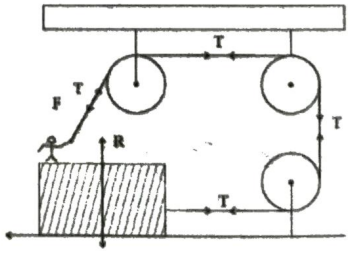
11. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is ($g = 10\ ms^{-2}$)
[*JEE Main-2020 (September)*]

(*a*) 0.45 m (*b*) 0.80 m
(*c*) 0.20 m (*d*) 0.60 m

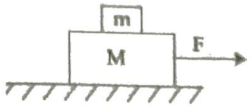
12. A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is '*x*' *N*. The value of '*x* ' to the nearest integer is [*JEE Main-2021 (March)*]

13. A body of mass 1kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force FN. The value of F will be (Round off to the Nearest Integer) [Take $g = 10\,ms^{-2}$]
[*JEE Main-2021 (March)*]

14. A boy of mass 4kg is standing on a piece of wood having mass 5kg. If the coefficient of friction between the wood and the floor is \$0.5\$, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is N. (Round off to the Nearest Integer) [Take $g = 10\,ms^{-2}$] [*JEE Main-2021 (March)*]



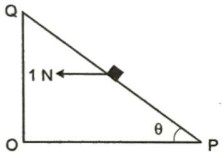
15. Two blocks (m = 0.5 kg and M = 4.5 kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is N. (Round off to the Nearest Integer) [Take g as $9.8\,ms^{-2}$] [*JEE Main-2021 (March)*]



16. A body of mass 2kg moves under a force of $(2\hat{i}+3\hat{t}j+5\hat{k})$N. It starts from rest and was at the origin initially. After $4s$, its new coordinates are $(8,b,20)$. The value of b is: (Round off to the Nearest Integer). [*JEE Main-2021 (March)*]

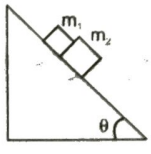
JEE-ADVANCED PREVIOUS YEAR'S

1. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N on the block through its center of mass as shown in the figure. The block remains stationary if (take g = 10 m/s²) [*IIT-JEE-2012*]



(*a*) θ = 45°
(*b*) θ > 45° and a frictional force acts on the block towards P.
(*c*) θ > 45° and a frictional force acts on the block towards
(*d*) θ < 45° and a frictional force acts on the block towards

2. A block of mass m_1 = 1 kg another mass m_2 = 2 kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to μ = 0.3. In List II expression for the friction on block m_2 given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g [*JEE Advanced-2014*]
[Useful information: tan (5.5°) ≈ 0.1; tan (11.5°) ≈ 0.2; tan (16.5° ≈ 0.3)]

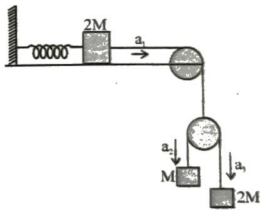


List-I	List-II
P. θ = 5°	1. $m_2 g \sin\theta$
Q. θ = 10°	2. $(m_1+m_2)g\sin\theta$
R. θ = 15°	3. $\mu m_2 g\cos\theta$
S. θ = 20°	4. $\mu(m_1+m_2)g\cos\theta$

Code :
(*a*) P-1, Q-1, R-1,S-3 (*b*) P-2, Q-2, R-2,S-3
(*c*) P-2, Q-2, R-2,S-4 (*d*) P-2, Q-2, R-3,S-3

3. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as dK/dt = γt, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true? [*JEE Advanced-2018*]
(*a*) The force applied on the particle is constante
(*b*) The speed of the particle is proportional to time
(*c*) The distance of the particle from the origin increases linerarly with time
(*d*) The force is conservative

4. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct? [*JEE Advanced-2019*]
[g is the acceleration due to gravity. Neglect friction]

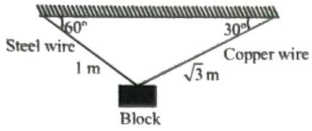


(*a*) $x_0 = \frac{4Mg}{k}$
(*b*) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$
(*c*) $a_2 - a_1 = a_1 - a_3$
(*d*) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

5. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm² and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is

($\Delta\ell_C$) and elongation in steel wire is ($\Delta\ell_s$), then the ratio $\frac{\Delta\ell_C}{\Delta\ell_S}$ is _____. [JEE Advanced-2019]

[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2 respectively]



Paragraph: A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}$ m/s. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x metres from the point O. The acceleration due to gravity g = 10 m/s^2. [JEE Advanced-2021]

6. The value of t is
7. The value of x is

ANSWER KEY

Topicwise Questions

1. (*c*)	**2.** (*c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*c*)	**6.** (*b*)	**7.** (*b*)	**8.** (*c*)	**9.** (*c*)	**10.** (*a*)
11. (*d*)	**12.** (*b*)	**13.** (*c*)	**14.** (*b*)	**15.** (*a*)	**16.** (*a*)	**17.** (*c*)	**18.** (*c*)	**19.** (*a*)	**20.** (*a*)
21. (*b*)	**22.** (*a*)	**23.** (*a*)	**24.** (*b*)	**25.** (*b*)	**26.** (*b*)	**27.** (*c*)	**28.** (*d*)	**29.** (*c*)	**30.** (*a*)
31. (*c*)	**32.** (*c*)	**33.** (*c*)	**34.** (*b*)	**35.** (*d*)	**36.** (*a*)	**37.** (*b*)	**38.** (*a*)	**39.** (*c*)	**40.** (*d*)
41. (*a*)	**42.** (*c*)	**43.** (*a*)	**44.** (*c*)	**45.** (*a*)	**46.** (*c*)	**47.** (*a*)	**48.** (*a*)	**49.** (*a*)	**50.** (*d*)
51. (*b*)	**52.** (*c*)	**53.** (*a*)	**54.** (*a*)	**55.** (*d*)	**56.** (*a*)				

Learning Plus

1. (*d*)	**2.** (*b*)	**3.** (*c*)	**4.** (*c*)	**5.** (*b*)	**6.** (*b*)	**7.** (*d*)	**8.** (*c*)	**9.** (*b*)	**10.** (*b*)
11. (*a*)	**12.** (*b*)	**13.** (*a*)	**14.** (*c*)	**15.** (*c*)	**16.** (*c*)	**17.** (*b*)	**18.** (*d*)	**19.** (*c*)	**20.** (*c*)
21. (*a*)	**22.** (*d*)	**23.** (*c*)	**24.** (*a*)	**25.** (*a*)	**26.** (*d*)	**27.** (*d*)	**28.** (*b*)	**29.** (*c*)	**30.** (*a*)
31. (*d*)	**32.** (*c*)	**33.** (*c*)	**34.** (*b*)	**35.** (*b*)	**36.** (*d*)	**37.** (*a*)	**38.** (*a*)	**39.** (*c*)	**40.** (*b*)
41. (*b*)	**42.** (*a*)	**43.** (*a*)	**44.** (*c*)	**45.** (*a*)					

Advanced Level Multiconcept Questions

1. (*a, b, c*) **2.** (*a, c*) **3.** (*b, d*) **4.** (*a, b, d*) **5.** (*a, b, d*) **6.** (*a, c*) **7.** (*b, c*) **8.** (*c, d*) **9.** (*a, b, c*)
10. (*a, b, c, d*) **11.** (*b, c*) **12.** (*b*) **13.** (*d*) **14.** (*a*) **15.** (*c*) **16.** (*b*) **17.** (*c*) **18.** (*a*)
19. (*b*) **20.** (*a*) **21.** (*a, d*) **22.** (*b, c*) **23.** (*a*) p, r (*b*) p, r (*c*) q, s (*d*) q, r **24.** (*a*) q (*b*) r (*c*) q (*d*) r
25. [0005] **26.** [0010] **27.** [0050] **28.** [0360] **29.** [0070] **30.** [0270] **31.** [0880] **32.** 4 m/s^2 **33.** [0002] **34.** [0200]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*a*) **2.** (*d*) **3.** (*a*) **4.** (*a*) **5.** (*b*) **6.** (*a*) **7.** (*b*) **8.** (*c*) **9.** (*c*) **10.** [346]
11. (*c*) **12.** [10] **13.** [5] **14.** [30] **15.** [21] **16.** [12]

JEE-ADVANCED PREVIOUS YEAR'S

1. (*a, c*) **2.** (*d*) **3.** (*a, b, d*) **4.** (*c*) **5.** [2.00] **6.** [0.50] **7.** [7.50]

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5 LAWS OF MOTION

Topicwise Questions

1. (*c*) **2.** (*c*) **3.** (*b*)

4. (*b*) Horizontal velocity of apple will remain same but due to retardation of train, velocity of train and hence velocity of boy w.r.t. ground decreases, so apple falls away from the hand of boy in the direction of motion of the train.

5. (*c*) Acceleration $\alpha = \frac{F}{m} = \frac{100}{5} = 20\ cm/s^2$

Now $v = \alpha t = 20 \times 10 = 200\ cm/s$

6. (*b*) u = velocity of bullet

$\frac{dm}{dt}$ = Mass thrown per second by the machine gun

= Mass of bullet × Number of bullet fired per second

= 10 g × 10 bullet/sec = 100 g/sec = 0.1 kg/sec

$\therefore$ Thrust $= \frac{udm}{dt} = 500 \times 0.1 = 50\ N$

7. (*b*) Force on particle at 20 cm away F = kx

$F = 15 \times 0.2 = 3\ N$[As k = 15 N/m]

$\therefore$ Acceleration $= \frac{Force}{Mass} = \frac{3}{0.3} = 10\ m/s^2$

8. (*c*) Thrust $F = u\left(\frac{dm}{dt}\right) = 5 \times 10^4 \times 40 = 2 \times 1 \times 10^6\ N$

9. (*c*) We know that in the given condition $s \propto \frac{1}{m^2}$

$\therefore \quad \frac{s_2}{s_1} = \left(\frac{m_1}{m_2}\right)^2 \Rightarrow s^2 = \left(\frac{m_1}{m_2}\right)^2 \times s_1$

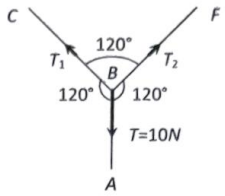
10. (*a*) $m = \frac{F}{\alpha} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = \sqrt{200} = 10\sqrt{2}\ kg.$

11. (*d*) Force

$= m\left(\frac{dv}{dt}\right) = \frac{0.25 \times [(10) - (-10)]}{0.01} = 25 \times 20 = 500\ N.$

12. (*b*) $F = ma = \frac{m(u - v)}{t} = \frac{2 \times (8 - 0)}{4} = 4\ N.$

13. (*c*) By drawing the free body diagram of point B



Let the tension in the section BC and BF are T_1 and T_2 respectively.

From Lami's theorem

$$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{T}{\sin 120^\circ}$$

$\Rightarrow T = T_1 = T_2 = 10\ N.$

14. (*b*) $\vec{p} = m\vec{v}$

15. (*a*) For exerted by ball on wall

= rate of change in momentum of ball

$= \frac{mv - (-mv)}{t} = \frac{2mu}{t}.$

16. (*a*) Given that $\vec{p} = p_x\hat{i} + p_y\hat{j} = 2\cos t\hat{i} + 2\sin t\hat{j}$

$\therefore \vec{F} = \frac{d\vec{p}}{dt} = -2\sin t\hat{i} + 2\cos t\hat{j}$

Now, $\vec{F}.\vec{p} = 0$ i.e. angle between $\vec{F}$ and $\vec{p}$ is 90°.

17. (*c*) Swimming is a result of pushing water in the opposite direction of the motion.

18. (*c*)

19. (*a*) Change in momentum = Impulse

$\Rightarrow \Delta p = F \times \Delta t \Rightarrow \Delta t = \frac{\Delta p}{F} = \frac{125}{250} = 0.5\ sec.$

20. (*a*)

21. (*b*) $F = \sqrt{(F)^2 + (F)^2 + 2F.F\cos\theta} \Rightarrow \theta = 120°$.

22. (*a*) FBD of mass 2 kg FBD of mass 4kg

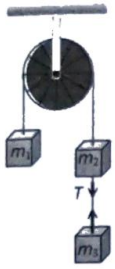


$T - T' - 20 = 4$ ....(i) $T' - 40 = 8$ ...(ii)

By solving (i) and (ii) $T' = 47.23$ N and $T = 70.8$ N

23. (*a*) **24.** (*b*)

25. (*b*) Tension between m_2 and m_3 is given by



$$T = \frac{2m_1m_3}{m_1+m_2+m_3} \times g$$

$$= \frac{2\times2\times2}{2+2+2} \times 9.8 = 13\text{N}.$$

26. (*b*) $a = \frac{m_2}{m_1+m_2} \times g = \frac{5}{4\times5} \times 9.8 = \frac{49}{9} = 5.44\ \text{m/s}^2$

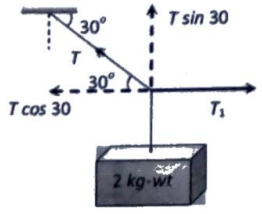
27. (*c*)



$$T' = (m_1+m_2) \times \frac{T}{m_1+m_2+m_3}.$$

28. (*d*) $T = \frac{2m_1m_2}{m_1+m_2} g = \frac{2\times10\times6}{10+6} \times 9.8 = 73.5\text{N}.$

29. (*c*) T sin 30 = 2kg wt



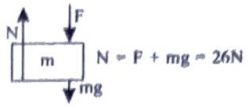
$\Rightarrow T = 4$ kg wt

$T_1 = T\cos 30°$

$= 4\cos 30° = 2\sqrt{3}$.

30. (*a*) For jumping he presses the spring platform, so the reading of spring balance increases first and finally it becomes zero.

31. (*c*)



$F = ma = 1$N

32. (*c*) 5N force will not produce any tension in spring without support of other 5N force. So here the tension in the spring will be 5N only.

33. (*c*) As the spring balances are massless therefore the reading of both balance should be equal.

34. (*b*)

35. (*d*) As the apparent weight increase therefore we can say that acceleration of the lift is in upward direction.

$R = m(g + a) \Rightarrow 4.8\ g = 4\ (g + a)$

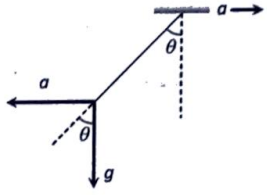
$\Rightarrow a = 0.2g = 1.96\ \text{m/s}^2$

36. (*a*) $T = m(g + a) = 500\ (10 + 2) = 6000$ N

37. (*b*) Rate of flow will be more when lift will move in upward direction with some acceleration because the net downward pull will be more and vice-versa.

$F_{upward} = m(g + a)$ and $F_{downward} = m(g - a)$ F_{upward}

38. (*a*) When car moves towards right with acceleration a then due to pseudo force the plumb line will tilt in backward direction making an angle θ with vertical.



From the figure,

$\tan\theta = a / g$

$\therefore \quad \theta = \tan^{-1}(a/g)$.

39. (*c*)

40. (*d*) $\mu = \frac{F}{R} = \frac{F}{mg} = \frac{98}{100\times9.8} = \frac{1}{10} = 0.1$

41. (*a*)

42. (*c*) $F_1 = \mu_s R = 0.4 \times mg = 0.4 \times 10 = 4$N i.e. minimum 4N force is required to start the motion of a body. But applied force is only 3N. So the block will not move.

43. (*a*) For limiting condition

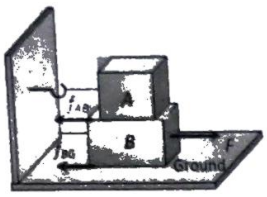
$$\mu = \frac{m_B}{m_A + m_C} \Rightarrow 0.2 = \frac{5}{10 + m_C}$$

$\Rightarrow 2 + 0.2m_C = 5 \Rightarrow m_C = 15$ kg

44. (*c*) Sliding friction is greater than rolling friction.

45. (*a*) $\mu = \frac{m_B}{m_A} \Rightarrow 0.2 = \frac{m_B}{10} \Rightarrow m_B = 2kg$

46. (*c*)



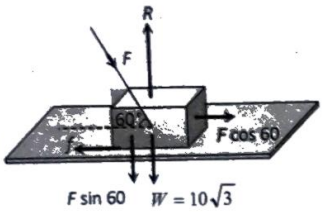
$F = f_{AB} + F_{BG}$

$= \mu_{AB} m_A g + \mu_{BG} (m_A + m_B) g$

$= 0.2 \times 100 \times 10 + 0.3\ (300) \times 10$

$= 200 + 900 = 1100$ N

47. (*a*)



$f = \mu R$

$F \cos 60° = \mu(W + F \sin 60°)$

Substituting

$\mu = \frac{1}{2\sqrt{3}}$ & $W = 10\sqrt{3}$ we get $F = 20$ N

48. (*a*) Retarding force $F = ma = \mu R = \mu\ mg \therefore a = \mu g$

Now from equation of motion $v^2 = u^2 - 2as$

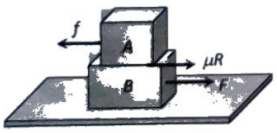
$\Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a} = \frac{u^2}{2\mu g}$.

49. (*a*) There is no friction between the body B and surface of the table. If the body B is pulled with force F then

$F = (m_A + m_B)\ a$

Due to this force upper body A will feel the pseudo force in a backward direction.

$F = m_A \times a$



But due to friction between A and B, body will not move. The body A will start moving when pseudo force is more than friction force.

i.e. for slipping, $m_A a = \mu m_A g \therefore a = \mu g$

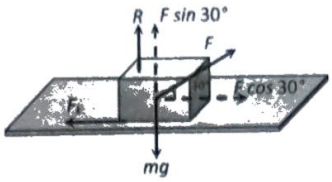
50. (*d*)

51. (*b*) From the relation $F - \mu mg = ma$

$a = \frac{F - \mu mg}{m} = \frac{129.4 - 0.3 \times 10 \times 9.8}{10} = 10\,m/s^2$.

52. (*c*)

53. (*a*)



Kinetic friction $= \mu_k R = 0.2\ (mg - F \sin 30°)$

$= 0.2\left(5 \times 10 - 40 \times \frac{1}{2}\right) = 0.2(50 - 20) = 6N$

Acceleration of the block

$= \frac{F\cos 30° - \text{Kinetic friction}}{\text{Mass}}$

$= \frac{40 \times \frac{\sqrt{3}}{2} - 6}{5} = 5.73\,m/s^2$.

54. (*a*) $a = g\ (\sin\theta - \mu \cos\theta) = 9.8\ (\sin 45° - 0.5 \cos 45°)$

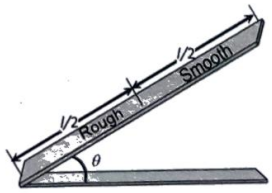
$= \frac{4.9}{\sqrt{2}} m/sec^2$.

55. (*d*) For upper half

$v^2 = u^2 + 2al\ /\ 2 = 2\ (g \sin\theta)\ l\ /\ 2 = gl \sin\theta$

For lower half

$\Rightarrow 0 = u^2 + 2g(\sin\theta - \mu\cos\theta)\frac{l}{2}$



$\Rightarrow -gl \sin\theta = gl\ (\sin\theta - \mu \cos\theta)$

$\Rightarrow \mu \cos\theta = 2 \sin\theta \Rightarrow \mu = 2 \tan\theta$.

56. (*a*) Limiting friction between block and slab $= \mu_s m_A g$

$= 0.6 \times 10 \times 9.8 = 58.8$ N

But applied force on block A is 100 N. So the block will slip over a slab.

Now kinetic friction works between block and slab

$FK = \mu_k m_A g = 0.4 \times 10 \times 9.8 = 39.2$ N

This kinetic friction helps to move the slab

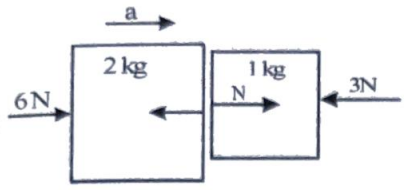
$\therefore$ Acceleration of slab $= \frac{39.2}{m_B} = \frac{39.2}{40} = 0.98\,m/s^2$.

1. (*d*) Experimental fact.

2. (*b*)

3. (*c*) Force exerted by string is always along the string and of pull type.
When there is a contact between a point and a surface the normal reaction is perpendicular to the surface and of push type.

4. (*c*)



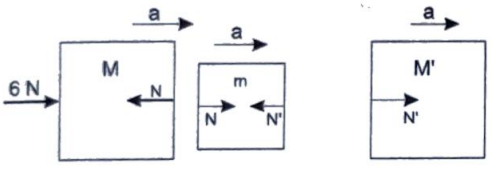
Both blocks are constrained to move with same acceleration.

$6 - N = 2a$ [Newtons II law for 2 kg block]

$N - 3 = 1a$ [Newtons II law for 1 kg block]

$\Rightarrow N = 4$ Newton

5. (*b*)



$F - N = Ma$ [Newtons law for block of mass M]

$N - N' = ma$ [Newtons law for block of mass m]

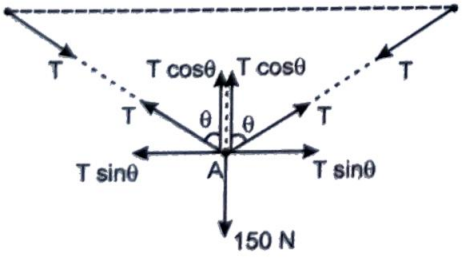
$N' = M'a$ [Newtons law for block of mass M']

$$\Rightarrow N' = M' \frac{F}{M + m + M'}$$

$$N = (m + M') \frac{F}{M + m + M'} \Rightarrow N > N'$$

6. (*b*)In free fall gravitation force acts.

7. (*d*)



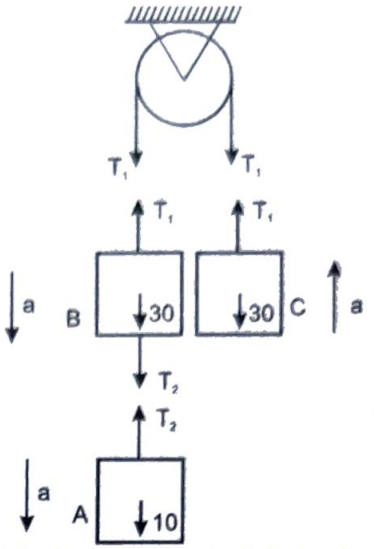
$T\cos\theta + T\cos\theta - 150 = 0$ [Equilibrium of point A]

$$2T\cos\theta = 150 T = \frac{75}{\cos\theta}$$

When string become straight θ becomes 90°

$\Rightarrow T = \infty$

8. (*c*)



$10 - T_2 = 1\,a$ [Newton's II law for A]

$T_2 + 30 - T_1 = 3\,a$ [Newton's II law for B]

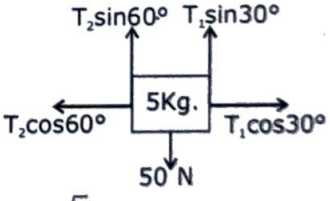
$T_1 - 30 = 3a$ [Newton's II law for C]

$$\Rightarrow a = \frac{g}{7}$$

$$\Rightarrow T_2 = \frac{6g}{7}$$

9. (*b*) From horizontal equilibrium

$$\frac{T_2}{2} = \frac{T_1\sqrt{3}}{2}$$



$T_2 = \sqrt{3}T_1$

From vertical equilibrium

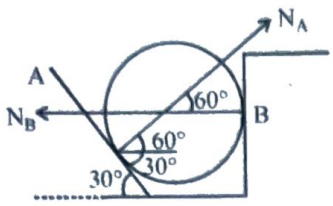
$$\frac{T_2\sqrt{3}}{2} + \frac{T_1}{2} = 50$$

$\Rightarrow T_1 = 25\text{N}$, $T_2 = 25\sqrt{3}\text{N}$

10. (*b*) Component of force in y direction is

$N_A \sin 60° = 500$

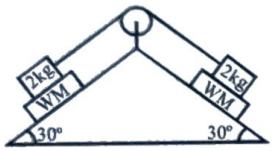
$$N_A = \frac{1000}{\sqrt{3}}$$



Component of force in x direction is

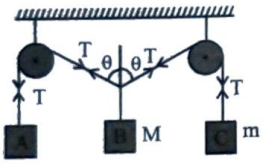
$N_A \cos 60° = N_B \Rightarrow N_B = \frac{500}{\sqrt{3}}$

11. (*a*) Weighing Machine always Measure Normal force



$N = 20 \cos 30° = 10\sqrt{3}$

12. (*b*)



$T = mg$ ...(i)

$2T \cos \theta = Mg$ ...(ii)

From equation (i) and (ii)

$\Rightarrow 2mg \cos \theta = Mg$

θ always > 0 so $M < 2m$

13. (*a*) At $t = 2$ sec $\Rightarrow a = \frac{10}{2} = 5$ m/s^2

So, $F = ma = \frac{50}{1000} \times 5 = 0.25$ N

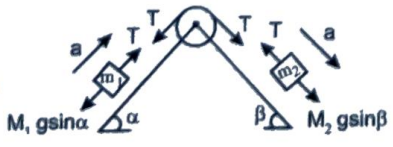
At $t = 4$ sec

$a = 0$ So $F = 0$

$\Rightarrow$ At $t = 6$ sec,

$\Rightarrow a = -5$ m/s^2 $\Rightarrow F = -0.25$ N

14. (*c*)



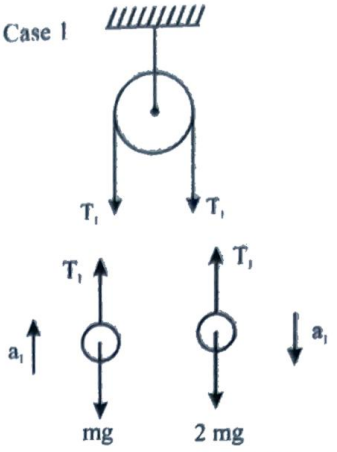
$M_2 g \sin\beta - T = M_2 a$ [Newton's II law for M_2]

$T - M_1 g \sin\alpha = M_1 a$ [Newton's II law for M_1]

By adding both equations

$$a = \left[\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_1 + M_2}\right] g$$

15. (*c*) Case 1

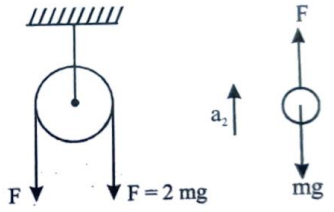


$T_1 - mg = ma_1$ [Newton's II law for m]

$2mg - T_1 = 2ma_1$ [Newton's II law for 2m]

$\Rightarrow a_1 = \frac{g}{3}$

Case 2



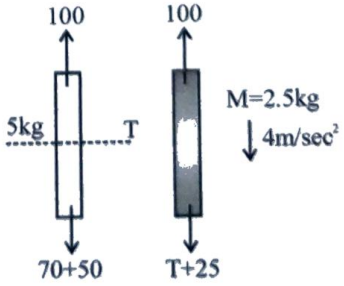
$F - mg = ma_2$[Newton's II law for m]

$\Rightarrow 2mg - mg = ma_2 \Rightarrow a_2 = g \Rightarrow a_2 > a_1$

16. (*c*) $mg - \frac{3}{4}mg = ma$ [Newton's II law for man]

$\Rightarrow a = \frac{g}{4}$

17. (*b*)



$120 - 100 = 5a$ ....(i)

$a = \frac{20}{5} \Rightarrow a = 4$m/s^2

$T + 25 - 100 = 2.5 \times 4$....(ii)

$T = 85$ N

18. (*d*) $v^2 = v_0^2 + 2as = 1^2 + 2\frac{F}{m}x$

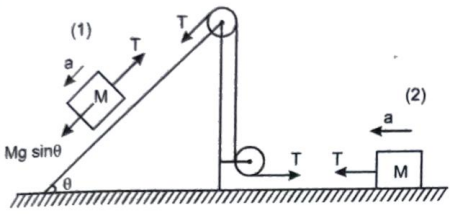
$$x = \frac{-m}{2F}$$

$$0^2 = 3^2 + \frac{2F^1}{m}x$$

$$\Rightarrow 0 = 9 + \frac{2F^1}{m}\left(\frac{-m}{2F}\right)$$

$$\Rightarrow F^1 = 9F$$

19. (*c*)



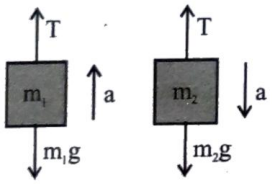
$Mg\sin\theta - T = Ma$ [Newton's II law for block 1]

$T = Ma$ [Newton's II law for block 2]

By dividing both equations

$$2T = Mg\sin\theta \quad T = \frac{Mg\sin\theta}{2}$$

20. (*c*)



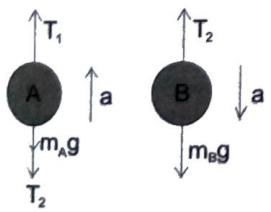
$T - m_1g = m_1a$ ....(i)

$m_2g - T = m_2a$ ....(ii)

On solving equation (i) and (ii)

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$

21. (*a*)



$T_1 - T_2 - m_Ag = m_Aa$

$T_2 - m_Bg = m_Ba$

$T_1 - (m_A + m_B)g = (m_A + m_B)$

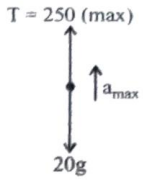
$T_2 = m_B(a + g)$

$$\frac{T_1}{T_2} = \frac{m_A + m_B}{m_B}$$

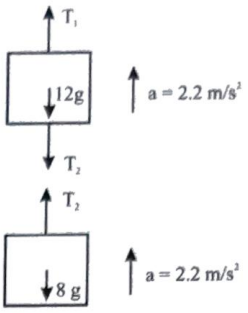
22. (*d*) $T_{max} = m_{max}g = 25 \times 10 = 250$

$250 - 200 = 20\ a_{max}$

$a_{max} = 2.5\ m/s^2$



23. (*c*)



$T_2 - 8g = 8a$ [Newton's II law for 8 kg block]

$\Rightarrow T_2 = 8 \times 2.2 + 8 \times 9.8$

$= 96\ N$

$T_1 - 12\ g - T_2 = 12\ a$

$\Rightarrow T_1 = 12 \times 2.2 + 12 \times 9.8 + 96$

$T_1 = 240\ N$

24. (*a*) $-v_A - v_A - v_A + v_B = 0$

From constrained

$-5 - 5 - 5 + v_B = 0$

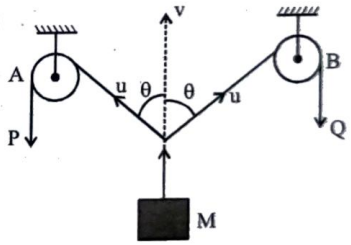
$v_B = 15\ m/s \downarrow$

25. (*a*)From constrained

$+2 - v_B - v_B + 1 = 0$

$v_B = 3/2\ m/s \uparrow$

26. (*d*)



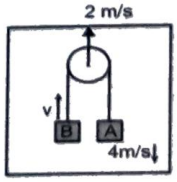
From constrained

The resultant vel. of the Block M is v in vertical direction.

So component of 'v' is direction of u is

$$v\cos\theta = u \Rightarrow v = \frac{u}{\cos\theta}$$

27. (*d*)



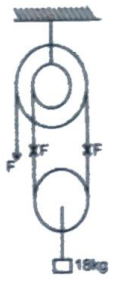
V = (velocity of B w.r.t. ground)

$\frac{V-4}{2} = 2, V = 8$ m/s (velcoity of B w.r.t. ground)

V' = 6 m/s (velocity of B w.r.t. lift)

28. (*b*) 18kg at rest => 180 = 2F

F = 90N



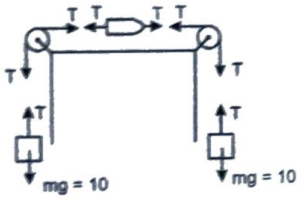
29. (*c*) T = mg + ma

T = mg – ma

T = mg

a = 0

30. (*a*)



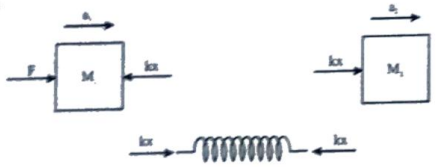
T – mg = 0 [Equilibrium of block]

T – 10 = 0

T = 10

Reading of spring balance is same as tension is spring balance.

31. (*d*)



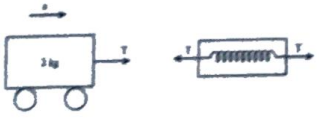
$F - kx = m_1 a_1$ [Newton's II law for M_1]

$kx = m_2 a_2$ [Newton's II law for M_2]

By adding both equations.

$$F = m_1 a_1 + m_2 a_2 \Rightarrow a_2 = \frac{F - m_1 a_1}{m_2}$$

32. (*c*)



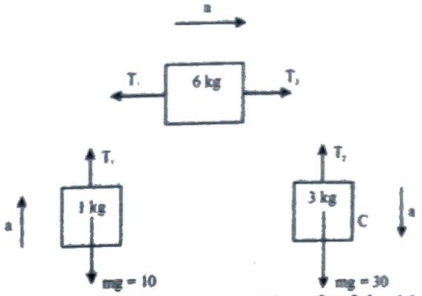
Reading of spring balance is same as tension in the balance.

$\Rightarrow$ T = 10 g = 98 N

T = 2 a [Newton's II law for 2 kg block]

$\Rightarrow$ a = 49 m/s²

33. (*c*)



$30 - T_2 = 3a$ [Newton's II law for 3 kg block]

$T_2 - T_1 = 6a$ [Newton's II law for 6 kg block]

$T_1 - 10 = 1a$ [Newton's II law for 1 kg block]

By adding three equations

$30 - 10 = 10a \Rightarrow a = 2\ m/s^2$.

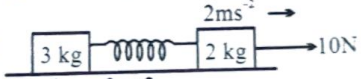
34. (*b*) $T = \frac{2m_1m_2g}{(m_1+m_2)} \Rightarrow T = \frac{2\times5\times1\times10}{6} = \frac{50}{3}$

$$2T = \frac{100}{3} \approx 33.3kg$$

The spring balance reads

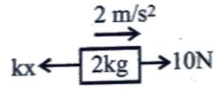
2T = 33.33kgwt < 60kgwt

35. (*b*)



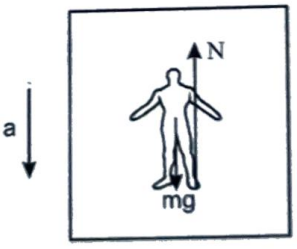
10 – kx = 2 × 2

kx = 6 N



$\therefore$ Acceleration of 3 kg = $\frac{6}{3} = 2\ m/s^2$

36. (*d*) Weight of man in stationary lift is mg.



mg – N = ma [Newton's II law for man]

$\Rightarrow$ N =m (g – a)

Weight of man in moving lift is equal to N.

$$\Rightarrow \frac{mg}{m(g-a)} = \frac{3}{2} \Rightarrow a = \frac{g}{3}$$

37. (*a*) $F < f_{max}$



friction=F

For $F > f_{max}$

friction constant

38. (*a*) Monkey is moving up due to friction force

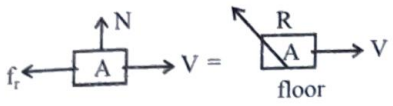


$f_r - mg = ma$

$f_r = m(a+g)$

towards up.

39. (*c*) Floor will provide the normal force and friction force the net reaction is provide by the floor is R.



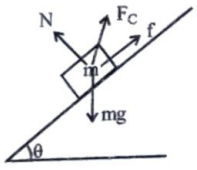
40. (*b*) $f_{max} > mg \sin\theta$

$\theta\uparrow$ $\sin\theta\uparrow$

at this condition block remains rest when

$mg \sin\theta > f_{max}$

sliping slant



For $\theta <$ angle of repose

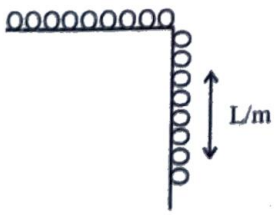
$F_c = mg$

For $\theta >$ angle of repose

as $\theta\uparrow$ $f = \mu mg \cos\theta\downarrow$

$N = mg \cos\theta\downarrow$

41. (*b*) $\mu\lambda L\left(1-\frac{1}{n}\right)g = \lambda\frac{L}{n}g$

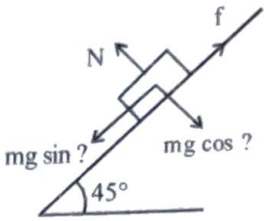


$\mu = \frac{1}{n-1}$

42. (*a*) $N - mg\cos\theta$

$f_s \le \mu N$

$mg \sin\theta \le \mu\, m\, g \cos\theta$

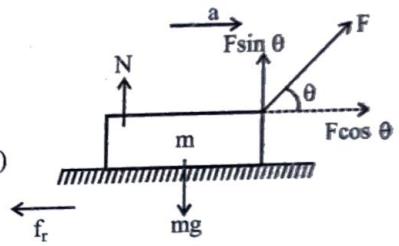


$\mu \ge 1$

43. (*a*)Friction not depend on surface Area so angle remain same.

$\therefore$ Angle = 30°

44. (*c*)



$F \sin\theta + N = mg$

or $N = mg - F \sin\theta$ ...(*a*)

$f_r = \mu N$ ...(*b*)

$F \cos\theta - f_r = ma$ ...(*c*)

on solving (*a*), (*b*) & (*c*)

$$a = \frac{F\cos\theta - \mu(mg - F\sin\theta)}{m}$$

$$a = \frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$$

45. (*a*) $V^2 = 2 \times g \sin\theta \times l$

$$\frac{v^2}{n^2} = 2\times(g\sin\theta - \mu g\cos\theta)\times l$$

$$\sin\theta\left(1-\frac{1}{n^2}\right) = \mu\cos\theta$$

$$\mu = \tan\theta\left(1-\frac{1}{n^2}\right)$$

Advanced Level Multiconcept Questions

1. (a,b,c) $F = \alpha t$

$$a = \frac{dv}{dt} = \frac{\alpha}{m} t \quad ....(i)$$

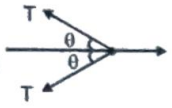
straight line curve 1 $dv = \frac{\alpha}{m} tdt$

$$v = \frac{\alpha}{m} \frac{t^2}{2} \text{ curve 2} \qquad ...(ii)$$

divide (ii) by

$$v = \frac{t}{2} a = \frac{a}{2} \times \frac{am}{\alpha} = \frac{a^2 m}{2\alpha} \rightarrow \text{Parabola curve 2.}$$

2. (ac)

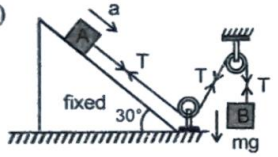


$F = 2T\cos\theta$ $T = \frac{F}{2\cos\theta}$

$\theta \uparrow$ $\cos\theta \downarrow$ $T \uparrow$

on increasing θ, $\cos\theta$ decreases and hence T increases.

3. (bd)



$T + mg\sin\theta = ma$ ....(a)

$mg - T = ma$ ....(b)

on solving (a) & (b) $a = \frac{3g}{4}$

$$T = \frac{3g}{4} = m\frac{mg}{4}$$

4. (a,b,d)



(a) $T = m_1 g < m_2 g$

$\therefore$ Acceleration of m_2 is $\downarrow$

(b) $T = m_2 g > m_1 g$

$\therefore$ acceleation of m_1 is $\uparrow$

(c) Masses is different

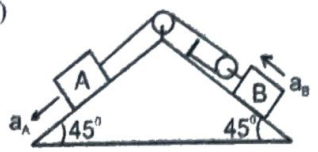
$\therefore$ Not possible

(d) $T - m_1 g = m_1 a$

$m_2 g - T = m_2 a$

on solving $a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$ Possible

5. (abd)



By string constraint

$a_A = 2a_B$ .....(a)

equation for block A.

$10 \times 10 \times \frac{1}{\sqrt{2}} - T = 10\, a_A$.....$(b)$

equation for block B.

$2T - \frac{400}{\sqrt{2}} = 40\, a_B$.....$(c)$

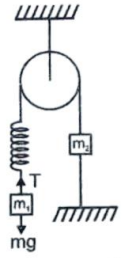
Solving equation (a), (b) & (c) we get

$$a_A = \frac{-5}{\sqrt{2}} \text{ m/s}^2$$

$$a_B = \frac{-5}{2\sqrt{2}} \text{ m/s}^2$$

$$T = \frac{150}{\sqrt{2}} \text{ N}$$

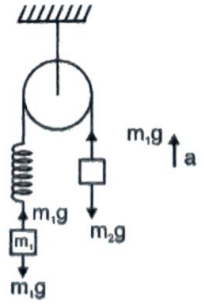
6. (ac)

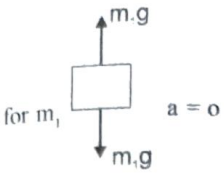


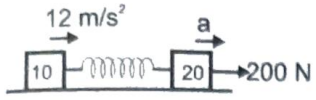
$T = m_1 g$

when thread is burnt, tension in spring remains same $= m_1 g$.

$m_1 g - m_2 g = m_2 a_1$ $\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g =$ a = upwards







7. (*bc*)

Apply NLM on the system

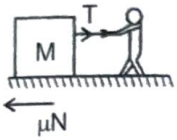
$200 = 20\,a + 12 \times 10$

$\frac{80}{20} = a = 4\ m/s^2$

spring Force $= 10 \times 12 = 120$ N

8. (*cd*) Pseudo force depends on acceleration of frame and mass of object

9. (*a,b,c*)

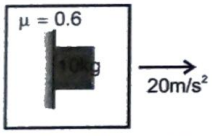


$T - \mu N = ma$

As $T \uparrow$ $ma\uparrow$

Can have tendency to move

10. (*a,b,c,d*)



(*a*) Acceleration of box = 20 m/s²

(when consider as system)

Force on Box

F = 200 N

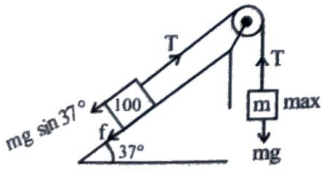
N = 200 N

$f_{max} = \mu N = 0.6 \times 200 = 120$ N

(*b*) $f_{required} = mg = 10 \times 10 = 100$ N

(*c*) $f_c = \sqrt{f^2 + N^2} = \sqrt{(100)^2 + (200)^2} = 100\sqrt{5}$ N

11. (*b,c*)



$T = mg$

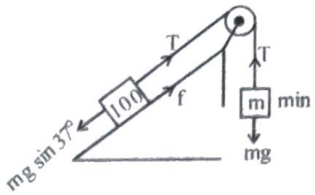
$T = 100\ mg \sin 37° + 0.3 \times 100\ g \cos 37°$

[Put g = 9.8]

$T = 588 + 235.2$

$mg = 823.2 \Rightarrow m = 82.33 = 83$ kg

(b)



$T + f = ma$

$T + 235.2 = 588$

$T = 588 - 235.2 = 352.8$

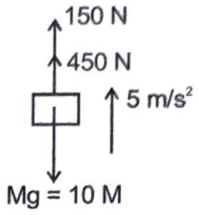
$m = 35.28$ kg

12. (*b*) FBD of Block in ground frame :

applying N.L.

$150 + 450 - 10\,M = 5M$

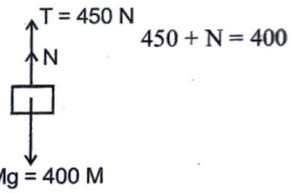
$\Rightarrow 15\,M = 600 \Rightarrow M = \frac{600}{15}$



$\Rightarrow$ M = **40 Kg Ans.**

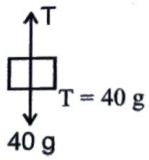
Normal on block is the reading of weighing machine i.e. 150 N.

13. (*d*) If lift is stopped & equilibrium is reached then



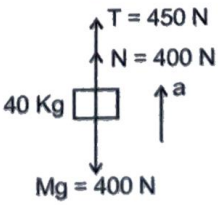
$\Rightarrow N = -50$

So block will lose the contact with weighing machine thus reading of weighing machine will be zero.



So reading of spring balance will be 40 Kg.

14. (*a*)



$a = \frac{950 - 400}{40} \Rightarrow a = \frac{450}{40} = \frac{45}{4}\ m/s^2$

15. (*c*) $a_P = \frac{10t}{10} = t$

$\therefore \quad \frac{dv}{dt} = t \Rightarrow \int_0^v dv = \int_0^t t\,dt \Rightarrow v = \frac{t^2}{2}$

Putting $v = 2$ we have $t = 2$ sec.

Now $\frac{dx}{dt} = \frac{t^2}{2} \therefore x_P = \left[\frac{t^3}{6}\right]_0^2 = \frac{4}{3}$

$x_B = 2 \times 2 = 4$ m

Hence relative displacement $= 4 - \frac{4}{3} = \frac{8}{3}$ m

16. (*b*) From above

$2t = \frac{t^3}{6} \Rightarrow t^2 = 12 \quad \Rightarrow t = 2\sqrt{3}$ sec.

17. (*c*) $a = t = 4 \therefore$ after 4 seconds $V_B = 2$ m/s

$V_P = \frac{4^2}{2} = 8$ m/s $\therefore V_{rel} = 8 - 2 = 6$ m/s.

18. (*a*) S_1 is accelerating frame so psuedo force act opposite to frame acceleration

F_{Pseudo} = mass of analyzing body × acceleration of frame

$= 2(-5\hat{i} - 10\hat{j}) = -10\hat{i} - 20\hat{j}$

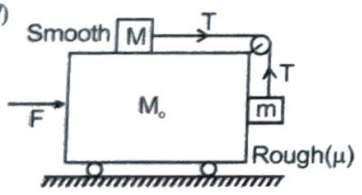
19. (*b*) S_2 is inertial frame

$F = ma$

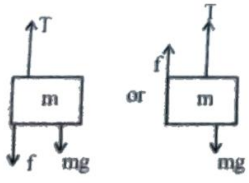
So $F = 10\hat{i} + 20\hat{j}$

20. (*a*) With respect to S_1 frame
Net force = zero.

21. (*a,d*)

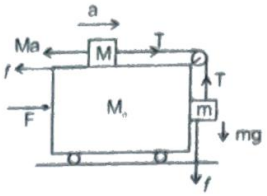


(*a*) When $F = 0$
No frictin b/w m & M_0 so system will move.
(*b*) When F is applied then friction develop a range for which M and m are stationary w.r.t M_0, such that



(*c*) Limiting friction between M_0 & m is μ ma
$\therefore$ Dependent on a
(*d*) When Pseudo acts on M is equal to T, then $f = 0$

22. (*b,c*)



Use Pseudo concept
$T = Ma$ ...(*a*)
$T = f + mg$
$\Rightarrow T = \mu ma + ma$ ...(*b*)
On using (*a*) & (*b*)
$Ma = \mu ma + mg$

23. (*a*) p,r (*b*) p, r (*c*) q,s (*d*) q,r

(*a*) Let the horizontal component of velocity be u_x. Then between the two instants (time interval T) the projectile is at same height, the net displacement ($u_x T$) is horizontal

$\therefore$ average velocity $= \frac{u_x T}{T} = u_x \Rightarrow (a)$ p, r

(*b*) Let $\hat{i}$ and $\hat{j}$ be unit vectors in direction of east and north respectively.

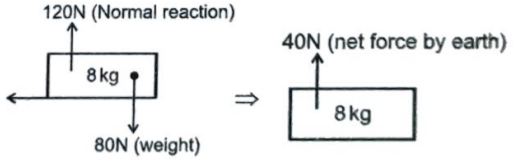
$\therefore \quad \vec{V}_{DC} = 20\hat{j}, \ \vec{V}_{BC} = 20\hat{i}$ and $\vec{V}_{BA} = -20\hat{j}$

$\therefore \quad -\vec{V}_{AD} = \vec{V}_{DC} + \vec{V}_{CB} + \vec{V}_{BA} = 20\hat{j} - 20\hat{i} - 20\hat{j}$

$= -20\hat{i}$

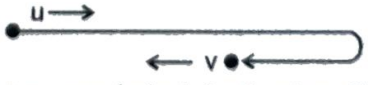
$\therefore \quad \vec{V}_{AD} = 20\hat{i}$ Hence $\vec{V}_{AD} = \vec{V}_{BC} \Rightarrow (b)$ p, r

(*c*) Net force exerted by earth on block of mass 8 kg is shown in FBD and normal reaction exerted by 8 kg block on earth is 120 N downwards.



Hence both forces in the statement are different in magnitude and opposite in direction. $\Rightarrow (c)$ q,s

(*d*) For magnitude of displacement to be less than distance, the particle should turn back. Since the magnitude of final velocity (v) is less than magnitude of initial velocity (u), the nature of motion is as shown.



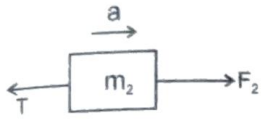
$\therefore$ Average velocity is in direction of initial velocity and magnitude of average velocity $= \frac{u - v}{2}$ is less than u because $v < u$. $\Rightarrow (c)$ q, r

24. (a) q (b) r (c) q (d) r

Let a be acceleration of two block system towards right

$\therefore a = \dfrac{F_2 - F_1}{m_1 + m_2}$

The F.B.D. of m_2 is



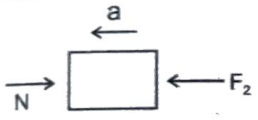
$\therefore F_2 - T = m_2 a$

Solving $T = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} + \dfrac{F_1}{m_1}\right)$

(b) Replace F_1 by $-F_1$ is result of A

$\therefore T = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} - \dfrac{F_1}{m_1}\right)$

(c) Let a be acceleration of two block system towards left



$\therefore a = \dfrac{F_2 - F_1}{m_1 + m_2}$

The FBD of m_2 is

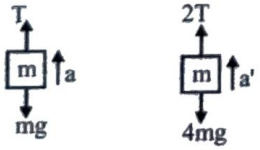
$\therefore F_2 - N_2 = m_2 a$

Solving $N = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_1}{m_1} + \dfrac{F_2}{m_2}\right)$

(d) Replace F_1 by $-F_1$ in result of C

$N = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} - \dfrac{F_1}{m_1}\right)$

25. [0005] $T - mg = ma$ ...(i)



$4mg - 2T = 4ma' \Rightarrow 2mg - T = 2ma'$ ...(ii)

constraint relation : $Ta - 2Ta' \Rightarrow a' = a/2$

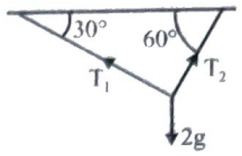
$\therefore$ from equation (ii) we get

$2mg - T = 2m\,(a/2)$ ...(iii)

solving (i) and (ii), we get

$a = \dfrac{g}{2} = 5\ m/s^2$

26. [0010] $T_1 \sin 30° + T_2 \sin 60° = 20$



$T_1 \cos 30° = T_2 \cos 60°$

$T_2 = T_1\sqrt{3} \qquad \Rightarrow \dfrac{4T_1}{2} = 20$

$T_1 = 10\ N$

27. [0050] $100 = k \times (40 - \ell_0)$

$200 = k \times (60 - \ell_0)$

$2 = \dfrac{60 - \ell_0}{40 - \ell_0}$

$80 - 2\ell_0 = 60 - \ell_0$

$20 = \ell_0 \qquad \Rightarrow \qquad 20k = 100$

$\Rightarrow k = 5$

$x = k \times (30 - 20) = 50\ N$

28. [0360] $a = \dfrac{90 - 50}{5} = 8$, after 2 sec

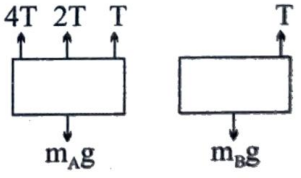
$S = 16m,\ V = 16\ m/s$

3 kg part hits the ground $t = 4$ sec.

remaining 2 kg has $a = 35$

$S_{rel} = 360$

29. [0070]



$7T = m_A g \qquad T = m_B g$

$\therefore m_A = 7m_B = 70$

30. [0270] $F = 2 \times 75 \cos 37° + 150$

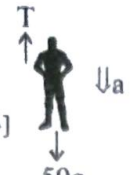
$= 150 \times \dfrac{4}{5} + 150 = 270$

31. [0880] $m = 16,\ M = 80,\ x = 6,\ L = 12$

$T = \left(\dfrac{mx}{L} + M\right)g$

$\Rightarrow \left[\dfrac{16 \times 6}{12} + 80\right] \times 10 = 880\ N$

32 [4]



$50g - T = 50a$

$$a = 10 - \frac{300}{50} = 4\ m/s^2$$

33. [0002]

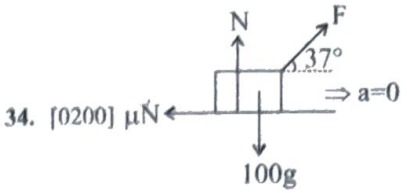
$a_{max} = \mu g = 0.2 \times 10 = 2 m/sec^2$

$v = u + at$

$0 = 4 - 2t$

$t = 2 sec$

34. [0200]



$F \cos 37° = \mu N$

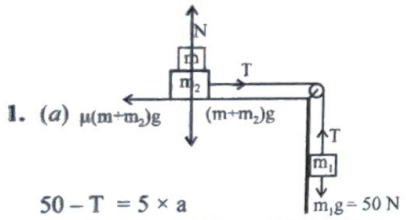
$F \sin 37° + N = 100\ g$

$$F = \frac{\mu mg}{\cos\theta + \mu \sin\theta} = 200\ N$$

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*a*)



$50 - T = 5 \times a$

$T - 0.15\ (m + 10)\ g = (10 + m)a$

$a = 0$ for rest

$50 = 0.15\ (m + 10)\ 10$

$$5 = \frac{3}{20}(m + 10)$$

$$\frac{100}{3} = (m + 10)$$

$m = 23.3$ kg

2. (*d*) $T \cos 45° = mg$

$T \sin 45° = mg$

$\Rightarrow F = mg\ 100$ N.

3. (*a*) For equilibrium of the block net force be zero. Hence we can write.

$mg \sin\theta + 3 = P + \text{friction}$

$mg \sin\theta + 3 = P + \mu mg \cos\theta$

After solving , we get, $P = 32$ N.

4. (*a*) $mg \sin\theta + 2 = \mu\ mg \cos\theta$ ...(*a*)

$10 - mg \sin\theta = \mu\ mg \cos\theta$ ...(*b*)

On adding (*a*) + (*b*)

$$12 = 2\mu mg\frac{\sqrt{3}}{2};\quad \mu mg = \frac{12}{\sqrt{3}}$$

On (*a*) – (*b*) :

$$8 = 2mg \times \frac{1}{2}; mg = 8;\ \mu = \frac{\sqrt{3}}{2}$$

5. (*b*) $-(g + \gamma v^2) = \frac{dv}{dt}$

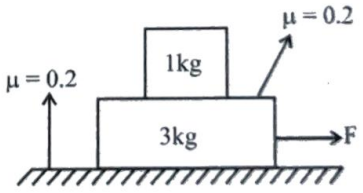
$$-gdt = \frac{g}{\gamma}\left(\frac{dv}{\frac{g}{\gamma} + v^2}\right)$$

Intergrating $0 \to t$ & $V_0 \to 0$:-

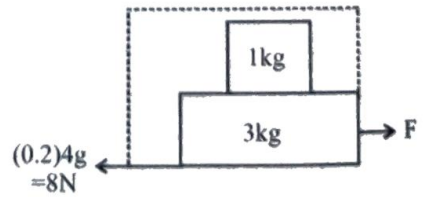
$$-gt = -\sqrt{\frac{g}{\gamma}}\tan^{-1}\left(\frac{V_0}{\sqrt{\frac{g}{\gamma}}}\right)$$

$$t = \frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$$

6. (*a*)



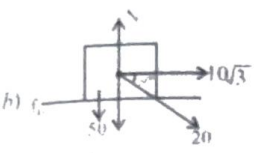
$a_{Amax} = \mu g = 2\ m/s^2$

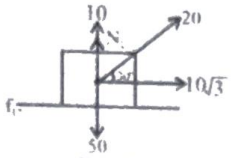


$F - 8 = 4 \times 2$

$F = 16$ N

7. (b)





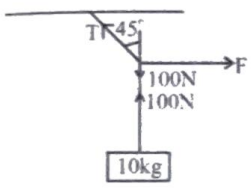
$N_1 = 60$ $\qquad N_2 = 40$

$$a_1 = \frac{10\sqrt{3} - 0.2\times 60}{5}$$

$$a_2 = \frac{10\sqrt{3} - 0.2\times 40}{5}$$

$a_1 - a_2 = 0.8$

8. (c)

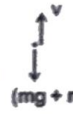


$$\frac{T}{\sqrt{2}} = 100$$

$$\frac{T}{\sqrt{2}} = F$$

$F = 100$ N

9. (c)



$|a| = g + kv^2$

$$\Rightarrow -\frac{vdv}{dh} = g + kv^2$$

$$\Rightarrow \int_u^0 \frac{vdv}{g+kv^2} = \int_0^{H_{max}} -dh$$

On solving

$$H_{max} = \frac{1}{2K}\ln\left(1+\frac{ku^2}{g}\right)$$

10. [346] For upward motion



$$\frac{1}{2}mv_0^2 = \left(mg\frac{S}{2} + \mu mg\frac{\sqrt{3}}{2}S\right)$$

$$\frac{1}{2}\frac{mv_0^2}{4} = \left(mg\frac{S}{2} - \mu mg\frac{\sqrt{3}}{2}S\right)$$

For downward

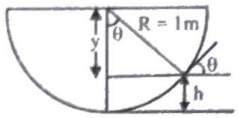
$$4 = \frac{(1+\mu\sqrt{3})}{(1-\mu\sqrt{3})}$$

$\Rightarrow 4 - 4\mu\sqrt{3} = 1 + \mu\sqrt{3}$

$\Rightarrow 3 = 5\mu\sqrt{3} \quad \therefore \mu = \frac{\sqrt{3}}{5} \approx \frac{346.41}{1000}$

$I = 346$

11. (c)



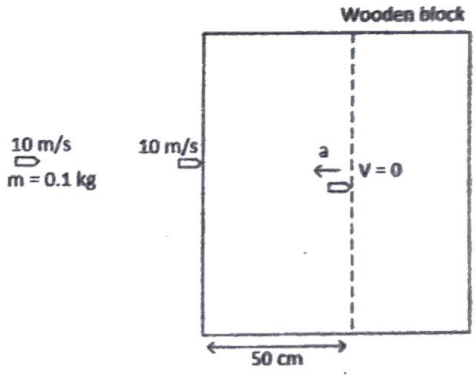
$\mu = \tan\theta$

$$\Rightarrow \frac{3}{4} = \tan\theta$$

$\Rightarrow \theta = 37°$

$$\therefore h = R - R\cos\theta = 1 - 1\times\frac{4}{5} = 0.2 \text{ m}$$

12. [10]



$M_{bullet} = 0.1\text{kg}$

$v^2 = 4^4 + 2(-a)\times 5$

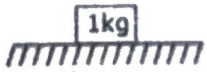
$\Rightarrow 0 = (10)^2 - 2a\times(0.5)$

Retardation $(a) = \frac{1000}{2\times 5} = 100\,\text{m/s}^2$

Retarding force $(F) = ma = 0.1\times 100$

$F_R = 10\,\text{N}$

13. [5]



$$\mu = \frac{1}{\sqrt{3}}$$

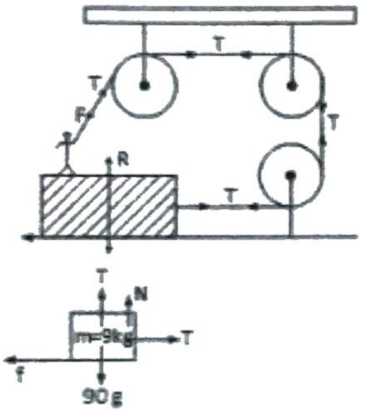
Minimum possible force

$$\Rightarrow F = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$F_{min} = \frac{1}{\sqrt{1+\frac{1}{3}}}$$

$$F_{min} = 5\,N3$$

14. [30]



$\because f = T$

$\Rightarrow \mu N = T$

$\Rightarrow \mu(90 - T) = T$

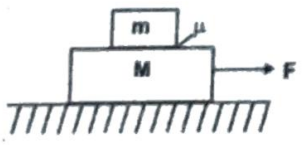
$\Rightarrow 0.5(90 - T) = T$

$\Rightarrow 90 - T = 2T$

$\Rightarrow 3T = 90$

$\Rightarrow T = 30\,N$

15. [21]



$$a = \frac{F}{M+m}$$

$$f = ma = m\frac{F}{M+m}$$

$$m\frac{F}{M+m} \le \mu mg \text{ (for no slipping)}$$

$\Rightarrow F \le \mu(m+M)g$

$$\therefore F_{max} = \frac{3}{7}(0.5+4.5)\times 9.8 = 21\,N$$

16. [12]

$\vec{F} = (2\hat{i}+3\hat{j}+5\hat{k})N$ time $= 4sec$ As body start from rest therefore position vector initially $\vec{r_i} = (0\hat{i}+0\hat{j}+0\hat{k})$ & u (initial velocity) $= 0$ given, $r_f = (x\hat{i}+y\hat{j}+z\hat{k})$ position vector initially $\vec{r_i} = (0\hat{i}+0\hat{j}+0\hat{k})$ u (initial velocity) $= 0$

given, $r_f = (x\hat{i}+y\hat{j}+z\hat{k})$

Now, from second equation of motion

$$\$\ \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \Rightarrow r_f - r_i = \frac{1}{2}\times\left(\frac{2\hat{i}+3\hat{j}+5\hat{k}}{2}\right)\times(4)^2$$

$$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}) - (0\hat{i}+0\hat{j}+0\hat{k})$$

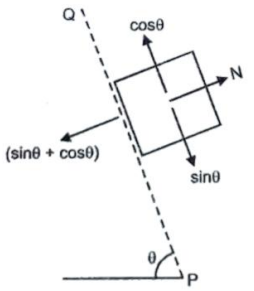
$$= 8\hat{i}+12\hat{j}+20\hat{k}$$

$$\Rightarrow x\hat{i}+y\hat{j}+z\hat{k} = 8\hat{i}+12\hat{j}+20\hat{k}$$

$\therefore$ The value of $b = 12$

JEE-ADVANCED PREVIOUS YEAR'S

1. (a,c)



f = 0, If $\sin\theta = \cos\theta \Rightarrow \theta = 45°$

f towards Q, $\sin\theta > \cos\theta \Rightarrow \theta > 45°$

f towards P, $\sin\theta < \cos\theta \Rightarrow \theta < 45°$

2. (d) Block will not slip if

$$(m_1 + m_2)\,g\sin\theta \le \mu\, m_2\, g\cos\theta$$

$$3\sin\theta \le \left(\frac{3}{10}\right)(b)\cos\theta$$

$$\tan\theta \le \frac{1}{5} \quad \Rightarrow \boxed{\theta \le 11.5°}$$

(P) $\theta = 5°$ friction is static $f = (m_1 + m_2)g\sin\theta$

(Q) $\theta = 10°$ friction is static $f = (m_1 + m_2)g\sin\theta$

(R) $\theta = 15°$ friction is kinetic $f = \mu m_2 g\cos\theta$

(S) $\theta = 20°$ friction is kinetic $f = \mu m_2 g\cos\theta$

3. (a,b,d) $\dfrac{dk}{dt} = \gamma t$ as $k = \dfrac{1}{2}mv^2$

$$\therefore \frac{dk}{dt} = mv\frac{dv}{dt} = \gamma t$$

$$\therefore m\int_0^v vdv = \gamma\int_0^t tdt$$

$$\frac{mv^2}{2} = \frac{\gamma t^2}{2}$$

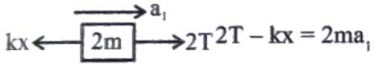
$$v = \sqrt{\frac{\gamma}{m}}t \qquad ...(i)$$

$$a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}} = \text{constant}$$

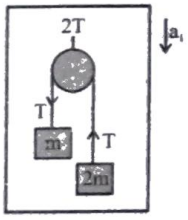
since $F = ma$

$$\therefore F = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m} = \text{constant}$$

4. (c)



$2T - kx = 2ma_1$



$$T = \frac{2(2m)(m)}{3m}(g - a_1)$$

$$= \frac{4m}{3}(g - a_1)$$

$$\frac{8m}{3}(g - a_1) - kx = 2ma_1$$

$$\frac{8Mg}{3} - \frac{8ma_1}{3} - kx = 2ma_1$$

$$\frac{8Mg}{3} - kx = \frac{14ma_1}{3}$$

$$\frac{8Mg - 3kx}{14m} = a_1$$

$$a_1 = \frac{8mg - 3kx}{14m}$$

$$\frac{vdv}{dx} = \left(\frac{8Mg}{14m} - \frac{3kx}{14m}\right)$$

$$\int vdv = \frac{1}{14m}\int(8Mg - 3kx)dx$$

for max elongation

$$0 = \frac{1}{14m}\int_0^{x_0}(8mg - 3kx)dx$$

$$= \frac{1}{14m}\left(8Mgx_0 - \frac{3kx_0^2}{2}\right)$$

$$8Mgx_0 = \frac{3kx_0^2}{2}$$

$$x_0 = \frac{16Mg}{3k}$$

at $x = \dfrac{x_0}{2}$

$$\int_0^v vdv = \frac{1}{14m}\int_0^{x_0/2}(8Mg - 3kx)dx$$

$$\frac{v^2}{2} = \frac{1}{14m}\left(\frac{8Mgx_0}{2} - \frac{3kx_0^2}{2\times 4}\right)$$

$$v^2 = \frac{1}{7m}\left(\frac{8Mg}{2}\times\frac{16Mg}{3x} - \frac{3x}{8}\times\frac{16M^2g^2}{3x\times 3x}\right)$$

$$= \frac{1}{7m}\left(\frac{64M^2g^2}{3x} - \frac{2M^2g^2}{3x}\right)$$

$$v^2 = \frac{62Mg^2}{21k}$$

For acc. $2a_1 = a_2 + a_3$ therefore

$a_2 - a_1 = a_1 - a_3$

$$a_1 = \frac{8Mg - 3kx_0/4}{14m}$$

$$= \frac{8g}{14} - \frac{3kx_0}{14m\times 4}$$

$$= \frac{8g}{14} - \frac{3x}{14m\times 4}\times\frac{16Mg}{3x}$$

$$= \frac{8g}{14} - \frac{4g}{14}$$

$$= \frac{4g}{14} = \frac{2g}{7}$$

OR

$$\frac{8mg}{3} - \frac{8m}{3}a_1 - kx = 2ma_1$$

$$\frac{14m}{3}a_1 = -k\left[x - \frac{8mg}{3k}\right]$$

$$a_1 = -\frac{3k}{14m}\left[x - \frac{8mg}{3k}\right] \quad ...(i)$$

that means, block 2m (connected with the spring) will per form SHM about $x_1 = \frac{8mg}{3k}$ therefore.

maximum elongation in the spring $x_0 = 2x_1 = \frac{16mg}{3k}$

on comparing equation (*a*) with
$a = -\omega^2(x - x_0)$

$$\omega = \sqrt{\frac{3k}{14m}}$$

at $\left(\frac{x_0}{2}\right)$, block will be passing through its mean position therefore at mean position

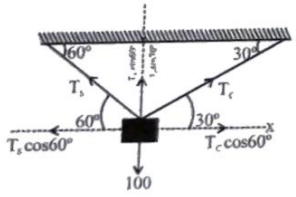
$$v_0 = A\omega = \frac{8mg}{3k}\sqrt{\frac{3k}{14m}}$$

At. $\frac{x_0}{4} \Rightarrow x = \frac{A}{2}$

$$\therefore a_{cc} = -\frac{A}{2}\omega^2$$

$$= -\frac{4mg}{3k}\cdot\frac{3h}{14m} = \frac{2g}{7}$$

5. (2.00) Let T_s = tension in steel wire
T_C = Tension in copper wire in x direction



$$T_C \cos30° = T_S \cos60°$$

$$T_C \times \frac{\sqrt{3}}{2} = T_S \times \frac{1}{2}$$

$$\sqrt{3}\,T_C = T_S \quad ...(i)$$

in y direction

$$T_C \sin30° + T_S \sin60° = 100$$

$$\frac{T_C}{2} + \frac{T_S\sqrt{3}}{2} = 100 \quad ...(ii)$$

Solving equation (i) & (ii)
$T_C = 50$ N

$T_S = 50\sqrt{3}$ N
We know

$$\Delta L = \frac{FL}{AY} = \frac{\Delta L_C}{\Delta L_S} = \frac{T_C L_C}{A_C Y_C} \times \frac{A_S Y_S}{T_S L_S}$$

On solving above equation

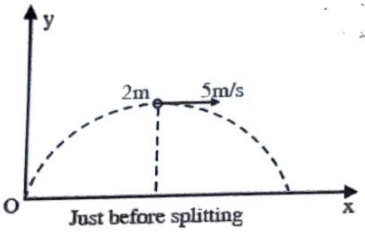
$$\frac{\Delta L_C}{\Delta L_S} = 2$$

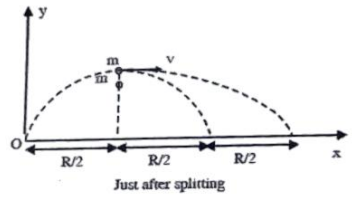
2.00

6. (0.50) and **7.** (7.50)

$$\text{Range } R = \frac{2u_x u_y}{g} = \frac{2\times5\times5}{10} = 5\,m$$

$$\text{Time of flight } T = \frac{2u_y}{g} = \frac{2\times5}{10} = 1\,sec$$





$\because$ Time of motion of one part falling vertically downwards is $= 0.5\text{sec} = \frac{T}{2}$

$\Rightarrow$ Time of motion of another part, $t = \frac{T}{2} = 0.5\text{sec}$

From momentum conservation $\Rightarrow P_i = P_f$

$2m\times5 = m\times v$

$v = 10\,m/s$

Displacement of other part in 0.5sec in horizontal direction $= v\frac{T}{2}$

$= 10\times0.5 = 5\,m = R$

Total distance of second part from point 'o' is,

$$x = \frac{3R}{2} = 3\times\frac{5}{2}$$

$x = 7.5\,m$

$\Rightarrow t = 0.5\text{sec}$

6 WORK, ENERGY, POWER AND COLLISIONS

t.me/realprayasians

WORK

Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

Work is the product of the applied force and the displacement of the body in the direction of force.

$W = F\,s\cos\theta = \vec{F}.\vec{s}$

If $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$

Then $W = \vec{F}.\vec{s} = F_x x + F_y y + F_z z$

NOTE:

(i) **Dimension :** $M^1L^2T^{-2}$

(ii) **SI Unit :** joule C.G.S. Unit : erg

1 joule = 10^7 erg

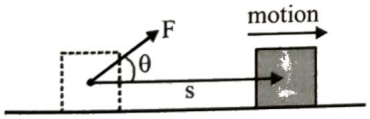
(iii) **Positive work :**

$W = Fs\cos\theta$

If the angle θ is acute ($\theta < 90°$) then the work is said to be positive.

The positive work signifies that the external force favours the motion of the body.

$\theta < 90°$ Positive work



(a) When a body falls freely under the action of gravity ($\theta = 0°$), the work done by gravity is positive.



Positive work done by gravitational force

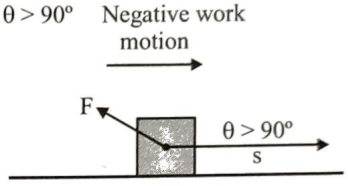
(b) When a spring is stretched, stretching force and the displacement both are in the same direction. So work done by stretching force is positive.

(iv) **Negative work :**

If the angle θ is obtuse ($\theta > 90°$).

Then the work is said to be negative.

$\theta > 90°$ Negative work



It signifies that the direction of force is such that it opposes the motion of the body.

(a) Work done by frictional force is negative when it opposes the motion.



Negative work done by gravitational force

(b) Work done by breaking force on the car is negative.

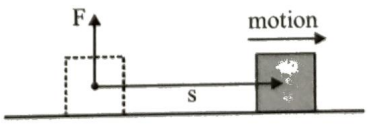
(v) **Zero work :**

$W = Fs\cos\theta$

Work done will be zero if

$F = 0$ or $s = 0$ or $\theta = 90°$

$\theta = 90°$ Work done is zero



(a) Work done by coolie is zero (against gravity).

(b) In circular motion work done by centripetal force is zero.

(c) When $\theta = 0$: A force does maximum positive work.

(d) When $\theta = 180$ force does maximum negative work.

(vi) Work done by a variable force :

When magnitude and direction of the force varies with position, the work done by force for infinitesimal displacement ds is $dW = \vec{F}.d\vec{s}$

The total work done for dispacement from A to B is W_{AB}

$$= \int_A^B \vec{F}.d\vec{s} = \int_A^B (F\cos\theta)\,ds$$

In terms of rectangular components

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}, \quad d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W_{AB} = \int_A^B \left(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right)\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

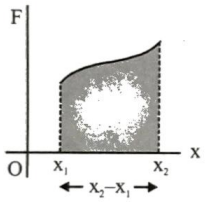
$$= \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

(vii) Work done by several Forces :

When several forces act on a body then the net work done on the body is the algebraic sum of work done by individual forces.

$$W_{net} = \vec{F}_1.\vec{s}_1 + \vec{F}_2.\vec{s}_2 + \ldots\ldots \vec{F}_n.\vec{s}_n$$

here $\vec{s}_1, \vec{s}_2, \ldots\ldots \vec{s}_n$ are the displacement of points of application of forces $\vec{F}_1, \vec{F}_2, \ldots\ldots \vec{F}_n$ respectively.



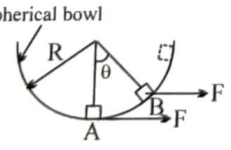
If a graph is constructed of the component F cos θ of a variable force, then the work done by the force can be determined by measuring the area between the curve and the displacement axis.

NOTE: Displacement depends on referance frame so work done by a force is referance frame dependent. Work done by a force can be different in different reference frame.

TRAIN YOUR BRAIN

Example 1: A block of mass m is taken from A to B along spherical bowl.

Work Done by gravity = $-mgR(1-\cos\theta)$



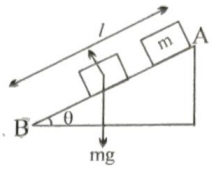
Work Done by force F = $FR(\sin\theta)$

Work Done by normal = 0

TRAIN YOUR BRAIN

Example 2: Find work done by gravity and normal when block comes from A to B

$W_g = mg(l\sin\theta) = (mg\sin\theta)l$



$W_N = 0$

(*because displacement is perpendicular to Force*)

Units of work:

I. In cgs system, the unit of work is erg. One erg of work is said to be done when a force of one dyne displaces a body through one centimetre in its own direction.

$\therefore$ 1 erg = 1 dyne × 1 cm = 1g cm s^{-2} × 1 cm = 1 g cm^2 s^{-2}

Note, Erg is also called dyne centimetre.

II. In SI i.e., International System of units, the unit of work is joule (abbreviated as J). It is named after the famous British physicist James Prescott Joule (1818 – 1869). One joule of work is said to be done when a force of one Newton displaces a body through one metre in its own direction.

1 joule = 1 Newton × 1 metre = 1 kg × 1 m/s^2 × 1 m = 1kg m^2 s^{-2}

Note Another name for joule is Newton metre.

Relation between joule and erg

1 joule = 1 Newton × 1 metre

1 joule = 10^5 dyne × 10^2 cm = 10^7 dyne cm

1 joule = 10^7 erg

1 erg = 10^{-7} joule

Dimensions of Work:

[Work] = [Force] [Distance] = $[MLT^{-2}]$ [L] = $[ML^2T^{-2}]$

Work has one dimension in mass, two dimensions in length and '–2' dimensions in time,

On the basis of dimensional formula,

the unit of work is kg m^2 s^{-2}.

Note that 1 kg m^2 s^{-2} = (1 kg m s^{-2}) m = 1 N m = 1 J.

WORK DONE BY MULTIPLE FORCES

If several forces act on a particle, then we can replace $\vec{F}$ in equation $W = \vec{F}\,.\,\vec{S}$ by the net force $\Sigma\vec{F}$ where

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots..$$

$\therefore$ $W = [\Sigma\vec{F}]\,.\,\vec{S}$ ...(i)

This gives the work done by the net force during a displacement $\vec{S}$ of the particle.

We can rewrite equation (i) as :

$W = \vec{F}_1 \cdot \vec{S} + \vec{F}_2 \cdot \vec{S} + \vec{F}_3 \cdot \vec{S} + ...$

or $W = W_1 + W_2 + W_3 + ...$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.

TRAIN YOUR BRAIN

Example 3: There is an elastic ball and a rigid wall. Ball is thrown towards the wall. The work done by the normal reaction exerted by the wall on the ball is -

(*a*) +ve(*b*) – ve

(*c*) zero (*d*) None of these

Sol. (c) As the point of application of force does not move, the work done by normal reaction is zero.

Example 4: Work done by the normal reaction when a person climbs up the stairs is -

(*a*) +ve(*b*) – ve

(*c*) zero (*d*) None of these

Sol. (c) As the point of application of force does not move, the work done by normal reaction is zero.

Example 5: A particle is displaced from point A (1, 2) to B(3, 4) by applying force $\vec{F} = 2\hat{i} + 3\hat{j}$. Find the work done by $\vec{F}$ to move the particle from point A to B.

Sol. $W = \vec{F} \cdot \Delta\vec{S}$



$\Delta S = (3-1)\hat{i} + (4-2)\hat{j}$

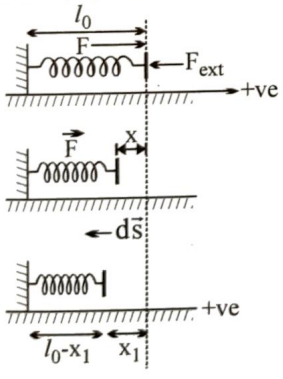
$W = (2\hat{i} + 3\hat{j}) \cdot (2\hat{i} + 2\hat{j})$

$= 2 \times 2 + 3 \times 2 = 10$ units

SPRING FORCE

Natural length of spring is l_0.

When we **compress** spring by x_1 from natural length, then work done by spring force.



$\vec{F} = kx\hat{i}$

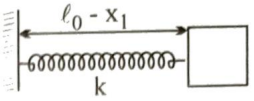
$d\vec{S} = (dx)(-\hat{i})$ {dx is +ve as x is increasing}

$dW = \vec{F}\, d\vec{S}$

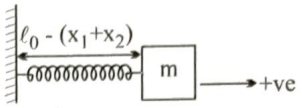
$$\int dW = -\int_0^{x_1} kx\, dx = -\frac{1}{2} k x_1^2$$

TRAIN YOUR BRAIN

Example 6: Find work done by spring if we compress it further by x_2.



Sol. $\vec{F} = kx\hat{i}$ $\quad d\vec{S} = dx(-\hat{i})$



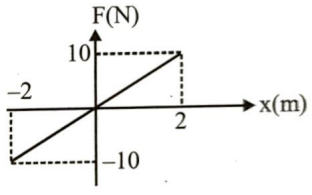
$dW = \vec{F}\, d\vec{S} = -kx\, dx$

$$W = -k \int_{x_1}^{(x_2+x_1)} x\, dx = -\frac{1}{2} k[(x_2 + x_1)^2 - x_1^2]$$

Example 7: A force F acting an a particle varies with the position x as shown in figure. Find the total work done by this force in displacing the particle from

(*a*) x = – 2 m to x = 0

(*b*) x = 0 to x = 2 m



Sol. (*a*) $W = -\frac{1}{2}(2)\,(10) = -10$ J

(*b*) $W = \frac{1}{2}(2)\,(10) = 10$ J

Example 8: A spring of force constant 100 N/m is stretched upto 5 cm. Find out work done.

Sol. Spring is stretched by length $l = 5$ cm $= 0.05$ m

Work done for small displacement dx is $dW = Fdx = kx\, dx$

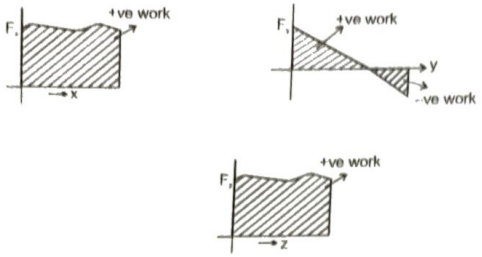
Total work done for length l is

$$W = \int_0^l F\, dx = \int_0^l kx\, dx = \frac{1}{2} kl^2 = \frac{1}{2} 100\,(0.05)^2$$

$= 0.125$ J

AREA UNDER THE FORCE DISPLACEMENT CURVE

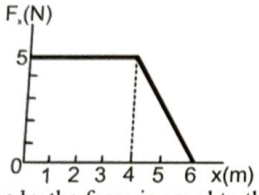
Graphically area under the force-displacement is the work done



The work done can be positive or negative as per the area above the x-axis or below the x-axis respectively.

TRAIN YOUR BRAIN

Example 9: Force acting on a particle varies with x as shown in figure. Calculate the work done by the force as the particle moves from x = 0 to x = 6.0 m.



Sol. The work done by the force is equal to the area under the curve from
x = 0 to x = 6.0 m.

This area is equal to the area of the rectangular section from x = 0 to x = 4.0 m plus the area of the triangular section from x = 4.0 m to x = 6.0 m. The area of the rectangle is (4.0) (5.0) N.m = 20 J, and the area of the triangle is $\frac{1}{2}$ (2.0), (5.0) N.m = 5.0 J. Therefore, the total work done is 25 J.

POWER

Rate of work done by a force is called power delivered by the force.

$$\text{Average power} = \frac{\text{work done}}{\text{time}} \text{ or } \quad P = \frac{W}{t}$$

W is the amount of the work done in total time t.

$$\text{Instantaneous Power (P)} = \frac{dW}{dt} \text{ or } \quad P = \frac{\vec{F}.d\vec{s}}{dt}$$

$(\because\ dW = \vec{F}.d\vec{s}\ )$

$$\text{or } \quad P = \vec{F}.\frac{d\vec{s}}{dt} \qquad \text{or } \quad P = \vec{F}.\vec{v}$$

NOTE :

(i) Dimensions : $[M^1L^2T^{-3}]\left(\text{from } P = \frac{W}{t}\right)$

(ii) *SI* unit : watt = $\frac{\text{joule}}{\text{second}}$

(iii) Bigger Unit of power :
1 kilo watt (kW) = 1000 watt = 10^3 W
1 maga watt = 1000, 000 watt or 1MW = 10^3kW
1 horse power (h.p.) = 746 W = 550 ft. lb/s

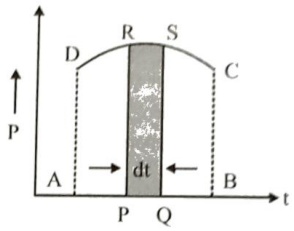
FPS Unit : $\frac{(\text{foot} \times \text{pound})}{\text{second}}$, 1W = 0.738 ft. lb/s

(iv) When an agent delivers power at a uniform rate, the average power is equal to the instantaneous power.

(v) The slope of work – time graph gives the instantaneous power

$$\text{slope} = \tan\theta = \frac{dW}{dt} = P \text{ (instantaneous power)}$$

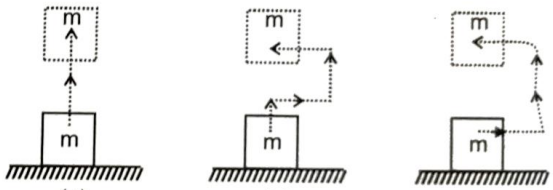
(vi) Area under power – time graph gives the work done.
Area under P-t graph = Area ABCD = work done from A to B.



Area under power – time graph = $\int Pdt = W$

(1) CONSERVATIVE FORCE :

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and does not depend on the nature of path followed between the initial and final positions.



Consider a body of mass m being raised to a height h vertically upwards as shown in above figure. The work done is mgh. Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the paths, we get the result mgh once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal path is zero. The work done along the vertical parts add up to mgh. Thus we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the intial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative forces.

(i) Gravitational force, not only due to the Earth, but due in its general form as given by the universal law of gravitation, is a conservative force.

(ii) Elastic force in a stretched or compressed spring is a conservative force.

(iii) Electrostatic force between two electric charges is a conservative force.

(iv) Magnetic force between two magnetic poles is a conservative force.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and Electrosatic forces are two important examples of central forces. Central forces are conservative forces.

Properties of Conservative forces

- **Work done by or against a conservative force depends only on the initial and the final position of the body.**
- Work done by or against a conservative force does not depend upon the nature of the path between initial and final position of the body.
- Work done by or against a conservative force in a round trip is zero.

  If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise it is non-conservative.

  The concept of potential energy exists only in the case of conservative forces.
- The work done by a conservative force is completely recoverable.

  Complete recoverability is an important aspect of the work done by a conservative force.

(2) NON-CONSERVATIVE FORCES :

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by fricitional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force etc., are non conservative forces.

Difference between conservative and Non-conservative forces

S. No.	Conservative forces	Non-Conservative forces
1	Work done does not depend upon path	Work done depends on path.
2	Work done in a round trip is zero.	Work done in a round trip is not zero.
3	Central in nature.	Forces are velocity-dependent and retarding in nature.
4	When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5	Work done is completely recoverable.	Work done in not completely recoverable.

ENERGY

A body is said to possess energy if it has the capacity to do work. When a body possessing energy does some work, part of its energy is used up. Conversely if some work is done upon an object, the object will be given some energy. Energy and work are mutually convertible.

There are various forms of energy. Heat, electricity, light, sound and chemical energy are all familiar forms. In studying mechanics, we are however concerned chiefly with mechanical energy. This type of energy is a property of configuration or position.

(1) Kinetic Energy

Kinetic energy (K.E.), is the capacity of a body to do work by virtue of its motion.

If a body of mass m has velocity v, its kinetic energy is equivalent to the work, which an external force would have to do to bring the body from rest up to its velocity v. The numerical value of the kinetic energy can be calculated from the formula

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{m\vec{v}.\vec{v}}{2} \qquad ...(8)$$

- Since both m and v^2 are always positive, K.E. is always positive and does not depend upon the direction of motion of the body.

(2) Potential Energy

Potential energy is energy of the body by virtue of its position. A body is capable to do work by virtue of its position, configuration or state of strain.

Now relation between Potential energy and work done is

$W.D = -\Delta U$

where ΔU is change in potential energy

There are two common forms of potential energy, gravitational and elastic.

Important points related to Potential energy:

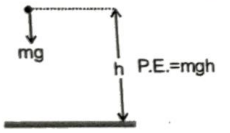
1. Potential energy is a straight function (defined only for position)
2. Potential energy of a point depends on a reference point
3. Potential energy difference between two position doesn't depend on the frame of reference.
4. Potential energy is defined only for conservative force because work done by conservative force is path independent.
5. If we define Potential energy for non conservative force then we have to define P.E. of a single point through different path which gives different value of P.E. at single point, this doesn't make any sense.

(a) Gravitational Potential Energy (GPE):

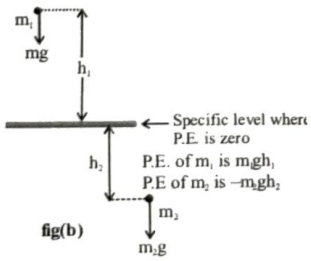
It is possessed by virtue of height.

When an object is allowed to fall from one level to a lower level it gains speed due to gravitational pull, i.e., it gains kinetic energy. Therefore, in possessing height, a body has the ability to convert its gravitational potential energy into kinetic energy. The gravitational potential energy is equivalent to the negative of the amount of work done by the weight of the body in causing the descent. If a mass m is at a height h above a lower level the P.E. possessed by the mass is (mg) (h). Since h is the height of an object above a specified level, an object below the specified level has negative potential energy.

Therefore GPE = ± mgh ...(9)



fig(a)



fig(b)

- The chosen level from which height is measured has no absolute position. It is important therefore to indicate clearly the zero P.E. level in any problem in which P.E. is to be calculated.
- GPE = ± mgh is applicable only when h is very small in comparison to the radius of the earth. We have discussed GPE in detail in 'GRAVITATION'.

(b) Elastic Potential Energy : It is a property of stretched or compressed springs.

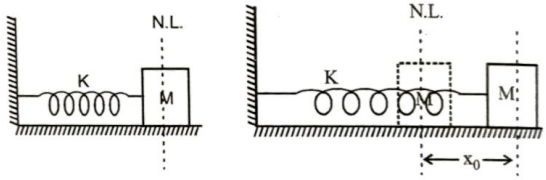
The end of a stretched elastic spring will begin to move if it is released. The spring. therefore possesses potential energy due to its elasticity. (i.e., due to change in its configuration)

The amount of elastic potential energy stored in a spring of natural length a and spring constant k when it is extended by a length x **(from the natural length)** is equivalent to the amount of work necessary to produce the extension.

$$\text{Elastic Potential Energy} = \frac{1}{2}kx^2 \qquad ...(10)$$

It is never negative whether the spring is extended or compressed.

Proof :



Consider a spring block system as shown in the figure and let us calculate work done by spring when the block is displaceed by x_0 from the natural length.

At any moment if the elongation in spring is x, then the force on the block by the spring is kx towards left. Therefore, the work done by the spring when block further displaces by dx

$dW = -kx\,dx$

$\therefore$ Total work done by the spring,

$$W = -\int_0^{x_0} kx\,dx = -\frac{1}{2}kx_0^2$$

Similarly, work done by the spring when it is given a compression x_0 is $-\frac{1}{2}kx_0^2$.

We assume zero potential energy at natural length of the spring:

CONSERVATIVE FORCE AND POTENTIAL ENERGY

$$F_s = -\frac{\partial U}{\partial s}$$

The projection of the force field , the vector F, at a given point in the direction of the displacement r equals the derivative of the potential energy U with respect to a given direction, taken with the opposite sign. The designation of a partial derivative $\partial/\partial s$ emphasizes the fact of deriving with respect to a definite direction. So, having reversed the sign of the partial derivatives of the function U with respect to x, y, z, we obtain the projection F_x, F_y and F_z of the vector F on the unit vectors i, j and k. Hence, one can readily find the vector itself:

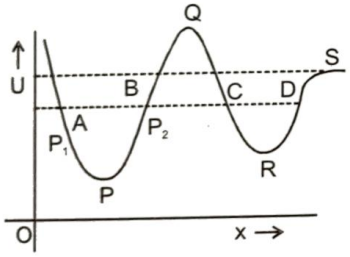
$$\vec{F} = F_x\hat{i} + F_y\vec{j} + F_z\hat{K}, \text{ or } F = -\left(\frac{\partial U}{\partial x}i + \frac{\partial U}{\partial y}j + \frac{\partial U}{\partial z}k\right)$$

The quantity in parentheses is referred to as the scalar gradient of the function U and is denoted by grad U or ∇ U. We shall use the second, more convenient, designation where ∇ ("nabla") signifies the symbolic vector or operator

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Potential Energy curve:

- A graph plotted between the PE of a particle and its displacement from the centre of force field is called PE curve.
- Using graph, we can predict the rate of motion of a particle at various positions.
- Force on the particle is $F_{(x)} = -\frac{dU}{dx}$



Case : I On increasing x, if U increases, force is in (–) ve x direction i.e. attraction force.

Case : II On increasing x, if U decreases, force is in (+) ve x-direction i.e. repulsion force.

DIFFERENT POSITIONS OF A PARTICLE

Position of equilibrium

If net force acting on a body is zero, it is said to be in equilibrium. For equilibrium $\frac{dU}{dx} = 0$. Points P, Q, R and S are the states of equilbrium positions.

Types of equilirbium:

- **Stable equilibrium:**
  When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.
  Necessary conditions:– $\frac{dU}{dx} = 0$, and $\frac{d^2U}{dx^2} = +\text{ ve}$
  In figure P and R point shows stable equilibrium point.
- **Unstable Equilibrium:**
  When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.
  Condition: $\frac{dU}{dx} = 0$ potential energy is maximum i.e. $= \frac{d^2U}{dx^2} = -\text{ ve}$
  Q point in figure shows unstable equilibrium point
- **Neutral equilibrium**:
  In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.
  In figure S is the neutral point
  Condition : $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$

TRAIN YOUR BRAIN

Example 10: The potential energy between two atoms in a molecule is given by, $U_{(x)} = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The system is in stable equilibrium when -

(*a*) $x = 0$ (*b*) $x = \frac{a}{2b}$

(*c*) $x = \left(\frac{2a}{b}\right)^{1/6}$ (*d*) $x = \left(\frac{11a}{5b}\right)$

Sol. (*c*)

Given that, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

We, know $F = -\frac{du}{dx}$

$= (-12)\, a\, x^{-13} - (-6\, b)\, x^{-7} = 0$

or $\frac{6b}{x^7} = \frac{12a}{4x^{13}}$

or $x^6 = 12a/6b = 2a/b$

or $x = \left(\frac{2a}{b}\right)^{1/6}$

Example 11: The potential energy of a conservative system is given by U = ax^2 − bx where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable, unstable or neutral.

Sol. In a conservative field $F = -\frac{dU}{dx}$

$\therefore F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$

For equilibrium F = 0 or b − 2ax = 0 $\therefore x = \frac{b}{2a}$

From the given equation we can see that $\frac{d^2U}{dx^2} = 2a$ (positive), i.e., U is minimum.

Therefore, $x = \frac{b}{2a}$ is the stable equilibrium positon.

WORK ENERGY THEOREM

If the resultant or net force acting on a body is F_{net} then Newton's second law states that

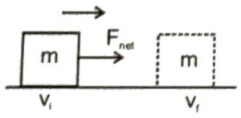
$F_{net} = ma$ ...(1)

If the resultant force varies with x, the acceleration and speed also depend on x.

then $a = v\frac{dv}{dx}$ ...(2)

from eq. (1)

$F_{net} = mv\frac{dv}{dx} \Rightarrow F_{net}.dx = m\,v\,dv$



$$\int F_{net}.dx = \int_{v_i}^{v_f} mvdv$$

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$W_{net} = K_f - K_i$

$W_{net} = \Delta K$ ...(3)

Work done by net force F_{net} in displacing a particle equals to the change in kinetic energy of the particle i.e.

we can write eq. (3) in following way

$(W.D)_c + (W.D)_{N.C} + (W.D)_{ext.} + (W.D)_{pseudo} = \Delta K$ ...(4)

where $(W.D)_c$ = work done by conservative force

$(W.D)_{N.C}$ = work done by non conservative force.

$(W.D)_{ext}$ = work done by external force

$(W.D)_{pseudo}$ = work done by pseudo force.

we know that

$(W.D)_c = -\Delta U$

$\Rightarrow -\Delta U + (W.D)_{N.C} + (W.D)_{ext} + (W.D)_{pseudo} = \Delta K$

$\Rightarrow (W.D)_{N.C} + (W.D)_{ext.} + (W.D)_{pseudo} = (k_f + u_f) - (k_i + u_i)$

$\because$ k + u = Mechanical energy.

$\Rightarrow$ work done by forces (except conservative forces) = change is mechanical energy.

If $(W.D)_{N.C} = (W.D)_{ext} = (W.D)_{pseudo} = 0$

$K_f + U_f = K_i + U_i$

Initial mechanical energy = final mechanical energy. This is called mechanical energy conservation law.

TRAIN YOUR BRAIN

Example 12: A truck and a car moving with the same kinetic energy are stopped by applying same retarding force by means of brakes. Which one will stop at a smaller distance?

Sol. Let m_1, m_2 be the masses and u_1, u_2 be the speeds of the truck and the car respectively.

Then, $\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_2u_2^2$

or $\frac{m_1}{m_2} = \frac{u_2^2}{u_1^2}$ ... (1)

Suppose, on applying the brakes the retardations are a_1 and a_2 respectively, then

$m_1a_1 = m_2a_2$

or $\frac{m_1}{m_2} = \frac{a_2}{a_1}$ ... (2)

From equations (1) and (2) we have,

$\frac{u_1^2}{a_1} = \frac{u_2^2}{a_2}$ ... (3)

If the distances moved by the truck and the car before coming to rest are s_1 and s_2 then,

using, $v^2 = u^2 + 2as$, we get

$0 = u_1^2 - 2a_1s_1$

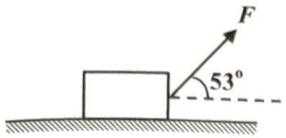
or $s_1 = \frac{u_1^2}{2a_1}$

Similarly, $s_2 = \frac{u_2^2}{2a_2}$

Using equation (3) we get $s_1 = s_2$

$\therefore$ Both will stop at the same distance.

Example 13: A block of mass m = 4 kg is dragged 2m along a horizontal surface by a force F = 30 N acting at 53° to the horizontal. The initial speed is 3 m/s and $\mu_k = 1/8$.



(a) Find the change in kinetic energy of the block

(b) Find its final speed

Sol. The forces acting on the block are shown in the figure.

Clearly, $W_N = 0$ and $W_g = 0$, whereas $W_F = Fs\cos\theta$

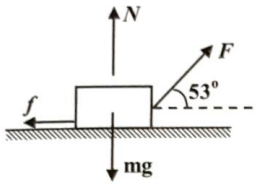
$W_f = -fs = -\mu_k Ns$ where $N = mg - F\sin\theta$

From the work-energy theorem,

$\Delta K = W_{net} = W_F + W_f$

therefore,

$\Delta K = F s \cos\theta - \mu_k(mg - F\sin\theta)s$



$$= (30)(2)(0.6) - \frac{1}{8}(40 - 24)(2) = 32 \text{ J}$$

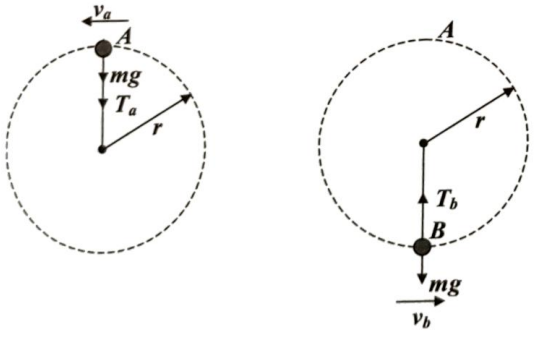
(b) Now $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 32$ J

Since $v_i = 3$ m/s.

Therefore, $v_f = 5$ m/s

MOTION IN A VERTICAL CIRCLE

When a body tied to the end of a string is rotated in a vertical circle, the speed of the body is different at different points of the circular path.



(a) **(b)**

Therefore, the centripetal force on the body and the tension in the string change continuously.

In fig. (a) a body is being rotated in a vertical circle of radius r. When the body is at the highest position A, then it is acted upon by two forces: (i) the weight mg of the body and (ii) the tension T_a in the string. Both of these forces are directed downward. Hence their resultant is $T_a + mg$, and this provides the required centripetal force to the body. Thus, if v_a is the velocity of the body at the point A, then

$$T_a + mg = \frac{mv_a^2}{r}$$

If the speed v_a is decreased, the tension T_a in the string also decreases and becomes zero at a certain minimum value of the speed. Let v_c be that minimum value of speed. Putting $T_a = 0$ and $v_a = v_c$ in the above equation, we get

$$0 + mg = \frac{mv_c^2}{r}$$

or $v_c = \sqrt{gr}$

This speed of the body is called the 'critical speed'. In this state the centripetal force is provided simply by the weight of the body. If the speed of the body at the highest point A is less than the critical speed $\sqrt{gr}$, then the required centripetal force would be less than the weight of the body which will therefore fall down and the string would slack.

The speed v_b of the body at the lowest point B (fig. b) can be calculated by the law of conservation of energy. When the body moves from B to A, it rises through a distanc 2r and so acquires a potential energy of mg × 2r. Thus

kinetic energy at B = kinetic energy at A + potential energy at A

$$\frac{1}{2}mv_b^2 = \frac{1}{2}mv_a^2 + 2mgr$$

If $v_o = \sqrt{gr}$, critical speed, then

$$\frac{1}{2}mv_b^2 = \frac{1}{2}m(gr) + 2mgr = (5/2)mgr$$

or $v_b = \sqrt{5gr}$

If v_b is less than $\sqrt{5gr}$, then v_a will be less than $\sqrt{gr}$, and the string will become slack at the highest point.

At the point B, the weight mg acts downward while the tension T_b acts upward. Their resultant $T_b - mg$ provides the necessary centripetal force. That is,

$$T_b - mg = \frac{mv_b^2}{r}$$

or $T_b = mg + \frac{mv_b^2}{r}$

If $v_b = \sqrt{5gr}$, then

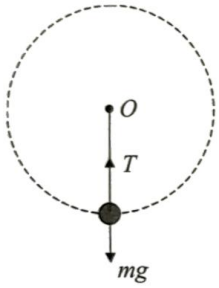
$$T_b = mg + \frac{5mgr}{r} = 6mg$$

Thus, if on rotating a body in a vertical circle, the tension in the string at the highest point A is zero; then the tension at the lowest point would be six times the weight of the body.

TRAIN YOUR BRAIN

Example 14: A child revolves a stone of mass 0.5 kg tied to the end of a string of length 40 cm in a vertical circle. The speed of the stone at the lowest point of the circle is 3 m/s. Calculate the tension in the string at this point.

Sol. Let T be the tension in the string at the lowest point of circle. Then the net force on the stone is T – mg, which is directed towards the centre of the circle. This is the centripetal force acting on the stone. Thus



$$T - mg = \frac{mv^2}{r},$$

where v is the speed of the stone at the lowest point of the circle of radius r.

$$\therefore\ T = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right)$$

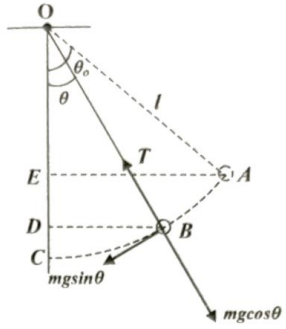
Putting the given values

$$T = (0.5)\left[\frac{3^2}{0.40} + 9.8\right] = 16.15\ N$$

Example 15: A 40-kg sphere suspended by a thread of length ℓ is oscillating in a vertical plane, the angular amplitude being θ_o. What is the tension in the thread when it makes an angle θ with the vertical during oscillations? If the thread can support a maximum tension of 80 kg, then what can be the maximum angular amplitude of oscillations of the sphere?

Sol. Let A be the position of the maximum displacement of the sphere when the thread makes an angle θ_o with the vertical. At A the speed of the sphere is zero. Let v be the speed of the sphere at B when the thread makes an angle θ with the vertical. The centripetal force is mv^2/ℓ, which is provided by the resultant force along the thread. Thus

$$T - mg\cos\theta = \frac{mv^2}{\ell} \quad ...(i)$$



where T is the tension in the thread and mg is the weight of the sphere. v can be calculated by the law of conservation of energy. The sphere, in moving from A to B, looses potential energy mg × ED which is converted into kinetic energy. Thus

$$\frac{1}{2}mv^2 = mg \times ED$$

But $ED = OD - OE = \ell\cos\theta - \ell\cos\theta_o$

$$\therefore\ \frac{1}{2}mv^2 = mg\,\ell\,(\cos\theta - \cos\theta_o)$$

Substituting this value of $\frac{1}{2}mv^2$ in equation (1),

we get

$T - mg\cos\theta = 2\,mg\,(\cos\theta - \cos\theta_o)$

or $T = 3\,mg\cos\theta - 2\,mg\cos\theta_o$ ....(ii)

This is the tension in the string at the position B. As the sphere returns to its mean position C, the angle θ decreases and the tension T increases. At the mean position ($\theta = 0$), the tension is maximum. Putting $\theta = 0$ in equation (ii), we have

$T_{max} = 3\,mg - 2\,mg\cos\theta_o$

The maximum tension T_{max} would increase with increasing angular amplitude θ_o.

The maximum tension for the thread is 80 kg i.e. 2mg. Putting

$T_{max} = 2\,mg$ in the last expression, we get

$2\,mg = 3\,mg - 2\,mg\cos\theta_o$

or $\cos\theta_o = \frac{1}{2}$

$\therefore\ \theta_o = 60^o$

The sphere cannot be given an angular amplitude of more than 60°, otherwise the thread would break.

CONDITIONS FOR OSCILLATION OR LEAVING THE CIRCLE

In case of non uniform circular motion in a vertical plane if speed of the body at lowest point is lesser than $\sqrt{5gr}$, the particle will not complete the circle in vertical plane. Now it can either oscillate about the lowest point or after reaching a certain height may loose the circular track.

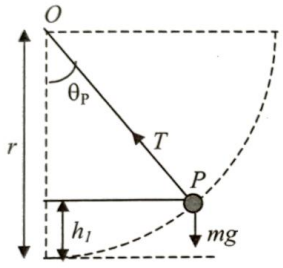
If h_1 is the height where v becomes zero, by conservation of mechanical energy

$$\frac{1}{2}mv_L^2 = mgh_1$$

$$\text{i.e. } h_1 = \frac{v_L^2}{2g} \qquad \text{....(i)}$$

And if h_2 is the height where tension T become zero then

$$\frac{mv_P^2}{r} + mg\cos\theta_P = 0$$



And by conservation of mechanical energy

$$\frac{1}{2}mv_L^2 = \frac{1}{2}mv_P^2 + mgh_2$$

$$\text{i.e. } v_P^2 = v_L^2 - 2gh_2$$

Substituting the value of v_P and $\cos\theta_P = (r - h_2)/r$ we get

$$\frac{\left[v_L^2 - 2gh_2\right]}{r} + g\frac{\left[r - h_2\right]}{r} = 0$$

Which on solving for h_2 gives

$$h_2 = \frac{v_L^2 + gr}{3g} \qquad \text{....(ii)}$$

From equation (i) and (ii) it is clear that

1. If v_p becomes zero before T vanishes, the body will oscillate and will not leave the path, i.e., for oscillation $h_1 < h_2$

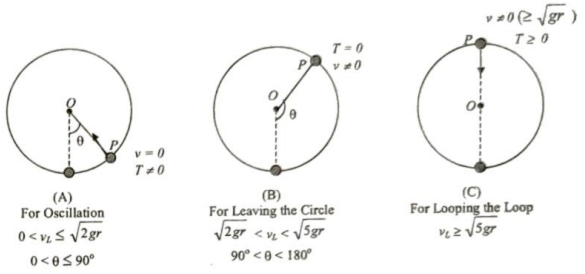
$$\text{i.e., } \frac{v_L^2}{2g} < \frac{v_L^2 + gr}{3g} \text{ or } v_L < \sqrt{2gr}$$

Now for $v_L = \sqrt{2gr}$, from equation (i) and (ii), we get $h_1 = h_2 = r$ which implies $\theta = 90°$. So if the velocity of the body at lowest point is $0 < v_L \le \sqrt{2gr}$ (or $0 < \theta \le 90°$), the body will oscillate about lowest position.

2. If T becomes zero before v_p vanishes, the body will leave the circular path with v_p along the tangent. So for leaving the circle $h_2 < h_1$

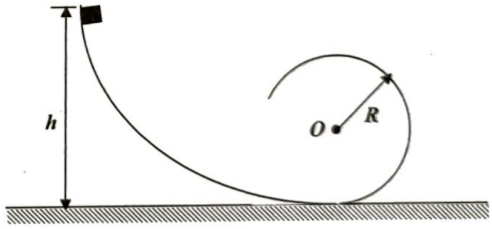
$$\text{i.e., } \frac{v_L^2 + gr}{3g} < \frac{v_L^2}{3g} \text{ or } v_L > \sqrt{2gr}$$

Now we know that if $v_L \ge \sqrt{5gr}$, the body will loop the loop. So if the velocity of a body at lowest point is such that $\sqrt{2gr} < v_L < \sqrt{5gr}$ the body will move along the circle for $\theta > 90°$ and will not reach upto highest point but will leave the circle somewhere between $90° < \theta < 180°$. Here it should be noted that at the point of leaving circle $T = 0$ but $v_p \neq 0$. The above discussion is summarized in the following figure.



TRAIN YOUR BRAIN

Example 16: A small body is allowed to slide on a frictionless track from rest position as shown in figure. What must be the minimum height in terms of R, so that body may successfully complete the loop.



Sol. For the body to be able to successfully complete the loop, its velocity at the bottom of the loop must be greater than or equal to $\sqrt{5gr}$.

Now from energy conservation

$$mgh = = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = 2gh \ge 5gR$$

$$\Rightarrow h \ge \frac{5}{2}R$$

$\therefore$ The minimum height required is $\frac{5}{2}R$.

WORK

Definition of Work

1. A man pushes wall and fails to displace it. He does
   (*a*) Negative work
   (*b*) Positive but not maximum work
   (*c*) No work at all
   (*d*) Maximum work

2. If the unit of force and length each be increased by four times, then the unit of energy is increased by
   (*a*) 16 times (*b*) 8 times
   (*c*) 2 times (*d*) 4 times

Work done by Constant Force, Variable Force

3. A body of mass m is moving in a circle of radius r with a constant speed v The force on the body is $\frac{mv^2}{r}$ and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle

   (*a*) $\frac{mv^2}{\pi r^2}$ (*b*) Zero

   (*c*) $\frac{mv^2}{r^2}$ (*d*) $\frac{\pi v^2}{mv^2}$

4. You lift a heavy book from the floor of the room and keep it in the book-shelf having a height 2 m. In this process you take 5 seconds. The work done by you will depend upon
   (*a*) Mass of the book and time taken
   (*b*) Weight of the book and height of the book-shelf
   (*c*) Height of the book-shelf and time taken
   (*d*) Mass of the book, height of the book-shelf and time taken

5. A body of mass m kg is lifted by a man to a height of one metre in 30 sec. Another man lifts the same mass to the same height in 60 sec. The work done by them are in the ratio
   (*a*) 1 : 2 (*b*) 1 : 1
   (*c*) 2 : 1 (*d*) 4 : 1

6. The work done against gravity in taking 10 kg mass at 1m height in 1sec will be
   (*a*) 49 J (*b*) 98 J
   (*c*) 196 J (*d*) None of these

7. A particle moves from position $\vec{r}_1 = 3\hat{i} + 2\hat{j} - 6\hat{k}$ to position $\vec{r}_2 = 14\hat{i} + 13\hat{j} + 9\hat{k}$ under the action of force $4\hat{i} + \hat{j} + 3\hat{k}$ N. The work done will be
   (*a*) 100 J (*b*) 50 J
   (*c*) 200 J (*d*) 75 J

8. A force $(\vec{F}) = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle causes a displacement: $(\vec{s}) = -4\hat{i} + 2\hat{j} + 3\hat{k}$ in its own direction. If the work done is 6 J, then the value of 'c' is
   (*a*) 0 (*b*) 1
   (*c*) 6 (*d*) 12

9. A body of mass 6kg is under a force which causes displacement in it given by $S = \frac{t^2}{4}$ metres where t is time. The work done by the force in 2 seconds is
   (*a*) 12 J (*b*) 9 J
   (*c*) 6 J (*d*) 3 J

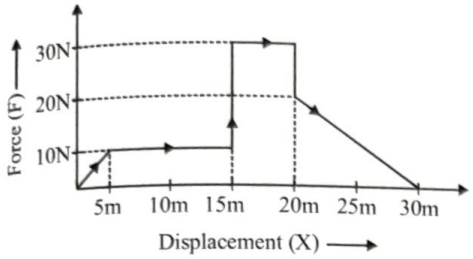
10. A cord is used to lower vertically a block of mass M by a distance d with constant downward acceleration $\frac{g}{4}$. Work done by the cord on the block is

    (*a*) $Mg\frac{d}{4}$ (*b*) $3Mg\frac{d}{4}$

    (*c*) $-3Mg\frac{d}{4}$ (*d*) Mgd

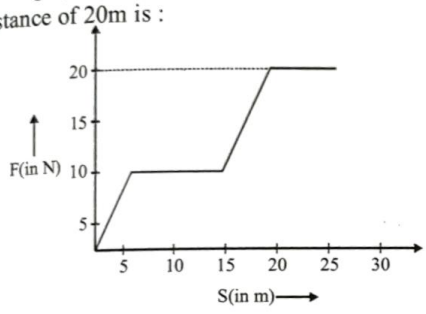
11. The force constant of a wire is k and that of another wire is 2k. When both the wires are stretched through same distance, then the work done
    (*a*) $W_2 = 2W_1^2$
    (*b*) $W_2 = 2W_1$
    (*c*) $W_2 = W_1$
    (*d*) $W_2 = 0.5\ W_1$

12. Given below is a graph between a variable force (F) (along y-axis) and the displacement (X) (along x-axis) of a particle is one dimension. The work done by the force in the displacement interval between 0m and 30 m is:



(a) 275 J (b) 375 J
(c) 400 J (d) 300 J

13. The work done by a force acting on a body is as shown in the graph. The total work done in covering an initial distance of 20m is :



(a) 225 J (b) 200 J
(c) 400 J (d) 175 J

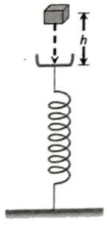
Work Energy Theorem

14. A block of mass m initially at rest is dropped from a height h on to a spring of force constant k. The maximum compression in the spring is x then

(a) $mgh = \frac{1}{2}kx^2$

(b) $mg(h+x) = \frac{1}{2}kx^2$

(c) $mgh = \frac{1}{2}k(x+h)^2$



(d) $mg(h+x) = \frac{1}{2}k(x+h)^2$

15. A block of mass 3 kg starts from rest and slides down a curved path in the shape of a quarter-circle of radius 2 m and reaches the bottom of path with a 4 m/s speed. If g is 10 m/s², the amount of work done against friction is
(a) 60 J (b) 36 J
(c) 24 J (d) 12 J

16. A truck accelerates from speed v to 2v. Work done in doing this is
(a) Three times as the work done in accelerating it from rest to v.
(b) Same as the work done in accelerating it from rest to v.
(c) Four times as the work done in accelerating it from rest to v.
(d) Less than the work done in accelerating it from rest to v.

17. If K_i and K_f are the initial and final values of kinetic energy of a body respectively, then the work done by the net force on the body is equal to

(a) $\frac{K_f K_i}{K_f - K_i}$ (b) $K_f + K_i$

(c) $\frac{K_f + K_i}{2}$ (d) $K_f - K_i$

18. A particle of mass 10g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion?
(a) 0.18 m/s² (b) 0.2 m/s²
(c) 0.1 m/s² (d) 0.15 m/s²

19. A body possessing kinetic energy T moving on a rough horizontal surface is stopped in a distance y. The frictional force exerted on the body is

(a) Ty (b) $\frac{\sqrt{T}}{y}$

(c) $\frac{T}{y}$ (d) $\frac{T}{\sqrt{y}}$

20. A force of 20 N is applied on a body of mass 5 kg resting on a horizontal plane. The body gains a kinetic energy of 10 J after it moves a distance 2m. The frictional force is
(a) 10 N (b) 15 N
(c) 20 N (d) 30 N

Power

21. An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg to a height of 40 m? (Take g = 10 ms⁻²) :
(a) 4s (b) 5s
(c) 8s (d) 10s

22. A body of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of time t, the instantaneous power delivered to the body is

(a) $\frac{mv_1 t}{t_1}$ (b) $\frac{mv_1^2 t}{t_1}$

(c) $\frac{mv_1 t^2}{t_1}$ (d) $\frac{mv_1^2 t}{t_1^2}$

23. A man is riding on a cycle with velocity 7.2 km/hr up a hill having a slope 1 in 20. The total mass of the man and cycle is 100 kg. The power of the man is
(a) 200 W (b) 175 W
(c) 125 W (d) 98 W

24. An electric motor creates a tension of 4500 newton in a hoisting cable and reels it in at the rate of 2 m/sec. What is the power of electric motor

(*a*) 15 kW (*b*) 9 kW

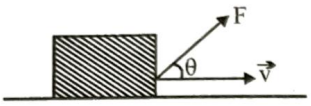
(*c*) 225 W (*d*) 9000 HP

25. A weight lifter lifts 300 kg from the ground to a height of 2 meter in 3 second. The average power generated by him is

(*a*) 5880 watt (*b*) 4410 watt

(*c*) 2205 watt (*d*) 1960 watt

26. A constant force $\vec{F}$ is acting on a body of mass m with constant velocity $\vec{v}$ as shown in the figure. The power P exerted is :



(*a*) F cos θv (*b*) F cos θ/mg

(*c*) F mg cos θ/v (*d*) mg sin θ/F

27. The power of a pump, which can pump 200kg of water to a height of 200m in 10sec is ($g = 10m / s^2$)

(*a*) 40 kW (*b*) 80 kW

(*c*) 400 kW (*d*) 960 kW

28. A force applied by an engine of a train of mass 2.05×10^6 kg changes its velocity from 5 m/s to 25 m/s in 5 minutes. The power of the engine is

(*a*) 1.025 MW (*b*) 2.05 MW

(*c*) 5 MW (*d*) 6 MW

Idea of Potential Energy Equilibrium

29. In which case does the potential energy decrease

(*a*) On compressing a spring

(*b*) On stretching a spring

(*c*) On moving a body against gravitational force

(*d*) On the rising of an air bubble in water

30. A 2g ball of glass is released from the edge of a hemispherical cup whose radius is 20 cm. How much work is done on the ball by the gravitational force during the ball's motion to the bottom of the cup?



(*a*) 1.96 mJ (*b*) 3.92 mJ

(*c*) 4.90 mJ (*d*) 5.88 mJ

31. A uniform chain of length L and mass m is kept on a smooth table. It is released from rest when the over-hanging part was n^{th} fraction of total length. Find the kinetic energy of the chain as it completely slips off the table :

(*a*) $\frac{1}{2}mgL(1-n^2)$ (*b*) $2mgL(1-n^2)$

(*c*) $\frac{1}{2}mgL(n^2-1)$ (*d*) $2mgL\,(n^2-1)$

32. In the stable equilibrium position, a body has

(*a*) maximum potential energy

(*b*) minimum potential energy

(*c*) minimum kinetic energy

(*d*) zero kinetic energy

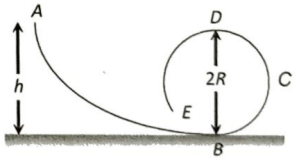
33. The potential energy of a conservative system is given by $V(x) = (x^2 - 3x)$ joule, where x is measured in metre. Then its equilibrium position is at

(*a*) 1.5 m (*b*) 2 m

(*c*) 3 m (*d*) 1 m

Vertical Circular Motion & Power

34. A frictionless track ABCDE ends in a circular loop of radius R. A body slides down the track from point A which is at a height h = 5 cm. Maximum value of R for the body to successfully complete the loop is



(*a*) 5 cm (*b*) $\frac{15}{4}$cm

(*c*) $\frac{10}{3}$cm (*d*) 2 cm

Learning Plus

1. A rigid body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done by this force on the body is 25 joules, the angle which the force makes with the direction of motion of the body is

(*a*) 0° (*b*) 30°

(*c*) 60° (*d*) 90°

2. The work done in slowly pulling up a block of wood weighing 2 kN for a length of 10m on a smooth plane inclined at an angle of 15° with the horizontal by a force parallel to the incline is (5 in 15° = 0.26)

(*a*) 4.36 kJ (*b*) 5.17 kJ

(*c*) 8.91 kJ (*d*) 9.82 kJ

3. A particle moves from position $\vec{r}_1 = 6\hat{i} + 4\hat{j} - 12\hat{k}$ to position $\vec{r}_2 = 12\hat{i} + 8\hat{j} - 6\hat{k}$ under the action of force $4\hat{i} + \hat{j} + 3\hat{k}$ N. The work done by this force will be

(a) 46 J (b) 50 J

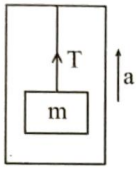
(c) 200 J (d) 75 J

4. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is

(a) 1 : 2 : 3 (b) 1 : 4 : 9

(c) 1 : 3 : 5 (d) 1 : 5 : 3

5. A block of mass m is suspended by a light thread from an elevator. The elevator is accelerating upward with uniform acceleration a. The work done by tension on the block during t seconds is (u = 0) :



(a) $\frac{m}{2}(g + a)\,at^2$ (b) $\frac{m}{2}(g - a)at^2$

(c) $\frac{m}{2}gat^2$ (d) 0

6. The work done in pushing a block of mass 10 kg from bottom to the top of a frictionless inclined plane 5 m long and 3 m high is- ($g = 9.8$ m/sec^2)

(a) 392 J (b) 294 J

(c) 98 J (d) 0.98 J

7. Two springs have their force constant as k_1 and $k_2 (k_1 > k_2)$. When they are stretched by the same force up to equilibrium-

(a) No work is done by this force in case of both the springs

(b) Equal work is done by this force in case of both the springs

(c) More work is done by this force in case of second spring

(d) More work is done by this force in case of first spring

8. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass during the above pulling is

(a) $\frac{1}{2}kx^2$ (b) $-\frac{1}{2}kx^2$

(c) $\frac{1}{4}kx^2$ (d) $-\frac{1}{4}kx^2$

9. A force acting on a particle varies with the displacement x as $F = ax - bx^2$. Where a = 1 N/m and b = 1 N/m^2. The work done by this force for the first one meter (F is in newtons, x is in meters) is :

(a) $\frac{1}{6}$ J (b) $\frac{2}{6}$ J

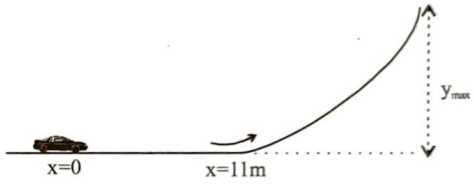
(c) $\frac{3}{6}$ J (d) None of these

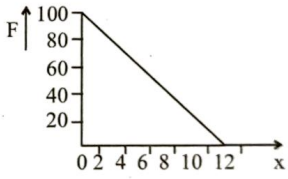
10. A force $F = (3x\,\hat{i} + 4\,\hat{j})$ Newton (where x is in metres) acts on a particle which moves from a position (2m, 3m) to (3m, 0m). Then the work done is

(a) 7.5J (b) –12J

(c) –4.5 J (d) +4.5 J

11. A toy car of mass 5 kg starts from rest and moves up a ramp under the influence of force F (F is applied in the direction of velocity) plotted against displacement x. The maximum height attained is given by





(a) $y_{max} = 20$ m (b) $y_{max} = 15$ m

(c) $y_{max} = 11$ m (d) $y_{max} = 5$ m

12. The kinetic energy of a body of mass 2 kg and momentum of 2 Ns is

(a) 1 J (b) 2J

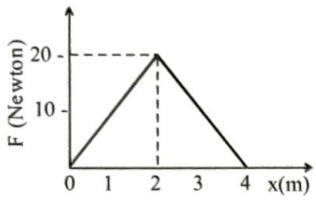
(c) 3 J (d) 4 J

13. A particle of mass m at rest is acted upon by a force F for a time t. Its kinetic energy after an interval t is :

(a) $\frac{F^2t^2}{m}$ (b) $\frac{F^2t^2}{2m}$

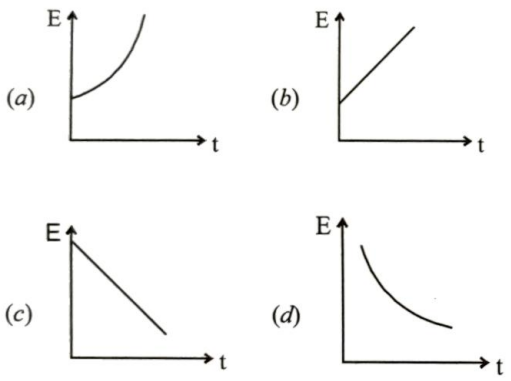
(c) $\frac{F^2t^2}{3m}$ (d) $\frac{Ft}{2m}$

14. The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4m, its kinetic energy would be



(a) 50 J (b) 40 J
(c) 20 J (d) 10 J

15. A particle is dropped from a height h. A constant horizontal velocity is given to the particle. Taking g to be constant every where, kinetic energy E of the particle with respect to time t is correctly shown in



16. A body starts from rest with uniform acceleration and acquires a velocity V in time T. The instantaneous kinetic energy of the body after any time t is proportional to:
(a) (V/T)t (b) $(V^2/T)t^2$
(c) $(V^2/T^2)t$ (d) $(V^2/T^2)t^2$

17. If v, p and E denote the velocity, momentum and kinetic energy of the particle, then:
(a) p = dE/dv (b) p = dE/dt
(c) p = dv/dt (d) None of these

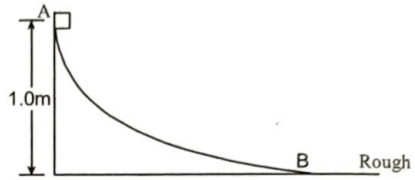
18. A body of mass m accelerates uniformly from rest to a speed v_0 in time t_0. The work done on the body till any time t is

(a) $\frac{1}{2}mv_0^2\left(\frac{t^2}{t_0^2}\right)$ (b) $\frac{1}{2}mv_0^2\left(\frac{t_0}{t}\right)$

(c) $mv_0^2\left(\frac{t}{t_0}\right)$ (d) $mv_0^2\left(\frac{t}{t_0}\right)^3$

19. A body moving at 2 m/s can be stopped over a distance x. If its kinetic energy is doubled, how long will it go before coming to rest, if the retarding force remains unchanged?
(a) x (b) 2x
(c) 4x (d) 8x

20. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface (figure). The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, the distance it will move on the rough surface is:

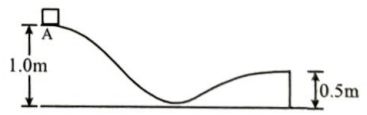


(a) 5.0 m (b) 10.0 m
(c) 15.0 m (d) 20.0 m

21. A light spring of length 20 cm and force constant 2 kg/cm is placed vertically on a table. A small block of mass 1 kg falls on it. The length h from the surface of the table at which the ball will have the maximum velocity is -
(a) 20 cm (b) 15 cm
(c) 10 cm (d) 5 cm

22. The work done in joules in increasing the extension of a spring of stiffness 10 N/cm from 4 cm to 6 cm is :
(a) 1 (b) 10
(c) 50 (d) 100

23. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?



(a) At a horizontal distance of 1 m from the end of the track.
(b) At a horizontal distance of 2 m from the end of the track.
(c) At a horizontal distance of 3 m from the end of the track.
(d) Insufficient information

24. A block of mass m is moving with a constant acceleration 'a' on a rough horizontal plane. If the coefficient of friction between the block and plane is μ. The power delivered by the external agent at a time t from the beginning is equal to:
(a) ma^2t (b) μmgat
(c) μm(a + μg) gt (d) m(a + μg) at

25. A particle moves with a velocity $\vec{v} = (5\hat{i} - 3\hat{j} + 6\hat{k})$ m/s under the influence of a constant force $\vec{F} = (10\hat{i} + 10\hat{j} + 20\hat{k})$N. The instantaneous power applied to the particle is :

(a) 200 J/s (b) 40 J/s
(c) 140 J (d) 170 J/s

26. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the stair case in 20 s. The ratio of the power developed by them (P_1 / P_2) will be:

(a) 1 (b) 4/3
(c) 16/9 (d) None of the above

27. A pump ejects 12000 kg of water at speed of 4 m/s in 40 second. Find the average rate at which the pump is working

(a) 0.24 KW (b) 2.4 W
(c) 2.4 KW (d) 24 W

28. When a conservative force does positive work on a body
(a) the potential energy increases
(b) the potential energy decreases
(c) total energy increases
(d) total energy decreases

29. _____ of a two particle system depends only on the separation between the two particles. The most appropriate choice for the blank space in the above sentence is

(a) kinetic energy (b) total mechanical energy
(c) potential energy (d) total energy

30. A body is dropped from a certain height. When it loses U amount of its energy it acquires a velocity 'v'. The mass of the body is :

(a) $2U/v^2$ (b) $2v/U^2$
(c) $2v/U$ (d) $U^2/2v$

31. Two springs A and B ($k_A = 2k_B$) are stretched by applying forces of equal magnitudes at the four ends. If the energy stored in A is E, that in B is

(a) E/2 (b) 2E
(c) E (d) E/4

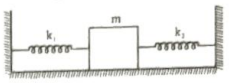
32. The P.E. of a certain spring when stretched from natural length through a distance 0.3 m is 10 J. The amount of work in joule that must be done on this spring to stretch it through an additional distance 0.15 m will be

(a) 10 J (b) 20 J
(c) 7.5 J (d) 12.5 J

33. When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by

(a) 100 J (b) 200 J
(c) 300 J (d) 400 J

34. A block of mass m is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure. The block is displaced towards right through a distance x and is released. Find the speed of the block as it passes through the mean position shown.



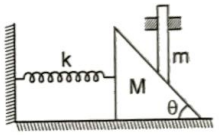
(a) $\sqrt{\frac{k_1 + k_2}{m}}\, x$ (b) $\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}\, x$

(c) $\sqrt{\frac{k_1^2 k_2^2}{m(k_1^2 + k_2^2)}}\, x$ (d) $\sqrt{\frac{k_1^3 k_2^3}{m(k_1^3 + k_2^3)}}$

35. A spring when stretched by 2 mm its potential energy becomes 4 J. If it is stretched by 10 mm, its potential energy is equal to

(a) 4 J (b) 54 J
(c) 415 J (d) 100 J

36. A wedge of mass M fitted with a spring of stiffness 'K' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is :



(a) $\frac{m g^2 \tan^2\theta}{2K}$ (b) $\frac{m^2 g \tan^2\theta}{2K}$

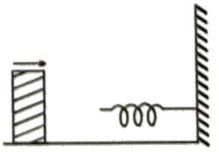
(c) $\frac{m^2 g^2 \tan^2\theta}{2K}$ (d) $\frac{m^2 g^2 \tan^2\theta}{K}$

37. The spring extends by x on loading, then energy stored by the spring is (if T is the tension in spring and k is spring constant).

(a) $\frac{T^2}{2k}$ (b) $\frac{T^2}{2k^2}$

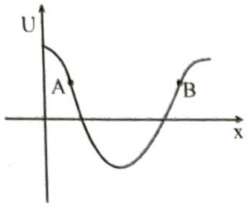
(c) $\frac{2k}{T^2}$ (d) $\frac{2T^2}{k}$

38. A 1.0 kg block collides with a horizontal weightless spring of force constant 2.75 Nm^{-1} as shown in figure. The block compresses the spring 4.0 m from the rest position. If the coefficient of kinetic friction between the block and horizontal surface is 0.25, the speed of the block at the instant of collision is



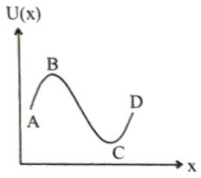
(a) 0.4 ms^{-1} (b) 4 ms^{-1}
(c) 0.8 ms^{-1} (d) 8 ms^{-1}

39. Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at A and B is respectively.



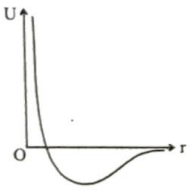
(*a*) Positive, Positive (*b*) Positive, Negative
(*c*) Negative, Positive (*d*) Negative, Negative

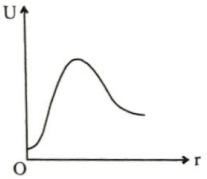
40. The potential energy of a particle varies with distance x as shown in the graph.



The force acting on the particle is zero at
(*a*) C (*b*) B
(*c*) B and C (*d*) A and D.

41. The diagrams represent the potential energy U of a function of the inter-atomic distance r. Which diagram corresponds to stable molecules found in nature.

(*a*)


(*b*)


(*c*)

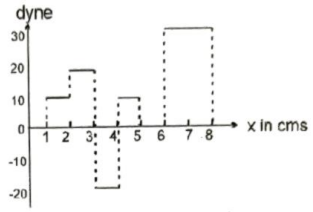

(*d*)


42. The potential energy for a force field $\vec{F}$ is given by $U(x, y) = \sin(x + y)$. The force acting on the particle of mass m at $\left(0, \frac{\pi}{4}\right)$ is

(*a*) 1 (*b*) $\sqrt{2}$
(*c*) $\frac{1}{\sqrt{2}}$ (*d*) 0

43. A particle is taken from point A to point B under the influence of a force field. Now it is taken back from B to A and it is observed that the work done in taking the particle from A to B is not equal to the work done in taking it from B to A. If W_{nc} and W_c is the work done by non-conservative forces and conservative forces present in the system respectively, ΔU is the change in potential energy, Δk is the change in kinetic energy, then
(*a*) $W_{nc} - \Delta U = \Delta k$ (*b*) $W_c = -\Delta U$
(*c*) $W_{nc} + W_c = \Delta k$ (*d*) $W_{nc} - \Delta U = -\Delta k$

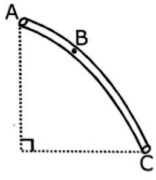
44. The relationship between force and position is shown in fig (in one dimensional case). The work done in displacing a body from x = 1cm to x = 5 cm is



(*a*) 20 erg (*b*) 60 erg
(*c*) 70 erg (*d*) 700 erg

45. $F = 2x^2 - 3x - 2$. Choose correct option
(*a*) x = –1/2 is position of stable equilibrium
(*b*) x = 2 is position of stable equilibrium
(*c*) x = –1/2 is position of unstable equilibrium
(*d*) x = 2 is position of neutral equilibrium

46. The tube AC forms a quarter circle in a vertical plane. The ball B has an area of cross-section slightly smaller than that of the tube, and can move without friction through it. B is placed at A and displaced slightly. It will



(*a*) always be in contact with the inner wall of the tube
(*b*) always be in contact with the outer wall of the tube
(*c*) initially be in contact with the inner wall and later with the outer wall
(*d*) initially be in contact with the outer wall and later with the inner wall

47. A particle is rotated in a vertical circle by connecting it to a light rod of length *l* and keeping the other end of the rod fixed. The minimum speed of particle when the light rod is horizontal for which the particle will complete the circle is

(*a*) $\sqrt{gl}$ (*b*) $\sqrt{2gl}$
(*c*) $\sqrt{3gl}$ (*d*) None of these

Advanced Level Multiconcept Questions

JEE-ADVANCED
MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. No work is done by a force on an object if
   (*a*) the force is always perpendicular to its velocity
   (*b*) the force is always perpendicular to its acceleration
   (*c*) the object is stationary but the point of application of the force moves on the object
   (*d*) the object moves in such a way that the point of application of the force remains fixed.

2. The total work done on a particle is equal to the change in its kinetic energy
   (*a*) if the forces acting on it are conservative
   (*b*) if gravitational force alone acts on it
   (*c*) if elastic force alone acts on it.
   (*d*) none of these

3. Work done by force of friction
   (*a*) can be zero (*b*) can be positive
   (*c*) can be negative (*d*) information insufficient

4. One end of a light spring of spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2}kx^2$. The possible cases are:
   (*a*) the spring was initially compressed by a distance x and was finally in its natural length
   (*b*) it was initially stretched by a distance x and finally was in its natural length
   (*c*) it was initially in its natural length and finally in a compressed position.
   (*d*) it was initially in its natural length and finally in a stretched position.

5. The kinetic energy of a particle continuously increases with time
   (*a*) the resultant force on the particle must be parallel to the velocity at all instants.
   (*b*) the resultant force on the particle must be at an angle less than 90° with the velocity all the time
   (*c*) its height above the ground level must continuously decrease
   (*d*) the magnitude of its linear momentum is increasing continuously

6. If force is always perpendicular to motion
   (*a*) KE remains constant (*b*) work done = 0
   (*c*) speed is constant (*d*) velocity is constant

7. A ball of mass m is attached to the lower end of light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstreched) length, comes to rest again after descending through a distance x.
   (*a*) x = mg/k
   (*b*) x = 2 mg/k
   (*c*) The ball will have no acceleration at the position where it has descended through x/2.
   (*d*) The ball will have an upward acceleration equal to g at its lowermost position.

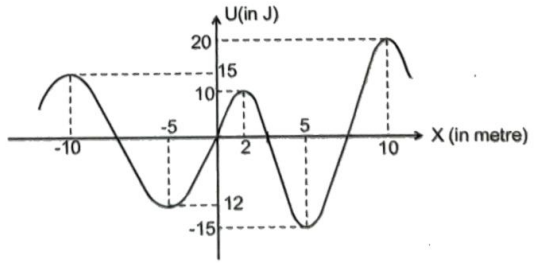
8. When total work done on a particle is positive
   (*a*) KE remains constant (*b*) Momentum increases
   (*c*) KE decreases (*d*) KE increases

9. When work done by force of gravity is negative (Assume only gravitational force to be acting)
   (*a*) KE increases (*b*) KE decreases
   (*c*) PE increases (*d*) PE stays constant

10. The potential energy in joules of a particle of mass 1 kg moving in a plane is given by U = 3x + 4y, the position coordinates of the point being x and y, measured in meters. If the particle is initially at rest at (6, 4), then
    (*a*) its acceleration is of magnitude 5 m/s^2
    (*b*) its speed when it crosses the y-axis is 10 m/s
    (*c*) it crosses the y-axis (x = 0) at y = –4
    (*d*) it moves in a straight line passing through the origin (0, 0)

Comprehension – 1 (No. 11 to 13)

In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



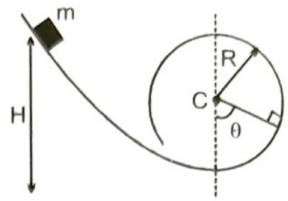
11. If the particle is released at the origin then:
(*a*) it will move towards positive x-axis.
(*b*) it will move towards negative x-axis.
(*c*) it will remain stationary at the origin.
(*d*) its subsequent motion cannot be decided due to lack of information.

12. If the particle is released at $x = 2 + \Delta$ where $\Delta \to 0$ (it is positive) then its maximum speed in subsequent motion will be :
(*a*) $\sqrt{10}$ m/s (*b*) 5 m/s
(*c*) $5\sqrt{2}$ (*d*) 7.5 m/s

13. $x = -5$ m and $x = 10$ m positions of the particle are respectively of
(*a*) neutral and stable equilibrium.
(*b*) neutral and unstable equilibrium.
(*c*) unstable and stable equilibrium.
(*d*) stable and unstable equilibrium.

Comprehension – 2 (No. 14 to 16)

A particle of mass m is released from a height H on a smooth curved surface which ends into a vertical loop of radius R, as shown

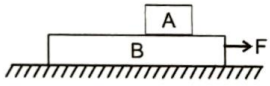


14. Choose the correct alternative(s) if $H = 2R$
(*a*) The particle reaches the top of the loop with zero velocity
(*b*) The particle cannot reach the top of the loop
(*c*) The particle breaks off at a height $H = R$ from the base of the loop
(*d*) The particle break off at a height $R < H < 2R$

15. If θ is instantaneous angle which the line joining the particle and the centre of the loop makes with the vertical, then identify the correct statement(s) related to the normal reaction N between the block and the surface
(*a*) The maximum value N occurs at $\theta = 0$
(*b*) The minimum value of N occurs at $\theta = \pi$ for $H > 5R/2$
(*c*) The value of N becomes negative for $\pi/2 < \theta < 3\pi/2$
(*d*) The value of N becomes zero only when $\theta \geq \pi/2$

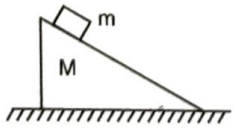
16. The minimum value of H required so that the particle makes a complete vertical circle is given by
(*a*) 5 R (*b*) 4 R
(*c*) 2.5 R (*d*) 2 R

17. A block A of mass m kg lies on block B of mass m kg. B in turn lies on smooth horizontal plane. The coefficient of friction between A and B is μ. Both the blocks are initially at rest. A horizontal force F is applied to lower block B at $t = 0$ such that there is relative motion between A and B. In the duration from $t = 0$ second till the lower block B undergoes a displacement of magnitude L, match the statements in Column-I with results in Column-II.



Column-I	Column-II
(*a*) Work done by friction force on block A is	(p) positive
(*b*) Work done by friction force on block B is	(q) negative
(*c*) Work done by friction on block A plus work done by friction on block B is	(r) less than μ mgL in magnitude
(*d*) Work done by force F on block B is	(s) equal to μ mgL in magnitude

18. A block of mass m lies on wedge of mass M. The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column-I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge). Match the statement in column-I with the results in column-II. (g is acceleration due to gravity)

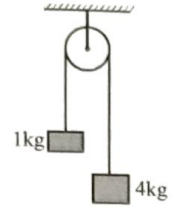


Column I	Column II
(*a*) Work done by normal reaction acting on the block is	(p) positive
(*b*) Work done by normal reaction (exerted by block) acting on wedge is	(q) negative
(*c*) The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is	(r) zero
(*d*) Net work done by all forces on block is	(s) less than mgh in magnitude

NUMERICAL VALUE BASED

19. The combined frictional and air resistance on a bicyclist has the force $F = av$, where v is his velocity and $a = 4$ newton-sec/m. At maximum effort, the cyclist can generate 400 watts propulsive power. What is his maximum speed (in m/s) on level ground with no wind?

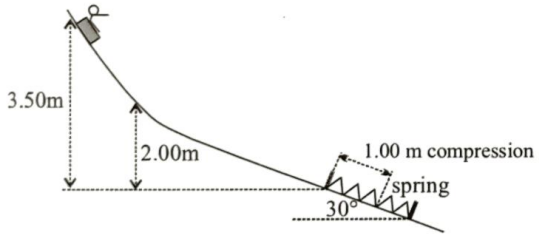
20. In the ideal atwood machine arrangement shown, what is the change in kinetic energy (in Joule) of the system during 3rd second of their motion starting from rest ?



21. A 300 kg piano is being lifted at a constant speed by a crane from ground to an apartment 24m high. The crane being used has a constant power of 960 W. What is the time (in sec.) taken for the lift ?

22. Two cars A and B are travelling at the same speed of 40 m/s each starting from rest. Car A has mass of 1200 kg and B has mass of 2000 kg. Compared to work required to bring car A to the required speed, how much additional work (in kJ) is required to be done on B ?

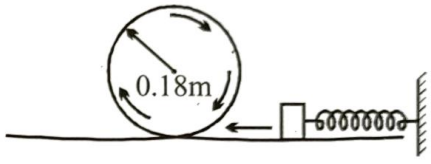
23. A person in a sledge (total mass of 120 kg) slides down a frictionless icy slope from a height of 3.50 m. The sledge is initially at rest and comes to a stop momentarily after compressing a coil spring of negligible mass by 1.00 m. Consider the sledge as a point. Assume elastic collision between the sledge and the spring. Calculate the spring constant of the spring (N/m).



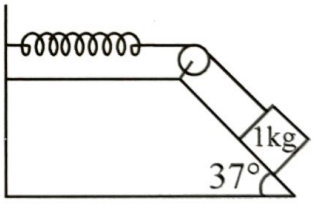
24. A skier starts from rest at the top of a hill. The skier coasts down the hill and up a second hill, as the drawing illustrates. The crest of the second hill is circular, with a radius of r = 36m. Neglect friction and air resistance. What must be the height h (in m) of the first hill so that the skier just loses contact with the snow at the crest of the second hill?



25. A block of mass 200 gm is pushed against spring (k = 4500 N/m). The entire track is smooth. What should be the minimum compression (in cm) of the spring so that the block does not lose contact with the loop any time?



26. A 1 kg block and a rough fixed incline is connected to spring of constant 100 N/m. Block is released from rest with spring unstretched. The block moves 10 cm down the incline before coming to instantaneous rest. Find the value of $\frac{1}{\mu}$?

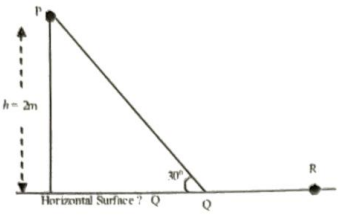


JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. A point particle of mass *m*, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals i. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.
The values of the coefficient of friction i and the distance x(= QR), are, respectively close to :

[JEE Main-2016]



(*a*) 0.2 and 6.5 m (*b*) 0.2 and 3.5 m
(*c*) 0.29 and 3.5 m (*d*) 0.29 and 6.5 m

2. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$: **[*JEE Main-2016*]**

(*a*) 2.45×10^{-3} kg (*b*) 6.45×10^{-3} kg
(*c*) 9.89×10^{-3} kg (*d*) 12.89×1^{-3} kg

3. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is $v_0 = 10\ ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be: **[*JEE Main-2017*]**

(*a*) 10^{-4} kg m^{-1} (*b*) 10^{-1} Kg m^{-1} s^{-1}
(*c*) 10^{-3} kg m^{-1} (*d*) 10^{-3} Kg s^{-1}

4. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be: **[*JEE Main-2017*]**

(*a*) 9 J (*b*) 18 J
(*c*) 4.5 J (*d*) 22 J

5. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is : **[*JEE Main-2018*]**

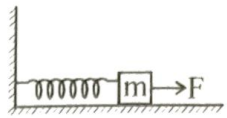
(*a*) $\frac{k}{2a^2}$ (*b*) zero

(*c*) $-\frac{3}{2}\frac{k}{a^2}$ (*d*) $-\frac{k}{4a^2}$

6. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? **[*JEE Main-2019 (January)*]**

(*a*) 850 J (*b*) 950 J
(*c*) 875 J (*d*) 900 J

7. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in a equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is **[*JEE Main-2019 (January)*]**



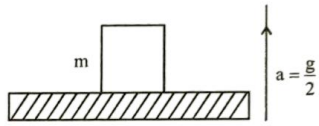
(*a*) $\frac{2F}{\sqrt{mk}}$ (*b*) $\frac{F}{\pi\sqrt{mk}}$

(*c*) $\frac{\pi F}{\sqrt{mk}}$ (*d*) $\frac{F}{\sqrt{mk}}$

8. A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ? **[*JEE Main-2019 (January)*]**

(*a*) 9 J (*b*) 12 K
(*c*) 10 J (*d*) 15 J

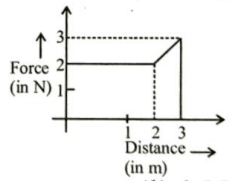
9. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in figure. Work done by normal reaction on block in time t is: **[*JEE Main-2019 (January)*]**



(*a*) $-\frac{mg^2t^2}{8}$ (*b*) $\frac{mg^2t^2}{8}$

(*c*) 0 (*d*) $\frac{3mg^2t^2}{8}$

10. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3m is **[*JEE Main-2019 (April)*]**



(*a*) 6.5 J (*b*) 2.5 J
(*c*) 4 J (*d*) 5 J

11. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be **[*JEE Main-2019 (April)*]**

(*a*) $\frac{MgL}{n^2}$ (*b*) $\frac{MgL}{2n^2}$

(*c*) $\frac{2MgL}{n^2}$ (*d*) nMgL

12. A spring whose unstreched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be: [JEE Main-2019 (April)]

(a) $\frac{1}{n^2}$ (b) n^2

(c) $\frac{1}{n}$ (d) n

13. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $\ell(\ell << L)$, is close to: [JEE Main-2019 (April)]

(a) $Mg\ell$ (b) $Mg\ell(1+\theta_0^2)$

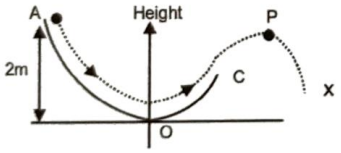
(c) $Mg\ell\left(1-\theta_0^2\right)$ (d) $Mg\ell\left(1+\frac{\theta_0^2}{2}\right)$

14. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to : (1 HP = 746 W, g = 10 ms^{-2}) [JEE Main-2020 (January)]

(a) 1.5 ms^{-1} (b) 1.9 ms^{-1}

(c) 2.0 ms^{-1} (d) 1.7 ms^{-1}

15. A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point A(height 2m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P(height 1m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; Take g = 10 ms^{-2})..... [JEE Main-2020 (January)]

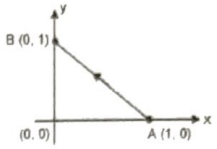


16. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10\ m/s^2$) must be at least: ($g = 10\ m/s^{-2}$) [JEE Main-2020 (January)]

(a) 62360 W (b) 56300W

(c) 48000W (d) 66000W

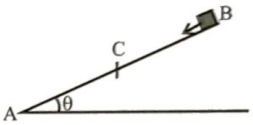
17. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1,0) to B(0,1) along the line segment is : [JEE Main-2020 (January)] (All quantities are in SI units)



(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) 2 (d) 1

18. A small block starts slipping down from a point B on an inclined plane AB, which is making an



angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found thatt the block comes to rest as it reaches the bottom (point A) of the incl i ned pl ane. If BC = 2AC, the coefficient of friction is given by $\mu = k\tan\theta$. The value of K is _____. [JEE Main-2020 (September)]

19. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching) the ball, the value of F (in N) is ($g = 10\ ms^{-2}$) ____. [JEE Main-2020 (September)]

20. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box? [JEE Main-2020 (September)]

(a) 5690 J (b) 3280 J

(c) 5250 J (d) 2780 J

21. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) _____. [JEE Main-2020 (September)]

22. If the potential energy between two molecules is given by $U = -\frac{A}{r^6}+\frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are

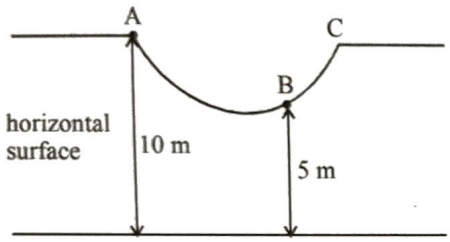
[*JEE Main-2020 (September)*]

23. A ball of mass 4 kg, moving with a velocity of 10 ms $^{-1}$, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is × m. The value of x, to the nearest integer, is

[*JEE Main-2021 (March)*]

24. As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take g = 10 m/s^2) The value of 'x' to the nearest integer is

[*JEE Main-2021 (March)*]



JEE-ADVANCED PREVIOUS YEAR'S

1. The work done on a particle of mass m by a force, K $\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is:

[*JEE Advanced-2013*]

(*a*) $\frac{2K\pi}{a}$ (*b*) $\frac{K\pi}{a}$

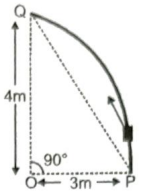
(*c*) $\frac{K\pi}{2a}$ (*d*) 0

2. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5s is:

[*JEE Advanced-2013*]

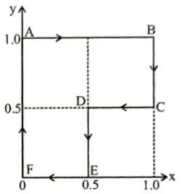
3. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3m and OQ= 4m. A block of mass 1kg is pulled along the rail from P to Q with a force of 18 N, Which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})

[*JEE Advanced-2014*]



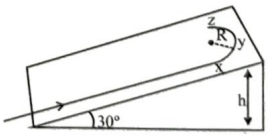
4. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F}=(\alpha y\hat{i}+2\alpha x\hat{j})$N where x and y are in meter and a = –1N/m–1. The work done on the particle by this force $\vec{F}$ will be ____ Joule

[*JEE Advanced-2019*]



5. A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity)

[*JEE (Advanced)-2020*]



(*a*) $v_0^2-2gh=\frac{1}{2}gR$

(*b*) $v_0^2-2gh=\frac{\sqrt{3}}{2}gR$

(*c*) the centripetal force required at points x and z is zero

(*d*) the centripetal force required is maximum at points x and z

ANSWER KEY

Topicwise Questions

1. (*c*)	**2.** (*a*)	**3.** (*b*)	**4.** (*b*)	**5.** (*b*)	**6.** (*b*)	**7.** (*a*)	**8.** (*c*)	**9.** (*d*)	**10.** (*c*)
11. (*b*)	**12.** (*b*)	**13.** (*b*)	**14.** (*b*)	**15.** (*b*)	**16.** (*a*)	**17.** (*d*)	**18.** (*c*)	**19.** (*c*)	**20.** (*b*)
21. (*c*)	**22.** (*d*)	**23.** (*d*)	**24.** (*b*)	**25.** (*d*)	**26.** (*a*)	**27.** (*a*)	**28.** (*b*)	**29.** (*d*)	**30.** (*b*)
31. (*a*)	**32.** (*b*)	**33.** (*a*)	**34.** (*d*)						

Learning Plus

1. (*c*)	**2.** (*b*)	**3.** (*a*)	**4.** (*c*)	**5.** (*a*)	**6.** (*b*)	**7.** (*c*)	**8.** (*d*)	**9.** (*a*)	**10.** (*c*)
11. (*c*)	**12.** (*a*)	**13.** (*b*)	**14.** (*d*)	**15.** (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*a*)	**19.** (*b*)	**20.** (*a*)
21. (*b*)	**22.** (*a*)	**23.** (*a*)	**24.** (*d*)	**25.** (*c*)	**26.** (*b*)	**27.** (*c*)	**28.** (*b*)	**29.** (*c*)	**30.** (*a*)
31. (*b*)	**32.** (*d*)	**33.** (*c*)	**34.** (*a*)	**35.** (*d*)	**36.** (*c*)	**37.** (*a*)	**38.** (*d*)	**39.** (*b*)	**40.** (*c*)
41. (*a*)	**42.** (*a*)	**43.** (*a*, *b*, *c*)	**44.** (*a*)	**45.** (*a*)	**46.** (*c*)	**47.** (*b*)			

Advanced Level Multiconcept Questions

1. (*a*, *c*, *d*) **2.** (*a*, *b*, *c*) **3.** (*a*, *b*, *c*) **4.**(*a*, *b*) **5.**(*b*, *d*) **6.**(*a*, *b*, *c*) **7.**(*b*, *c*, *d*) **8.**(*b*, *d*) **9.**(*b*, *c*) **10.**(*a*, *b*, *c*)

11. (*b*) **12.** (*b*) **13.** (*d*) **14.** (*b*, *d*) **15.** (*a*, *b*, *d*) **16.** (*c*) **17.** (*a*) p, r (*b*) q, s (*c*) q, r (*d*) p

18. (*a*) q, s (*b*) p, s (*c*) r, s (*d*) p, s **19.** 0010 **20.** 0450 **21.** 75 sec. **22.** 0640 kJ **23.** 9600 **24.** 0018

25. 0002 cm **26.** 0008

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*d*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** (*d*)	**8.** (*d*)	**9.** (*d*)	**10.** (*a*)
11. (*b*)	**12.** (*c*)	**13.** (*b*)	**14.** (*b*)	**15.** (10)	**16.** (*d*)	**17.** (*d*)	**18.** (*c*)	**19.** (150.00)	**20.** (*c*)
21. (18.00)	**22.** (*c*)	**23.** [6]	**24.** [10]						

JEE-ADVANCED PREVIOUS YEAR'S

1. (*d*) **2.** [5] **3.** [5] **4.** (0.75) **5.** (*a*, *d*)

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7 SYSTEM OF PARTICLES AND ROTATIONAL MOTION

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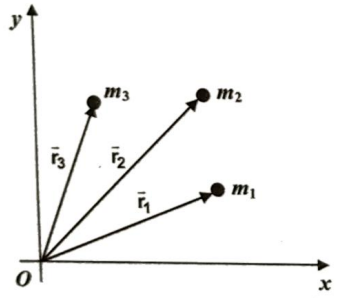
CENTRE OF MASS

In a system of extended bodies there is one special point that has some interesting and simple properties no matter how complicated the system is. This point is called the **center of mass**.

For a system of n particles whose position vectors are $\vec{r}_1, \vec{r}_2, \ldots\ldots\ldots, \vec{r}_n$ as shown in the figure, the position vector for the center of mass $\vec{r}_{cm}$ is defined as

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots\ldots + m_n}$$

or $$\vec{r}_{cm} = \frac{\sum m_i\vec{r}_i}{M}$$



where $M = \Sigma m_i$ is the total mass of the system.

The components of the above equation may be written as

$$x_{cm} = \frac{\sum m_i x_i}{M}; \quad y_{cm} = \frac{\sum m_i y_i}{M}; \quad z_{cm} = \frac{\sum m_i z_i}{M}$$

The location of the centre of mass is independent of the reference frame used to locate it. The centre of mass of the system of particles depends only on the masses of the particles and the positions of the particles relative to one another.

A rigid body, such as a meter stick, can be thought of as a system of closely packed particles. Hence it also has the centre of mass. The number of particles in the body is so large and their spacing so small, however, that we can treat the body as though it has a continuous distribution of mass. For continuous distribution of mass, the centre of mass is defined as

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$

where $\vec{r}$ is the position vector of the centre of mass of a small mass element dm.

The components of this equation are

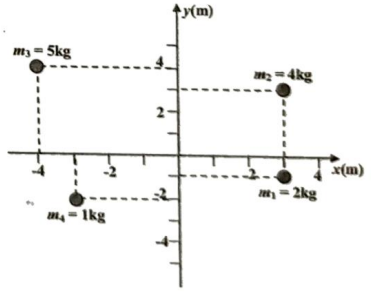
$$x_{cm} = \frac{1}{M}\int x\,dm, \quad y_{cm} = \frac{1}{M}\int y\,dm$$

and $$z_{cm} = \frac{1}{M}\int z\,dm$$

Often we deal with homogeneous objects having a point, a line, or a plane of symmetry. Then the centre of mass will lie at the point, on the line, or in the plane of symmetry. For example, the centre of mass of a homogeneous rod will lie at the centre of the rod, the centre of mass of a homogenous sphere will be at the centre of the sphere, the centre of mass of a cone will be at the axis of the cone, etc.

TRAIN YOUR BRAIN

Example 1: Find the center of mass of the four point masses as shown in figure.



Sol. The total mass M = 12 kg, from the component equations, we have

$$x_{cm} = \frac{(2kg)(3m)+(4kg)(3m)+(5kg)(-4m)+(1kg)(-3m)}{12kg} = -\frac{5}{12}m$$

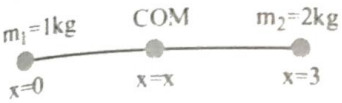
$$y_{cm} = \frac{(2kg)(-1m)+(4kg)(3m)+(5kg)(4m)+(1kg)(-2m)}{12kg}$$

$$= \frac{28}{12}m$$

The position vector of the centre of mass is

$$\vec{r}_{cm} = -0.42\,\hat{i} + 2.3\,\hat{j}\ m$$

Example 2: Two particles of masses 1 kg and 2 kg are located at x = 0 and x = 3 m. Find the position of their centre of mass.



Sol.



Since, both the particles lie on x-axis, the COM will also lie on x-axis. Let the COM is located at x = x, then

r_1 = distance of COM from the particle of mass 1 kg = x

and r_2 = distance of COM from the particle of mass 2 kg = (3 – x)

Using $\frac{r_1}{r_2} = \frac{m_2}{m_1}$ or $\frac{x}{3-x} = \frac{2}{1}$ or x = 2 m

thus, the COM of the two particles is located at x = 2m

Example 3: The position vector of three particles of masses m_1 = 1 kg, m_2 = 2 kg and m_3 = 3 kg are $\vec{r}_1 = (\hat{i}+4\hat{j}+\hat{k})$ m, $\vec{r}_2 = (\hat{i}+\hat{j}+\hat{k})$ m and $\vec{r}_3 = (2\hat{i}-\hat{j}-2\hat{k})$ m respectively. Find the position vector of their center of mass.

Sol. The position vector of COM of the three particles will be given by

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

Substituting the values, we get

$$\vec{r}_{COM} = \frac{(1)(\hat{i}+4\hat{j}+\hat{k}) + (2)(\hat{i}+\hat{j}+\hat{k}) + (3)(2\hat{i}-\hat{j}-2\hat{k})}{1+2+3}$$

$$= \frac{1}{2}(3\hat{i}+\hat{j}-\hat{k})\,m \qquad \textbf{Ans.}$$

CENTRE OF MASS OF CONTINUOUS MASS SYSTEM

(i) Center of Mass of a Uniform Rod :

Suppose a rod of mass M and length L is lying along the x-axis with its one end at x = 0 and the other at x = L. Mass per unit length of the rod = $\frac{M}{L}$

Hence, dm, (the mass of the element dx situated at x = x is) = $\frac{M}{L}dx$

The coordinates of the element dx are (x, 0, 0). Therefore, x-coordinate of COM of the rod will be



$$x_{COM} = \frac{\int_0^L x\,dm}{\int dm} = \frac{\int_0^L (x)\left(\frac{M}{L}dx\right)}{M}$$

$$= \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$

The y-coordinate of COM is

$$y_{COM} = \frac{\int y\,dm}{\int dm} = 0$$

Similarly, $z_{COM} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$,

i.e. it lies at the center of the rod.

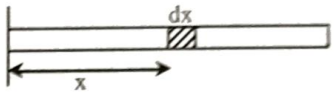
TRAIN YOUR BRAIN

Example 4: Mass is non uniformly distributed over the rod of length 'ℓ'. Its linear mass density varies linearly with length as $\lambda = kx$. The position of centre of mass is given by–

(a) $\frac{2\ell}{5}$ (b) $\frac{\ell}{3}$

(c) $\frac{3\ell}{4}$ (d) $\frac{2\ell}{3}$

Sol. (d)



$\lambda = kx$

Let the cross section = A

mass of element dx at distance x

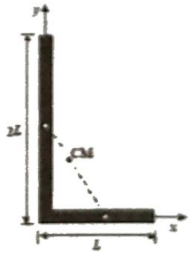
dm = (kx) Adx

$$X_{cm} = \frac{\int_0^\ell x(kx)\,Adx}{\int_0^\ell (kx)\,Adx} = \frac{\int_0^\ell x^2dx}{\int_0^\ell xdx} = \frac{\left[\frac{x^3}{3}\right]_0^\ell}{\left(\frac{x^2}{2}\right)_0^\ell}$$

$$= \frac{2\ell}{3}$$

Example 5: A thin rod of length 3L is bent at right angles at a distance L from one end. Locate the centre of mass with respect to the corner. Take L = 1.2 m

Sol. The centre of mass of each arm is at its midpoint. The centre of mass of the two arms can be found by treating each arm as a point particle at its centre of mass. From the diagram we see that x_1 = L/2, y_1 = 0 and x_2 = 0,



$y_2 = L$. If we take $m_1 = m$, then $m_2 = 2m$. From equation, we have

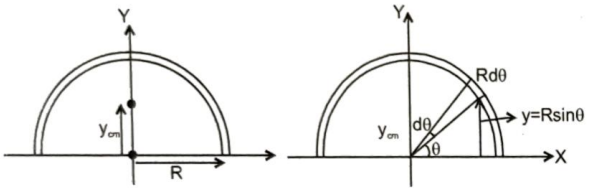
$$x_{c.m.} = \frac{m_1x_1 + m_2x_2}{M} = \frac{L}{6}$$

$$y_{c.m.} = \frac{m_1y_1 + m_2y_2}{M} = \frac{2L}{3}$$

The position of the centre of mass is

$\vec{r}_{c.m.} = 0.2\ \hat{i} + 0.8\ \hat{j}$ m. **Note** that the centre of mass does not lie within the body itself.

(ii) Center of mass of a Semicircular Ring: Figure shows the object (semi circular ring). By observation we can say that the x-coordinate of the center of mass of the ring is zero as the half ring is symmetrical about y-axis on both sides of the origin. Only we are required to find the y-coordinate of the center of mass.



To find y_{cm} we use

$$y_{cm} = \frac{1}{M}\int dm\, y \qquad ...(i)$$

Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width dθ. If radius of the ring is R then its y coordinate will be R sinθ, here dm is given as

$$dm = \frac{M}{\pi R} \times R\, d\theta$$

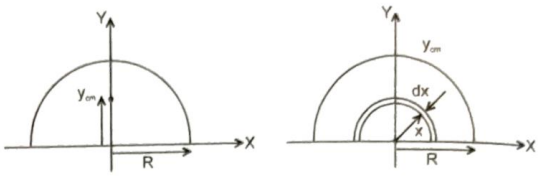
So from equation .....(ii)
we have

$$y_{cm} = \frac{1}{M}\int_0^{\pi} \frac{M}{\pi R} R d\theta\,(R\sin\theta) = \frac{R}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

$$y_{cm} = \frac{2R}{\pi} \qquad .....(iii)$$

(iii) Center of mass of Semicircular Disc: Figure shows the half disc of mass M and radius R. Here, we are only required to find the y-coordinate of the center of mass of this disc as center of mass will be located on its half vertical diameter. Here to find y_{cm}, we consider a small elemental ring of mass dm of radius x on the disc (disc can be considered to be made up such thin rings of increasing radii) which will be integrated from 0 to R. Here dm is given as

$$dm = \frac{2M}{\pi R^2}(\pi x)dx$$



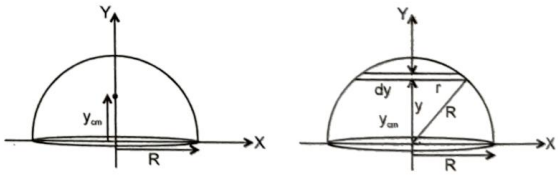
Now the y-coordinate of the element is taken as $\frac{2x}{\pi}$, as in previous section, we have derived that the center of mass of a semi circular ring is concentrated at $\frac{2R}{\pi}$

Here y_{cm} is given as

$$y_{cm} = \frac{1}{M}\int_0^R dm\ \frac{2x}{\pi} = \frac{1}{M}\int_0^R \frac{4M}{\pi R^2}x^2\, dx$$

$$y_{cm} = \frac{4R}{3\pi}$$

(iv) Center of mass of a solid Hemisphere: The hemisphere is of mass M and radius R. To find its center of mass (only y-coordinate), we consider an element disc of width dy, mass dm at a distance y from the center of the hemisphere. The radius of this elemental disc will be given as, $r = \sqrt{R^2 - y^2}$



The mass dm of this disc can be given as

$$dm = \frac{3M}{2\pi R^3} \times \pi r^2\, dy = \frac{3M}{2R^3}(R^2 - y^2)\, dy$$

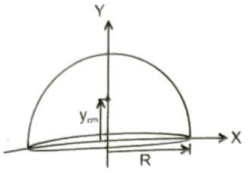
y_{cm} of the hemisphere is given as

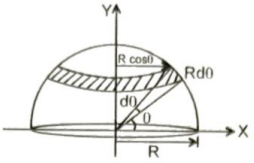
$$y_{cm} = \frac{1}{M}\int_0^R dm\, y = \frac{1}{M}\int_0^R \left[\frac{3M}{2R^3}(R^2 - y^2)\, dy\right] y$$

$$= \frac{3}{2R^3}\int_0^R (R^2 - y^2)\, y\, dy \qquad y_{cm} = \frac{3R}{8}$$

(v) **Center of mass of a Hollow Hemisphere:** A hollow hemisphere of mass M and radius R. Now we consider an elemental circular strip of angular width dθ at an angular distance θ from the base of the hemisphere. This strip will have an area.

$dS = 2\pi R \cos\theta \, R d\theta$





Its mass dm is given as

$$dm = \frac{M}{2\pi R^2} 2\pi R \cos\theta \, R d\theta$$

Here y-coordinate of this strip of mass dm can be taken as R sinθ. Now we can obtain the center of mass of the system as.

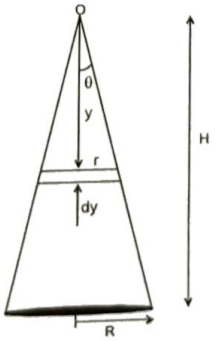
$$y_{cm} = \frac{1}{M}\int_0^{\frac{\pi}{2}} dm \, R\sin\theta$$

$$= \frac{1}{M}\int_0^{\frac{\pi}{2}} \left(\frac{M}{2\pi R^2} 2\pi R^2 \cos\theta \, d\theta\right) R\sin\theta$$

$$= R\int_0^{\frac{\pi}{2}} \sin\theta \cos\theta \, d\theta \Rightarrow y_{cm} = \frac{R}{2}$$

(vi) **Center of mass of a Solid Cone:** A solid cone has mass M, height H and base radius R. Obviously the center of mass of this cone will lie somewhere on its axis, at a height less than H/2. To locate the center of mass we consider an elemental disc of width dy and radius r, at a distance y from the apex of the cone. Let the mass of this disc be dm, which can be given as

$$dm = \frac{3M}{\pi R^2 H} \times \pi r^2 \, dy$$



here y_{cm} can be given as

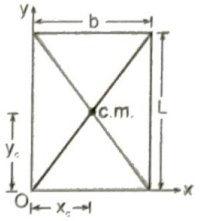
$$y_{cm} = \frac{1}{M}\int_0^H y \, dm$$

$$= \frac{1}{M}\int_0^H \left(\frac{3M}{\pi R^2 H}\pi\left(\frac{Ry}{H}\right)^2 dy\right) y$$

$$= \frac{3}{H^3}\int_0^H y^3 \, dy = \frac{3H}{4} \text{ (from verfex)}$$

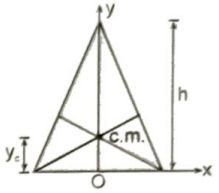
(vii) **Center of mass of some common systems**

⇒ Rectangular plate (By symmetry)



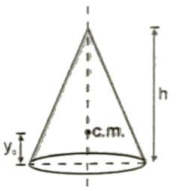
$$x_c = \frac{b}{2}; \quad y_c = \frac{L}{2}$$

⇒ A triangular plate (By qualitative argument)



at the centroid : $y_c = \frac{h}{3}$

⇒ A circular cone (hollow)

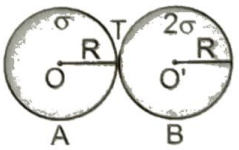


$$y_c = \frac{h}{3}$$

CENTER OF MASS OF COMBINATION OF MASSES

TRAIN YOUR BRAIN

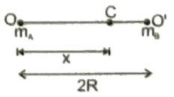
Example 6: Two circular disc having radius R and mass density σ and 2σ respectively are placed as shown in figure. Then find out the position of COM of the system (from point 'O').



Sol. Mass of disc A $m_A = \sigma\pi R^2$

Mass of disc B $m_B = 2\sigma\pi R^2$

Due to symmetry the COM of disc A lie at point O and COM of disc B lie at point O'. So we realize the above problem in a following way



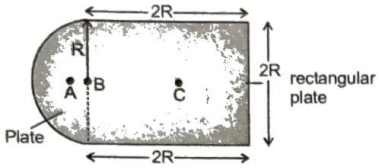
Centre of mass due to both the disc lie at point C (assume), having distance x from m_A

$$\Rightarrow x = \frac{m_B(2R)}{m_A + m_B} \; ; \; x = \frac{2\sigma\pi R^2(2R)}{\sigma(\pi R^2 + 2\pi R^2)}$$

$$x = \frac{4R}{3}$$

So the centre of mass lie in the disc B having distance $\frac{4R}{3}$ from O.

Example 7: Find out the position of centre of mass of the figure shown below.



Sol. We divide the above problem in two parts

(i) First find out position of centre of mass of both semicircular plate and rectangular plate separately.

(ii) Then find the position of centre of mass of given structure.

Centre of mass of semicircular disc lie at $\frac{4R}{3\pi}$

$$\Rightarrow AB = \frac{4R}{3\pi}$$

Centre of mass of rectangular plate lie at the centre of plate at point C

$$\Rightarrow BC = R \qquad \Rightarrow \; m_{SC} \text{ ——— } m_R$$

$$m_{SC} = \frac{\sigma\pi R^2}{2} \; ; \; m_R = \sigma 4R^2$$

$$\Rightarrow \; m_{SC} \text{ ——— C ——— } m_R \quad (r_1)$$

$\Rightarrow$ Let us assume COM is at r_1 distance from m_R

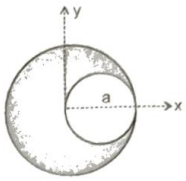
$$\Rightarrow r_1 = \frac{\sigma.\frac{\pi R^2}{2}\left(R + \frac{4R}{3\pi}\right)}{\sigma.\frac{\pi R^2}{2} + \sigma 4R^2}$$

$$\Rightarrow r_1 = \frac{R(3\pi+4)}{3(\pi+8)} \quad \textbf{Ans.}$$

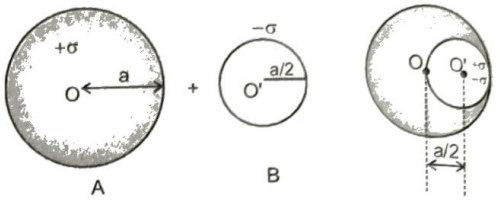
CAVITY PROBLEMS

If some mass or area is removed from a rigid body then the position of centre of mass of the remaining portion is obtained by assuming that in a remaining part +m & – m mass is there. Further steps are explained by following example.

Example 8: Find the position of centre of mass of the uniform lamina shown in figure. If the mass density of the lamina is σ.



Sol. We assume that in remaining portion a disc of radius a having mass density $+\sigma$ is there then we also include one disc of a/2 radius having $-\sigma$ mass density. So now the problem change in following form



So the centre of mass of both disc A & B lie in their respective centre such as O & O'.

Now

$$\Rightarrow \text{C.O.M. of the lamina} = \frac{m_B\, a/2}{m_A + m_B}$$

O ——— C ——— O'

m_A ——— a/2 ——— m_B

$m_A = \sigma(\pi a^2)$

$m_B = -\sigma(\pi)(a/2)^2$

$$= -\sigma\,\pi\frac{a^2}{4}$$

$$\Rightarrow X_{com} = \frac{-\left(\sigma\pi\frac{a^2}{4}\right)\times\frac{a}{2}}{\sigma\pi a^2 - \sigma\pi\frac{a^2}{4}}$$

$$= \frac{2a}{3} = \frac{-a}{6}$$

i.e., C.O.M lie on leftward side from point O.

MOTION OF THE CENTRE OF MASS

Now we can discuss the physical importance of the centre of mass concept. Consider the motion of a group of particles m_1, m_2, ..., m_n and whose total mass is M which is a constant. From the definition of the centre of mass, we have

$$M\,\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots\ldots + m_n\vec{r}_n$$

Differentiating this equation w.r.t. time, we obtain

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n$$

where $\vec{v}_1\left(=\frac{d\vec{r}_1}{dt}\right)$ is the velocity of the first particle, etc., and $\vec{v}_{cm}\left(=\frac{d\vec{r}_{cm}}{dt}\right)$ is the velocity of the centre of mass.

Differentiating the above equation w.r.t. time, we obtain

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots\ldots\ldots + m_n\vec{a}_n$$

where $\vec{a}_1$ is the acceleration of the first particle, etc., and $\vec{a}_{cm}$ is the acceleration of the centre of mass. Now, from Newton's second law the force $\vec{F}_1$ acting on the first particle is given by $\vec{F}_1 = m_1\vec{a}_1$. Likewise, $\vec{F}_2 = m_2\vec{a}_2$, etc. We can then write the above equation as $M\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \ldots\ldots\ldots + \vec{F}_n$

Among all these forces the internal forces are exerted by the particles on each other. However, from Newton's third law, these internal forces will occur in equal and opposite pairs, so that they contribute nothing to the sum. The right hand sum in the above equation then represents the sum of only the external forces acting on all the particles (system). We can then rewrite the above equation as

$$M\vec{a}_{cm} = \Sigma\vec{F}_{external}$$

This states that the centre of mass of a system of particles moves as though all the mass of the system is concentrated at the centre of mass and all the external forces were applied at that point.

One important situation is that in which $\Sigma\vec{F}_{external} = \vec{0}$ then $\vec{v}_{cm}$ = constant.

TRAIN YOUR BRAIN

Example 9: A dog of mass 10 kg stands on a stationary boat of mass 40 kg so that he is 20 m from the shore. He then walks 8 m on the boat towards the shore and halts. How far is he from the shore now? Assume that there is no friction between the boat and water.

Sol. In the horizontal direction, there is no force on the system (dog + boat). Therefore, the centre of mass of the system does not move in the horizontal direction. Since the dog moves towards the shore, the boat moves away from the shore to keep centre of mass stationary. Let d be the distance by which the boat moves backwards and let x be the initial distance of the boat from the shore. The initial x-coordinate of the centre

of mass $= \dfrac{10\times 20 + 40.x}{10+40}$

The final x-coordinate of the centre of mass

$$= \frac{10(20-8+d)+40(x+d)}{10+40}$$

Equating the two, we get

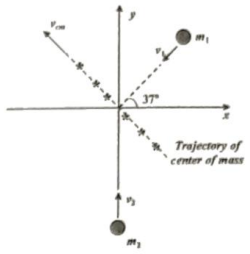
$$d = \frac{5}{8}\text{m}$$

$\therefore$ The dog is $20 - 8 + \dfrac{8}{5} = 13.6$ m from the shore.

Example 10: Two balls with masses $m_1 = 3$ kg and $m_2 = 5$ kg have initial velocities $v_1 = v_2 = 5$ m/s in the directions as shown in the figure. They collide at the origin.

(a) Find the velocity of the center of mass 3 s before the collision

(b) Find the position of the center of mass 2 s after the collision



Sol. (a) The given time is of no consequence since v_{cm} is constant for all times.

From equation, in component form

$$v_{cmx} = \frac{m_1v_{1x} + m_2v_{2x}}{M}$$

$$= \frac{(3\text{kg})(-5\cos 37^\circ\,\text{m/s}) + (5\text{kg})(0\,\text{m/s})}{8\text{kg}} = -1.5$$

$$v_{cmy} = \frac{m_1v_{1y} + m_2v_{2y}}{M}$$

$$= \frac{(3\text{kg})(-5\sin 37^\circ\,\text{m/s}) + (5\text{kg})(5\,\text{m/s})}{8\text{kg}} = +2 \text{ m/s}$$

Thus, $\vec{v}_{cm} = -1.5\,\hat{i} + 2\,\hat{j}$ m/s

(b) Since the collision occurs at the origin, the position of the center of mass 2s later is

$$\vec{r}_{c.m.} = \vec{v}_{cm}\,t = -3\,\hat{i} + 4\,\hat{j}\text{ m}$$

Example 11: A 75 kg man stands at the rear end of a platform of mass 25 kg and length 4m, which moves initially at 4 m/s over a frictionless surface. At t = 0, he walks at 2 m/s relative to the platform and then stops at the front end. During the period of walking, find the displacement of

(a) the platform,

(b) the man

(c) the center of mass

Sol. Initially the man, the platform, and the cm have the same velocity, 4m/s. When he begins to walk forward, his increase in momentum must be compensated by a decrease in the platform's momentum. Let us say that the velocity of the platform relative to the ground while he is walking i.e.

$$\vec{v}_{PG} = v_p\hat{i}$$

The velocity of man relative to the ground is then

$$\vec{v}_{MG} = \vec{v}_{MP} + \vec{v}_{PG} = (2 + v_p)\,\hat{i}.$$

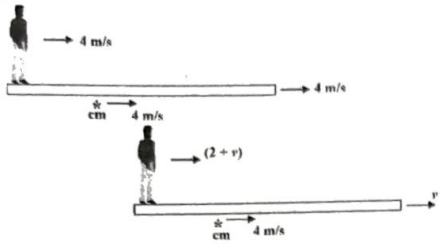
From the conservation of momentum, we have

$$(75 + 25) \times 4 = 75(2 + v_p) + 25\,v_p$$

Thus, the velocity of the platform is $v_p = 2.5$ m/s, and the velocity of the man is $v_m = 4.5$ m/s.

Since the velocity of man relative to platform is 2m/s, therefore, it takes $t = \dfrac{4\text{m}}{2\text{m/s}} = 2$ s for him to walk from the rear to the front.

Displacement of platform is $x_p = v_p t = (2.5)(2) = 5m$
Displacement of the man is $x_m = v_m t = (4.5)(2) = 9m$
Displacement of the center of mass $x_{cm} = v_{cm} t$
$= (4)(2) = 8$ m



Example 12: A particle of mass $m_1 = 4$ kg moves at $5\hat{i}$ m/s, while $m_2 = 2$ kg moves at $2\hat{i}$ m/s. Find

(a) the kinetic energy of center of mass

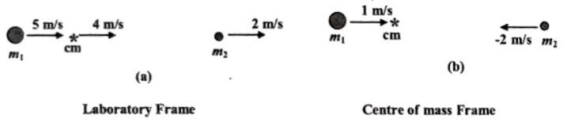
(b) the kinetic energy with respect to center of mass.

Sol. The velocity of the centre of mass is obtained as

$$v_{cm} = \frac{(4kg)(5m/s)+(2kg)(2m/s)}{6kg} = 4 \text{ m/s}$$

The following figure shows the velocities relative to the centre of mass

$v_1' = v_1 - v_{cm} = +1$ m/s
$v_2' = v_2 - v_{cm} = -2$ m/s



Thus

(a) $K_{cm} = \frac{1}{2}(m_1 + m_2)v_{cm}^2 = 48$ J

(b) $K' = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 = 6$ J

IMPULSE

In the previous chapter, we have learnt the concept of work which was an integral of force with respect to displacement. Now we are going to learn another concept, called impulse.

Impulse is defined as the integral of force with respect to time.

$$\vec{I} = \int_{t_i}^{t_f} \vec{F} dt$$

Since force is a vector and time is a scalar, the result of the integral in above equation is a vector. If the force is constant (both in magnitude and direction), it may be removed from the integral so that the integral is reduced to

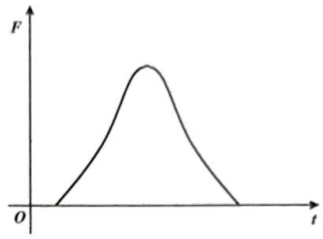
$$I = F\int_{t_i}^{t_f} dt = F(t_f - t_i) = F\Delta t$$

Graphically, the impulse is the area between the force curve and the F = 0 axis, as shown in figure.

The SI unit of impulse is Ns.

If more than one force are acting on a particle, then the net impulse is given by the time integral of the net force.

$$\vec{I}_{net} = \int_{t_i}^{t_f} \vec{F}_{net} dt$$



TRAIN YOUR BRAIN

Example 13: Find the impulse due to the force $\vec{F} = a\hat{i} + bt\hat{j}$, where a = 2 N and b = 4 N/s, if this force acts from $t_i = 0$ to $t_f = 0.3$ s.

Sol. $\vec{I} = \int_{t_i}^{t_f} \vec{F} dt = \int_0^{0.3} (a\hat{i} + bt\hat{j}) dt$

or $\vec{I} = a\hat{i}\int_0^{0.3} dt + b\hat{j}\int_0^{0.3} t dt$

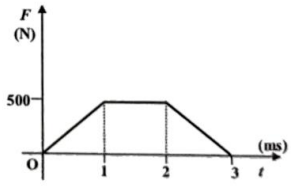
or $\vec{I} = at\hat{i}\Big|_0^{0.3} + \frac{bt^2}{2}\hat{j}\Big|_0^{0.3}$

or $\vec{I} = (2)(0.3)\,\hat{i} + \frac{(4)(0.3)^2}{2}\hat{j}$

thus $\vec{I} = (0.6\,\hat{i} + 0.18\,\hat{j})$ Ns

Example 14: Figure shows the variation of force acting on a body with time. Calculate the impulse of this force.

Sol. Impulse of a force is the area under the graph.



Thus

$$I = \left[\frac{1}{2}(500)(1) + (500)(2-1) + \frac{1}{2}(500)(3-2)\right] \times 10^{-3}$$

or $I = [250 + 500 + 250] \times 10^{-3}$
or $I = 1$ Ns

IMPULSE – MOMENTUM THEOREM

In the previous chapter, we have learnt that work done by a force brings about change in kinetic energy of a particle. Let us see what physical quantity changes due to impulse of a force.

According to Newton's second law, the net force acting on a particle is equal to the product of mass and acceleration.

$$\vec{F}_{net} = m\vec{a}$$

Since $\vec{a} = \frac{d\vec{v}}{dt}$, therefore

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt}$$

Substituting the value of net force, we get

$$\vec{I}_{net} = \int_{t_i}^{t_f}\left(m\frac{d\vec{v}}{dt}\right)dt$$

or $$\vec{I}_{net} = \int_{v_i}^{v_f} m\,d\vec{v}$$

Notice when we change the variable of integration from t to v, we must also change the time limits of the integral to the corresponding limits of v.

For constant mass,

$$\vec{I}_{net} = m\int_{v_i}^{v_f} d\vec{v}$$

or $$\vec{I}_{net} = m(\vec{v}_f - \vec{v}_i) = m\vec{v}_f - m\vec{v}_i$$

The quantity

$$\vec{p} = m\vec{v}$$

is a vector and is called the momentum of a particle of mass m moving with velocity $\vec{v}$.

Thus $$\vec{I}_{net} = \vec{p}_f - \vec{p}_i$$

or $$\vec{I}_{net} = \Delta\vec{p}$$

The above equation shows that the net impulse of forces acting on a particle is equal to the change in momentum of the particle. This is called the **Impulse-Momentum Theorem.**

TRAIN YOUR BRAIN

Example 15: A 2 kg bock is moving at a speed of 6 m/s. How large a force F is needed to stop the block in a time of 0.5 ms?

Sol. Impulse on block = Change in momentum of block

$Ft = mv_f - mv_i$

$F(5 \times 10^{-4}) = 2(0) - (2)(6)$

or $F = -2.4 \times 10^4$ N

The negative sign indicates that the force opposes the motion.

Example 16: A ball falling with velocity $\vec{v}_i = \left(-0.65\hat{i} - 0.35\hat{j}\right)$ m/s is subjected to a net impulse $\vec{I} = (0.6\,\hat{i} + 0.18\,\hat{j})$ Ns. If the ball has a mass of 275 g, calculate its velocity immediately following the impulse.

Sol. Using Impulse - Momentum Theorem

$$m\vec{v}_f - m\vec{v}_i = \vec{I}$$

or $$\vec{v}_f = \vec{v}_i + \frac{\vec{I}}{m}$$

Thus, $$\vec{v}_f = -0.65\,\hat{i} - 0.35\,\hat{j} + \frac{0.6\,\hat{i} + 0.18\,\hat{j}}{0.275}$$

or $$\vec{v}_f = (-0.65\,\hat{i} - 0.35\,\hat{j}) + \left(2.18\hat{i} + 0.655\hat{j}\right)$$

or $$\vec{v}_f = (1.53\,\hat{i} + 0.305\,\hat{j})\ \text{m/s}$$

NEWTON'S SECOND LAW AND THE RATE OF CHANGE OF MOMENTUM

From the impulse momentum theorem

$$\vec{I}_{net} = \Delta\vec{p}$$

Using definition of Impulse, we get

$$\int_{t_i}^{t_f}\vec{F}_{net}\,dt = \Delta\vec{p}$$

For constant net force,

$$\vec{F}_{net}\int_{t_i}^{t_f} dt = \Delta\vec{p}$$

or $$\vec{F}_{net}\,\Delta t = \Delta\vec{p}$$

If the force is not constant, then we can find the average force as

$$\vec{F}_{net\ av} = \frac{\Delta\vec{p}}{\Delta t}$$

For short intervals $\frac{\Delta\vec{p}}{\Delta t}$ approximates the instantaneous force. In the limit as Δt approaches zero the fraction $\frac{\Delta\vec{p}}{\Delta t}$ becomes the derivative.

Thus, $$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

The above equation is the momentum statement of the Newton's Second Law.

The net force acting on a particle is equal to the time rate of change of its linear momentum.

For a particle of constant mass the above equation reduces to the common statement of Newton's Second Law.

$$\vec{F}_{net} = \frac{d}{dt}(m\vec{v})$$

or $$\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

TRAIN YOUR BRAIN

Example 17: An 150 g ball is thrown at 30 m/s. It is struck by a bat, which gives it a velocity of 40 m/s in the opposite direction. If the time of contact is 10^{-2}s, what is the average force on the ball?

Sol. If we choose the original direction as +x axis, then

$$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = m(-40\,\hat{i} - 30\,\hat{i})$$

The average force is

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{-0.15kg \times 70\hat{i}}{10^{-2}} = -1050\hat{i}\,N$$

Notice that this is much larger than the weight (1.5 N) of the ball.

CONSERVATION OF LINEAR MOMENTUM

When the sum of the forces on an object is zero, the equation

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

tells us that the time derivative of momentum is zero.

That is, $\frac{d\vec{p}}{dt} = 0$

This implies $\vec{p}$ = constant

Consequently, one can state a conservation law for momentum:

When the net force on a particle is zero, its momentum is constant.

The real utility of the momentum conservation concept comes about when it is applied to a collection of particles. For a system of particles, the total momentum is simply the vector sum of the momentum of each of the particles in the system. That is

$$\vec{P} = \sum_{i=1}^{N}\vec{p}_i$$

Now consider the net force on a system of particles.

There are two kinds of forces:

(i) **Internal forces**, resulting from the forces between the particles within the system, and

(ii) **External forces,** arising from the forces between the particles in the system and objects outside the system.

For example, consider a system of two blocks joined together with a spring. If the system is allowed to fall freely under gravity, then the gravitational force acting on each block is the external force and the spring force acting on each block is internal.

When we calculate the net force on a system of particles by performing the vector sum, then the summation of all the internal forces is zero.

$$\sum_{\bullet}\vec{F}_{int} = 0$$

Thus the momentum statement of the Newton 's second law may be written as

$$\vec{F}_{net} = \sum\left(\vec{F}_{ext} + \vec{F}_{int}\right) = \frac{d\vec{P}}{dt}$$

or $$\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

Now, if the summation of external forces is zero, then

$$\frac{d\vec{P}}{dt} = 0$$

Thus, $\vec{P}$ = constant

In the absence of a net external force, the momentum of a system is conserved.

The conservation of momentum law can be used to relate the initial motion of particles within a system to the motion of those same particles some time later. The law emphasizes the equality of momentum before and after something happens within the system.

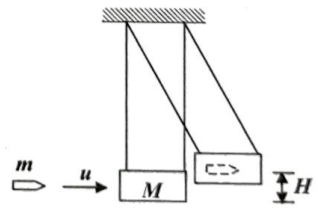
Thus, the conservation law is usually written as

$$\vec{P}_{initial} = \vec{P}_{final}$$

$$\sum_{i=1}^{N}\vec{p}_i = \sum_{f=1}^{N}\vec{p}_f$$

TRAIN YOUR BRAIN

Example 18: Suppose that a bullet, of mass m = 10 g and speed u, is fired into a block of mass M = 2 kg suspended as in figure. The bullet embeds in the block and raises it by a height H = 5 cm.



(a) How can one determine u from H?

(b) What is the thermal energy generated?

Sol. If the collision occurs within a very short time interval, the ropes will remain essentially vertical as the bullet comes to rest. Thus, there will be no external horizontal force. We may apply the conservation of linear momentum along this direction.

$$mu = (m + M)V \qquad ...(i)$$

where V is the common velocity after the collision. Since the collision is completely inelastic part of the bullet's initial kinetic energy, $\frac{1}{2}mu^2$, is converted into thermal energy. Only the kinetic energy of the bullet-block system that remains after the collision is available to raise the system by height H. From the conservation of mechanical energy we have

$\frac{1}{2}(m + M)V^2 = (m + M)gH$ ...(ii)

Hence, $V = (2gH)^{1/2}$. Substituting this into equation (i), we find

$u = \frac{(m+M)\sqrt{2gH}}{m}$

Putting $m = 10^{-2}$ kg; $M = 2$ kg; $H = 0.05$ m; $g = 10 m/s^2$

We get $u = \left(1+\frac{2}{10^{-2}}\right)\sqrt{2(10)(0.05)}$

or $u = 201$ m/s

(b) The kinetic energies before and after the collision are

$K_i = \frac{1}{2}mu^2 = 202$ J and $K_f = \frac{1}{2}(m + M)V^2 = 1$ J

The change in the kinetic energy as a result of the collision is –201J. Virtually all the bullet's kinetic energy is converted into thermal energy.

COLLISION

Collision is a brief event between objects that contact each other. The interaction between two or more objects is called a collision if there exists three identifiable stages to this interaction: before, during and after. In the before and after stage the interaction forces are zero or approaches zero asymptotically. Between these two states the interaction forces are large and often the dominating forces governing the object's motion. The magnitude of the interacting force is often unknown. Therefore, the conservation of momentum statement is useful for relating the initial velocities before the interaction to the final velocities after the interaction without requiring a detailed knowledge of the interaction forces.

Types of Collision

Collisions may be either elastic or inelastic. Linear momentum is conserved in both cases. A perfectly elastic collision is defined as one in which the total kinetic energy of the particles is also conserved.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

In an inelastic collision, the total kinetic energy of the particle changes. Some of the kinetic energy is stored as potential energy associated with a change in internal structure or state, and is not immediately recovered. Some of the energy may be used to raise the system (e.g. an atom) to a state with higher energy. Or, it may be converted into thermal energy of vibrating atoms and molecules or into light, sound or some other form of energy.

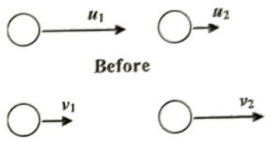
In a completely inelastic collision, the two bodies couple or stick together.

Coefficient of Restitution (e)

The elasticity of collision may be measured in terms of a dimensionless parameter called the coefficient of restitution (e).

It is defined as the ratio of velocity of separation to the velocity of approach of the two colliding bodies

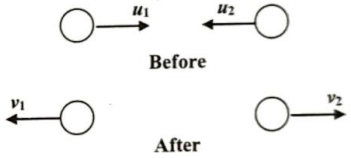
$$e = \frac{\text{velocity of separation}}{\text{velocity of appraoch}}$$



Velocity of separation $= v_2 - v_1$

Velocity of approach $= u_1 - u_2$

$\therefore e = \frac{v_2 - v_1}{u_1 - u_2}$



Velocity of separation $= v_1 + v_2$

Velocity of approach $= u_1 + u_2$

$\therefore e = \frac{v_1 + v_2}{u_1 + u_2}$

For an elastic collision: $e = 1$

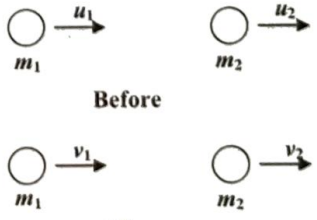
For an inelastic collision: $0 < e < 1$

For completely inelastic collision: $e = 0$

ELASTIC COLLISION IN ONE DIMENSION

Given figure shows a one - dimensional elastic collision between two balls.

Applying momentum conservation,



$P_i = P_f$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or $m_1(u_1 - v_1) = m_2(v_2 - u_2)$

Taking e = 1, velocity of separation = velocity of approach

or $v_2 - v_1 = u_1 - u_2$

Let us consider two special cases of elastic collisions: First, when the particles have equal mass, and second when one of them, say m_2, is initially at rest.

(i) Equal masses: $m_1 = m_2 = m$

The above equations become $u_1 + u_2 = v_1 + v_2$ and $v_2 - v_1 = u_1 - u_2$

Solving above equations

$v_1 = u_2$ and $v_2 = u_1$

i.e. in case of one dimensional elastic collision of particles of equal mass, the particles exchange their velocities.



(ii) Unequal masses

$m_1 \neq m_2$. Target at Rest: $u_2 = 0$

In this case, we get

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} \text{ and } v_2 = \frac{2m_1u_1}{m_1 + m_2}$$

(a) When $m_1 >> m_2$, we may ignore the mass of m_2 in comparison with m_1. This leads to $v_1 = u_1$ and $v_2 = 2u_1$, which means that m_1 maintains (approximately) its initial velocity u_1 but it imparts double this value to m_2.

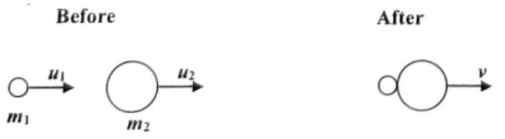


(b) When $m_1 << m_2$ we may ignore m_1 in comparison with m_2. We then find that $v_1 = u_1$ and $v_2 = 0$. Thus, m_1 reverses its velocity, leaving m_2 essentially unmoved as in figure.



INELASTIC COLLISION IN ONE DIMENSION

In case of inelastic collision, after collision the two bodies move with same velocity (or stick together).



If two particles of masses m_1 and m_2 moving with velocities u_1 and u_2 ($< u_1$) respectively along the same line collide "head-on" and after collision they have a common velocity v, then by conservation of linear momentum,

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

or $$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

The kinetic energy of the system before collision is

$$K_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

and kinetic energy after collision is

$$K_f = \frac{1}{2}(m_1 + m_2)v^2$$

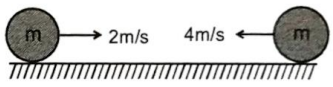
Therefore, loss in kinetic energy during the collision is

$$K_i - K_f = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

TRAIN YOUR BRAIN

Example 19: Two identical balls are approaching towards each other on a straight line with velocity 2 m/s and 4 m/s respectively. Find the final velocities, after elastic collision between them.



Sol. The two velocities will be exchanged and the final motion is reverse of initial motion for both.



Example 20: Two elastic bodies P and Q having equal masses are moving along the same line with velocities of 16 m/s and 10m/s respectively. Their velocities after the elastic collision will be in m/s -

(*a*) 0 and 25 (*b*) 5 and 20

(*c*) 10 and 16 (*d*) 20 and 5

Sol. (*c*)

Since collision is elastic so from linear momentum conservation

$m \times 26 = m(v_1 + v_2)$; $v_1 + v_2 = 26$ .....(i)

From K.E. conservation

$$\frac{1}{2}m(16^2 + 10^2) = \frac{1}{2}m(v_1^2 + v_2^2);$$

$356 = v_1^2 + v_2^2$ .....(ii)

From equation (i) & (ii)

$v_1 = 10$ m/s

$v_2 = 16$ m/s & option (C) is correct

Example 21: Two solid balls of rubber A and B whose masses are 200gm and 400gm respectively, are moving in mutually opposite directions. If the velocity A is 0.3 m/s and both the balls come to rest after collision, then the velocity of ball B is-

(*a*) 0.15 ms^{-1} (*b*) –0.15 ms^{-1}

(*c*) 1.5 ms^{-1} (*d*) none of these

Sol. (*b*)

From momentum conservation

$P_1 + P_2 = 0$ [Given : $m_1 = 200$ gm $m_2 = 400$ gm]

$m(0.3) + 2m.V_B = 0$

$V_B = -\frac{0.3}{2} = -0.15$ ms^{-1} so correct options is (B)

Example 22: As shown in figure A, B and C are identical balls B and C are at rest and, the ball A is moving with velocity v collides elastically with ball B, then after collision –



(*a*) All the three balls move with velocity v/2

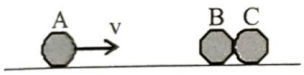
(*b*) A comes to rest and (B + C) moves with velocity $v/\sqrt{2}$

(*c*) A moves with velocity v and (B + C) moves with velocity v

(*d*) A and B come to rest and C moves with velocity v

Sol. (*d*)

From Question figure:



It should be follow momentum conservation

$p_A = p_B' + p_C'$

from option (A), $mv = m\left(\frac{v}{2}+\frac{v}{2}+\frac{v}{2}\right)=\frac{3}{2}mv$

$mv \neq \frac{3}{2}mv$ so its wrong

Option (B), $mv = m \times 0 + m\left(\frac{v}{\sqrt{2}}+\frac{v}{\sqrt{2}}\right)$

$mv \neq \sqrt{2}\,mv$ & its wrong

Option (C), $mv = mv + 2mv$

$= 3mv$ (wrong)

Option (D), $mv = m \times 0 + m \times 0 + mv$

$mv = mv$; or $p_i = p_f$

So option (D) is correct option

Example 23: A sphere of mass m moving with velocity u hits another stationary sphere of same mass. If e is the coefficient of restitution, what is the ratio of velocities of two spheres after the collision?

Sol. By definition of coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

i.e. $v_2 - v_1 = eu$ (i)

And by conservation of momentum

$mu = mv_1 + mv_2$

or $v_2 + v_1 = u$ (ii)

Solving equations (i) and (ii) for v_1 and v_2, we get

$v_1 = u(1 - e)/2$ and $v_2 = u(1 + e)/2$

so $\frac{v_1}{v_2} = \left[\frac{1-e}{1+e}\right]$

Example 24: A moving particle of mass m makes a head on elastic collision with a particle of mass 2m which is initially at rest. Show that the colliding particle losses (8/9)th of its energy after collision.

Sol. Let u be the initial velocity of body of mass m and v_1 and v_2 be the velocities of bodies of mass m and 2m respectively after the collision. As in collision momentum is always conserved

$mu = mv_1 + 2mv_2$

i.e. $v_1 + 2v_2 = u$ or $(u - v_1) = 2v_2$ ...(i)

And as the collision in elastic

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2$$

i.e. $v_1^2 + 2v_2^2 = u^2$ or $\left(u^2 - v_1^2\right) = 2v_2^2$ ...(ii)

Dividing equation (ii) by (i), we get

$u + v_1 = v_2$

Substituting this value of v_2 in equation (i),

$u - v_1 = 2(u + v_1)$, i.e., $v_1 = -(u/3)$

So $K_f = \frac{1}{2}mv_1^2 = \frac{1}{2}m\left(-\frac{u}{3}\right)^2 = \frac{1}{2}mu^2 \times \frac{1}{9} = \frac{1}{9}K_i$

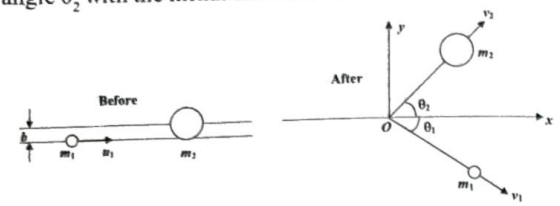
[as $K_i = \frac{1}{2}mu^2$]

$\therefore \quad \frac{K_i - K_f}{K_i} = \frac{8}{9}$

COLLISION IN TWO DIMENSIONS

In two or three dimensions (except for a completely inelastic collision) the conservation laws alone cannot tell us the motion of particles after a collision, if we know the motion before the collision. For example, for a two-dimensional elastic collision, which is simplest case, we have four unknowns, namely the two components of velocity for each of two particles after collision, but we have only three known relations between them, one for the conservation of kinetic energy and a conservation of momentum relation for each of the two dimensions. Hence we need more information than just the initial conditions. When we do not know the actual forces of interaction, as is often the case, the additional information must be obtained from experiment. It is simplest to specify the angle of recoil of one of the colliding particles.

A typical situation is shown in figure. The distance b between the initial line of motion and a line parallel to it through the center of the target particle is called the impact parameter. This is a measure of the directness of the collision, b = 0, corresponding to a head-on collision. The direction of motion of the incident particle m_1 after collision makes an angle θ_1 with the initial direction, and the target projectile m_2, initially at rest, moves in a direction after collision making an angle θ_2 with the initial direction of the incident projectile.



Applying the conservation of momentum, which is a vector relation, we obtain two scalar equations; for the x-component of motion we have

$$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

and for the y-component

$$0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2$$

If the collision is an elastic one we have,

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

We have four unknowns v_1, v_2, θ_1 and θ_2 and only three equations relating them.

We can determine the motion after collision only if we specify a value for one of these quantities such as θ_1.

TRAIN YOUR BRAIN

Example 25: A gas molecule having a speed of 300 m/s collides elastically with another molecule of the same mass which is initially at rest. After the collision the first molecule moves at an angle of 30° to its initial direction. Find the speed of each molecule after collision and the angle made with the incident direction by the recoiling target molecule.

Sol. This application corresponds exactly to the situation just discussed, with $m_1 = m_2$, the equations become

$u_1 = v_1\cos\theta_1 + v_2\cos\theta_2$ ...(1)

$v_1\sin\theta_1 = v_2\sin\theta_2$ ...(2)

and $u_1^2 = v_1^2 + v_2^2$ ...(3)

Squaring and adding the first two equations, we get

$$u_1^2 + v_1^2 - 2u_1v_1\cos\theta_1 = v_2^2$$

Combining this with the third equation, we obtain

$$2v_1^2 = 2u_1v_1\cos\theta_1$$

or (since $u_1 \neq 0$) $v_1 = u_1\cos\theta_1 = 260$ m/s

From the third equation, $v_2 = 150$ m/s

and finally from the second equation,

$\theta_2 = 60°$.

The molecules move apart at right angles ($\theta_1 + \theta_2 = 90°$). The student should be able to show that in an elastic collision between particles of equal mass, one of which is initially at rest, the recoiling particles always move off at right angles to one another.

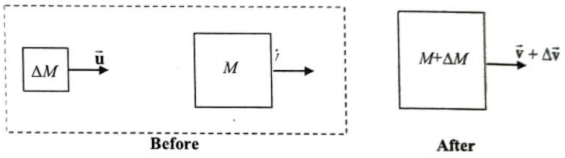
SYSTEM OF VARIABLE MASS

So far we have dealt with the dynamics of a system whose mass is constant. We now discuss the dynamics of a system, such as a rocket, whose mass varies. One may try to apply the following equation in such a case.

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} = M\frac{d\vec{v}}{dt} + \vec{v}\ \frac{dM}{dt}$$

However, this equation is **correct** in only a few very special cases. This approach is **not correct** when mass actually enters or leaves the system.

Let us examine the motion of a body of mass M, moving with velocity **v**. Another small body of mass ΔM approaches with velocity $\vec{u}$ along the same line. We assume $\vec{u} > \vec{v}$ and that after the collision the two bodies stick together and move at velocity $(\vec{v} + \Delta\vec{v})$, as shown in the figure.



By defining the system comprising of both the bodies we can use $\vec{F}_{ext} = \frac{d\vec{P}}{dt}$, where $\vec{p}$ is the total momentum of the system of constant mass, M + ΔM. The change in momentum of the system in time Δt is

$$\Delta\vec{P} = (M+\Delta M)(\vec{v}+\Delta\vec{v}) - M\vec{v} - \Delta M\vec{u} = M\Delta\vec{v} - (\vec{u}-\vec{v})\Delta M$$

Now $(\vec{u}-\vec{v}) = \vec{v}_{rel}$ is the velocity of ΔM relative to M before the collision.

We divide both sides of this equation by Δt and take the limit as $\Delta t \to 0$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} = M\frac{d\vec{v}}{dt} - \vec{v}_{rel}\frac{dM}{dt}$$

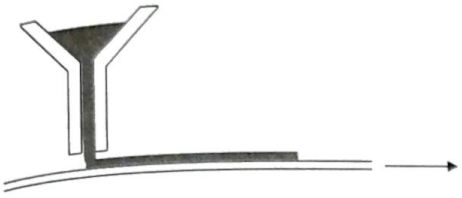
It is convenient to rewrite it in the form

$$M\frac{d\vec{v}}{dt} = \vec{F}_{ext} + \vec{v}_{rel}\frac{dM}{dt}$$

The last term in the above equation $\left(\vec{v}_{rel}\frac{dM}{dt}\right)$, is the rate at which momentum is being transferred into (or out of) the system by the mass that system has collected (or ejected). It can be interpreted as the force exerted on the system by the mass that leaves or joins it. This force is referred to as the reaction force.

TRAIN YOUR BRAIN

Example 26: A hopper releases grain at a rate dm/dt onto a conveyor belt that moves at a constant speed v. What is the power of the motor driving the belt?



Sol.

Let the system has some arbitrary length of belt whose mass we can call M. This mass of the system increases at the same rate as that the grain falls,

so $dM/dt = dm/dt$.

Since the grain falls vertically, $\vec{u} = 0$ and $\vec{v}_{rel} = \vec{u} - \vec{v} = -\vec{v}$. Since the speed is constant,

$dv/dt = 0$. Thus from equation for the system of variable mass

$$\vec{F}_{ext} - \vec{v}\frac{dm}{dt} \Rightarrow \vec{F}_{ext} = \vec{v}\frac{dm}{dt}$$

where $\vec{F}_{ext}$ is the force needed to maintain constant speed because the mass is increasing. The power required ($P = \vec{F} \cdot \vec{v}$) is

$$P = V^2\frac{dm}{dt}$$

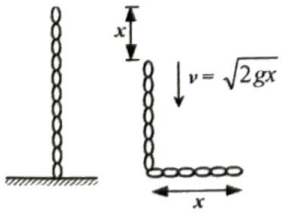
It is interesting to compare this with the rate at which the kinetic energy of the grain increases.

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}V^2\frac{dm}{dt}$$

This is only half the power input. The other half is dissipated as heat when the grain lands on the belt and slips relative to it.

Example 27: A uniform chain of mass m and length l hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the chain on the surface when half of its length has fallen on the table. The fallen part does not form heap.

Sol. Let N be the normal reaction acting on the chain.



It consists of two parts

$N = N_{wt} + N_{thrust}$

$$N_{wt} = \frac{m}{l}xg$$

$$N_{thrust} = v_{rel}\frac{dm}{dt}$$

If v be the instantaneous velocity of the particle falling on the floor, then

$v_{rel} = v - 0 = v$

$$\frac{dm}{dt} = \frac{dm}{dx}\frac{dx}{dt} \text{ or } \frac{dm}{dt} = v\left(\frac{m}{l}\right)$$

Thus $N_{th} = v^2\left(\frac{m}{l}\right)$

Since $v = \sqrt{2gx}$

$$\therefore N_{th} = \frac{2mg}{l}x$$

Total reaction is

$$N = \frac{mgx}{l} + \frac{2mgx}{l}$$

$$N = 3mg\left(\frac{x}{l}\right)$$

for $x = l/2$

$$N = \frac{3}{2}mg$$

ROCKET PROPULSION

Consider a rocket of mass M with fuel of mass Δm. Their common velocity is **v** relative to some inertial frame. When the rocket engines are fired, the gases are expelled backward with an exhaust velocity $\vec{v}_{ex} = -v_{ex}\,\hat{i}$ relative to the rocket. This is a fixed quantity determined by the design of the engine and the type of fuel. If the rocket's velocity changes to $\vec{v} + \Delta\vec{v}$ relative to the inertial frame, then the velocity of gas with respect to the frame will be

$$\vec{v}_{gas} = \vec{v}_{ex} + \vec{v} + \Delta\vec{v} = (-v_{ex} + v + \Delta v)\,\hat{i}$$

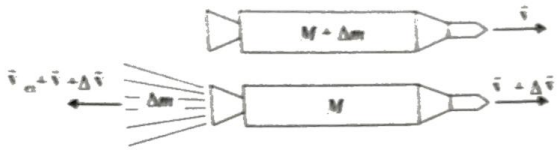
Applying the law of conservation of momentum, we get

$(M + \Delta m)\,v$

$= M(v + \Delta v) + \Delta m(-v_{ex} + v + \Delta v)$

After some cancellation we find

$0 = M\Delta v + \Delta m(-v_{ex} + \Delta v)$



If both Δv and Δm are small quantities relative to v and M, respectively, their product, $\Delta v \Delta m$, is negligible in comparison with the other terms, and we are left with

$$\Delta v = v_{ex} \frac{\Delta m}{M}$$

Since an increase in the mass of the expelled gases corresponds exactly to the loss in mass of the rocket system, we have $\Delta m = -\Delta M$. In the limit as $\Delta M \to 0$ the above equation becomes

$$dv = -v_{ex} \frac{dM}{M}$$

On integrating both sides,

$$\int_{v_i}^{v_f} dv = -\int_{M_i}^{M_f} v_{ex} \frac{dM}{M}$$

We find

$$v_f - v_i = v_{ex} \ln \frac{M_i}{M_f}$$

TRAIN YOUR BRAIN

Example 28: The mass of a rocket is 2.8×10^6 kg at the launch time. Of this, 2×10^6 kg is fuel. The exhaust speed is 2500 m/s and the fuel is ejected at the rate of 1.4×10^4 kg/s.

(a) Find the thrust of the rocket

(b) What is its initial acceleration at launch time? Ignore air resistance.

Sol. (a) The magnitude of the thrust is given by

$$\text{Thrust} = v_{ex} \frac{dM}{dt} = (2500)(1.4 \times 10^4) = 3.5 \times 10^7 \text{ N}$$

(b) Using equation

$$M \frac{dv}{dt} = F_{ext} + v_{rel} \frac{dM}{dt}$$

We get the acceleration

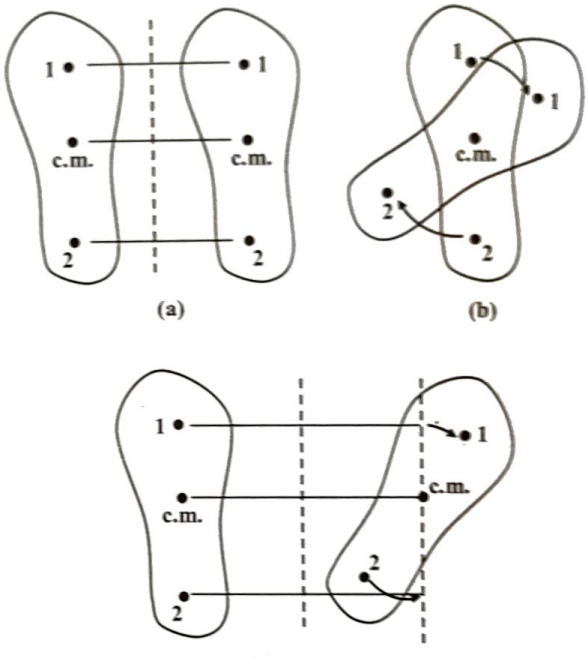
$$a = \frac{dv}{dt} = \frac{F_{ext}}{M} + \frac{v_{rel}}{M} \frac{dM}{dt}$$

$$\frac{dv}{dt} = -g + \frac{1}{M} v_{ex} \frac{dM}{dt} \quad = -9.8 + 12.5 = +2.7 \text{ m/s}^2$$

ROTATIONAL KINEMATICS

In the previous chapters we have studied the translatory motion. In this chapter we will study the rotational motion of a rigid body about a fixed axis. A rigid body is defined as an object that has fixed size and shape. In other words, the relative positions of its constituent particles remain constant.

Actually a rigid body does not exist – it is an useful idealization. By the term **fixed axis**, we mean that the axis must be fixed relative to the body and fixed in direction relative to an inertial position. In figure (a) below, translatory motion of a rigid body is shown. In figure (b) rotation about an axis which is fixed both in space and direction is shown. In figure (c), combined translation and rotation is shown.

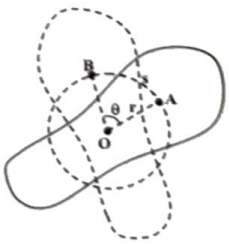


VARIABLES OF ROTATIONAL MOTION

Consider a body of arbitrary shape rotating about a fixed axis 'O' as shown in figure. In a given interval all the particles lying on the line OA move to their corresponding positions lying on OB.

Although the particles of the body have different linear displacements, they all have the same angular displacement

θ, which is given by $\theta = \frac{s}{r}$



The average angular velocity of the body for a finite time interval is given by

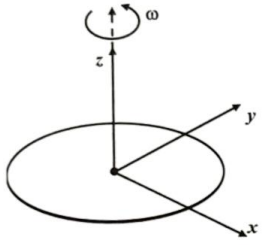
$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

The unit of angular velocity is radian per second (rad/s). The instantaneous angular velocity is defined as

$$\omega = \underset{\Delta t \to 0}{\text{Lim}} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

It is the rate of change of the angular position θ with respect to time. It is a vector quantity.

The direction of angular velocity is given by the right-hand rule. We hold the right hand such that when the fingers of the right hand curve in the sense of rotation, the thumb points in the direction of ω.



PERIOD AND FREQUENCY OF REVOLUTION

The **period T** is the time for one revolution and the **frequency** f is the number of revolutions per second (rev/s). The period and frequency are related as

$$f = \frac{1}{T}.$$

If the angular velocity is constant, the instantaneous and average angular velocities are equal. In terms of period T and frequency f the angular velocity is given by $\omega = \frac{2\pi}{T} = 2\pi f$

The relation between **linear speed** and **angular speed** is obtained by differentiating equation

$\theta = \frac{s}{r}$ with respect to time. $\frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt}$ or $\omega = \frac{v}{r}$

or $v = \omega r$

Although all particles have the same angular velocity, their speeds increase linearly with distance from the axis of rotation.

The average angular acceleration is defined as

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

and the instantaneous angular acceleration is defined as

$$\alpha = \underset{\Delta t \to 0}{\text{Lim}} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Angular acceleration is a vector quantity measured in rad/s².

Rotation with Constant Angular Acceleration

When the angular acceleration is constant, we can find the change in angular velocity by integrating equation $\alpha = \frac{d\omega}{dt}$

$d\omega = \alpha\, dt$

$$\text{or} \int_{\omega_o}^{\omega} d\omega = \int_0^t \alpha dt$$

or $\omega - \omega_o = at$ or $\omega = \omega_o + \alpha t$

Substituting the value of ω in equation $\omega = \frac{d\theta}{dt}$,

we get

$d\theta = \omega dt = (\omega_o + \alpha t)dt$

On integrating $\int_{\theta_o}^{\theta} d\theta = \int_0^t (\omega_o + \alpha t)dt$

$$\theta - \theta_o = \omega_o t + \frac{1}{2}\alpha t^2$$

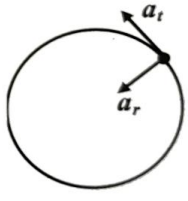
or $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

On eliminating t from the above equations, we get

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

TOTAL ACCELERATION IN ROTATIONAL MOTION

A particle moving in a circular path with speed v has a centripetal (or radial) acceleration $a_t = \frac{v^2}{r} = \omega^2 r$



If there is angular acceleration, the speed of the particle changes and thus we can find the tangential acceleration

$$a_t = \frac{dv}{dt} = \left(\frac{d\omega}{dt}\right) r = \alpha r$$

The net linear acceleration is

$$\vec{a} = \vec{a}_r + \vec{a}_t$$

The magnitude of the net acceleration is given by

$$a = \sqrt{a_r^2 + a_t^2}$$

Analogy Between Rotational Kinematics and Linear Kinematics

Quantity	Linear	Rotational
1. Position	x	θ
2. Displacement	Δx	$\Delta\theta$
3. Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
4. Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$
	$v = \int a dt + c$	$\omega = \int \alpha dt + c$
	$x = \int v dt + c'$	$\theta = \int \omega dt + c'$
5. Equations of Kinematics for constant linear acceleration / angular acceleration	$v = v_o + at$	$\omega = \omega_o + \alpha t$
	$x = x_o + v_o t + \frac{1}{2}at^2$	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$
	$v^2 - v_o^2 = 2ax$	$\omega^2 - \omega_o^2 = 2\alpha\theta$

TRAIN YOUR BRAIN

Example 29: A disc starts rotating with constant angular acceleration of p rad/s^2 about a fixed axis perpendicular to its plane and through its centre at t = 0.

(a) Find the angular velocity of the disc at t = 4 s.

(b) Find the angular displacement of the disc in 4 s and

(c) Find number of turns accomplished by the disc in 4 s.

Sol. Here a = p rad/sec^2

$\omega_0 = 0$

t = 4 sec

(a) $\omega = 0 + (\pi \text{ rad/s}^2) \times 4 \text{ sec} = 4\pi \text{ rad/s}.$

(b) $\theta = 0 + \frac{1}{2}(\pi \text{ rad/s}^2) \times (16 \text{ s}^2) = 8\pi \text{ rad}.$

(c) Let the number of turns be n

$\Rightarrow$ n (2 π) rad = 8 π rad $\Rightarrow$ n = 4

Example 30: A wheel rotates with an angular acceleration given by $\alpha = 4at^3 - 3bt^2$, where t is the time and a and b are constants. If the wheel has initial angular speed ω_0, write the equations for the:

(i) angular speed

(ii) angular displacement.

Sol. (i) We know

$d\omega = \alpha\, dt$

Integrating both sides, we get

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\, dt = \int_0^t (4at^3 - 3bt^2)\, dt$$

or $\omega = \omega_0 + at^4 - bt^3$

(ii) We know

$d\theta = \omega dt$

On integrating both the sides, we get

$$\int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_O^t (\omega_0 + at^4 - bt^3)\, dt$$

or $\theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$

TRAIN YOUR BRAIN

Example 31: A flywheel of radius 20 cm starts from rest, and has a constant angular acceleration of 60 rad/s^2. Find

(a) the magnitude of the net linear acceleration of a point on the rim after 0.15 s

(b) the number of revolutions completed in 0.25 s

Sol. (a) The tangential acceleration is constant and given by

$a_t = \alpha r = (60 \text{ rad/s}^2)(0.2 \text{ m}) = 12 \text{ m/s}^2$

In order to calculate the radial acceleration we first need to find the angular velocity at the given time. We have

$\omega = \omega_0 + \alpha t = 0 + (60 \text{ rad/s}^2)(0.15 \text{ s}) = 9 \text{ rad/s}$

$\therefore$ $a_r = \omega^2 r = (81 \text{ rad}^2/\text{s}^2)(0.2 \text{ m}) = 16.2 \text{ m/s}^2$

The magnitude of the net linear acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = 20.2\ m/s^2$$

(b) Finally

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(60\ rad/s^2)(0.25s)^2 = 1.88\ rad$$

This corresponds to (1.88 rad) (1 rev/2π rad) = 0.3 rev

MOMENT OF INERTIA

Like the centre of mass, the moment of inertia is a property of an object that is related to its mass distribution. The moment of inertia (denoted by I) is an important quantity in the study of system of particles that are rotating. The role of the moment of inertia in the study of rotational motion is analogous to that of mass in the study of linear motion. Moment of inertia gives a measurement of the resistance of a body to a change in its rotaional motion.

If a body is at rest, the larger the moment of inertia of a body the more difficuilt it is to put that body into rotational motion. Similarly, the larger the moment of inertia of a body, the more difficult to stop its rotational motion. The moment of inertia is calculated about some axis (usually the rotational axis).

Moment of inertia depends on :

(i) density of the material of body
(ii) shape & size of body
(iii) axis of rotation

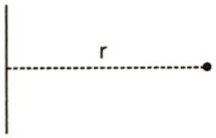
In totality we can say that it depends upon distribution of mass relative to axis of rotation.

Note

Moment of inertia does not change if the mass :
(i) is shifted parallel to the axis of the rotation
(ii) is rotated with constant radius about axis of rotation

Moment of Inertia of a Single Particle

For a very simple case the moment of inertia of a single particle about an axis is given by,



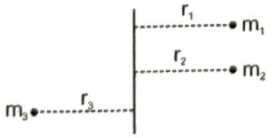
$$I = mr^2 \qquad ...(i)$$

Here, m is the mass of the particle and r its distance from the axis under consideration.

MOMENT OF INERTIA OF A SYSTEM OF PARTICLES

The moment of inertia of a system of particles about an axis is given by,

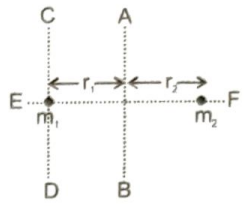
$$I = \sum_i m_i r_i^2 \qquad ...(ii)$$



where r_i is the perpendicular distance from the axis to the ith particle, which has a mass m_i.

TRAIN YOUR BRAIN

Example 32: Two heavy particles having masses m_1 & m_2 are situated in a plane perpendicular to line AB at a distance of r_1 and r_2 respectively.



(i) What is the moment of inertia of the system about axis AB?
(ii) What is the moment of inertia of the system about an axis passing through m_1 and perpendicular to the line joining m_1 and m_2?
(iii) What is the moment of inertia of the system about an axis passing through m_1 and m_2?

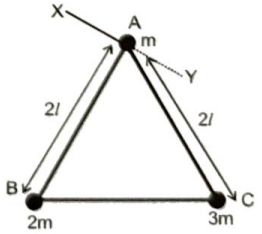
Sol. (i) Moment of inertia of particle on left is $I_1 = m_1r_1^2$.
Moment of Inertia of particle on right is $I_2 = m_2r_2^2$.
Moment of Inertia of the system about AB is
$I = I_1 + I_2 = m_1r_1^2 + m_2r_2^2$

(ii) Moment of inertia of particle on left is $I_1 = 0$
Moment of Inertia of the system about CD is
$I = I_1 + I_2 = 0 + m_2(r_1 + r_2)^2$

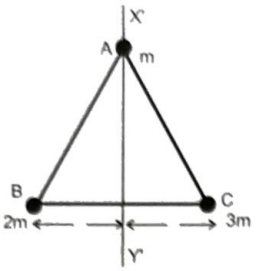
(iii) Moment of inertia of particle on left is $I_1 = 0$
Moment of inertia of particle on right is $I_2 = 0$
Moment of Inertia of the system about EF is
$I = I_1 + I_2 = 0 + 0$

Example 33: Three light rods, each of length 2ℓ, are joined together to form a triangle. Three particles A, B, C of masses m, 2m, 3m are fixed to the vertices of the triangle. Find the moment of inertia of the resulting body about
(a) an axis through A perpendicular to the plane ABC,
(b) an axis passing through A and the midpoint of BC.

Sol.

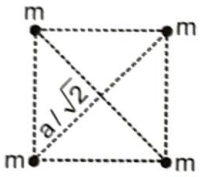


(a) B is at a distant 2ℓ from the axis XY so the moment of inertia of B (I_B) about XY is $2\,m\,(2\ell)^2 + 3m\,(2\ell)^2 = 20\,m\ell^2$
The moment of inertia of the body about X' Y' is
$m(0)^2 + 2m(\ell)^2 + 3m(\ell)^2 = 5\,m\ell^2$



Example 34: Four particles each of mass m are kept at the four corners of a square of edge a. Find the moment of inertia of the system about a line perpendicular to the plane of the square and passing through the centre of the square.

Sol. The perpendicular distance of every particle from the given line is $a/\sqrt{2}$. The moment of inertia of one particle is, therefore, $m(a/\sqrt{2})^2 = \frac{1}{2}ma^2$. The moment of inertia of the system is, therefore, $4 \times \frac{1}{2}ma^2 = 2\,ma^2$.

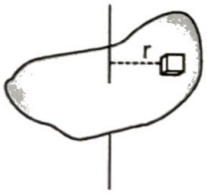


MOMENT OF INERTIA OF RIGID BODIES

For a continuous mass distribution such as found in a rigid body, we replace the summation of $I = \sum_i m_i r_i^2$ by an integral.

If the system is divided into infinitesimal element of mass dm and if r is the distance from a mass element to the axis of rotation, the moment of inertia is,

$$I = \int r^2\,dm$$

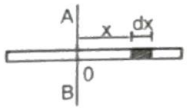


(A) Uniform rod about a perpendicular bisector

Consider a uniform rod of mass M and length l figure and suppose the moment of inertia is to be calculated about the bisector AB. Take the origin at the middle point O of the rod. Consider the element of the rod between a distance x and x + dx from the origin. As the rod is uniform,

Mass per unit length of the rod = M/l

so that the mass of the element = (M/l)dx.



The perpendicular distance of the element from the line AB is x. The moment of inertia of this element about AB is

$$dI = \frac{M}{l}dx\ x^2.$$

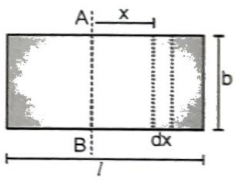
When $x = -l/2$, the element is at the left end of the rod. As x is changed from $-l/2$ to $l/2$, the elements cover the whole rod.

Thus, the moment of inertia of the entire rod about AB is

$$I = \int_{-l/2}^{l/2} \frac{M}{l}x^2dx = \left[\frac{M}{l}\frac{x^3}{3}\right]_{-l/2}^{l/2} = \frac{Ml^2}{12}$$

(B) Moment of inertia of a rectangular plate about a line parallel to an edge and passing through the centre

The situation is shown in figure. Draw a line parallel to AB at a distance x from it and another at a distance x + dx. We can take the strip enclosed between the two lines as the small element.



It is "small" because the perpendiculars from different points of the strip to AB differ by not more than dx. As the plate is uniform,

its mass per unit area = $\frac{M}{bl}$

Mass of the strip = $\frac{M}{bl}b\,dx = \frac{M}{l}dx$.

The perpendicular distance of the strip from AB = x.

The moment of inertia of the strip about

AB = $dI = \frac{M}{l}dx\,x^2$. The moment of inertia of the given plate is, therefore,

$$I = \int_{-l/2}^{l/2} \frac{M}{l}x^2dx = \frac{Ml^2}{12}$$

The moment of inertia of the plate about the line parallel to the other edge and passing through the centre may be obtained from the above formula by replacing l by b and thus,

$$I = \frac{Mb^2}{12}.$$

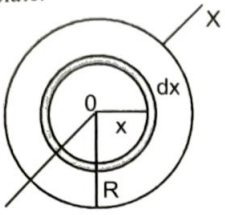
(C) Moment of inertia of a circular ring about its axis (the line perpendicular to the plane of the ring through its centre)

Suppose the radius of the ring is R and its mass is M. As all the elements of the ring are at the same perpendicular distance R from the axis, the moment of inertia of the ring is

$$I = \int r^2\, dm = \int R^2\, dm = R^2 \int dm = MR^2.$$

(D) Moment of inertia of a uniform circular plate about its axis

Let the mass of the plate be M and its radius R. The centre is at O and the axis OX is perpendicular to the plane of the plate.



Draw two concentric circles of radii x and x + dx, both centred at O and consider the area of the plate in between the two circles.

This part of the plate may be considered to be a circular ring of radius x. As the periphery of the ring is 2π x and its width is dx, the area of this elementary ring is $2\pi x dx$. The area of the plate is πR^2. As the plate is uniform,

$$\text{Its mass per unit area} = \frac{M}{\pi R^2}$$

$$\text{Mass of the ring} = \frac{M}{\pi R^2} 2\pi x\, dx = \frac{2Mx\, dx}{R^2}$$

Using the result obtained above for a circular ring, the moment of inertia of the elementary ring about OX is

$$dI = \left[\frac{2Mx\, dx}{R^2}\right] x^2.$$

The moment of inertia of the plate about OX is

$$I = \int_0^R \frac{2M}{R^2} x^3\, dx = \frac{MR^2}{2}.$$

(E) Moment of inertia of a hollow cylinder about its axis

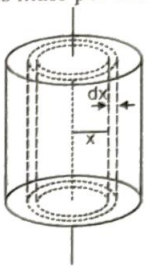
Suppose the radius of the cylinder is R and its mass is M. As every element of this cylinder is at the same perpendicular distance R from the axis, the moment of inertia of the hollow cylinder about its axis is

$$I = \int r^2\, dm = R^2 \int dm = MR^2$$

(F) Moment of inertia of a uniform solid cylinder about its axis

Let the mass of the cylinder be M and its radius R. Draw two cylindrical surface of radii x and x + dx coaxial with the given cylinder. Consider the part of the cylinder in between the two surface. This part of the cylinder may be considered to be a hollow cylinder of radius x. The area of crosssection of the wall of this hollow cylinder is 2π x dx. If the length of the cylinder is l, the volume of the material of this elementary hollow cylinder is 2π x dxl.

The volume of the solid cylinder is $\pi R^2 l$ and it is uniform, hence its mass per unit volume is



$$\rho = \frac{M}{\pi R^2 l}$$

The mass of the hollow cylinder considered is

$$\frac{M}{\pi R^2 l} 2\pi x\, dx\, l = \frac{2M}{R^2} x\, dx.$$

As its radius is x, its moment of inertia about the given axis is

$$dI = \left[\frac{2M}{R^2} x\, dx\right] x^2.$$

The moment of inertia of the solid cylinder is, therefore,

$$I = \int_0^R \frac{2M}{R^2} x^3 dx = \frac{MR^2}{2}.$$

Note that the formula does not depend on the length of the cylinder.

(G) Moment of inertia of a uniform hollow sphere about a diameter

Let M and R be the mass and the radius of the sphere, O its centre and OX the given axis (figure). The mass is spread over the surface of the sphere and the inside is hollow.

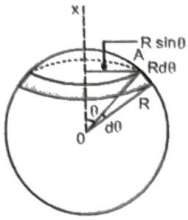
Let us consider a radius OA of the sphere at an angle θ with the axis OX and rotate this radius about OX. The point A traces a circle on the sphere. Now change θ to $\theta + d\theta$ and get another circle of somewhat larger radius on the sphere. The part of the sphere between these two circles, shown in the figure, forms a ring of radius R $\sin\theta$. The width of this ring is $Rd\theta$ and its periphery is $2\pi R \sin\theta$.

Hence, the area of the ring = $(2\pi R \sin\theta)(Rd\theta)$.

$$\text{Mass per unit area of the sphere} = \frac{M}{4\pi R^2}.$$

The mass of the ring

$$= \frac{M}{4\pi R^2}(2\pi R \sin\theta)(Rd\theta) = \frac{M}{2} \sin\theta\, d\theta.$$



The moment of inertia of this elemental ring about OX is

$$dI = \left(\frac{M}{2}\sin\theta\, d\theta.\right)(R\sin\theta)^2 = \frac{M}{2}R^2\sin^3\theta\, d\theta$$

As θ increases from 0 to π, the elemental rings cover the whole spherical surface. The moment of inertia of the hollow sphere is, therefore,

$$I = \int_0^{\pi}\frac{M}{2}R^2\sin^3\theta\, d\theta = \frac{MR^2}{2}\left[\int_0^{\pi}(1-\cos^2\theta)\sin\theta\, d\theta\right]$$

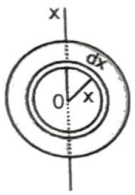
$$= \frac{MR^2}{2}\left[\int_{\theta=0}^{\pi} -(1-\cos^2\theta)\, d(\cos\theta)\right]$$

$$= \frac{-MR^2}{2}\left[\cos\theta - \frac{\cos^3\theta}{3}\right]_0^{\pi} = \frac{2}{3}MR^2$$

(H) Moment of inertia of a uniform solid sphere about a diameter

Let M and R be the mass and radius of the given solid sphere. Let O be centre and OX the given axis. Draw two spheres of radii x and x + dx concentric with the given solid sphere. The thin spherical shell trapped between these spheres may be treated as a hollow sphere of radius x.

The mass per unit volume of the solid sphere



$$= \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$$

The thin hollow sphere considered above has a surface area $4\pi x^2$ and thickness dx. Its volume is $4\pi x^2\, dx$ and hence its mass is

$$= \left(\frac{3M}{4\pi R^3}\right)(4\pi x^2\, dx) = \frac{3M}{R^3}x^2\, dx$$

Its moment of inertia about the diameter OX is, therefore,

$$dI = \frac{2}{3}\left[\frac{3M}{R^3}x^2 dx\right]x^2 = \frac{2M}{R^3}x^4 dx$$

If x = 0, the shell is formed at the centre of the solid sphere. As x increases from 0 to R, the shells cover the whole solid sphere.

The moment of inertia of the solid sphere about OX is, therefore,

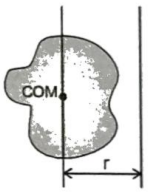
$$I = \int_0^R \frac{2M}{R^3}x^4 dx = \frac{2}{5}MR^2.$$

THEOREMS OF MOMENT OF INERTIA

There are two important theorems on moment of inertia, which, in some cases enable the moment of inertia of a body to be determined about an axis, if its moment of inertia about some other axis is known. Let us now discuss both of them.

Theorem of parallel axes

A very useful theorem, called the parallel axes theorem relates the moment of inertia of a rigid body about two parallel axes, one of which passes through the centre of mass.



Two such axes are shown in figure for a body of mass M. If r is the distance between the axes and I_{COM} and I are the respective moments of inertia about them, these moments are related by,

$I = I_{COM} + Mr^2$

- Theorem of parallel axis is applicable for any type of rigid body whether it is a two dimensional or three dimensional

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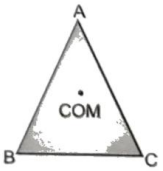
Example 35: Three rods each of mass m and length l are joined together to form an equilateral triangle as shown in figure. Find the moment of inertia of the system about an axis passing through its centre of mass and perpendicular to the plane of triangle.



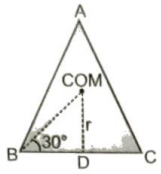
Sol. Moment of inertia of rod BC about an axis perpendicular to plane of triangle ABC and passing through the mid-point of rod BC (i.e., D) is

$$I_1 = \frac{ml^2}{12}$$

From theorem of parallel axes, moment of inertia of this rod about the asked axis is

$$I_2 = I_1 + mr^2$$

$$\tan 30° = \frac{r}{BD} = \frac{r}{L/2}$$



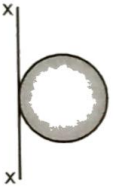
$$r = \frac{L}{2\sqrt{3}}$$

$$= \frac{ml^2}{12} + m\left(\frac{l}{2\sqrt{3}}\right)^2 = \frac{ml^2}{6}$$

∴ Moment of inertia of all the three rod is

$$I = 3I_2 = 3\left(\frac{ml^2}{6}\right) = \frac{ml^2}{2}$$

Example 36: Find the moment of inertia of a solid sphere of mass M and radius R about an axis XX shown in figure.



Sol.



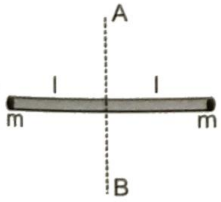
From theorem of parallel axis,

$$I_{XX} = I_{COM} + Mr^2$$

$$= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

Example 37: Consider a uniform thin wire of mass m and length 2l with two particles of mass m each at its ends. Let AB be a line perpendicular to the length of the rod passing through its centre. Find the moment of inertia of the system about AB.

Sol.



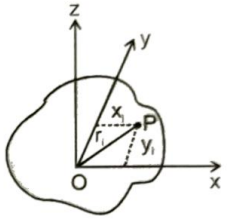
$$I_{AB} = I_{rod} + I_{both\ particles}$$

$$= \frac{m(2l)^2}{12} + 2(ml^2) = \frac{7}{3}ml^2$$ **Ans.**

THEOREM OF PERPENDICULAR AXES

The theorem states that the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about two axes perpendicular to each other, in its own plane and intersecting each other, at the point where the perpendicular axis passes through it.

Let x and y axes be chosen in the plane of the body and z-axis perpendicular, to this plane, three axes being mutually perpendicular, then the theorem states that.



$$I_z = I_x + I_y$$

Important point in perpendicular axis theorem

(i) This theorem is applicable only for the plane bodies (two dimensional).

(ii) In theorem of perpendicular axes, all the three axes (x, y and z) intersect each other and this point may be any point on the plane of the body (it may even lie outside the body).

(iii) Intersection point may or may not be the centre of mass of the body.

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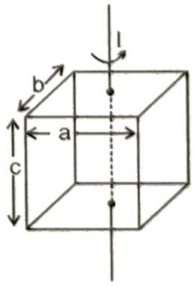
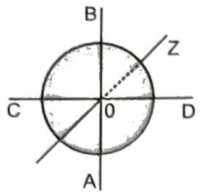
Example 38: Find the moment of Inertia of a cuboid along the axis as shown in the figure.



Sol. After compressing the cuboid parallel to the axis

$$I = \frac{M(a^2 + b^2)}{12}$$

Example 39: Find the moment of inertia of uniform ring of mass M and radius R about a diameter.

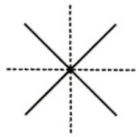


Sol. Let AB and CD be two mutually perpendicular diameters of the ring. Take them as X and Y-axes and the line perpendicular to the plane of the ring through the centre as the Z-axis. The moment of inertia of the ring about the Z-axis is $I = MR^2$. As the ring is uniform, all of its diameter equivalent and so $I_x = I_y$, From perpendicular axes theorem,

$$I_z = I_x + I_y \text{ Hence } I_x = \frac{I_z}{2} = \frac{MR^2}{2}$$

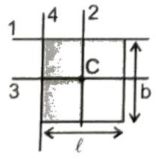
Similarly, the moment of inertia of a uniform disc about a diameter is $MR^2/4$

Example 40: Two uniform identical rods each of mass M and length ℓ are joined perpendicularly to form a cross as shown in figure. Find the moment of inertia of the cross about a bisector as shown dotted in the figure.



Sol. Consider the line perpendicular to the plane of the figure through the centre of the cross. The moment of inertia of each rod about this line is $\frac{M\ell^2}{12}$ and hence the moment of inertia of the cross is $\frac{M\ell^2}{6}$. The moment of inertia of the cross about the two bisector are equal by symmetry and according to the theorem of perpendicular axes, the moment of inertia of the cross about the bisector is $\frac{M\ell^2}{12}$.

Example 41: In the figure shown find moment of inertia of a plate having mass M, length ℓ and width b about axis 1, 2, 3 and 4. Assume that C is centre and mass is uniformly distributed



Sol. Moment of inertia of the plate about axis 1 (by taking rods perpendicular to axis 1)

$$I_1 = \frac{mb^2}{12} + n\left(\frac{b}{2}\right)^2 = \frac{mb^2}{3}$$

Moment of inertia of the plate about axis 2 (by taking rods perpendicular to axis 2)

$I_2 = M\ell^2/12$

Moment of inertia of the plate about axis 3 (by taking rods perpendicular to axis 3)

$$I_3 = \frac{Mb^2}{12}$$

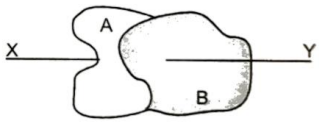
Moment of inertia of the plate about axis 4(by taking rods perpendicular to axis 4)

$$I_4 = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{3}$$

MOMENT OF INERTIA OF COMPOUND BODIES

Consider two bodies A and B, rigidly joined together. The moment of inertia of this compound body, about an axis XY, is required. If I_A is the moment of inertia of body A about XY. I_B is the moment of inertia of body B about XY. Then, moment of Inertia of compound body $I = I_A + I_B$

Extending this argument to cover any number of bodies rigidly joined together, we see that the moment of inertia of the compound body, about a specified axis, is the sum of the moments of inertia of the separate parts of the body about the same axis.



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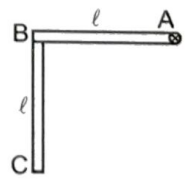
Example 42: Two rods each having length l and mass m joined together at point B as shown in figure. Then findout moment of inertia about axis passing thorugh A and perpendicular to the plane of page as shown in figure.

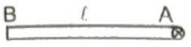


Sol. We find the resultant moment of inertia I by dividing in two parts such as

I = M.I of rod AB about A + M.I of rod BC about A

$I = I_1 + I_2$ ... (1)

first calculate I_1:

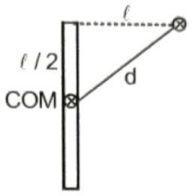


$$I_1 = \frac{m\ell^2}{3} \qquad ...(2)$$

Calculation of I_2:

Use parallel axis theorem $I_2 = I_{CM} + md^2$

$$= \frac{m\ell^2}{12} + m\left(\frac{\ell^2}{4} + \ell^2\right)$$



$$= \frac{m\ell^2}{12} + \frac{5\ell^2}{4}m \quad ...(3)$$

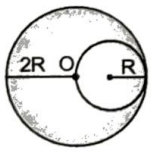
Put value from eq. (2) & (3) into (1)

$$\Rightarrow I = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + \frac{5\ell^2 m}{4}$$

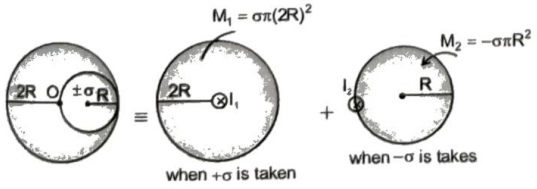
$$I = \frac{m\ell^2}{12}(4+1+15)$$

$$\Rightarrow I = \frac{5m\ell^2}{3}$$

Example 43: A uniform disc having radius 2R and mass density σ as shown in figure. If a small disc of radius R is cut from the disc as shown. Then find out the moment of inertia of remaining disc around the axis that passes through O and is perpendicular to the plane of the page.



Sol. We assume that in remaning part a disc of radius R and mass density $\pm\sigma$ is placed. Then



Total Moment of Inertia $I = I_1 + I_2$

$$I_1 = \frac{M_1(2R)^2}{2}$$

$$I_1 = \frac{\sigma\pi 4R^2 . 4R^2}{2} = 8\pi\sigma R^4$$

To calculate I_2 we use parallel axis theorem.

$$I_2 = I_{CM} + M_2R^2$$

$$I_2 = \frac{M_2R^2}{2} + M_2R^2$$

$$I_2 = \frac{3}{2}M_2R^2 = \frac{3}{2}(-\sigma\pi R^2)R^2 \qquad I_2 = -\frac{3}{2}\sigma\pi R^4$$

Now $I = I_1 + I_2$

$$I = 8\pi\sigma R^4 - \frac{3}{2}\sigma\pi R^4$$

$$I = \frac{13}{2}\sigma\pi R^4$$

Example 44: A uniform disc of radius R has a round disc of radius R/3 cut as shown in Fig. The mass of the remaining (shaded) portion of the disc equals M. Find the moment of inertia of such a disc relative to the axis passing through geometrical centre of original disc and perpendicular to the plane of the disc.



Sol. Let the mass per unit area of the material of disc be σ. Now the empty space can be considered as having density $-\sigma$ and σ.

Now $I_0 = I_\sigma + I_{-\sigma}$

$I_\sigma = (\sigma\pi R^2)R^2/2$ = M.I of σ about O

$$I_{-\sigma} = \frac{-\sigma\pi(R/3)^2(R/3)^2}{2} + [-\sigma\pi(R/3)^2](2R/3)^2$$

= M.I of $-\sigma$ about 0

$$\therefore I_0 = \frac{4}{9}\sigma\pi R^4$$

where $\sigma = \dfrac{M}{\pi R^2 - \pi\left(\dfrac{R^2}{9}\right)}$

$$\sigma = \frac{9M}{8\pi R^2}$$

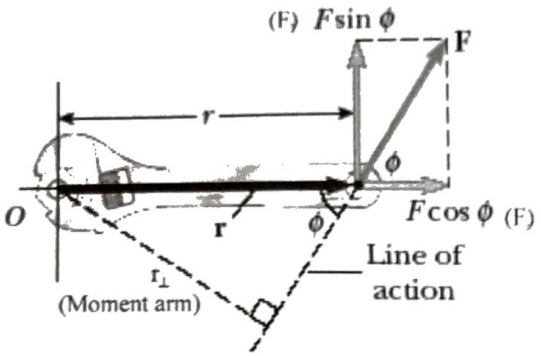
MOMENT OF FORCE OR TORQUE

Rotational Analogue of Force

Let us take the example of opening or closing of a door. A door is a rigid body which can rotate about a fixed vertical axis passing through the hinges. What makes the door rotate? It is clear that unless a force is applied the door does not rotate. But any force does not do the job. A force applied to the hinge line cannot produce any rotation at all, whereas a force of given magnitude applied at right angles to the door at its outer

edge is most effective in producing rotation. It is not the force alone, but how and where the force is applied is important in rotational motion. The quantitative measure of the tendency of a force to cause or change the rotational motion of a body is called torque. Consider an example to understand this.

In the figure below, the wrench is trying to open the nut. Now the ability of wrench to open the nut will depend not only on the applied force, but the distance at which force is applied. This gives birth to a **new physical quantity** called torque.



If only radial force F_r were present, the nut could not be turned. Thus the force causing the rotation is tangential force F_T only. The magnitude of the torque about an axis due to a force is given by

τ = (Force causing the rotation)×(distance of point of application of force from the axis)

$= (F \sin \phi)\, r$

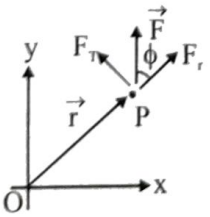
we may also write $\tau = F\,(r \sin\phi) = F\,(r_\perp)$

$\therefore \quad \tau = rF\sin\phi = (r \sin\phi)F$

$= rF_T = r_\perp F$

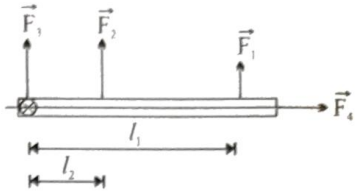
where $r_\perp$ is known as moment arm (lever arm)

Thus, if a force acts on a single particle at a point P whose position with respect to the origin O is given by the position vector $\vec{r}$, (see figure), the moment of force (torque) acting on the particle with respect to the origin O is defined as the vector product;

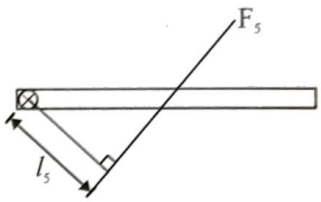


$\vec{\tau} = \vec{r} \times \vec{F}$

Direction of torque is found by sliding the force vector at the axis of rotation and using right hand thumb rule.



Talk about F_1, F_2, F_3, F_4 and F_5 and their moment arm or force arm



(Torque) $\tau_1 = F_1 l_1$

$\tau_2 = F_2 l_2$

$\tau_3 = 0$

$\tau_4 = 0$

$\tau_5 = F_5 l_5$

Important Note

(1) The SI unit of torque is newton-metre. This is also the unit of work and energy. But torque is not work or energy. Usually, the unit of work or energy is written as "joule". But the torque should not be expressed in "joule". However, both have same dimensions ML^2T^{-2}.

(2) Torque is always defined with reference to a given point (or given line). On changing the referenece point (or line) the torque may change.

(3) Torque is a vector quantity, whose direction is perpendicular to the plane of force and position vector and its direction is given by right hand screw rule.

(4) If the torque rotates the body in anticlock wise direction, the torque is positive and if the torque rotates the body in clock-wire direction, the torque will be negative.

(5) As a shortcut the magnitude of torque can be calculated as the product of the force and the length of line perpendicular to the force starting from the point about which the torque is to be calculated.

(6) $\tau = 0$ if $r = 0$, if $F = 0$, or if $\theta = 0°$ or $180°$. Hence if the line of action of the force passes through the reference point, the torque vanishes.

(7) The more is the value of r, the more will be torque and easier to rotate the body.

(i) The handle of screw driver is taken thick.

(ii) In villages the handle of flour-mill is placed near the circumference.

(iii) The handle of handpump is kept-long.

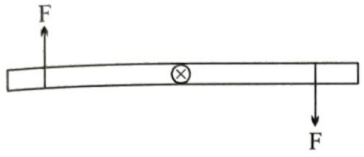
(iv) The rinch used for opening the tap, is kept-long.

(8) If a body is acted upon by more than one force, the total torque is the vector sum of each torque.

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_2 + \ldots\ldots\ldots + \vec{\tau}_n$$

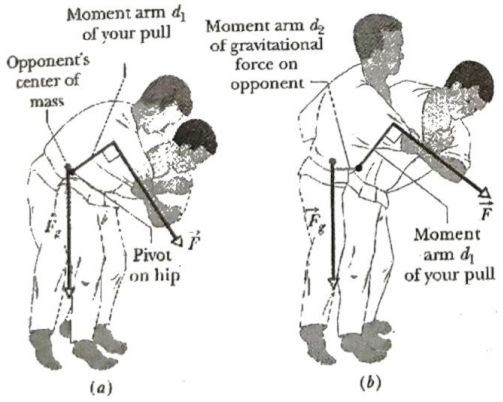
Thus, torque also follows superposition principle.

(9) There can be non-zero torque on body even when net force is zero (couple) and there can be force producing zero torque.



(10) **Force couple:** A pair of forces each of same magnitude and in opposite direction is called a force couple. Torque due to a couple about any point is same and its magnitude = Magnitude of one force X distance between their lines of action

A PRACTICAL EXAMPLE OF TORQUE



A judo hip throw

In judo, a weaker and smaller fighter who understands physics can defeat a stronger and larger fighter who does not. This fact is demonstrated by the basic "hip throw," in which a fighter rotates the fighter's opponent around his hip and if the throw is successful onto the mat. Without the proper use of physics, the throw requires considerable strength and can easily fail.

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Example 45: Given that, $\vec{r} = 2\hat{i} + 3\hat{j}$ and $\vec{F} = 2\hat{i} + 6\hat{k}$. The magnitude of torque will be-

(a) $\sqrt{405}$ N.m (b) $\sqrt{410}$ N.m

(c) $\sqrt{504}$ N.m (d) $\sqrt{510}$ N.m

Sol. We know that, $\vec{\tau} = \vec{r} \times \vec{F}$

$$\Rightarrow \vec{\tau} = (2\hat{i} + 3\hat{j}) \times (2\hat{i} + 6\hat{k})$$

$$= 12(-\hat{j}) + 6(-\hat{k}) + 18\hat{i}$$

$$= -12\hat{j} - 6\hat{k} + 18\hat{i}$$

[Note : $\hat{i} \times \hat{i} = 0$, $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$ etc]

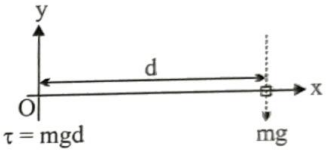
Now, $|\vec{\tau}| = \sqrt{(-12)^2 + (-6)^2 + (18)^2}$

$$= \sqrt{144 + 36 + 324} = \sqrt{504}$$

Example 46: A particle is falling freely along line x = d. Find torque on this particle due to gravity, about origin when it

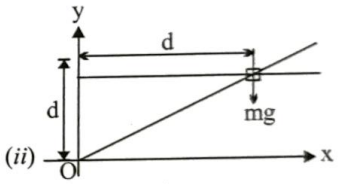
(i) Crosses x axis (ii) is at y = d

Sol. (*i*)



$\tau = mgd$

(Clockwise)



$\tau = mgd$

(clockwise)

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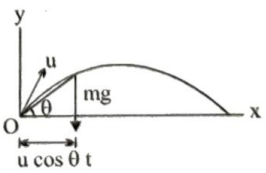
Example 47: Find torque due to gravity at any time t about pt. of projection, if a body is projected with velocity u at an angle θ.

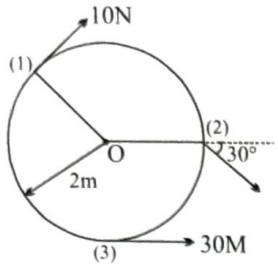
Sol.



At every instant force mg is in downward direction and perpendicular distance will be displacement along x-axis i.e. x = (u cosθ)t

τ_0 = mg u cos θ tit is increasing with time

Example 48: Find net torque about axis of rotation passing through O.



Sol. $\tau_1 = 10 \times 2$ $\quad \otimes = 20 \otimes$

$\tau_2 = 20 \times 2 \sin 30°$ $\quad \otimes = 20 \otimes$

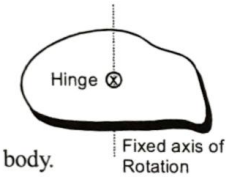
$\tau_3 = 30 \times 2$ $\quad \odot = 60 \odot$

$\Sigma\vec{\tau} = 20 \odot$

ROTATION ABOUT A FIXED AXIS

If I_{Hinge} = moment of inertia about the axis of rotation (since this axis passes through the hinge, hence the name I_{Hinge}).

$\vec{\tau}_{ext}$ = resultant external torque acting on the body about axis of rotation



α = angular acceleration of the body.

$(\vec{\tau}_{ext})_{Hinge} = I_{Hinge}\ \vec{\alpha}$

Rotational Kinetic Energy $= \frac{1}{2}.I.\omega^2$

$\vec{P} = M\vec{v}_{CM}$; $\vec{F}_{external} = M\vec{a}_{CM}$

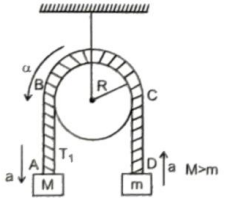
Net external force acting on the body has two component tangential and centripetal.

$\Rightarrow F_C = ma_C = m\frac{v^2}{r_{CM}} = m\omega^2 r_{CM} \Rightarrow F_t = ma_t = m\alpha r_{CM}$

PULLEY BLOCK SYSTEM

If there is friction between pulley and string and pulley have some mass then tension is different on two sides of the pulley.

Reason: To understand this concept we take a pulley block system as shown in figure.

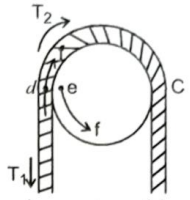


Let us assume that tension induced in part AB of the string is T_1 and block M move downward. If friction is present between pulley and string then it opposes the relative slipping between pulley and string, take two point e and f on pulley and string respectively. If friction is there then due to this, both points wants to move together. So friction force act on e and d in the direction as shown is figure

This friction force f acting on point d increases the tension T_1 by a small amount dT.

Then $T_1 = T_2 + dT$

or we can say $T_2 = T_1 - f$



In this way the tension on two side of pulley is different If there is no relative slipping between pulley and string then

$\alpha = \frac{a_t}{R} = \frac{a}{R}$

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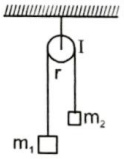
Example 49: The pulley shown in figure has a moment of inertia I about its axis and its radius is r. Calculate the magnitude of the acceleration of the two blocks. Assume that the string is light and does not slip on the pulley.



Sol. Suppose the tension in the left string is T_1 and that in the right string in T_2. Suppose the block of mass m_1 goes down with an acceleration α and the other block moves up with the same acceleration a. This is also the tangential acceleration of the rim of the wheel as the string does not slip over the rim. The angular acceleration of the wheel is, therefore, $\alpha = a/r$. The equations of motion for the mass m_1, the mass m_2 and the pulley are as follows:

$m_1g - T_1 = m_1a$ .....(i)

$T_2 - m_2g = m_2a$ .....(ii)

$T_1r - T_2r = I\alpha = Ia/r$ .....(iii)

Putting T_1 and T_2 from (i) and (ii) into (iii),

$[(m_1g - a) - m_2(g + a)]\ r = I\frac{a}{r}$

which gives $a = \frac{(m_1 - m_2)gr^2}{I + (m_1 + m_2)r^2}$.

Example 50: A uniform rod of mass m and length ℓ can rotate in vertical plane about a smooth horizontal axis hinged at point H.



(i) Find angular acceleration α of the rod just after it is released from initial horizontal position from rest ?

(ii) Calculate the acceleration (tangential and radial) of point A at this moment.

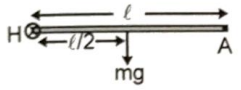
(iii) Calculate net hinge force acting at this moment.

(iv) Find α and ω when rod becomes vertical.

(v) Find hinge force when rod become vertical.

Sol. (i) $\tau_H = I_H \alpha$

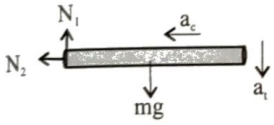
$$mg.\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha \Rightarrow \alpha = \frac{3g}{2\ell}$$



(ii) $a_{tA} = \alpha\ell = \frac{3g}{2\ell}.\ell = \frac{3g}{2}$

$a_{CA} = \omega^2 r = 0.\ell = 0 (\because \omega = 0$ just after release)

(iii) Suppose hinge exerts normal reaction in component form as shown



In vertical direction

$F_{ext} = ma_{CM}$

$\Rightarrow mg - N_1 = m.\frac{3g}{4}$

(we get the value of a_{CM} from previous example)

$\Rightarrow N_1 = \frac{mg}{4}$

In horizontal direction

$F_{ext} = ma_{CM}$

$\Rightarrow N_2 = 0$

($\because a_{CM}$ in horizontal = 0 as $\omega = 0$ just after release).

(iv) Torque = 0 when rod becomes vertical.

so $\alpha = 0$

using energy conservation

$$\frac{mg\ell}{2} = \frac{1}{2}I\omega^2 \left(I = \frac{m\ell^2}{3}\right)$$

$$\omega = \sqrt{\frac{3g}{\ell}}$$

(v) When rod becomes vertical

$$\alpha = 0, \omega = \sqrt{\frac{3g}{\ell}}$$

$$F_H - mg = \frac{m\omega^2\ell}{2}$$

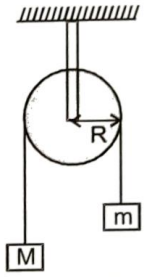
$$F_H = \frac{5mg}{2}$$

Example 51: The pulley shown in figure has moment of inertia I about its axis and radius R. Find the acceleration of the two blocks. Assume that the string is light and does not slip on the pulley.

Sol. Suppose the tension in the left string is T_1 and that in the right string is T_2. Suppose the block of mass M goes down with an acceleration a and the other block moves up with the same acceleration. This is also the tangential acceleration of the rim of the wheel as the string does not slip over the rim.

The angular acceleration of the wheel $\alpha = \frac{a}{R}$.

The equations of motion for the mass M, the mass m and the pulley are as follows;



$Mg - T_1 = Ma$ ...(i)

$T_2 - mg = ma$ ...(ii)

$$T_1R - T_2R = I\alpha = \frac{Ia}{R} \quad ...(iii)$$

Substituting for T_1 and T_2 from equations (i) and (ii) in equation (iii)

$$[M(g-a) - m(g+a)]R = \frac{Ia}{R}$$

Solving, we get, $a = \frac{(M-m)gR^2}{I+(M+m)R^2}$

KINETIC ENERGY OF A RIGID BODY ROTATING ABOUT A FIXED AXIS

Suppose a rigid body is rotating about a fixed axis with angular speed ω.

Then, kinetic energy of the rigid body will be :

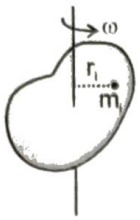
Here, $\vec{\tau}$ is the resultant torque acting on the body.

Further, since

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \therefore \vec{\tau}\,dt = d\vec{L}$$

or $\int_{t_1}^{t_2} \vec{\tau}\, dt$ = angular impulse = $\vec{L}_2 - \vec{L}_1$

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2$$



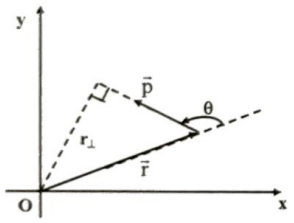
$$= \frac{1}{2}\omega^2 \sum_i m_i r_i^2 = \frac{1}{2} I\omega^2 \quad (\text{as } \sum_i m_i r_i^2 = I)$$

Thus, $KE = \frac{1}{2} I\omega^2$

Sometimes it is called the rotational kinetic energy.

ANGULAR MOMENTUM

Consider a particle that has linear momentum $\vec{p}$ and is located at position $\vec{r}$ relative to an origin O, as shown in the figure. Its angular momentum about the origin is defined as



$$\vec{L} = \vec{r} \times \vec{p}$$

The magnitude of momentum is given by

$L = rp \sin\theta$

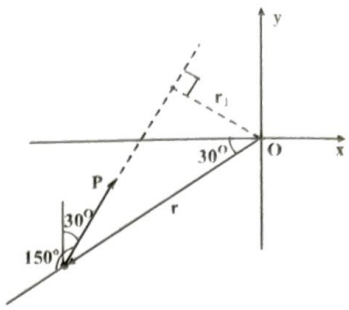
The direction of angular momentum is given by the right hand rule.

The SI unit of angular momentum is kg m²/s.

TRAIN YOUR BRAIN

Example 52: What is the angular momentum of a particle of mass m = 2 kg that is located 15 m from the origin in the direction 30° south of west and has a velocity v = 10 m/s in the direction 30° east of north?

Sol. In the figure (a), the x - axis points east. We know r = 15 m; p = mv = 20 kg m/s. The angle between r and p is $(180° - 30°) = 150°$



Thus,

$L = rp \sin\theta = (15)(20)\sin 150° = 150$ kg m²/s

In unit vector notation,

$\vec{r} = -15 \cos 30°\, \hat{i} - 15 \sin 30°\, \hat{j}$ m

$p = 20 \sin 30°\, \hat{i} + 20 \cos 30°\, \hat{j}$ kg m/s.

Therefore,

$$\vec{L} = (-\frac{15\sqrt{3}}{2}\, \hat{i} - y\, \frac{15}{2}\, \hat{j}) \times (10\, \hat{i} + 10\sqrt{3}\, \hat{j})$$

$$= -150\, \hat{k} \text{ kg m}^2/\text{s}$$

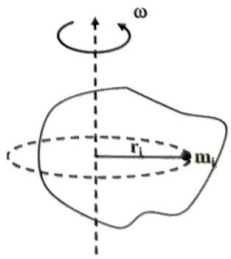
Note that angular momentum is defined always with respect to a point

The total angular momentum $\vec{L}$ of a system of particles relative to a given origin is the sum of the angular momentum of the particles.

$$\vec{L} = \sum \vec{r}_i \times \vec{p}_i$$

Consider an object rotating about a fixed axis as shown in the figure.

Since $\vec{r}_i$ and $\vec{p}_i$ are perpendicular, therefore,



$L = \Sigma r_i p_i$

Also $p_i = m_i v_i = m_i r_i \omega$

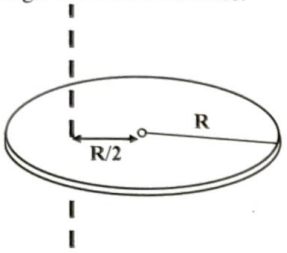
or $L = \Sigma m_i r_i^2\, \omega$ or $L = I\omega$

where $I = \Sigma m_i r_i^2$

TRAIN YOUR BRAIN

Example 53: A disc of mass M and radius R is rotating at angular velocity ω about an axis perpendicular to its plane at a distance R/2 from the center, as shown in the figure. What is its angular momentum? The moment of inertia of a disc about the central axis is $\frac{1}{2}MR^2$.

Sol. The moment of inertia of the disc about the given axis may be found from the parallel axes theorem, equation $I = I_{cm} + Mh^2$, where h is the distance between the given axis and a parallel axis through the center of mass.



Here $h = \frac{R}{2}$, therefore,

$$\frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2 = \frac{3}{4}MR^2$$

The angular momentum is

$$L = I\omega = \frac{3}{4}MR^2\omega$$

TORQUE AND RATE OF CHANGE OF ANGULAR MOMENTUM

As force changes the linear momentum of a particle, torque changes the angular momentum of a particle. The rate ofchange of angular momentum with time is given by

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times m\vec{v})$$

$$\text{or } \frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$\text{or } \frac{d\vec{L}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times \vec{F}$$

$$\text{or } \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau} \text{ because } \vec{v} \times m\vec{v} \text{ is zero}$$

Thus, $\vec{\tau} = \frac{d\vec{L}}{dt}$

The above equation is the rotational analog of equation

$$\vec{F} = \frac{d\vec{P}}{dt}$$

TRAIN YOUR BRAIN

Example 54: Show that the above equation can be applied to the motion of a projectile.

Sol. In figure, we take the initial point as the origin. At a later time, $\vec{r} = x\hat{i} + y\hat{j}$.

Since the force on the particle is $\vec{F} = -mg\,\hat{j}$, the gravitational torque on it is

$$\vec{\tau} = (x\,\hat{i} + y\hat{j}) \times (-mg\,\hat{j}) = -mgx\,\hat{k}$$

The rate of change of the angular momentum

$\vec{L}$ $\vec{r} \times \vec{p}$ = is



$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} = m\vec{r} \times \frac{d\vec{v}}{dt}$$

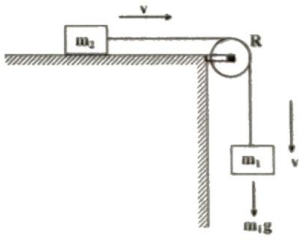
But the acceleration is $\frac{d\vec{v}}{dt} = -g\,\hat{j}$. Thus

$$\frac{d\vec{L}}{dt} = m\vec{r} \times \frac{d\vec{v}}{dt} = m\left(x\hat{i} + y\hat{j}\right) \times \left(-g\,\hat{j}\right) = -mgx\,\hat{k}$$

Hence the equation $\vec{\tau} = \frac{d\vec{L}}{dt}$ is applicable.

TRAIN YOUR BRAIN

Example 55: Two blocks with masses m_1 = 3 kg and m_2 = 1 kg are connected by a rope that passes over a pulley of radius R = 0.2 m and mass M = 4 kg. The moment of inertia of the pulley about its center is $I = \frac{1}{2}MR^2$. Use the concept of angular momentum to find the linear acceleration of the blocks. There is no friction. Assume that the c.m. of the block of mass m_2 is at a distance R above the center of the pulley.



Sol. If we take the origin at the center of the pulley, the angular momenta of the blocks are m_1vR and m_2vR and that of the pulley is $I\omega$. Therefore, the total angular momentum is

$L = m_1vR + m_2vR + I\omega$ ....(i)

If the rope does not slip, then $v = \omega R$.

$$\therefore \quad L = (m_1 + m_2)vR + \frac{MR}{2}v$$

The net external torque about the center of the pulley is due to the weight of m_1

$t_{ext} = r_\perp F = R(m_1g)$ .....(ii)

Applying equation $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$

We obtain

$$Rm_1g = (m_1 + m_2)Ra + \frac{MR}{2}a$$

$$\text{or } a = \frac{m_1g}{m_1 + m_2 + M/2}$$

putting $m_1 = 3$ kg; $m_2 = 1$ kg; $M = 4$ kg; $R = 0.2$ m

$$a = \frac{(3)(10)}{3+1+4/2} = 5 \text{ m/s}^2$$

CONSERVATION OF ANGULAR MOMENTUM

If the net external torque on a system is zero, the total angular momentum is constant in magnitude and direction.

That is, if $\vec{\tau}_{ext} = \vec{0}$ $\frac{d\vec{L}}{dt} = \vec{0}$

Thus, $\vec{L}$ = constant

For a rigid body rotating about a fixed axis,

$L_f = L_i$

or $I_f\omega_f = I_i\omega_i$

TRAIN YOUR BRAIN

Example 56: A disc of moment of inertia 4 kg m^2 is spinning freely at 3 rad/s. A second disc of moment of inertia 2 kg m^2 slides down the spindle and they rotate together.

(a) What is the angular velocity of the combination?

(b) What is the change in kinetic energy of the system?

Sol. (a) Since there are no external torques acting, we may apply the conservation of angular momentum.

$I_f\omega_f = I_i\omega_i$

(6 kg m^2) ω_f = (4 kg m^2)(3 rad/s)

Thus, $\omega_f = 2$ rad/s

(b) The kinetic energies before and after the collision are

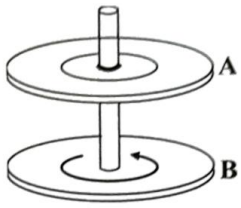
$$K_i = \frac{1}{2}I_i\omega_i^2 = 18 \text{ J};$$

$$K_f = \frac{1}{2}I_f\omega_f^2 = 12 \text{ J}$$

The change is

$\Delta K = K_f - K_i = -6$ J.

In order for the two discs to spin together at the same rate, there had to be friction between them. The lost kinetic energy is converted to thermal energy.



ANGULAR IMPULSE

In complete analogy with the linear momentum, angular impulse is defined as

$$\vec{J} = \int \vec{\tau}_{ext}\,dt$$

Using Newton's second law for rotation motion,

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

$$\therefore \quad \vec{J} = \Delta\vec{L} = \vec{L}_f - \vec{L}_i$$

The net angular impulse acting on a rigid body is equal to the change in angular momentum of the body. This is called the impulse – momentum theorem for rotational dynamics.

SPIN AND ORBITAL ANGULAR MOMENTUM

For a rigid body undergoing linear and rotational motion, the total angular momentum may be split into two parts – the **orbital angular momentum** and the **spin angular momentum**. The orbital angular momentum $\vec{L}_o$ is the angular momentum of the center of mass motion about an origin O in an inertial frame.

The spin angular momentum $\vec{L}_{cm}$ is the angular momentum relative to the center of mass.

The orbital term treats the system as a point particle at the center of mass, whereas the spin term is the sum of the angular momenta of the particles relative to the center of mass. The total angular momentum relative to the origin O in an inertial frame is the sum:

$$\vec{L} = \vec{L}_o + \vec{L}_{cm}$$

TRAIN YOUR BRAIN

Example 57: A solid sphere of mass M and radius R rolls without slipping on a horizontal surface as shown in the figure. Find the total angular momentum of the sphere with respect to the origin O fixed on the ground.

Sol. Let us assume the clockwise sense of rotation positive.

Orbital angular momentum about O is

$$L_o = MvR$$

Spin angular momentum about c.m. is

$$L_{c.m} = I\omega = \frac{2}{5}MR^2\omega$$

The total angular momentum is

$$L = L_o + L_{cm} = MvR + \frac{2}{5}MR^2\omega$$

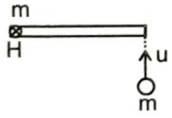
For pure rolling, $v = \omega R$, therefore, $L = \frac{7}{5}MvR$

Finally, we present an analogy between rotational dynamics and linear dynamics.

	Quantity	Linear	Rotational
1.	Inertia	m	$I = \sum m_i r_i^2$ or $I = \int r^2 dm$
2.	Newton's Second Law	$F_{ext} = ma$ $\vec{F}_{ext} = \frac{d\vec{p}}{dt}$	$\tau_{ext} = I\alpha$ $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$
3.	Work	$W_{lin} = \int \vec{F}.d\vec{s}$	$W_{rot} = \vec{\tau}.d\vec{\theta}$
4.	Kinetic Energy	$K_{lin} = \frac{1}{2}mv^2$	$K_{rot} = \frac{1}{2}I\omega^2$
5.	Work Energy Theorem	$W_{lin} = \Delta K_{lin}$	$W_{rot} = \Delta K_{rot}$
6.	Impulse	$I = \int F_{ext} dt$	$\vec{J} = \int \vec{\tau}_{ext} dt$
7.	Momentum	$p = mv$	$L = I\omega$
8.	Impulse Momentum Theorem	$\vec{I} = \Delta\vec{p}$	$\vec{J} = \Delta\vec{L}$
9.	Power	$P = \vec{F}.\vec{v}$	$P = \vec{\tau}.\vec{\omega}$

TRAIN YOUR BRAIN

Example 58: A uniform rod of mass m and length ℓ can rotate freely on a smooth horizontal plane about a vertical axis hinged at point H. A point mass having same mass m coming with an initial speed u perpendicular to the rod, strikes the rod inelastically at its free end. Find out the angular velocity of the rod just after collision?



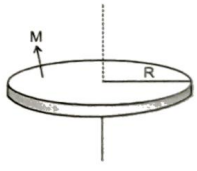
Sol. Angular momentum is conserved about H because no external force is present in horizontal plane which is producing torque about H.

$$mu\ell = \left(\frac{m\ell^2}{3} + m\ell^2\right)\omega \Rightarrow \omega = \frac{3u}{4\ell}$$

Example 59: A person of mass m stands at the edge of a circular platform of radius R and moment of inertia I. A platform is at rest initially. But the platform rotate when the person jumps off from the platform tangentially with velocity u with respect to platfrom. Determine the angular velocity of the platform.

Sol. Let the angular velocity of platform is ω.

Then the velocity of person with respect to ground v.



$$v_{mD} = v_{mG} - V_{DG}$$

$$u = v_m + \omega R$$

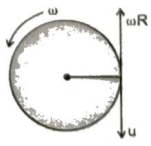
$$v_m = u - \omega R$$

Now from angular momentum conservation

$$L_i = L_f$$

$$0 = mv_m R - I\omega$$

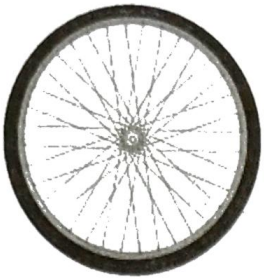
$$\Rightarrow I\omega = m(u - \omega R).R$$



$$\Rightarrow \omega = \frac{muR}{I + mR^2}$$

ROLLING - COMBINATION OF TRANSLATIONAL AND ROTATIONAL MOTION

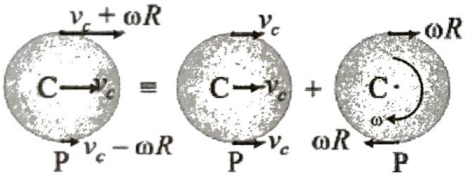
Rolling Motion



A photograph of a rolling bicycle wheel. The spokes near the top of the wheel are more blurred than those near the bottom of the wheel because they are moving faster

Pure rolling means no sliding. Now, the motion of any body can be divided into pure translation & pure rotation. And we can see rotation about any axis. So; for a wheel rolling on a horizontal surface, we are taking its COM as reference point to study its motion. If C is the reference point, then the wheel can be considered rotating about C. And the point C would be translating with velocity v_c.

Sliding refers to the condition under which two bodies in contact have relative velocity. And under pure rolling, the relative velocity at point of contact should be zero.



Rolling = Transalation + Rotation

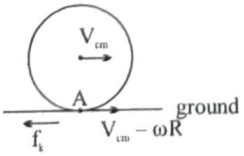
Now for the wheel shown, point P is in contact with the ground. The point P on the ground has zero velocity. Thus, the point P on the wheel should also have zero velocity.



A time exposure photograph of a rolling disk. Small lights have been attached to the disk, one at its center and one at its edge. The latter traces out a curve called a **cycloid**.

Rolling with sliping:

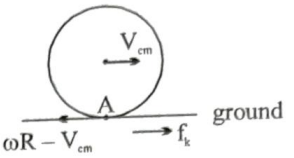
Case-1 $V_{cm} > \omega R$



NOTE: The friction will be kinetic in nature & its magnitude can be determined using $f_k = \mu_k N$. Direction will be opposite to V_{cm} (because pt. of contact A is moving forward w.r.t. ground)

(This is how brakes stop a car)

Case-2 $V_{cm} < \omega R$



(This is how a car accelerate because of friction)

Case-3 $V_{cm} = \omega r$

Known as perfect rolling

Maximum problems we come across this situation

NOTE :

1. The friction will be be static in nature. Its magnitude be determined, it will very from O to $\mu_s N$.
2. Even its direction can not be perdicted
3. The total work done by static friction is zero. Thus mechanical energy of the system will remain conserved.

Kinetic energy of a rolling Body:

Since the rolling motion is a combination of linear velocity of the center and rotational motion about the center. Therefore, the total kinetic energy of a rolling body is given by

$$K = \frac{1}{2}mv_c^2 + \frac{1}{2}I_c\omega^2 \quad .....(i)$$

Where $\frac{1}{2}mv_c^2$ is the translational kinetic energy and

$\frac{1}{2}I_c\omega^2$ is the rotational kinetic energy about the center of mass

In pure rolling motion, $v_c = \omega R$

$$\therefore K = \frac{1}{2}m(\omega R)^2 + \frac{1}{2}I_c\omega^2$$

or $K = (I_c + mR^2)\,\omega^2$

Using parallel axes theorem, the term $I_c + mR^2$ gives the moment of inertia about the point of contact, therefore, $I_0 = I_c + mR^2$

and $$K = \frac{1}{2}I_0\omega^2 \quad ....(ii)$$

Note that equation (ii) give the rotational kinetic energy of the wheel about the point of contact.

Application of Newton's Second Law in Rolling Motion

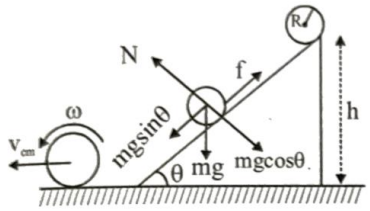
1. Write $F_{net} = M a_{cm}$ for the object as if it were a point-mass, that is, ignoring ratation.
2. Write $\tau = I_{cm} \alpha$ as if the object were only rotating about the centre of mass, that is ignoring translation.
3. Use of no-slip condition
4. Solve the resulting equations simultaneously for any unknown.

Caution:

- In general, it is not the case that $f = \mu N$
- Be certain that the sign convention of forces and torques are consistent.

ROLLING ON THE INCLINED PLANE

(a) **Velocity :** When a rigid body of mass M rolls on an inclined plane without slipping, the friction force 'f' is static, and no work is done by the friction force. Hence from the conservation of mechanical energy,



$$Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}MK^2\omega^2$$

(where K is the radius of gyration)

$$= \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\cdot\frac{K^2}{R^2}\cdot MR^2\omega^2$$

$$= \frac{1}{2}M\left(1+\frac{K^2}{R^2}\right)v_{cm}^2, \text{ [where } v_{cm} = R\omega \text{ for pure rolling]}$$

$$\text{or, } v_{cm} = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}},$$

[where $\frac{K^2}{R^2}$ is a pure number between 0 and 1 that depends on the shape of the body]

It is important to note that the velocity of a rolling body is independent of its mass (M) and radius (R). All uniform solid spheres have the same speed at the bottom even if their masses and radii are different because they have same $\frac{K^2}{R^2} = \left(\frac{2}{5}\right)$. All solid cylinders have the same speed $\left(\frac{K^2}{R^2} = \frac{1}{2}\right)$, all hollow cylinders have the same speed $\left(\frac{K^2}{R^2} = 1\right)$ and so on. Smaller the value of $\frac{K^2}{R^2}$, faster the body is moving.

(b) **Acceleration :** Let the linear acceleration of the rolling body be a_{cm}. Then for the linear motion, the net force = $mg\sin\theta - f = ma_{cm}$ ...(i)

For the rotational motion, the net torque

$$fR = I_{cm}.\alpha = MK^2.\frac{a_{cm}}{R}$$

$$\text{or, } f = \frac{MK^2}{R^2}a_{cm} \quad \text{...(ii)}$$

solving (i) and (ii) for a_{cm}, we get

$$\boxed{a_{cm} = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}}$$

It is also independent of mass (M) and radius R.

(c) **Time of descent (t) :** $\frac{1}{2}a_{cm}.t^2 = \frac{h}{\sin\theta}$

$$\text{or, } t = \sqrt{\frac{2h}{a_{cm}\cdot\sin\theta}}$$

$$\boxed{t = \frac{1}{\sin\theta}\sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g}}}$$

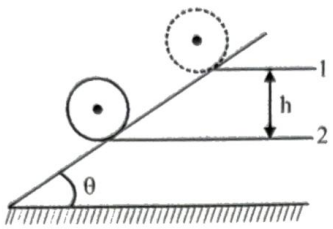
It is also independent of mass and radius. Clearly, a body of smaller $\frac{K^2}{R^2}$, takes less time.

TRAIN YOUR BRAIN

Example 60: A solid cylinder of mass m and radius r starts rolling down an inclined plane of inclination θ. Friction is enough to prevent slipping. Find the speed of tis centre of mass when, its centre of mass has fallen a height h.

Sol. Consider the two shown positions of the cylinder. As it does not slip, total mechanical energy will be conserved.

Energy at position 1 is $E_1 = mgh$



Energy at position 2 is $E_2 = \frac{1}{2}mv^2_{c.m.} + \frac{1}{2}I_{c.m.}\omega^2$

$\because \frac{V_{c.m.}}{r} = \omega$, and $I_{c.m.} = \frac{mr^2}{2}$

$$E_2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v_c}{r}\right)^2$$

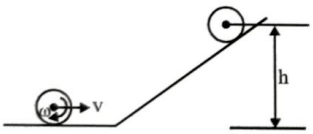
$$\Rightarrow E_2 = \frac{3}{4}mv^2_{c.m.}$$

From COE, $E_1 = E_2$

$$\Rightarrow V_{c.m.} = \sqrt{\frac{4}{3}gh}$$

Example 61: A ball of radius R and mass m is rolling without slipping on a horizontal surface with velocity of its centre of mass v. It then rolls without sloping up a hill to a height h before momentarily coming to rets. Find h.

Sol. $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

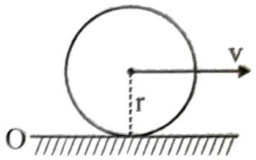


$v = \omega R$, $I = \frac{2}{5}mR^2$

$$h = \frac{7v^2_{CM}}{10g}$$

Example 62: Figure shows a solid sphere rolling without slipping on a horizontal surface. Its radius is r, mass is m and the velocity of its cenre is v. Find the angular momentum about any point O on the horizontal surface along the line parallel to its velocity $\vec{v}$

Sol. The angular momentum about O is



$$L = L_{cm} + I_{cm}\omega = \left|m\vec{r}_{cm} \times \vec{v}_{cm}\right| + I_{cm}\omega$$

$$= mvr + \frac{2}{5}mr^2 \cdot \frac{v}{r} = \frac{7}{5}mvr$$

Example 63: A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is-

(*a*) 2/5 (*b*) 3/5

(*c*) 2/7 (*d*) 3/7

Sol. Total energy,

$E = (1/2)\, I\omega^2 + (1/2)\, mv^2$

$= (1/2)\,(2/5\, mr^2)\,\omega^2 + (1/2)\, mr^2\omega^2$

$= (1/5)\, mr^2\omega^2 + (1/2)\, mr^2\omega^2 = (7/10)\, mr^2\omega^2$

Rotational energy $= (1/5)\, mr^2\omega^2$

$$\therefore \frac{\text{Rotational energy}}{\text{Total energy}} = \frac{\frac{1}{5}mr^2\omega^2}{\frac{7}{10}mr^2\omega^2} = \frac{2}{7}$$

Topicwise Questions

CENTER OF MASS

Calculation of COM

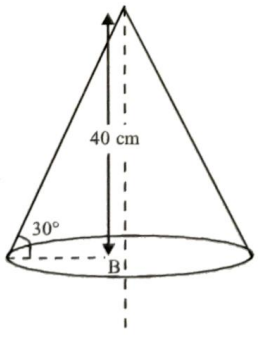
1. The centre of mass of a body:
   (*a*) Lies always at the geometrical centre
   (*b*) Lies always inside the boy
   (*c*) Lies always outside the body
   (*d*) Lies within or outside the body

2. A body has its centre of mass at the origin. The x-coordinates of the particles
   (*a*) may be all positive
   (*b*) may be all negative
   (*c*) must be all non-negative
   (*d*) may be positive for some particles and negative in other particles

3. All the particles of a body are situated at a distance R from the origin. The distance of the centre of mass of the body from the origin is
   (*a*) = R (*b*) ≤ R
   (*c*) > R (*d*) ≥ R

4. Where will be the centre of mass on combining two masses m and M(M > m) :
   (*a*) towards m (*b*) towards M
   (*c*) between m and M (*d*) anywhere

5. A uniform solid cone of height 40 cm is shown in figure. The distance of centre of mass of the cone from point B (centre of the base) is :



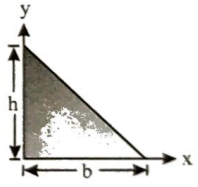
   (*a*) 20 cm (*b*) 10/3 cm
   (*c*) 20/3 cm (*d*) 10 cm

6. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m). The approximate location of the centre of mass of the molecule, distance from hydrogen atom assuming the chlorine atom to be about 35.5 times massive as hydrogen is
   (*a*) 1Å (*b*) 2.5 Å
   (*c*) 1.24 Å (*d*) 1.5 Å

7. Centre of mass is a point
   (*a*) Which is geometric centre of a body
   (*b*) From which distance of particles are same
   (*c*) Where the whole mass of the body is supposed to concentrated
   (*d*) Which is the origin of reference frame

8. Choose the correct statement about the centre of mass (CM) of a system of two particles
   (*a*) The CM lies on the line joining the two particles midway between them
   (*b*) The CM lies on the line joining them at a point whose distance form each particle is inversely proportional to the mass of that particle
   (*c*) The CM lies on the line joining them at a point whose distance from each particle is proportional to the square of the mass of that particle
   (*d*) The CM is on the line joining them at a point whose distance from each particle is proportional to the mass of that particle

9. The centre of mass of triangle shown in figure has coordinates



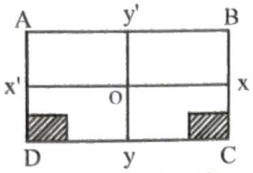
   (*a*) $x = \frac{h}{2}, y = \frac{b}{2}$ (*b*) $x = \frac{b}{2}, y = \frac{h}{2}$
   (*c*) $x = \frac{b}{3}, y = \frac{h}{3}$ (*d*) $x = \frac{h}{3}, y = \frac{b}{3}$

10. Three identical spheres, each of mass 1 kg are kept as shown in figure, touching each other, with their centres on a straight line. If their centres are marked P, Q, R respectively, the distance of centre of mass of the system from P (origin) is



   (*a*) $\frac{PQ + PR + QR}{3}$ (*b*) $\frac{PQ + PR}{3}$
   (*c*) $\frac{PQ + QR}{3}$ (*d*) $\frac{PR + QR}{3}$

11. A uniform square plate ABCD has a mass of 10 kg. If two point masses of 3 kg each are placed at the corners C and D as shown in the adjoining figure, then the centre of mass shifts to the point which is lie on -



(*a*) OC (*b*) OD
(*c*) OY (*d*) OX

Displacement, velocity, Acceleration of COM

12. A bomb travelling in a parabolic path under the effect of gravity, explodes in mid air. The centre of mass of fragments will:
(*a*) Move vertically upwards and then downwards
(*b*) Move vertically downwards
(*c*) Move in irregular path
(*d*) Move in the parabolic path which the unexploded bomb would have travelled.

13. A body at rest may have
(*a*) Energy (*b*) Momentum
(*c*) Speed (*d*) Velocity

14. Internal forces can change :
(*a*) the linear momentum but not the kinetic energy of the system.
(*b*) the kinetic energy but not the linear momentum of the system.
(*c*) linear momentum as well as kinetic energy of the system.
(*d*) neither the linear momentum nor the kinetic energy of the system.

15. Two balls are thrown in air. The acceleration of the centre of mass of the two balls while in air (neglect air resistance)
(*a*) depends on the direction of the motion of the balls
(*b*) depends on the masses of the two balls
(*c*) depends on the speeds of the two balls
(*d*) is equal to g

16. The motion of the centre of mass of a system of two particles is unaffected by their internal forces :
(*a*) irrespective of the actual directions of the internal forces
(*b*) only if they are along the line joining the particles
(*c*) only if they are at right angles to the line joining the particles
(*d*) only if they are obliquely inclined to the line joining the particles.

17. Two objects of masses 200 gm and 500 gm posses velocities $10\hat{i}$ m/s and $3\hat{i}+5\hat{j}$ m/s respectively. The velocity of their centre of mass in m/s is :

(*a*) $5\hat{i}-25\hat{j}$ (*b*) $\frac{5}{7}\hat{i}-25\hat{j}$

(*c*) $5\hat{i}+\frac{25}{7}\hat{j}$ (*d*) $25\hat{i}-\frac{5}{7}\hat{j}$

18. 2 bodies of different masses of 2kg and 4kg are moving with velocities 20m/s and 10m/s towards each other due to mutual gravitational attraction. What is the velocity of their centre of mass?
(*a*) 5 m/s (*b*) 6 m/s
(*c*) 8 m/s (*d*) zero

19. Two bodies of masses 2 kg and 4 kg are moving with velocities 2 m/s and 10m/s respectively along same direction. Then the velocity of their centre of mass will be
(*a*) 8.1 m/s (*b*) 7.3 m/s
(*c*) 6.4 m/s (*d*) 5.3 m/s

20. Two particle of masses m_1 and m_2 initially at rest start moving towards each other under their mutual force of attraction. The speed of the centre of mass at any time t, when they are at a distance r apart, is

(*a*) Zero (*b*) $\left(G\frac{m_1m_2}{r^2}.\frac{1}{m_1}\right)t$

(*c*) $\left(G\frac{m_1m_2}{r^2}.\frac{1}{m_2}\right)t$ (*d*) $\left(G\frac{m_1m_2}{r^2}.\frac{1}{m_1+m_2}\right)t$

21. The two particles X and Y, initially at rest, start moving towards each other under mutual attraction. If at any instant the velocity of X is V and that of Y is 2V, the velocity of their centre of mass will be
(*a*) Zero (*b*) V
(*c*) 2V (*d*) V/2

22. Two particles whose masses are 10 kg and 30 kg and their position vectors are $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}-\hat{j}-\hat{k}$ respectively would have the centre of mass at -

(*a*) $-\frac{(\hat{i}+\hat{j}+\hat{k})}{2}$ (*b*) $\frac{(\hat{i}+\hat{j}+\hat{k})}{2}$

(*c*) $-\frac{(\hat{i}+\hat{j}+\hat{k})}{4}$ (*d*) $\frac{(\hat{i}+\hat{j}+\hat{k})}{4}$

23. Two balls A and B of masses 100 gm and 250 gm respectively are connected by a stretched spring of negligible mass and placed on a smooth table. When the balls are released simultaneously the initial acceleration of B is 10 cm/sec² west ward. What is the magnitude and direction of initial acceleration of the ball A -
(*a*) 25 cm/sec² East ward
(*b*) 25 cm/sec² North ward
(*c*) 25 cm/sec² West ward
(*d*) 25 cm/sec² South ward

LAW OF CONSERVATION OF MOMENTUM

24. Two bodies of masses m_1 and m_2 have equal kinetic energies. If p_1 and p_2 are their respective momentum, then ratio $p_1 : p_2$ is equal to

(*a*) $m_1 : m_2$ (*b*) $m_2 : m_1$

(*c*) $\sqrt{m_1} : \sqrt{m_2}$ (*d*) $m_1^2 : m_2^2$

25. A bullet of mass m is being fired from a stationary gun of mass M. If the velocity of the bullet is v, the velocity of the gun is-

(*a*) $\frac{Mv}{m+M}$ (*b*) $\frac{-mv}{M}$

(*c*) $\frac{(M+m)v}{M}$ (*d*) $\frac{M+m}{Mv}$

26. Two identical blocks A and B, each of mass 'm' resting on smooth floor are connected by a light spring of natural length L and spring constant K, with the spring at its natural length. A third identical block 'C' (mass m) moving with a speed v along the line joining A and B collides with A. the maximum compression in the spring is (consider all collision are elastic)

(*a*) $v\sqrt{\frac{m}{2k}}$ (*b*) $m\sqrt{\frac{v}{2k}}$

(c) $\sqrt{\frac{mv}{k}}$ (*d*) $\frac{mv}{2k}$

27. A ball of mass 3 kg collides with a wall with velocity 10 m/sec at an angle of 30° and after collision reflects at the same angle with the same speed. The change in momentum of ball in MKS unit is-

(*a*) 20 (*b*) 30

(*c*) 15 (*d*) 45

28. A bomb at rest has mass 60 kg. It explodes and a fragment of 40 kg has kinetic energy 96 joule. Then kinetic energy of other fragment is-

(*a*) 180 J (*b*) 190 J

(*c*) 182 J (*d*) 192 J

IDEA OF IMPULSIVE / NON IMPULSIVE FORCE

29. A force of 50 dynes is acted on a body of mass 5gm which is at rest for an interval of 3 sec, then impulse is-

(*a*) 0.16×10^{-3} N-S (*b*) 0.98×10^{-3} N-S

(*c*) 1.5×10^{-3} N-S (*d*) 2.5×10^{-3} N-S

30. The area of F-t curve is A, where 'F' is the force on one mass due to the other. If one of the colliding bodies of mass M is at rest initially, its speed just after the collision is :

(*a*) A/M (*b*) M/A

(*c*) AM (*d*) $\sqrt{\frac{2A}{M}}$

31. If two balls, each of mass 0.06 kg, moving in opposite directions with speed of 4m/s, collide and rebound with the same speed, then the impulse imparted to each ball due to other (in kg-m/s) is :

(*a*) 0.48 (*b*) 0.53

(*c*) 0.81 (*d*) 0.92

COLLISIONS (OBLIQUE & HEAD ON)

32. A block moving in air explodes in two parts then just after explosion

(*a*) the total momentum must be conserved

(*b*) the total kinetic energy of two parts must be same as that of block before explosion.

(*c*) the total momentum must change

(*d*) the total kinetic energy must not be increased

33. In head on elastic collision of two bodies of equal masses, it is not possible :

(*a*) the velocities are interchanged

(*b*) the speeds are interchanged

(*c*) the momenta are interchanged

(*d*) the faster body speeds up and the slower body slows down

34. A ball of mass 'm', moving with uniform speed, collides elastically with another stationary ball. The incident ball will lose maximum kinetic energy when the mass of the stationary ball is

(*a*) m (*b*) 2m

(*c*) 4m (*d*) infinity

35. The coefficient of restitution e for a perfectly elastic collision is

(*a*) 1 (*b*) 0

(*c*) ∞ (*d*) -1

36. A lead ball strikes a wall and falls down, a tennis ball having the same mass and velocity strikes the wall and bounces back. Check the correct statement

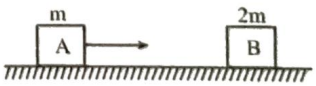
(*a*) The momentum of the lead ball is greater than that of the tennis ball

(*b*) The lead ball suffers a greater change in momentum compared with the tennis ball

(*c*) The tennis ball suffers a greater change in momentum as compared with the lead ball

(*d*) Both suffer an equal change in momentum

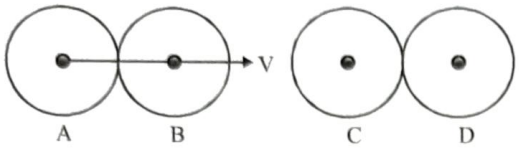
37. In the figure shown the block A collides head on with another block B at rest. Mass of B is twice the mass of A. The block A stops after collision. The co-efficient of restitution is :



(*a*) 0.5 (*b*) 1

(*c*) 0.25 (*d*) it is not possible

38. Two identical smooth spheres A and B are moving with same velocity and collides with similar spheres C and D, then after collision- (Consider one dimensional collision and all collision are elastic)



(*a*) D will move with greater speed

(*b*) C and D will move with same velocity v

(*c*) C will stop and D will move with velocity v

(*d*) All spheres A, B, C & D will move with velocity v/2

39. A body of mass m having an initial velocity v, makes head on collision with a stationary body of mass M. After the collision, the body of mass m comes to rest and only the body having mass M moves. This will happen only when (consider all collision are elastic)

(*a*) $m >> M$ (*b*) $m << M$

(*c*) $m = M$ (*d*) $m = \frac{1}{2}M$

40. Two equal masses m_1 and m_2 moving along the same straight line with velocities + 3 m/s and – 5 m/s respectively collide elastically. Their velocities after the collision will be respectively

(*a*) + 4 m/s for both (*b*) – 3 m/s and +5 m/s

(*c*) – 4 m/s and + 4 m/s (*d*) – 5 m/s and + 3 m/s

41. A body falls on a surface of coefficient of restitution 0.6 from a height of 1 m. Then the body rebounds to a height of

(*a*) 0.6 m (*b*) 0.4 m

(*c*) 1 m (*d*) 0.36 m

42. A ball of mass m falls vertically to the ground from a height h_1 and rebound to a height h_2. The change in momentum of the ball on striking the ground is

(*a*) $mg(h_1 - h_2)$ (*b*) $m(\sqrt{2gh_1} + \sqrt{2gh_2})$

(*c*) $m\sqrt{2g(h_1 + h_2)}$ (*d*) $m\sqrt{2g}(h_1 + h_2)$

43. Two spheres approaching each other collides elastically. Before collision the speed of A is 5m/s and that of B is 10m/s. Their masses are 1kg and 0.5kg. After collision velocities of A and B are respectively-

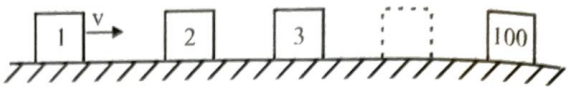
(*a*) 5 m/s –10 m/s

(*b*) 10 m/s, –5 m/s

(*c*) –10 m/s, –5 m/s

(*d*) –5 m/s, 10 m/s

44. There are hundred identical sliders equally spaced on a frictionless track as shown in the figure. Initially all the sliders are at rest. Slider 1 is pushed with velocity v towards slider 2. In a collision the sliders stick together. The final velocity of the set of hundred stucked sliders will be :



(*a*) $\frac{v}{99}$ (*b*) $\frac{v}{100}$

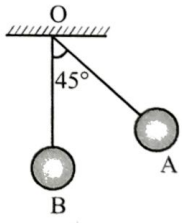
(*c*) zero (*d*) v

45. A space craft of mass M is moving with velocity V and suddenly explodes into two pieces. A part of it of mass m becomes at rest, then the velocity of other part will be

(*a*) $\frac{MV}{M-m}$ (*b*) $\frac{MV}{M+m}$

(*c*) $\frac{mV}{M-m}$ (*d*) $\frac{(M+m)V}{m}$

46. The bob A of a simple pendulum is released when the string makes an angle of 45°with the vertical. It hits another bob B of the same material and same mass kept at rest on the table. If the collision is elastic



(*a*) Both A and B rise to the same height

(*b*) Both A and B come to rest at B

(*c*) Both A and B move with the same velocity of A

(*d*) A comes to rest and B moves with the velocity of A

47. A body of mass M_1 collides elastically with another mass M_2 at rest. There is maximum transfer of energy when

(*a*) $M_1 > M_2$

(*b*) $M_1 < M_2$

(*c*) $M_1 = M_2$

(*d*) Same for all values of M_1 and M_2

48. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision



(a) $\frac{2}{\sqrt{3}}\upsilon$ (b) $\frac{\upsilon}{\sqrt{3}}$

(c) v (d) $\sqrt{3}\upsilon$

49. A completely inelastic collision is one in which the two colliding particles
(a) Are separated after collision
(b) Remain together after collision
(c) Split into small fragments flying in all directions
(d) None of the above

50. A body of mass 2kg is moving with velocity 10 m/s towards east. Another body of same mass and same velocity moving towards north collides with former and coalsces and moves towards north-east. Its velocity is
(a) 10 m/s (b) 5 m/s
(c) 2.5 m/s (d) $5\sqrt{2}$ m/s

51. A body of mass m_1 is moving with a velocity V. It collides with another stationary body of mass m_2. They get embedded. At the point of collision, the velocity of the system
(a) Increases
(b) Decreases but does not become zero
(c) Remains same
(d) Become zero

VARIABLE MASS SYSTEM

52. A rocket with a life-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s^2. The initial thrust of the blast is-
(a) 14.0×10^5 N (b) 1.76×10^5 N
(c) 3.5×10^5 N (d) 7.0×10^5 N

53. Fuel is consumed at the rate of 100 kg/sec. in a rocket. The exhaust gases are ejected as a speed of 4.5×10^4 m/s. What is the thrust experienced by the rocket-
(a) 3×10^6 N (b) 4.5×10^6 N
(c) 6×10^6 N (d) 9×10^6 N

54. A 6000 kg rocket is set for vertical firing. If the exhaust speed is 1000 m/sec. How much gas must be ejected each second to supply the thrust needed to give the rocket an initial upward acceleration of 20 m/sec^2- (consider g = 9.8 msec^{-2} acceleration due to gravity)
(a) 92.4 kg/sec (b) 178.8 kg/sec
(c) 143.2 kg/sec (d) 47.2 kg/sec

ROTATIONAL MOTION

Angular displacement, velocity & acceleration

55. The shaft of a motor rotates at a constant angular velocity of 3000 rpm. The radians it has turned in 1 sec are
(a) 1000 π (b) 100 π
(c) π (d) 10 π

56. The angular speed of second hand of a clock is
(a) (1/60) rad/s (b) (π/60) rad/s
(c) (2π/60) rad/s (d) (360/60) rad/s

57. Two bodies of mass 10 kg and 5 kg moving in concentric orbits of radius r_1 and r_2 such that their periods are same. The ratio of centripetal accelerations is
(a) r_1/r_2 (b) r_2/r_1
(c) $(r_1/r_2)3$ (d) $(r_2/r_1)^2$

58.. A wheel starts rotating from rest and attains an angular velocity of 60 rad/sec in 5 seconds. The total angular displacement in radians will be-
(a) 60 (b) 80
(c) 100 (d) 150

59. Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates uniformly about the common axis. The strings supporting A and B do not slip on the wheels. If x and y be the distances travelled by A and B in the same time interval, then-



(a) x = 2y (b) x = y
(c) y = 2x (d) None of these

60. The linear and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively. It will be at a distance from the axis of rotation.
(a) 50 m (b) $\frac{1}{2}$m
(c) 1 m (d) 2 m

61. A particle is moving with a constant angular velocity about an exterior axis. Its linear velocity will depend upon -

(*a*) perpendicular distance of the particle form the axis

(*b*) the mass of particle

(*c*) angular acceleration of the particle

(*d*) the linear acceleration of particle

62. A body is in pure rotation. The linear speed v of a particle, the distance r of the particle from the axis and the angular velocity ω of the body are related as $\omega = \frac{v}{r}$.

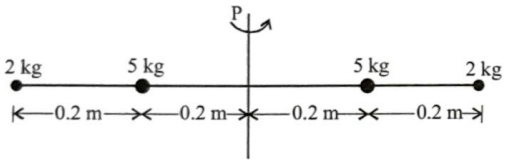
Thus-

(*a*) $\omega \propto \frac{1}{r}$ (*b*) $\omega \propto r$

(*c*) $\omega = 0$ (*d*) ω is independent of r.

Moment of Inertia

63. Four masses are fixed on a massless rod as shown in fig. The moment of inertia about the axis P is nearly



(*a*) 2 kg m^2 (*b*) 1 kg m^2

(*c*) 0.5 kg m^2 (*d*) 0.3 kg m^2

64. By the theorem of perpendicular axes, if a body be in X-Z-plane then :-

(*a*) $I_x - I_y = I_z$ (*b*) $I_x + I_z = I_y$

(*c*) $I_x + I_y = I_z$ (*d*) $I_y + I_z = I_x$

65. The axis X and Z in the plane of a disc are mutually perpendicular and Y-axis is perpendicular to the plane of the disc. If the moment of inertia of the body about X and Y axes is respectively 30 kg m^2 and 40 kg m^2 then M.I. about Z-axis in kg m^2 will be :

(*a*) 70 (*b*) 50

(*c*) 10 (*d*) Zero

66. Two rods each of mass m and length ℓ are joined at the centre to form a cross. The moment of inertia of this cross about an axis passing through the common centre of the rods and perpendicular to the plane formed by them, is :

(*a*) $\frac{m\ell^2}{12}$ (*b*) $\frac{m\ell^2}{6}$

(*c*) $\frac{m\ell^2}{3}$ (*d*) $\frac{m\ell^2}{2}$

67. For the same total mass which of the following will have the largest moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of the body

(*a*) A disc of radius a

(*b*) A ring of radius a

(*c*) A square lamina of side 2a

(*d*) Four rods forming a square of side 2a

68. The moment of inertia of a thin uniform circular disc about one of the diameters is I. Its moment of inertia about an axis perpendicular to the .plane of disc and passing through its centre is

(*a*) $(\sqrt{2})$I (*b*) 2I

(*c*) I/2 (*d*) $I/\sqrt{2}$

69. The moment of inertia of a uniformsemicircular wire of mass M and radius r about a line perpendicular to the plane of the wire through the centre is

(*a*) Mr^2 (*b*) $\frac{1}{2}Mr^2$

(*c*) $\frac{1}{4}Mr^2$ (*d*) $\frac{2}{5}Mr^2$

70. The density of a rod AB increases linearly from A to B. Its midpoint is O and its centre of mass is at C. Four axes pass through A, B, O and C, all perpendicular to the length of the rod. The moments of inertia of the rod about these axes are I_A, I_B, I_O and I_C respectively.

(*a*) $I_A > I_B$ (*b*) $I_A < I_B$

(*c*) $I_O = I_C$ (*d*) $I_O < I_C$

71. A stone of mass 4 kg is whirled in a horizontal circle of radius 1m and makes 2 rev/sec. The moment of inertia of the stone about the axis of rotation is-

(*a*) 64 kg × m^2 (*b*) 4 kg × m^2

(*c*) 16 kg × m^2 (*d*) 1 kg × m^2

72. In an arrangement four particles, each of mass 2 gram are situated at the coordinate points (3, 2, 0), (1, –1, 0), (0, 0, 0) and (–1, 1, 0). The moment of inertia of this arrangement about the Z-axis will be-

(*a*) 8 units (*b*) 16 units

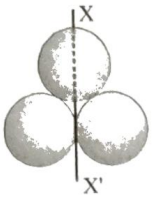
(*c*) 43 units (*d*) 34 units

73. A solid sphere and a hollow sphere of the same mass have the same M.I. about their respective diameters. The ratio of their radii will be :

(*a*) 1 : 2 (*b*) $\sqrt{3}:\sqrt{5}$

(*c*) $\sqrt{5}:\sqrt{3}$ (*d*) 5 : 4

74. Three solid spheres of mass M and radius R are shown in the figure. The moment of inertia of the system about XX' axis will be:



(a) $\frac{7}{2}MR^2$ (b) $\frac{14}{5}MR^2$

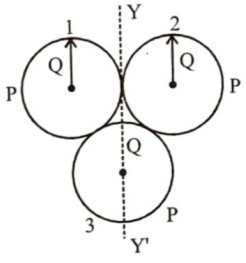
(c) $\frac{16}{5}MR^2$ (d) $\frac{21}{5}MR^2$

75. The moment of inertia of a square lamina about the perpendicular axis through its centre of mass is 20 kg-m². Then, its moment of inertia about an axis touching its side and in the plane of the lamina will be:-

(a) 10 kg-m² (b) 30 kg-m²

(c) 40 kg-m² (d) 25 kg-m²

76. Three rings, each of mass P and radius Q are arranged as shown in the figure. The moment of inertia of the arrangement about YY' axis will be-



(a) $\frac{7}{2}PQ^2$ (b) $\frac{2}{7}PQ^2$

(c) $\frac{2}{5}PQ^2$ (d) $\frac{5}{2}PQ^2$

77. The moment of inertia of a rod of mass M and length L about an axis passing through one edge and perpendicular to its length will be:–

(a) $\frac{ML^2}{12}$ (b) $\frac{ML^2}{6}$

(c) $\frac{ML^2}{3}$ (d) ML^2

78. Three thin uniform rods each of mass M and length L and placed along the three axis of a Cartesian coordinate system with one end of each rod at the origin. The M.I. of the system about z-axis is-

(a) $\frac{ML^2}{3}$ (b) $\frac{2ML^2}{3}$

(c) $\frac{ML^2}{6}$ (d) ML^2

79. Four particles each of mass m are placed at the corners of a square of side length ℓ.The radius of gyration of the system about an axis perpendicular to the square and passing through centre is :–

(a) $\frac{\ell}{\sqrt{2}}$ (b) $\frac{\ell}{2}$

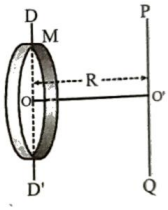
(c) ℓ (d) $\ell\sqrt{2}$

80. The M.I. of a thin rod of length ℓ about the perpendicular axis through its centre is I. The M.I. of the square structure made by four such rods about a perpendicular axis to the plane and through the centre will be :-

(a) 4 I (b) 8 I

(c) 12 I (d) 16 I

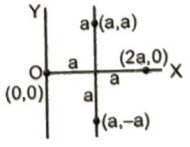
81. The moment of inertia of a ring of mass M and radius R about PQ axis will be :



(a) MR^2 (b) $\frac{MR^2}{2}$

(c) $\frac{3}{2}MR^2$ (d) 2MR

82. Four point masses (each of mass m) are arranged it the X-Y plane the moment of inertia of this array of masses about Y-axis is



(a) ma^2 (b) $2ma^2$

(c) $4ma^2$ (d) $6ma^2$

83. An equilateral traingular wire frame is made from 3 rods of equal mass and length ℓ each. The frame is rotated about an axis perpendicular to the plane of the frame and passing through its end. What is the radius of gyration of the frame?

(*a*) $\frac{\ell}{2}$ (*b*) ℓ

(*c*) $\frac{\ell}{\sqrt{2}}$ (*d*) $\frac{\ell}{2\sqrt{3}}$

Fixed Axis Rotation + Toppling

84. For a system to be in equilibrium, the torques acting on it must balance. This is true only if the torques are taken about

(*a*) the centre of the system
(*b*) the centre of mass of the system
(*c*) any point on the system
(*d*) any point on the system or outside it

85. A rectangular block has a square base measuring a × a, and its height is h. It moves on a horizontal surface in a direction perpendicular to one of the edges h being vertical. The coefficient of friction is μ. It will topple if

(*a*) $\mu > \frac{h}{a}$ (*b*) $\mu > \frac{a}{h}$

(*c*) $\mu > \frac{2a}{h}$ (*d*) $\mu > \frac{a}{2h}$

86. A force of $(2\hat{i} - 4\hat{j} + 2\hat{k})$ Newton acts at a point $(3\hat{i} + 2\hat{j} - 4\hat{k})$ metre from the origin. The magnitude of torque is -

(*a*) zero (*b*) 24.4 N-m
(*c*) 0.244 N-m (*d*) 2.444 N-m

87. Rate of change of angular momentum with respect to time is proportional to :

(*a*) angular velocity (*b*) angular acceleration
(*c*) moment of inertia (*d*) torque

88. When constant torque is acting on a body then :–

(*a*) body maintain its state or moves in straight line with same velocity
(*b*) acquire linear acceleration
(*c*) acquire angular acceleration
(*d*) rotates with a constant angular velocity

89. If $I = 50$ kg-m^2, then how much torque will be applied to stop it in 10 sec. Its initial angular speed is 20 rad/sec.:

(*a*) 100 N-m (*b*) 150 N-m
(*c*) 200 N-m (*d*) 250 N-m

90. A particle is at a distance r from the axis of rotation. A given torque τ produces some angular acceleration in it. If the mass of the particle is doubled and its distance from the axis is halved. the value of torque to produce the same angular acceleration is -

(*a*) τ/2 (*b*) τ
(*c*) 2τ (*d*) 4τ

Energy Analysis

91. A ring of radius r and mass m rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω. Its kinetic energy is

(*a*) mrω (*b*) $\frac{1}{2}mr\omega^2$

(*c*) $mr^2\omega^2$ (*d*) $\frac{1}{2}mr^2\omega^2$

92. A rod of length L is hinged at one end. It is brought to a horizontal position and released. The angular velocity of the rod when it is in vertical position is

(*a*) $\sqrt{2g/L}$ (*b*) $\sqrt{3g/L}$

(*c*) $\sqrt{g/2L}$ (*d*) $\sqrt{g/L}$

93. Two bodies A and B having same angular momentum and $I_A > I_B$, then the relation between $(K.E.)_A$ and $(K.E.)_B$ will be:

(*a*) $(K.E.)_A > (K.E.)_B$ (*b*) $(K.E.)_A = (K.E.)_B$
(*c*) $(K.E.)_A < (K.E.)_B$ (*d*) $(K.E.)_A \neq (K.E.)_B$

Angular Momentum

94. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects, each of mass m are attached gently to the opposite ends of a diameter of the ring. The wheel now rotates with an angular velocity

(*a*) $\frac{\omega M}{(M+m)}$ (*b*) $\frac{\omega(M-2m)}{(M+2m)}$

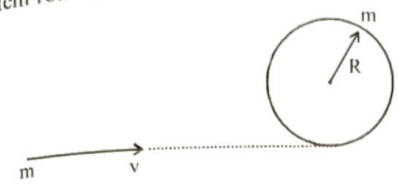
(*c*) $\frac{\omega M}{(M+2m)}$ (*d*) $\frac{\omega(M+2m)}{M}$

95. A rotating table completes one rotation is 10 sec. and its moment of ineratia is 100 kg-m^2. A person of 50 kg. mass stands at the centre of the rotating table. If the person moves 2m. from the centre, the angular velocity of the rotating table in rad/sec. will be:

(*a*) $\frac{2\pi}{30}$ (*b*) $\frac{20\pi}{30}$

(*c*) $\frac{2\pi}{3}$ (*d*) 2π

96. A circular hoop of mass m, and radius R rests flat on a horizontal frictionless surface. A bullet, also of mass m, and moving with a velocity v, strikes the hoop and gets embedded in it. The thickness of the hoop is much smaller than R. The angular velocity with which the system rotates after the bullet strikes the hoop is



(a) $\frac{V}{4R}$ (b) $\frac{V}{3R}$

(c) $\frac{2V}{3R}$ (d) $\frac{3V}{4R}$

97. A uniform heavy disc is rotating at constant angular velocity (ω) about a vertical axis through its centre O. Some wax W is dropped gently on the disc. The angular velocity of the disc-

(a) does not change (b) increases

(c) decreases (d) becomes zero

98. A girl sits near the edge of a rotating circular platform. If the girl moves from circumference towards the centre of the platform then the angular velocity of the platform will-

(a) decrease (b) increase

(c) remain same (d) becomes zero

99. Two wheels P and Q are mounted on the same axle. The moment of inertia of P is 6 kg-m^2 and it is rotating at 600 rotations per minute and Q is at rest. If the two are joined by means of a clutch then they combined and rotate at 400 rotations per minute. The moment of inertia of Q will be -

(a) 3 kg-m^2 (b) 4 kg-m^2

(c) 5 kg-m^2 (d) 8 kg-m^2

Combined Rotation and Translation

100. If a spherical ball rolls on a table without slipping, the fraction of its total kinetic energy associated with rotation is

(a) 3/5 (b) 2/7

(c) 2/5 (d) 3/7

101. The speed of a homogeneous solid sphere after rolling down an inclined plane of vertical height h, from rest without sliding is

(a) $\sqrt{gh}$ (b) $\sqrt{(g/5)gh}$

(c) $\sqrt{(4/3)gh}$ (d) $\sqrt{(10/7)gh}$

102. The rotational kinetic energy of a body is E. In the absence of external torque, if mass of the body is halved and radius of gyration doubled, then its rotational kinetic energy will be :-

(a) 0.5E (b) 0.25E

(c) E (d) 2E

103. A ring is rolling without slipping. Its energy of translation is E. Its total kinetic energy will be :-

(a) E (b) 2E

(c) 3E (d) 4E

104. One hollow and one solid cylinder of the same outer radius rolls down on a rough inclined plane. The foot of the inclined plane is reached by

(a) solid cylinder earlier

(b) hollow cylinder earlier

(c) simultaneously

(d) the heavier earlier irrespective of being solid or hollow

105. If a solid sphere, disc and cylinder are allowed to roll down an inclined plane from the same height

(a) cylinder will reach the bottom first

(b) disc will reach the bottom first

(c) sphere will reach the bottom first

(d) all will reach the bottom at the same time

106. A solid sphere rolls down without slipping two different inclined planes of the same height but of different inclinations

(a) in both cases the speeds and time of descend will be same

(b) the speeds will be same but time of descend will be different

(c) the speeds will be different but time of descend will be same

(d) speeds and time of descend both will be different

107. A solid homogeneous sphere is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere

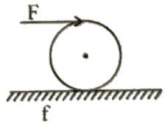
(a) total kinetic energy is conserved

(b) angular momentum of the sphere about the point of contact with the plane is conserved

(c) only the rotational kinetic energy about the centre of mass is conserved

(d) angular momentum about centre of mass is conversed

108. A ring of mass M is kept on a horizontal rough surface. A force F is applied tangentially at its rim as shown. The coefficient of friction between the ring and the surface is μ. Then



(a) friction will act in the forward direction

(b) friction will act in the backward direction

(c) frictional force will not act

(d) frictional force will be μMg.

109. A disc is rolling on an inclined plane without slipping then what fraction of its total energy will be in form of rotational kinetic energy :

(*a*) 1 : 3 (*b*) 1 : 2

(*c*) 2 : 7 (*d*) 2 : 5

110. A ring takes time t_1 and t_2 for sliding down and rolling down an inclined plane of length L respectively for reaching the bottom. The ratio of t_1 and t_2 is :-

(*a*) $\sqrt{2}$: 1 (*b*) 1 : $\sqrt{2}$

(*c*) 1 : 2 (*d*) 2 : 1

111. A ladder rests against a frictionless vertical wall, with its upper end 6m above the ground and the lower end 4m away from the wall. The weight of the ladder is 500 N and its C. G. at 1/3rd distance from the lower end. Wall's reaction will be, (in Newton)

(*a*) 111

(*b*) 333

(*c*) 222

(*d*) 129

Learning Plus

CENTRE OF MASS

1. A thin uniform wire is bent to form the two equal sides AB and AC of triangle ABC, where AB = AC = 5 cm. The third side BC, of length 6cm, is made from uniform wire of twice the density of the first. The distance of centre of mass from A is :

(*a*) $\frac{34}{11}$ cm (*b*) $\frac{11}{34}$ cm

(*c*) $\frac{34}{9}$ cm (*d*) $\frac{11}{45}$ cm

2. The centre of mass of a system of particles is at the origin. From this we conclude that

(*a*) the number of particles on positive x-axis is equal to the number of particles on negative x-axis

(*b*) the total mass of the particles on positive x-axis is same as the total mass on negative x-axis

(*c*) the number of particles on X-axis may be equal to the number of particles on Y-axis.

(*d*) if there is a particle on the positive X-axis, there must be at least one particle on the negative X-axis.

3. A body of mass 1 kg moving in the x-direction, suddenly explodes into two fragments of mass $\frac{1}{8}$ kg and $\frac{7}{8}$ kg. An instant later, the smaller fragment is 0.14 m above the x-axis. The position of the heavier fragment is -

(*a*) $\frac{1}{50}$ m above x-axis (*b*) $\frac{1}{50}$ m below x-axis

(*c*) $\frac{7}{50}$ m below x-axis (*d*) $\frac{7}{50}$ m above x-axis

4. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to other end of the plank. If the mass of the plank is $\frac{M}{3}$, then the distance that the man moves relative to ground is:

(*a*) $\frac{3L}{4}$ (*b*) $\frac{L}{4}$

(*c*) $\frac{4L}{5}$ (*d*) $\frac{L}{3}$

5. A particle of mass 3m is projected from the ground at some angle with horizontal. The horizontal range is R. At the highest point of its path it breaks into two pieces m and 2m. The smaller mass comes to rest and larger mass finally falls at a distance x from the point of projection where x is equal to

(*a*) $\frac{3R}{4}$ (*b*) $\frac{3R}{2}$

(*c*) $\frac{5R}{4}$ (*d*) 3R

6. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speed of 2m/s and 6m/s respectively on a smooth horizontal surface. The speed of centre of mass of the system is :

(*a*) $\frac{10}{3}$ m/s (*b*) $\frac{10}{7}$ m/s

(*c*) $\frac{11}{2}$ m/s (*d*) $\frac{12}{3}$ m/s

7. Two particles having mass ratio n : 1 are interconnected by a light inextensible string that passes over a smooth pulley. If the system is released, then the acceleration of the centre of mass of the system is :

(a) $(n-1)^2 g$ (b) $\left(\frac{n+1}{n-1}\right)^2 g$

(c) $\left(\frac{n-1}{n+1}\right)^2 g$ (d) $\left(\frac{n+1}{n-1}\right) g$

8. Internal forces in a system can change
(a) linear momentum only
(b) kinetic energy only
(c) both kinetic energy and linear momentum
(d) neither the linear momentum nor the kinetic energy of the system.

9. A man of mass 'm' climbs on a rope of length L suspended below a balloon of mass M. The balloon is stationary with respect to ground. If the man begins to climb up the rope at a speed v_{rel} (relative to rope). In what direction and with what speed (relative to ground) will the balloon move?

(a) downwards, $\frac{mv_{rel}}{m+M}$ (b) upwards, $\frac{Mv_{rel}}{m+M}$

(c) downwards, $\frac{mv_{rel}}{M}$ (d) downwards, $\frac{(M+m)v_{rel}}{M}$

10. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the compartment plus passengers system is C_2. If the passengers move about inside the compartment along the track.
(a) both C_1 and C_2 will move with respect to the ground
(b) neither C_1 nor C_2 will move with respect to the ground
(c) C_1 will move but C_2 will be stationary with respect to the ground
(d) C_2 will move but C_1 will be stationary with respect to the ground

11. A shell is fired from a canon with a velocity V at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces come to rest. The speed of the other piece immediately after the explosion is
(a) $3V\cos\theta$ (b) $2V\cos\theta$

(c) $\frac{3}{2} V\cos\theta$ (d) $V\cos\theta$

12. A small sphere is moving at a constant speed in a vertical circle. Below is a list of quantities that could be used to describe some aspect of the motion of the sphere
I. kinetic energy
II. gravitational potential energy
III. momentum
Which of these quantities will change as this sphere moves around the circle ?
(a) I and II only (b) I and III only
(c) III only (d) II and III only

13. A bomb at rest explodes into two parts of masses m_1 and m_2. If the momentums of the two parts be p_1 and p_2, then their kinetic energies will be in the ratio of-
(a) m_1 / m_2 (b) m_2 / m_1
(c) p_1 / p_2 (d) p_2 / p_1

14. A body of mass m collides against a wall with the velocity υ and rebounds with the same speed. Its change of momentum is-
(a) 2 mυ (b) mυ
(c) – mυ (d) 0

15. A bomb initially at rest explodes by it self into three equal mass fragments. The velocities of two fragments are $(3\hat{i}+2\hat{j})$ m/s and $(-\hat{i}-4\hat{j})$ m/s. The velocity of the third fragment is (in m/s) -
(a) $2\hat{i}+2\hat{j}$ (b) $2\hat{i}-2\hat{j}$
(c) $-2\hat{i}+2\hat{j}$ (d) $-2\hat{i}-2\hat{j}$

16. A stone of mass m_1 moving with a uniform speed v suddenly explodes on its own into two fragments. If the fragment of mass m_2 is at rest, the speed of the other fragment is-

(a) $\frac{m_1 v}{(m_1-m_2)}$ (b) $\frac{m_2 v}{(m_1-m_2)}$

(c) $\frac{m_1 v}{(m_1+m_2)}$ (d) $\frac{m_1 v}{m_2}$

17. A nucleus of mass number A originally at rest emits α-particle with speed v. The recoil speed of daughter nucleus is :

(a) $\frac{4v}{A-4}$ (b) $\frac{4v}{A+4}$

(c) $\frac{v}{A-4}$ (d) $\frac{v}{A+4}$

18. A super-ball is to bounce elastically back and forth between two rigid walls at a distance d from each other. Neglecting gravity and assuming the velocity of super-ball to be v_0 horizontally, the average force being exerted by the super-ball on each wall is :

(a) $\frac{1}{2}\frac{mv_0^2}{d}$ (b) $\frac{mv_0^2}{d}$

(c) $\frac{2mv_0^2}{d}$ (d) $\frac{4mv_0^2}{d}$

19. A force exerts an impulse I on a particle changing its speed from u to 2u. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

(a) $\frac{3}{2}$Iu (b) $\frac{1}{2}$Iu

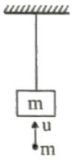
(c) Iu (d) 2Iu

20. A particle of mass 4m which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed 'v' each in mutually perpendicular directions. The minimum energy released in the process of explosion is :

(*a*) $(2/3)\,mv^2$ (*b*) $(3/2)\,mv^2$

(*c*) $(4/3)\,mv^2$ (*d*) $(3/4)\,mv^2$

21. A bullet of mass m moving vertically upwards instantaneously with a velocity 'u' hits the hanging block of mass 'm' and gets embedded in it. As shown in the figure the height through which block rises after the collision, assume sufficient space above block :



(*a*) $u^2/2g$ (*b*) u^2/g

(*c*) $u^2/8g$ (*d*) $u^2/4g$

22. In an inelastic collision-

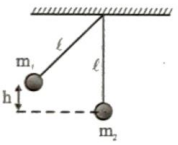
(*a*) momentum is conserved but kinetic energy is not

(*b*) momentum is not conserved but kinetic energy is conserved

(*c*) neighter momentum nor kinetic energy is conserved

(*d*) both the momentum and kinetic energy are conserved

23. In the arrangement shown, the pendulum on the left is pulled aside. It is then released and allowed to collide with other pendulum which is at rest. A perfectly inelastic collision occurs and the system rises to a height h/4. The ratio of the masses (m_1 / m_2) of the pendulum is :



(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

24. Two perfectly elastic balls of same mass m are moving with velocities u_1 and u_2. They collide elastically n times. The kinetic energy of the system finally is:

(*a*) $\frac{1}{2}\frac{m}{u}u_1^2$ (*b*) $\frac{1}{2}\frac{m}{u}(u_1^2+u_2^2)$

(*c*) $\frac{1}{2}m(u_1^2+u_2^2)$ (*d*) $\frac{1}{2}mn(u_1^2+u_2^2)$

25. A ball hits the floor and rebounds after an inelastic collision . In this case-

(*a*) the momentum of the ball just after the collision is the same as that just before the collision

(*b*) the mechanical energy of the ball remains the same in the collision

(*c*) the total momentum of the ball and the earth is conserved.

(*d*) the total energy of the ball and the earth is conserved

26. Six steel balls of identical size are lined up along a straight frictionless groove. Two similar balls moving with a speed V along the groove collide with this row on the extreme left hand then (all collision are ellastic)-



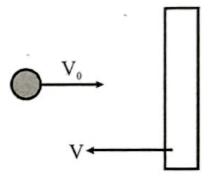
(*a*) all the balls will start moving to the right with speed 1/8 each

(*b*) all the six balls initially at rest will move on with speed V/6 each and two identical balls will come to rest

(*c*) two balls from the extreme right end will move on with speed V each and the remaining balls will remain at rest

(*d*) one ball from the right end will move on with speed 2V, the remaining balls will be at rest.

27. A particle of mass m moves with velocity $v_0 = 20$ m/sec towards a large wall that is moving with velocity v = 5 m/sec. towards a particle as shown. If the particle collides with the wall elastically, the speed of the particle just after the collision is :



(*a*) 30 m/s (*b*) 20 m/s

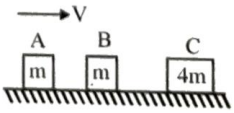
(*c*) 25 m/s (*d*) 22 m/s

28. A sphere of mass m moving with a constant velocity hits another stationary sphere of the same mass. If e is the coefficient of restitution, then ratio of speed of the first sphere to the speed of the second sphere after collision will be :

(*a*) $\left(\frac{1-e}{1+e}\right)$ (*b*) $\left(\frac{1+e}{1-e}\right)$

(*c*) $\left(\frac{e+1}{e-1}\right)$ (*d*) $\left(\frac{e-1}{e+1}\right)$

29. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity v to the right. Block B with mass m and block C with mass 4 m are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



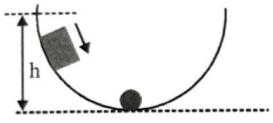
(*a*) 0.60 v to the left (*b*) 1.4 v to the left

(*c*) v to the left (*d*) 0.4 v to the left

30. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occured ?
(*a*) inelastic
(*b*) elastic
(*c*) completely inelastic
(*d*) Cannot be determined from the information given

31. An object of mass 5 kg and speed 10 ms^{-1} explodes into two pieces of equal mass. One piece comes to rest. The kinetic energy added to the system during the explosion is-
(*a*) Zero. (*b*) 50 J.
(*c*) 250 J. (*d*) 500 J.

32. A block of mass m starts from rest and slides down a frictionless semi-circular track from a height h as shown. When it reaches the lowest point of the track, it collides with a stationary piece of putty also having mass m. If the block and the putty stick together and continue to slide, the maximum height that the block-putty system could reach is



(*a*) h/4 (*b*) h/2
(*c*) h (*d*) independent of h

ROTATIONAL MOTION

33. A fan is running at 3000 rpm. It is switched off. It comes to rest by uniformly decreasing its angular speed in 10 seconds. The total number of revolutions in this period.
(*a*) 150 (*b*) 250
(*c*) 350 (*d*) 300

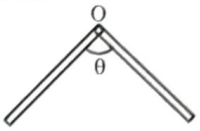
34. The moment of inertia of a non-uniform semicircular wire having mass m and radius r about a line perpendicular to the plane of the wire through the centre is

(*a*) mr^2 (*b*) $\frac{1}{2}mr^2$

(*c*) $\frac{1}{4}mr^2$ (*d*) $\frac{2}{5}mr^2$

35. Let I_A and I_B be the moments of inertia of two solid cylinders of identical geometrical shape and size about their axes, the first made of aluminium and the second of iron.
(*a*) $I_A < I_B$
(*b*) $I_A = I_B$
(*c*) $I_A > I_B$
(*d*) relation between I_A and I_B depends on the actual shapes of the bodies.

36. A unifrom thin rod of length L and mass M is bent at the middle point O as shown in figure. Consider an axis passing through its middle point O and perpendicular to the plane of the bent rod. Then moment of inertia about this axis is :



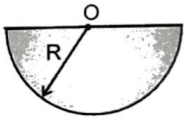
(*a*) $\frac{2}{3}mL^2$ (*b*) $\frac{1}{3}mL^2$

(*c*) $\frac{1}{12}mL^2$ (*d*) dependent on θ

37. Two spheres of same mass and radius are in contact with each other. If the moment of inertia of a sphere about its diameter is I, then the moment of inertia of both the spheres about the tangent at their common point would be -
(*a*) 3I (*b*) 7I
(*c*) 4I (*d*) 5I

38. The M.I. of a disc about its diameter is 2 units. Its M.I. about axis through a point on its rim and in the plane of the disc is
(*a*) 4 units. (*b*) 6 units
(*c*) 8 units (*d*) 10 units

39. Moment of inertia of a thin semicircular disc (mass = M & radius = R) about an axis through point O and perpendicular to plane of disc, is given by :



(*a*) $\frac{1}{4}MR^2$ (*b*) $\frac{1}{2}MR^2$

(*c*) $\frac{1}{8}MR^2$ (*d*) MR^2

40. A rigid body can be hinged about any point on the x-axis. When it is hinged such that the hinge is at x, the moment of inertia is given by $I = 2x^2 - 12x + 27$ The x-coordinate of centre of mass is
(*a*) x = 2 (*b*) x = 0
(*c*) x = 1 (*d*) x = 3

41. On applying a constant torque on a body-
(*a*) linear velocity may be increases
(*b*) angular velocity may be increases
(*c*) it will rotate with constant angular velocity
(*d*) it will move with constant velocity

42. A wheel starting with angular velocity of 10 radian/sec acquires angular velocity of 100 radian/sec in 15 seconds. If moment of inertia is 10kg-m^2, then applied torque (in newton-metre) is
(*a*) 900 (*b*) 100
(*c*) 90 (*d*) 60

43. A torque of 2 newton-m produces an angular acceleration of 2 rad/sec² a body. If its radius of gyration is 2m, its mass will be:

(*a*) 2kg (*b*) 4 kg

(*c*) 1/2 kg (*d*) 1/4 kg

44. A uniform circular disc A of radius r is made from a metal plate of thickness t and another uniform circular disc B of radius 4r is made from the same metal plate of thickness t/4. If equal torques act on the discs A and B, initially both being at rest. At a later instant, the angular speeds of a point on the rim of A and another point on the rim of B are ω_A and ω_B respectively. We have

(*a*) $\omega_A > \omega_B$

(*b*) $\omega_A = \omega_B$

(*c*) $\omega_A < \omega_B$

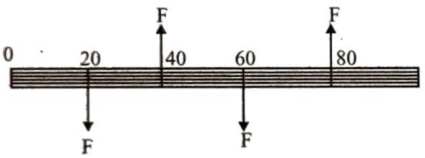
(*d*) the relation depends on the actual magnitude of the torques.

45. A force $\vec{F} = 4\hat{i} - 10\hat{j}$ acts on a body at a point having position vector $-5\hat{i} - 3\hat{j}$ relative to origin of co-ordinates on the axis of rotation . The torque acting on the body about the origin is :

(*a*) 38 $\hat{k}$ (*b*) $-25\ \hat{k}$

(*c*) 62 $\hat{k}$ (*d*) none of these

46. Four equal and parallel forces are acting on a rod (as shown in figure) at distances of 20 cm, 40 cm, 60 cm and 80 cm respectively from one end of the rod. Under the influence of these forces the rod :



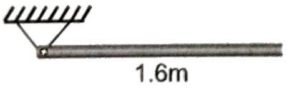
(*a*) is at rest

(*b*) experiences a torque

(*c*) experiences a linear motion

(*d*) experiences a torque and also a linear motion

47. The uniform rod of mass 20 kg and length 1.6 m is pivoted at its end and swings freely in the vertical plane. Angular acceleration of rod just after the rod is released from rest in the horizontal position as shown in figure is



(*a*) $\frac{15g}{16}$ (*b*) $\frac{17g}{16}$

(*c*) $\frac{16g}{15}$ (*d*) $\frac{g}{15}$

48. Two men support a uniform horizontal rod at its two ends. If one of them suddenly lets go, the force exerted by the rod on the other man will:

(*a*) remain unaffected

(*b*) increase

(*c*) decrease

(*d*) become unequal to the force exerted by him on the beam.

49. A Cubical bloc of mass M and edge a slides down a rough inclined plane of inclination θ with a uniform velocity. The torque of the normal force on the block about its centre has a magnitude.

(*a*) zero (*b*) Mga

(*c*) Mga sin θ (*d*) $\frac{1}{2}$Mga sin θ

50. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force 'F' is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. The minimum value of 'F' for which the cube begins to tilt about the edge is (assume that the cube does not slide).

(*a*) $\frac{2}{3}$mg (*b*) $\frac{4}{3}$mg

(*c*) $\frac{5}{4}$mg (*d*) $\frac{1}{2}$mg

51. The moment of inertia and rotational kinetic energy of a fly wheel are 20kg-m² and 1000 joule respectively. Its angular frequency per minute would be -

(*a*) $\frac{600}{\pi}$ (*b*) $\frac{25}{\pi^2}$

(*c*) $\frac{5}{\pi}$ (*d*) $\frac{300}{\pi}$

52. A circular ring of wire of mass M and radius R is making n revolutions/sec about an axis passing through a point on its rim and perpendicular to its plane. The kinetic energy of rotation of the ring is given by-

(*a*) $4\pi^2MR^2n^2$ (*b*) $2\pi^2MR^2n^2$

(*c*) $\frac{1}{2}\pi^2MR^2n^2$ (*d*) $8\pi^2MR^2n^2$

53. Rotational kinetic energy of a disc of constant moment of inertia is -

(*a*) directly proportional to angular velocity

(*b*) inversely proportional to angular velocity

(*c*) inversely proportional to square of angular velocity

(*d*) directly proportional to square of angular velocity

54. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 sec. the magnitude of this torque is :

(*a*) $4A_0$ (*b*) A_0

(*c*) $3A_0/4$ (*d*) $12A_0$

55. A particle moves with a constant velocity parallel to the Y-axis. Its angular momentum about the origin.
(*a*) is zero
(*b*) remains constant
(*c*) goes on increasing
(*d*) goes on decreasing.

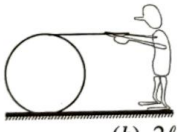
56. A boy sitting firmly over a rotating stool has his arms folded. If he stretches his arms, his angular momentum about the axis of rotation
(*a*) increases (*b*) decreases
(*c*) remains unchanged (*d*) doubles

57. The rotational kinetic energy of a rigid body of moment of inertia 5 kg-m^2 is 10 joules. The angular momentum about the axis of rotation would be -
(*a*) 100 joule-sec (*b*) 50 joule-sec
(*c*) 10 joule-sec (*d*) 2 joule -sec

58. The angular velocity of a body changes from one revolution per 9 second to 1 revolution per second without applying any torque. The ratio of its radius of gyration in the two cases is
(*a*) 1 : 9 (*b*) 3 : 1
(*c*) 9 : 1 (*d*) 1 : 3

59. A disc rolls on a table. The ratio of its K.E. of rotation to the total K.E. is -
(*a*) 2/5 (*b*) 1/3
(*c*) 5/6 (*d*) 2/3

60. A thin string is wrapped several times around a cylinder kept on a rough horizontal surface. A boy standing at a distance ℓ from the cylinder draws the string towards him as shown in figure. The cylinder rolls without slipping. The length of the string passed through the hand of the boy while the cylinder reaches his hand is



(*a*) ℓ (*b*) 2ℓ
(*c*) 3ℓ (*d*) 4ℓ

61. A solid sphere, a hollow sphere and a ring, all having equal mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are equal and but not sufficient to allow pure rolling. The greastest kinetic energy at the bottom of the incline will be achieved by
(*a*) the solid sphere
(*b*) the hollow sphere
(*c*) the ring
(*d*) all will achieve same kinetic energy.

62. A body is given translational velocity and kept on a surface that has sufficient friction. Then:
(*a*) body will move forward before pure rolling
(*b*) body will move backward before pure rolling
(*c*) body will start pure rolling immediately
(*d*) none of these

63. A disk and a ring of the same mass are rolling to have the same kinetic energy. What is ratio of their velocities of centre of mass
(*a*) $(4:3)^{1/2}$ (*b*) $(3:4)^{1/2}$
(*c*) $(b)^{1/2} : (c)^{1/2}$ (*d*) $(c)^{1/2} : (b)^{1/2}$

64. A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be :
(*a*) thin pipe
(*b*) uniform cylinder
(*c*) uniform sphere
(*d*) thin spherical shell

65. A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere :
(*a*) must be forward (in direction of v)
(*b*) must be backward (opposite to v)
(*c*) cannot be zero
(*d*) none of the above

66. A cylinder is pure rolling up an incline plane. It stops momentarily and then rolls back. The force of friction.
(*a*) on the cylinder is zero throughout the journey
(*b*) is directed opposite to the velocity of the centre of mass throughout the journey
(*c*) is directed up the plane throughout the journey
(*d*) is directed down the plane throughout the journey

67. A sphere is released on a smooth inclined plane from the top. When it moves down its angular momentum is:
(*a*) conserved about every point
(*b*) conserved about the point of contact only
(*c*) conserved about the centre of the sphere only
(*d*) conserved about any point on a fixed line parallel to the inclined plane and passing throughthe centre of the ball.

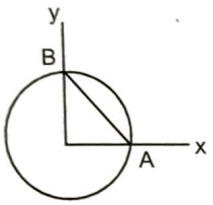
Advanced Level Multiconcept Questions

JEE-ADVANCED
COMPREHENSION/STATEMENT/MATCHING/MCQ

Centre of Mass

1. A system of particles has its centre of mass at the origin. The x-coordinates of all the particles
   (*a*) may be positive (*b*) may be negative
   (*c*) may be non-negative (*d*) may be non-positive

2. An object comprises of a uniform ring of radius R and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following can not be the centre of mass of the object.



   (*a*) (R/3, R/3) (*b*) (R3, R/2)
   (*c*) (R/4, R/4) (*d*) $(R/\sqrt{2}, R/\sqrt{2})$

3. If the net external forces acting on a system is zero, then the centre of mass
   (*a*) must not move (*b*) must not accelerate
   (*c*) may move (*d*) may accelerate

4. An external force $\vec{F}(\vec{F} \neq 0)$ acts on a system of particles.
   The velocity and the acceleration of the centre of mass are found to be v_{cm} and a_{cm}, then it is possible that
   (*a*) $v_{cm} = 0, a_{cm} = 0$ (*b*) $v_{cm} = 0, a_{cm} \neq 0$
   (*c*) $v_{cm} \neq 0, a_{cm} = 0$ (*d*) $v_{cm} \neq 0, a_{cm} \neq 0$

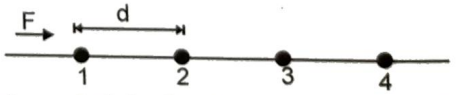
5. Two blocks A and B each of mass 'm' are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal plank block C also of mass m moves on the floor with a speed 'v' along the line joining A and B and collides elastically with A then which of the following is/are correct :
   (*a*) K E of the AB system at maximum compression of the spring is zero
   (*b*) The KE of AB system at maximum compression is (1/4) mv^2
   (*c*) The maximum compression of spring is $v\sqrt{m/k}$
   (*d*) The maximum compression of spring is $v\sqrt{m/2k}$

6. In an elastic collision in absence of external force, which of the following is/are correct :
   (*a*) The linear momentum is conserved
   (*b*) The potential energy is conserved in collision
   (*c*) The final kinetic energy is less than the initial kinetic energy
   (*d*) The final kinetic energy is equal to the initial kinetic energy

7. A block moving in air explodes in two parts then just after explosion (neglect change in momentum duet to gravity)
   (*a*) the total momentum of two parts must be same to the momentum of the block before explosion.
   (*b*) the total kinetic energy of two parts must be same as that of block before explosion.
   (*c*) the total momentum must change
   (*d*) the total kinetic energy must increase

8. A set of n-identical cubical blocks lie at rest parallel to each other along a line on a smooth horizontal surface. The separation between the near surfaces of any two adjacent blocks is L. The block at one end is given a speed V towards the next one at time t = 0. All collisions are completely inelastic, then
   (*a*) The last block starts moving at $t = n(n-1)\frac{L}{2V}$
   (*b*) The last block starts moving at $t = (n-1)\frac{L}{V}$
   (*c*) The centre of mass of the system will have a final speed v/n
   (*d*) The centre of mass of the system will have a final speed v

9. The fig. shows a string of equally spaced beads of mass m, separated by distance d. The beads are free to slide without friction on a thin wire. A constant force F acts on the first bead initially at rest till it makes collision with the second bead. The second bead then collides with the third and so on. Suppose all collisions are elastic, then:



   (*a*) speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\frac{2Fd}{m}}$ and zero respectively.

(*b*) speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\frac{2Fd}{m}}$ and $\frac{1}{2}\sqrt{\frac{2Fd}{m}}$ respectively.

(*c*) speed of the second bead immediately after its collision with third bead is zero.

(*d*) the average speed of the first bead is $\frac{1}{2}\sqrt{\frac{2Fd}{m}}$.

10. A shell explodes in a region of negligible gravitational field, giving out n fragments of equal mass m. Then its total

(*a*) Kinetic energy is smaller than that before the explosion

(*b*) Kinetic energy is greater than that before the explosion

(*c*) Momentum and kinetic energy depend on n

(*d*) Momentum is equal to that before the explosion.

11. Two identical balls are interconnected with a massless and inextensible thread. The system is in gravity free space with the thread just taut. Each ball is imparted a velocity v, one towards the other ball and the other perpendicular to the first, at t = 0. Then,

(*a*) the thread will become taut at t = (L/v)

(*b*) the thread will become taut at some time t < (L/v).

(*c*) the thread will always remain taut for t > (L/v)

(*d*) the kinetic energy of the system will always remain mv^2.

12. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle when has twice its mass during the impact.

(*a*) The minimum kinetic energy of the system is 1 joule

(*b*) The maximum elastic potential energy of the system is 2 joule.

(*c*) Momentum and total kinetic energy of the system are conserved at every instant.

(*d*) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

13. In an inelastic collision,

(*a*) the velocity of both the particles may be same after collision.

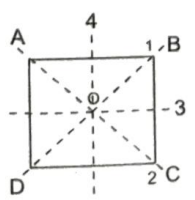
(*b*) kinetic energy is not conserved

(*c*) linear momentum of the system is conserved.

(*d*) velocity of separation will be less than velocity of approach.

ROTATIONAL MOTION

14. The moment of inertia of a thin uniform square plate ABCD of uniform thickness about an axis passing through the centre O and perpendicular to the plate is

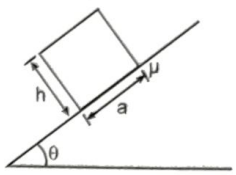


where I_1, I_2, I_3, and I_4 are respectively the moments of inertia about axes 1, 2, 3, and 4 which are in the plane of the plate.

(*a*) $I_1 + I_2$ (*b*) $I_3 + I_4$

(*c*) $I_1 + I_3$ (*d*) $I_1 + I_2 + I_3 + I_4$

15. A block with a square base measuring a × a and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will:



(*a*) topple before sliding if $\mu > \frac{a}{h}$

(*b*) topple before sliding if $\mu < \frac{a}{h}$

(*c*) slide before toppling if $\mu > \frac{a}{h}$

(*d*) slide before toppling if $\mu < \frac{a}{h}$

16. A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at a distance x from A.

(*a*) the normal reaction at A is $\frac{wx}{d}$

(*b*) the normal reaction at A is $\frac{w(d-x)}{d}$

(*c*) the normal reaction at B is $\frac{wx}{d}$

(*d*) the normal reaction at B is $\frac{w(d-x)}{d}$

17. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is

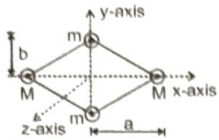
(*a*) 2, if their lines of action pass through the centre of mass of the body

(*b*) 3, if their lines of action are not parallel

(*c*) 3, if their lines of action are parallel

(*d*) 4, if their lines of action are parallel and all the forces have the same magnitude

18. Four point masses are fastened to the corners of a frame of negligible mass lying in the xy plane. Let w be the angular speed of rotation. Then



(*a*) rotational kinetic energy associated with a given angular speed depends on the axis of rotation.
(*b*) rotational kinetic energy about y-axis is independent of m and its value is $Ma^2\omega^2$
(*c*) rotational kinetic energy about z-axis depends on m and its value is $(Ma^2 + mb^2)\omega^2$
(*d*) rotational kinetic energy about z-axis is independent of m and its value is $Mb^2\omega^2$

19. In the given figure a ball strikes a rod elastically and rod is hinged at point A. Then which of the statement(s) is/are correct for the collision?



(*a*) linear momentum of system (ball + rod) is conserved
(*b*) angular momentum of system about hinged point A is conserved
(*c*) initial KE of the system is equal to final KE of the system
(*d*) linear momentum of ball is conserved.

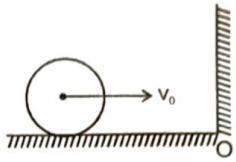
20. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground.
(*a*) Angular momentum of the particle about O is increasing
(*b*) Torque of the gravitational force on the particle about O is decreasing
(*c*) The moment of inertia of the particle about O is decreasing
(*d*) The angular velocity of the particle about O is increasing

21. A man spinning in free space changes the shape of his body, eg. by spreading his arms or curling up. By doing this, he can change his
(*a*) moment of inertia (*b*) angular momentum
(*c*) angular velocity (*d*) rotational kinetic energy

22. When a bicycle is in motion (accelerating) on a rough plane, the force of friction exerted by the plane on the two wheels is such that it acts :
(*a*) In the backward direction on the front wheel and in the forward direction on the rear wheel
(*b*) In the forward direction on the front wheel and in the backward direction on the rear wheel
(*c*) In the backward direction on both front and the rear wheels
(*d*) In the forward direction on both the front and the rear wheels

23. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is v.
(*a*) $0 \le v \le 2u$
(*b*) $v = u$, if CP is horizontal
(*c*) $v = u$, if CP makes an angle of 30° with the horizontal and P is below the horizontal level of C
(*d*) $v = \sqrt{2}\,u$, if CP is horizontal

24. Consider a sphere of mass 'm' radius 'R' doing pure rolling motion on a rough surface having velocity $\vec{v}_0$ as shown in the Figure. It makes an elastic impact with the smooth wall and moves back and starts pure rolling after some time again.

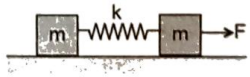


(*a*) Change in angular momentum about 'O' in the entire motion equals $2mv_0R$ in magnitude.
(*b*) Moment of impulse provided by wall during impact about O equals $2mv_0R$ in magnitude
(*c*) Final velocity of ball will be $\frac{3}{7}\vec{v}_0$
(*d*) Final velocity of ball will be $-\frac{3}{7}\vec{v}_0$

CENTRE OF MASS

Comprehension – 1 (No. 25 to 27)

Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on the first block pulling it away from the other as shown in figure.



25. Then the displacement of the centre of mass at time *t* is
(*a*) $\frac{Ft^2}{2m}$ (*b*) $\frac{Ft^2}{3m}$
(*c*) $\frac{Ft^2}{4m}$ (*d*) $\frac{Ft^2}{m}$

26. If the extension of the spring is x_0 at time t, then the displacement of the right block at this instant is:
(*a*) $\frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$ (*b*) $-\frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$
(*c*) $\frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$ (*d*) $\left(\frac{Ft^2}{2m} + x_0\right)$

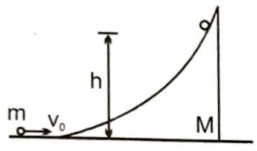
27. If the extension of the spring is x_0 at time t, then the displacement of the left block at this instant is:

(a) $\left(\frac{Ft^2}{2m} - x_0\right)$ (b) $\frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$

(c) $\frac{1}{2}\left(\frac{2Ft^2}{m} - x_0\right)$ (d) $\frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$

Comprehension – 2 (No. 28 to 34)

A particle of mass m moving horizontal with v_0 strikes a smooth wedge of mass M, as shown in figure. After collision, the ball starts moving up the inclined face of the wedge and rises to a height h.



28. The final velocity of the wedge v_2 is

(a) $\frac{mv_0}{M}$ (b) $\frac{mv_0}{M+m}$

(c) v_0 (d) insufficient data

29. When the particle has risen to a height h on the wedge, then choose the correct alternative(s)

(a) The particle is stationary with respect to ground

(b) Both are stationary with respect to the centre of mass

(c) The kinetic energy of the centre of mass remains constant

(d) The kinetic energy with respect to centre of mass is converted into potential energy

30. The maximum height h attained by the particle is

(a) $\left(\frac{m}{m+M}\right)\frac{v_0^2}{2g}$ (b) $\left(\frac{m}{M}\right)\frac{v_0^2}{2g}$

(c) $\left(\frac{M}{m+M}\right)\frac{v_0^2}{2g}$ (d) none of these.

31. Identify the correct statement(s) related to the situation when the particle starts moving downward.

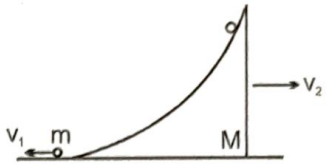
(a) The centre of mass of the system remains stationary

(b) Both the particle and the wedge remain stationary with respect to centre of mass

(c) When the particle reaches the horizontal surface its velocity relative to the wedge is v_0

(d) None of these

32. Suppose the particle when reaches the horizontal surfaces, its velocity with respect to ground is v_1 and that of wedge is v_2. Choose the correct statement (s)



(a) $mv_1 = Mv_2$ (b) $Mv_2 - mv_1 = mv_0$

(c) $v_1 + v_2 = v_0$ (d) $v_1 + v_2 < v_0$

33. Choose the correct statement(s) related to particle m

(a) Its kinetic energy is $K_f = \left(\frac{mM}{m+M}\right)gh$

(b) $v_1 = v_0\left(\frac{M-m}{M+m}\right)$

(c) The ratio of its final kinetic energy to its initial kinetic energy is $\frac{K_f}{K_i} = \left(\frac{M}{m+M}\right)^2$

(d) It moves opposite to its initial direction of motion

34. Choose the correct statement related to the wedge M

(a) Its kinetic energy is $K_f = \left(\frac{4m^2}{m+M}\right)gh$

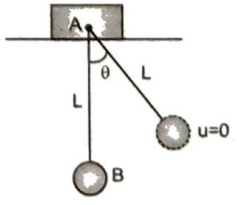
(b) $v_2 = \left(\frac{2m}{m+M}\right)v_0$

(c) Its gain in kinetic energy is $\Delta K = \left(\frac{4mM}{(m+M)^2}\right)\left(\frac{1}{2}mv_0^2\right)$

(d) Its velocity is more that the velocity of centre of mass

Comprehension – 3 (No. 35 to 38)

A small ball B of mass m is suspended with light inelastic string of length L from a block A of same mass m which can move on smooth horizontal surface as shown in the figure. The ball is displaced by angle θ from equilibrium position & then released.



35. The displacement of block when ball reaches the equilibrium position is

(a) $\frac{L\sin\theta}{2}$ (b) $L\sin\theta$

(c) L (d) none of these

36. Tension in string when it is vertical, is
(a) mg (b) $mg(2-\cos\theta)$
(c) $mg(3-2\cos\theta)$ (d) none of these

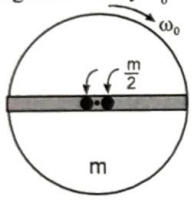
37. Maximum velocity of block during subsequent motion of the system after release of ball is
(a) $[g/(1-\cos\theta)]^{1/2}$
(b) $[2g/(1-\cos\theta)]^{1/2}$
(c) $[g/\cos\theta]^{1/2}$
(d) informations are insufficient to decide

38. The displacement of centre of mass of A + B system till the string becomes vertical is
(a) zero (b) $\frac{L}{2}(1-\cos\theta)$
(c) $\frac{L}{2}(1-\sin\theta)$ (d) none of these

ROTATIONAL MOTION

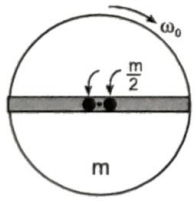
Comprehension – 04 (No. 39 to 41)

A uniform disc of mass 'm' and radius R is free to rotate in horizontal plane about a vertical smooth fixed axis passing through its centre. There is a smooth groove along the diameter of the disc and two small balls of mass $\frac{m}{2}$ each are placed in it on either side of the centre of the disc as shown in fig. The disc is given initial angular velocity ω_0 and released.



39. The angular speed of the disc when the balls reach the end of the disc is :
(a) $\frac{\omega_0}{2}$ (b) $\frac{\omega_0}{3}$
(c) $\frac{2\omega_0}{3}$ (d) $\frac{\omega_0}{4}$

40. The speed of each ball relative to ground just after they leave the disc is :



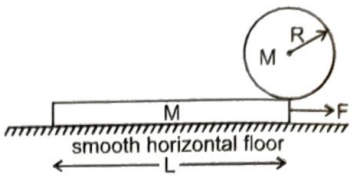
(a) $\frac{R\omega_0}{\sqrt{3}}$ (b) $\frac{R\omega_0}{\sqrt{2}}$
(c) $\frac{2R\omega_0}{3}$ (d) $\frac{R\omega_0}{3}$

41. The net work done by forces exerted by disc on one of the ball for the duration ball remains on the disc is
(a) $\frac{2mR^2\omega_0^2}{9}$ (b) $\frac{mR^2\omega_0^2}{18}$
(c) $\frac{mR^2\omega_0^2}{6}$ (d) $\frac{mR^2\omega_0^2}{9}$

Comprehension – 05 (No. 42 to 44)

A uniform disc of mass M and radius R initially stands vertically on the right end of a horizontal plank of mass M and length L, as shown in the figure.

The plank rests on smooth horizontal floor and friction between disc and plank is sufficiently high such that disc rolls on plank without slipping. The plank is pulled to right with a constant horizontal force of magnitude F.



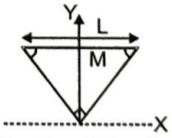
42. The magnitude of acceleration of plank is-
(a) $\frac{F}{8M}$ (b) $\frac{F}{4M}$
(c) $\frac{3F}{2M}$ (d) $\frac{3F}{4M}$

43. The magnitude of angular acceleration of the disc is -
(a) $\frac{F}{4mR}$ (b) $\frac{F}{8mR}$
(c) $\frac{F}{2mR}$ (d) $\frac{3F}{2mR}$

44. The distance travelled by centre of disc from its initial position till the left end of plank comes vertically below the centre of disc is
(a) $\frac{L}{2}$ (b) $\frac{L}{4}$
(c) $\frac{L}{8}$ (d) L

Comprehension – 06 (No. 45 to 48)

The figure shows an isosceles triangular plate of mass M and base L. The angle at the apex is 90°. The apex lies at the origin and the base is parallel to X - axis.



45. The moment of inertia of the plate about the z-axis is
(a) $\frac{ML^2}{12}$ (b) $\frac{ML^2}{24}$
(c) $\frac{ML^2}{6}$ (d) none of these

46. The moment of inertia of the plate about the x-axis is

(a) $\frac{ML^2}{8}$ (b) $\frac{ML^2}{32}$

(c) $\frac{ML^2}{24}$ (d) $\frac{ML^2}{6}$

47. The moment of inertia of the plate about its base parallel to the x-axis is

(a) $\frac{ML^2}{18}$ (b) $\frac{ML^2}{36}$

(c) $\frac{ML^2}{24}$ (d) none of these

48. The moment of inertia of the plate about the y-axis is

(a) $\frac{ML^2}{6}$

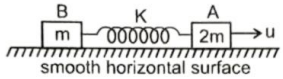
(b) $\frac{ML^2}{8}$

(c) $\frac{ML^2}{24}$

(d) none of these

CENTRE OF MASS EXERCISE–II

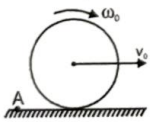
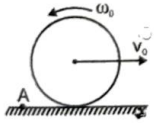
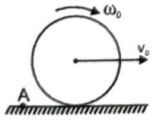
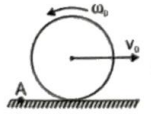
49. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of Column-I, certain statements are given and corresponding results are given in Column-II. Match the statements in Column-I corresponding results in Column-II.



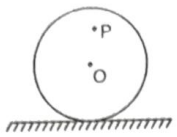
Column-I	Column-II
(a) The velocity of block A	(p) can never be zero
(b) The velocity of block B	(q) may be zero at certain instants of time
(c) The kinetic energy of system of two blocks	(r) is minimum at maximum compression of spring
(d) The potential energy of spring	(s) is maximum at maximum extension of spring

ROTATIONAL MOTION

50. In each situation of column-I, a uniform disc of mass m and radius R rolls on a rough fixed horizontal surface as shown in the figure. At t = 0 (initially) the angular velocity of disc is ω_0 and velocity of centre of mass of disc is v_0 (in horizontal direction). The relation between v_0 and ω_0 for each situation and also initial sense of rotation is given for each situation in Column-I. Then match the statements in Column-I with the corresponding results in Column-II.

Column-I	Column-II
(a)  $(v_0 > R\omega_0)$	(p) The angular momentum of disc about point A (as shown in figure) remains conserved.
(b)  $(v_0 > R\omega_0)$	(q) The kinetic energy of disc after it starts rolling without slipping is less than its initial kinetic energy.
(c)  $(v_0 < R\omega_0)$	(r) In the duration disc rolls with slipping, the friction acts on disc towards left.
(d)  $(v_0 < R\omega_0)$	(s) In the duration disc rolls with slipping, the friction acts on disc for some time towards right and for some time towards left.

51. A uniform disc rolls without slipping on a rough horizontal surface with uniform angular velocity. Point O is the centre of disc and P is a point on disc as shown in the figure. In each situation of column I a statement is given and the corresponding results are given in Column-II. Match the statements in Column-I with the results in Column-II.

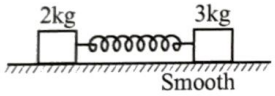


Column I	Column II
(*a*) The velocity of point P on disc	(p) Changes in magnitude with time.
(*b*) The acceleration of point P on disc	(q) is always directed from that point (the point on disc given in column-I) towards centre of disc.
(*c*) The tangential acceleration of point P on disc	(r) is always zero.
(*d*) The acceleration of point on disc which is in contact with rough horizontal surface in magnitude.	(s) is non-zero and remains constant

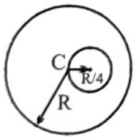
NUMERICAL BASED QUESTIONS

CENTER OF MASS

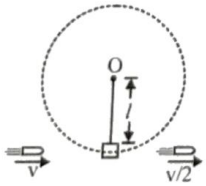
52. A 2kg mass and 3 kg mass are used to compress oppoite ends of a spring (k = 750 N/m) by a distance of 40 cm from natural length and released from rest. If the speeds of the two masses as they leave the spring are v_2 and v_3, find $v_2 + v_3$ (in m/s).



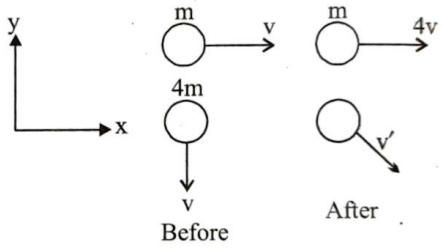
53. For years together, people thought that hell is located inside earth. Assume that hell is a spherical bubble created inside earth. It is completely evacuated to make sinful people feel suffocated. It has a radius R/4 and is located as shown. (C is earth's centre). What is the distance of centre of mass of the resulting body from C (in km). Assume that earth's radius is 6300 km.



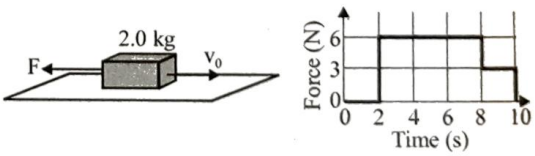
54. A bullet of mass m and velocity v passes through a pendulum bob of mass M and emerges with velocity v/2 (figure). The pendulum bob is at the end of a string of length *l*. What is the minimum value of v (in m/s) such that the pendulum bob will swing through a complete circle ? (Take : *l* = 2m, M = 1kg, m = 10 gm.)



55. Two particles of masses m and 4m, moving in vacuum at right angles to each other experience same force $\vec{F}$ for time T simultaneously. Consequently the particle m moves with velocity 4v in its original direction. Find the new magnitude of the velocity v' (in m/s) of the particle 4m. Given v = 100 m/s.



56. A block of mass 2kg is sliding on a smooth surface. At t = 0, its speed is v_0 = 2 m/s. At t = 0, a time - varying force starts acting on the block in the direction opposite to v_0. Find the speed of object (in m/s) at t = 10

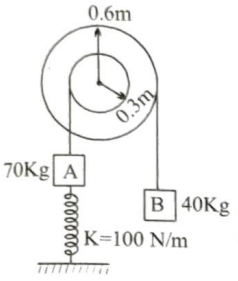


ROTATIONAL MOTION

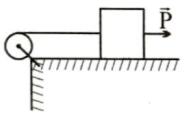
57. A merry-go-round is a common piece of playground equipment. A 4 m diameter merry-go-round with a moment of inertia of 500 kg-m^2 is spinning at 5.6 rad/s. John runs tangent to the merry-go-round at 5 m/s, in the same direction that it is turning, and jumps onto the outer edge. John's mass is 30 kg. What is the merry-go-round's angular velocity, (in rad/s), after John jumps on?

58. A solid Hemisphere rests in equilibrium on a rough ground and against a smooth wall. The curved surface touches the wall and the ground. The angle of inclination of the circular base to the horizontal is 30°, Find minimum coefficient of friction required between ground and hemisphere. Fill 32μ in OMR sheet.

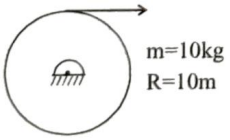
59. The figure shown is in equilibrium. Find out the extension in the spring (in cm).



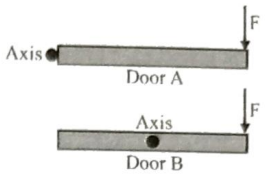
60. A wheel of radius 0.20 m is mounted on a frictionless horizontal axis. The rotational inertia of the wheel about the axis is 0.07 kgm^2. A massless cord wrapped around the wheel is attached to a 2.0 kg block that slides on a horizontal frictionless surface. If a horizontal force of magnitude P = 3.0 N is applied to the block as shown in figure, what is the magnitude of the angular acceleration (in r/s^2) of the wheel? Assume the string does not slip on the wheel.



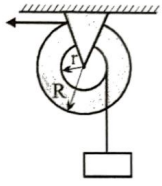
61. A force of 100 N is applied on a disc along the tangent for 10 sec. During this time, the disc attains an angular velocity of 10 rad/s. There is constant frictional torque at the axis which opposes the rotation of the disc. Now the force is removed. After what time (in sec) from the removal of force will the disc come to rest.



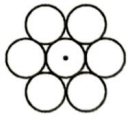
62. The drawing shows the top view of two doors. The doors are uniform and identical. Door A rotates about an axis through its left edge, while door B rotates about an axis through the center. The same force F is applied perpendicular to each door at its right edge, and the force remains perpendicular as the door turns. Starting from rest, door A rotates through a certain angle in 3 s. How long does it take door B to rotate through the same angle? Round off to nearest integer.



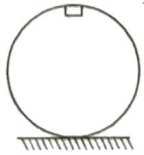
63. A yo-yo-shaped device mounted on a horizontal frictionless axis is used to lift a 30kg box as shown in figure. The outer radius R of the device is 0.50 m, an the radius r of the hub is 0.20 m. When a constant horizontal force of magnitude 152 N is applied in the left direction to a rope wrapped around the outside of the device, the box, which is suspended from a rope wrapped around the hub, has an upward acceleration of magnitude 0.80 m/s^2. What is the rotational inertia I (in kg-m^2) of the device about its axis of rotation?



64. Seven pennies are arranged in a hexagonal, planar pattern so as to touch each neighbor, as shown in the figure below. Each penny is a uniform disk of mass m = 2kg and radius r = 1m. What is the moment of inertia of the system (in kg-m^2) of seven pennies about an axis that passes through the center of the central penny and is normal to the plane of the pennies?



65. A ring of mass m = 1 kg and radius R = 1.25 m is kept on a rough horizontal ground. A small body of mass m is struck to the top of the ring. When it was given a slight push forward, the ring started rolling purely on the ground. What is the maximum speed of the centre of the ring (in m/s)?



JEE-MAIN PREVIOUS YEAR'S
CENTRE OF MASS

1. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d : while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c . The values of p_d and p_c are respectively: [*JEE Main-2018*]

(*a*) (·28, ·89) (*b*) (0, 0)

(*c*) (0, 1) (*d*) (·89, ·28)

2. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

[*JEE Main-2018*]

(*a*) $\sqrt{2}\,v_0$ (*b*) $\frac{v_0}{2}$

(*c*) $\frac{v_0}{\sqrt{2}}$ (*d*) $\frac{v_0}{4}$

3. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly: [*JEE Main-2018*]

(*a*) 4.70×10^3 N/m^2 (*b*) 2.35×10^2 N/m^2

(*c*) 4.70×10^2 N/m^2 (*d*) 2.35×10^3 N/m^2

4. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is :

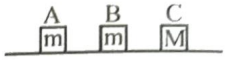
[*JEE Main-2019 (January)*]

(*a*) $2\sqrt{\frac{k}{p}}$ (*b*) $2\sqrt{\frac{p}{k}}$

(*c*) $\sqrt{\frac{2k}{p}}$ (*d*) $\sqrt{\frac{2p}{k}}$

5. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically if $\frac{5}{6}$ th of the initial kinetic energy is lost in whole process. What is value of M/m?

[*JEE Main-2019 (January)*]



(*a*) 5 (*b*) 2

(*c*) 4 (*d*) 3

6. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is ($g = 10\ ms^{-2}$)

[*JEE Main-2019 (January)*]

(*a*) 20 m (*b*) 30 m

(*c*) 40 m (*d*) 10 m

7. A particle of mass 'm' is moving with speed '2v' and collides with a mass '2m' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be : [*JEE Main-2019 (April)*]

(*a*) $v/\left(2\sqrt{2}\right)$ (*b*) $2\sqrt{2}v$

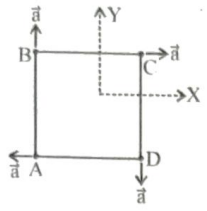
(*c*) $\sqrt{2}v$ (*d*) $v/\sqrt{2}$

8. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by:- [*JEE Main-2019 (April)*]

(*a*) $\frac{2v^2}{7g}$ (*b*) $\frac{v^2}{g}$

(*c*) $\frac{2v^2}{5g}$ (*d*) $\frac{v^2}{2g}$

9. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is : [JEE Main-2019 (April)]



(a) $\frac{a}{5}(\hat{i}-\hat{j})$ (b) $\frac{a}{5}(\hat{i}+\hat{j})$

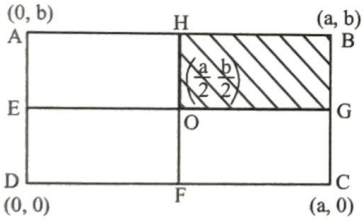
(c) Zero (d) $a(\hat{i}+\hat{j})$

10. If 10^{22} gas molecules each of mass 10^{-26} kg colide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of: [JEE Main-2019 (April)]

(a) 10^8 N/m^2 (b) 10^4 N/m^2

(c) 10^3 N/m^2 (d) 10^{16} N/m^2

11. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be :- [JEE Main-2019 (April)]



(a) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (b) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (d) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

12. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$, After colision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5\, m_1$ and $v_3 = 0.5\, v_1$, then v_1 is: [JEE Main-2019 (April)]

(a) $v_4 - \frac{v_2}{4}$ (b) $v_4 - \frac{v_2}{2}$

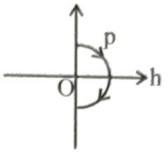
(c) $v_4 - v_2$ (d) $v_4 + v_2$

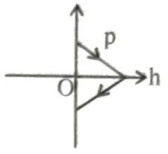
13. A body of mass 2 kg makes an eleastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body? [JEE Main-2019 (April)]

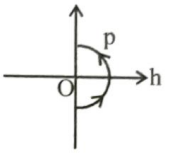
(a) 1.8 kg (b) 1.2 kg

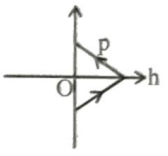
(c) 1.5 kg (d) 1.0 kg

14. A ball is thrown vertically up (taken as +z-axis) from the ground. The correct momentum-height (p-h) diagram is : [JEE Main-2019 (April)]

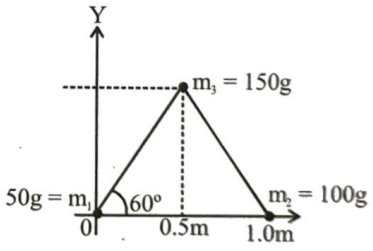
(a) 

(b) 

(c) 

(d) 

15. Three particles of masses 50 g, 100g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be : [JEE Main-2019 (April)]



(a) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$ (b) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$

(c) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$ (d) $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

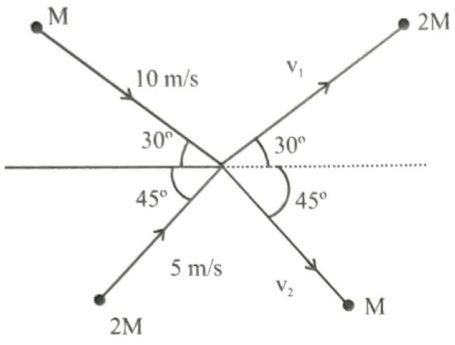
16. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is: [JEE Main-2019 (April)]

(a) 0.20 ms^{-1} (b) 0.14 ms^{-1}

(c) 0.47 ms^{-1} (d) 0.28 ms^{-1}

17. Two particles, of masses M and 2M, moving as shown, with speeds of 10 m/s and 5 ms $^{-1}$ collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2 respectively. The values of v_1 and v_2 are nearly :

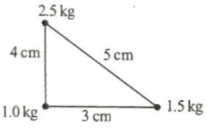
[*JEE Main-2019 (April)*]



(*a*) 3.2 m/s and 6.3 m/s (*b*) 3.2 m/s and 12.6 m/s
(*c*) 6.5 m/s and 6.3 m/s (*d*) 6.5 m/s and 3.2 m/s

18. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point :

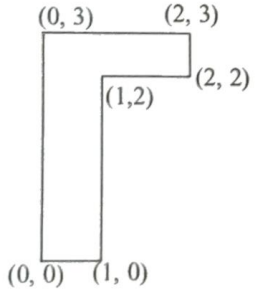
[*JEE Main-2020 (January)*]



(*a*) 0.6 cm right and 2.0 cm above 1kg mass
(*b*) 2.0 cm right and 0.9 cm above 1 kg mass
(*c*) 0.9 cm right and 2.0 cm above 1kg mass
(*d*) 1.5 cm right and 1.2 cm above 1kg mass

19. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in figure) are:

[*JEE Main-2020 (January)*]



(*a*) (1.25 m, 1.50 m) (*b*) (0.75 m, 0.75 m)
(*c*) (0.75 m, 1.75 m) (*d*) (1m, 1.75m)

20. A body A, of mass m = 0.1 kg has an initial velocity of $3\hat{i}$ ms $^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}$ms^{-1}. After collision, A moves with a velocity $\vec{v} = 4(\hat{i} + \hat{j})$. The energy of B after collision is written as $\frac{x}{10}$J. The value of x is_____.

[*JEE Main-2020 (January)*]

21. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms $^{-2}$) near the surface on that planet is _____

[*JEE Main-2020 (January)*]

22. As shown in figure. When a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e on the surface of the cavity. R can be determined by the equation:

[*JEE Main-2020 (January)*]



(*a*) $(R^2 - R + 1)(2 - R) = 1$
(*b*) $(R^2 + R - 1)(2 - R) = 1$
(*c*) $(R^2 - R - 1)(2 - R) = 1$
(*d*) $(R^2 + R - 1)(2 - R) = 1$

23. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely in elastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is: [*JEE Main-2020 (January)*]

(*a*) $\sqrt{\frac{3}{4}}$ (*b*) $\sqrt{\frac{3}{2}}$
(*c*) $\sqrt{\frac{1}{2}}$ (*d*) $\frac{1}{2}$

24. Two particles of equal mas m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely

[*JEE Main-2020 (January)*]

(*a*) $\frac{3}{4}mu^2$ (*b*) $\sqrt{\frac{2}{3}}mu^2$
(*c*) $\frac{1}{3}mu^2$ (*d*) $\frac{1}{8}mu^2$

25. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body:

[*JEE Main-2020 (January)*]

(*a*) Escapes from the Planet's Gravitational field
(*b*) Continues to move in a circular orbit
(*c*) Falls vertically downwards towards the planet
(*d*) Starts moving in an elliptical orbit around the planet

26. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \pi/3$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is:

[*JEE Main-2020 (January)*]

(*a*) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (*b*) $2\sqrt{2}\frac{u^2}{g}$

(*c*) $\frac{5}{8}\frac{u^2}{g}$ (*d*) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

27. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at:

[*JEE Main-2020 (January)*]

(*a*) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (*b*) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

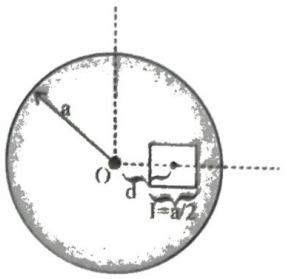
(*c*) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (*d*) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

28. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy(see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$, then value of n is _____.

[*JEE Main-2020 (September)*]

29. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{x}$, value of X (to the nearest integer) is __.

[*JEE Main-2020 (September)*]



30. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3 m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by

[*JEE Main-2020 (September)*]

(*a*) $v = \frac{1}{\sqrt{6}}u$ (*b*) $v = \frac{u}{\sqrt{3}}$

(*c*) $v = \sqrt{\frac{2}{3}}u$ (*d*) $v = \frac{u}{\sqrt{2}}$

31. A block of mass 1.9 kg is at rest at the edge of a table, of height 1m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is

[*JEE Main-2020 (September)*]

[Take $g = 10\ m/s^2$ Assume there is no rotational motion and loss of energy after the collision is negligable.]

(*a*) 19 J (*b*) 23 J
(*c*) 20 J (*d*) 21 J

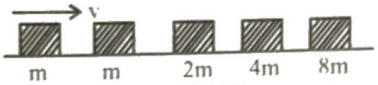
32. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I=M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio L/R for it to have minimum possible I is

[*JEE Main-2020 (September)*]

(*a*) $\sqrt{\frac{2}{3}}$ (*b*) $\frac{3}{2}$

(*c*) $\frac{2}{3}$ (*d*) $\sqrt{\frac{3}{2}}$

33. Blocks of masses m, 2m, 4m and 8m are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8 m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to [JEE Main-2020 (September)]



(a) 37 (b) 77
(c) 87 (d) 94

34. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where v(t) is its instantaneous velocity. The instantaneous acceleration of the satellite is [JEE Main-2020 (September)]

(a) $-bv^3(t)$ (b) $-\frac{2bv^3}{M(t)}$

(c) $-\frac{bv^3}{M(t)}$ (d) $-\frac{bv^3}{2M(t)}$

35. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is _____. [JEE Main-2020 (September)]

36. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1+\frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is [JEE Main-2020 (September)]

(a) $\frac{2}{5}ML^2$ (b) $\frac{5}{12}ML^2$

(c) $\frac{3}{7}ML^2$ (d) $\frac{7}{18}ML^2$

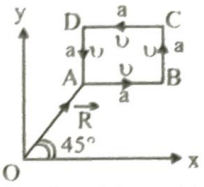
37. Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i}+\hat{j})\,ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V}_1 = (\hat{i}+\sqrt{3}\hat{j})\,ms^{-1}$, the angle between $\vec{V}_1$ and $\vec{V}_2$ is [JEE Main-2020 (September)]

(a) $-45°$ (b) $60°$
(c) $15°$ (d) $105°$

38. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is __. [JEE Main-2020 (September)]

ROTATIONAL MOTION

39. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed u in the x-y plane as shown in the figure : [JEE Main-2016]



Which of the following statement is false for the angular momentum $\vec{L}$ about the origin ?

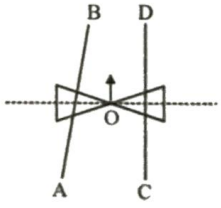
(a) $\vec{L} = \frac{mu}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A

(b) $\vec{L} = \frac{mu}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B

(c) $\vec{L} = mu\left(\frac{R}{\sqrt{2}}-a\right)\hat{k}$ when the particle is moving from C to D

(d) $\vec{L} = mu\left(\frac{R}{\sqrt{2}}+a\right)\hat{k}$ when the particle is moving from B to C

40. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :- [JEE Main-2016]



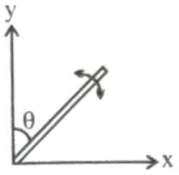
(a) turn left and right alternately
(b) turn left.
(c) turn right.
(d) go straight.

41. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum? [JEE Main-2017]

(a) 1 (b) $\frac{3}{\sqrt{2}}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$

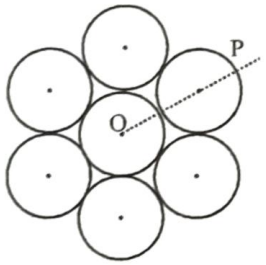
42. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is: [JEE Main-2017]



(a) $\frac{3g}{2\ell}\cos\theta$ (b) $\frac{2g}{3\ell}\cos\theta$

(c) $\frac{3g}{2\ell}\sin\theta$ (d) $\frac{2g}{3\ell}\sin\theta$

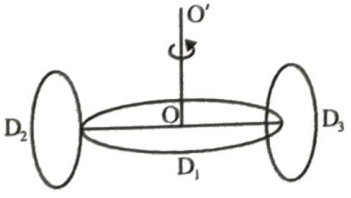
43. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is: [JEE Main-2018]



(a) $\frac{55}{2}MR^2$ (b) $\frac{73}{2}MR^2$

(c) $\frac{181}{2}MR^2$ (d) $\frac{19}{2}MR^2$

44. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly as its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1 as shown in the figure, will: [JEE Main-2019 (January)]



(a) MR^2 (b) $3MR^2$

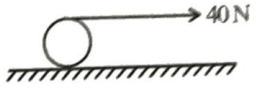
(c) $\frac{4}{5}MR^2$ (d) $\frac{2}{3}MR^2$

45. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians): [JEE Main-2019 (January)]

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

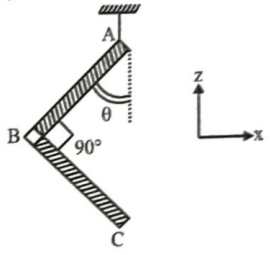
46. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) [JEE Main-2019 (January)]



(a) 20 rad/s² (b) 16 rad/s²

(c) 12 rad/s² (d) 10 rad/s²

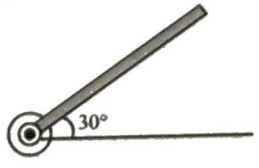
47. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then: [JEE Main-2019 (January)]



(a) $\tan\theta = \frac{1}{2\sqrt{3}}$ (b) $\tan\theta = \frac{1}{2}$

(c) $\tan\theta = \frac{2}{\sqrt{3}}$ (d) $\tan\theta = \frac{1}{3}$

48. A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° for the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10\ ms^{-2}$). [JEE Main-2019 (January)]



(a) $\sqrt{\frac{30}{2}}$ (b) $\sqrt{30}$

(c) $\sqrt{\frac{20}{2}}$ (d) $\sqrt{\frac{30}{2}}$

49. A simple pendulum, made of a string of length ℓ and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by :

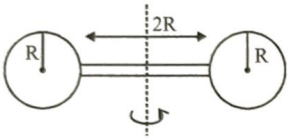
[JEE Main-2019 (January)]

(a) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (b) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

(c) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (d) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

50. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is:

[JEE Main - 2019 (January)]

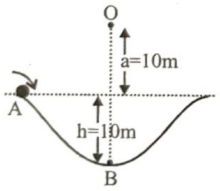


(a) $\frac{137}{15}MR^2$ (b) $\frac{17}{15}MR^2$

(c) $\frac{209}{15}MR^2$ (d) $\frac{152}{15}MR^2$

51. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the poitn A, as shwon in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : [Take g = 10 m/s^2]

[JEE Main-2019 (January)]

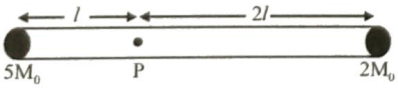


(a) 2 kg-m^2/s (b) 8 kg-m^2/s

(c) 6 kg-m^2/s (d) 3 kg-m^2/s

52. A rigid massless road of length 3l has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released form initial horizontal position, its instantaneous angular acceleration will be:

[JEE Main - 2019 (January)]



(a) $\frac{g}{13l}$ (b) $\frac{g}{3l}$

(c) $\frac{g}{2l}$ (d) $\frac{7g}{3l}$

53. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is :

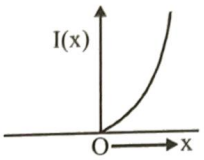
[JEE Main-2019 (January)]

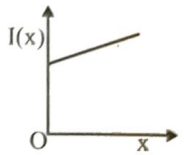
(a) $\frac{3F}{2mR}$ (b) $\frac{F}{3mR}$

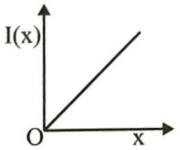
(c) $\frac{F}{2mR}$ (d) $\frac{2F}{3mR}$

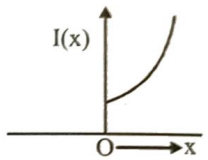
54. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is 'I(x)'. Which one of the graphs represents the variation of I(x) with x correctly?

[JEE Main - 2019 (January)]

(a) 

(b) 

(c) 

(d) 

55. To mop–clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and the floor is μ, the torque, applied by the machine on the mop is:

[JEE Main-2019 (January)]

(a) $\mu FR/3$ (b) $\mu FR/6$

(c) $\mu FR/2$ (d) $\frac{2}{3}\mu FR$

56. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of:

[JEE Main-2019 (April)]

(a) 2 s (b) 5s

(c) 2.5 s (d) 3 s

57. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be:

[JEE Main-2019 (April)]

(a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+m}$

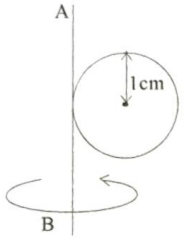
(c) $\frac{M\omega_0}{M+2m}$ (d) $\frac{M\omega_0}{M+6m}$

58. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is:

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$

(c) $\frac{3}{5}$ (d) $\frac{8}{5}$

59. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is close to: [JEE Main-2019 (April)]



(a) 4.0×10^{-6}Nm (b) 2.0×10^{-5}Nm

(c) 1.6×10^{-5}Nm (d) 7.9×10^{-6}Nm

60. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is : [JEE Main-2019 (April)]

(a) $36\hat{k}$ (b) $-34(\hat{k} - \hat{i})$

(c) $48\ (\hat{i} + \hat{j})$ (d) $-48\hat{k}$

61. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by : [JEE Main-2019 (April)]

(a) 185 (b) 65

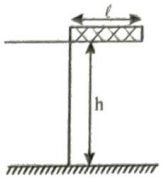
(c) 285 (d) 140

62. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by : [JEE Main-2019 (April)]

(a) $\frac{14}{15}$ (b) $\frac{4}{5}$

(c) 1 (d) $\frac{2}{\sqrt{5}}$

63. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time $\tau = 0.01$s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to :- [JEE Main-2019 (April)]



(a) 0.02 (b) 0.28

(c) 0.5 (d) 0.3

64. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane: (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum height they climb is : [JEE Main-2019 (April)]

(a) 4 : 3 : 2 (b) 14 : 15 : 20

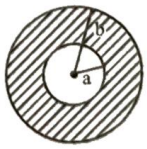
(c) 10 : 15 : 7 (d) 2 : 3 : 4

65. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a functionof θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is : [JEE Main-2019 (April)]

(a) $\frac{k}{2I}\theta$ (b) $\frac{k}{I}\theta$

(c) $\frac{k}{4I}\theta$ (d) $\frac{2k}{I}\theta$

66. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is : [JEE Main-2019 (April)]



(a) $\frac{a+b}{2}$ (b) $\frac{a+b}{3}$

(c) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (d) $\sqrt{\frac{a^2+b^2+ab}{3}}$

67. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is:

(a) $\frac{MR^2}{6}$ (b) $\frac{MR^2}{3}$

(c) $\frac{2MR^2}{3}$ (d) $\frac{MR^2}{2}$

68. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is :

[*JEE Main-2019 (April)*]

(a) $\frac{I_1\omega_1^2}{12}$ (b) $\frac{3}{8}I_1\omega_1^2$

(c) $\frac{I_1\omega_1^2}{6}$ (d) $\frac{I_1\omega_1^2}{24}$

69. A particle of mass m is moving along a trajectory given by [*JEE Main-2019 (April)*]

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

The torque, acting on the particle about the origin, at $t = 0$ is :

(a) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$

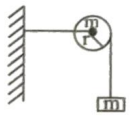
(b) $+ my_0 a\,\omega_1^2\hat{k}$

(c) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

(d) zero

ROTATIONAL MOTION

70.



As shown in the figure, a bob a mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be : [*JEE Main-2020 (January)*]

(a) $r\sqrt{\frac{3}{4gh}}$ (b) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

(c) $r\sqrt{\frac{3}{2gh}}$ (d) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

71. The radius of gyration of a uniform rod of length ℓ, about an axis passing through a point $\frac{\ell}{4}$ away from the centre of the rod, and perpendicular to it, is

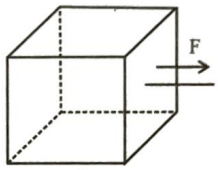
[*JEE Main-2020 (January)*]

(a) $\sqrt{\frac{7}{48}}\ell$ (b) $\frac{1}{8}\ell$

(c) $\sqrt{\frac{3}{8}}\ell$ (d) $\frac{1}{4}\ell$

72. Consider uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100\times\frac{b}{a}$ for box not to topple before moving is

[*JEE Main-2020 (January)*]



73. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane passing through its centre is:

[*JEE Main-2020 (January)*]

(a) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$ (b) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

(c) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (d) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

74. Consider a uniform rod of mass $M = 4m$ and length ℓ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is:

[*JEE Main-2020 (January)*]

(a) $\frac{4}{7}\frac{v}{\ell}$ (b) $\frac{3\sqrt{2}}{7}\frac{v}{\ell}$

(c) $\frac{3}{7}\frac{v}{\ell}$ (d) $\frac{3}{7\sqrt{2}}\frac{v}{\ell}$

75. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is :

[*JEE Main-2020 (January)*]

(a) 8.75×10^{-4} J (b) 8.75×10^{-3} J

(c) 1.13×10^{-3} J (d) 6.25×10^{-4} J

76. Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertial I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is : [*JEE Main-2020 (January)*]

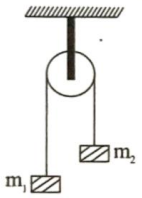
(*a*) $\frac{23}{13}$ (*b*) $\frac{13}{15}$

(*c*) $\frac{13}{23}$ (*d*) $\frac{15}{23}$

77. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\ s^{-1}$, where n is an integer. The value of n is ..... .

[*JEE Main-2020 (January)*]

78. A uniformly thick wheel with moment of inertial I and radius R is free to rotate about its centre of mass (see figure). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is : [*JEE Main-2020 (January)*]



(*a*) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}} gh$ (*b*) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

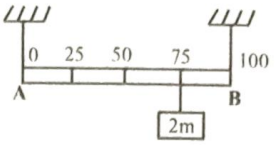
(*c*) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$ (*d*) $\left[\frac{(m_1+m_2)}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}} gh$

79. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is [*JEE Main-2020 (September)*]

(*a*) $\frac{20}{3}$J (*b*) $\frac{5}{3}$J

(*c*) $\frac{10}{3}$J (*d*) $\frac{2}{3}$J

80.



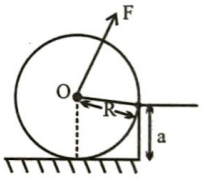
Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is [*JEE Main-2020 (September)*]

(*a*) 0.5 mg (*b*) 2 mg

(*c*) 0.75 mg (*d*) 1 mg

81. A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is

[*JEE Main-2020 (September)*]

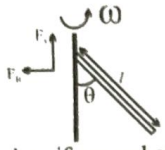


(*a*) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ (*b*) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(*c*) $Mg\sqrt{1-\frac{a^2}{R^2}}$ (*d*) $Mg\frac{a}{R}$

82. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}ma^2$ where N is an integer. The value of N is _____.

[*JEE Main-2020 (September)*]

83. 

A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that

[*JEE Main-2020 (September)*]

(*a*) $\cos\theta = \frac{g}{l\omega^2}$ (*b*) $\cos\theta = \frac{2g}{3l\omega^2}$

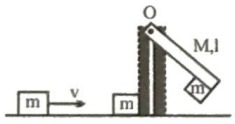
(*c*) $\cos\theta = \frac{g}{2l\omega^2}$ (*d*) $\cos\theta = \frac{3g}{2l\omega^2}$

84. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _____.

[*JEE Main-2020 (September)*]

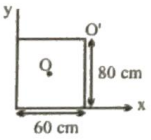
85. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg and length l = 1 m, the value of θ is approximately (take g = 10 m/s^2)

[*JEE Main-2020 (September)*]



(*a*) 49° (*b*) 55°

(*c*) 63° (*d*) 69°

86. 

For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is [*JEE Main-2020 (September)*]

(*a*) 1/2 (*b*) 1/4

(*c*) 1/8 (*d*) 2/3

87. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is [*JEE Main-2020 (September)*]

(*a*) 2 (*b*) 4

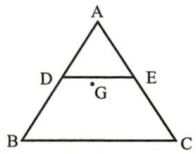
(*c*) $2\sqrt{2}$ (*d*) $\sqrt{2}$

88. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is _____.

[*JEE Main-2020 (September)*]

89. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is _____.

[*JEE Main-2020 (September)*]



90. A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be _____. [*JEE Main-2020 (September)*]

91. A force $\vec{F} = (\hat{i}+2\hat{j}+3\hat{k})$N acts at a point $(4\hat{i}+3\hat{j}-\hat{k})$m. Then the magnitude of torque about the point $(\hat{i}+2\hat{j}+\hat{k})$m will be $\sqrt{x}$ N – m. The value of x is —.

[*JEE Main-2020 (September)*]

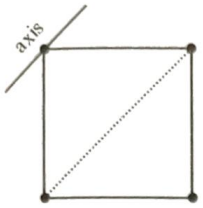
92. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is

[*JEE Main-2020 (September)*]

(*a*) $\frac{5}{6}$ (*b*) $\frac{1}{4}$

(*c*) 0 (*d*) $\frac{3}{4}$

93. Four point masses, each of mass m, are fixed at the corners of a square of side ℓ. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is [*JEE Main-2020 (September)*]



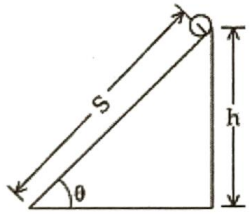
(*a*) $3\,m\ell^2\omega$ (*b*) $4\,m\ell^2\omega$
(*c*) $m\ell^2\omega$ (*d*) $2\,m\ell^2\omega$

94. Shown in the figure is a hollow icecream cone (it open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is [*JEE Main-2020 (September)*]

(*a*) $\frac{M(R^2+H^2)}{4}$ (*b*) $\frac{MR^2}{2}$

(*c*) $\frac{MR^2}{3}$ (*d*) $\frac{MH^2}{3}$

95. The following bodies, of same mass ' m ' and radius ' $R ' are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is [Mark the body as per their respective numbering given in the question] [*JEE Main-2021 (March)*]

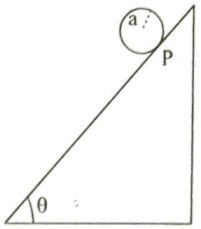


(*a*) a ring (*b*) a disc
(*c*) a solid cylinder (*d*) a solid sphere,

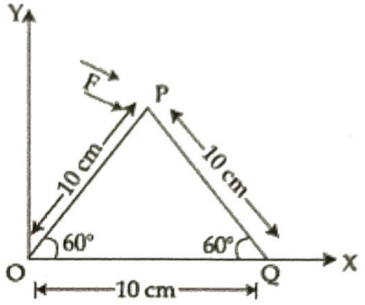
96. The angular speed of truck wheel is increased from 900rpm to 2460rpm in 26 seconds. The number of revolutions by the truck engine during this time is (Assuming the acceleration to be uniform).

[*JEE Main-2021 (March)*]

97. A solid disc of radius ' a ' and mass ' m ' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}g\sin\theta$ where b is (Round off to the Nearest Integer) (g = acceleration due to gravity, θ = angle as shown in figure) [*JEE Main-2021 (March)*]



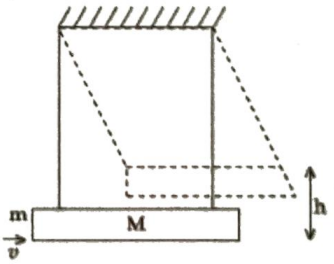
98. A triangular plate is shown. A force $\vec{F}=4\hat{i}-3\hat{j}$ is applied at point P. The torque at point P with respect to point ' O ' and ' Q ' are :



(*a*) $-15+20\sqrt{3},15+20\sqrt{3}$
(*b*) $15-20\sqrt{3},15+20\sqrt{3}$
(*c*) $15+20\sqrt{3},15-20\sqrt{3}$
(*d*) $-15-20\sqrt{3},15-20\sqrt{3}$

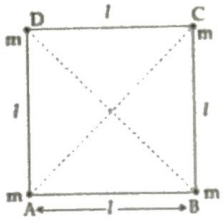
99. A large block of wood of mass $M=5.99\,kg$ is hanging from two long massless cords. A bullet of mass $m=10\ g$ is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance $h=9.8\,cm$ before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is:

(take $g=9.8\,ms^{-2}$) [*JEE Main-2021 (March)*]



(*a*) 831.4 m/s (*b*) 811.4 m/s
(*c*) 841.4 m/s (*d*) 821.4 m/s

100. Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be: [*JEE Main-2021 (March)*]



(a) $\sqrt{3}ml^2$ (b) $2ml^2$

(c) ml^2 (d) $3ml^2$

JEE-ADVANCED

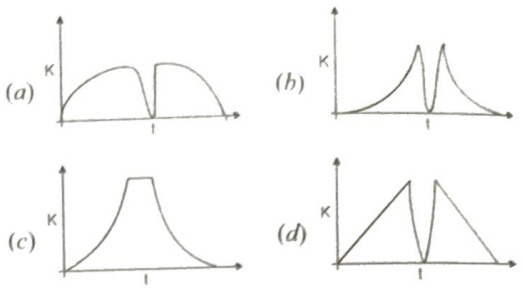
1. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane, At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is: [*JEE Advanced-2013*]

2. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is: [*JEE Advanced-2013*]

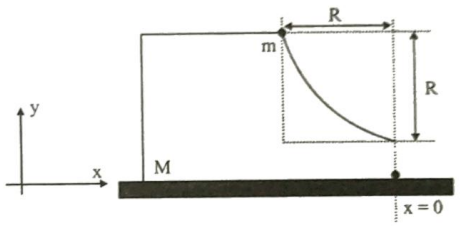
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}+a$

(c) $\frac{\pi}{4}-a$ (d) $-\frac{\pi}{4}$

3. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale. [*JEE Advanced-2014*]



4. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and its slides down. When the mass loses contact with the block, its position is x and the velocity is v. At the instant, which of the following options is/are correct? [*JEE Advanced-2017*]



(a) The velocity of the point mass m is : $v=\sqrt{\frac{2gR}{1+\frac{m}{M}}}$

(b) The x component of displacement of the center of mass of the block M is : $-\frac{mR}{M+m}$

(c) The position of the point mass is $x=-\sqrt{2}\frac{mR}{M+m}$

(d) The velocity of the block M is : $V=-\frac{m}{M}\sqrt{2gR}$

5. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is /are true? [*JEE Advanced -2017*]

(a) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

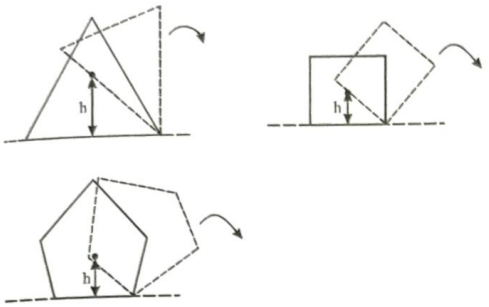
(b) At a later time the external force F balances the resistive force

(c) The resistive force experienced by the plate is proportional to v

(d) The plate will continue to move with constant non-zero acceleration, at all times

6. Consider regular polygon with number of sides n = 3, 4, 5..... as shown in the figure. The centre of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ.Then Δ depends on n and h as

[*JEE Advanced -2017*]



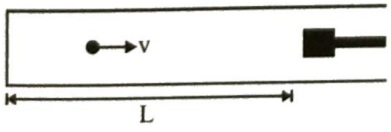
(*a*) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (*b*) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

(*c*) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (*d*) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4 kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and ∴ τ = 4s. The displacement of the block, in metres, at t = τ is ..... Take $e^{-1} = 0.37$?

[*JEE Advanced - 2018*]

8. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statemnt (s) is/are correct [*JEE Advanced - 2019*]



(*a*) The rate at which the particle strikes the piston is v/L

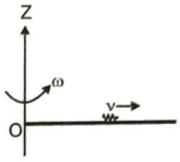
(*b*) After each collision with the piston, the particle speed increases by 2V

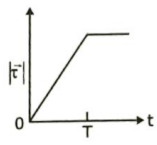
(*c*) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2} L_0$

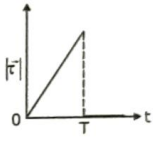
(*d*) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

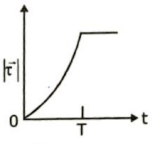
ROTATIONAL MOTION

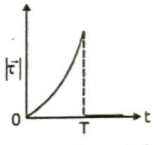
9. A thin uniform rod, pivoted at O is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time t = 0, small insect starts from O and moves with constant speed v with respect to the rod towards the other end. it reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot? [*IIT JEE-2012*]



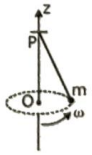
(*a*) 

(*b*) 

(*c*) 

(*d*) 

10. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω.If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_0$ and $\vec{L}_P$ respectively, then. [*IIT JEE-2012*]



(*a*) $\vec{L}_0$ and $\vec{L}_P$ do not vary with time

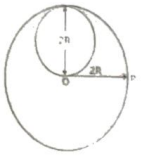
(*b*) $\vec{L}_0$ varies with time while $\vec{L}_P$ remains constant

(*c*) $\vec{L}_0$ remains constant while $\vec{L}_P$ varies with time

(*d*) $\vec{L}_0$ and $\vec{L}_P$ both vary with time.

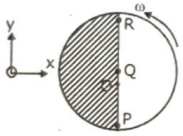
11. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_o and I_p, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_p}{I_o}$ to the nearest integer is

[*IIT JEE-2012*]



12. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then

[*IIT JEE-2012*]



(*a*) P lands in the shaded region and Q in the unshaded region

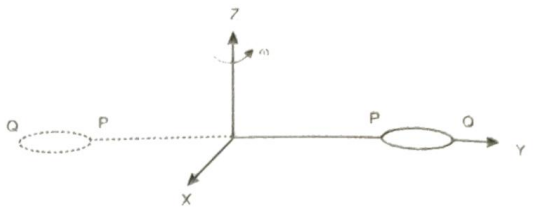
(*b*) P lands in the unshaded region and Q in the shaded region

(*c*) Both P and Q land in the unshaded region

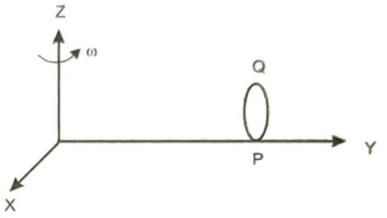
(*d*) Both P and Q land in the shaded region

Comprehension–1 (No. 13 and 14)

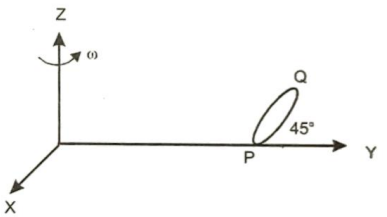
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantanneous axis passing through center of mass. These axes need not be stationary. Consider, for example, a thin uniform welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. Where disc-stick system is rotated about the origin ona horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both the motions have the same angular speed ω in the case.



Now consider two similar systems as shown in the figure: case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and systems are rotated with constant angular speed ω about the z-axis.



Case (a)



Case (b)

13. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

[*IIT JEE-2012*]

(*a*) It is vertical for both the cases (a) and (b).

(*b*) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(*c*) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(*d*) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

14. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct

[*IIT JEE-2012*]

(*a*) It is $\sqrt{2}\omega$ for both the cases.

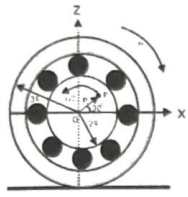
(*b*) It is ω for case (a); and $\omega/\sqrt{2}$ for case (b).

(*c*) It is ω for case (a); and $\sqrt{2}\omega$ for case (b).

(*d*) It is ω for both the cases.

15. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface.

[*IIT JEE-2012*]



(*a*) the point O has a linear velocity $3R\omega\hat{i}$.

(*b*) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(*c*) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(*d*) the point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

16. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct? [*IIT JEE-2012*]

(*a*) Both cylinders P and Q reach the ground at the same time.

(*b*) Cylinder P has larger linear acceleration than cylinder

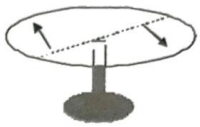
(*c*) Both cylinders reach the ground with same translational kinetic energy.

(*d*) Cylinder Q reaches the ground with larger angular speed.

17. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is:

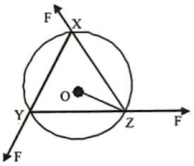
[*JEE Advanced-2013*]

18. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 005 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is [*JEE Advanced-2014*]

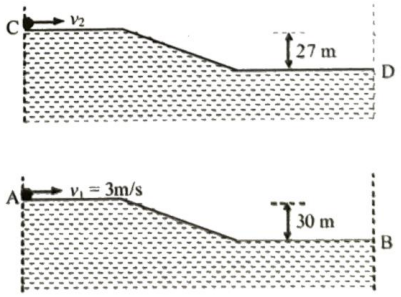


19. A uniform circular disc of mass 1.5 kg and radius 0.5 m is ini8tially at rest on a horizontal friction less surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad/s is

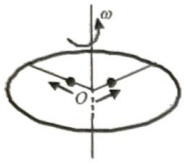
[*JEE Advanced-2014*]



20. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then v_2 in m/s is ($g = 10$ m/s^2) [*JEE Advanced-2015*]



21. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move readially outwards along two massless rods fixed on the ring as shown in the figure. At $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}$ R from O. At this instant the distance of the other mass from O is **[JEE Advanced-2015]**



(a) $\frac{2R}{3}$ (b) $\frac{R}{3}$

(c) $\frac{3R}{5}$ (d) $\frac{4R}{5}$

22. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k, is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is: **[JEE Advanced-2015]**

23. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h ($<\ell$) such that a small portion of the stick extends beyond the wall. The reactionforce of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick isequal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are: ($g = 10\ ms^{-2}$) **[JEE Advanced-2016]**

(a) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$ (b) $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}N$

(c) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}N$ (d) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

24. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$, where $\alpha = \frac{10}{3}ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle? **[JEE Advanced-2016]**

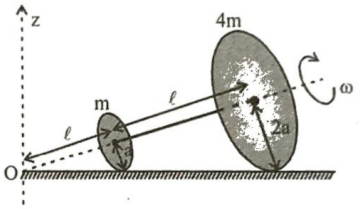
(a) The velocity $\vec{v}$ is given by $\vec{v} = \left(10\hat{i} + 10\hat{j}\right)ms^{-1}$

(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -\left(\frac{5}{3}\right)\hat{k}Nms$

(c) The force $\vec{F}$ is given by $\vec{F} = \left(\hat{i} + 2\hat{j}\right)N$

(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = \left(\frac{20}{3}\right)kNm$

25. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, right rod of length $\ell = \sqrt{24}a$ through their center. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true? **[JEE Advanced-2016]**



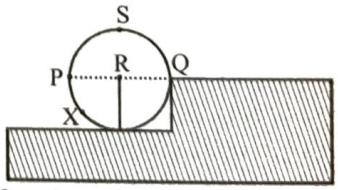
(a) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(b) The magnitude of the z-component of $\vec{L}$ is $55\ ma^2\omega$

(c) The magnitude of angular momentum of center of mass of the assembly about the point O is $81\ ma^2\omega$

(d) The center of mass of the assembly rotates about the z-axis with an angular speed of ω/5

26. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Which of the following options is/are correct? **[JEE Advanced-2017]**



(a) If the force is applied normal to the circumference at point P then τ is zero

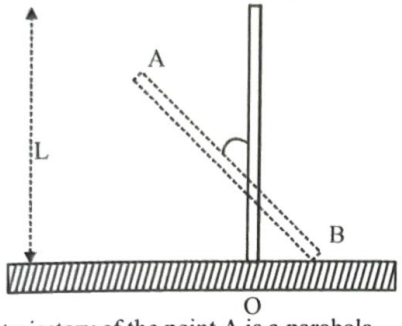
(b) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

(c) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

(d) If the force is applied normal to the circumference at point X then τ is constant

27. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionlesss floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?

[*JEE Advanced-2017*]



(*a*) The trajectory of the point A is a parabola

(*b*) Instantaneous torque about the point in contact with the floor is proportional to sin θ.

(*c*) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$

(*d*) The midpoint of the bar will fall vertically downward.

Comprehension 2 (No. 28 and 29)

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure-1. In the process the finger never loses contact with the inner rim of the ring . The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure-2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

[*JEE Advanced-2017*]

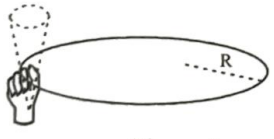


Figure-1

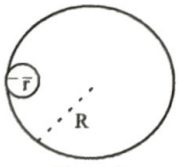


Figure-2

28. The total kinetic energy of the ring is :

(*a*) $\frac{1}{2}M\omega_0^2(R-r)^2$ (*b*) $\frac{3}{2}M\omega_0^2(R-r)^2$

(*c*) $M\omega_0^2R^2$ (*d*) $M\omega_0^2(R-r)^2$

29. The minimum value of ω_0 below which the ring will drop down is :

(*a*) $\sqrt{\frac{g}{\mu(R-r)}}$ (*b*) $\sqrt{\frac{g}{2\mu(R-r)}}$

(*c*) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (*d*) $\sqrt{\frac{2g}{\mu(R-r)}}$

30. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, what of the following statements is(are) true? [*JEE Advanced - 2018*]

(*a*) $v = \sqrt{\frac{k}{2m}}R$ (*b*) $v = \sqrt{\frac{k}{m}}R$

(*c*) $L = \sqrt{mk}R^2$ (*d*) $L = \sqrt{\frac{mk}{2}}R^2$

31. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0\ Ns^{-1}$ and $\beta = 1.0$ N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is(are) true?

[*JEE Advanced - 2018*]

(*a*) $|\vec{\tau}| = \frac{1}{3}$ Nm

(*b*) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(*c*) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}(i + 2ms^{-1})$

(*d*) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}$ m.

32. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in meters, is ____. Take g = 10 ms $^{-2}$.

[*JEE Advanced - 2018*]

33. In the List-I below, four different paths of a particle are given as functions of time. In these functions α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned; $\vec{p}$ is the linear momentum $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path

[*JEE Advanced - 2018*]

List - I	*List - II*
P. $\vec{r}(t)=\alpha t\hat{i}+\beta t\hat{j}$	1. $\vec{p}$
Q. $\vec{r}(t)=\alpha\cos\omega t\hat{i}+\beta\sin\omega t\hat{j}$	2. $\vec{L}$
R. $\vec{r}(t)=\alpha\left(\cos\omega t\hat{i}+\sin\omega t\hat{j}\right)$	3. K
S. $\vec{r}(t)=\alpha t\hat{i}+\frac{\beta}{2}t^2\hat{j}$	4. U
	5. E

(*a*) P → 1, 2, 3, 4, 5 ; Q → 2, 5; R → 2, 3, 4,5; S → 5
(*b*) P → 1, 2, 3, 4, 5; Q → 3, 5; R → 2, 3, 4, 5; S → 2, 5
(*c*) P → 2, 3, 4 ; Q → 5; R → 1, 2, 4; S → 2, 5
(*d*) P → 1, 2, 3, 5; Q → 2, 5 ; R → 2, 3, 4, 5; S → 2, 5

34. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ?
[g is the acceleration due to gravity] [*JEE Advanced - 2019*]

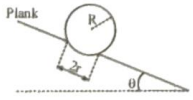
(*a*) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(*b*) The angular acceleration of the rod will be $\frac{2g}{L}$

(*c*) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(*d*) The normal reaction force from the floor on the will be $\frac{Mg}{16}$

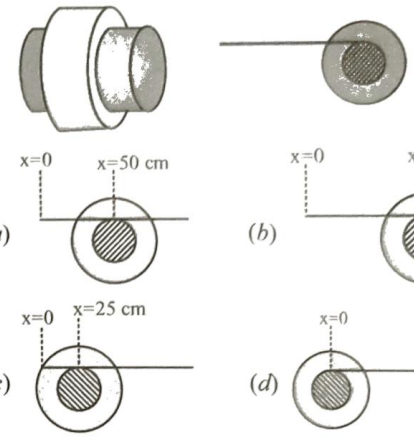
35. A football of radius R is kept on a hole of radius r (r < R) made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle θ from the horizontal as shown in the figure below. The maximum value of θ so that the football does not start rolling down the plank satisfies (figure is schematic and not drawn to scale) - [*JEE Advanced - 2020*]



(*a*) $\sin\theta=\frac{r}{R}$ (*b*) $\tan\theta=\frac{r}{R}$

(*c*) $\sin\theta=\frac{r}{2R}$ (*d*) $\cos\theta=\frac{r}{2R}$

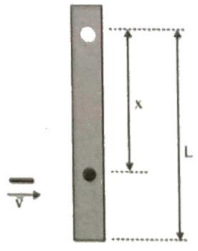
36. A small roller of diameter 20 cm has an axle of diameter 10 cm (see figure below on the left). It is on a horizontal floor and a meter scale is positioned horizontally on its axle with one edge of the scale on top of the axle (see figure on the right). The scale is now pushed slowly on the axle so that it moves without slipping on the axle, and the roller starts rolling without slipping. After the roller has moved 50 cm, the position of the scale will look like (figures are schematic and not drawn to scale)- [*JEE Advanced - 2020*]



37. Put a uniform meter scale horizontally on your extended index fingers with the left one at 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly towards the center, initially only the left finger slips with respect to the scale and the right finger does not. After some distance, the left finger stops and the right one starts slipping. Then the right finger stops at a distance x_R from the center (50.00 cm) of the scale and the left one starts slipping again. This happens because of the difference in the frictional forces on the two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of x_R (in cm) is .....

[*JEE Advanced-2020*]

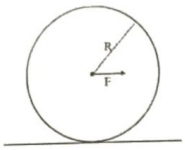
38. A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot. The maximum angular speed ω_m is achieved for $x=x_M$. Then [*JEE Advanced-2020*]



(*a*) $\omega = \dfrac{3vx}{L^2+3x^2}$ (*b*) $\omega = \dfrac{12vx}{L^2+12x^2}$

(*c*) $x_M = \dfrac{L}{\sqrt{3}}$ (*d*) $\omega_M = \dfrac{v}{2L}\sqrt{3}$

39. A horizontal force F is applied at the centre of mass of a cylindrical object of mass m and radius R, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ. The centre of mass of the object has an acceleration a. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct? *[JEE Advanced-2021]*



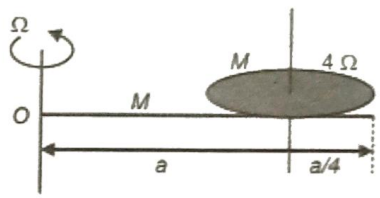
(*a*) For the same F, the value of a does not depend on whether the cylinder is solid or hollow

(*b*) For a solid cylinder, the maximum possible value of a is $2\mu g$

(*c*) The magnitude of the frictional force on the object due to the ground is always μmg

(*d*) For a thin-walled hollow cylinder, $a = \dfrac{F}{2m}$

40. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius $a/4$ is pivoted on this rod with its center at a distance $a/4$ from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω. The total angular momentum of the system about the point O is $\left(\dfrac{Ma^2\Omega}{48}\right)n$. The value of n is *[JEE Advanced-2021]*



ANSWER KEY

Topicwise Questions

1. (*d*)	**2.** (*d*)	**3.** (*b*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)	**7.** (*c*)	**8.** (*b*)	**9.** (*c*)	**10.** (*b*)
11. (*c*)	**12.** (*d*)	**13.** (*a*)	**14.** (*b*)	**15.** (*d*)	**16.** (*a*)	**17.** (*c*)	**18.** (*d*)	**19.** (*b*)	**20.** (*a*)
21. (*a*)	**22.** (*a*)	**23.** (*a*)	**24.** (*c*)	**25.** (*b*)	**26.** (*a*)	**27.** (*b*)	**28.** (*d*)	**29.** (*c*)	**30.** (*a*)
31. (*a*)	**32.** (*a*)	**33.** (*d*)	**34.** (*a*)	**35.** (*a*)	**36.** (*c*)	**37.** (*a*)	**38.** (*b*)	**39.** (*c*)	**40.** (*d*)
41. (*d*)	**42.** (*b*)	**43.** (*d*)	**44.** (*b*)	**45.** (*a*)	**46.** (*d*)	**47.** (*c*)	**48.** (*a*)	**49.** (*b*)	**50.** (*d*)
51. (*b*)	**52.** (*d*)	**53.** (*b*)	**54.** (*b*)	**55.** (*b*)	**56.** (*c*)	**57.** (*a*)	**58.** (*d*)	**59.** (*c*)	**60.** (*d*)
61. (*a*)	**62.** (*d*)	**63.** (*b*)	**64.** (*b*)	**65.** (*c*)	**66.** (*b*)	**67.** (*d*)	**68.** (*b*)	**69.** (*a*)	**70.** (*a*)
71. (*b*)	**72.** (*d*)	**73.** (*c*)	**74.** (*c*)	**75.** (*c*)	**76.** (*a*)	**77.** (*c*)	**78.** (*b*)	**79.** (*a*)	**80.** (*d*)
81. (*c*)	**82.** (*d*)	**83.** (*c*)	**84.** (*d*)	**85.** (*b*)	**86.** (*b*)	**87.** (*d*)	**88.** (*c*)	**89.** (*a*)	**90.** (*a*)
91. (*d*)	**92.** (*b*)	**93.** (*c*)	**94.** (*c*)	**95.** (*a*)	**96.** (*b*)	**97.** (*c*)	**98.** (*b*)	**99.** (*a*)	**100.** (*b*)
101. (*d*)	**102.** (*a*)	**103.** (*b*)	**104.** (*a*)	**105.** (*c*)	**106.** (*b*)	**107.** (*b*)	**108.** (*c*)	**109.** (*a*)	**110.** (*b*)
111. (*a*)									

Learning Plus

1. (*a*)	**2.** (*c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*c*)	**6.** (*a*)	**7.** (*c*)	**8.** (*b*)	**9.** (*a*)	**10.** (*c*)
11. (*b*)	**12.** (*d*)	**13.** (*b*)	**14.** (*a*)	**15.** (*c*)	**16.** (*a*)	**17.** (*a*)	**18.** (*b*)	**19.** (*b*)	**20.** (*b*)
21. (*c*)	**22.** (*a*)	**23.** (*a*)	**24.** (*c*)	**25.** (*c*)	**26.** (*c*)	**27.** (*a*)	**28.** (*a*)	**29.** (*a*)	**30.** (*b*)
31. (*c*)	**32.** (*a*)	**33.** (*b*)	**34.** (*a*)	**35.** (*a*)	**36.** (*c*)	**37.** (*b*)	**38.** (*d*)	**39.** (*b*)	**40.** (*d*)
41. (*b*)	**42.** (*d*)	**43.** (*d*)	**44.** (*a*)	**45.** (*c*)	**46.** (*b*)	**47.** (*a*)	**48.** (*c*)	**49.** (*d*)	**50.** (*a*)
51. (*d*)	**52.** (*a*)	**53.** (*d*)	**54.** (*c*)	**55.** (*b*)	**56.** (*c*)	**57.** (*c*)	**58.** (*b*)	**59.** (*b*)	**60.** (*b*)
61. (*a*)	**62.** (*a*)	**63.** (*a*)	**64.** (*a*)	**65.** (*d*)	**66.** (*c*)	**67.** (*d*)			

Advanced Level Multiconcept Questions

1. (*a, b, c, d*) **2.** (*b, d*) **3.** (*b, c*) **4.** (*b, d*) **5.** (*b, d*) **6.** (*a, d*) **7.** (*a, d*) **8.** (*a, c*) **9.** (*a, c, d*)
10. (*b, d*) **11.** (*a, c*) **12.** (*a, b, d*) **13.** (*a, b, c, d*) **14.** (*a, b, c*) **15.** (*a, d*) **16.** (*b, c*) **17.** (*b, c, d*) **18.** (*a, b, c*) **19.** (*b, c*)
20. (*a, c, d*) **21.** (*a, c, d*) **22.** (*a, c*) **23.** (*a, c, d*) **24.** (*a, b, d*) **25.** (*c*) **26.** (*a*) **27.** (*d*) **28.** (*b*) **29.** (*b, d*)
30. (*c*) **31.** (*c*) **32.** (*b, c*) **33.** (*b*) **34.** (*a, b, c, d*) **35.** (*a*) **36.** (*d*) **37.** (*a*) **38.** (*b*)
39. (*b*) **40.** (*c*) **41.** (*d*) **42.** (*d*) **43.** (*c*) **44.** (*a*) **45.** (*c*) **46.** (*a*) **47.** (*c*) **48.** (*c*)
49. (*a*) p (*b*) q (*c*) p,r (*d*) q,s **50.** (*a*) p,q,r (*b*) p,q,r (*c*) p,q (*d*) p,q,r **51.** (*a*) p (*b*) q,s (*c*) p (*d*) q,s **52.** [0010]
53. [0025] **54.** [2000] **55.** [125] **56.** [0019] **57.** [5 rad/s] **58.** [0006] **59.** [0100] **60.** [0004] **61.** [0010] **62.** [0002]
63. [28] **64.** [0055] **65.** [0005]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*d*)	**2.** (*a*)	**3.** (*d*)	**4.** (*b*)	**5.** (*c*)	**6.** (*c*)	**7.** (*b*)	**8.** (*c*)	**9.** (*a*)	**10.** (*c*)
11. (*d*)	**12.** (*c*)	**13.** (*b*)	**14.** (*a*)	**15.** (*c*)	**16.** (*a*)	**17.** (*c*)	**18.** (*c*)	**19.** (*c*)	**20.** [1]
21. [8.00]	**22.** (*b*)	**23.** (*b*)	**24.** (*d*)	**25.** (*d*)	**26.** (*a*)	**27.** (*b*)	**28.** [10]	**29.** [23]	**30.** (*d*)
31. (*d*)	**32.** (*d*)	**33.** (*d*)	**34.** (*c*)	**35.** [03]	**36.** (*d*)	**37.** (*d*)	**38.** [120.00]	**39.** (*b,d*)	**40.** (*b*)
41. (*c*)	**42.** (*c*)	**43.** (*c*)	**44.** (*b*)	**45.** (*a*)	**46.** (*b*)	**47.** (*b*)	**48.** (*b*)	**49.** (*c*)	**50.** (*a*)
51. (*c*)	**52.** (*a*)	**53.** (*d*)	**54.** (*d*)	**55.** (*d*)	**56.** (*a*)	**57.** (*d*)	**58.** (*d*)	**59.** (*b*)	**60.** (*d*)
61. (*d*)	**62.** (*a*)	**63.** (*c*)	**64.** (*b*)	**65.** (*d*)	**66.** (*d*)	**67.** (*c*)	**68.** (*d*)	**69.** (*b*)	**70.** (*d*)
71. (*a*)	**72.** [50.00]	**73.** (*b*)	**74.** (*b*)	**75.** (*a*)	**76.** (*c*)	**77.** [15]	**78.** (*b*)	**79.** (*a*)	**80.** (*d*)
81. (*a*)	**82.** [25]	**83.** (*d*)	**84.** [09.00]	**85.** (*c*)	**86.** (*b*)	**87.** (*a*)	**88.** [20.00]	**89.** [11]	**90.** [20.00]
91. [195.00]	**92.** (*d*)	**93.** (*a*)	**94.** (*b*)	**95.** (*d*)	**96.** (728)	**97.** [3]	**98.** (*d*)	**99.** (*a*)	**100.** (*d*)

JEE-ADVANCED
PREVIOUS YEAR'S

1. [5]	**2.** (*a*)	**3.** (*b*)	**4.** (*a, b*)	**5.** (*a, b, c*)	**6.** (*d*)	**7.** [6.30]	**8.** (*b, c*)	**9.** (*b*)	**10.** (*c*)
11. [3]	**12.** (*c*)	**13.** (*a*)	**14.** (*d*)	**15.** (*a, b*)	**16.** (*d*)	**17.** [8]	**18.** [4]	**19.** [2]	**20.** [7]
21. (*c, d*)	**22.** [6]	**23.** (*d*)	**24.** (*a, b, d*)	**25.** (*b*) or (*a, b*)		**26.** (*a*, or *ab*)		**27.** (*b, c, d*)	
28. (*b*onus)	**29.** (*a*)	**30.** (*b, c*)	**31.** (*a, c*)	**32.** [0.75 m]	**33.** (*a*)	**34.** (*a, c, d*)	**35.** (*a*)	**36.** (*b*)	
37. [25.60]	**38.** (*a,c,d*)	**39.** (*b, d*)	**40.** [49]						

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8 GRAVITATION

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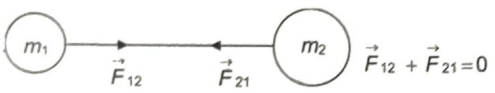
NEWTON'S LAW OF GRAVITATION

Newton's law of gravitation states that every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the particles.

Therefore from Newton's law of gravitation

$$\vec{F} = \frac{Gm_1m_2}{r^2}\hat{r} \qquad \ldots (i)$$

where G is called the gravitational constant and $\hat{r}$ is the unit vector along the line joining the two mass particles.



The gravitational force between two particles form an action reaction pair.

TRAIN YOUR BRAIN

Example 1: A mass M is split into two parts m and (M – m), which are then separated by a certain distance. What ratio (M/m) maximizes the gravitational force between the parts?

Sol. If 'r' is the distance between m and (M – m) , the gravitational force will be

$$F = G\frac{(M-m)m}{r^2} = \frac{G}{r^2}[Mm - m^2]$$

for F to be maximum, $\frac{dF}{dm} = 0$,

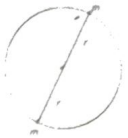
i.e., $\frac{d}{dm}\left[\frac{G}{r^2}(Mm - m^2)\right] = 0$

or, M – 2m = 0 $\quad [\because \frac{G}{r^2} \neq 0]$

or, M/m = 2, i.e., the force will be maximum when two parts are equal.

Example 2: Two particles of equal mass m = 10 kg are moving in a circle of radius r = 1.65m under the action of their mutual gravitational attraction. Find the speed (in μm/s) of each particle. ($g = 6.6 \times 10^{-11} Nm^2/kg^2$)

Sol. The particles will always remain diametrically opposite so that the force on each particle will be directed along the radius.



Considering the circular motion of one particle, we have,

$$\frac{mv^2}{r} = \frac{Gm.m}{(2r)^2}$$

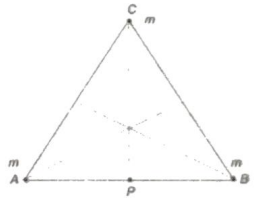
$\because$ $v = \sqrt{\frac{Gm}{4r}} = 10$ μm/s

Example 3: Three equal particles each of mass m =10 kg are placed at the three corners of an equilateral triangle of side a = 1mm. Find the force exerted (in CGS units) by this system on another particle of mass m placed at (a) the mid–point of a side (b) at the center of the triangle. ($G = 6.6 \times 10^{-11}$ Nm^2/kg^2)

Sol. As gravitational force is a two body interaction, the principle of superposition is valid, i.e., resultant force on the particle of mass m at P is

$$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C$$

(a) As shown in the above figure, when P is at the mid–point of a side, $\vec{F}_A$ and $\vec{F}_B$ will be equal in magnitude but opposite in direction. So they will cancel each–other. So the point mass m at P will experience a force due to C only, i.e.,

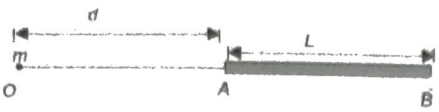


$$F = \frac{Gmm}{(CP)^2} = \frac{Gm^2}{(a\sin 60°)^2} = \frac{4Gm^2}{3a^2}$$

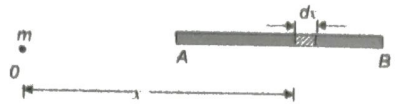
F = 880 dynes along PC

(b) From symmetry, the net force on the particle at the center of triangle = 0

Example 4: Find the gravitational force in micronewton of attraction on the point mass 'm' placed at O by a thin rod of mass M and length L as shown in figure. [take m = 10 kg, G = 6.6 × 10^{-11} Nm^2/kg^2, M = 1000 kg, L = 1m, d = 10 cm]



Sol.



First we need to find the force due to an element of length dx.

The mass of the element is dm = $\left(\frac{M}{L}\right)dx$. So,

$dF = G\frac{Mm}{L}\frac{dx}{x^2}$

∴ The net gravitational force is

$$F = \frac{GMm}{L}\int_{d}^{d+L}\frac{dx}{x^2} = \frac{GMm}{L}\left[\frac{1}{d} - \frac{1}{L+d}\right]$$

$$= \frac{GMm}{d(L+d)} = 66\ \mu N$$

Notice that when d >> L,

we find $F = \frac{GMm}{d^2}$,

the result for two point masses.

GRAVITATIONAL FIELD

Gravitational field due to a mass is defined as the region of space in which it interacts with other masses. In order to get the extent of interaction between two masses we defined another quantity called gravitational field intensity. Gravitational field intensity due to a mass m at a distance r is defined as the force acting on unit mass kept at a distance r. The gravitational field intensity is a vector quantity and its direction is the direction along which the unit mass has a tendency to move. The unit of gravitational field intensity is N/kg and its dimensions are [LT^{-2}].

CALCULATION OF GRAVITATIONAL FIELD

(a) Gravitational field intensity due to a point mass:

Consider a point mass M at O and let us calculate gravitational intensity at A due to this point mass.

Suppose a test mass is placed at A.

By Newton's law of gravitation, force on test mass

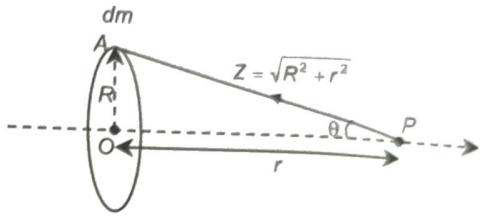
$F = \frac{GMm}{r^2}$ along $\overrightarrow{AO}$ ...(1)

$$E = \frac{F}{m} = \frac{GM}{r^2}\hat{e}_r \quad \ldots(2)$$



(b) Gravitational field intensity due to a uniform circular ring at a point on its axis.

Figure shows a ring of mass M and radius R. Let P is the point at a distance r from the centre of the ring. By symmetry the field must be towards the centre that is along $\overrightarrow{PO}$.



Let us assume that a particle of mass dm on the ring say, at point A. Now the distance AP is $\sqrt{R^2+r^2}$. Again the gravitational field at P due to dm is along $\overrightarrow{PA}$ and its magnitude is

$dE = \frac{Gdm}{Z^2}$

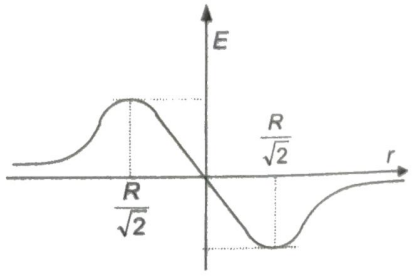
∴ $dE\cos\theta = \frac{Gdm}{Z^2}\cos\theta$

Net gravitational field $E = \frac{G\cos\theta}{Z^2}\int dm$

$$= \frac{GM}{Z^2}\frac{r}{Z}$$

$$= \frac{GMr}{(r^2+R^2)^{3/2}} \text{ along } \overrightarrow{PO} \quad \ldots(3)$$

Variation of gravitational field due to a ring as a function of its axial distance.



Important points:

(i) If $r >> R$, $r^2 + R^2 \simeq r^2$

$\therefore E = -\frac{GMr}{r^3} = \frac{GM}{r^2}$ [where negative sign is because of attraction]

Thus, for a distant point, a ring behaves as a point mass placed at the center of the ring.

(ii) If $r << R$, $r^2 + R^2 \simeq R^2$

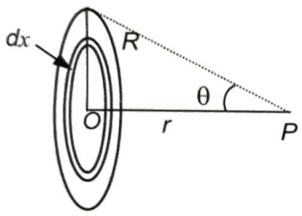
$\therefore E = -\frac{GMr}{R^3}$

i.e., $E \propto r$

(c) Gravitational field intensity due to a uniform disc at a point and its axis

Let the mass of disc be M and its radius is R and P is the point on its axis where gravitational field is to be calculated.

Let us draw a circle of radius x and centre at O. We draw another concentric circle of radius x + dx. The part of disc enclosed between two circle can be treated as a uniform ring of radius x. The area of this ring is $2\pi x dx$



Therefore mass dm of the ring

$= \frac{M}{\pi R^2} 2\pi x\, dx = \frac{2Mx dx}{R^2}$

gravitational field at P due to the ring is,

$$dE = \frac{G\left(\frac{2Mx\,dx}{R^2}\right)r}{(r^2+x^2)^{3/2}}$$

$$\int dE = \frac{2GMr}{R^2}\int_0^a \frac{x\,dx}{(r^2+x^2)^{3/2}}$$

$$= \frac{2GMr}{R^2}\left[-\frac{1}{\sqrt{r^2+x^2}}\right]_0^R$$

$$= \frac{2GMr}{R^2}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2+R^2}}\right]$$

in terms of θ

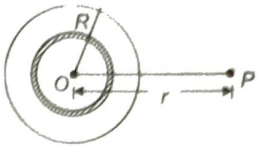
$$E = \frac{2GM}{R^2}(1-\cos\theta) \quad \ldots (4)$$

(d) Gravitational field due to a uniform solid sphere

Case-I:

Field at an external point

Let the mass of sphere is M and its radius is R we have to calculate the gravitational field at P.



$$\int dE = \int \frac{Gdm}{r^2}$$

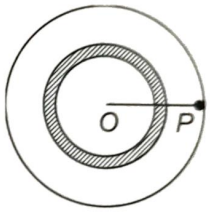
$$= \frac{G}{r^2}\int dm = \frac{Gm}{r^2} \quad \ldots (5)$$

Thus, a uniform sphere may be treated as a single particle of equal mass placed at its centre for calculating the gravitational field at an external point.

Case-II:

Field at an internal point

Suppose the point P is inside the solid sphere, in this case $r < R$ the sphere may be divided into thin spherical shells all centered at O. Suppose the mass of such a shell is dm. Then



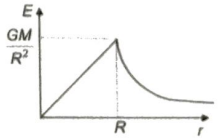
$$dE = \frac{Gdm}{r^2} \text{ along PO} = \frac{G}{r^2}\int dm$$

where $\int dm = \frac{M}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3 = \frac{Mr^3}{R^3}$

$\therefore E = \frac{GM}{R^3} r \quad \ldots (6)$

Therefore gravitational field due to a uniform sphere at an internal point is proportional to the distance of the point from the centre of the sphere. At the centre r = 0 the field is zero. At the surface of the sphere r = R

$$E = \frac{GM}{R^2} \quad \ldots (7)$$



Gravitational field due to solid sphere is continuous but it is not differentiable function.

(e) Field due to uniform thin spherical shell

Case-I:

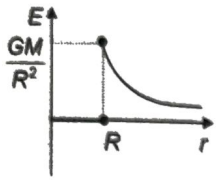
When point lies inside the spherical shell

$$\int dE = \frac{G}{r^2}\int m \quad = 0 \quad \ldots (8)$$

Case-II: enclosed

When point P lies outside the spherical shell

$$\int dE = \frac{G}{r^2}\int dm = \frac{GM}{r^2} \quad \ldots (9)$$

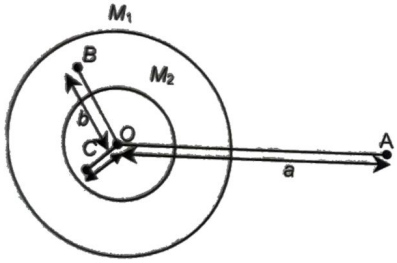


Gravitational field due to thin spherical shell is both discontinuous and non-differentiable function.

TRAIN YOUR BRAIN

Example 5: Two concentric shells of masses M_1 and M_2 are situated as shown in figure. Find the force on a particle of mass m when the particle is located at (a) r = a (b) r = b (c) r ≤ c. The distance r is measured from the center of the shell.

[M_1 = 500 kg, M_2 = 1500kg, a = $\sqrt{6.6}\,m$, b = $\sqrt{3.3}$, G = 6.6 × 10^{-11} kgm^2/kg^2, Take 10^{-8} N as unit]



Sol. We know that attraction at an external point due to spherical shell of mass M is $\frac{GM}{r^2}$ while at an internal point is zero. So

(a) for r = a, the point is external for both the shell ; so

$$E_A = \frac{G(M_1 + M_2)}{a^2}$$

$$\therefore F_A = mE_A = \frac{G[M_1 + M_2]m}{a^2} = 2 \text{ units}$$

(b) For r = b, the point is external to the shell of mass M_2 and internal to the shell of mass M_1 ; so

$$E_B = \frac{GM_2}{b^2} + O$$

$$\therefore F_B = mE_B = \frac{GM_2m}{b^2} = 3 \text{ units}$$

(c) For r = c , the point is internal to both the shells, so

$E_C = 0 + 0 = 0$

$\therefore F_C = m E_C = 0$

GRAVITATIONAL FIELD OR GRAVITATIONAL FIELD STRENGTH

All the bodies on or above earth's surface experience gravitational force known as the weight of the bodies. Therefore the space surrounding the earth, where the gravitational force (weight) is experienced is known as the gravitational field of the earth. Similarly the space surrounding each and every material particle is known as gravitational field of that particle. Gravitational field is a vector field.

Gravitational field strength at any point is defined as gravitational force exerted on a unit point mass. It is numerically equal to acceleration due to gravity. Its unit is N/kg and dimension $[M^0LT^{-2}]$.

Now if we want to measure the strength of the gravitational field at any point we will have to calculate the force acting on a point mass placed at that point. We see that , different masses experience different forces. The larger the mass, the larger the force it will experience. When we take the ratio of the gravitational force $\vec{F}_g$ and the point mass m we obtain a constant value for that point. This constant is known as the strength of the gravitational field. If the field exerts a large force on the point mass, we say that the strength of the gravitational field is stronger at that point and vice-versa.

The strength of the gravitational field $\vec{E}_g = (\vec{F}_g / m)$

Gravitational field strength is defined as gravitational force per unit mass.

In earth's gravitational field

$$\vec{E}_g = \frac{\text{weight of the particle}}{\text{mass of the particle}} = \frac{\vec{W}}{m}$$

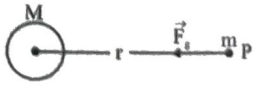
The above expression is equal to the acceleration due to gravity '$\vec{a}_g$'

ACCELERATION DUE TO GRAVITY OF THE EARTH

We want to find a_g due to M at point P. As per the procedure, place a point mass m at P, measure the force imparted by M on the test mass m. That is equal to $F_g = \frac{GMm}{r^2}$

r = distance between the particle and centre of M

$$\Rightarrow a_g = \frac{F_g}{m} = \frac{1}{m}\left(\frac{GMm}{r^2}\right)$$



$$\Rightarrow a_g = \frac{GM}{r^2}$$ and it is directed towards the mass M.

ACCELERATION DUE TO GRAVITY ON SURFACE OF EARTH

$$a_g = g = \frac{GM_e}{R_e^2}$$

Where $G = 6.67 \times 10^{-11} \frac{N - m^2}{Kg^2}$

(universal gravitational constant)

$M_e = 5.983 \times 10^{24}$ Kg (mass of earth)

$R_e = 6.378 \times 10^6$ m (radius of earth)

Also, if the particle is very close to the earth's surface then $r = R_e + h \simeq R_e$

Putting all the values we obtain

$g = 9.8$ m/sec^2

This value near the surface of the earth can be determined by various experiments.

From $g = \frac{GM_e}{R_e^2}$, we get $M_e = \frac{gR_e^2}{G}$.

Using this formula, Cavendish computed the mass M of the Earth when he measured $G = 6.67 \times 10^{-11}$ N.m^2/kg^2. The mass of the Earth comes out to be 5.98×10^{24} kg

Variation in the value of a_g

The acceleration due to gravity is given by

$$a_g = \frac{F}{m} = \frac{GM_e}{r^2}$$

where F is the force exerted by the earth on an object of mass m. This force is affected by a number of factors and hence a_g also depends on these factors.

(a) Variation of a_g with altitude or height from the surface of the Earth

If the object is placed at a distance h above the surface of the earth, the force of gravitation on it due to the earth is

$$F = \frac{GM_e m}{(R_e + h)^2}$$

where M_e is the mass of the earth and R_e is its radius.

Thus, $a_g = \frac{F}{m} = \frac{GM_e}{(R_e + h)^2}$

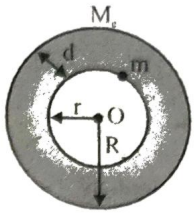
We see that the value of g decreases as one goes up. We can write,

$$a_g = \frac{GM_e}{R_e^2\left(1 + \frac{h}{R_e}\right)^2} = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

where $g = \frac{GM_e}{R_e^2}$ is the value of a_g at the surface of the earth. If $h << R_e$,

$$a_g = g\left(1 + \frac{h}{R_e}\right)^{-2} \approx g\left(1 - \frac{2h}{R_e}\right).$$

(b) Variation of a_g with depth:



The figure shows the Earth with mass M_e and radius R_e. A small body of mass m has been shown to be located at a distance r from the centre O of the Earth. If we assume the Earth as a homogenous sphere (of uniform density), the mass M' of the sphere of radius r concentric with the Earth is given by

$$M' = \frac{M_e}{\frac{4}{3}\pi R_e^3} \cdot \frac{4}{3}\pi r^3 = \frac{M_e r^3}{R_e^2}$$

According to Newton's shell theory, the shell of thickness $d = R_e - r$ exterior to the mass m exerts no force on this mass. Hence the gravitational force on the mass m is the force due to the sphere of mass M' (radius r) which is interior of the mass. So, its weight at depth d is

$$W = \frac{GM'm}{r^2} \text{ or } ma_g = \frac{GM'm}{r^2}$$

or, $a_g = \frac{GM'}{r^2}$ at distance r from the Earth's centre

or, $a_g = \frac{G}{r^2}\frac{M_e r^3}{R_e^3} = \frac{GM_e}{R_e^3} r$

or, $a_g = kr$,

where $k = \frac{GM_e}{R_e^3}$ is constant for the Earth for any position of the body..

Thus $\boxed{a_g = \frac{GM_e}{R_e^3} r = kr}$

Thus, inside the Earth, $a_g \propto r$ but outside it $a_g \propto \frac{1}{r^2}$.

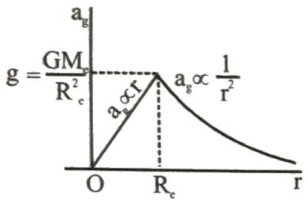
Hence the value of a_g decreases as we go inside or outside the Earth's surface, i.e. a_g is maximum at the surface of the Earth.

We can also write,

$$a_g = \frac{GM_e}{R_e^3} r = \frac{GM_e}{R_e^3}(R_e - d) = \frac{GM_e}{R_e^2}\left(1 - \frac{d}{R_e}\right)$$

or, $\boxed{a_g = g\left(1 - \frac{d}{R_e}\right)}$, ...

[acceleration due to gravity at a depth d]

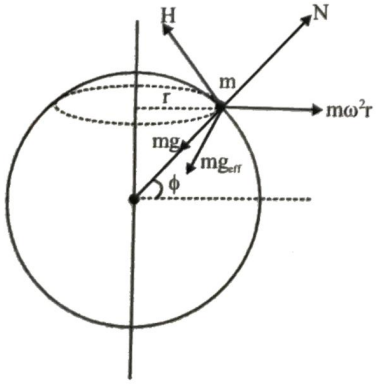


Figure, shows the graph of the value "a_g" at a distance 'r' from the centre

Here $g = \frac{GM_e}{R_e^2}$ is the value of g at the surface of the Earth.

At the centre of the Earth, $d = R_e$, so $a_g = 0$ at the centre

(c) Variation with latitude (Due to rotation of earth)



In figure m is a mass placed on a weighing machine situated at a latitude of ϕ the real forces acting on it are:

(i) the gravitational force mg towards the center of earth.

(ii) the normal reaction N of the weighing machine directed away from the center of earth,

(iii) The horizontal reaction H of the weighing machine directed along the tangent as shown.

In the reference frame fixed to the earth's surface the body would also be acted upon by the pseudo force (centrifugal force) $m\omega^2 r$ directed as shown,

where r is the distance of P from earth's axis, $r = R_e\cos\phi$, where R_e is radius of earth.

Now in this reference frame m is at rest. Resolving forces along the radial and normal direction we get,

$mg = m\omega^2 r\cos\phi + N$

$= m\omega^2 R_e\cos^2\phi + N \Rightarrow N = mg - m\omega^2 R_e\cos^2\phi$

and $H = m\omega^2 R_e\cos\phi\sin\phi$

Now the effective weight of the body is the net force experienced by the weighing machine which is equal and opposite to the force exerted by the weighing machine on the body.

$\therefore\ m\vec{g}_{eff} = -(\vec{N} + \vec{H}) \Rightarrow g_{eff} = |\vec{g}_{eff}| = \frac{1}{m}\sqrt{N^2 + H^2}$

$= \sqrt{(g - \omega^2 R_e\cos^2\varphi)^2 + (\omega^2 R_e\cos\varphi\sin\varphi)^2}$

Now $\omega^2 R_e \simeq 0.0337$, hence $(\omega^2 R_e\cos\phi\sin\phi)^2$ may be neglected and

thus $\boxed{g_{eff} \simeq g - \omega^2 R_e\cos^2\phi}$

where g_{eff} = apparent acceleration due to gravity at a latitude of ϕ

* At poles, $\phi = \pi/2$ and hence $g_{eff} = g$

* At the equator, $\phi = 0$ and hence $g_{eff} = g - \omega^2 R_e$

Important Points

(i) Non-sphericity of the Earth

All formulae and equations have been derived by assuming that the earth is a uniform solid sphere. The shape of the earth slightly deviates from the perfect sphere. It is an ellipsoid, flattened at the poles and bulging at the equator. The radius in the equatorial plane is about 21 km larger than the radius along the poles. Due to this the force of gravity is more at the poles and less at the equator. The value of g is accordingly larger at the poles and less at the equator. Note that due to rotation of earth also, the value of g is smaller at the equator than that at the poles.

(ii) Non-uniformity of the Earth

The earth is not a uniformly dense object. There are a veriety of minerals, metals, water, oil, etc., inside the earth. Then at the surface there are mountains, seas, etc. Due to these nonuniformities in the mass distribution, the value of g is locally affected.

TRAIN YOUR BRAIN

Example 6: If the radius of the earth were to shrink by one percent, its mass remaining the same. What would happen to the acceleration due to gravity on the earth's surface.

Sol. Consider the case of a body of mass m placed on the earth's surface (mass of the earth M and radius R).

$$g = \frac{GM_e}{R_e^2} \qquad ...(i)$$

Now, when the radius reduced by 1%, i.e. become 0.99 R, let acceleration due to gravity be g', then

$$g' = \frac{GM}{(0.99R)^2} \qquad ...(ii)$$

From equation (i) and (ii), we get

$$\frac{g'}{g} = \frac{R^2}{(0.99)R^2} = \frac{1}{(0.99)^2}$$

$$\text{or } g' = g \times \left(\frac{1}{(0.99)}\right)^2$$

or $g' = 1.02\ g$

Thus the value of g is increased by 2%

Example 7: At what rate should the earth rotate so that the apparent g at the equator becomes zero ? What will be the length of the day in this situation?

Sol. At earth's equator effective value of gravity is

$$g_{eq} = g_s - \omega^2 R_e$$

If g_{eff} at equator to be zero, we have

$$g_s - \omega^2 R_e = 0$$

$$\text{or } \omega = \sqrt{\frac{g_s}{R_e}}$$

Thus length of the day will be

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R_e}{g_s}}$$

$$= 2 \times 3.14 \sqrt{\frac{6.4\times10^6}{9.8}} = 5074.77\text{s} \simeq 84.57 \text{ min.}$$

Example 8: Calculate the value of acceleration due to gravity at a point (a) 5.0 km above the earth's surface and (b) 5.0 km below the earth's surface. Radius of earth = 6400 km and the value of g at the surface of the earth is 9.80 m s^{-2}.

Sol. (a) The value of g at a height h is (for $h << R$)

$$g_0 = g_0\left(1 - \frac{2h}{R}\right)$$

$$= (9.80 \text{ m s}^{-2})\left(1 - \frac{2\times5.0 \text{ km}}{6400 \text{ km}}\right)$$

$$= 9.78 \text{ ms}^{-2}$$

(b) The value at a depth h is

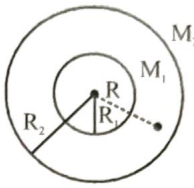
$$g = g_0\left(1 - \frac{h}{R}\right)$$

$$= (9.8 \text{ m s}^{-2})\left(1 - \frac{5.0 \text{ km}}{6400 \text{ km}}\right)$$

$$= 9.79 \text{ m s}^{-2}$$

Example 9: Two concentric spherical shells have masses M_1, M_2 and radii R_1, $R_2 (R_1 < R_2)$. What is the force exerted by this system on a particle of mass m if it is placed at a distance $\frac{(R_1 + R_2)}{2}$ from the centre ?

Sol.



$$R = \frac{R_1 + R_2}{2}$$

As the particle 'm' is inside shell M_2 no force due to it and M_1 will exert force as if whole of the mass is concentrated at its centre.

$$\therefore \text{ F on 'm'} = \frac{GM_1m}{R^2} = \frac{4GM_1m}{(R_1 + R_2)^2}$$

GRAVITATIONAL POTENTIAL ENERGY

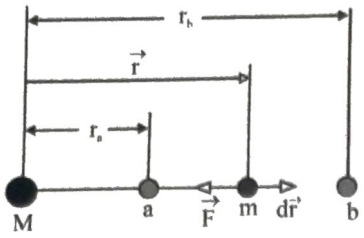
In analyzing the motion of planets and satellites, it is often easier and more informative to use energy rather than force. In this section we shall evaluate the potential energy of a system consisting of two bodies that interact through the gravitational force. We obtained the potential energy change due to gravity for a body that moves through a height y near the Earth's surface : $\Delta U = mgy$. However, this applies only near the Earth's surface, (for changes in height that are small compared with the distance from the center of the Earth) where we can regard the gravitational force as approximately constant. Our goal here is to find a general expression that applies at all location, such as at the altitude of an orbiting satellite.

The potential energy difference can be found from equation:

$\Delta U = U_b - U_a = -W_{ab}$, where W_{ab} is the work done by gravitational force to change configuration a to configuration b. However, this equation applies only if the force is conservative.

CALCULATING THE POTENTIAL ENERGY

The gravitational force is conservative so we can calculate the potential energy. Figure shows a particle of mass m moving from a to b along a radial path. A particle of mass M, which we assume to be at rest at the origin, exerts a gravitational force on m. The vector $\vec{r}$ locates the position of m relative to M.



The work done by the gravitational force is

$$W_{ab} = \int_a^b \vec{F}.d\vec{r} = -\int_a^b F dr$$

($\because \vec{F}$ and $d\vec{r}$ are in opposite direction)

$$= -\int_{r_a}^{r_b} \frac{GMm}{r^2} dr = -GMm \int_{r_a}^{r_b} \frac{dr}{r^2}$$

$$= -GMm\left(-\frac{1}{r}\right)\Big|_{r_a}^{r_b} = GMm\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Applying equation ($\Delta U = - W_{ab}$), we can find the change in the potential energy of the system as m moves between points a and b

$\Delta U = U_b - U_a = - W_{ab}$

$$= GMm\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

= Negative of work done by gravitational force

Instead of differences in potential energy, we can consider the value of the potential energy at a single point if we define a reference point. We choose our reference point to be an infinite separation of the particles, and the we define the potential energy to be zero in that point. Let us evaluate equation for $r_b = \infty$ and $U_b = 0$. If a represents any arbitrary point, where the separation between the particles is r, then the above equation becomes

$$U(\infty) - U(r) = GMm\left(\frac{1}{r} - 0\right)$$

or $$U(r) = -\frac{GMm}{r}$$

The potential energy given by this equation is a property of the system of two particles rather than of either particle alone. There is no way to divide this energy and say that so much belongs to one particle and so much to the other. Nevertheless, if M >> m, as is true for Earth and a baseball, we often speak of "the potential energy of the baseball". We can get away with this because, when a baseball moves in the vicinity of Earth, changes in the potential energy of the baseball. Earth system appear almost entirely as changes in the kinetic energy of the baseball, since changes in the kinetic energy of Earth are too small to be measured.

In the situation discussed by us, a particle of mass M exerts a gravitational force $\vec{F}$ on a particle of mass m that moves from a to b. If m moves outward from a to b, the change in potential energy is positive ($U_b > U_a$). That is, if the particle passes through point a with a certain kinetic energy K_a, as it travels to b its gravitational potential energy increases as its kinetic energy decreases ($K_b < K_a$). Conversely, if the particle is moving inward, its potential energy decreases as its kinetic energy increases.

Importants points

1. We can reverse the previous calculation and derive the gravitational force from the potential energy. For spherically symmetric potential energy functions, the relation F = – dU/dr gives the radial component of the force; see equation.

$$F = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{GMm}{r}\right) = -\frac{GMm}{r^2}$$

The minus sign here shows that the force is attractive, directed inward along a radius.

2. We can show that the potential energy defined according to equation $U_{(r)} = -\frac{GMm}{r}$ leads to the familiar 'mgy' for a small difference in elevation y near the surface of the Earth. Let us evaluate equation for the difference in potential energy between the location at a height y above the surface (that is, $r_b = R_e + y$, where R_e is the radius of the Earth) and the surface ($r_a = R_e$)

$$\Delta U = U(R_e + y) - U(R_e) = GM_E m\left(\frac{1}{R_e} - \frac{1}{R_e + y}\right)$$

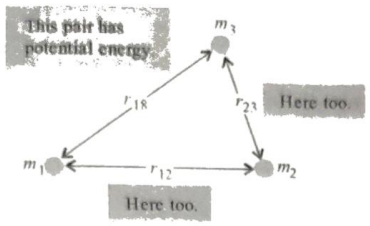
$$= \frac{GM_E m}{R_e}\left(1 - \frac{1}{1 + y/R_e}\right)$$

When $y << R_e$, which would be the case for small displacements of bodies near the Earth's surface, we can use the binomial expansion to approximate the last term as $(1 + x)^{-1} = 1 - x + ... \approx 1 - x$, which gives

$$\Delta U \approx \frac{GM_E m}{R_e}\left[1 - \left(1 - \frac{y}{R_e}\right)\right] = \frac{GM_E my}{R_e^2} = mgy$$

using equation to replace GM_E / R_e^2 with g.

3. If our system contains more than two particles, we consider each pair of particles in turn, calculate the gravitational potential energy of that pair with equation as if the other particles were not there, and then algebraically sum the results. Each of the three pairs of figure, for example, gives the potential energy of the system as



$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right)$$

TRAIN YOUR BRAIN

Example 10: An object is dropped from height $h = 2R_e$ on the surface of earth. Find the speed with which it will collide with ground

[where R is radius of earth, take mass of earth M]

Sol. The initial potential energy (U_i) of object is

$$U_i = -\frac{GMm}{3R_e} \quad r = R_e + 2R_e = 3R_e$$

Final potential energy $U_f = -\frac{GMm}{R}$

By law of conservation of mechanical energy

$\Delta KE = -\Delta P.E.$

$$\Rightarrow \frac{1}{2}mv^2 = -[U_f - U_i] = U_i - U_f$$

$$\Rightarrow \frac{1}{2}mv^2 = -\frac{GMm}{3R_e} + \frac{GMm}{R_e}$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{2GM}{3R_e} \Rightarrow v = \sqrt{\frac{4GM}{3R_e}} = 2\sqrt{\frac{GM}{3R_e}}$$

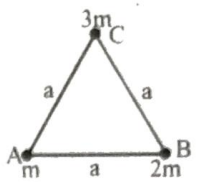
Example 11: Three particles of masses m, 2 m and 3m are placed at the corners of an equilateral triangle of side a. Calculate (i) The potential energy of the system (ii) The work done on the system if the side of the triangle is changed from a to 2a, Assume the potential energy to be zero when the separation is infinity.

Sol. (i) The potential energy of the system = Sum of the potential energies of all the three possible distinct pairs.

i.e., $U = U_{AB} + U_{BC} + U_{CA}$

$$= \frac{-G(m)(2m)}{a} + \frac{-G(2m)(3m)}{a} + \frac{-G(3m)(m)}{a}$$

$$= \frac{-11Gm^2}{a}$$



(ii) When the side a is changed to 2a, the potential energy

$$U' = \frac{-11Gm^2}{2a}$$

The work done on the system = $U'-U = \frac{11Gm^2}{2a}$

NOTE:

The work done by the system = The work done by the gravitational

$$\text{force} = -(U' - U) = \frac{-11Gm^2}{2a}$$

The work done on the system = The work done against the gravitational force = The work done by the external force

$$= U'-U = \frac{11Gm^2}{2a}$$

GRAVITATIONAL POTENTIAL

"The gravitational potential at a point is defined as the work done per unit mass by an external agent (or against the gravitational force) in bringing a particle slowly from the reference point to the given point ".

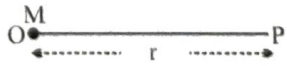
This definition is equivalent to the definition given below:

The gravitational potential at a point is equal to the change in potential energy per unit mass, as the mass is brought from the reference point to the given point.

The SI unit of gravitational potential is $\frac{J}{kg}$.

(a) Potential due to a point mass:

Suppose a paricle of mass M lies at O. We want to know the gravitaitional potential at a point P distant r from O If we take the reference point of zero potential at infinity, then on placing another particle of mass m from infinity to P, the change in potential energy.



$$U - U_\infty = \frac{-GMm}{r}, \quad (\because U_\infty = 0)$$

$\therefore$ The potential at P is

$$V = \frac{U - U_\infty}{m} = \frac{-GM}{r}$$

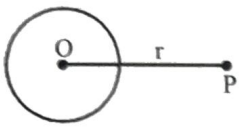
Remember that potential is a scalar quantity.

(b) Potential due to a uniform thin spherical shell :

Let us take a thin uniform spherical shell of mass M and radius R with centre at O.

Case-I: When P is outside the shell

$$V = -\frac{GM}{r}$$



(As if whole mass M is concentrated at the centre)

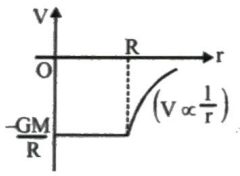
Case-II: When P is inside the shell

$$V = -\frac{GM}{R}$$



This is independent of the location of P inside the spherical shell.

Variation of V with r



(c) Potential due to a uniform solid sphere : Let us consider a uniform solid of mass M and radius R.

Case-I:

when P is outside the sphere

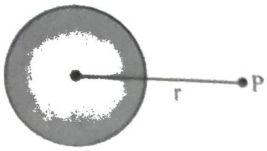
$$V = -\frac{GM}{r}$$

(As if whole mass is concentrated at its centre)

Case-II:

when P is inside the sphere

$$V = -\frac{GM}{2R^3}[3R^2 - r^2]$$

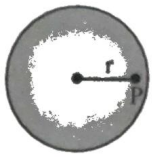


Special case

(i) At the centre of sphere ; r = 0

$$V_{centre} = -\frac{3GM}{2R}$$

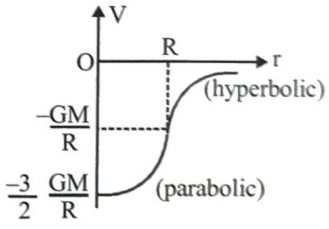
(ii) At the surface of sphere



$$V_{surface} = \frac{-GM}{R}$$

$$\therefore \quad V_{centre} = -\frac{3}{2}V_{surface}$$

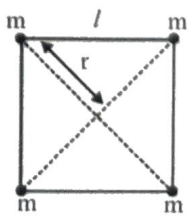
Variation of V with r



TRAIN YOUR BRAIN

Example 12: Find the potential energy of a system of four particles placed at the vertices of a square of side ℓ. Also obtain the potential at the centre of the square.

Sol. Consider four masses each of mass m at the corners of a square of side ℓ; See figure. We have four mass pairs at distance ℓ and two diagonal pairs at distance $\sqrt{2}\ \ell$



Hence,

$$U = -4\frac{Gm^2}{\ell} - 2\frac{Gm^2}{\sqrt{2}\ell}$$

$$= -\frac{2Gm^2}{\ell}\left(2+\frac{1}{\sqrt{2}}\right)$$

The gravitational potential at the centre of the square $(r = \sqrt{2}\ell/2)$ is

$$V = -4\sqrt{2}\frac{Gm}{\ell}$$

ESCAPE SPEED

It is the minimum speed with which a body must be projected from the surface of a planet (usually the earth) so that it permanently overcomes and escapes from the gravitational field of the planet (the earth). We can also say that a body projected with escape speed will be able to go to a point which is at infinite distance from the earth.

If a body of mass m is projected with speed v from the surface of a planet of mass M and radius R, then

$$\text{K.E.} = \frac{1}{2}mv^2 \text{ ; G.P.E.} = -\frac{GMm}{R}$$

Total mechanical energy (T.M.E.) of the body

$$= \frac{1}{2}mv^2 - \frac{GMm}{R}$$

If the v ′ is the speed of the body at infinity, then

$$\text{T.M.E. at infinity} = 0 + \frac{1}{2}mv'^2 = \frac{1}{2}mv'^2$$

Applying the principle of conservation of mechanical energy, we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mv'^2$$

$$\text{or, } v^2 = \frac{2GM}{R} + v'^2$$

v will be minimum when $v' \to o$, i.e.,

$$V_e = V_{min} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$$\left[\because g = \frac{GM}{R^2}\right]$$

Important points

(i) Escape speed is independent of the mass and direction of projection of the body.

(ii) For earth as $g = 9.8\ m/s^2$ and $R = 6400$ km

$$V_e = \sqrt{2\times 9.8\times 6.4\times 10^6} = 11.2\,km/s$$

Escape speed depends on:

(i) Mass (M_e) and size (R) of the planet

(ii) Position from where the particle is projected.

Escape speed does not depend on:

(i) Mass of the body which is projected (m_0)

(ii) Angle of projection.

If a body is thrown from Earth's surface with escape speed, it goes out of earth's gravitational field and never returns to the earth's surface. But it starts revolving around the sun.

PLANETS AND SATELLITES

(a) Planets

Planets move around the sun due to the gravitational attraction of the sun. The path of these planets are elliptical with the sun at a focus. However, the difference in major and minor axis is not large. The orbits can be treated as nearly circular for not too advanced calculations. The motion of planets is largely governed by Kepler's Laws.

(b) Satellite

Satellites are launched from the earth so as to move around it. A number of rockets are fired from the satellite at proper time to establish the satellite in the desired orbit. Once the satellite is placed in the desired orbit with the correct speed for that orbit, it will continue to move in that orbit under gravitational attraction of the earth.

We make two assumptions that simplify the analysis:

(1) We consider the gravitational force only between the orbiting body and the central body, ignoring the perturbing effect of the gravitational force of other bodies (such as other planets)

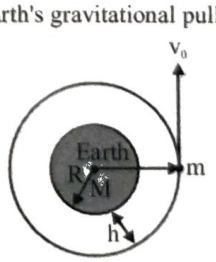
(2) We assume that the central body is so much more massive than the orbiting body that we can ignore its motion under their mutual interaction.

Let the speed of an artificial earth satellite in its orbits of radius r be v_0. The satellite is accelerating towards the centre of earth by earth's gravitational pull $\frac{GMm}{r^2}$

$$\Rightarrow F_{CP} = \frac{GMm}{r^2}$$

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow v_0 = \sqrt{\frac{GM}{r}} \quad \text{...(i)}$$



Putting $\frac{GM}{r^2} = a_g$ (acceleration due to gravity at the orbit), we obtain,

$\Rightarrow v_0 = \sqrt{a_g r}$ ...(ii)

When its orbit at an altitude h, putting r = (R+h)

$$\Rightarrow v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{R(1+h/R)}}$$

$$= \sqrt{\frac{gR}{(1+h/R)}}$$

Angular speed

The angluar speed

$$\omega = \frac{v_0}{r}$$

Putting $v_0 = \sqrt{\frac{GM}{r}}$, we obtain

$$\omega = \sqrt{\frac{GM}{r^3}} = \sqrt{\frac{GM}{(R+h)^3}}$$

Angular momentum

The angular momentum of an earth satellite or a planet is given as

$L = mv_0 r$

$$= m\sqrt{\frac{GM}{r}} \times r$$

$$= m\sqrt{GMr}$$

Time period of Revolution

The period of revolution

$$T = \frac{2\pi}{\omega}$$

Putting $\omega = \sqrt{\frac{GM}{r^3}}$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

ENERGY CONSIDERATION IN PLANETARY AND SATELLITE MOTION

$$U(r) = -\frac{GMm}{r}$$

Where r is the radius of the circular orbit.

The kinetic energy of the system is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 r^2$$

the Earth being at rest

$$\omega^2 r^2 = \frac{GM}{r}$$

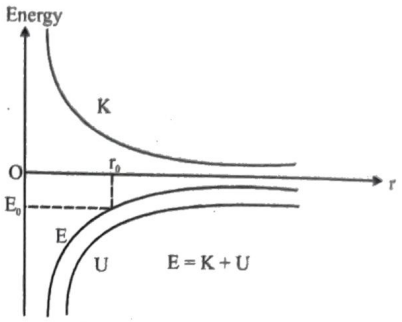
so that (with $v = \omega r$)

$$K = \frac{GMm}{2r}$$

The total mechanical energy is

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

This energy is constant and negative. The kinetic energy can never be negative.



Importants points

(1) Kinetic energy K, potential energy U, and total energy E = K + U of a body in circular planetary motion A satellite with total energy E < 0 will remain in a orbit with radius r_0. The greater the distance from the planet, the greater (that is, less negative) its total energy E. To escape from the center of force and still have kinetic energy at infinity, the planet would need positive total energy.

(2) **Gravitational binding energy**

We have seen that if a particle of mass m placed on the earth is given an energy $\frac{1}{2}mv^2 = \frac{GMm}{R}$ or more, it finally escapes from earth. The minimum energy needed to take the particle infinitely away from the earth is called the binding energy of the earth-particle system. Thus, the binding energy of the earth-particle system is $\frac{GMm}{R}$

GEOSTATIONARY SATELLITE

A satellite which appears to be stationary when seen from earth is called a Geostationary satellite.

For a satellite to be geostationary.

(i) Its orbit must be circular

(ii) It must rotate about the same axis as earth, i.e. it must move in the equatorial plane.

(iii) It must revolve from west to east.

(iv) Its time period must be 24 hours.

$$\Rightarrow \quad m\omega^2(R+h) = \frac{GMm}{(R+h)^2}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{(R+h)^3}$$

$$(R+h) = \left\{GM\left(\frac{T}{2\pi}\right)^2\right\}^{1/3}$$

where $T = 24 \times 3600$ sec.

Solving $h = 35800$ km and $r = R + h = 42200$ km

(v) The orbital speed of geostationary satellite is

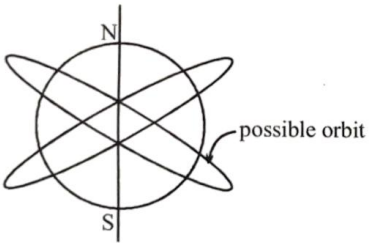
$$v_0 = \frac{2\pi r}{T} = 3.07\,\text{km/s}$$

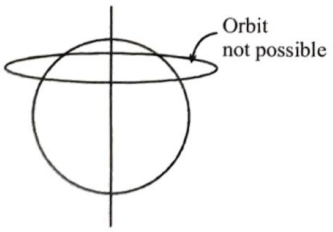
(vi) The angular orbital speed of geostationary satellite is

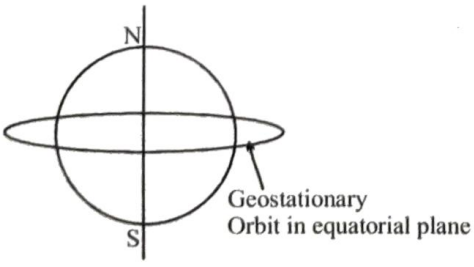
$$\omega = \frac{2\pi}{T} = 7.3\times10^{-5}\,\text{rad/s}$$

(vii) The INSAT group of satellites sent up by India are one such group of Geostationary satellites widely used for telecommunications in India.

* Any satellite must rotate about the center of earth.







TRAIN YOUR BRAIN

Example 13: Estimate the mass of the sun, assuming the orbit of the earth round the sun to be a circle. The distance between the sun and earth is 1.49×10^{11} m and $G = 6.66\times10^{-11}$ Nm^2/kg^2.

Sol. Here the revolving speed of earth can be given as

$$v = \sqrt{\frac{GM}{R}} \qquad \text{[Orbital speed]}$$

Where M is the mass of sun and r is the orbit radius of earth.

We know time period of earth around sun is T = 365 days, thus we have

$$T = \frac{2\pi r}{v}$$

$$\text{or} \quad T = 2\pi r\sqrt{\frac{r}{GM}}$$

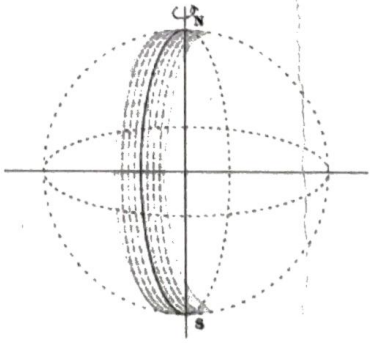
$$\text{or} \quad M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\times(3.14)^2\times(1.49\times1011)^3}{(365\times24\times3600)^2\times(6.66\times10^{-11})} = 1.972\times10^{22}\,\text{kg}$$

POLAR SATELLITE

These are low altitude ($h \approx 500$ to 800 km) satellites, but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Since its time period is around 100 minutes it crosses any altitude many times a day. However, since its height h above the earth is about 500-800 km, a camera fixed on it can view only small strips of the earth in one orbit. Adjacent strips are viewed in the next orbit, so that in effect the whole earth can be viewed strip by strip during the entire day. These satellites can view polar and equatorial regions at close distances with good resolution.

Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.

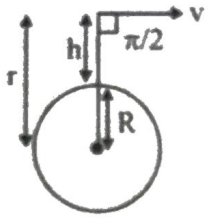


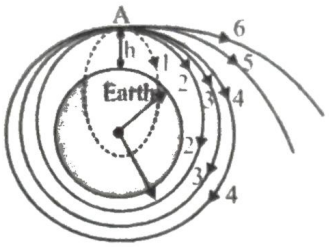
A Polar satellite. A strip on earth's surface (shown shaded) is visible from the satellite during one cycle. For the next revolution of the satellite, the earth has rotated a little on its axis so that an adjacent strip becomes visible.

The PSLV (polar satellite Launch vehicle) is an expandable launch system developed and operated by ISRO (Indian space research organisation). PSLV is designed and developed at VSSC [Vikram Sarabhai space centre]. Thiruvananthapuram, Kerala. The PSLV was first time successfully launched in 1996.

CONDITIONS FOR DIFFERENT TRAJECTROY

For a body being projected tangentially from above earth's surface, say at a distance r from earth's center, the trajectory would depend on the velocity of projection v.





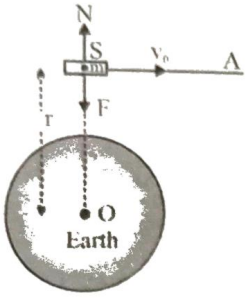
Velocity	Orbit
1. Velocity, $v << \sqrt{\frac{GM}{r}}$	Body returns to earth following elliptical Path.
2. $v < \sqrt{\frac{GM}{r}}$	Body acquires an elliptical orbit with earth as the far-focus w.r.t. the point of projection.
3. Velocity is equal to the critical velocity of the orbit, $v = \sqrt{\frac{GM}{r}}$	Circular orbit with radius r
4. Velocity is between the critical and escape velocity of the orbit $\sqrt{\frac{2GM}{r}} > v > \sqrt{\frac{GM}{r}}$	Body acquires an elliptical orbit with earth as the near focus w.r.t. the point of projection.
5. $v = v_{esc} = \sqrt{\frac{2GM}{r}}$	Body just escapes earth's gravity, along a parabolic path.
6. $v > v_{esc} = \sqrt{\frac{2GM}{r}}$	Body escape earth's gravity along a hyperbolic path.

Importants Note:

(1) Closed orbits are always elliptical and total mechanical energy is always negative.
(2) Open orbits are either parabolic or hyperbolic and total mechanical energy are always non-negative.
(3) When the mechanical energy of a satellite is zero or positive, it escapes away from the gravitational field of the central planet.
(4) When a satellite is orbiting then no energy is required to keep it moving in its orbit.
(5) When the speed of satellite in circular orbit is increased, then its energy increases. It starts moving in an elliptical path of greater semi-major axis.
(6) When the height of a satellite is increased, its potential energy increases and kinetic energy decreases.
(7) The numerical value of negative potential energy of a satellite orbiting in circular orbit is always more than its kinetic energy.

WEIGHTLESSNESS

Life of astronauts inside an orbiting satellite is very different. Absence of gravity has severe physiological effects. They develop puffy faces, their noses get blocked by fluids, their height may grow by few centimetre, the cardiovascular system does less work and causes headaches. Bones grow weak; muscles face deleterious effects in the absence of gravity which may cripple their day to day activity once they come back on the Earth.



Now, consider a body of mass m in a satellite S orbiting the Earth in a circular orbit of radius r with orbital speed v_0.

You know $v_0 = \sqrt{\frac{GM}{r}}$, where M is the mass of the Earth.

The body of mass m is also orbiting with the satellite with speed v_0. Suppose the normal reaction of the floor of the satellite on the body is N. The gravitational force F on the body is $\frac{GMm}{r^2}$. The net force towards the centre will be equal to the centripetal force on the body.

$$\therefore \quad F - N = \frac{mv_0^2}{r}$$

$$\text{or,} \quad \frac{GMm}{r^2} - N = \frac{mv_0^2}{r}$$

$$\text{or,} \quad N = \frac{GMm}{r^2} - \frac{m}{r}.\frac{GM}{r} = 0$$

Thus, the normal reaction becomes zero. Hence apparent weight of the body is zero. Thus **"everybody inside the satellite is in a sate of weightlessness"**.

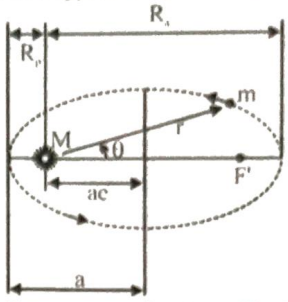
If we suspend a body by a spring balance in the satellite, the balance will read zero.

Figure shows that the force of the Earth's gravitation causes the satellite S to deviate from its natural straight line path SA towards the Earth. Hence a satellite may be called to **"fall towards the Earth freely because its acceleration is g".**

KEPLER'S LAWS

The empirical basis for understanding the motion of the planets is three laws deduced by Kepler (1571 - 1630, well before Newton) from studies of the motion of the planet Mars.

1. **The law of orbits : All planets move in elliptical orbits having the Sun at one focus.** Newton was the first to realize that there is a direct mathematical relationship between inverse-square ($1/r^2$) force and elliptical orbits. Figure shows a typical elliptical orbit.



A planet of mass m moving in a elliptical orbit around the Sun. The Sun, of mass M, is at one focus of the ellipse. F' marks the other or "empty" focus. The semimajor axis a of the ellipse, the perihelion distance "R_p", and the aphelion distance "R_a" are also shown. The distance "ae" locates the focal points, e being the eccentricity of the orbit.

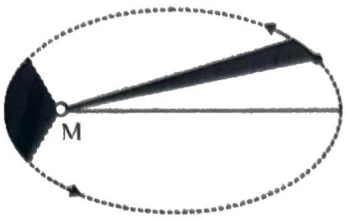
For other planets in the solar system, the eccentricities are small and the orbits are nearly circular.

The maximum distance R_a of the orbiting body from the central body is indicated by the prefix (ap-), as in aphelion (the maximum distance from the Sun) or (apo-) apogee. Similarly, the closest distance R_p is indicated by the prefix (peri-), as in perihelion or perigee. As you can see from figure $R_a = a(1 + e)$ and $R_p = a(1 - e)$. For circular orbits: $R_a = R_p = a$

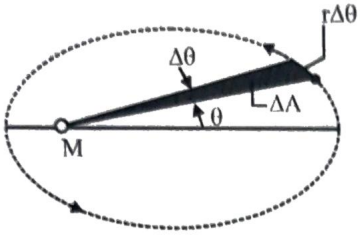
2. **The Law of Areas : A line joining any planet to the Sun sweeps out equal areas in equal times.** Figure illustrates this law : in effect it says that the orbiting body moves more rapidly when it is close to the central body than it does when it is far away. We now show that the law of areas is identical with the law of conservation of angular momentum.

Consider the small area increament ΔA covered in a time interval Δt, as shown in figure. The area of this approximately triangular wedge is one-half its base, $r\Delta\theta$, times its height r. The rate at which this area is swept out is $\Delta A/\Delta t = \frac{1}{2}(r\Delta\theta)(r)/\Delta t$. In the instantaneous limit this becomes

$$\frac{dA}{dt} = \lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \lim_{\Delta t \to 0} \frac{1}{2} r^2 \frac{\Delta \theta}{\Delta t} = \frac{1}{2} r^2 \omega$$



(a)



(b)

(a) The equal shaded areas are covered in equal times by a line connecting the planet to the Sun, demonstrating the law of areas.

(b) The area ΔA is covered in a time Δt, during which the line sweeps through an angle $\Delta\theta$.

Assuming we can regard the more massive body M as at rest, the angular momentum of the orbiting body m relative to the origin at the central body is , according to equation $L_z = I\,\omega = mr^2\omega$ (choosing the z axis perpendicular to the plane of the orbit). Thus

$$\frac{dA}{dt} = \frac{L_z}{2m}$$

If the system of M and m is isolated, meaning that there is no net external torque on the system, then L_z is a constant; therefore dA/dt is also constant. That is in every interval dt in the orbit, the line connecting m and M sweeps out equal areas dA, which verifies kepler's second law.

The speeding up of a comet as it passes close to the Sun is an example of this effect and is thus a direct consequence of the law of conservation of angular momentum.

3. **The law of Periods : The square of the period of any planet about the Sun is proportional to the cube of seminajor axis of the elliptical orbit.** If 'T' is the period of revolution and 'a' be the semi-major axis of the path of planet then according to kepler's III Law, we have $T^2 \propto a^3$

Let us prove this result for circular orbits. The gravitational force provides the necessary centripetal acceleration for circular motion.

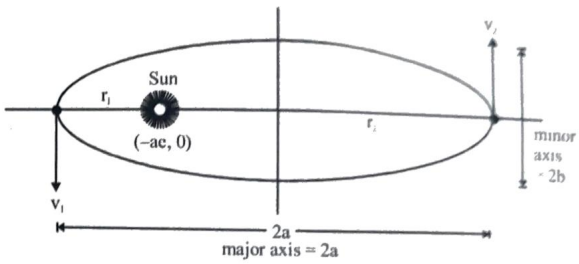
For circular orbits, it is a special case of ellipse when its major and minor axis are equal. If a planet is in a circular orbit of radius r around the sun then its revolution speed must be given as

$$v = \sqrt{\frac{GM_s}{r}}$$

Where M_s is the mass of sun. There you can recall that this speed is independent from the mass of planet. Here the time period of revolution can be given as

$$T = \frac{2\pi r}{v} \text{ or } T = \frac{2\pi r}{\sqrt{\frac{GM_s}{r}}}$$

Squaring equation $T^2 = \frac{4\pi^2}{GM_s} r^3$



Here r_1 and r_2 are the shortest and farthest distance of planet from sun during its motion, which are given as

$r_1 = a(1 - e)$

and $r_2 = a(1 + e)$

From geometry we know that the relation in semi major axis 'a' and semiminor axis 'b' is given as

$b = a\sqrt{1-e^2}$

If v_1 and v_2 are the planet speeds at perihelion and aphelion points then from conservation of momentum we have $L_z = mv_1r_1 = m\,v_2r_2$

TRAIN YOUR BRAIN

Example 14: If the earth be on-half of its present distance from the sun, how many days will be there in one year ?

Sol. If orbit of earth is r, we've discussed that time period is given as

$$T = 2\pi r \sqrt{\frac{r}{GM}} = \frac{2\pi}{\sqrt{GM}} r^{3/2} \quad ...(i)$$

If radius changes to $r' = \frac{r}{2}$, new time period become

$$T' = \frac{2\pi}{\sqrt{GM}} r'^{3/2} \qquad ...(ii)$$

From equation (i) and (ii) we have

$$\frac{T}{T'} = \left(\frac{r}{r'}\right)^{3/2}$$

or $$T' = T\left(\frac{r'}{r}\right)^{3/2}$$

$$= 365\left(\frac{1}{2}\right)^{3/2} = \left(\frac{365}{2\sqrt{2}}\right) \text{ days} \simeq 129 \text{ days}$$

Example 15: Find the energy required to launch a satellite of mass m from earth's surface in a circular orbit at an altitude 3 R. (where R radius of earth)

Sol. Circular orbit's radius = 4R

$\therefore$ Total mechanical energy of satellite is $-\frac{GM_em}{2(4R)}$

Thus; Energy required

$$\frac{GM_em}{R} - \frac{GM_em}{8R}$$

$\Rightarrow$ Energy required

$$\frac{GM_em}{R} - \frac{GM_em}{8R} = \frac{7}{8}\frac{GM_em}{R}$$

$$= \frac{7}{8}mgR \text{ (in the form of Kinetic Energy)}$$

Example 16: A satellite revolve around an imaginary planet, on the surface of which gravitational force (F) varies with distance (r) from its centre as $F \propto r^{-1}$. Find

(i) The propartionality relation between its orbital speed and radius of its orbit.

(ii) The proportionality relation between its time period and radius of its orbit.

Sol. $F \propto r^{-1}$, but this force is responsible for providing centripetal acceleration

$$\therefore \quad \frac{v_0^2}{r} \propto r^{-3}$$

$$\Rightarrow \quad v_0^2 \propto r^{-2} \qquad \Rightarrow v_0 \propto r^{-1}$$

and $$T = \frac{2\pi r}{v_0} \propto \frac{r}{r^{-1}} = r^2$$

$$\Rightarrow \quad T \propto r^2$$

Topicwise Questions

FORCE / FIELD

1. If the distance between two masses is doubled, the gravitational attraction between them
   (*a*) Is doubled (*b*) Becomes four times
   (*c*) Is reduced to half (*d*) Is reduced to a quarter

2. Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is
   (*a*) $v = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (*b*) $v = \sqrt{\frac{Gm}{2R}}$
   (*c*) $v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$ (*d*) $v = \sqrt{\frac{4Gm}{R}}$

3. Gravitational mass is proportional to gravitational
   (*a*) Field (*b*) Force
   (*c*) Intensity (*d*) All of these

4. Earth binds the atmosphere because of
   (*a*) Gravity
   (*b*) Oxygen between earth and atmosphere
   (*c*) Both (*a*) and (*b*)
   (*d*) None of these

5. Which of the following statements about the gravitational constant is true?
   (*a*) It is a force
   (*b*) It has no unit
   (*c*) It depends on the value of the masses
   (*d*) It does not depend on the nature of the medium in which the bodies are kept.

6. Two identical solid copper spheres of radius R placed in contact with each other. The gravitational attracton between them is proportional to
   (*a*) R^2 (*b*) R^{-2}
   (*c*) R^4 (*d*) R^{-4}

VARIATION OF *g*

7. The time period of a simple pendulum on a freely moving artificial satellite is
   (*a*) Zero (*b*) 2 sec
   (*c*) 3 sec (*d*) Infinite

8. A solid metallic sphere is placed in vacuum. It is given a small charge. Due to the sphere, the gravitational acceleration at a distance x from the centre (inside the sphere) is
   (*a*) Zero
   (*b*) Proportional to x
   (*c*) Inversely proportional to x^2
   (*d*) Inversely proportional to x

9. When a body is taken from the equator to the poles, its weight
   (*a*) Remains constant
   (*b*) Increases
   (*c*) Decreases
   (*d*) Increases at N-pole and decreases at S-pole

10. A body weighs 700 gm wt on the surface of the earth. How much will it weigh on the surface of a planet whose mass is $\frac{1}{7}$ and radius is half that of the earth
    (*a*) 200 gm wt (*b*) 400 gm wt
    (*c*) 50 gm wt (*d*) 300 gm wt

11. The radius of the earth is 6400 km and $g = 10\ m/sec^2$. In order that a body of 5 kg weighs zero at the equator, the angular speed of the earth is
    (*a*) 1/80 radian/sec (*b*) 1/400 radian/sec
    (*c*) 1/800 radian/sec (*d*) 1/1600 radian/sec

12. Select the wrong statement :
    The acceleration due to gravity 'g' decreases if
    (*a*) We go down from the surface of the earth towards its centre
    (*b*) We go up from the surface of the earth
    (*c*) We go from the equator towards the poles on the surface of the earth
    (*d*) The rotational velocity of the earth is increased

13. If the earth suddenly shrinks (without changing mass) to half of its present radius, the acceleration due to gravity will be
    (*a*) g/2 (*b*) 4g
    (*c*) g/4 (*d*) 2g

14. If the angular speed of the earth is doubled, the value of acceleration due to gravity (g) at the north pole
    (*a*) Doubles (*b*) Becomes half
    (*c*) Remains same (*d*) Becomes zero

15. Assume that the acceleration due to gravity on the surface of the moon is 0.2 times the acceleration due to gravity on the surface of the earth. If R_e is the maximum range of a projectile on the earth's surface, what is the maximum range on the surface of the moon for the same velocity of projection
    (*a*) 0.2 R_e (*b*) 2 R_e
    (*c*) 0.5 R_e (*d*) 5 R_e

16. What will be the acceleration due to gravity at height h if h >> R. Where R is radius of earth and g is acceleration due to gravity on the surface of earth

(a) $\frac{g}{\left(1+\frac{h}{R}\right)^2}$ (b) $g\left(1-\frac{2h}{R}\right)$

(c) $\frac{g}{\left(1-\frac{h}{R}\right)^2}$ (d) $g\left(1-\frac{h}{R}\right)$

17. Acceleration due to gravity on moon is 1/6 of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_e) and moon (ρ_m) is $\left(\frac{\rho_e}{\rho_m}\right)=\frac{5}{3}$ then radius of moon R_m in terms of R_e will be

(a) $\frac{5}{18}R_e$ (b) $\frac{1}{6}R_e$

(c) $\frac{3}{18}R_e$ (d) $\frac{1}{2\sqrt{3}}R_e$

GRAVITATION POTENTIAL AND POTENTIAL ENERGY AND ESCAPE VELOCITY

18. A body of mass m rises to height h = R/5 from the earth's surface, where R is earth's radius. If g is acceleration due to gravity at earth's surface, the increase in potential energy is

(a) mgh (b) $\frac{4}{5}$mgh

(c) $\frac{5}{6}$mgh (d) $\frac{6}{7}$mgh

19. In a gravitational field, at a point where the gravitational potential is zero
(a) The gravitational field is necessarily zero
(b) The gravitational field is not necessarily zero
(c) Nothing can be said definitely about the gravitational field
(d) None of these

20. The mass of the earth is 6.00×10^{24} kg and that of the moon is 7.40×10^{22} kg . The constant of gravitation $G = 6.67 \times 10^{-11}$ N–m²/kg². The potential energy of the system is -7.79×10^{28} joules . The mean distance between the earth and moon is
(a) 3.80×10^8 metres (b) 3.37×10^6 metres
(c) 7.60×10^4 metres (d) 1.90×10^2 metres

21. There are two bodies of masses 100 kg and 10000 kg separated by a distance 1 m. At what distance from the smaller body, the intensity of gravitational field will be zero?

(a) $\frac{1}{9}$m (b) $\frac{1}{10}$m

(c) $\frac{1}{11}$m (d) $\frac{10}{11}$m

22. The gravitational potential energy of a body of mass 'm' at the earth's surface $-mgR_e$. Its gravitational potential energy at a height R_e from the earth's surface will be (Here R_e is the radius of the earth)

(a) $2mgR_e$ (b) $2mgR_e$

(c) $\frac{1}{2}mgR_e$ (d) $-\frac{1}{2}mgR_e$

23. A body is projected vertically upwards from the surface of a planet of radius R with a velocity equal to half the escape velocity for that planet. The maximum height attained by the body is
(a) R/3 (b) R/2
(c) R/4 (d) R/5

24. Energy required to move a body of mass m from an orbit of radius 2R to 3R is
(a) GMm/12R² (b) GMm/3R²
(c) GMm/8R (d) GMm/6R

25. The escape velocity from the earth is about 11 km/second. The escape velocity from a planet having twice the radius and the same mean density as the earth, is
(a) 22 km/sec (b) 11 km/sec
(c) 5.5 km/sec (d) 15.5 km/sec

26. A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energy is
(a) Positive
(b) Negative
(c) Zero
(d) May be positive or negative depending upon its initial velocity

27. Four particles each of mass M, are located at the vertices of a square with side L. The gravitational potential due to this at the centre of the square is

(a) $-\sqrt{32}\frac{GM}{L}$ (b) $-\sqrt{64}\frac{GM}{L^2}$

(c) Zero (d) $\sqrt{32}\frac{GM}{L}$

SATELLITE MOTION

28. The escape velocity of a projectile from the earth is approximately
(a) 11.2 m/sec (b) 112 km/sec
(c) 11.2 km/sec (d) 11200 km/sec

29. If the radius of a planet is R and its density is ρ, the escape velocity from its surface will be

(a) $v_e \propto \rho R$ (b) $v_e \propto \sqrt{\rho}R$

(c) $v_e \propto \frac{\sqrt{\rho}}{R}$ (d) $v_e \propto \frac{1}{\sqrt{\rho R}}$

30. Which is constant for a satellite in orbit?
(a) Velocity (b) Angular momentum
(c) Potential energy (d) Acceleration

31. Select the correct statement from the following
(a) The orbital velocity of a satellite increases with the radius of the orbit
(b) Escape velocity of a particle from the surface of the earth depends on the speed with which it is fired
(c) The time period of a satellite does not depend on the radius of the orbit
(d) The orbital velocity is inversely proportional to the square root of the radius of the orbit

32. The ratio of the K.E. required to be given to the satellite to escape earth's gravitational field to the K.E. required to be given so that the satellite moves in a circular orbit just above earth atmosphere is
(a) One (b) Two
(c) Half (d) Infinity

33. For a satellite escape velocity is 11 km/s. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be
(a) 11 km/s (b) $11\sqrt{3}$ km/s
(c) $\frac{11}{\sqrt{3}}$ km/s (d) 33 km/s

34. Which of the following statements is correct in respect of a geostationary satellite?
(a) It moves in a plane containing the Greenwich meridian
(b) It moves in a plane perpendicular to the celestial equatorial plane
(c) Its height above the earth's surface is about the same as the radius of the earth
(d) Its height above the earth's surface is about six times the radius of the earth

35. An earth satellite S has an orbit radius which is 4 times that of a communication satellite C. The period of revolution of S is
(a) 4 days (b) 8 days
(c) 16 days (d) 32 days

36. If v_e and v_0 represent the escape velocity and orbital velocity of a satellite corresponding to a circular orbit of radius R, then
(a) $v_e = v_0$
(b) $\sqrt{2}v_0 = v_e$
(c) $v_e = v_0/\sqrt{2}$
(d) v_e and v_0 are not related

KEPLER'S LAW

37. A satellite which is geostationary in a particular orbit is taken to another orbit. Its distance from the centre of earth in new orbit is 2 times that of the earlier orbit. The time period in the second orbit is
(a) 4.8 hours (b) $48\sqrt{2}$ hours
(c) 24 hours (d) $24\sqrt{2}$ hours

38. Kepler discovered
(a) Laws of motion
(b) Laws of rotational motion
(c) Laws of planetary motion
(d) Laws of curvilinear motion

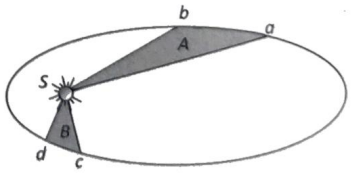
39. The period of a satellite in a circular orbit of radius R is T, the period of another satellite in a circular orbit of radius 4R is:
(a) 4T (b) T/4
(c) 8T (d) T/8

40. Orbit of a planet around a star is
(a) A circle (b) An ellipse
(c) A parabola (d) A straight line

41. Two planets move around the sun. The periodic times and the mean radii of the orbits are T_1, T_2 and r_1, r_2 respectively. The ratio T_1/T_2 is equal to
(a) $(r_1/r_2)^{1/2}$ (b) r_1/r_2
(c) $(r_1/r_2)^2$ (d) $(r_1/r_2)^{3/2}$

42. A satellite of mass m is circulating around the earth with constant angular velocity. If radius of the orbit is R_0 and mass of the earth M, the angular momentum about the centre of the earth is
(a) $m\sqrt{GMR_0}$ (b) $M\sqrt{GMR_0}$
(c) $m\sqrt{\frac{GM}{R_0}}$ (d) $M\sqrt{\frac{GM}{R_0}}$

43. The figure shows the motion of a planet around the sun in an elliptical orbit with sun at the focus. The shaded areas A and B are also shown in the figure which can be assumed to be equal. If t_1 and t_2 represent the time for the planet to move from a to b and d to c respectively, then



(a) $t_1 < t_2$ (b) $t_1 > t_2$
(c) $t_1 = t_2$ (d) $t_1 \leq t_2$

44. The distance of a planet from the sun is 5 times the distance between the earth and the sun. The time period of the planet is
(a) $5^{3/2}$ years (b) $5^{2/3}$ years
(c) $5^{1/3}$ years (d) $5^{1/2}$ years

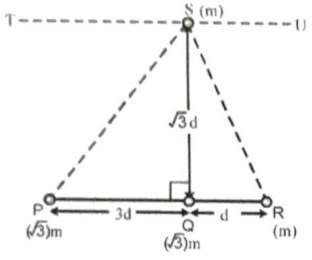
Learning Plus

1. If the distance between sun and earth is made 3 times of the present value then gravitational force between them will become :

(*a*) 9 times (*b*) $\frac{1}{9}$ times

(*c*) $\frac{1}{3}$ times (*d*) 3 times

2. Three particles P, Q and R are placed as per given figure. Masses of P, Q and R are $\sqrt{3}$ m, $\sqrt{3}$ m and m respectively. The gravitational force on a fourth particle 'S' of mass m is equal to



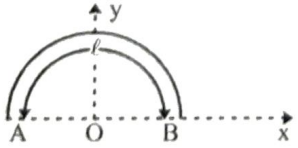
(*a*) $\frac{\sqrt{3}GM^2}{2d^2}$ in ST direction only

(*b*) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}Gm^2}{2d^2}$ in SU direction

(*c*) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction only

(*d*) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}Gm^2}{2d^2}$ in ST direction

3. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass m and length ℓ as shown in the figure. is:



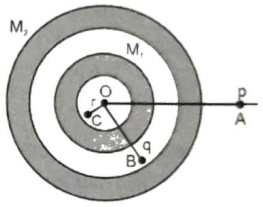
(*a*) $\frac{Gm}{\ell^2}$ along +x axis

(*b*) $\frac{Gm}{\pi\ell^2}$ along +y axis

(*c*) $\frac{2\pi Gm}{\ell^2}$ along +x axis

(*d*) $\frac{2\pi Gm}{\ell^2}$ along +y axis

4. Two concentric shells of uniform density of mass M_1 and M_2 are situated as shown in the figure. The forces experienced by a particle of mass m when placed at positions A, B and C respectively are (given OA = p, OB = q and OC = r).



(*a*) 0, $G\frac{M_1m}{q^2}$ and $G\frac{(M_1+m_2)m}{p^2}$

(*b*) $G\frac{(M_1+M_2)m}{p^2}$, $G\frac{(M_1+M_2)m}{q^2}$ and $G\frac{M_1m}{r^2}$

(*c*) $G\frac{M_1m}{q^2}$, $\frac{G(M_1+M_2)m}{p^2}$, $G\frac{M_1m}{q^2}$ and 0

(*d*) $\frac{G(M_1+M_2)m}{p^2}$, $G\frac{M_1m}{q^2}$ and 0

5. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is

(*a*) $4\pi G/3gR$ (*b*) $3\pi R/4gG$

(*c*) $3g/4\pi RG$ (*d*) $\pi Rg/12G$

6. The height above surface of earth where the value of gravitational acceleration is one fourth of that at surface, will be
(a) $R_e/4$ (b) $R_e/2$
(c) $3R_e/4$ (d) R_e

7. The decrease in the value of g on going to a height R/2 above the earth's surface will be -
(a) g/2 (b) $\frac{3g}{4}$
(c) $\frac{4g}{9}$ (d) $\frac{g}{3}$

8. If the rotational motion of earth increases, then the weight of the body -
(a) will remain same (b) will increase
(c) will decrease (d) none of these

9. If the acceleration due to gravity inside the earth is to be kept constant, then the relation between the density d and the distance r from the centre of earth will be -
(a) $d \propto r$ (b) $d \propto r^{1/2}$
(c) $d \propto 1/r$ (d) $d \propto \frac{1}{r^2}$

10. If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same?
(a) 1/25 (b) 1/5
(c) $1/\sqrt{5}$ (d) 5

11. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is two seconds pendulum on earth?
(a) $\sqrt{2}$ second (b) $2\sqrt{2}$ seconds
(c) $\frac{1}{\sqrt{2}}$ second (d) $\frac{1}{2\sqrt{2}}$ second

12. If a body is carried from surface of earth to moon, then
(a) the weight of a body will continuously increase
(b) the mass of a body will continuously increase
(c) the weight of a body will decrease first, become zero and then increase,
(d) the mass of a body will decrease first, become zero and then increase.

13. Let gravitation field in a space be given as $E = -(k/r)$. If the reference point is at distance d_i where potential is V_i then relation for potential is :
(a) $V = k \ln \frac{1}{V_i} + 0$ (b) $V = k \ln \frac{r}{d_i} + V_i$
(c) $V = \ln \frac{r}{d_i} + kV_i$ (d) $V = \ln \frac{r}{d_i} + \frac{V_i}{k}$

14. A very large number of particles of same mass m are kept at horizontal distances of 1m, 2m, 4m, 8m and so on from (0,0) point. The total gravitational potential at this point is:
(a) $-8Gm$ (b) $-3Gm$
(c) $-4Gm$ (d) $-2Gm$

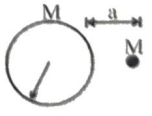
15. A body starts from rest at a point, distance R_0 from the centre of the earth of mass M, radius R. The velocity acquired by the body when it reaches the surface of the earth will be
(a) $GM\left(\frac{1}{R} - \frac{1}{R_0}\right)$ (b) $2GM\left(\frac{1}{R} - \frac{1}{R_0}\right)$
(c) $\sqrt{2GM\left(\frac{1}{R} - \frac{1}{R_0}\right)}$ (d) $2GM\sqrt{\left(\frac{1}{R} - \frac{1}{R_0}\right)}$

16. Three equal masses each of mass 'm' are placed at the three-corners of an equilateral triangle of side 'a'. If a fourth particle of equal mass is placed at the centre of triangle, then net force acting on it, is equal to :
(a) $\frac{Gm^2}{a^2}$ (b) $\frac{4Gm^2}{3a^2}$
(c) $\frac{3Gm^2}{a^2}$ (d) zero

17. If above given three particles system of equilateral triangle side a is to be changed to side of 2a, then work done on the system is equal to :
(a) $\frac{3Gm^2}{a}$ (b) $\frac{3Gm^2}{2a}$
(c) $\frac{4Gm^2}{3a}$ (d) $\frac{Gm^2}{a}$

18. In the above given three particle system, if two particles are kept fixed and third particle is released. Then speed of the particle when it reaches to the mid-point of the side connecting other two masses:
(a) $\sqrt{\frac{2Gm}{a}}$ (b) $2\sqrt{\frac{Gm}{a}}$
(c) $\sqrt{\frac{Gm}{a}}$ (d) $\sqrt{\frac{Gm}{2a}}$

19. A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a.



(a) Gravitational field and potential both are zero at centre of the shell
(b) Gravitational field is zero not only inside the shell but at a point outside the shell also
(c) Inside the shell, gravitational field alone is zero
(d) Neither gravitational field nor gravitational potential is zero inside the shell

20. If the kinetic energy of a satellite orbiting around the earth is doubled then -
(*a*) the satellite will escape into the space
(*b*) the satellite will fall down on the earth
(*c*) radius of its orbit will be doubled
(*d*) radius of its orbit will become half

21. Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity $\frac{v_A}{v_B}$ is

(*a*) 2 (*b*) $\sqrt{2}$
(*c*) $1/\sqrt{2}$ (*d*) 1/2

22. A projectile is fired from the surface of earth of radius R with a speed kv_e in radially outward direction (where v_e is the escape velocity and $k < 1$). Neglecting air resistance, the maximum distance of rise from centre of earth is

(*a*) $\frac{R}{k^2+1}$ (*b*) $k^2 R$
(*c*) $\frac{R}{1-k^2}$ (*d*) kR

23. Two different masses are dropped from same heights, then just before these strike the ground, the following is same:
(*a*) kinetic energy (*b*) potential energy
(*c*) linear momentum (*d*) Acceleration

24. Work done in taking a body of mass m to a height nR above surface of earth will be : (R = radius of earth)
(*a*) mgnR (*b*) mgR (n/n + 1)
(*c*) $mgR\frac{(n+1)}{n}$ (*d*) $\frac{mgR}{n(n+1)}$

25. The potential energy of a body of mass 3kg on the surface of a planet is 54 joule. The escape velocity will be -
(*a*) 18m/s (*b*) 162 m/s
(*c*) 36 m/s (*d*) 6 m/s

26. A body of mass m is situated at a distance $4R_e$ above the earth's surface, where R_e is the radius of earth. How much minimum energy be given to the body so that it may escape-
(*a*) mgR_e (*b*) $2mgR_e$
(*c*) $\frac{mgR_e}{5}$ (*d*) $\frac{mgR_e}{16}$

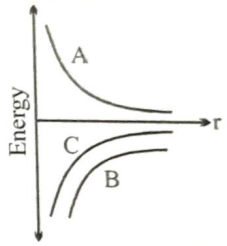
27. Periodic-time of satellite revolving around the earth is - (ρ is density of earth)
(*a*) Proportional to $\frac{1}{\rho}$ (*b*) Proportional to $\frac{1}{\sqrt{\rho}}$
(*c*) Proportional ρ (*d*) does not depend on ρ

28. A satellite of the earth is revolving in circular orbit with a uniform velocity V. If the gravitational force suddenly disappears, the statellite will
(*a*) continue to move with the same velocity in the same orbit
(*b*) move tangentially to the original orbit with velocity V
(*c*) fall down with increasing velocity
(*d*) come to a stop somewhere in its original orbit

29. A satellite revolves in the geostationary orbit but in a direction east to west. The time interval between its successive passing about a point on the equator is
(*a*) 48 hrs (*b*) 24 hrs
(*c*) 12 hrs (*d*) never

30. Two point masses of mass 4m and m respectively separated by d distance are revolving under mutual force of attraction. Ratio of their kinetic energies will be
(*a*) 1 : 4 (*b*) 1 : 5
(*c*) 1 : 1 (*d*) 1 : 2

31. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



(*a*) A shows the kinetic energy, B the total energy and C the potential energy of the system
(*b*) C shows the total energy, B the kinetic energy and A the potential energy of the system
(*c*) C and A are kinetic and potential energies respectively and B is the total energy of the system
(*d*) A and B are kinetic and potential energies and C is the total energy of the system

32. A planet of mass m revolves around the sun of mass M in an elliptical orbit. The minimum and maximum distance of the planet from the sun are r_1 & r_2 respectively. If the minimum velocity of the planet is $\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$ then it's maximum velocity will be :

(*a*) $\sqrt{\frac{2GMr_2}{(r_1+r_2)r_1}}$ (*b*) $g\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$
(*c*) $\sqrt{\frac{2Gmr_2}{(r_1+r_2)r_1}}$ (*d*) $\sqrt{\frac{2GM}{r_1+r_2}}$

33. A planet has mass 1/10 of that of earth, while radius is 1/3 that of earth. If a person can throw a stone on earth surface to a height of 90m, then he will be able to throw the stone on that planet to a height

(*a*) 90m (*b*) 40 m

(*c*) 100 m (*d*) 45 m

34. The ratio of distances of satellites A and B from the centre of the earth is 1.4 : 1, then the ratio of energies of satellites B and A will be –

(*a*) 1.4 : 1 (*b*) 2 : 1

(*c*) 1 : 3 (*d*) 4 : 1

35. A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the center is

(*a*) $\frac{r}{2}$ (*b*) $\frac{r}{2\sqrt{2}}$

(*c*) $\frac{r}{(4)^{1/3}}$ (*d*) $\frac{r}{(2)^{1/3}}$

36. A body is dropped by a satellite in its geostationary orbit-

(*a*) it will burn on entering into the atmosphere

(*b*) it will remain in the same place with respect to the earth

(*c*) it will reach the earth in 24 hours

(*d*) it will perform uncertain motion

37. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is υ. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is

(*a*) $\frac{3}{2}\upsilon$ (*b*) $\sqrt{\frac{3}{2}}\upsilon$ (*c*) $\upsilon\sqrt{\frac{2}{3}}$ (*d*) $\upsilon\frac{2}{3}$

38. When a satellite moves around the earth in a certain orbit, the quantity which remains constant is

(*a*) angular velocity (*b*) kinetic energy

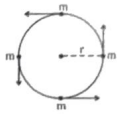
(*c*) areal velocity (*d*) potential energy

39. A planet of mass m is in an elliptical orbit about the sun ($m << M_{sun}$) with an orbital period T. If A be the area of orbit, then its angular momentum would be

(*a*) $\frac{2mA}{T}$ (*b*) mAT

(*c*) $\frac{mA}{2T}$ (*d*) 2mAT

40. Four similar particles of mass m are orbiting in a circle of radius r in the same direction and same speed because of their mutual gravitational attractive force as shown in the figure. Velocity of a particle is given by



(*a*) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ (*b*) $\sqrt[3]{\frac{Gm}{r}}$

(*c*) $\sqrt{\frac{Gm}{r}\left(1+2\sqrt{2}\right)}$ (*d*) zero

Advanced Level Multiconcept Questions

JEE-ADVANCED MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. Assuming the earth to be a sphere of uniform density the acceleration due to gravity

(*a*) at a point outside the earth is inversely proportional to the square of its distance from the center

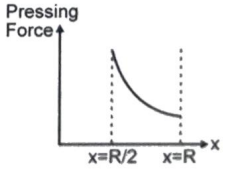
(*b*) at a point outside the earth is inversely proportional to its distance from the centre

(*c*) at a point inside is zero

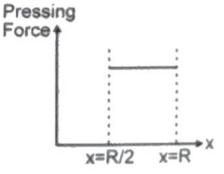
(*d*) at a point inside is proportional to its distance from the centre

2. A tunnel is dug along a chord of the earth at a perpendicular distance R/2 from the earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall and the acceleration of the particle varies with x (distance of the particle from the centre) according to :

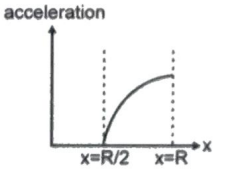
(*a*)



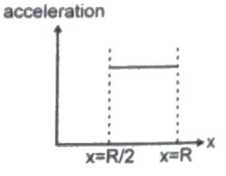
(*b*)



(*c*)

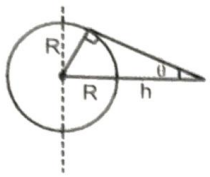


(*d*)



3. In side a hollow spherical shell
   (*a*) everywhere gravitational potential is zero
   (*b*) everywhere gravitational field is zero
   (*c*) everywhere gravitational potential is same
   (*d*) everywhere gravitational field is same

4. A geostationary satellite is at a height h above the surface of earth. If earth radius is R



   (*a*) The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1}(R/R+h)$
   (*b*) The maximum colatitudes on earth upto which the satellite can be used for communication is $\sin^{-1}(R/R+h)$
   (*c*) The area on earth escaped from this satellite is given as $2\pi R^2(1+\sin\theta)$
   (*d*) The area on earth escaped from this satellite is given as $2\pi R^2(1+\cos\theta)$

5. Inside an isolated uniform spherical shell :
   (*a*) The gravitation potential is not zero
   (*b*) The gravitational field is not zero
   (*c*) The gravitational potential is same everywhere
   (*d*) The gravitational field is same everywhere

6. A satellite close to the earth is in orbit above the equator with a period of revolution of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time
   (*a*) 1.5 hours
   (*b*) 1.6 hours if it is rotating from west to east
   (*c*) 24/17 hours if it is rotating from east to west
   (*d*) 24/17 hours if it is rotating from west to east

7. In case of earth :
   (*a*) field is zero, both at centre and infinity
   (*b*) potential is zero, both at centre and infinity
   (*c*) potential is same, both at centre and infinity but not zero
   (*d*) potential is minimum at the centre

8. A communications Earth satellite
   (*a*) goes round the earth from east to west
   (*b*) can be in the equatorial plane only
   (*c*) can be vertically above any place on the earth
   (*d*) goes round the earth from west to east

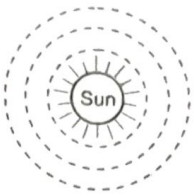
9. For a satellite to orbit around the earth, which of the following must be true ?
   (*a*) It must be above the equator at some time
   (*b*) It cannot pass over the poles at any time
   (*c*) Its height above the surface cannot exceed 36,000 km
   (*d*) Its period of rotation must be $> 2\pi\sqrt{R/g}$ where R is radius of earth

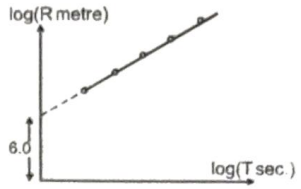
Comprehension – 1 (No. 10 to 12)

Many planets are revolving around the fixed sun, in circular orbits of different radius (R) and different time period (T). To estimate the mass of the sun, the orbital radius (R) and time period (T) of planets were noted. Then $\log_{10}$ T v/s $\log_{10}$ R curve was plotted.

The curve was found to be approximately straight line (as shown in figure) having y intercept = 6.0 (Neglect the gravitational interaction among the planets

[Take $G = \frac{20}{3}\times 10^{-11}$ in MKS, $\pi^2 = 10$]

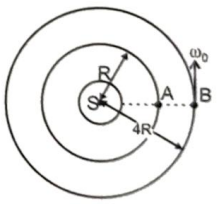




10. The slope of the line should be :
   (*a*) 1 (*b*) $\frac{3}{2}$
   (*c*) $\frac{2}{3}$ (*d*) $\frac{19}{4}$

11. Estimate the mass of the sun :
   (*a*) 6×10^{29} kg (*b*) 5×10^{20} kg
   (*c*) 8×10^{25} kg (*d*) 3×10^{35} kg

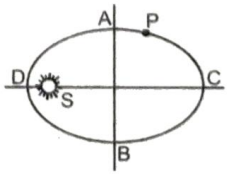
12. Two planets A and B, having orbital radius R and 4R are initially at the closest position and rotating in the same direction. If angular velocity of planet B is ω_0, then after how much time will both the planets be again in the closest position? (Neglect the interaction between planets).



   (*a*) $\frac{2\pi}{7\omega_0}$ (*b*) $\frac{2\pi}{9\omega_0}$
   (*c*) $\frac{2\pi}{\omega_0}$ (*d*) $\frac{2\pi}{5\omega_0}$

Comprehension – 2 (No 13 and 14)

Figure shows the orbit of a planet P round the sun S. AB and CD are the minor and major axes of the ellipse.



13. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then
(a) $t_1 = t_2$ (b) $t_1 > t_2$
(c) $t_1 < t_2$ (d) nothing can be concluded

14. If U is the potential energy and K kinetic energy then $|U| > |K|$ at
(a) Only D (b) Only C
(c) Both D & C (d) neither D nor C

15. A particle is taken to a distance r (> R) from centre of the earth. R is radius of the earth. It is given velocity V which is perpendicular to $\vec{r}$. With the given values of V in Column-I you have to match the values of total energy of particle in Column-II and the resultant path of particle in Column-III. Here 'G' is the universal gravitational constant and 'M' is the mass of the earth.

Column-I (Velocity)	Column-II (Total energy)	Column-III (Path)
(a) $V = \sqrt{GM/r}$	(p) Negative	(t) Elliptical
(b) $V = \sqrt{2GM/r}$	(q) Positive	(u) Parabolic
(c) $V > \sqrt{2GM/r}$	(r) Zero	(v) Hyperbolic
(d) $\sqrt{GM/r} < V < \sqrt{2GM/r}$	(s) Infinite	(w) Circular

16. Let V and E denote the gravitational potential and gravitational field respectively at a point due to certain uniform mass distribution described in four different situations of Column-I. Assume the gravitational potential at infinity to be zero. The value of E and V are given in Column-II. Match the statement in Column-I with results in Column-II.

Column-I	Column-II
(a) At centre of thin spherical shell	(p) $E = 0$
(b) At centre of solid sphere	(q) $E \neq 0$
(c) A solid sphere has a non-concentric spherical cavity At the centre of the spherical cavity	(r) $V \neq 0$
(d) At centre of line joining two point masses of equal magnitude	(s) $V = 0$

NUMERICAL VALUE BASED

17. A satellite of earth is in near earth circular orbit. What is its speed (in km/s). Round off to nearest integer?

18. Gravitational potential difference between a point on surface of planet and another point 10 m above is 4J/kg. Considering gravitaional field to be uniform , how much work is done by an external agent in moving a mass of 2.0 kg from the surface to a point 5.0 m above the surface slowly.

19. You are at a distance of $R = 1.5 \times 10^6$ m from the centre of an unknown planet. You notice that if you throw a ball horizontally it goes completely around the planet hitting you in the back 90,000 seconds later with exactly the same speed that you originally threw it. If the length of semi major axis of ball is 2R, what is the mass of the planet. Express in form $a \times 10^b$ kg and fill a + b in OMR sheet. [Take: $G = \frac{20}{3} \times 10^{-11}$ Nm²/kg², $\pi^2 = 10$]

20. Imagine a frictionless tunnel along a chord of earth having length equal to radius of earth R. Particle is thrown from surface of earth inside it along its length at a velocity $\sqrt{\frac{3gR}{4}}$ where g is acceleration due to gravity on surface of earth. Find the time (in sec) taken by particle to cross the tunnel.
Take $g = 10$ m/s², $\pi = 3.14$ and $R = 6400$ km.

21. A spherical planet has no atmosphere and consists of pure gold. Find the minimum orbital period T (in sec.) for a satellite circling the planet.
Take density of gold as $5\pi \times 10^3$ kg/m³ and $G = \frac{20}{3} \times 10^{-11} Nm^2/kg^2$

22. Two particles of mass 'm' and 3m are initially at rest an infinite distance apart. Both the particles start moving due to gravitational attraction. At any instant their relative velocity of approach is $\sqrt{\frac{\eta Gm}{d}}$, where 'd' is their separation at that instant. Find η.

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R ; $h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to: (Neglect the effect of atmosphere.) **[JEE Main-2016, JEE Main-2019 (January)]**

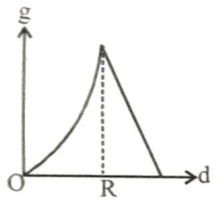
(*a*) $\sqrt{2gR}$

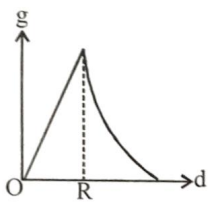
(*b*) $\sqrt{gR}$

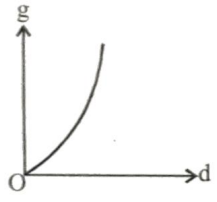
(*c*) $\sqrt{gR/2}$

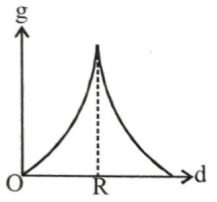
(*d*) $\sqrt{gR}(\sqrt{2}-1)$

2. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius): **[JEE Main-2017]**

(*a*)


(*b*)


(*c*)


(*d*)


3. A satellite is moving with a constant speed υ in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is: **[JEE Main-2019 (January)]**

(*a*) $2m\upsilon^2$ (*b*) $m\upsilon^2$

(*c*) $\frac{1}{2}m\upsilon^2$ (*d*) $\frac{3}{2}m\upsilon^2$

4. A heavy ball of mass M is suspended from the ceiling of car by a light strong of mass m ($m << M$). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g is closest to : **[JEE Main-2019 (January)]**

(*a*) $\frac{g}{30}$ (*b*) $\frac{g}{5}$

(*c*) $\frac{g}{10}$ (*d*) $\frac{g}{20}$

5. If the angular momentum of a planet of mass m, moving a round the Sun in a circular orbit is L, about the center of the Sun, its a real velocity is : **[JEE Main-2019 (January)]**

(*a*) $\frac{L}{m}$ (*b*) $\frac{4L}{m}$

(*c*) $\frac{L}{2m}$ (*d*) $\frac{2L}{m}$

6. Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B is : **[JEE Main-2019 (January)]**

(*a*) $\frac{1}{2}$ (*b*) 1

(*c*) 2 (*d*) $\sqrt{\frac{1}{2}}$

7. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be: **[JEE Main-2019 (January)]**

(*a*) such that it escapes to infinity
(*b*) in an elliptical orbit
(*c*) in the same circular orbit of radius R
(*d*) in a circular orbit of a different radius

8. A straight rod of length L extends from x = a to x = L + a. The gravitational force it exerts on a point mass 'm' at x = 0, if the mass per unit length of the rod is $A + Bx^2$, is given by: [*JEE Main-2019 (January)*]

(*a*) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

(*b*) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

(*c*) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(*d*) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

9. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O.A meteorite passes through O moving perpendicular to the star's rotation plane. In order ot escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is: (Take Gravitational constant $G = 6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$) [*JEE Main-2019 (January)*]

(*a*) 2.4×10^4 m/s (*b*) 1.4×10^5 m/s

(*c*) 3.8×10^4 m/s (*d*) 2.8×10^5 m/s

10. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T: [*JEE Main-2019 (April)*]

(*a*) T/R^2 is a constant (*b*) TR is a constant

(*c*) T^2/R^3 is a constant (*d*) T\R is a constant

11. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square? [*JEE Main-2019 (April)*]



(*a*) $1.21\sqrt{\frac{GM}{a}}$ (*b*) $1.41\sqrt{\frac{GM}{a}}$

(*c*) $1.16\sqrt{\frac{GM}{a}}$ (*d*) $1.35\sqrt{\frac{GM}{a}}$

12. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet ?
[**Given :** Mass of planet = 8×10^{22} kg ;
Radius of planet = 2×10^6 m,
Gravitational constant G = 6.67×10^{-11} Nm^2/kg^2] [*JEE Main-2019 (April)*]

(*a*) 9 (*b*) 11

(*c*) 13 (*d*) 17

13. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon ? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon: [*JEE Main-2019 (April)*]

(*a*) $\frac{E}{4}$ (*b*) $\frac{E}{16}$

(*c*) $\frac{E}{32}$ (*d*) $\frac{E}{64}$

14. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be: [*JEE Main-2019 (April)*]

(*a*) $\frac{2GM}{9a^2}$ (*b*) $\frac{GM}{3a^2}$

(*c*) $\frac{GM}{9a^2}$ (*d*) $\frac{2GM}{3a^2}$

15. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density) [*JEE Main-2019 (April)*]

(*a*) $\frac{R}{3}$ (*b*) $\frac{R}{2}$

(*c*) $\frac{R}{4}$ (*d*) $\frac{R}{9}$

16. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to : (Radius of earth = 6.4×10^6 m) [*JEE Main-2019 (April)*]

(*a*) 1.6×10^6 m (*b*) 6.4×10^6 m

(*c*) 9.0×10^6 m (*d*) 2.6×10^6 m

17. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R(R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):

[*JEE Main-2020 (January)*]

(*a*) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$ (*b*) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$

(*c*) $\frac{3m}{8}\left(u+\sqrt{\frac{5GM}{6R}}\right)^2$ (*d*) $\frac{m}{2}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$

18. A box weights 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ ms^{-2}$ at the north pole and the radius of the earth = 6400 km)

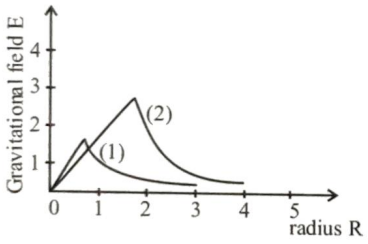
[*JEE Main-2020 (January)*]

(*a*) 194.66 N (*b*) 195.66 N

(*c*) 195.32 N (*d*) 194.32 N

19. Consider two solid spheres of radii $R_1 = 1m$, $R_2 = 2m$ and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is:

[*JEE Main-2020 (January)*]



(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{1}{2}$ (*d*) $\frac{1}{6}$

20. An asteroid is moving directly towards the centre of the earth. when at a distance of 10R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s^{-1})? Give your answer to the nearest integer in kilometer/s.

[*JEE Main-2020 (January)*]

21. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are V_A and V_B, respectively, then $V_A/V_B = n/4$. The value of n is :

[*JEE Main-2020 (January)*]

(*a*) 3 (*b*) 2

(*c*) 1 (*d*) 4

22. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected)

[*JEE Main-2020 (September)*]

(*a*) $\frac{\sqrt{5}R-R}{2}$ (*b*) $\frac{\sqrt{3}R-R}{2}$

(*c*) $\frac{R}{2}$ (*d*) $\frac{\sqrt{5}}{2}R-R$

23. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to

[*JEE Main-2020 (September)*]

(*a*) 1.2×10^{-28} kg (*b*) 9.1×10^{-31} kg

(*c*) 4.8×10^{-27} kg (*d*) 9.7×10^{-28} kg

24. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from it s centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as

[*JEE Main-2020 (September)*]

(*a*) $T^2 \propto \frac{1}{R^3}$ (*b*) $T^2 \propto R$

(*c*) $T \propto R$ (*d*) $T^2 \propto R^3$

25. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1-\frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at

[*JEE Main-2020 (September)*]

(*a*) $r = \frac{1}{\sqrt{3}}R$ (*b*) $r = \sqrt{\frac{3}{4}}R$

(*c*) $r = \sqrt{\frac{5}{9}}R$ (*c*) $r = R$

26. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is

[*JEE Main-2020 (September)*]

(*a*) $3R_e$ (*b*) $4R_e$

(*c*) $2.5R_e$ (*d*) $2R_e$

27. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is

[*JEE Main-2020 (September)*]

(*a*) 1 (*b*) $\sqrt{2}$

(*c*) 2 (*d*) $\frac{1}{\sqrt{2}}$

28. On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is

[*JEE Main-2020 (September)*]

(*a*) $A(x^2+a^2)^{3/2}$ (*b*) $\frac{A}{(x^2+a^2)^{3/2}}$

(*c*) $\frac{A}{(x^2+a^2)^{1/2}}$ (*d*) $A(x^2+a^2)^{1/2}$

29. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is (h << R, where R is the radius of the earth)

[*JEE Main-2020 (September)*]

(*a*) $\frac{R^2\omega^2}{2g}$ (*b*) $\frac{R^2\omega^2}{g}$

(*c*) $\frac{R^2\omega^2}{8g}$ (*d*) $\frac{R^2\omega^2}{4g}$

30. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals

[*JEE Main-2020 (September)*]

(*a*) $\frac{5}{9}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{7}{9}$ (*d*) $\frac{4}{9}$

31. Two planets have masses M and 16M and their radii are a and 2a, respectively. The separation between the centres of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is

[*JEE Main-2020 (September)*]

(*a*) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$ (*b*) $\sqrt{\frac{GM^2}{ma}}$

(*c*) $2\sqrt{\frac{GM}{ma}}$ (*d*) $4\sqrt{\frac{GM}{a}}$

32. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is $\frac{1}{6}$ times of that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is

[*JEE Main-2020 (September)*]

(*a*) 1 : 6 (*b*) 1 : 2

(*c*) 3 : 4 (*d*) 1 : 3

33. The radius in kilometre to which the present radius of earth ($R = 6400\,km$) to be compressed so that the escape velocity is increased 10 times is [*JEE Main-2021 (March)*]

34. If one wants to remove all the mass of the earth to infinity in order to break it up completely. [*JEE Main-2021 (March)*] The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is (Round off to the Nearest Integer)

(*M* is the mass of earth, *R* is the radius of earth, *G* is the gravitational constant)

JEE-ADVANCED PREVIOUS YEAR'S

1. Two spherical planets P and Q have the same unfirom density ρ, masses M_P and M_Q, with surface areas A and 4A, respectively. A spherical planet R also has unfirom density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V respectivley. Then [JEE-2012]

(a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$

(c) $V_R/V_P = 3$ (d) $V_P/V_Q = \frac{1}{2}$

2. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) : [JEE Advanced-2013]

(a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(d) The energy of the mass m remains constant.

3. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity on Earth is 10 ms^{-2}) [JEE Advanced-2014]

(a) 96 N (b) 108N

(c) 120N (d) 150N

4. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to : (Ignore the rotation and revolution of the Earth and the presence of any other planet) [JEE Advanced-2017]

(a) $v_s = 72$ km s^{-1} (b) $v_s = 22$ km s^{-1}

(c) $v_s = 42$ km s^{-1} (d) $v_s = 62$ km s^{-1}

5. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force betweent he satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. [JEE Advanced-2018]

List-I	List-II
P. $\frac{v_1}{v_2}$	1. $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	2. 1
R. $\frac{K_1}{K_2}$	3. 2
S. $\frac{T_1}{T_2}$	4. 8

(a) P $\rightarrow$ 4; Q $\rightarrow$ 2; R $\rightarrow$ 1; S $\rightarrow$ 3

(b) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 1

(c) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 1; S $\rightarrow$ 4

(d) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 4; S $\rightarrow$ 1

6. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force.
If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is :
[G is universal gravitational constant] [JEE Advanced-2019]

(a) $\frac{K}{\pi r^2 m^2 G}$ (b) $\frac{K}{6\pi r^2 m^2 G}$

(c) $\frac{3K}{\pi r^2 m^2 G}$ (d) $\frac{K}{2\pi r^2 m^2 G}$

7. The distance between two stars of masses $3M_S$ and $6M_S$ is 9 R. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period n T, where T is the period of Earth's revolution around the Sun. The value of n is [JEE Advanced-2021]

ANSWER KEY

Topicwise Questions

1. (*d*)	**2.** (*c*)	**3.** (*d*)	**4.** (*a*)	**5.** (*d*)	**6.** (*c*)	**7.** (*d*)	**8.** (*b*)	**9.** (*b*)	**10.** (*b*)
11. (*c*)	**12.** (*c*)	**13.** (*b*)	**14.** (*c*)	**15.** (*d*)	**16.** (*a*)	**17.** (*a*)	**18.** (*c*)	**19.** (*a*)	**20.** (*a*)
21. (*c*)	**22.** (*d*)	**23.** (*a*)	**24.** (*d*)	**25.** (*a*)	**26.** (*b*)	**27.** (*a*)	**28.** (*c*)	**29.** (*b*)	**30.** (*b*)
31. (*d*)	**32.** (*b*)	**33.** (*a*)	**34.** (*d*)	**35.** (*b*)	**36.** (*b*)	**37.** (*b*)	**38.** (*c*)	**39.** (*c*)	**40.** (*b*)
41. (*d*)	**42.** (*a*)	**43.** (*c*)	**44.** (*a*)						

Learning Plus

1. (*b*)	**2.** (*c*)	**3.** (*d*)	**4.** (*d*)	**5.** (*c*)	**6.** (*d*)	**7.** (*b*)	**8.** (*c*)	**9.** (*c*)	**10.** (*b*)
11. (*b*)	**12.** (*c*)	**13.** (*b*)	**14.** (*d*)	**15.** (*c*)	**16.** (*d*)	**17.** (*b*)	**18.** (*b*)	**19.** (*d*)	**20.** (*a*)
21. (*a*)	**22.** (*c*)	**23.** (*d*)	**24.** (*b*)	**25.** (*d*)	**26.** (*c*)	**27.** (*b*)	**28.** (*b*)	**29.** (*c*)	**30.** (*a*)
31. (*d*)	**32.** (*a*)	**33.** (*c*)	**34.** (*a*)	**35.** (*c*)	**36.** (*b*)	**37.** (*c*)	**38.** (*c*)	**39.** (*a*)	**40.** (*a*)

Advanced Level Multiconcept Questions

1. (*a, d*) **2.** (*b, c*) **3.** (*b, c, d*) **4.** (*a, c*) **5.** (*a, c, d*) **6.** (*b, c*) **7.** (*a, d*) **8.** (*b, d*) **9.** (*a, d*) **10.** (*c*)

11. (*a*) **12.** (*a*) **13.** (*b*) **14.** (*c*) **15.** (A-p,w ; B-r,u ; C-q,v ; D-p,t) **16.** (*a*) p, r (*b*) p, r (*c*) q, r (*d*) p, r

17. [0008] **18.** [0004] **19.** [0023] **20.** [837.33] **21.** [3000] **22.** [8]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*d*)	**2.** (*b*)	**3.** (*b*)	**4.** (*b*)	**5.** (*c*)	**6.** (*b*)	**7.** (*b*)	**8.** (*d*)	**9.** (*d*)	**10.** (*d*)
11. (*c*)	**12.** (*b*)	**13.** (*b*)	**14.** (*b*)	**15.** (*b*)	**16.** (*d*)	**17.** (*b*)	**18.** (*c*)	**19.** (*d*)	**20.** (16)
21. (*d*)	**22.** (*a*)	**23.** (*d*)	**24.** (*b*)	**25.** (*c*)	**26.** (*a*)	**27.** (*d*)	**28.** (*c*)	**29.** (*a*)	**30.** (*a*)
31. (*a*)	**32.** (*a*)	**33.** (64)	**34.** (3)						

JEE-ADVANCED
PREVIOUS YEAR'S

1. (*b,d*)	**2.** (*b*)	**3.** (*b*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** [9]

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9 MECHANICAL PROPERTIES OF SOLIDS

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ELASTICITY

If the modified shape is retained by the solid only so long as the forces act, and the original shape is regained when they cease to act, the solid is said to possess elasticity.

PLASTICITY

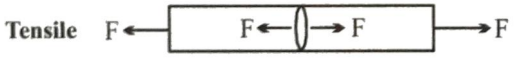
If the modified shape is retained after the forces cease to act, the solid is said to be plastic. The volume of matter, whether this matter be solid or fluid, can be altered by the application of suitable forces, and in most cases the original volume is regained when the forces cease to act ; hence, all matter possesses volume elasticity.

Any alteration produced in the shape or volume of matter is called a strain.

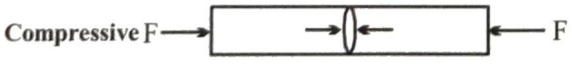
STRESS

Restoring force per unit area set up inside the body is called stress and is measured by the magnitude of force acting on unit area of the body in equilibrium. If F is the force applied and A is the area of cross section of the body, Magnitude of the stress = F/A. The SI unit of stress is N m^{-2} or pascal (Pa) and its dimensional formula is [$ML^{-1}T^{-2}$].

Stress (Tensile/Compressive)



$$\sigma = \frac{F}{A} \text{ as } F \propto A$$



Strain (Longitudinal)

Suppose we stretch a wire by applying tensile forces of magnitude F to each end. The length of the wire increases from L to L + ΔL. The fractional length change is called the strain; it is a dimensionless measure of the degree of deformation.

$$\text{Strain} = \frac{\Delta L}{L}$$

HOOKE'S LAW FOR TENSILE AND COMPRESSIVE FORCES

Hooke's Law

stress ∝ strain

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

Equation still says that the length change (ΔL) is proportional to the magnitude of the deforming forces (F). Stress and strain account for the effects of length and cross-sectional area ; the proportionality constant Y depends only on the inherent stiffness of the material from which the object is composed ; it is independent of the length and cross-sectional area.

Comparing equation $F = k\Delta L$ and $\frac{F}{A} = Y \frac{\Delta L}{L}$ $k = Y \frac{\Delta L}{L} A$.

Y is called the **elastic modulus or Young's modulus**, Y has the same units as those of stress (Pa or N/m^2) since strain is dimensionless.

Young's modulus can be through of as the inherent stiffness of a material ; it measures the resistance of the material to elongation or compression. Material that is flexible and stretches easily (for example, rubber) has a low Young's modulus. A rigid material (such as steel) has a high Young's modulus ; it takes a larger stress to produce the same strain.

Hooke's law holds up to a maximum stress called the proportional limit. For many materials, Young's modulus has the same value for tension and compression. Some composite materials, such as bone and concrete, have significantly different Young's moduli for tension and compression. The different properties of these two substances lead to different values of Young's modulus for tension and compression.

TRAIN YOUR BRAIN

Example 1: A light wire of length 4m is suspended to the ceiling by one of its ends. If its crossectional area is19.6 mm^2, what is its extension under a load of 10kg? Young's modulus of steel = 2×10^{11} Pa.

Sol. Given ; original length $\ell = 4$m ;
force $F = 10 \times 9.8 = 98$ N ;
and $Y = 2 \times 10^{11}$ Nm^{-2} ; $\ell = ?$
Using the relation,

$$\text{Young's modulus (Y)} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

We have

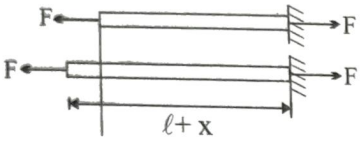
$$Y = \frac{F/A}{\ell/L} \Rightarrow \ell = \frac{FL}{YA}$$

$$\therefore \quad \ell = \frac{98 \times 4}{2 \times 10^{11} \times 19.6 \times 10^{-6}} = 1 \times 10^{-4} \text{ m}$$

$$= 0.1 \text{ mm}$$

ELASTIC POTENTIAL ENERGY

(where F is applied in equilibrium)



$$dw = Fdx \qquad \left(F = \frac{AY}{L}x\right)$$

$$= \frac{AY}{L} x\, dx$$

$$w = \frac{AY}{L} \int_0^{\ell} x\, dx$$

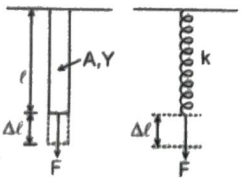
$$U = \frac{AY\ell^2}{2L} = \frac{1}{2}\left(\frac{y\ell}{L}\right)\left(\frac{\ell}{L}\right)(AL)$$

$$U = \frac{1}{2} \text{ (stress) (strain) (volume)}$$

ANALOGY OF ROD AS A SPRING

$$Y = \frac{\text{stress}}{\text{strain}} \Rightarrow Y = \frac{F\ell}{A\Delta\ell}$$

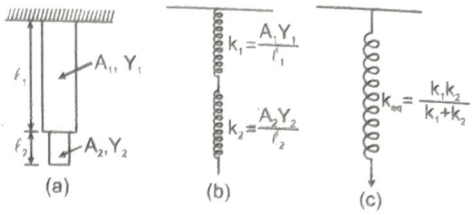
or $F = \frac{AY}{\ell}\Delta\ell$



$\frac{AY}{\ell}$ = constant, depends on type of material and geometry of rod.

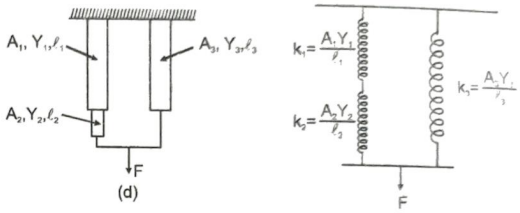
$F = k\Delta\ell$

where $k = \frac{AY}{\ell}$ = equivalent spring constant.



for the system of rods shown in figure (a), the replaced spring system is shown in figure (b) two spring in series]. Figure (c) represents equivalent spring system.

Figure (d) represents another combination of rods and their replaced spring system.



TRAIN YOUR BRAIN

Example 2: A wire having a length $l = 2$ m and cross sectional area $A = 5$ mm^2 is suspended at one of its ends from a ceiling. What will be its strain energy due to its own weight, if the density and Young's modulus of the material of the wire be $\rho = 9$g/cm^3 and $Y = 1.5 \times 10^{11}$ Nm^{-2}?

Sol. Consider an elemental length of the wire of length dx, at a distance x from the lower end.

Clearly, this length is acted upon by the external force equal to the weight of the portion of wire below it = xAρg. In equilibrium, the restoring force f = xAρg.

$$\therefore \text{ stress} = \frac{f}{A} = x\rho g.$$

Now, elastic potential energy stored in the elemental length will be

$$dU = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

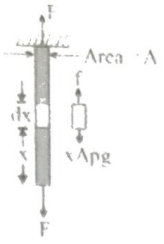
$$= \frac{1}{2} \times \frac{(\text{stress})^2}{Y} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{(x\rho g)^2}{Y} \times A dx = \frac{1}{2} \frac{\rho^2 g^2 A}{Y} x^2\, dx$$

$\therefore$ Total elastic potential energy $U = \int dU$

$$= \int_0^l \frac{1}{2} \frac{\rho^2 g^2 A}{Y} x^2 \, dx$$

$$= \frac{1}{6} \rho^2 g^2 \frac{Al^3}{Y}$$



Substituting the values,

$$U = \frac{1}{6} \times \frac{(9\times10^{-1})\,(9{\cdot}8)^2 \times 5 \times 10^{-6} \times 2^3}{1.5 \times 10^{11}}$$

$$= 3.841 \times 10^{-5}\ \text{J}$$

STRESS-STRAIN CURVE

The relation between the stress and the strain for a given material under tensile stress can be found experimentally. The applied force is gradually increased in steps and the change in length is noted. A graph is plotted between the stress and the strain produced. The stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads. From the graph, we can see that in the region between O to A, the curve is linear. In this region, Hooke's law is obeyed. The body regains its original dimensions when the applied force is removed. In this region, the solid behaves as an elastic body.

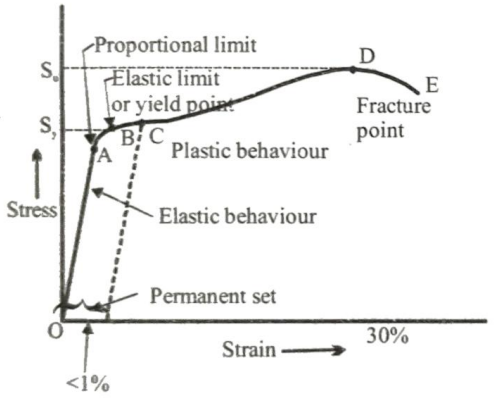


Figure: Stress-strain curve for a metal.

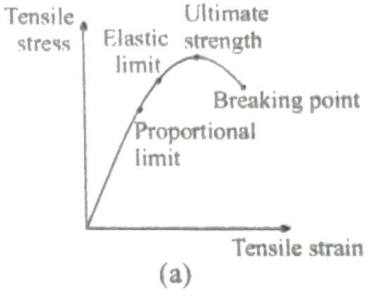
BEYOND HOOKE'S LAW

In the region from A to B, stress and strain are not proportional. Nevertheless, the body still returns to its original dimension when the load is removed. The point B in the curve is known as **yield point** (also known as **elastic limit**) and the corresponding stress is known as **yield strength** (S_y) of the material.

If the tensile or compressive stress exceeds the proportional limit, the strain is no longer proportional to the stress (figure). The solid still returns to its original length when the stress is removed as long as the stress does not exceed the elastic limit.

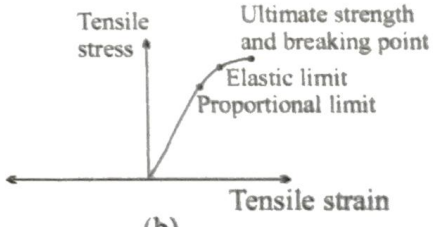
If the stress exceeds the elastic limit, the material is permanently deformed. For still larger stresses, the solid fractures when the stress reaches the breaking point. The maximum stress that can be withstood without breaking is called the ultimate strength. The ultimate strength can be different for compression and tension ; then we refer to the compressive strength or the tensile strength of the material. A ductile material continues to stretch beyond its ultimate tensile strength without breaking ; the stress then decreases from the ultimate strength (fig. (a)). Examples of ductile solids are relatively soft metals, such as gold, silver, copper, and lead. These metals can be pulled like taffy, becoming thinner and thinner until finally reaching the breaking point.

Ductile



(a)

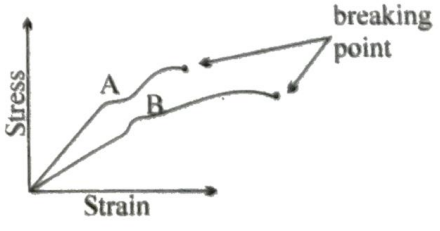
Brittle



(b)

TRAIN YOUR BRAIN

Example 3: Select the correct statement on the basis of the given graph :



(*a*) Young's modulus of A is greater but it is less ductile

(*b*) Young's modulus of A is greater and it is more ductile

(*c*) Young's modulus of A is less and it is less ductile

(*d*) Young's modulus of A is less but it is more ductile

Ans. (a)

Sol. $Y = \frac{\text{stress}}{\text{strain}}$ for same strain $(\text{stress})_A > (\text{stress})_B$

$Y_A > Y_B$

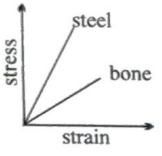
Breaking point of B > Breaking point of A

So, A is less ductile.

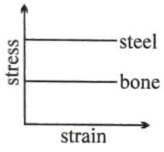
Example 4: Which of the following graphs of stress versus strain is consistent with the information presented in Table?

Material	Young's modulus $\times 10^{10}$ Pa	Bulk modulus $\times 10^{10}$ Pa	Shear modulus $\times 10^{10}$ Pa
Aluminum	7.0	7.5	2.5
Bone (tension)	1.6	0.8	8.0
Brass	9.0	6.0	3.5
Steel	20.0	16.0	7.5

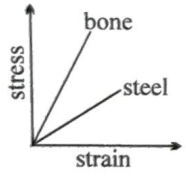
(*a*)



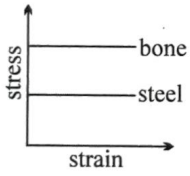
(*b*)



(*c*)



(*d*)

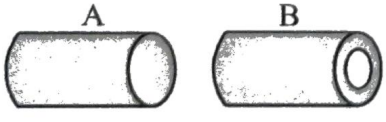


Ans. (a)

Sol. $Y_{steel} > Y_{bone}$

$$\left(\frac{\text{stress}}{\text{strain}}\right)_{steel} > \left(\frac{\text{stress}}{\text{strain}}\right)_{bone}$$

Example 5: The drawing shows two cylinders. They are identical in all respects, except one is hollow. In a setup like that in figure, identical forces are applied to the right end of each cylinder while the left end is fixed.



(*a*) The elongation of A is more than that of B

(*b*) The elongation of B is more than that of A

(*c*) The energy stored in B is more than that in A

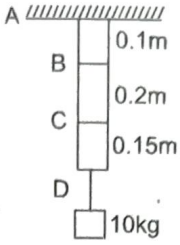
(*d*) The energy stored in A is more than that in B

Ans. (*a*, *c*)

Sol. $\frac{F}{A} = y\frac{\Delta\ell}{\ell} \Rightarrow \Delta\ell = \left(\frac{\ell F}{y}\right)\frac{1}{A}$

$$\Rightarrow U = \frac{1}{2}\left(\frac{Ay}{\ell}\right)\Delta\ell^2 = \frac{1}{2}\frac{Ay}{\ell}\left(\frac{\ell F}{r}\right)^2\frac{1}{A^2} = U \propto \frac{1}{A}$$

Example 6: Find out the shift in point B, C and D



$Y_{AB} = 2.5 \times 10^{10}$ N/m²

$Y_{BC} = 4 \times 10^{10}$ N/m²

$Y_{CD} = 1 \times 10^{10}$ N/m²

$A = 10^{-7}$ m²

Sol. $\Delta L_B = \Delta L_{AB} = \frac{FL}{AY} = \frac{MgL}{AY} = \frac{10\times10\times0.1}{10^{-7}\times2.5\times10^{10}}$

$= 4 \times 10^{-3}$ m $= 4$ mm

$$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4 \times 10^{-3} + \frac{100\times0.2}{10^{-7}\times4\times10^{10}}$$

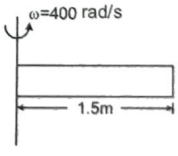
$= 4 \times 10^{-3} + 5 \times 10^{-3} = 9$ mm

$$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9 \times 10^{-3} + \frac{100\times0.15}{10^{-7}\times1\times10^{10}}$$

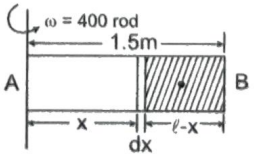
$= 9 \times 10^{-3} + 15 \times 10^{-3} = 24$ mm

Example 7: A road having uniform mass distribution is rotated with constant angular velocity ω (400 rad/s) as shown in fig. Find out elongation in rod.

Given $Y = 2 \times 10^{11}$ N/m²; density of rod $= 10^4$ kg/m³



Sol.



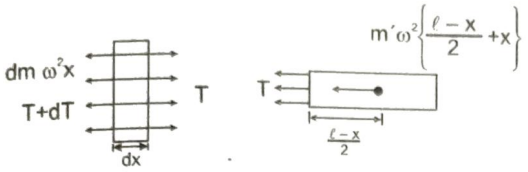
mass of shaded portion

$m' = \frac{m}{\ell}(\ell - x)$ [where m = total mass = ρ A 1]

$$T = m'\omega^2\left[\frac{\ell - x}{2} + x\right]$$

$$\Rightarrow T = \frac{m}{\ell}(1 - x)\,\omega^2\left(\frac{\ell + x}{2}\right)$$

$$T = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$



this tension will be maximum at A $\left(\frac{m\omega^2\ell}{2}\right)$ and minimum

at 'B' (zero), elongation in element of width 'dy' $= \frac{Tdx}{AY}$

Total elongation

$$y = \int \frac{Tdx}{AY} = \int_0^{\ell} \frac{m\omega^2(\ell^2 - x^2)}{2\ell\ AY}\ dx$$

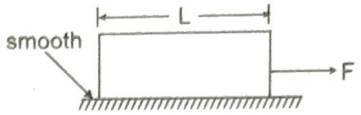
$$y = \frac{m\omega^2}{2\ell\ AY}\left[\ell^2 x - \frac{x^3}{3}\right]_0^{\ell}$$

$$= \frac{m\omega^2 \times 2\ell^3}{2\ell AY \times 3} = \frac{m\omega^2\ell^2}{3AY} = \frac{\rho A\ell\omega^2\ell^2}{3AY}$$

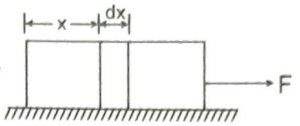
$$y = \frac{\rho\omega^2\ell^3}{3Y} = \frac{10^4 \times (400) \times (1.5)^3}{3 \times 2 \times 10^{11}}$$

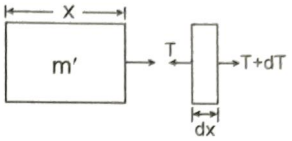
$= 9 \times 10^{-3}$ m = 9mm

Example 8: Find out the elongation in block. If mass, area of cross-section and young modulus of block are m, A and Y respectively.



Sol.





Acceleration, $a = \frac{F}{m}$ then $T = m'a$

where $\Rightarrow m' = \frac{m}{\ell}x$

$$T = \frac{m}{\ell}x\,\frac{F}{m} = \frac{F\,x}{\ell}$$

Elongation in element

$$'d\ell' = \frac{Tdx}{AY}$$

Total elongation,

$$\delta = \int_0^{\ell} \frac{Tdx}{AY}\quad d = \int_0^{\ell} \frac{Fxdx}{A\ell Y} = \frac{F\ell}{2AY}$$

Different Types of Stress

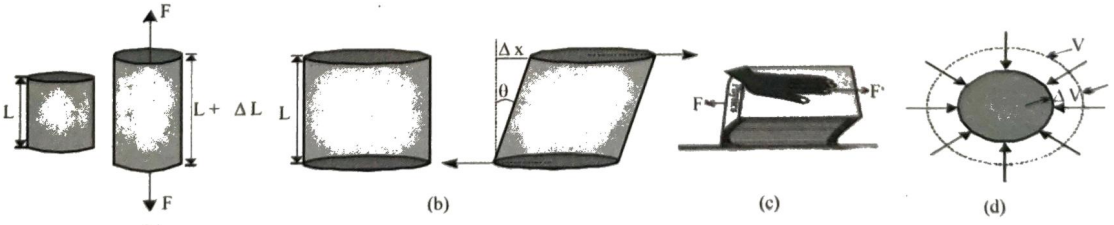


Figure (a) Cylinder subjected to tensile stress stretches it by an amount ΔL. (b) A cylinder subjected to shearing (tangential) stress deforms by an angle θ. (c) A book subjected to a shearing stress (d) A solid sphere subjected to a uniform hydraulic stress shrinks in volume by an amount ΔV.

There are three ways in which a solid may change its dimensions when an external force acts on it. These are shown in Fig.(a), a cylinder is stretched by two equal forces applied normal to its cross-sectional area. The restoring force per unit area in this case is called **tensile stress**. If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as **compressive stress**. Tensile or compressive stress can also be termed as longitudinal stress. In both the cases, there is a change in the length of the cylinder. The change in the length ΔL to the original length L of the body (cylinder in this case) is known as **longitudinal strain**. Longitudinal strain $= \frac{\Delta L}{L}$.

SHEARING STRESS

However, if two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder, as shown in Fig. (b), there is relative displacement between the opposite faces of the cylinder. The restoring force per unit area developed due to the applied tangential force is known as **tangential** or **shearing stress**.

As a result of applied tangential force, there is a relative displacement Δx between opposite faces of the cylinder as shown in the Fig. (b). The strain so produced is known as **shearing strain** and it is defined as the ratio of relative displacement of the faces Δx to the length of the cylinder L.

Shearing strain $\frac{\Delta x}{L} = \tan\theta$

where θ is the angular displacement of the cylinder from the vertical (original position of the cylinder). Usually θ is very small, $\tan\theta$ is nearly equal to angle θ, (if $\theta = 10°$, for example, there is only 1% difference between θ and $\tan\theta$).

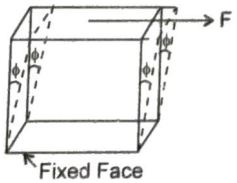
It can also be visualised, when a book is pressed with the hand and pushed horizontally, as shown in Fig. (c). Thus, shearing strain $= \tan\theta \approx \theta$.

Modulus of Rigidity:

It is defined as the ratio of the tangential stress to the shear strain. Let us consider a cube whose lower face is fixed and a tangential force F acts on the upper face whose area is A.

$\therefore$ Tangential stress = F/A.

Let the vertical sides of the cube shifts through an angle θ, called shear strain



$\therefore$ Modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}}$$

$$\text{or } \eta = \frac{F/A}{\phi} = \frac{F}{A\phi}$$

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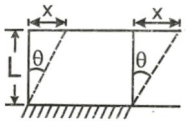
Example 9: A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4×10^6 N/m^2.

Sol. $L = 5 \times 10^{-2}$ m $\Rightarrow \frac{F}{A} = \eta \frac{x}{L}$



$$\text{strain} = \tan\theta = \frac{F}{A\eta} = \frac{1800}{25\times10^{-4}\times2.4\times10^{6}}$$

$$= \frac{180}{25\times24} = \frac{3}{10} = 0.3 \text{ radian}$$

$$\frac{x}{L} = 0.3 \Rightarrow x = 0.3 \times 5 \times 10^{-2}$$

$= 1.5 \times 10^{-2}$ m $= 1.5$ cm **Ans.**

BULK MODULUS

It is defined as the ratio of the normal stress to the volume strain

$$\text{i.e. } B = \frac{\text{Pressure}}{\text{Volume strain}}$$

The stress being the normal force applied per unit area and is equal to the pressure applied (p).

$$B = \frac{p}{\frac{-\Delta V}{V}} = -\frac{pV}{\Delta V}$$

Negative sign shows that increase in pressure (p) causes decrease in volume (ΔV).

COMPRESSIBILITY

The reciprocal of bulk modulus of elasticity is called compressibility. Unit of compressibility in SI is N^{-1} m^2 or pascal^{-1} (Pa^{-1}).

Bulk modulus of solids is about fifty times that of liquids, and for gases it is 10^{-8} times of solids.

$$B_{solids} > B_{liquids} > B_{gases}$$

Isothermal bulk modulus of elasticity of gas

$B = P$ (pressure of gas)

Adiabatic bulk modulus of elasticity of gas

$$B = \gamma \times P \qquad \text{where } \gamma = \frac{C_p}{C_v}$$

In Fig. (d), a solid sphere placed in the fluid under high pressure is compressed uniformly on all sides. The force applied by the fluid acts in perpendicular direction at each point of the surface and the body is said to be under hydraulic compression. This leads to decrease in its volume without any change of its geometrical shape. The body develops internal restoring forces that are equal and opposite to the forces applied by the fluid (the body restores its original shape and size when taken out from the fluid). The internal restoring force per unit area in this case is equal to the hydraulic pressure (applied force per unit area). The strain produced by a hydraulic pressure is called **volume strain** and is defined as the ratio of change in volume (ΔV) to the original volume (V).

$$\text{Volume strain} = \frac{\Delta V}{V}$$

We have seen that when a body is submerged in a fluid, it undergoes a hydraulic stress (equal in magnitude to the hydraulic pressure). This leads to the decrease in the volume of the body thus producing a strain called volume strain.

VOLUME DEFORMATION

Since the fluid presses inward on all sides of the object (figure), the solid is compressed-its volume is reduced. The fluid pressure P is the force per unit surface area ; it can be through of as the volume stress on the solid object. Pressure has the same units as the other kinds of stress: N/m^2 or Pa.

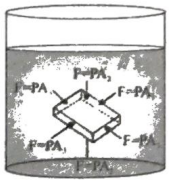


Figure L Forces on an object when submerged in a fluid

$$\text{volume stress} = \text{pressure} = \frac{F}{A} = P$$

The resulting deformation of the object is characterized by the volume strain, which is the fractional change in volume :

Volume strain

$$= \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

Unless the stress is too large, the stress and strain are proportional within a constant of proportionally called the bulk modulus B. A substance with a large bulk modulus is more difficult to compress than a substance with a small bulk modulus. For solids and liquids, the volume strain due to atmospheric pressure is, for most purposes, negligibly small (5×10^{-5} for water).

Hooke's law for volume deformation

$$\Delta P = -B \frac{\Delta V}{V}$$

where V is the volume at atmospheric pressure.

The negative sign. equation $\Delta P = -B \frac{\Delta V}{V}$ allows the bulk modulus to be positive-an increase in the volume stress causes a decrease in volume, so ΔV is negative.

The reciprocal of the bulk modulus is called compressibility and is denoted by k. It is defined as the fractional change in volume per unit increase in pressure.

$k = (1/B) = -(1/\Delta p) \times (\Delta V/V)$

Bulk modulus (all three states of matter)

$$B = \frac{\text{volume stress}}{\text{strain}}$$

$$B = \frac{\Delta P}{\frac{-\Delta V}{V}} = -V \frac{\Delta P}{\Delta V}$$

$$C = \frac{1}{B} = -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)$$

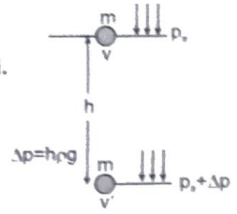
The bulk moduli of liquids are generally not much less than those of solids, since the atoms in liquids are nearly as close together as those in solids.

Gases are much easier to compress than solids or liquids, so their bulk moduli are much smaller.

TRAIN YOUR BRAIN

Example 10: Find the depth of lake at which density of water is 1% greater than at the surface. Given compressibility $K = 50 \times 10^{-6}$ /atm.

Sol.



m = constant

ρV = constant

$$d\rho\, V + dV \,.\, \rho = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V}$$

$$\text{i.e. } \frac{\Delta\rho}{\rho} = \frac{\Delta p}{B} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{1}{100}$$

$$\frac{1}{100} = \frac{h\rho g}{B} \text{ [assuming } \rho = \text{const.]}$$

$$h\,\rho\, g = \frac{B}{100} = \frac{1}{100K}$$

$$\Rightarrow h\,\rho\, g = \frac{1\times10^5}{100\times50\times10^{-6}}$$

$$h = \frac{10^5}{5000\times10^{-6}\times1000\times10}$$

$$= \frac{100\times10^3}{50} = 2\text{km}$$

POISSONS RATIO

$$\sigma = \frac{-\text{lateral strain}}{\text{longitudinal strain}} = \frac{-\delta r/r}{\delta L/L}$$

TRAIN YOUR BRAIN

Example 11: A uniform bar of length L and cross sectional area A is subjected to a tensile load F. If Y be the Young's modulus of the material of the bar and σ be its poisson's ratio, then determine the volumetric strain.

Sol. Longitudinal stress $= \frac{F}{A}$

Longitudinal strain $= \frac{F}{AY} = \varepsilon_1$ (say) ... (i)

Now, by definition of Poisson's ratio,

$$\sigma = \frac{-\text{lateral strain}}{\text{longitudinal strain}} = \frac{-\delta r/r}{\delta L/L}$$

or $\delta r/r = -\sigma\delta L/L$

$$\Rightarrow -\frac{\sigma F}{AY} \text{ [From eqn. (i)]}$$

Since,

Volumetric strain = Strain in length + Twice strain in radius.

$$\therefore \text{ Volumetric strain} = \frac{\delta L}{L} + \frac{2\delta r}{r}$$

$$= \frac{F}{AY} + 2\left(-\frac{\sigma F}{AY}\right) = \frac{F}{AY}(1-2\sigma)$$

Topicwise Questions

ELASTIC BEHAVIOUR, LONGITUDINAL STRESS YOUNG MODULUS

1. An elastic rod will not change its length when it is -
   (*a*) Pulled along the rough surface
   (*b*) Pulled along the smooth surface
   (*c*) Is hanging under gravity
   (*d*) Freely falls vertically under gravity

2. The length of an iron wire is L and area of cross-section is A. The increase in length is ℓ on applying the force F on its two ends. Which of the statement is correct?
   (*a*) Increase in length is inversely proportional to its length L
   (*b*) Increase in length is proportional to area of cross-section A
   (*c*) Increase in length is inversely proportional to A
   (*d*) Increase in length is proportional to Young's modulus

3. Two wires have the same diamter and length. One is made of copper and the other brass. They are connected together at one end. When free ends are pulled in opposite direction by same force.
   (*a*) the wires will have same strain
   (*b*) the wires will have same stress
   (*c*) Both the wires will break at the same force
   (*d*) Both wires will have same elogation

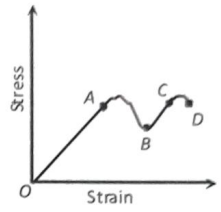
4. The ratio of the lengths of two wires A and B of same material is 1 : 2 and the ratio of their diameter is 2 : 1. They are stretched by the same force, then the ratio of increase in length will be-
   (*a*) 2 : 1 (*b*) 1 : 4
   (*c*) 1 : 8 (*d*) 8 : 1

5. The area of cross-section of a wire of length 1.1 metre is 1 mm^2. It is loaded with 1 kg. If Young's modulus of copper is 1.1×10^{11} N/m^2, then the increase in length will be- (If $g = 10$ m/s^2)
   (*a*) 0.01 mm
   (*b*) 0.075 mm
   (*c*) 0.1 mm
   (*d*) 0. 15 mm

6. If the temperature increases, the modulus of elasticity-
   (*a*) Decreases (*b*) Increases
   (*c*) Remains constant (*d*) Becomes zero

7. A force F is needed to break a copper wire having radius R. The force needed to break a copper wire of radius 2R will be-
   (*a*) F/2 (*b*) 2F
   (*c*) 4F (*d*) F/4

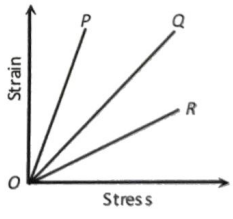
8. A graph is shown between stress and strain for a metal. The part in which Hooke's law holds good is -



   (*a*) OA (*b*) AB
   (*c*) BC (*d*) CD

9. A force of 400 kg. weight can break a wire. The force required to break a wire of double the area of cross-section will be-
   (*a*) 800 kg-wt (*b*) 200 kg-wt
   (*c*) 1600 kg-wt (*d*) 100 kg-wt

10. The strain-stress curves of three wires of different materials are shown in the figure. P, Q and R are the elastic limits of the wires. The figure shows that-



   (*a*) Elasticity of wire P is maximum
   (*b*) Elasticity of wire Q is maximum
   (*c*) Tensile strength of R is maximum
   (*d*) None of these

11. A metal wire of length L area of cross-section A and Young modulus Y behaves as a spring of spring constant k, where k is equal to-
   (*a*) $\frac{YL}{A}$ (*b*) $\frac{YA}{L}$
   (*c*) $\frac{2YA}{L}$ (*d*) $\frac{YA}{2L}$

TANGENTIAL STRESS AND STRAIN, SHEAR MODULUS

12. Modulus of rigidity of diamond is-
(*a*) Too less
(*b*) Greater than all matters
(*c*) Less than all matters
(*d*) Zero

13. Modulus of rigidity of a liquid-
(*a*) Non zero constant
(*b*) Infinite
(*c*) Zero
(*d*) Can not be predicted

14. For a given material, the Young's modulus is 2.4 times that of rigidity modulus. Its Poisson's ratio is-
(*a*) 2.4 (*b*) 1.2
(*c*) 0.4 (*d*) 0.2

15. Shearing stress causes change in-
(*a*) Length (*b*) Breadth
(*c*) Shape (*d*) Volume

PRESSURE AND VOLUMETRIC STRAIN, BULK MODULUS OF ELASTICITY

16. The isothermal elasticity of a gas is equal to-
(*a*) Density (*b*) Volume
(*c*) Pressure (*d*) Specific heat

17. The adiabatic elasticity of a gas is equal to-
(*a*) $\gamma \times$ density (*b*) $\gamma \times$ volume
(*c*) $\gamma \times$ pressure (*d*) $\gamma \times$ specific heat

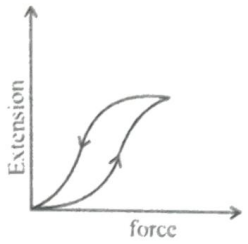
18. The only elastic modulus that applies to fluids is-
(*a*) Young's modulus
(*b*) Shear modulus
(*c*) Modulus of rigidity
(*d*) Bulk modulus

19. A ball falling in a lake of depth 200 m shows 0.1% decrease in its volume at the bottom. What is the bulk modulus of the material of the ball-
(*a*) $19.6 \times 10^{8}\ N/m^2$ (*b*) $19.6 \times 10^{-10}\ N/m^2$
(*c*) $19.6 \times 10^{10}\ N/m^2$ (*d*) $19.6 \times 10^{-8}\ N/m^2$

ELASTIC POTENTIAL ENERGY

20. An iron bar of length 'L' cross section 'A' and Young's modulus 'Y' is pulled by a force 'F', from both the ends so as to produce an elongation 'e'. The elastic energy stored in the deformed rod will be-
(*a*) $\frac{F^2L}{AY}$ (*b*) $\frac{F^2L}{2AY}$
(*c*) $\frac{F^2}{2A^2Y}$ (*d*) None of these

21. The diagram shows a force-extension graph for a rubber band. Consider the following statements.



I. It will be easier to compress this rubber than expand it
II. Rubber does not return to its original length after it is stretched
III. The rubber band will get heated if it is stretched and released
Which of these can be deduced from the graph?
(*a*) III only (*b*) II and III
(*c*) I and III (*d*) I only

22. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is-
(*a*) 20 J (*b*) 0.1 J
(*c*) 0.2 J (*d*) 10 J

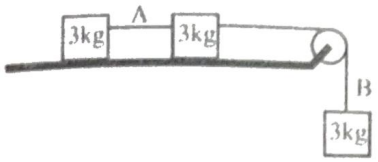
23. A 5 metre long wire is fixed to the ceiling. A weight of 10 kg is hung at the lower end and is 1 metre above the floor. The wire was elongated by 1 mm. The energy stored in the wire due to stretching is-
(*a*) Zero
(*b*) 0.05 joule
(*c*) 100 joule
(*d*) 500 joule

24. If the tension on a wire is removed at once, then-
(*a*) It will break
(*b*) Its temperature will reduce
(*c*) There will be no change in its temperature
(*d*) Its temperature increases

25. A wire is suspended by one end. At the other end, a weight equivalent to 20 N force is applied. If the increase in length is 1.0 mm, the increase in energy of the wire will be-
(*a*) 0.01 J (*b*) 0.02 J
(*c*) 0.04 J (*d*) 1.00 J

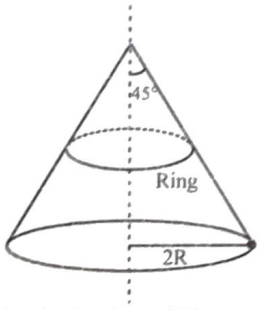
Learning Plus

1. Three equal masses of 3 kg are connected by two massless strings of cross sectional area 0.005 cm^2 and Young modulus is 2×10^{11} N/m^2 each The longitudinal strain in the wires-



(*a*) are equal
(*b*) cannot be different
(*c*) must be different
(*d*) may or may not be different

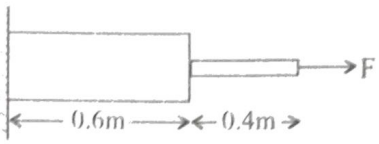
2. A ring of mass m, radius R, cross-sectional area a and Young's modulus Y is kept on a smooth cone of radius 2R and semi-vertical angle 45°, as shown in the figure. Assume that the extension in the ring is small. Then choose the wrong statement-



(*a*) The tension in the ring will be same throughout.
(*b*) The tension in the ring will be independent of the radius of ring if radius of the ring is less than 2R.
(*c*) The extension in the ring will be $\frac{mgR}{aY}$
(*d*) Elastic potential energy stored in the ring will be $\frac{m^2g^2R}{8\pi Ya}$

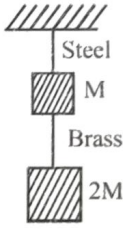
3. The diameter of a brass rod is 4 mm and Young's modulus of brass is 9×10^{10} N/m^2. The force required to stretch it by 0.1% of its length is -
(*a*) $360\,\pi$ N
(*b*) 36 N
(*c*) $144\,\pi \times 10^3$ N
(*d*) $36\,\pi \times 10^5$ N

4. Two bars of steel ($Y = 2 \times 10^{11}$ N/m^2) are joined together as shown. The area of cross section of the left bar is 15 cm^2 and the area of right bar is unknown. The extension in both bars is the same.



(*a*) The area of right bar is 10 cm^2
(*b*) The stresses in left and right bar are in ratio 3 : 2
(*c*) The decrease in thickness of bar is more for the left
(*d*) The decrease in thickness of bar is less for right bar

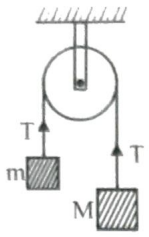
5. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increases in their lengths is :



(*a*) $\frac{2a^2c}{b}$ (*b*) $\frac{3a}{2b^2c}$
(*c*) $\frac{2ac}{b^2}$ (*d*) $\frac{3ac}{2ab^2}$

6. The dimensions of four wires of the same material are given below. In which wire the increase in length will be maximum when the same tension is applied-
(*a*) Length 100 cm, Diameter 1 mm
(*b*) Length 200 cm, Diameter 2 mm
(*c*) Length 300 cm, Diameter 3 mm
(*d*) Length 50 cm, Diameter 0.5 mm

7. Two blocks of masses m and M are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in the figure. The system is then released. If M = 2m, then the stress produced in the wire is-



(a) $\frac{2mg}{3A}$ (b) $\frac{3mg}{4A}$

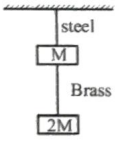
(c) $\frac{mg}{A}$ (d) $\frac{4mg}{3A}$

8. Two bodies of masses 2 kg and 3 kg are connected by a metal wire of cross-section 0.04 mm^2 and are placed on a frictionless horizontal surface. Breaking stress of metal wire is 2.5 GPa. The maximum force F that can be applied to 3 kg block so that wire does not break is

(a) 100 N (b) 150 N

(c) 200 N (d) 250 N

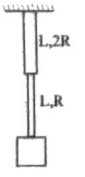
9. If the ratio of lengths, radii and young's modulus of steel and brass wires in figure are 2 : 1, 2 : 1, 3 : 1 respectively. Then corresponding ratio of increase in their length would be. (Neglect the mass of the wires and take g = 10 m/s^2)



(a) 1 : 4 (b) 8 : 1

(c) 1 : 3 (d) 2 : 4

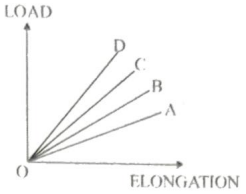
10. Two light wires of the same material (Young's modulus Y) and same length L but different radii R and 2R, as shown in the figure, are joined end to end and supported from a fixed support. A weight W is suspended from the combination. The elastic potential energy in the system is



(a) $\frac{3W^2L}{4\pi R^2Y}$ (b) $\frac{3W^2L}{8\pi R^2Y}$

(c) $\frac{5W^2L}{8\pi R^2Y}$ (d) $\frac{W^2L}{\pi R^2Y}$

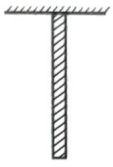
11. The load versus elongation graph for four wires of the same materials is shown in the figure. The thinnest wire is represented by the line :



(a) OC (b) OD

(c) OA (d) OB

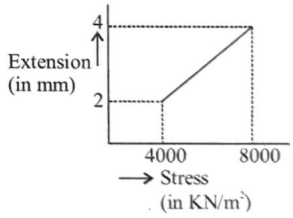
12. The extension in a uniform rod of length ℓ, mass m, cross-section radius r and young's modulus Y when it is suspended at one of its end is :



(a) $\frac{mg\ell}{\pi r^2Y}$ (b) $\frac{mg\ell}{2\pi r^2Y}$

(c) $\frac{2mg\ell}{\pi r^2Y}$ (d) $\frac{mg\ell}{4\pi r^2Y}$

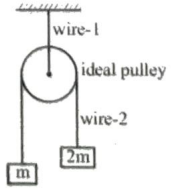
13. In determination of young modulus of elasticity of wire, a force is applied and extension is recorded. Initial length of wire is 1 m. The curve between extension and stress is depicted then young modulus of wire will be:



(a) 2×10^9 N/m^2 (b) 1×10^9 N/m^2

(c) 2×10^{10} N/m^2 (d) 1×10^{10} N/m^2

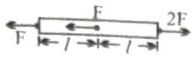
14. Find the ratio of stress present in wire-1 to that in wire-2 shown in the diagram while both wires have same material & thickness.



(a) 1 (b) 2

(c) 3 (d) 4

15. Forces acting on a uniform rod having length $2l$, area of cross-section A and Young's modulus Y are shown in the figure. Elongation of the rod is-



(a) $\frac{Fl}{AY}$ (b) $\frac{3Fl}{2AY}$

(c) $\frac{2Fl}{AY}$ (d) $\frac{3Fl}{AY}$

16. One end of a cylindrical solid rod of length L and radius r is clamped in a fixed positon. The other end is turned by an external torque τ resulting in a twist θ. The sheer modulus is given by η. The twist angle θ is proportional to-

(a) $\tau r^1\eta/\pi L^2$ (b) $\tau r^4/\pi \eta L$

(c) $\tau \eta L/\pi r^4$ (d) $\tau L/\pi \eta r^4$

17. The upper end of a wire of radius 4 mm and length 100 cm is clamped and its other end is twisted through an angle of 30°. Then angle of shear is-

(a) 12° (b) 0.12°

(c) 1.2° (d) 0.012°

18. A uniform wire (Young's modulus 2×10^{11} Nm^{-2}) is subjected to longitudinal tensile stress of 5×10^7 Nm^{-2}. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to-

(a) 0.25×10^{-4} (b) 1.0×10^{-4}

(c) 5×10^{-4} (d) 1.5×10^{-4}

19. A metal block is experiencing an atmospheric pressure of 1×10^5 N/m^2, when the same block is placed in a vacuum chamber, the fractional change in its volume is- (the bulk modulus of metal is 1.25×10^{11} N/m^2)

(a) 4×10^{-7} (b) 2×10^{-7}

(c) 8×10^{-7} (d) 1×10^{-7}

20. Expansion during heating

(a) occurs only in a solid

(b) increases the density of the material

(c) decreases the density of the material

(d) occurs at the same rate for all liquids and solids

21. For a constant hydraulic stress on an object, the fractional change in the object's volume $\left(\frac{\Delta V}{V}\right)$ and its bulk modulus (B) are related as -

(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto \frac{1}{B}$

(c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto B^2$

22. If work done in stretching a wire by 1mm is 2J, the work necessary for stretching another wire of same material, but with double the radius and half the length by 1mm in joule is -

(a) 1/4 (b) 4

(c) 8 (d) 16

23. If the potential energy of a spring is V on stretching it by 2 cm, then its potential energy when it is stretched by 10 cm will be-

(a) V/25 (b) 5V

(c) V/5 (d) 25V

24. Two wires of same diameter of the same material having the length l and $2l$. If the force F is applied on each, the ratio of the work done in the two wires will be-

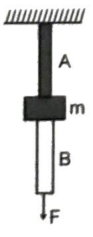
(a) 1 : 2 (b) 1 : 4

(c) 2 : 1 (d) 1 : 1

Advanced Level Multiconcept Questions

MCQ/STATEMENT/COMPREHENSION/MATCHING

1. The wires A and B shown in the figure, are made of the same material and have radii r_A and r_B. A block of mass m kg is tied between them: If the force F is mg/3, one of the wires breaks. Then,



(a) A will break before B if $r_A < 2r_B$

(b) A will break before B if $r_A = r_B$

(c) Either A or B will break if $r_A = 2r_B$

(d) The lengths of A and B must be known to decide which wire will break

2. A body of mass M is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is l.

(a) Loss in gravitational potential energy of M is Mgl

(b) The elastic potential energy stored in the wire is Mgl

(c) The elastic potential energy stored in the wire is $1/2\ Mgl$

(d) Heat produced is $1/2\ Mgl$

Comprehension Type Questions – 1 (No.3 to 7)

When a tensile or compressive load 'P' is applied to rod or cable, its length changes. The change in length x for an elastic material is proportional to the force (Hook's law).

$P \alpha x$ or $P = kx$

The above equation is similar to the equation of spring. For a rod of length L, area A and young modulus Y, the extension x can be expressed as -

$$x = \frac{PL}{AY} \text{ or } P = \frac{AY}{L}x, \text{ hence } K = \frac{AY}{L}$$

Thus rods or cables attached to lift can be treated as springs. The energy stored in rod is called strain energy & equal to $\frac{1}{2}Px$. The loads placed or dropped on the floor of lift cause stresses in the cables and can be evaluated by spring analogy. If the cable of lift is previously stressed and load is placed or dropped, then maximum extension in cable can be calculated by energy conservation.

3. If rod of length 4 m, area $4cm^2$ and young modules 2×10^{16} N m^{-2} is attached with mass 200 kg, then angular frequency of SHM (rad/sec.) of mass is equal to -

(*a*) 1000 (*b*) 10

(*c*) 100 (*d*) 10 π

4. In above problem if mass of 10 kg falls on the massless collar attached to rod from the height of 99cm then maximum extension in the rod is equal to- ($g = 10$ m/sec^2)

(*a*) 9.9 cm (*b*) 10 cm

(*c*) 0.99 cm (*d*) 1 cm

5. In the above problem, the maximum stress developed in the rod is equal to - (N/m^2)

(*a*) 5×10^7 (*b*) 5×10^8

(*c*) 4×10^7 (*d*) 4×10^8

6. If two rods of same length (4m) and cross section areas 2 cm^2 and 4 cm^2 with same young modulus 2×10^{16} N/m^2 are attached one after the other with mass 600 kg then angular frequency is -

(*a*) $\frac{1000}{3}$ (*b*) $\frac{10}{3}$

(*c*) $\frac{100}{3}$ (*d*) $\frac{10\pi}{3}$

7. Four identical rods of geometry as described in problem (*b*) are attached with lift. If weight of the lift cage is 1000 N, and elastic limit of each rod is taken as 9×10^6 N/m^2 then the number of persons it can carry safely is equal to- ($g = 10$m/sec^2 , assume average mass of a person as 50 kg and lift moves with constant speed)

(*a*) 7 (*b*) 26

(*c*) 24 (*d*) 25

8. A metal wire of length L is suspended vertically from a rigid support. When a bob of mass M is attached to the lower end of wire, the elongation of the wire is ℓ :

Column - I

(*a*) The loss in gravitational potential energy of mass M is equal to

(*b*) The elastic potential energy stored in the wire is equal to

(*c*) The elastic constant of the wire is equal to

(*d*) Heat produced during extension is equal to

Column - II

(p) $Mg\ell$

(q) $\frac{1}{2}Mg\ell$

(r) Mg/ℓ

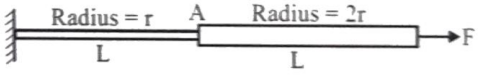
(s) $\frac{1}{4}Mg\ell$

INTEGER TYPE

9. Find the greatest length (in km) of a steel wire of uniform cross-section that can hang vertically without breaking. Breaking stress of steel = 8.0×10^8 N/m^2. Density of steel = 8.0×10^3 kg/m^3.

10. The elastic limit of a steel cable is 2.40×10^8 Pa and the cross-section area is 4.00 cm^2. Find the maximum upward acceleration (in m/s^2) that can be given to a 800 kg elevator supported by the cable if the stress should not exceed one third of the elastic limit.

11. Two steel wires of same length but radii r and 2r are connected together end to end and tied to a wall as shown.



The force stretches the combination by 10 mm. How far does the midpoint A move. (in mm)

12. What is the maximum height (in m) of a brick column of uniform cross section for which column deformation due to its own weight is within the elastic limit?

$P_{atmospheric} = 100$ kPa, $\rho = 1.8 \times 10^3$ kg/m^3. Elastic limit, $\sigma = 3.7$ M Pa.

13. A substance breaks down under a stress of 10^5 Pa. If the density of the wire is 2×10^3 kg/m^3, find the minimum length of the wire which will break under its own weight ($g = 10$ m/s^2)

14. A steel wire, 3.2 m long, has a diameter of 1.2 mm. The wire stretches by 1.6 mm when it bears a load. Young's modulus for steel is 2.0×10^{11} Pa. Find the mass (in kg) of the load closest?

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of : [*JEE Main-2017*]

(*a*) 81 (*b*) 1/81

(*c*) 9 (*d*) 1/9

2. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is : [*JEE Main-2018*]

(*a*) $\frac{Ka}{3mg}$ (*b*) $\frac{mg}{3Ka}$

(*c*) $\frac{mg}{Ka}$ (*d*) $\frac{Ka}{mg}$

3. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is [*JEE Main-2019 (January)*]

(*a*) 3.0 mm (*b*) 4.0 mm

(*c*) 5.0 mm (*d*) zero

4. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to : [*JEE Main-2019 (April)*]

(*a*) $10^4\ Nm^{-2}$ (*b*) $10^8\ Nm^{-2}$

(*c*) $10^6\ Nm^{-2}$ (*d*) $10^3\ Nm^{-2}$

5. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1 $\pi\ ms^{-2}$, what will be the tensile stress that would be developed in the wire? [*JEE Main-2019 (April)*]

(*a*) $4.8 \times 10^6\ Nm^{-2}$ (*b*) $5.2 \times 10^6\ Nm^{-2}$

(*c*) $6.2 \times 10^6\ Nm^{-2}$ (*d*) $3.1 \times 10^6\ Nm^{-2}$

6. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit? [*JEE Main-2019 (April)*]

(*a*) 1.16 mm (*b*) 0.90 mm

(*c*) 1.36 mm (*d*) 1.00 mm

7. In an experiment, brass and steel wires of length 1 m each with areas of cross section 1 mm^2 are used. the wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is : [*JEE Main-2019 (April)*]

(Given, the Young's Modulus for steel and brass are respectively, $120 \times 10^9\ N/m^2$ and $60 \times 10^9\ N/m^2$)

(*a*) $0.2 \times 10^6\ N/m^2$ (*b*) $4.0 \times 10^6\ N/m^2$

(*c*) $1.8 \times 10^6\ N/m^2$ (*d*) $1.2 \times 10^6\ N/m^2$

8. Young's moduli of two wires A and B in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to :- [*JEE Main-2019 (April)*]

(*a*) 1.9 mm (*b*) 1.7 mm

(*c*) 1.5 mm (*d*) 1.3 mm

9. A non-isotropic solid metal cube has coefficients of linear expansion as: 5×10^{-5} /°C along the x-axis and 5×10^{-6} /°C along the y and the z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6}$ /°C then the value of C is .... [*JEE Main-2020 (January)*]

10. A leak proof cylinder of length 1m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to: [*JEE Main-2020 (January)*]

(*a*) 1.0 (*b*) 1.04

(*c*) 1.26 (*d*) 1.01

11. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (Breaking stress of wire $= 4.8 \times 10^7$ Nm^{-2} and area of cross section of the wire $= 10^{-2}$ cm^2)

[*JEE Main-2020 (January)*]

12. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is

[*JEE Main-2020 (January)*]

(*a*) 1 : 2 (*b*) $1:\sqrt{2}$

(*c*) 2 : 1 (*d*) $\sqrt{2}:1$

13. An object of mass m is suspended at the end of a massless wire of length L and area of cross-section A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is [*JEE Main-2020 (September)*]

(*a*) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$ (*b*) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

(*c*) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$ (*d*) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$

14. A stone of mass 20 g is projected from a rubber catapult of length 0.1m and area of cross section 10^{-6} m^2 stretched by an amount 0.04m. The velocity of the projected stone is m/s.

(Young's modulus of rubber $= 0.5 \times 10^9$ N/m^2)

[*JEE Main-2021 (July)*]

15. The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track is 10^{-5}/°C. The energy stored per meter in the track is J/m. (Young's modulus of material of track is 10^{11} Nm^{-2}) [*JEE Main-2021 (July)*]

JEE-ADVANCED PREVIOUS YEAR'S

1. One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by a applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is : [*JEE Advanced-2013*]

(*a*) 0.25 (*b*) 0.50

(*c*) 2.00 (*d*) 4.00

2. A cubical solid aluminium (bulk modulus $= -V\frac{dP}{dV} = 70$ GPa) block has an edge length of 1 m on the surface of the earth. It is kept on the floor of a 5 km deep ocean. Taking the average density of water and the acceleration due to gravity to be 10^3 kg m^{-3} and 10 ms^{-2}, respectively, the change in the edge length of the block in mm is

[*JEE (Advanced) - 2020*]

ANSWER KEY

Topicwise Questions

1. (*d*)	**2.** (*c*)	**3.** (*b*)	**4.** (*c*)	**5.** (*c*)	**6.** (*a*)	**7.** (*c*)	**8.** (*a*)	**9.** (*a*)	**10.** (*d*)
11. (*b*)	**12.** (*b*)	**13.** (*c*)	**14.** (*d*)	**15.** (*c*)	**16.** (*c*)	**17.** (*c*)	**18.** (*d*)	**19.** (*a*)	**20.** (*b*)
21. (*a*)	**22.** (*b*)	**23.** (*b*)	**24.** (*d*)	**25.** (*a*)					

Learning Plus

1. (*c*)	**2.** (*d*)	**3.** (*a*)	**4.** (*a*)	**5.** (*b*)	**6.** (*d*)	**7.** (*d*)	**8.** (*d*)	**9.** (*a*)	**10.** (*c*)
11. (*c*)	**12.** (*b*)	**13.** (*a*)	**14.** (*a*)	**15.** (*d*)	**16.** (*d*)	**17.** (*b*)	**18.** (*a*)	**19.** (*c*)	**20.** (*c*)
21. (*b*)	**22.** (*d*)	**23.** (*d*)	**24.** (*a*)						

Advanced Level Multiconcept Questions

1. (*a*, *b*, *c*)	**2.** (*a*, *c*, *d*)	**3.** (*c*)	**4.** (*d*)	**5.** (*a*)	**6.** (*c*)	**7.** (*b*)	**8.** (*a*) p (*b*) q ; (*c*) r ; (*d*) q
9. [0010]	**10.** [0030]	**11.** [8]	**12.** [200]	**13.** [5]	**14.** [0011]		

JEE Mains & Advanced Past Years Questions

JEE-MAIN

1. (*c*)	**2.** (*b*)	**3.** (*a*)	**4.** (*c*)	**5.** (*d*)	**6.** (*a*)	**7.** (BONUS)	**8.** (*b*)	**9.** [60]	**10.** (*d*)
11. [4]	**12.** (*d*)	**13.** (*b*)	**14.** [20]	**15.** [05]					

JEE-ADVANCED

1. (*c*) **2.** [0.23 to 0.24]

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10 MECHANICAL PROPERTIES OF FLUIDS

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DEFINITION OF FLUID

The term fluid refers to a substance that can flow and does not have a shape of its own. For example liquid and gases.

Fluid includes property → (A) Density (B) Viscosity (C) Bulk modulus of elasticity (D) pressure (E) specific gravity

Density

The **density** ρ of a substance is defined as the mass per unit volume of a sample of the substance.

If a small mass element Δm occupies a volume ΔV, the density is given by

$$\rho = \frac{\Delta m}{\Delta V}$$

In general, the density of an object depends on position, so that

$$\rho = f(x, y, z)$$

If the object is homogeneous, its physical parameters do not change with position throughout its volume. Thus, for a homogeneous object of mass M and volume V, the density is defined as

$$\rho = \frac{M}{V}$$

The SI unit of density is kg m^{-3} and dimension is $[ML^{-3}]$

Relative Density/Specific Gravity

The **specific gravity** of a substance is the ratio of its density to that of water at 4°C, which is 1000 kg/m^3. Specific gravity is a dimensionless quantity numerically equal to the density quoted in g/cm^3. For example, the specific gravity of mercury is 13.6, and the specific gravity of water at 100°C is 0.998.

TRAIN YOUR BRAIN

Example 1: Find the density and specific gravity of gasoline if 51 g occupies 75 cm^3.

Sol. Density = $\frac{\text{mass}}{\text{volume}} = \frac{0.051\,\text{kg}}{75\times10^{-6}\,\text{m}^3} = 680$ kg/m^3

$$\text{Sp. gravity} = \frac{\text{density of gasoline}}{\text{density of water}}$$

$$= \frac{680\,\text{kg/m}^3}{1000\,\text{kg/m}^3}$$

$$= 0.68$$

$$\text{or Sp. gravity} = \frac{\text{mass of } 75\,\text{cm}^3 \text{ gasoline}}{\text{mass of } 75\,\text{cm}^3 \text{ water}}$$

$$= \frac{51\,\text{g}}{75\,\text{g}} = 0.68$$

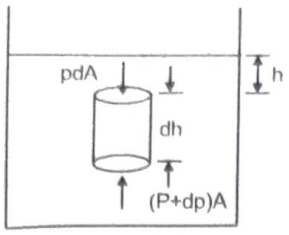
THRUST AND PRESSURE

A **perfect fluid** resists forces normal to its surface and offers no resistance to forces acting tangential to its surface. A heavy log of wood can be drawn along the surface of water with very little effort because the force applied on the log of wood is horizontal and parallel to the water surface. Thus a fluid is capable of exerting normal stresses on a surface with which it is in contact.

Force exerted perpendicular to a surface is called thrust and thrust per unit area is called pressure.

Variation of pressure with Height

In all fluids at rest the pressure is a function of vertical dimension. To determine this consider the forces acting on a vertical column of fluid of cross sectional-area dA as shown in figure. The positive direction of vertical measurement h is taken downward. The pressure on the upper side is P, and that on the lower face is P + dP. The weight of the element is ρgdhdA. The normal forces on the vertical surfaces of the column do not affect the balance of forces in the vertical direction. Equilibrium of the fluid element in the vertical direction requires.



$$pdA + \rho g dA dh - (p + dp)\, dA = 0$$

$$\Rightarrow dP = \rho g dh$$

$$\int_{P_0}^{P} dP = \int_{0}^{h} \rho g dh$$

$$P = P_0 + \rho g h$$

This differential relation shows that the pressure in a fluid increases with depth or decreases with increased elevation. Above equation holds for both liquids and gases and agrees with our common observations of air and water pressure.

Liquids are generally treated as incompressible and we may consider their density ρ constant for every part of the liquid. With ρ as constant equation may be integrated as it stands, and the result is

$$P = P_0 + \rho g h \ldots (10)$$

The pressure P_0 is the pressure at the surface of the liquid where $h = 0$.

Characteristics of Fluid Pressure

(i) Pressure at a point acts equally in all directions.

(ii) Liquids at rest exerts lateral pressure, which increases with depth.

(iii) Pressure acts normally on any area in whatever orientation the area may be held.

(iv) Free surface of a liquid at rest remains horizontal.

(v) Pressure at every point in the same horizontal line is the same inside a liquid at rest.

(vi) Liquid at rest stands at the same height in communicating vessels.

Force due to fluid on a plane submerged surface

For system, which is open to earth's atmosphere, the atmospheric pressure P_0 acts over all surfaces. In such cases, then we need to consider only fluid pressure $P = \rho g h$ which is increment above atmospheric pressure.

In general pressure at different point on the submerged surface varies so to calculate resultant force, we divide the surface into number of elementary areas and we calculate force on it first by treating pressure as constant then we integrate it to get net force.

$$\text{i.e.,} F_R = \int P dA$$

The point of application of resultant force must be such that the moment of the resultant force about any axis is equal to the moment of the distributed force about the axis.

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Example 2: A liquid is filled upto the top in a rectangular tank of square cross section. The sides of cross section is a = 1m and height of the tank is H = 3m. If density of liquid is $\rho = 100\ kgm^{-3}$, find force on the bottom of the tank and on one of its wall. Also calculate the position of point of application of the force on the wall.

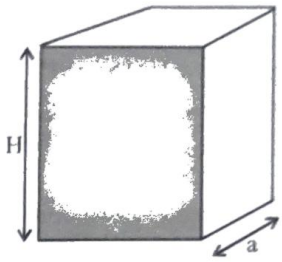
Sol. Force on the bottom of tank: pressure at the bottom of tank due to liquid is uniform of magnitude ρgh.

Area of the bottom of the tank = a^2

$\therefore$ force on the bottom = pressure × area

$= \rho g H a^2 = 3000$ **N**

Force on the wall and its points of application:



Force on the wall of the tank can't be calculated using pressure × area as pressure is not uniform over the surface. The problem can be solved by finding force dF on a thin strip of thickness dh at a depth h below the free surface and then taking its integral.

Pressure of liquid at depth h,

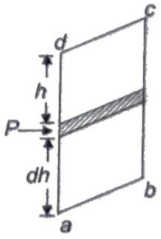
$P = \rho g h$

Area of thin strip = adh

$\therefore$ Force of the strip, $dF = P\, adh = \rho h \cdot g a dh$

$\therefore$ Total force on the wall = $\int_0^H \rho g a h dh$

$$F = \rho g a \frac{H^2}{2} = 4500\ \text{N}$$



The point of application of force on the wall can be calculated by equating the moment of resultant force about any line say dc to the moment of distributed force about the same line dc.

Moment of dF about line cd = dFh

$= \rho gh.\ adh.\ h$

$\therefore$ Net moment of distributed forces $= \rho ga\int_0^H h^2 dh$

$= \rho ga\frac{H^3}{3}$

Let the point of application of net force is at a depth x from the line cd

Then torque of resultant force about line

$$cd = Fx = \rho ga\frac{H^2}{2}x$$

From the above discussion,

$$\rho ga\frac{H^2}{2}x = \rho ga\frac{H^3}{3}$$

$$\Rightarrow x = \frac{2H}{3} = \mathbf{2\ m}$$

Hence, resultant force on the vertical wall of the tank will act at a depth **2m** from the free surface of water or at a height of **1m** from the bottom of tank.

- **Absolute Pressure and Gauge Pressure:** Absolute pressure is the total pressure at a point while gauge pressure is relative to the local atmospheric pressure. Gauge pressure may be positive or negative depending upon the fact whether the pressure is more or less than the atmospheric pressure.

$$p_{gauge} = p_{absolute} - p_{atm}$$

PASCAL'S LAW

According to equation

$p = p_o + \rho gh$

pressure at any depth h in a fluid may be increased by increasing the pressure p_o at the surface. Pascal given this law so we now call **Pascal's Law**.

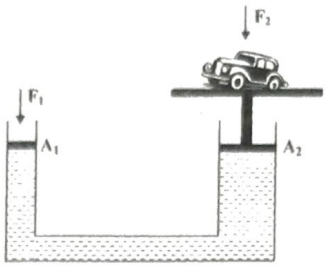
A pressure applied to a confined fluid at rest is transmitted equally undiminished to every part of the fluid and the walls of the container.

This principle is used in a hydraulic jack or lift, as shown in the figure.

The pressure due to a small force F_1 applied to a piston of area A_1 is transmitted to the larger piston of area A_2. The pressure at the two pistons is the same because they are at the same level.

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \text{ or } F_2 = \left(\frac{A_2}{A_1}\right)F_1$$

Consequently, the force on the larger piston is large Thus, a small force F_1 acting on a small area A_1 results in a larger force F_2 acting on a larger area A_2.



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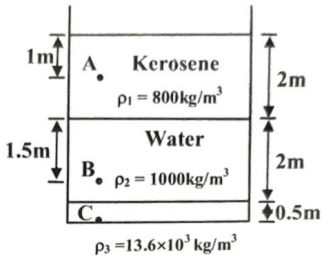
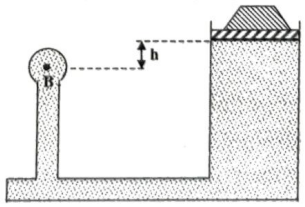
Example 3: Find the absolute pressure and gauge pressure at points A, B and C as shown in the figure. (1 atm = 10^5 Pa)



Sol. $p_{atm} = 10^5$ Pa

Points	Gauge Pressure	Absolute Pressure
A	$p_A = \rho_1 gh_A = (800)(10)1 = 8kPa$	$p'_A = p_A + p_{atm}$ = 108 kPa
B	$p_B = \rho_1 g(2) + \rho_2 g(1.5) = (800)(10)(2) + (10^3)(10)(1.5)$ = 31 kPa	$p'_B = p_B + p_{atm}$ = 131 kPa
C	$p_C = \rho_1 g(2) + \rho_2 g(2) + \rho_3 g(0.5)$ = (800)(10)(2) + (10^3)(10)(2) + (13.6 × 10^3)(10)(0.5) = 104 kPa	$p'_C = p_C + p_{atm}$ = 204 kPa

Example 4: A weighted piston confines a fluid of density ρ in a closed container, as shown in the figure. The combined weight of piston and weight is W = 200 N, and the cross-sectional area of the piston is A = 8 cm^2. Find the total pressure at point B if the fluid is mercury and h = 25 cm (ρ_m = 13600 kg/m^3). What would an ordinary pressure gauge read at B?



Sol. Notice what Pascal's principle tells us about the pressure applied to the fluid by the piston and atmosphere. This added pressure is applied at all points within the fluid. Therefore, the total pressure at B is composed of three parts:

Pressure of atmosphere = 1.0 × 10^5 Pa

Pressure due to piston and weight = $\frac{W}{A} = \frac{200N}{8\times10^{-4}m^2}$

$= 2.5 \times 10^5$ Pa

Pressure due to height h of fluid = $h\rho g = 0.33 \times 10^5$ Pa

In this case, the pressure of the fluid itself is relatively small. We have

Total pressure at B $\approx 3.8 \times 10^5$Pa = 380 kPa

The gauge pressure does not include atmospheric pressure. Therefore,

Gauge pressure at B = 280 kPa

VARIATION OF ATMOSPHERIC PRESSURE WITH HEIGHT

For gases, the constant density assumed in the compressible model is often not adequate. However, an alternative simplifying assumption can be made that the density is proportional to the pressure, i.e.,

$\rho = kp$

Let ρ_o be the density of air at the earth's surface where the pressure is atmospheric p_o, then

$\rho_o = kp_o$

After eliminating k, we get

$$\rho = \frac{\rho_o}{p_o} p$$

Substituting the value of ρ in equation,

$dp = -\rho g dy$

or $$dp = -\left(\frac{\rho_o}{p_o} p\right) g dy$$

On rearranging, we get

$$\int_{p_o}^{p} \frac{dp}{p} = -\frac{\rho_o}{p_o} g \int_0^h dy$$

where p is the pressure at a height y = h above the earth's surface.

After integrating, we get

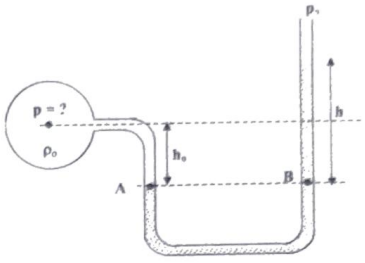
$$ln\left|\frac{p}{p_o}\right| = -\frac{\rho_o}{p_o} gh \qquad \text{or } p = p_0 e^{\frac{-\rho_0}{p_0} gh}$$

Note that instead of a linear decrease in pressure with increasing height as in the case of an incompressible fluid, in this case pressure decreases exponentially.

Pressure Measuring Devices

A **manometer** is a tube open at both the ends and bent into the shape of a "U" and partially filled with mercury. When one end of the tube is subjected to an unknown pressure p, the mercury level drops on that side of the tube and rises on the other so that the difference in mercury level is h as shown in the figure.

According to Pascal's Law, when we move down in a fluid pressure increases with depth and when we move up the pressure decreases with depth. When we move horizontally in a fluid pressure remains constant. Therefore $p + \rho_0 gh_0 - \rho_m gh = p_0$ where p_0 the atmospheric pressure, and ρ_0 is the density of the fluid inside the vessel



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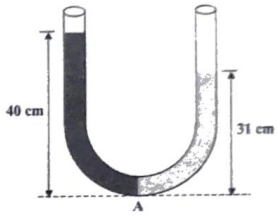
Example 5: As shown in the figure, a column of water 40 cm high supports a 31 cm of an unknown fluid. What is the density of the unknown fluid?

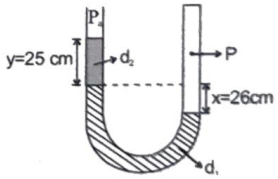


Sol. The pressure at point A due to the two fluids must be equal (or the one with the higher pressure would push the lower pressure fluid away). Therefore, Pressure due to water = pressure due to known fluid $h_1\rho_1 g = h_2\rho_2 g$

from which $\rho_2 = \frac{h_1}{h_2}\rho_1 = \frac{40}{31}(1000 \text{ kg/m}^2) = 1290 \text{ kg/m}^3$

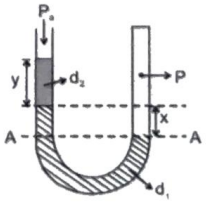
Example 6: In a given U-tube (open at one-end) find out relation between P and P_a.

Given $d_2 = 2 \times 13.6$ gm/cm^3 $d_1 = 13.6$ gm/cm^3



Sol. Pressure in a liquid at same level is same i.e. at A– A–,

$P_a + d_2 yg + xd_1 g = P$



In C.G.S

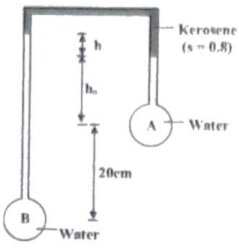
$P_a + 13.6 \times 2 \times 25 \times g + 13.6 \times 26 \times g = P$

$P_a + 13.6 \times g [50 + 26] = P$

$2P_a = P$

$[P_a = 13.6 \times g \times 76]$

Example 7: For the arrangement shown in the figure, determine h if the pressure difference between the vessels A and B is 3 kN/m².



Sol. Let p_A and p_B be the pressure in the vessels A and B, respectively.

Using Pascal's law

$p_B - \rho_w (h + h_o + 0.2]g = p_A - \rho_w gh_o - \rho_k gh$

$[p_B - p_A = \rho_w - \rho_k)gh + \rho_w gh \times 0.2$

here, $p_B - p_A = 3 \times 10^3$ N/m²,

$\rho_w = 10^3$ kg/m³

$\rho_k = 800$ kg/m³.

Thus, h = 0.5 m = 50 cm

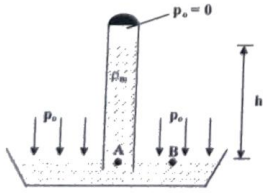
The Mercury Barometer: It is a straight glass tube (closed at one end) completely filled with mercury and inserted into a dish which is also filled with mercury as shown in the figure. Atmospheric pressure supports the column of mercury in the tube to a height h. The pressure between the closed end of the tube and the column of mercury is zero, p = 0.

Therefore, pressure at points **A** and **B** are equal and thus

$p_o = 0 + \rho_m gh$

At the sea level, p_o can support a column of mercury about 76 cm in height.

Hence $p_o = (13.6 \times 10^3)(9.81)(0.76) = 1.01 \times 10^5$ Nm^{-2} (or Pa)

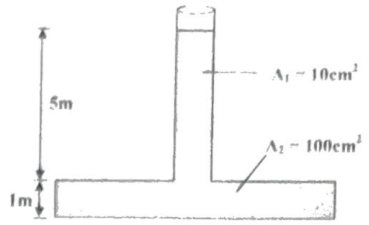


TRAIN YOUR BRAIN

Example 8: In the figure shown, find

(a) the total force on the bottom of the tank due to the water pressure.

(b) the total weight of water.



Sol. (a) Pressure at the base due to water is

$p = \rho_w g [5 + 1] = (10^3)(10)(5 + 1) = 6 \times 10^4$ N/m²

$\therefore$ Force $= pA_2 = (6 \times 10^4)(100 \times 10^{-4}) = 600$ N

(b) Weight of water $= \rho_w g [5A_1 + A_2]$

$= 10^4 [5 \times 10 \times 10^{-4} + 100 \times 10^{-4}] = 150$ N

Water Barometer

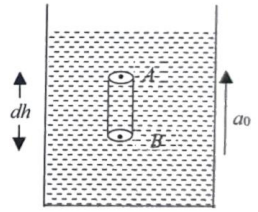
Let us suppose water is used in the barometer instead of mercury.

$h\rho g = 1.013 \times 10^5$ or $h = \dfrac{1.013 \times 10^5}{\rho g}$

The height of the water column in the tube will be 10.3 m. Such a long tube cannot be managed easily, thus water barometer is not feasible.

Liquid in Accelerated Vessel

A Liquid in accelerated vessel can be considered as in the rigid body motion i.e. motion without deformation as though it were a solid body. As in case of static liquid to determine the pressure variation we apply Newton's law, the same is applicable in case of liquid in accelerated vessel also.



$P = P_0 + \rho g_{eff} h$

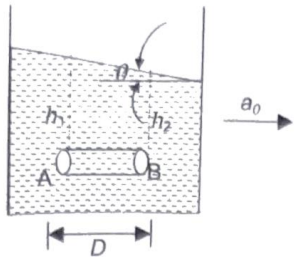
Where $g_{eff} = g + a_0$ in case of upward acceleration and $g_{eff} = g - a_0$ in case of downward acceleration

Also we can say the force of buoyancy F_B on a body in liquid in vertically accelerated vessel is given by $F_B = V \rho g_{eff}$

Where V is volume of liquid displaced by the body.

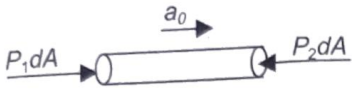
Shape of free surface of liquid in horizontally accelerated vessel

When a vessel filled with liquid accelerates horizontally. We observe its free surface inclined at some angle with horizontal. To find angle θ made by free surface with horizontal consider a horizontal liquid column including two points, A and B at depths h_1 and h_2 from the inclined free surface of liquid as shown in figure. D is length of liquid column. Horizontal forces acting on the liquid column are as shown in diagram, for horizontal motion of column we can write,



$P_1 dA - P_2 dA = \rho(dA)\, D(a_0)$

$(h_1 - h_2)\, g = D\, a_0$



$$\Rightarrow \frac{h_1 - h_2}{D} = \frac{a_0}{g}$$

$$\Rightarrow \tan\theta = \frac{a_0}{g} \Rightarrow \theta = \tan^{-1}\frac{a_0}{g}$$

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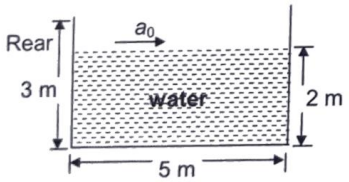
Example 9: An open rectangular tank 5 m × 4 m × 3 m high containing water upto a height of 2 m is accelerated horizontally along the longer side.

(a) Determine the maximum acceleration that can be given without spilling the water.

(b) Calculate the percentage of water spilt over, if this acceleration is increased by 20%

(c) If initially, the tank is closed at the top and is accelerated horizontally by 9 m/s², find the gauge pressure at the bottom of the front wall of the tank. (Take g = 10 m/s²).



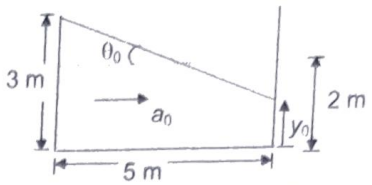
Sol. (a) Volume of water inside the tank remains constant

$$\frac{3+y_0}{2}\times 5\times 4 = 5\times 2\times 4$$

or $y_0 = 1$ m

$$\therefore \tan\theta_0 = \frac{3-1}{5} = 0.4$$

since, $\tan\theta_0 = \frac{a_0}{g}$, therefore $a_0 = 0.4\ g = 4$ m/s²



(b) When acceleration increased by 20%

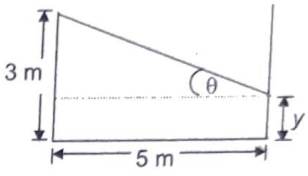
$a = 1.2\ a_0 = 0.48\ g$

$$\therefore \tan\theta = \frac{a}{g} = 0.48$$

Now, $y = 3 - 5\tan\theta = 3 - 5\,(0.48) = 0.6$ m

$$= \frac{4\times 2\times 5 - \frac{(3+0.6)}{2}\times 5\times 4}{2\times 5\times 4} = 0.1$$

percentage of water spilt over = 10%



(c) $a' = 0.9\ g$

$$\tan\theta' = \frac{a'}{g} = 0.9$$

volume of air remains constant

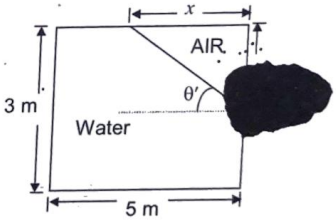
$$4 \times \frac{1}{2}\, yx = (5)\,(1) \times 4$$

since $y = x \tan\theta'$

$$\therefore \frac{1}{2}x^2 \tan\theta' = 5$$

or $x = 3.33$ m; $y = 3.0$ m (so front wall contact with air)

Gauge pressure at the bottom of the front wall p_f = zero



Example 10: A liquid of density ρ is in a bucket that spins with angular velocity ω as shown in figure. Show that the pressure at a radial distance r from the axis is

$$P = P_0 + \frac{\rho\omega^2 r^2}{2}$$



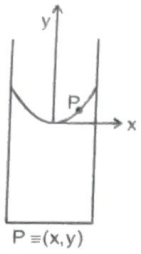
where P_0 is the atmospheric pressure.

Sol. Consider a fluid particle P of mass m at coordinates (x, y). From a non-inertial rotating frame of reference two forces are acting on it,

(i) pseudo force ($m x \omega^2$)

(ii) weight (mg)

in the directions shown in figure.

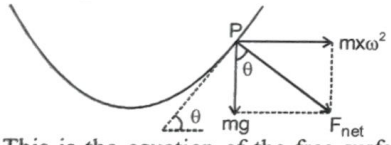


Net force on it should be perpendicular to the free surface (in equilibrium). Hence.

$$\tan\theta = \frac{m x\omega^2}{mg} = \frac{x\omega^2}{g} \quad \text{or} \quad \frac{dy}{dx} = \frac{x\omega^2}{g}$$

$$\therefore \int_0^y dy = \int_0^x \frac{x\omega^2}{g}.dx$$

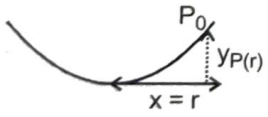
$$\therefore \quad y = \frac{x^2\omega^2}{2g}$$



This is the equation of the free surface of the liquid, which is a parabola.

$$\text{At } x = r, \; y = \frac{r^2\omega^2}{2g}$$

$$\therefore \quad P(r) = P_0 + \rho g y$$



$$\text{or} \quad P(r) = P_0 + \frac{\rho\omega^2 r^2}{2}$$

BUOYANCY: ARCHIMEDES' PRINCIPLE

If a body is partially or wholly immersed in a fluid, it experiences an upward force due to the fluid surrounding it.

The phenomenon of force exerted by fluid on the body is called **buoyancy** and the force is called **buoyant force**.

A body experiences buoyant force whether it floats or sinks, under its own weight or due to other forces applied on it.

Archimedes Principle

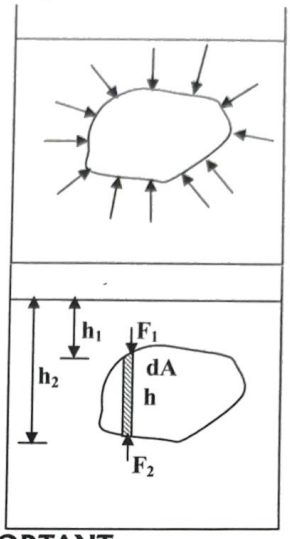
A body immersed in a fluid experiences an upward buoyant force equivalent to the weight of the fluid displaced by it.

$\therefore$ The net upward force is

$$F = \int \rho g dV = \rho V g$$

Hence, for the entire body, **the buoyant force is the weight of the volume of the fluid displaced.**

The buoyant force acts through the centroid of the displaced fluid.



IMPORTANT

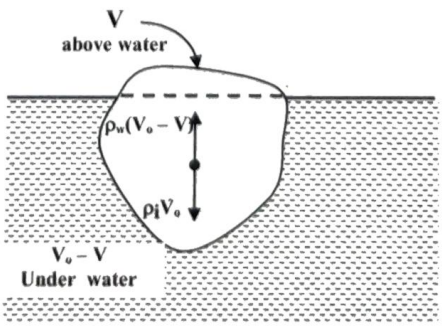
The buoyant force arises because the pressure in the fluid is not uniform; it increases with depth.

An object floats on water if it can displace a volume of water whose weight is greater than that of the object. If the density of the material is less than that of the liquid, it will float even if the material is a uniform solid, such as a block of wood floats on water surface.

If the density of the material is greater than that of water, such as iron, the object can be made to float provided it is not a uniform solid. An iron hulled ship is an example to this case.

TRAIN YOUR BRAIN

Example 11: An iceberg with a density of 920 kgm^{-3} floats on an ocean of density 1025 kg m^{-3}. What fraction of the iceberg is visible?



Sol. Let V be the volume of the iceberg above the water surface, then the volume under water will be $V_o - V$. Under floating conditions, the weight ($\rho_i V_o g$) of the iceberg is balanced by the buoyant force $\rho_w(V_o - V)g$. Thus,

$$\rho_i V_o g = \rho_w(V_o - V)g$$

or $\rho_w V = (\rho_w - \rho_i)V_o$

or $$\frac{V}{V_o} = \left(\frac{\rho_w - \rho_i}{\rho_w}\right)$$

Since $\rho_w = 1025$ kg m^{-3} and $\rho_i = 920$ kg m^{-3}, therefore,

$$\frac{V}{V_o} = \frac{1025 - 920}{1025} = 0.10$$

Hence 10 % of the total volume is visible.

Example 12: When a 2.5 kg crown is immersed in water, it has an apparent weight of 22 N. What is the density of the crown?

Sol. Let W = actual weight of the crown

W′ = apparent weight of the crown

ρ = density of crown

ρ_o = density of water

The buoyant force is given by

$F_B = W - W'$

or $\rho_o Vg = W - W'$

Since $W = \rho Vg$, therefore, $V = \dfrac{W}{\rho g}$

Eliminating V from the above two equations, we get

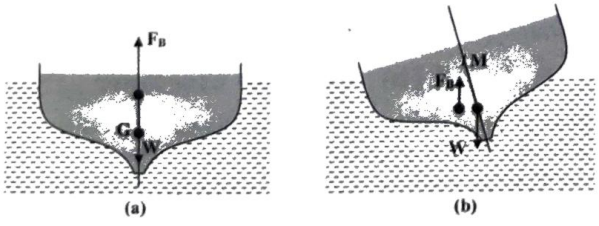
$$\rho = \frac{\rho_o W}{W - W'}$$

Here W = 25 N; W′ = 22 N; $\rho_o = 10^3$ kg m^{-3}

$$\therefore \rho = \frac{(10^3)(25)}{25 - 22} = 8.3 \times 10^3 \text{ kg m}^{-3}$$

STABILITY OF A FLOATING BODY

The stability of a floating body depends on the effective point of application of the buoyant force. The weight of the body acts at its centre of gravity. The buoyant force acts at the centre of gravity of the displaced liquid. This is called the **centre of buoyancy**. Under equilibrium condition the centre of gravity G and the centre of buoyancy B lies along the vertical axis of the body as shown in the figure (a).



(a) (b)

When the body tilts to one side, the centre of buoyancy shifts relative to the centre of gravity as shown in the figure(b). The two forces act along different vertical lines. As a result the buoyant force exerts a torque about the centre of gravity. The line of action of the buoyant force crosses the axis of the body at the point M, called the **metacentre**.

If G is below M, the torque will tend to restore the body to its equilibrium position.

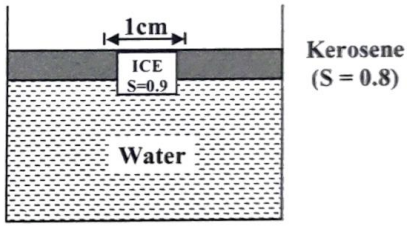
If G is above M, the torque will tend to rotate the body away from its equilibrium position and the body will be unstable.

TRAIN YOUR BRAIN

Example 13: An ice cube of side 1cm is floating at the interface of kerosene and water in a beaker of base area 10 cm^2. The level of kerosene is just covering the top surface of the ice cube.

(a) Find the depth of submergence in the kerosene and that in the water.

(b) Find the change in the total level of the liquid when the whole ice melts into water.



Sol. (a) Condition of floating

$0.8\,\rho_w g h_k + \rho_w g h_w = 0.9\,\rho_w g h$

or $0.8h_k + h_w = 0.9h$ .....(1)

where h_k and h_w be the submerged depth of the ice in the kerosene and water, respectively.

Also, $h_k + h_w = h$ .....(2)

Solving equations (1) and (2), we get

$h_k = 0.5$ cm , $h_w = 0.5$ cm

(b) **1cm^3 (Ice) $\xrightarrow{\text{melts}}$ 0.9cm^3 (Water)**

Fall in the level of kerosene $\Delta h_k = \dfrac{0.5}{A}$

Rise in the level of water $\Delta h_w = \dfrac{0.9 - 0.5}{A} = \dfrac{0.4}{A}$

Net fall in the overall level

$$\Delta h = \frac{0.1}{A} = \frac{0.1}{10} = 0.01cm = 0.1mm$$

FLUID DYNAMICS

In order to describe the motion of a fluid, in principle one might apply Newton's laws to a particle (a small volume element of fluid) and follow its progress in time. This is a difficult approach. Instead, we consider the properties of the fluid, such as velocity and pressure, at fixed points in space.

In order to simplify the discussion we make several assumptions:

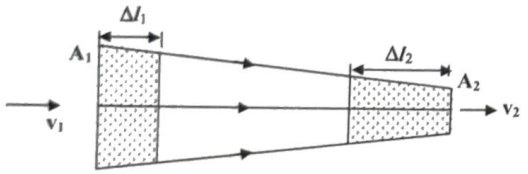
(i) The fluid is non viscous:
There is no loss of energy due to internal friction between adjacent layer in the fluid.

(ii) The flow is steady:
The velocity and pressure at each point are constant in time

(iii) The flow is irrotational:
A tiny paddle wheel placed in the liquid will not rotate. In rotational flow, for example, in eddies, the fluid has net angular momentum about a given point.

The Principle of Continuity

In general, the velocity of a particle will not be constant along a streamline. The density and the cross-sectional area of a tube of flow will also change. Consider two sections of a tube of flow, as shown in figure. The mass of fluid contained in a small cylinder of length Δl_1 and area A_1 is $\Delta m_1 = \rho_1 A_1 \Delta l_1$.



Since fluid does not leave the tube of flow, this mass will later pass through a cylinder of length Δl_2 and area A_2. The mass in this cylinder is $\Delta m_2 = \rho_2 A_2 \Delta l_2$. The lengths Δl_1 and Δl_2 are related to the speeds at the respective locations: $\Delta l_1 = v_1 \Delta t$ and $\Delta l_2 = v_2 \Delta t$. Since no mass is lost or gained.

$\Delta m_1 = \Delta m_2$, and

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

This is called the **equation of continuity**. It is a statement of the conservation of mass.

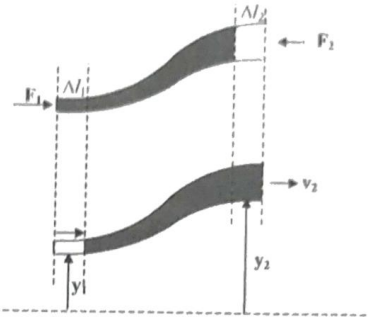
If the fluid is incompressible, its density remains unchanged. This is a good approximation for liquid, but not for gases. If $\rho_1 = \rho_2$, the equation becomes,

$A_1 v_1 = A_1 v_2$

The product Av is the volume rate of flow (m^3/s). Figure shows a pipe whose cross section narrows. From equation we conclude that the speed of a fluid is greatest where the cross-sectional area is the least. **Notice** that the streamlines are close together where the speed is higher.



Bernoulli's Equation



The changes in the potential and kinetic energies are

$\Delta U = \Delta m\, g(y_2 - y_1)$

$$\Delta K = \frac{1}{2}\Delta m\,(v_2^2 - v_1^2)$$

These changes are brought about by the net work done on the system,

$W = \Delta U + \Delta K$

$$(P_1 - P_2)\Delta V = \Delta mg(y_2 - y_1) + \frac{1}{2}\Delta m\,(v_2^2 - v_1^2)$$

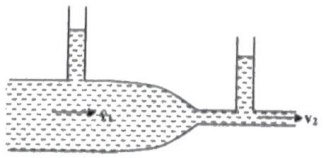
Since the density is $\rho = \Delta m/\Delta V$, we have

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Since the points 1 and 2 can be chosen arbitrarily, we can express this result as **Bernoulli's Equation**

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

Daniel Bernoulli derived this equation in 1738. It is applied to all points along a streamline in a nonviscous, incompressible fluid.



TRAIN YOUR BRAIN

Example 14: A siphon tube is discharging a liquid of specific gravity 0.9 from a reservoir as shown in the figure.

(a) Find the velocity of the liquid through the siphon

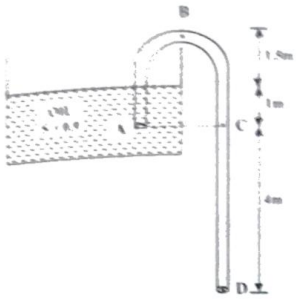
(b) Find the pressure at the highest point B.

(c) Find the pressure at the points A (out side the tube) and C. State and explain the following.

(d) Would the rate of flow be more, less or the same if the liquid were water ?

(e) Is there a limit on the maximum height of B above the liquid level in the reservoir ?

(f) Is there a limit on the vertical depth of the right limb of the siphon ?



Sol. Assume datum at the free surface of the liquid.

(a) Applying Bernoulli's equation on point 1 and 2 , as shown in the figure.

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+y_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+y_2$$

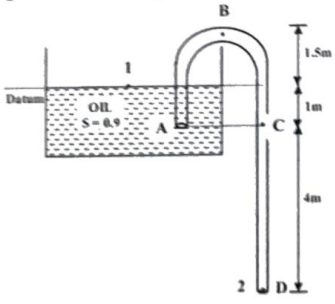
Here $p_1 = p_2 = p_0 = 10^5$ N/m^2, $y_1 = 0$, $y_2 = -5$ m

Since area of the tube is very small as compared to that of the reservoir, therefore,

$v_1 << v_2$, thus $\frac{v_1^2}{2g} \approx 0$

$\therefore\ v_2=\sqrt{2g(y_1-y_2)}=\sqrt{2(10)(5)}=10$ m/s

(b) Applying Bernoulli's equation at D and B.



$\frac{p_B}{\rho g}+\frac{v_B^2}{2g}+y_B=\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+y_1$ Here, $p_1 = 10^5$ N/m^2 ;

$\frac{v_1^2}{2g}\approx 0$;

$y_1 = 0$, $v_B = v_2 = 10$m/s, $y_B = 1.5$ m

$\therefore\ p_B = p_1 - \frac{1}{2}\rho v_2^2 - \rho g y_B$

or $p_B = 10^5 - \frac{1}{2}(900)(10)^2 - (900)(10)(1.5)$

$= 41.5$ kN m^{-2}

(c) Applying Bernoulli's equation at 1 and A

$p_A = p_1 + \rho g (y_1 - y_A)$

or $p_A = 10^5 + (900)(10)(1) = 109$ kN m^{-2}

Applying Bernoulli's equation at 1 and C,

$p_C = p_1 - \frac{1}{2}\rho v_C^2 - \rho g y_C$

$= 10^5 - \frac{1}{2}(900)(10)^2 - (900)(10)(-1)$

$= 10^5 - 45000 + 9000 = 64$ kN/m^2.

(d) The velocity of flow is independent of the density of the liquid , therefore, the discharge would remain the same.

(e) Since the pressure at B is less than atmospheric, the liquid, therefore, has a tendency to get vapourised if the pressure becomes equal to the vapour pressure of it. Thus , $p_B > p_{vapour}$.

(f) The velocity of flow depends on the depth of the point D, below the free surface

$\frac{v^2}{2g} = y_1 - y_2 = H$ and

$p_B = p_1 - \frac{1}{2}\rho v^2 - \rho g y_B = p_1 - \rho g H - \rho g y_B$

For working of siphon , $H \neq 0$, and H should not be high enough so that p_B may not reduce to vapour pressure.

Example 15: A garden hose has an inside cross-sectional area of 3.60 cm^2, and the opening in the nozzle is 0.250 cm^2. If the water velocity is 50 m/s in a segment of the hose that lies on the ground

(a) with what velocity does the water come from the nozzle when it is held 1.50 m above the ground and

(b) What is the water pressure in the hose on the ground?



Sol. (a) We first apply the Equation of Continuity, to find the velocity of the fluid at the nozzle.

$$v_2 = \frac{A_1}{A_2}v_1 = \left(\frac{3.6\,\text{cm}^2}{0.25\,\text{cm}^2}\right)(50\,\text{cms}^{-1}) = 720\ \text{cms}^{-1} = 7.2\,\text{ms}^{-1}$$

(b) We next apply Bernoulli's Equation to find the pressure p_1. We know that

$h_1 = 0$ and $h_2 = 1.5$ m.

The pressure at the nozzle is atmospheric pressure

$p_2 = 1.01 \times 10^5$ Pa.

Solving for p_1 and using the density of water $\rho = 1 \times 10^3$ kg/m^3, we have

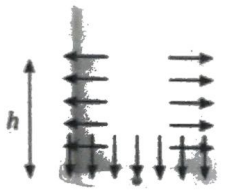
$$p_1 = p_2 + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g(h_2 - h_1)$$

$$= (1.01 \times 10^5) + \frac{1}{2}(1 \times 10^3)[(7.2)^2 - (0.50)^2] +$$

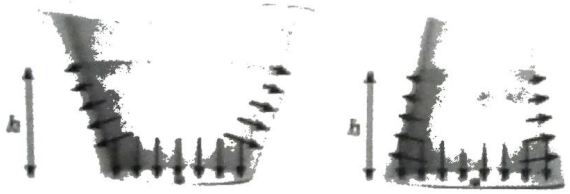
$$(1 \times 10^3)(9.8)(1.5 - 0) = 1.41 \times 10^5 \text{ Pa}$$

HYDROSTATIC PARADOX

Three water containers of different shapes. The base of the containers is of the same area, A_o.



A

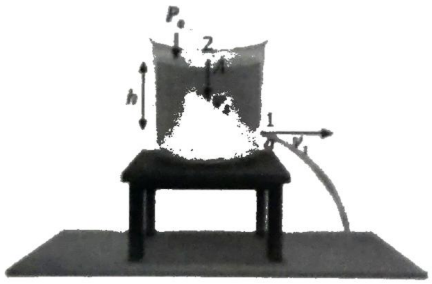


B C

The normal reactions on the bottom of the containers are the same for the containers filled with liquids up to the same heights.

Speed of Efflux

The speed with which the liquid is streaming out from the orifice is known as the speed of efflux.



$\Rightarrow v_1 = \sqrt{2gh}$ = Speed of efflux

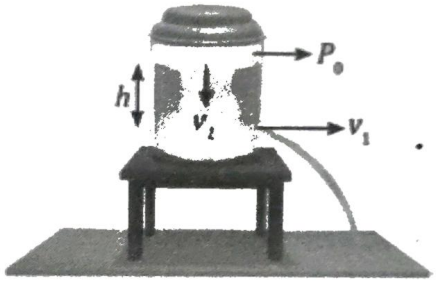
Torricelli's Theorem

The velocity of efflux of a liquid streaming out of an orifice is the same as it would attain if it is allowed to fall freely through the vertical height between the free surface of the liquid and the orifice.

As we know, the free fall velocity is only a function of height, i.e., $v = \sqrt{2gh}$

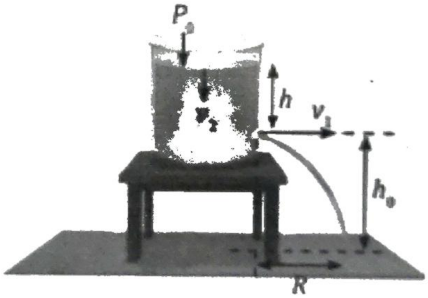
(a) This relation is valid only if the area of the orifice is very less as compared to the area of the container ($A >> a$).

(b) Here, 'h' **(height of the water level above the orifice)** is a function of time. Hence, the velocity of the efflux is also a function of time.



Horizontal Range of a Projectile

We know that the speed of efflux is given by, $v_1 = \sqrt{2gh}$ when the area of cross-section is very small ($a << A$).



Since the height of the orifice from the ground is h_0, the time of flight of the water droplets is given by,

$$T = \sqrt{\frac{2h_0}{g}}$$

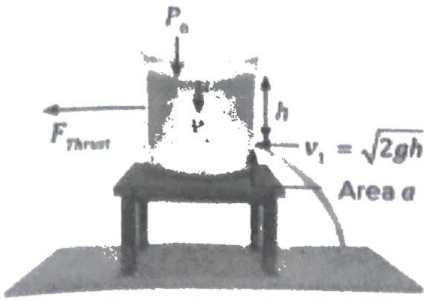
Therefore,

Range (R) = Speed of efflux (v_1) × Time of flight (T)

$$R = \sqrt{2gh} \times \sqrt{\frac{2h_0}{g}}$$

$$\Rightarrow R = 2\sqrt{hh_0}$$

Thrust Force on the Water Tank and its Acceleration



As the water streams out of the orifice, there is a force on the tank known as thrust force (F_{thrust}), which is opposite to the velocity of the water. Therefore, the relative velocity of the water tank is given by,

$$\Rightarrow F_T = 2\rho agh \quad \left[Since\ v_1 = \sqrt{2gh}\right]$$

The acceleration of the water tank becomes,

$$a = \frac{F_T}{m_{total}}$$

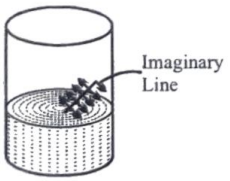
$$\Rightarrow a = \frac{2\rho agh}{\left[m_{water} + m_{tank}\right]}$$

SURFACE TENSION

Surface tension is a property of liquid at rest by virtue of which a liquid surface gets contracted to a minimum area and behaves like a stretched membrane.

Surface tension of a liquid is measured by force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line as shown in figure i.e. surface tension.

$$(T) = \frac{\text{Total force on either of the imaginary line (F)}}{\text{Length of the line } (l)}$$

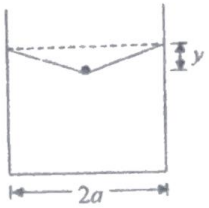


UNIT OF SURFACE TENSION

In C.G. S. system the unit of surface tension is dyne/cm (dyne cm^{-1}) and SI system its units in Nm^{-1} and Dimensions is $[M^1 T^{-2}]$.

TRAIN YOUR BRAIN

Example 16: A container of width 2a is filled with a liquid. A thin wire of weight per unit length λ is gently placed over the liquid surface in the middle of the surface as shown in the figure. As a result the liquid surface is depressed by a distance y (y <<a). Determine the surface tension of the liquid.



Sol. By force balance on the wire,

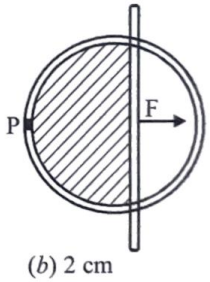
$$2T\cos\theta\ \ell = \lambda\ell g$$

$$\frac{2Ty}{\sqrt{a^2+y^2}}\ell = \lambda\ell g$$

Taking approximation

$$\left[T = \frac{\lambda ag}{2y}\right]$$

Example 17: A circular wire, 10 cm in diameter, with a slider wire on it, is in a horizontal plane. A liquid film is formed, bounded by the wires, on the left side of the slider, as shown. The surface tension of the liquid is 0.1 N/m. An applied force 16 mN, perpendicular to the slider, maintains the film in equilibrium. Ignore the sag in the film. What can be the distance between point P and slider.



(*a*) 8 cm (*b*) 2 cm
(*c*) 5cm (*d*) slider cannot be in equilibrium

Sol. $16\times10^{-3} = 2T \times l$

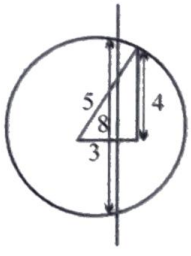
$$l = \frac{16\times10^{-3}}{2\times0.1}$$

$= 8$ cm

$\Rightarrow$ distance from centre = 3 cm

$\Rightarrow$ 5 + 3 = 8cm

or 5 – 3 = 2cm



Example 18: The average force that acts on any molecule from the side of all the others, however, is always equal to zero if the liquid is in equilibrium. This is why the work done to move the liquid from a depth to the surface should also be zero. What is the origin, in this case, of the surface energy?

Sol. The forces of attraction acting on a molecule in the surface layer from all the other molecules produce a resultant directed

downward. The closest neighbours, however, exert a force of repulsion on the molecule which is therefore in equilibrium

Owing to the forces of attraction and repulsion, the density of the liquid is smaller in the surface layer than inside. Indeed, molecule 1 (figure) is acted upon by the force of repulsion from molecule 2 and the forces of attraction from all the other molecules (3, 4, ...). Molecule 2 is acted upon by the forces of repulsion from 3 and 1 and the forces of attraction from the molecules in the deep layers.

As a result, distance 1-2 should be greater than 2-3etc.

(1)

(2)

(3)

(4)

(5)

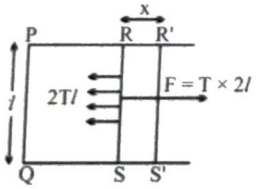
(6)

This course of reasoning is quite approximate (thermal motion, etc is disregarded), but nevertheless it gives a qualitatively correct energy.

The potential energy of surface molecules per unit area of the surface is called surface energy. Unit of surface energy is erg cm^{-2} in C.G. S. system and Jm^{-2} in SI system. Dimensional formula of surface energy is $[ML^0T^{-2}]$ surface energy depends on number of surfaces e.g. a liquid drop is having one liquid air surface while bubble is having two liquid air surface.

So more is surface area more is the surface energy as the molecules on the surface has more energy and as the system tries to remain in minimum energy stage and hence surface area must be minimum.

RELATION BETWEEN SURFACE TENSION AND SURFACE ENERGY



or $T = E = \frac{\Delta W}{A}$

Thus, surface tension is numerically equal to surface energy or work done per unit increase surface area. So, if area is increased surface energy is increased so this is not a natural process.

TRAIN YOUR BRAIN

Example 19: A film of water is formed between two straight parallel wires each 10 cm long and at a separation 0.5 cm. Calculate the work required to increase 1 mm distance between them. Surface tension of water 72×10^{-3} N/m.

Sol. Since this a water film, it has two surface, therefore increase in area

$\Delta S = 2 \times 10 \times 0.1 = 2cm^2$.

$\therefore$ Work required to be done

$W = \Delta S \times T$

$= 2 \times 10^{-4} \times 72 \times 10^{-3}$

$= 144 \times 10^{-7}$ joule

$= 1.44 \times 10^{-5}$ joule.

Example 20: Find the radius of the drop that is formed due to combining of 8 drop of radius R. If surface tension is T. Find the energy released in the process.

Sol. $8 \times \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R'^3 \Rightarrow 8R^3 = R'^3 \Rightarrow R' = 2R$

$A_1 = 8 \times 4\pi R^2 = 32\pi R^2$.

$A_2 = 4\pi(2R)^2 = 16\pi R^2$

$\Delta A = 16\pi R^2$

So, work done $= T \times 16\pi R^2$.]

- Surface area of final drop is less than initial drop.
- Energy is released in this process. So, combining of drop is a natural process.
- If you want to break a drop you have to increase the surface area. So you have to do work so breaking of drop is not a natural process.

Example 21: In the above question if energy released goes in increasing the temperature of the liquid find temperature rise. Given that density of liquid is ρ, specific heat is S.

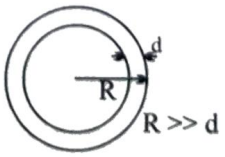
Sol. Heat released $= T \times 16\pi R^2$

$Q = mS\Delta\theta$

$T \times 16\pi R^2 = 8 \times \frac{4}{3}\pi R^3 \rho \times S\Delta\theta$

$\Delta\theta = \frac{T \times 16 \times 3 \times R^2}{8 \times 4R^3 \times \rho \times S} = \frac{3T}{2R\rho S}$

Example 22: A soap bubble has radius R and thickness d (<< R) as shown. It collapse into a spherical drop. Then to energy released, and pressure in that process is



Sol. $2 \times 4\pi R^2 \times d = \frac{4}{3}\pi r^3$

$6R^2d = r^3 \Rightarrow r = \left(6R^2d\right)^{\frac{1}{3}}$

$S' = 4\pi r^2 = \left(6R^2d\right)^{\frac{2}{3}}$

$\Delta U = T \times (2 \times 4\pi R^2 - 4\pi r^2) \quad [\because DU = T \times DA]$

$= 4T\pi \left[2R^2 - \left(6R^2d\right)^{\frac{2}{3}}\right]$

EXCESS PRESSURE INSIDE A LIQUID DROP AND A BUBBLE

1. **Inside a bubble:** Consider a soap bubble of radius r. Let p be the pressure inside the bubble and p_a outside. The

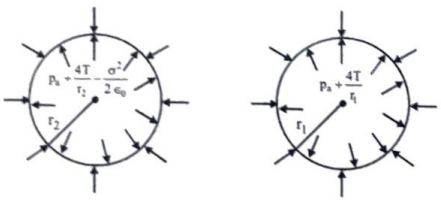
   excess pressure $= p - p_a$ or $(p - p_a) = \frac{4T}{r} = p_{excess}$

   above expression can also be obtained by equation of excess pressure of curve surface by putting $R_1 = R_2$.
2. **Inside the drop:** In a drop, there is only one surface and hence excess pressure can be written as

   $(p - p_a) = \frac{2T}{r} = p_{excess}$
3. **Inside air bubble in a liquid:**

   $(p - p_a) = \frac{2T}{r} = p_{excess}$
4. **A charged bubble:** If bubble is charged, it's radius increases. Bubble has pressure excess due to charge too.

   Initially pressure inside the bubble $= p_a + \frac{4T}{r_1}$



For charge bubble, pressure inside $= p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}$,

where σ surface is surface charge density.

Taking temperature remains constant, then from Boyle's law

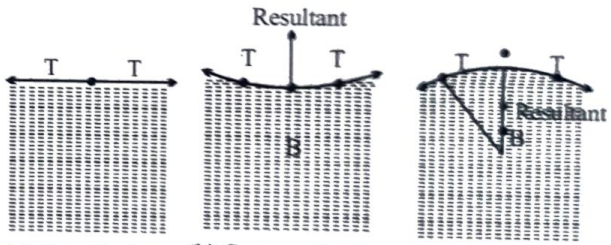
$$\left(p_a + \frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 = \left[p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}\right]\frac{4}{3}\pi r_2^3$$

From above expression the radius of charged drop may be calculated. It can conclude that radius of charged bubble increases i.e. $r_2 > r_1$.

EXCESS OF PRESSURE INSIDE A CURVED SURFACE

1. **Plane Surface:** If the surface of the liquid is plane [as shown in figure (a)], the molecule on the liquid surface is attracted equally in all directions. The resultant force due to surface tension is zero. The pressure, therefore on the liquid surface is normal.
2. **Concave Surface:** If the surface is concave upward [as shown in figure (b)], there will be upward resultant force due to surface tension acting on the molecule. Since the molecule on the surface is in equilibrium, there must be an excess of pressure on the concave side in the downward direction to balance the resultant force of surface tension $p_A - p_B = \frac{2T}{r}$.
3. **Convex Surface:** If the surface is convex [as shown figure (c)], the resultant force due to surface tension acts in the downward direction. Since the molecule on the surface are in equilibrium, there must be an excess of pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of surface tension. Hence, there is always in excess of pressure on concave side of a curved surface over that on the convex side.

   $p_B - p_A = \frac{2T}{r}$



(a) Plane Surface (b) Concave Surface (c) Convex Surface

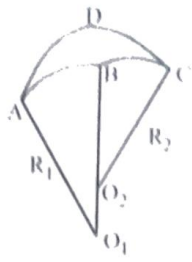
RELATION BETWEEN SURFACE TENSION, RADII OF CURVATURE AND EXCESS PRESSURE ON A CURVED SURFACE

Let us consider a small element ABCD (figure) of a curved liquid surface which is convex on the upper side. R_1 and R_2 are the maximum and minimum radii of curvature respectively. They are called the "principal radii of curvature" of the surface. Let p be the pressure on the concave side.

Then $p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$.

If instead of a liquid surface, we have a liquid film, the above expression will be

$p = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, because a film has two surface.



TRAIN YOUR BRAIN

Example 23: A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76 cm of mercury.

Sol. The total pressure inside the bubble at depth h_1 is (P atmospheric pressure)

$$= (P + h_1\rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is

$$= (P + h_2\rho g) + \frac{2T}{r_2} = P_2$$

Now, according to Boyl's Law

$P_1V_1 = P_2V_2$ where $V_1 = \frac{\pi}{3}\pi r_1^3$ and $V_2 = \frac{\pi}{3}\pi r_2^3$

Hence, we get $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^3$

$$= \left[(P + h_2\rho g) + \frac{2T}{r_2}\right]\frac{4}{3}\pi r_2^3$$

or $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right] r_1^3$

$$= \left[(P + h_2\rho g) + \frac{2T}{r_2}\right] r_2^3$$

Given that: $h_1 = 100$ cm,

$r_1 = 0.1$ mm $= 0.01$ cm,

$r_2 = 0.126$ cm, $T = 567$ dyne/cm,

$P = 76$ cm of mercury

Substituting all the values, we get $h_2 = 9.48$ cm.

THE FORCE OF COHESION

The force of attraction between the molecules of the same substance is called cohesion.

In case of solids, the force of cohesion is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

Example:

(i) Two drops of a liquid combine into one when brought in mutual contact because of the cohesive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

FORCE OF ADHESION

The force of attraction between molecules of different substance is called adhesion.

Example:

(i) Adhesive force enables us to write on the black board with a chalk.

(ii) Adhesive force helps us to write on the paper with ink.

(iii) Large force of adhesion between cement and bricks helps us in construction work.

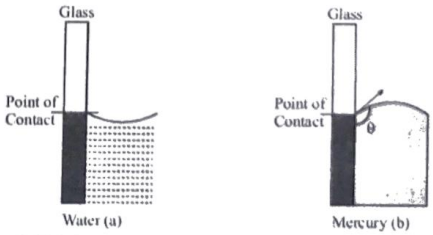
(iv) Due to force of adhesive, water wets the glass plate.

(v) Fevicol and gum are used in joint two surfaces together because of adhesive force.

ANGLE OF CONTACT

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact.

Those liquids which we the wall of the container (say in case of water and glass) have meniscus concave upwards and their value of angle of contact is less than 90° (also called acute angle). However, those liquids which don't wet the walls of the container (say in case of mercury and glass) have meniscus convex upwards and their value of angle of contact is greater than 90° (also called obtuse angle). The angle of contact of mercury with glass about 140°, whereas the angle of contact of water with glass is about 8°. But, for pure water, the angle of contact θ with glass is taken as 0°.



SHAPE OF LIQUID MENISCUS

When a capillary tube or a tube is dipped in a liquid, the liquid surface becomes curved near the point of contact. This curved surface is due to the two forces i.e.

(i) due to the force of cohesion and

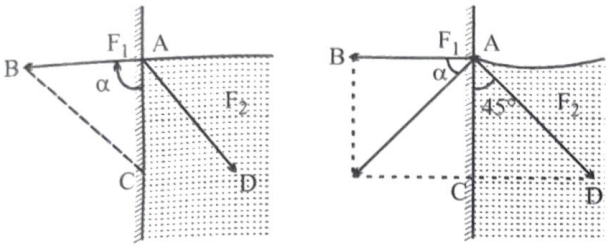
(ii) due to the force of adhesion. The curved surface of the liquid is called meniscus of the liquid. Various forces acting on molecule A are

(iii) Force F_1 due to force of adhesion which acts outwards at right angle to the wall of the tube. This force is represented by AB.

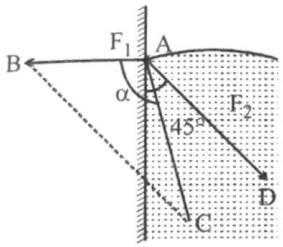
(iv) Force F_2 due to force of cohesion which acts at an angle of 45° to the vertical. This force is represented by AD.

(v) The weight of the molecule A which acts vertically downward along the wall of the tube.

Since the weight of the molecule is negligible as compared to F_1 and F_2 and hence can be neglected. Thus, there are only two forces (F_1 and F_2) acting on the liquid molecules. These forces are inclined at an angle of 135°.



(a) (b)



(c)

The resultant force represented by AC will depend upon the values of F_1 and F_2. Let the resultant force makes an angle α with F_1.

According to parallelogram law of vectors

$$\tan\alpha = \frac{F_2 \sin 135°}{F_1 + F_2 \cos 135°} = \frac{\frac{F_2}{\sqrt{2}}}{F_1 - \frac{F_2}{\sqrt{2}}} = \frac{F_2}{\sqrt{2}F_1 - F_2}$$

SPECIAL CASES

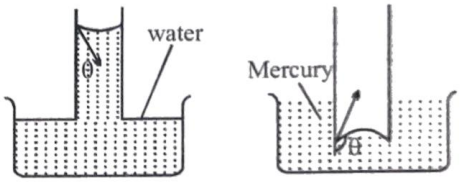
(i) If $F_2 = \sqrt{2}F_1$, then $\tan\alpha = \infty$. $\therefore \alpha = 90°$.
Then the resultant force will act vertically downward and hence the meniscus will be plane or horizontal shown in figure (a). Example: pure water contained in silver capillary tube.

(ii) If $F_2 < \sqrt{2}F_1$, then $\tan\alpha$ is positive. $\therefore$ α is acute angle.
Thus, the resultant will be directed outside the liquid and hence the meniscus will be concave upward shown in figure (b). This is possible is case of liquids which wet the walls of the capillary tube. Example: water in glass capillary tube.

(iii) If $F_2 > \sqrt{2}F_1$, then $\tan\alpha$ is negative.
$\therefore$ α is obtuse angle.
Thus, the resultant will be directed inside the liquid and hence the meniscus will be convex upward shown in figure (c). This is possible in case of liquid which do not wet the walls of the capillary tube. Example: mercury in glass capillary tube.

CAPILLARITY

A glass tube of very fine bore throughout the length of the tube is called capillary tube. If the capillary tube is dipped in water, the water wets the inner side of the tube and rises in it [shown in figure (a)]. If the same capillary tube is dipped in the mercury, then the mercury is depressed [shown in figure (b)]. The phenomenon of rise or fall of liquids in a capillary tube is called capillarity.



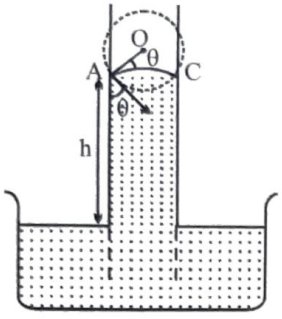
(a) (b)

PARTICLE APPLICATIONS OF CAPILLARITY

1. The oil in a lamp rises in the wick by capillary action.
2. The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tin or nib continuously.
3. Sap and water rise upto the top of the leaves of the tree by capillary action.
4. If one end of the towel dips into a bucket of water and the other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.
5. Ink is absorbed by the blotter due to capillary action.
6. Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.
7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture in the soil, capillaries must be broken up. This is done by ploughing and leveling the fields.
8. Bricks are porous and behave like capillaries.

CAPILLARY RISE (HEIGHT OF A LIQUID IN A CAPILLARY TUBE) (ASCENT FORMULA)

Consider the liquid which wets the wall of the tube, forms a concave meniscus shown in figure. Consider a capillary tube of radius r dipped in a liquid of surface tension T and density p. Let h be the height through which the liquid rises in the tube. Let p be the pressure on the concave side of the meniscus and p_a be the pressure on the convex side of the meniscus. The excess pressure $(p - p_a)$ is given by $(p - p_a) = \frac{2T}{R}$.



Where R is the radius of the meniscus. Due to this excess pressure, the liquid will rise in the capillary tube till it becomes equal to the hydrostatic pressure hρg. Thus in equilibrium state.

Excess pressure = Hydrostatic pressure or $\frac{2T}{R} = h\rho g$.

Let θ be the angle of contact and r be the radius of the capillary tube shown in figure.

$$\text{From}\Delta OAC = \frac{OC}{OA} = \cos\theta \text{ or } R = \frac{r}{\cos\theta}$$

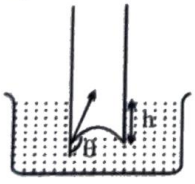
$$\Rightarrow h = \frac{2T\cos\theta}{r\rho g}$$

The expression is called **ASCENT** formula.

Discussion:

(i) For liquids which wet the glass tube or capillary tube, angle of contact $\theta < 90°$. Hence $\cos\theta$ = positive $\Rightarrow$ h = positive.
It means that these liquids rise in the capillary tube. Hence, the liquids which wet capillary tube rise in the capillary tube. For example, water milk, kerosene oil, Petrol etc.

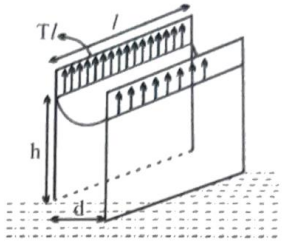
(ii) For liquids which do not wet the glass tube or capillary tube, angle of contact $\theta > 90°$. Hence, $\cos\theta$ = negative $\Rightarrow$ n = negative. Hence, the liquids which do not wet capillary tube are depressed in the capillary tube. For example, mercury.



(iii) T, θ, ρ and g are constant and hence $h \propto \frac{1}{r}$. Thus, the liquid rises more in a narrow tube and less in a wider tube. This is called Jurin's Law.

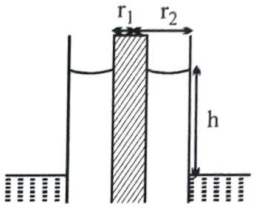
(iv) If two parallel plates with the spacing 'd' are placed in water, then height of rise

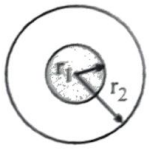
$$\Rightarrow 2Tl = \rho lhdg \quad \Rightarrow h = \frac{2T}{\rho dg}$$



(v) If two concentric tube of radius 'r_1' and 'r_2' (inner one is solid) are placed in water reservoir, then height of rise

$\Rightarrow T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\rho g$

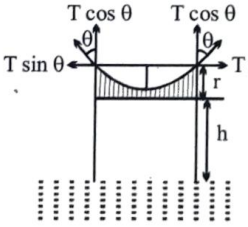




$$\Rightarrow h = \frac{2T}{(r_2 - r_1)\rho g}$$

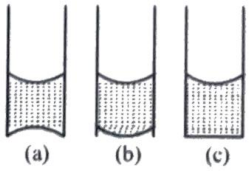
(vi) If weight of the liquid in the meniscus is to be consider:

$$T\cos\theta \times 2\pi r = \left[\pi r^2 h + \frac{1}{3}\pi r^2 \times r\right]\rho g$$



$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}$$

(vii) When capillary tube (radius 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = 2T/R_1$ where R_1 = radius of curvature of upper meniscus.



The hydrostatic pressure $p_2 = h\rho g$ is always directed downwards.

If $p_1 > p_2$ i.e. resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward [figure (a)]. The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$.

If $p_1 < p_2$ i.e. resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward [Figure (b)].

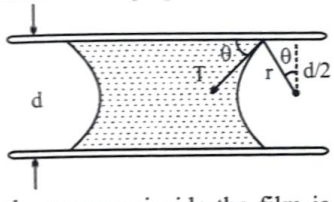
The radius of lower meniscus can be given by

$$\frac{2T}{R_2} = (p_2 - p_1).$$

If $p_1 = p_2$, then is no resulting pressure, then

$p_1 - p_2 = \frac{2T}{R_2} = 0$ or $R_2 = \infty$ i.e. lower surface will be flat [Figure (c)].

(viii) **Liquid between two plates** : When a small drop of water is placed between two glass plates put face to face, it forms a thin film which is concave outward along its boundary. Let 'R' and 'r' be the radii of curvature of the enclosed film in two perpendicular directions.



Hence, the pressure inside the film is less than the atmospheric pressure outside it by an amount p given by

$$p = T\left(\frac{1}{r} + \frac{1}{r=\infty}\right) \text{ and we have, } p = \frac{T}{r}.$$

If d be the distance between the two plates and θ the angle of contact for water and glass, then from the figure

$$\cos\theta = \frac{\frac{d}{2}}{r} \text{ or } \frac{1}{r} = \frac{2\cos\theta}{d}.$$

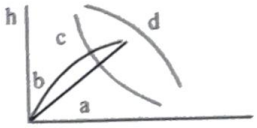
Substituting for $\frac{1}{r}$ in, we get $p = \frac{2T}{d}\cos\theta$.

θ can be taken zero water and glass, i.e. $\cos\theta = 1$. Thus, the upper plate is pressure downward by the atmospheric pressure minus $\frac{2T}{d}$. Hence, the resultant downward pressure acting on the upper plate is $\frac{2T}{d}$. If A be the arc of the plate wetted by the film, the resultant force F pressing the upper plate downward is given by

F = resultant pressure × area = $\frac{2TA}{d}$. For very nearly plane surface, d will be very small and hence the pressing force F very large. Therefore, it will be difficult to separate the two plates normally.

TRAIN YOUR BRAIN

Example 24: Which of the following graphs may represent the relation between the capillary rise h and the radius r of the capillary?



(*a*) a (*b*) b (*c*) c (*d*) d

Ans. (c)

Sol. $h = \frac{2T\cos\theta}{r\rho g}$; $h \propto \frac{1}{r}$

Example 25: A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be–

(*a*) 8 cm (*b*) 6 cm (*c*) 10 cm (*d*) 20 cm

Ans. (d)

Sol. In the freely falling elevator. The water column in capillary tube will be 20 cm full length of capillary.

Example 26: A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. If the height of the liquid in the tube is 0.2 m more than that in the wide tube, calculate the true pressure difference. Density of liquid $= 10^3$ kg/m^3, surface tension $= 72 \times 10^{-3}$ N/m and g = 9.8 m/s^2.

Sol. Let the pressure in the wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2. then pressure just below the meniscus in the wide and narrow tubes respectively are:

$$\left(P_1 - \frac{2T}{r_1}\right) \text{ and } \left(P_2 - \frac{2T}{r_2}\right)$$

[Excess pressure = $\frac{2T}{r}$]

Difference in these pressure

$$= \left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$$

∴ True pressure difference

$$= P_1 - P_2 = h\rho g + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}$$

$$\left[\frac{1}{1.44\times10^{-3}} - \frac{1}{7.2\times10^{-4}}\right]$$

$$= 1.86 \times 10^3 = 1860 \text{ N/m}^2.$$

Example 27: A glass tube of circular cross-section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Give outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension of water 73 dyne/cm and g = 980 cm/sec^2.

Sol. Let l be the length of the tube inside water. The forces acting on the tube are :

$\rho = 10^3 kg/m^3 = gm/cm^3$

(i) Upthrust of water acting upward = $\pi r^2 l \times 1 \times 980$

$= \frac{22}{7} \times (0.14)^2 \times l \times 980 = 60.368\, l$ dyne.

(ii) Weight of the system acting downward = mg $= 0.2 \times 980 = 196$ dyne.

(iii) Force of surface tension acting downward = $2\pi rxt$

$= 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24$ dyne.

Since, the tube is in equilibrium, the upward force is balanced by the downward forces.

i.e. $60.368\, l = 196 + 64.24 = 260.24$

$\therefore\ l = \frac{260.24}{60.368} = 4.31$ cm.

CAPILLARY RISE IN A TUBE OF INSUFFICIENT LENGTH

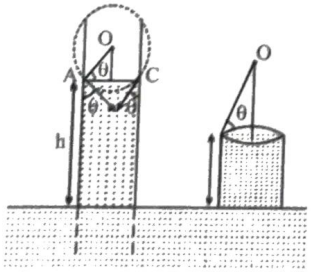
We know, then height through which a liquid rises in the capillary tube of radius r is given by

$\therefore\ h = \frac{2T}{R\rho g}$ or $h\,R = \frac{2T}{\rho g}$ = constant

When the capillary tube is cut and its length is less than h (i.e. h') then the liquid rises upto the top of the tube and spreads in such a way that the radius (R') of the liquid meniscus increases and it becomes more flat so that hR = h'R' = constant. Hence the liquid does not overflow.

If $h' < h$ then $R' > R$

or $\frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$



$\Rightarrow\ \cos\theta' < \cos\theta$

$\Rightarrow\ \theta' > \theta$

TRAIN YOUR BRAIN

Example 28: If a 5 cm long capillary tube with 0.1 mm internal diameter open at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary. (ii) water will rise up to the upper end of capillary (iii) what will overflow out of the upper end of capillary. Explain your answer.

Sol. Given that surface tension of water, T = 75 dyne/cm

Radius $r = \frac{0.1}{2}$ mm = 0.05 mm = 0.005 cm

density $\rho = 1$ gm/cm^3, angle of contact $\theta = 0°$

Let h be the height to which water rise in the capillary tube. Then

$h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times 75\times\cos 0°}{0.005\times 1\times 981}$ cm = 30.58 cm

But length of capillary tube h' = 5 cm

(i) Because $h > \frac{h'}{2}$ therefore the first possibility does not exist.

(ii) Because the tube is of insufficient length therefore the water will rise upto the upper end of the tube.

(iii) The water will not overflow out of the upper end of the capillary. It will rise only upto the upper end of the capillary.

The liquid meniscus will adjust its radius of curvature R' in such a way that

R'h' = Rh

$\left[\because hR = \frac{2T}{\rho g} = \text{constant}\right]$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length.

$\therefore\ R' = \frac{Rh}{h'} = \frac{rh}{h'}\left[\because R = \frac{r}{\cos\theta} = \frac{r}{\cos 0°} = r\right]$

$= \frac{0.005\times 30.58}{5} = 0.0306$ cm.

EFFECT OF TEMPERATURE AND IMPURITIES ON SURFACE TENSION

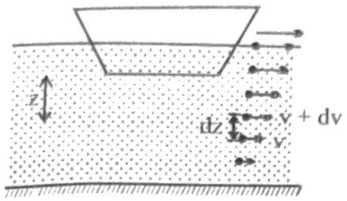
The surface tension of a liquid decreases with the rise in temperature and vice versa. According to Ferguson,

$T = T_0\left(1-\frac{\theta}{\theta_c}\right)^n$ where T_0 is surface tension at 0°C, θ is absolute temperature of the liquid, θ_c is the critical temperature and n is a constant varies slightly from liquid and has means value 1.21. This formula shows that the surface tension becomes zero at the critical temperature, where the while machinery parts get jammed in winter.

The surface tension of a liquid change appreciably with addition of impurities. For example, surface tension of water increases with addition of highly soluble substance like NaCl, $ZnSO_4$ etc. On the other hand surface tension of water gets reduced with addition of sparingly soluble substances like phenol, soap etc.

VISCOSITY

When a layer of a fluid slips or tend to slip on another layer in contact, the two layers exert tangential forces on each other. The directions are such that the relative motion between the layers is opposed, this property of a fluid to oppose relative motion between its layers is called viscosity.



MORE IS VISCOSITY MORE IS RATE OF CHANGE OF VELOCITY

The force of viscosity between two layers of a fluid is proportional to the velocity gradient in the direction perpendicular to the layers. Also the force is proportional to the area of the layer.

Thus, if F is the force exerted by a layer of area A on a layer in contact.

$$F \propto A \text{ and } F \propto \frac{dv}{dz}$$

$$\text{or } F = -\eta A \frac{dv}{dz}$$

The negative sign is included as the force is force is tional in nature and opposes relative motion. The constant proportionality η is called the coefficient of viscosity.

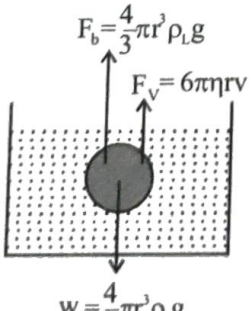
SI unit - N-s/m^2

CGS unit - dyne-s/cm^2 or poise

1 poise = 0.1 N-s/m^2

RMINAL VELOCITY

e viscous force on a ball moving through a fluid is portional to its velocity. When a solid is dropped in a fluid, forces acting on it area



weight W acting vertically downward,
the viscous force F acting vertically upward and
the buoyancy force B acting vertically upward.

The weight W and the buoyancy B are constant but the force F is proportional to the velocity v, initially, the velocity and hence the viscous force F is zero and the solid is accelerated due to the net force W-B. Because of the acceleration, the velocity increases. Accordingly, the viscous force also increases. At a certain instant the viscous force becomes equal to W-B, the net force then becomes zero and the solid falls with constant velocity. The constant velocity is known as the terminal velocity. Consider a spherical body falling through a liquid. Suppose the density of the body = ρ, density of the liquid = σ, radius of the sphere = r and the terminal velocity = v_0. The viscous force is

$F = 6\pi\eta r v_0$

the weight $W = \frac{4}{3}\pi r^3 \rho g$

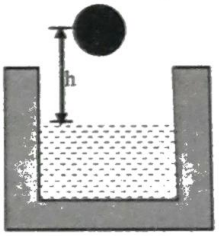
and the buoyancy force $B = \frac{4}{3}\pi r^3 \sigma g$

We have $6\pi\eta r v_0 = W - B = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$

or $v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta}$.

TRAIN YOUR BRAIN

Example 29: A ball of radius r and density ρ falls freely under gravity through a distance h before entering water. Velocity of ball does not change even on entering water. If viscosity of water is η, the value of h is given by:



(a) $\frac{2}{9}r^2\left(\frac{1-\rho}{\eta}\right)g$ (b) $\frac{2}{81}r^2\left(\frac{\rho-1}{\eta}\right)g$

(c) $\frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$ (d) $\frac{2}{9}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$

Ans. (c)

Sol. Refer to figure given in question

Velocity of ball does not change on entering water

Velocity attained $v = \sqrt{2gh} = V_T$

$$\therefore \sqrt{2gh} = \frac{2}{9}\frac{r^2(\rho-1)g}{\eta} \; ; \; \sigma = 1 \text{ for water}$$

$$\Rightarrow 2gh = \frac{4}{81} r^4 \left[\frac{(\rho-1)}{\eta}\right]^2 g^2$$

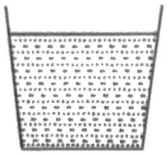
$$h = \frac{2}{81} r^4 \left[\frac{(\rho-1)}{\eta}\right]^2 g$$

Topicwise Questions

FLUID MECHANICS

Measurement and Calculation of Pressure

1. A liquid of mass 1 kg is filled in a flask as shown in figure. The force exerted by the flask on the liquid is ($g = 10\ m/s^2$) [Neglect atmospheric pressure]



(*a*) 10 N (*b*) greater than 10 N
(*c*) less than 10 N (*d*) zero

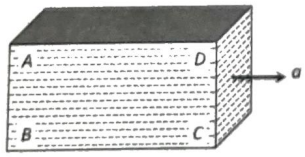
2. Density of ice is ρ water is σ. What will be the decrease in volume when a mass M of ice melts

(*a*) $\frac{M}{\sigma-\rho}$ (*b*) $\frac{\sigma-\rho}{M}$

(*c*) $M\left[\frac{1}{\rho}-\frac{1}{\sigma}\right]$ (*d*) $\frac{1}{M}\left[\frac{1}{\rho}-\frac{1}{\sigma}\right]$

3. The volume of an air bubble becomes three times as it rises from the bottom of a lake to its surface. Assuming atmospheric pressure to be 75 cm of Hg and the density of water to be 1/10 of the density of mercury, the depth of the lake is
(*a*) 5 m (*b*) 10 m
(*c*) 15 m (*d*) 20 m

4. A closed rectangular tank is completely filled with water and is accelerated horizontally with an acceleration *a* towards right. Pressure is (i) maximum at, and (ii) minimum at

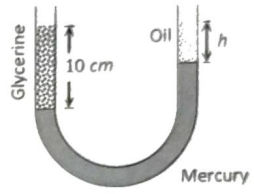


(*a*) (i) B (ii) D (*b*) (i) C (ii) D
(*c*) (i) B (ii) C (*d*) (i) *B* (ii) *A*

5. Why the dam of water reservoir is thick at the bottom?
(*a*) Quantity of water increases with depth
(*b*) Density of water increases with depth
(*c*) Pressure of water increases with depth
(*d*) Temperature of water increases with depth

PASCAL LAW

6. A vertical U-tube of uniform inner cross section contains mercury in both sides of its arms. A glycerin (density = 1.3 g/cm^3) column of length 10 cm is introduced into one of its arms. Oil of density 0.8 gm/cm^3 is poured into the other arm until the upper surfaces of the oil and glycerin are in the same horizontal level. Find the length of the oil column. Density of mercury = 13.6 g/cm^3



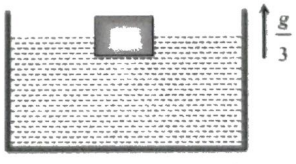
(*a*) 10.4 cm (*b*) 8.2 cm
(*c*) 7.2 cm (*d*) 9.7 cm

Archemedies Principle and Force of Buoyancy

7. A concrete sphere of radius R has a cavity of radius r which is packed with sawdust. The specific gravities of concrete and sawdust are respectively 2.4 and 0.3 for this sphere to float with its entire volume submerged under water. Ratio of mass of concrete to mass of sawdust will be
(*a*) 8 (*b*) 4
(*c*) 3 (*d*) Zero

8. A metallic block of density 5 gm cm^{-3} and having dimensions 5 cm × 5 cm × 5 cm is weighed in water. Its apparent weight will be
(*a*) 5 × 5 × 5 × 5 gf (*b*) 4 × 4 × 4 × 4 gf
(*c*) 5 × 4 × 4 × 4 gf (*d*) 4 × 5 × 5 × 5 gf

9. A cubical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upwards with acceleration of g/3, the fraction of volume immersed in the liquid will be



(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{8}$
(*c*) $\frac{2}{3}$ (*d*) $\frac{3}{4}$

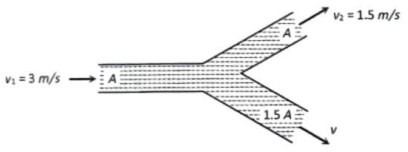
10. A boat carrying steel balls is floating on the surface of water in a tank. If the balls are thrown into the tank one by one, how will it affect the level of water?
(*a*) It will remain unchanged
(*b*) It will rise
(*c*) It will fall
(*d*) First it will first rise and then fall

Continuity Equation & Bernoulli Theorem and Their Application

11. In which one of the following cases will the liquid flow in a pipe be most streamlined?
(*a*) Liquid of high viscosity and high density flowing through a pipe of small radius
(*b*) Liquid of high viscosity and low density flowing through a pipe of small radius
(*c*) Liquid of low viscosity and low density flowing through a pipe of large radius
(*d*) Liquid of low viscosity and high density flowing through a pipe of large radius

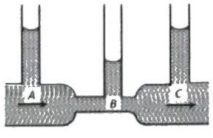
12. Two water pipes of diameters 2 cm and 4 cm are connected with the main supply line. The velocity of flow of water in the pipe of 2 cm diameter is
(*a*) 4 times that in the other pipe
(*b*) $\frac{1}{4}$ times that in the other pipe
(*c*) 2 times that in the other pipe
(*d*) $\frac{1}{2}$ times that in the other pipe

13. An incompressible liquid flows through a horizontal tube as shown in the following fig. Then the velocity v of the fluid is



(*a*) 3.0 m/s (*b*) 1.5 m/s
(*c*) 1.0 m/s (*d*) 2.25 m/s

14. In the following fig. is shown the flow of liquid through a horizontal pipe. Three tubes A, B and C are connected to the pipe. The radii of the tubes A, B and C at the junction are respectively 2 cm, 1 cm and 2 cm. It can be said that the

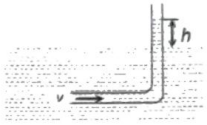


(*a*) Height of the liquid in the tube A is maximum
(*b*) Height of the liquid in the tubes A and B is the same
(*c*) Height of the liquid in all the three tubes is the same
(*d*) Height of the liquid in the tubes A and C is the same

15. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in m/s) through a small hole on the side wall of the cylinder near its bottom is
(*a*) 10 (*b*) 20
(*c*) 25.5 (*d*) 5

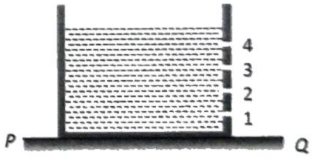
16. Fig. represents vertical sections of four wings moving horizontally in air. In which case the force is upwards
(*a*) 
(*b*) 
(*c*) 
(*d*) 

17. An L-shaped glass tube is just immersed in flowing water such that its opening is pointing against flowing water. If the speed of water current is v, then



(*a*) The water in the tube rises to height $\frac{v^2}{2g}$
(*b*) The water in the tube rises to height $\frac{g}{2v^2}$
(*c*) The water in the tube does not rise at all
(*d*) None of these

18. A cylindrical vessel of 90 cm height is kept filled upto the brim. It has four holes 1, 2, 3, 4 which are respectively at heights of 20 cm, 30 cm, 45 cm and 50 cm from the horizontal floor P. The water falling at the maximum horizontal distance from the vessel comes from



(*a*) Hole number 4 (*b*) Hole number 3
(*c*) Hole number 2 (*d*) Hole number 1

19. A streamlined body falls through air from a height h on the surface of a liquid. If d and D(D > d) represents the densities of the material of the body and liquid respectively, then the time after which the body will be instantaneously at rest, is
(*a*) $\sqrt{\frac{2h}{g}}$ (*b*) $\sqrt{\frac{2h}{g}\cdot\frac{D}{d}}$
(*c*) $\sqrt{\frac{2h}{g}\cdot\frac{d}{D}}$ (*d*) $\sqrt{\frac{2h}{g}}\left(\frac{d}{D-d}\right)$

SURFACE TENSION

Surface Tension, Surface Energy

20. The value of surface tension of a liquid at critical temperature is
(a) Zero (b) Infinite
(c) Between 0 and ∞ (d) Can not be determined

21. Soap helps in cleaning clothes, because
(a) Chemicals of soap change
(b) It increases the surface tension of the solution
(c) It absorbs the dirt
(d) It lowers the surface tension of the solution

22. A pin or a needle floats on the surface of water, the reason for this is
(a) Surface tension (b) Less weight
(c) Upthrust of liquid (d) None of the above

23. Coatings used on raincoat are waterproof because
(a) Water is absorbed by the coating
(b) Cohesive force becomes greater
(c) Water is not scattered away by the coating
(d) Angle of contact decreases

24. The surface tension of a liquid
(a) Increases with area
(b) Decreases with area
(c) Increase with temperature
(d) Decrease with temperature

25. A thin metal disc of radius r floats on water surface and bends the surface downwards along the perimeter making an angle θ with vertical edge of the disc. If the disc displaces a weight of water W and surface tension of water is T, then the weight of metal disc is
(a) $2\pi rT + W$ (b) $2\pi rT \cos\theta - W$
(c) $2\pi rT \cos\theta + W$ (d) $W - 2\pi rT \cos\theta$

26. A 10 cm long wire is placed horizontally on the surface of water and is gently pulled up with a force of 2×10^{-2} N to keep the wire in equilibrium. The surface tension, in Nm^{-1}, of water is
(a) 0.1 (b) 0.2
(c) 0.001 (d) 0.002

27. Two droplets merge with each other and forms a large droplet. In this process
(a) Energy is liberated
(b) Energy is absorbed
(c) Neither liberated nor absorbed
(d) Some mass is converted into energy

28. Radius of a soap bubble is 'r', surface tension of soap solution is T. Then without increasing the temperature, how much energy will be needed to double its radius?
(a) $4\pi r^2T$ (b) $2\pi r^2T$
(c) $12\pi r^2T$ (d) $24\pi r^2T$

29. If T is the surface tension of soap solution, the amount of work done in blowing a soap bubble from a diameter D to 2D is
(a) $2\pi D^2T$ (b) $4\pi D^2T$
(c) $6\pi D^2T$ (d) $8\pi D^2T$

30. The radius of a soap bubble is increased from $\frac{1}{\sqrt{\pi}}$ cm to $\frac{2}{\sqrt{\pi}}$ cm. If the surface tension of water is 30 dynes per cm, then the work done will be
(a) 180 ergs (b) 360 ergs
(c) 720 ergs (d) 960 ergs

CAPILLARY RISE

31. Water does not wet an oily glass because
(a) Cohesive force of oil >> adhesive force between oil and glass
(b) Cohesive force of oil > cohesive force of water
(c) Oil repels water
(d) Cohesive force for water > adhesive force between water and oil molecules

32. A water drop takes the shape of a sphere in a oil while the oil drop spreads in water, because
(a) C.F. for water > A.F. for water and oil
(b) C.F. for oil > A.F. for water and oil
(c) C.F. for oil < A.F. for water and oil
(d) None of these
(A.F. = adhesive force C.F. = cohesive force)

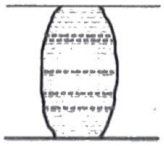
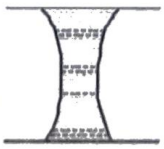
33. Which of the fact is not due to surface tension?
(a) Dancing of a camphor piece over the surface of water
(b) Small mercury drop itself becomes spherical
(c) A liquid surface comes at rest after stirring
(d) Mercury does not wet the glass vessel

34. Mercury does not wet glass, wood or iron because
(a) Cohesive force is less than adhesive force
(b) Cohesive force is greater than adhesive force
(c) Angle of contact is less than 90°
(d) Cohesive force is equal to adhesive force

35. A mercury drop does not spread on a glass plate because the angle of contact between glass and mercury is
(a) Acute (b) Obtuse
(c) Zero (d) 90°

36. A glass plate is partly dipped vertically in the mercury and the angle of contact is measured. If the plate is inclined, then the angle of contact will
(a) Increase (b) Remain unchanged
(c) Increase or decrease (d) Decrease

37. If a water drop is kept between two glass plates, then its shape is

(a) 
(b) 
(c) 
(d) none of these

EXCESS PRESSURE IN DROP AND BUBBLE

38. A soap bubble assumes a spherical surface. Which of the following statement is wrong?
(a) The soap film consists of two surface layers of molecules back to back
(b) The bubble encloses air inside it
(c) The pressure of air inside the bubble is less than the atmospheric pressure; that is why the atmospheric pressure has compressed it equally from all sides to give it a spherical shape
(d) Because of the elastic property of the film, it will tend to shrink to as small a surface area as possible for the volume it has enclosed

39. If two soap bubbles of different radii are in connected with each other
(a) Air flows from larger bubble into the smaller one
(b) The size of the bubbles remains the same
(c) Air flows from the smaller bubble into the large one and the larger bubble grows at the expense of the smaller one
(d) The air flows from the larger

40. The excess of pressure inside a soap bubble than that of the outer pressure is
(a) $\frac{2T}{r}$ (b) $\frac{4T}{r}$
(c) $\frac{T}{2r}$ (d) $\frac{T}{r}$

41. A soap bubble in vacuum has a radius of 3 cm and another soap bubble in vacuum has a radius of 4 cm. If the two bubbles coalesce under isothermal condition, then the radius of the new bubble is
(a) 2.3 cm (b) 4.5 cm
(c) 5 cm (d) 7 cm

42. Excess pressure of one soap bubble is four times more than the other. Then the ratio of volume of first bubble to another one is
(a) 1 : 64 (b) 1 : 4
(c) 64 : 1 (d) 1 : 2

Learning Plus

FLUID MECHANICS

1. The density of ice is x gm/cc and that of water is y gm/cc. What is the change in volume in cc, when m gm of ice melts?
(a) M (y – x) (b) (y – x)/m
(c) mxy (x – y) (d) m (1/y – 1/x)

2. A boy carries a fish in one hand and a bucket (not full) of water in the other hand. If he places the fish in the bucket, the weight now carried by him (assume that water does not spill) :
(a) is less than before
(b) is more than before
(c) is the same as before
(d) depends upon his speed

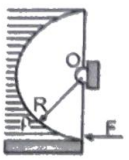
3. A uniformly tapering vessel shown in Fig. is filled with liquid of density 900 kg/m^3. The force that acts on the base of the vessel due to liquid is (Take g = 10 m/s^2) -



(a) 3.6 N (b) 7.2 N
(c) 9.0 N (d) 12.6 N

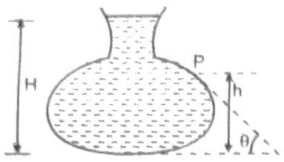
4. An inverted bell lying at the bottom of a lake 47.6 m deep has 50 cm^3 of air trapped in it. The bell is brought to the surface of the lake. The volume of the trapped air will be (atmospheric pressure = 70 cm of Hg and density of Hg = 13.6 g/cm^3)
(a) 350 cm^3 (b) 300 cm^3
(c) 250 cm^3 (d) 22 cm^3

5. A light semi cylindrical gate of radius R is piovted at its mid point O, of the diameter as shown in the figure holding liquid of density r. The force F required to prevent the rotation of the gate is equal to



(a) 2pR^{3}rg
(b) 2rgR^3/
(c) $\frac{2R^2/\rho g}{3}$
(d) none of these

6. Figure here shows the vertical cross-section of a vessel filled with a liquid of density r. The normal thrust per unit area on the walls of the vessel at point. P, as shown, will be

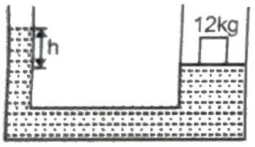


(a) h r g (b) H r g
(c) (H – h) r g (d) (H – h) r g cosq

7. Two stretched membranes of areas 2 and 3 m^2 are placed in a liquid at the same depth. The ratio of the pressure on them is -
(a) 1 : 1 (b) 2 : 3
(c) $\sqrt{2} : \sqrt{3}$ (d) $2^2 : 3^2$

8. In a hydraulic lift, used at a service station the radius of the large and small piston are in the ratio of 20 : 1. What weight placed on the small piston will be sufficient to lift a car of mass 1500 kg ?
(a) 3.75 kg (b) 37.5 kg
(c) 7.5 kg (d) 75 kg.

9. The area of cross-section of the wider tube shown in figure is 800 cm^2. If a mass of 12 kg is placed on the massless piston, the difference in heights h in the level of water in the two tubes is:



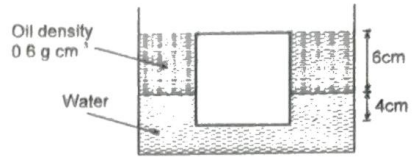
(a) 10 cm (b) 6 cm
(c) 15 cm (d) 2 cm

10. The reading of a spring balance when a block is suspended from it in air is 60 newton. This reading is changed to 40 newton when the block is submerged in water. The specific gravity of the block must be therefore:
(a) 3 (b) 2
(c) 6 (d) 3/2

11. An ice block floats in a liquid whose density is less than water. A part of block is outside the liquid. When whole of ice has melted, the liquid level will -
(a) rise
(b) go down
(c) remain same
(d) first rise then go down

12. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 36 g and its density is 9 g/cc. If the mass of the other is 48 g, its density in g/cc is :
(a) 4/3 (b) 3/2
(c) 3 (d) 5

13. A cubical block of wood 10 cm on a side, floats at the interface of oil and water as shown in figure. The density of oil is 0.6 g cm^{-3} and density of water is 1 g cm^{-3}. The mass of the block is



(a) 706 g (b) 607 g
(c) 760 g (d) 670 g

14. A tank with length 10 m, breadth 8 m and depth 6m is filled with water to the top. If g = 10 m s^{-2} and density of water is 1000 kg m^{-3}, then the thrust on the bottom is
(a) $6 \times 1000 \times 10 \times 80$ N (b) $3 \times 1000 \times 10 \times 48$ N
(c) $3 \times 1000 \times 10 \times 60$ N (d) $3 \times 1000 \times 10 \times 80$ N

15. A piece of steel has a weight W in air, W_1 when completely immersed in water and W_2 when completely immersed in an unknown liquid. The relative density (specific gravity) of liquid is :
(a) $\frac{W - W_1}{W - W_2}$ (b) $\frac{W - W_2}{W - W_1}$
(c) $\frac{W_1 - W_2}{W - W_1}$ (d) $\frac{W_1 - W_2}{W - W_2}$

16. A metal ball of density 7800 kg/m^3 is suspected to have a large number of cavities. It weighs 9.8 kg when weighed directly on a balance and 1.5 kg less when immersed in water. The fraction by volume of the cavities in the metal ball is approximately :
(a) 20% (b) 30%
(c) 16% (d) 11%

17. A sphere of radius R and made of material of relative density σ has a concentric cavity of radius r. It just floats when placed in a tank full of water. The value of the ratio R/r will be
(a) $\left(\frac{\sigma}{\sigma - 1}\right)^{1/3}$ (b) $\left(\frac{\sigma - 1}{\sigma}\right)^{1/3}$
(c) $\left(\frac{\sigma + 1}{\sigma}\right)^{1/3}$ (d) $\left(\frac{\sigma - 1}{\sigma + 1}\right)^{1/3}$

18. A beaker containing water is placed on the platform of a spring balance. The balance reads 1.5 kg. A stone of mass 0.5 kg and density 500 kg/m^3 is immersed in water without touching the walls of beaker. What will be the balance reading now?

(*a*) 2 kg (*b*) 2.5 kg

(*c*) 1 kg (*d*) 3 kg

19. A cylindrical block of area of cross-section A and of material of density ρ is placed in a liquid of density one-third of density of block. The block compresses a spring and compression in the spring is one-third of the length of the block. If acceleration due to gravity is g, the spring constant of the spring is



(*a*) ρAg (*b*) $2\rho Ag$

(*c*) $2\rho Ag/3$ (*d*) $\rho Ag/3$

20. Ram and Shyam are having the same weight when measured outside the water. When measured under water, it is found that weight of Ram is more than that of Shyam, then we can say that

(*a*) Ram is having more fat content than Shyam

(*b*) Shyam is having more fat content that Ram

(*c*) Ram and Shyam both are having the same fat content

(*d*) None of these

21. Salt water is denser than fresh water. If you were immersed fully first in salt water and then in fresh water and weighed, then

(*a*) Reading would be less in salt water

(*b*) Reading would be more in salt water

(*c*) Reading would be the same in both the cases

(*d*) reading could be less or more

22. A person of mass 165 Kg having one fourth of his volume consisting of fat (relative density 0.4) and rest of the volume consisting of everything else (average relative density $\frac{4}{3}$) is weighed under water by the spring balance. The reading shown by the spring balance is -

(*a*) 15 N (*b*) 65 N

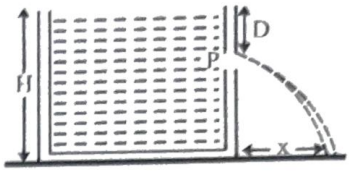
(*c*) 150 N (*d*) 165 N

23. In the above question if the spring is cut, the acceleration of the person just after cutting the spring is

(*a*) zero (*b*) 1 m/s^2

(*c*) 9.8 m/s^2 (*d*) 0.91 m/s^2

24. A tank is filled with water up to height H. Water is allowed to come out of a hole P in one of the walls at a depth D below the surface of water as shown in the figure. Express the horizontal distance x in terms of H and D:

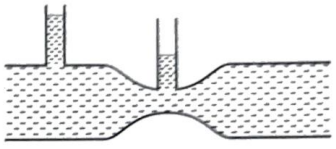


(*a*) $x = \sqrt{D(H-D)}$ (*b*) $x = \sqrt{\frac{D(H-D)}{2}}$

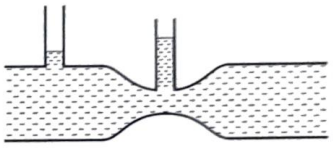
(*c*) $x = 2\sqrt{D(H-D)}$ (*d*) $x = 4\sqrt{D(H-D)}$

25. For a fluid which is flowing steadily in the figure shown, the level in the vertical tubes is best represented by

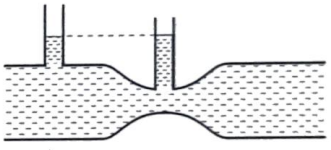
(*a*)



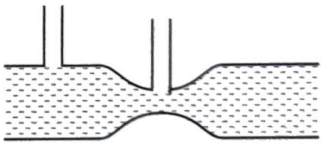
(*b*)



(*c*)



(*d*)



26. A cylindrical tank of height 0.4 m is open at the top and has a diameter 0.16 m. Water is filled in it up to a height of 0.16 m. How long it will take to empty the tank through a hole of radius 5×10^{-3} m in its bottom ?

(*a*) 46.26 sec. (*b*) 4.6 sec.

(*c*) 462.6 sec. (*d*) 0.46 sec.

27. Water is flowing in a horizontal pipe of non-uniform cross-section. At the most contracted place of the pipe-

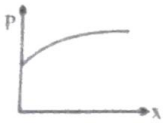
(*a*) Velocity of water will be maximum and pressure minimum

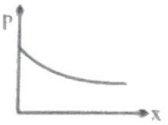
(*b*) Pressure of water will be maximum and velocity minimum

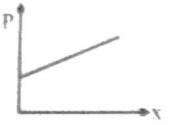
(*c*) Both pressure and velocity of water will be maximum

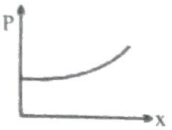
(*d*) Both pressure and velocity of water will be minimum

28. The cross sectional area of a horizontal tube increases along its length linearly, as we move in the direction of flow. The variation of pressure, as we move along its length in the direction of flow (x-direction), is best depicted by which of the following graphs

(*a*) 

(*b*) 

(*c*) 

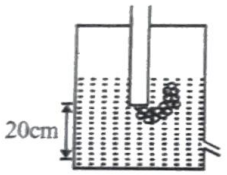
(*d*) 

29. Water is flowing steadily through a horizontal tube of non uniform cross-section. If the pressure of water is 4×10^4 N/m^2 at a point where cross-section is 0.02 m^2 and velocity of flow is 2 m/s, what is pressure at a point where cross-section reduces to 0.01 m^2

(*a*) 1.4×10^4 N/m^2 (*b*) 3.4×10^4 N/m^2

(*c*) 2.4×10^{-4} N/m^2 (*d*) none of these

30. A tube is attached as shown in closed vessel containing water. The velocity of water coming out from a small hole is:



(*a*) $\sqrt{2}$ m/s

(*b*) 2 m/s

(*c*) depends on pressure of air inside vessel

(*d*) None of these

31. A cylindrical vessel open at the open at the top is 20 cm high and 10 cm in diameter. A circular hole whose cross-sectional area 1 cm^2 is cut at the centre of the bottom of the vessel. Water flows from a tube above it into the vessel at the rate 100 cm^{3}s^{-1}. The height of water in the vessel under state is (Take g = 1000 cms^{-2})

(*a*) 20 cm (*b*) 15 cm

(*c*) 10 cm (*d*) 5 cm

32. A fire hydrant delivers water of density ρ at a volume rate L. The water travels vertically upward through the hydrant and then does 90° turn to emerge horizontally at speed V. The pipe and nozzle have uniform cross-section throughout. The force exerted by the water on the corner of the hydrant is:



(*a*) ρVL (*b*) zero

(*c*) 2ρVL (*d*) $\sqrt{2}\,\rho$VL

33. A vertical tank, open at the top, is filled with a liquid and rests on a smooth horizontal surface. A small hole is opened at the centre of one side of the tank. The area of cross-section of the tank is N times the area of the hole, where N is a large number. Neglect mass of the tank itself. The initial acceleration of the tank is

(*a*) $\frac{g}{2N}$ (*b*) $\frac{g}{\sqrt{2}\,N}$

(*c*) $\frac{g}{N}$ (*d*) $\frac{g}{2\sqrt{N}}$

34. A cyclindrical vessel of cross-sectional area 1000 cm^2, is fitted with a frictionless piston of mass 10 kg, and filled with water completely. A small hole of cross-sectional area 10 mm^2 is opened at a point 50 cm deep from the lower surface of the piston. The velocity of efflux from the hole will be

(*a*) 10.5 m/s (*b*) 3.4 m/s

(*c*) 0.8 m/s (*d*) 0.2 m/s

35. A horizontal right angle pipe bend has cross-sectional area = 10 cm^2 and water flows through it at speed = 20 m/s. The force on the pipe-bend due to the turning of water is :

(*a*) 565.7 N (*b*) 400 N

(*c*) 20 N (*d*) 282.8 N

36. Water is pumped from a depth of 10 m and delivered through a pipe of cross section 10^{-2}m^2. If it is needed to deliver a volume of 10^{-1}m^3 per second the power required will be:

(*a*) 10 kW (*b*) 9.8 kW

(*c*) 15 kW (*d*) 4.9 kW

37. Which of the following is not an assumption for an ideal fluid flow for which Bernoulli's principle is valid?

(*a*) Steady flow (*b*) Incompressible

(*c*) Viscous (*d*) Irrotational

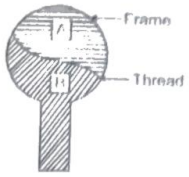
38. Fountains usually seen in gardens are generated by a wide pipe with an enclosure at one end having many small holes. Consider one such fountain which is produced by a pipe of internal diameter 2 cm in which water flows at a rate 3ms^{-1}. The enclosure has 100 holes each of diameter 0.05 cm. The velocity of water coming out of the holes is (in ms^{-1}):

(*a*) 0.48 (*b*) 96

(*c*) 24 (*d*) 48

39. The rate of flowing of water from the orifice in a wall of a tank will be more if the orific is
(*a*) Near the bottom
(*b*) Near the upper end
(*c*) Exactly in the middle
(*d*) Does not depend upon the position of orific

40. A thread is tied slightly loose to a wire frame as shown in the figure. And the frame is dipped into a soap solution and taken out. The frame is completely covered with the film. When the portion A is punctured with a pin, the thread:



(*a*) becomes convex towards A
(*b*) becomes concave towards A
(*c*) remains in the initial position
(*d*) either (*a*) or (*b*) depending on size of A w.r.t. B

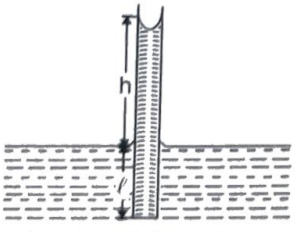
41. The work done to get n smaller equal size spherical drops from a bigger size spherical drop of water is proportional to :

(*a*) $\left(\frac{1}{n^{2/3}}\right)-1$ (*b*) $\left(\frac{1}{n^{1/3}}\right)-1$

(*c*) $n^{1/3}-1$ (*d*) $n^{4/3}-1$

42. A capillary tube of radius R is immersed in water and water rises in it to a height H. Mass of water in capillary tube is M. if the radius of the tube is doubled, mass of water that will rise in capillary tube will be
(*a*) 2M (*b*) M
(*c*) $\frac{M}{2}$ (*d*) 4M

43. Water rises to a height h in a capillary tube lowered vertically into water to a depth ℓ as shown in the figure. The lower end of the tube is closed, the tube is then taken out of the water and opened again. The length of the water column remaining in the tube will be :



(*a*) 2h if $\ell \geq h$ and $\ell + h$ if $\ell \leq h$
(*b*) h if $\ell \geq h$ and $\ell + h$ if $\ell \leq h$
(*c*) 4h if $\ell \geq h$ and $\ell - h$ if $\ell \leq h$
(*d*) h/2 if $\ell \geq h$ and $\ell + h$ if $\ell \leq h$

44. Two parallel glass plates are dipped partly in the liquid of density 'd', keeping them vertical. If the distance between the plates is 'x'. Surface tension for liquid is T & angle of contact is θ then rise of liquid between the plates due to capillary will be

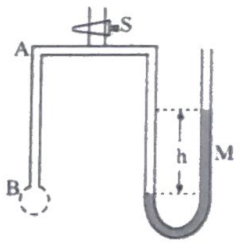
(*a*) $\frac{T\cos\theta}{x\,d}$ (*b*) $\frac{2T\cos\theta}{x\,d\,g}$

(*c*) $\frac{2T}{x\,d\,g\cos\theta}$ (*d*) $\frac{T\cos\theta}{x\,d\,g}$

45. In a surface tension experiment with a capillary tube water rises upto 0.1 m. If the same experiment is repeated in an artificial satellite, which is revolving around the earth ; water will rise in the capillary tube upto a height of
(*a*) 0.1 m (*b*) 0.2 m
(*c*) 0.98 m (*d*) full length of tube

46. Water rises in a capillary tube to a height h. It will rise to a height more than h
(*a*) on the surface of sun
(*b*) in a lift moving down with an acceleration
(*c*) at the poles
(*d*) in a lift moving up with an acceleration.

47. A tube of fine bore AB is connected to a manometer M as shown. the stop cock S controls the flow of air. AB is dipped into a liquid whose surface tension is σ. On opening the stop cock for a while, a bubble is formed at B and the manometer level is recorded, showing a difference h in the levels in the two arms. if ρ be the density of manometer liquid and r the radius of curvature of the bubble, then the surface tension σ of the liquid is given by



(*a*) phrg (*b*) 2phgr
(*c*) 4phrg (*d*) $\frac{rh\rho g}{4}$

48. When charge is given to a soap bubble, it shows :
(*a*) a decrease in size
(*b*) no change in size
(*c*) an increase in size
(*d*) sometimes an increase and sometimes a decreases in size

49. A water drop is divided into 8 equal droplets. The pressure difference between the inner and outer side of the big drop will be :

(*a*) same as for smaller droplet

(*b*) 1/2 of that for smaller droplet

(*c*) 1/4 of that for smaller droplet

(*d*) twice that for smaller droplet

50. An air bubble of radius r in water is at a depth h below the water surface at some instant. If P is atmospheric pressure, d and T are density and surface tension of water respectively, the pressure inside the bubble will be:

(*a*) $P + h\,dg - \frac{4T}{r}$ (*b*) $P + h\,dg + \frac{2T}{r}$

(*c*) $P + h\,dg - \frac{2T}{r}$ (*d*) $P + h\,dg + \frac{4T}{r}$

51. A cylinder with a movable piston contains air under a pressure p_1 and a soap bubble of radius 'r'. The pressure p_2 to which the air should be compressed by slowly pushing the piston into the cylinder for the soap bubble to reduce its size by half will be : (The surface tension is σ, and the temperature T is maintained constant)

(*a*) $\left[8p_1 + \frac{24\sigma}{r}\right]$ (*b*) $\left[4p_1 + \frac{24\sigma}{r}\right]$

(*c*) $\left[2p_1 + \frac{24\sigma}{r}\right]$ (*d*) $\left[2p_1 + \frac{12\sigma}{r}\right]$

52. A soap bubble of radius r_1 is placed on another soap bubble of radius $r_2 (r_1 < r_2)$. The radius R of the soapy film separating the two bubbles is :

(*a*) $r_1 + r_2$ (*b*) $\sqrt{r_1^2 + r_2^2}$

(*c*) $(r_1^3 + r_2^3)$ (*d*) $\frac{r_2 r_1}{r_2 - r_1}$

53. Two unequal soap bubbles are formed one on each side of a tube closed in the middle by a tap. What happens when the tap is opened to put the two bubbles in communication ?

(*a*) No air passes in any direction as the pressure are the same on two sides of the tap

(*b*) Larger bubble shrinks and smaller bubble increases in size till they become equal in size

(*c*) Smaller bubble gradually collapses and the bigger one increases in size

(*d*) None of these

54. A Newtonian fluid fills the clearance between a shaft and a sleeve. When a force of 800N is applied to the shaft, parallel to the sleeve, the shaft attains a speed of 1.5 cm/sec. If a force of 2.4 kN is applied instead, the shaft would move with a speed of

(*a*) 1.5 cm/sec

(*b*) 13.5 cm/sec

(*c*) 4.5 cm/sec

(*d*) None of these

55. A solid metallic sphere of radius r is allowed to fall freely through air. If the frictional resistance due to air is proportional to the cross-sectional area and to the square of the velocity, then the terminal velocity of the sphere is proportional to which of the following?

(*a*) r^2 (*b*) r

(*c*) $r^{3/2}$ (*d*) $r^{1/2}$

56. Two drops of same radius are falling through air with steady velocity of v cm/s. If the two drops coalesce, what would be the terminal velocity?

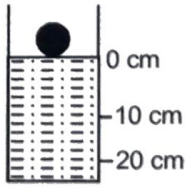
(*a*) 4 v

(*b*) $(d)^{1/3}$v

(*c*) 2 v

(*d*) 64 v

57. A spherical ball of density ρ and radius 0.003m is dropped into a tube containing a viscous fluid filled up to the 0 cm mark as shown in the figure. Viscosity of the fluid = 1.260 $N.m^{-2}$ and its density $\rho_L = \rho/2 = 1260$ $kg.m^{-3}$. Assume the ball reaches a terminal speed by the 10 cm mark. The time taken by the ball to traverse the distance between the 10 cm and 20 cm mark is.
(g = acceleration due to gravity = 10 ms^{-2})



(*a*) 500 μs

(*b*) 50 ms

(*c*) 0.5 s

(*d*) 5 s

JEE-ADVANCED COMPREHENSION/STATEMENT/MATCHING/ MCQ

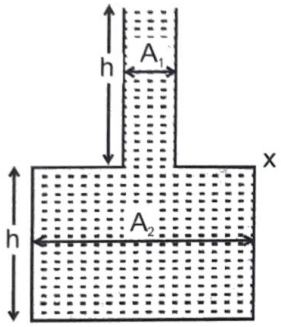
FLUID MECHANICS

1. Pressure gradient in a static fluid is represented by (z–direction is vertically upwards, and x-axis is along horizontal,d is density of fluid) :

   (*a*) $\frac{\partial p}{\partial z} = -\,dg$ (*b*) $\frac{\partial p}{\partial x} = dg$

   (*c*) $\frac{\partial p}{\partial x} = 0$ (*d*) $\frac{\partial p}{\partial z} = 0$

2. The vessel shown in Figure has two sections of area of cross-section A_1 and A_2. A liquid of density r fills both the sections, up to height h in each. Neglecting atomospheric pressure,



   (*a*) the pressure at the base of the vesel is 2 h r g
   (*b*) the weight of the liquid in vessel in equal to 2 h r g
   (*c*) the force exerted by the liquid on the base of vessel is 2 h r g A_2
   (*d*) the walls of the vessel at the level X exert a force h r g ($A_2 - A_1$) downwards on the liquid.

3. A cubical block of wood of edge 10cm and mass 0.92kg floats on a tank of water with oil of rel. density 0.6. Thickness of oil is 4cm above water. When the block attains equilibrium with four of its side edges vertical:
   (*a*) 1 cm of it will be above the free surface of oil
   (*b*) 5 cm of it will be under water
   (*c*) 2 cm of it will be above the common surface of oil and water
   (*d*) 8 cm of it will be under water

4. An air bubble in a water tank rises from the bottom to the top. Which of the following statements are true?
   (*a*) Bubble rises upwards because pressure at the bottom is less than that at the top
   (*b*) Bubble rises upwards because pressure at the bottom is greater than that at the top
   (*c*) As the bubble rises, its size increases
   (*d*) As the bubble rises, its size decreases

SURFACE TENSION

5. If for a liquid in a vessel, force of a cohesion is twice of adhesion:
   (*a*) the meniscus will be convex upwards
   (*b*) the angle of contact will be obtuse
   (*c*) the liquid will descend in the capillary tube
   (*d*) the liquid will wet the solid

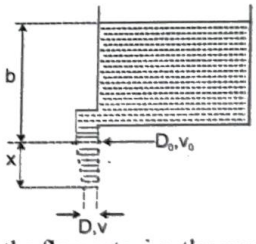
6. The rise of liquid in a capillary tube depends on :
   (*a*) the material of tube and nature of liquid
   (*b*) the length of tube
   (*c*) the outer radius
   (*d*) the inner radius of the tube

7. Which of the following statements are true in case when two water drops coalesce and make a bigger drop:
   (*a*) Energy is released
   (*b*) Energy is absorbed
   (*c*) The surface area of the bigger drop is greater than the sum of the surface areas of both the drops
   (*d*) The surface area of the bigger drop is smaller than the sum of the surface areas of both the drops

Comprehension – 1 (No. 8 to 12)

The figure shows the commonly observed decrease in diameter of a water stream as it falls from a tap. The tap has internal diameter D_0 and is connected to a large tank of water. The surface of the water is at a height b above the end of the tap.

By considering the dynamics of a thin "cylinder" of water in the stream answer the following: (Ignore any resistance to the flow and any effects of surface tension, given ρ_w = density of water)



8. Equation for the flow rate, i.e. the mass of water flowing through a given point in the stream per unit time, as function of the water speed v will be
   (*a*) $v\rho_w \pi D^2/4$ (*b*) $v\rho_w(\pi D^2/4 - \pi D_0^2/4)$
   (*c*) $v\rho_w \pi D^2/2$ (*d*) $v\rho_w \pi D_0^2/4$

9. Which of the following equation expresses the fact that the flow rate at the tap is the same as at the stream point with diameter D and velocity v (i.e. D in terms of D_0 , v_0 and v will be) :

   (*a*) $D = \frac{D_0 v_0}{v}$ (*b*) $D = \frac{D_0 v_0^2}{v^2}$

   (*c*) $D = \frac{D_0 v}{v_0}$ (*d*) $D = D_0\sqrt{\frac{v_0}{v}}$

10. The equation for the water speed v as a function of the distance x below the tap will be :

(*a*) $v = \sqrt{2gh}$ (*b*) $v = [2g(h + x)]^{1/2}$

(*c*) $v = \sqrt{2gx}$ (*d*) $v = [2g(h - x)]^{1/2}$

11. Equation for the stream diameter D in terms of x and D_0 will be :

(*a*) $D = D_0\left(\frac{h}{h+x}\right)^{1/4}$ (*b*) $D = D_0\left(\frac{h}{h+x}\right)^{1/2}$

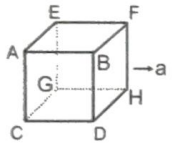
(*c*) $D = D_0\left(\frac{h}{h+x}\right)$ (*d*) $D = D_0\left(\frac{h}{h+x}\right)^{2}$

12. A student observes after setting up this experiment that for a tap with $D_0 = 1$ cm at $x = 0.3$ m the stream diameter $D = 0.9$ cm. The heights b of the water above the tap in this case will be:

(*a*) 5.7 cm (*b*) 57 cm

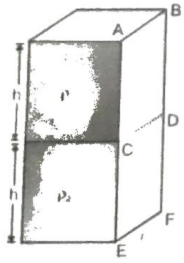
(*c*) 27 cm (*d*) 2.7 cm

13. A cubical box is completely filled with mass m of a liquid and is given horizontal acceleration a as shown in the figure. Match the force due to fluid pressure on the faces of the cube with their appropriate values (assume zero pressure as minimum pressure)



Column-I	Column-II
(*a*) force on face ABFE	(p) $\frac{ma}{2}$
(*b*) force on face BFHD	(q) $\frac{mg}{2}$
(*c*) force on face ACGE	(r) $\frac{ma}{2}+\frac{mg}{2}$
(*d*) force on face CGHD	(s) $\frac{ma}{2}$ +mg
	(t) $\frac{mg}{2}$ + ma

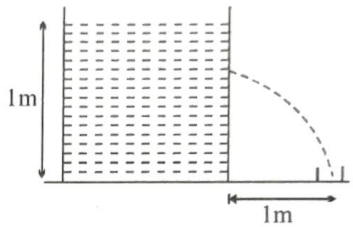
14. A cuboid is filled with liquid of density ρ_2 upto height h & with liquid of density ρ_1, also upto height h as shown in the figure



Column-I	Column-II
(*a*) Force on face ABCD due to liquid of density ρ_1	(p) zero
(*b*) Force on face ABCD due to liquid of density ρ_2	(q) $\frac{\rho_1 gh^2\ell}{2}$
(*c*) Force on face CDEF transferrred due to liquid of density ρ_1	(r) $\rho_1 gh^2\ell$
(*d*) Force on face CDEF due to liquid of density ρ_2 only	(s) $\frac{\rho_2 gh^2\ell}{2}$

NUMERICAL VALUE BASED

15. A tank is filled with water upto height of 1m. A hole is to be made in the side wall so that the liquid coming out from the side falls first in a cup kept 1 m away from the right side. How high (in cm) should the hole be made?

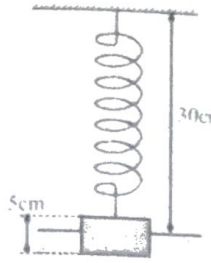


16. The cross-sectional area of the aorta (the major blood vessel emerging from the heart) of a normal resting person is 3 cm^2, and the speed of blood through it is 30 cm/s. A typical capillary has cross-sectional area of 3×10^{-7} cm^2 and a flow speed of 0.05 cm/s. How many capillaries (in multiple of 10^9) does such a person have? (Assume that all the blood that passes through the capillaries must have passed through the aorta).

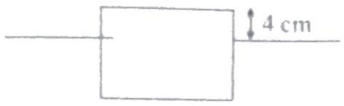
17. A horizontal pipe has a cross section of 10 cm^2 in one region and of 5 cm^2 in another. The water velocity at the first is 5 ms^{-1} and the pressure in the second is 2×10^5 Nm^{-2}. Find the pressure of water in 1st region (in 10^2 Pa).

18. A uniform block of wood placed in water floats with exactly 2/3 of its volume submerged. Assume that the water has a density of 1×10^3 kg/m^3. The exact same block of wood, placed in oil, floats with $\frac{5}{6}$ of its volume submerged. Using this information, what is the density of the oil in kg/m^3?

19. The original length of a spring is 25 cm. It elongates 2 cm if a force of 0.96 N is exerted on it. A container is filled with water and one end of the spring is fixed 30 cm above the surface of the water in the container. A wooden block of mass 32 g and of density 0.4 g/cm^3 is hanged onto the other end of the spring. The height of the block is 5 cm. To what depth (in mm) does the block sink into the water?



20. A wooden block of mass 0.6 kg of size 10 cm × 10 cm × 10 cm is floating over an unknown liquid as shown in the figure



How much minimum mass (in gm) should be kept on the wooden block so that it completely submerges into liquid?

21. A microscope slide measures 6.0 cm × 1.5 cm × 0.20 cm. It is suspended with its face vertical and with its longest side horizontal and is lowered into a liquid until it is half immersed. Its apparent weight is then found to be the same as its weight in air. Calculate the surface tension of liquid in 10^{-3} N m^{-1}, assuming the angle of contact to be zero. Density of liquid is 1000 kg m^{-3}. Round off to nearest integer.

22. Two solid balls have different radii but are made of same material. The balls are linked together with a long thin thread and released from a large height. At the terminal velocity, the thread is under tension. The larger ball has a fixed mass, but we have choice of the smaller ball with different masses. At what ratio of larger and smaller mass will this tension be maximum?

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be: [*JEE Main-2019 (January)*]

(*a*) $\frac{1}{4}\rho v^2$ (*b*) $\frac{3}{4}\rho v^2$

(*c*) $\frac{1}{2}\rho v^2$ (*d*) ρv^2

2. A cylindrical plastic bottle of negligible mass of filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to: (density of water = 10^3 kg/m^3) [*JEE Main-2019 (January)*]

(*a*) 3.75 rad s^{-1} (*b*) 1.25 rad s^{-1}
(*c*) 2.50 rad s^{-1} (*d*) 5.00 rad s^{-1}

3. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second. then the difference in the heights between the centre and the sides, in cm, will be: [*JEE Main-2019 (January)*]

(*a*) 2.0 (*b*) 0.1
(*c*) 0.4 (*d*) 1.2

4. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^{3}s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is: [*JEE Main-2019 (January)*]

(*a*) 5.1 cm (*b*) 1.7 cm
(*c*) 4 cm (*d*) 2.9 cm

5. The top of a water is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to: [*JEE Main-2019 (January)*]

(*a*) 6.0 m (*b*) 4.8 m
(*c*) 9.6 m (*d*) 2.9 m

6. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is: [*JEE Main-2019 (April)*]

(*a*) 0.5 (*b*) 0.7
(*c*) 0.6 (*d*) 0.8

7. Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg m^{-3}, coefficient of viscosity of water = 1mPas) [*JEE Main-2019 (April)*]

(*a*) 10^6 (*b*) 10^3
(*c*) 10^4 (*d*) 10^2

8. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2})

[*JEE Main-2019 (April)*]

(*a*) 500 m (*b*) 400 m
(*c*) 300 m (*d*) 600 m

9. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The crosssectional area of the tap is 10^{-4}m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be: [*JEE Main-2019 (April)*]
(Take g = 10 ms^{-2})

(*a*) 1×10^{-5} m^2 (*b*) 5×10^{-5} m^2
(*c*) 2×10^{-5} m^2 (*d*) 5×10^{-4} m^2

10. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water ? (Take density of water = 10^3 kg/m^3)

[*JEE Main-2019 (April)*]

(*a*) 65.4 kg (*b*) 87.5 kg
(*c*) 30.1 kg (*d*) 46.3 kg

11. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. the maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is:

[*JEE Main-2020 (January)*]

(*a*) $\frac{9}{16}$ (*b*) $\frac{\sqrt{3}}{2}$

(*c*) $\frac{81}{256}$ (*d*) $\frac{3}{4}$

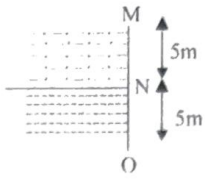
12. Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1-\frac{r^2}{R^2}\right) 0 < r \le R$. The minimum density of a liquid in which it will float is:

[*JEE Main-2020 (January)*]

(*a*) $\frac{\rho_0}{3}$ (*b*) $\frac{2\rho_0}{3}$

(*c*) $\frac{\rho_0}{5}$ (*d*) $\frac{2\rho_0}{5}$

13. Two liquids of densities ρ_1 and ρ_2 ($\rho_2 = 2\rho_1$) are filled up in a closed vessel behind a square wall of side 10m as shown in figure. Each liquid has a height of 5m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing): [*JEE Main-2020 (January)*]

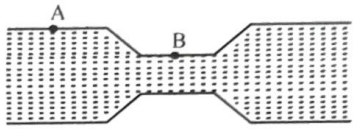


(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{1}{4}$ (*d*) $\frac{2}{3}$

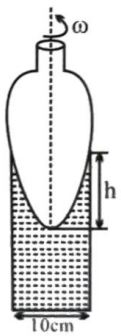
14. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water thorugh the tube. (density of water = 1000 kgm^{-3})

[*JEE Main-2020 (January)*]



(*a*) 2720 cm^3/s (*b*) 2420 cm^3/s
(*c*) 3020 cm^3/s (*d*) 1810 cm^3/s

15. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height. h (in cm) of liquid at the centre of vessel and at the side will be [*JEE Main-2020 (September)*]



(*a*) $\frac{2\omega^2}{25g}$ (*b*) $\frac{5\omega^2}{2g}$

(*c*) $\frac{2\omega^2}{5g}$ (*d*) $\frac{25\omega^2}{2g}$

16. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is: [JEE Main-2020 (September)]

(a) $gdS(x_2 + x_1)^2$

(b) $\frac{1}{4}gdS(x_2 - x_1)^2$

(c) $\frac{3}{4}gdS(x_2 - x_1)^2$

(d) $gdS(x_2^2 + x_1^2)$

17. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is

[JEE Main-2020 (September)]

(a) $\frac{2}{3}R$ (b) $\frac{4}{9}R$

(c) $\frac{1}{3}R$ (d) $\frac{8}{9}R$

18. A fluid is flowing through a horizontal pipe of varying cross-section, with speed v ms^{-1} at a point where the pressure is P pascal.

At another point where pressure is $\frac{P}{2}$ pascal its speed is V ms^{-1}. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to :

[JEE Main-2020 (September)]

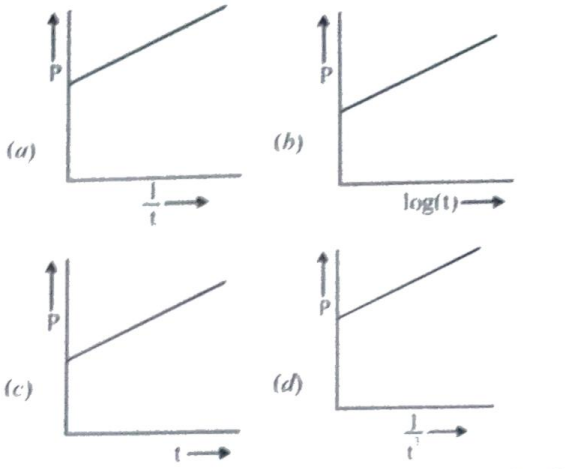
(a) $\sqrt{\frac{P}{\rho}+v^2}$ (b) $\sqrt{\frac{2P}{\rho}+v^2}$

(c) $\sqrt{\frac{P}{\rho}+v}$ (d) $\sqrt{\frac{P}{2\rho}+v^2}$

SURFACE TENSION AND VISCOSITY

19. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure isnide the bubble is given by:

[JEE Main-2019 (January)]



20. If 'M' is the mass of water that rises in a capillary tube of radius 'r'. then mass of water which will rise in a capillary tube of radius '2r' is : [JEE Main-2019 (April)]

(a) 4M (b) M

(c) 2M (d) $\frac{M}{2}$

21. A solid sphere, of radius R acquires a terminal velocity V_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ration (v_1/v_2) equals.

[JEE Main-2019 (April)]

(a) 1/27 (b) 1/9

(c) 27 (d) 9

22. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of thier densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1. while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2). is then close to:

(a) 2/3 (b) 3/5

(c) 2 / 5 (d) 4 / 5

SURFACE TENSION

23. A small spherical droplet of density d is floating exactly half immerse in a liquid ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet);

[JEE Main-2020 (January)]

(a) $r = \sqrt{\frac{T}{(d+\rho)g}}$ (b) $r = \sqrt{\frac{3T}{(2d-\rho)g}}$

(c) $r = \sqrt{\frac{T}{(d-\rho)g}}$ (d) $r = \sqrt{\frac{2T}{3(d-\rho)g}}$

24. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used Then the value of λ is (in nm)

[*JEE Main-2020 (January)*]

25. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm,density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to ($g = 10\ ms^{-2}$)

[*JEE Main-2020 (September)*]

(*a*) 0.049 m (*b*) 0.087 m

(*c*) 0.137 m (*d*) 0.172 m

26. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m^{-1}, is [*JEE Main-2020 (September)*]

[$\rho_{(liquid)} = 900\ kgm^{-3}$, $g = 10\ ms^{-2}$]

(Give answer in closest integer) ____.

27. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is

[*JEE Main-2020 (September)*]

(*a*) 2 : 1 (*b*) 0.8 : 1

(*c*) 4 : 1 (*d*) 8 : 1

28. A air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is ($g = 980\ cm/s^2$)

[*JEE Main-2020 (September)*]

(*a*) 1.52 gm (*b*) 4.51 gm

(*c*) 3.15 gm (*d*) 4.15 gm

29. In an experiment to verify Stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to (ignore viscosity of air) [*JEE Main-2020 (September)*]

(*a*) r^4 (*b*) r^3

(*c*) r (*d*) r^2

30. The water is filled up to height of 12 m in a tank having vertical sidewalls. A hole is made in one of the walls at a depth 'h' below the water level. The value of 'h' for which the emerging steam of water strikes the ground at the maximum range is m. [*JEE Main-2021 (July)*]

JEE-ADVANCED PREVIOUS YEAR'S

1. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) [*JEE Advanced-2013*]

(*a*) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

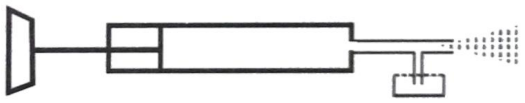
(*b*) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(*c*) the light sphere is partially submerged.

(*d*) the light sphere is completely submerged.

Comprehension – 1(No. 2 to 3)

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1mm respectively. The upper end of the container is open to the atmosphere.



2. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of:

[*JEE Advanced-2014*]

(*a*) 0.1 ms^{-1} (*b*) 1 ms^{-1}

(*c*) 2 ms^{-1} (*d*) 8 ms^{-1}

3. If the density of air is ρ_a and that of the liquid ρ_ℓ, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

[*JEE Advanced-2014*]

(*a*) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (*b*) $\sqrt{\rho_a \rho_\ell}$

(*c*) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (*d*) ρ_ℓ

4. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List - I and the distance where the water jet hits the floor of the lift is given in List - II. Match the statements from List - I with those in List- II and select the correct answer using the code given below the lists.

[*JEE Advanced-2014*]

List -I

P. Lift is accelerating vertically up.

Q. Lift is accelerating vertically down with an accelerating less than the gravitational acceleration.

R. List is moving vertically up with constant Speed

S. Lift is falling freely.

List -II

1. d = 1.2 m
2. d > 1.2m
3. d < 1.2 m
4. No water leaks out of the jar

Code :

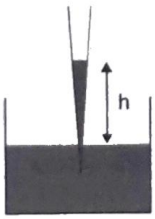
(*a*) P-2, Q-3, R-2, S-4 (*b*) P-2, Q-3, R-1, S-4

(*c*) P-1, Q-1, R-1, S-4 (*d*) P-2, Q-3, R-1, S-1

SURFACE TENSION AND VISCOSITY

5. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)

[*JEE Advanced-2014*]



(*a*) $\frac{2S}{b\rho g}\cos(\theta-\alpha)$

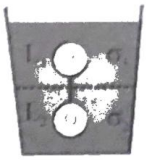
(*b*) $\frac{2S}{b\rho g}\cos(\theta+\alpha)$

(*c*) $\frac{2S}{b\rho g}\cos(\theta-\alpha/2)$

(*d*) $\frac{2S}{b\rho g}\cos(\theta+\alpha/2)$

6. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then

[*JEE Advanced -2015*]



(*a*) $\frac{|\vec{V}_P|}{|\vec{V}_Q|}=\frac{\eta_1}{\eta_2}$ (*b*) $\frac{|\vec{V}_P|}{|\vec{V}_Q|}=\frac{\eta_2}{\eta_1}$

(*c*) $\vec{V}_P\cdot\vec{V}_Q>0$ (*d*) $\vec{V}_P\cdot\vec{V}_Q<0$

7. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density of 0.8 gm cm^{-3} and viscosity $\eta = 3$ pioseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity η = 2 poiseulles. The ratio of the terminal velocities of P and Q is [*JEE Advanced-2016*]

8. A drop of liquid of radius R = 10^{-2} m having surface tension $S=\frac{0.1}{4\pi}\,Nm^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is :

[*JEE Advanced-2017*]

9. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is(are) true? [*JEE Advanced-2018*]

(*a*) For a given material of the capillary tube, h decreases with increase in r.

(*b*) For a given material of the capillary tube, h is independent of σ.

(*c*) If this experiment is performed in a lift going up with a constant acceleration, then h decreases.

(*d*) h is proportional to contact angel θ.

10. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true?

[*JEE Advanced-2018*]

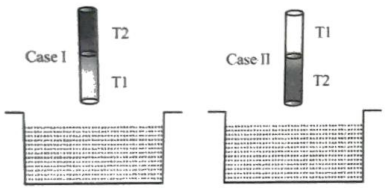
(*a*) The resistive force of liquid on the plate is inversely proportional to h

(*b*) The resistive force of liquid on the plate is independent of the area of the plate

(*c*) The tangential (shear) stress on the floor of the tank increases with u_0

(*d*) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid.

11. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct ?
(Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10m/s^2) [*JEE Advanced-2019*]



(*a*) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

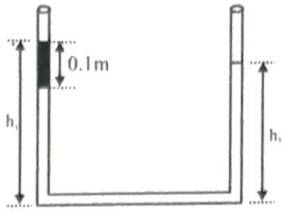
(*b*) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

(*c*) For case I, if the joint is kept at 8 cm above the water surface, t he height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

(*d*) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

12. An open-ended U-tube of uniform cross-sectional area contains water (density 10^3 kg m^{-3}).Initially the water level stands at 0.29 m from the bottom in each arm. Kerosene oil (a water-immiscible liquid) of density 800 kg m^{-3} is added to the left arm until its length is 0.1 m, as shown in the schematic figure below. The ratio $\left(\frac{h_1}{h_2}\right)$ of the heights of the liquid in the two arms is:

[*JEE (Advanced)-2020*]



(*a*) $\frac{15}{14}$ (*b*) $\frac{35}{33}$

(*c*) $\frac{7}{6}$ (*d*) $\frac{5}{4}$

13. A train with cross-sectional area S_t is moving with speed v_t inside a long tunnel of cross-sectional area $S_0(S_0=4S_t)$. Assume that almost all the air (density ρ) in front of the train flows back between its sides and the walls of the tunnel. Also, the air flow with respect to the train is steady and laminar. Take the ambient pressure and that inside the train to be p_0. If the pressure in the region between the sides of the train and the tunnel walls is p, then $p_0 - p = \frac{7}{2N}\rho v_t^2$. The value of N is .....

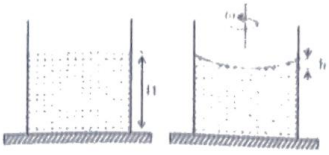
[*JEE (Advanced)-2020*]

14. A hot air balloon is carrying some passengers, and a few sandbags of mass 1 kg each so that its total mass is 480 kg. Its effective volume giving the balloon its buoyancy is V. The balloon is floating at an equilibrium height of 100 m. When N number of sandbags are thrown out, the balloon rises to a new equilibrium height close to 150 m with its volume V remaining unchanged. If the variation of the density of air with height h from the ground is $\rho(h)=\rho_0 e^{\frac{h}{h_0}}$, where ρ_0=1.25 kg m^{-3} and h_0=6000 m, the value of N is _____.

[*JEE (Advanced)-2020*]

15. A beaker of radius r is filled with water (refractive index $\frac{4}{3}$) up to a height H as shown in the figure on the left. The beaker is kept on a horizontal table rotating with angular speed ω. This makes the water surface curved so that the difference in the height of water level at the center and at the circumference of the beaker is h (h<<H, h<<r), as shown in the figure on the right. Take this surface to be approximately spherical with a radius of curvature R. Which of the following is/are correct?
(g is the acceleration due to gravity)

[*JEE (Advanced)-2020*]



(*a*) $R = \frac{h^2 + r^2}{2h}$

(*b*) $R = \frac{3r^2}{2h}$

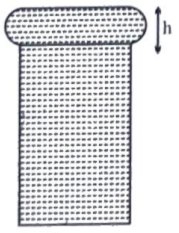
(*c*) Apparent depth of the bottom of the beaker is close to $\frac{3H}{2}\left(1+\frac{\omega^2 H}{2g}\right)$

(*d*) Apparent depth of the bottom of the beaker is close to $\frac{3H}{4}\left(1+\frac{\omega^2 H}{4g}\right)$

SURFACE TENSION

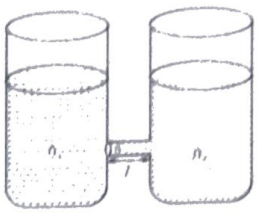
16. When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in the figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are 10^3 kg m^{-3}, 0.07 Nm^{-1} and 10 ms^{-2}, respectively, the value of h (in mm) is _____

[JEE (Advanced) - 2020]



17. As shown schematically in the figure, two vessels contain water solutions (at temperature T) of potassium permanganate ($KMnO_4$) of different concentrations n_1 and n_2 ($n_1 > n_2$) molecules per unit volume with $\Delta n = (n_1 - n_2) << n_1$. When they are connected by a tube of small length ℓ and cross-sectional area S, $KMnO_4$ starts to diffuse from the left to the right vessel through the tube. Consider the collection of molecules to behave as dilute ideal gases and the difference in their partial pressure in the two vessels causing the diffusion. The speed v of the molecules is limited by the viscous force βv on each molecule, where β is a constant. Neglecting all terms of the order $(\Delta n)^2$, which of the following is/are correct? (k_B is the Boltzmann constant)-

[JEE (Advanced)-2020]



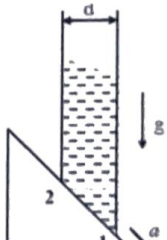
(*a*) the force causing the molecules to move across the tube is $\Delta n k_B T S$

(*b*) force balance implies $n_1 \beta v \ell = \Delta n k_B T$

(*c*) total number of molecules going across the tube per sec is $\left(\frac{\Delta n}{\ell}\right)\left(\frac{k_B T}{\beta}\right) S$

(*d*) rate of molecules getting transferred through the tube does not change with time

18. A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration a along a fixed inclined plane with angle $\theta = 45°$. P_1 and P_2 are pressures at points 1 and 2, respectively, located at the base of the tube. Let $\beta = (P_1 - P_2)/(\rho g d)$, where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct? *[JEE (Advanced) - 2021]*



(*a*) $\beta = 0$ when $a = g/\sqrt{2}$

(*b*) $\beta > 0$ when $a = g/\sqrt{2}$

(*c*) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when $a = g/2$

(*d*) $\beta = \frac{1}{\sqrt{2}}$ when

ANSWER KEY

Topicwise Questions

1. (*a*)	**2.** (*c*)	**3.** (*c*)	**4.** (*a*)	**5.** (*c*)	**6.** (*d*)	**7.** (*b*)	**8.** (*d*)	**9.** (*a*)	**10.** (*c*)
11. (*b*)	**12.** (*a*)	**13.** (*c*)	**14.** (*d*)	**15.** (*b*)	**16.** (*a*)	**17.** (*a*)	**18.** (*b*)	**19.** (*d*)	**20.** (*a*)
21. (*d*)	**22.** (*a*)	**23.** (*b*)	**24.** (*d*)	**25.** (*c*)	**26.** (*a*)	**27.** (*a*)	**28.** (*d*)	**29.** (*c*)	**30.** (*c*)
31. (*d*)	**32.** (*a*)	**33.** (*c*)	**34.** (*b*)	**35.** (*b*)	**36.** (*b*)	**37.** (*c*)	**38.** (*c*)	**39.** (*c*)	**40.** (*b*)
41. (*c*)	**42.** (*a*)								

Learning Plus

1. (*d*)	**2.** (*c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)	**7.** (*a*)	**8.** (*a*)	**9.** (*c*)	**10.** (*a*)
11. (*b*)	**12.** (*c*)	**13.** (*c*)	**14.** (*a*)	**15.** (*b*)	**16.** (*c*)	**17.** (*a*)	**18.** (*b*)	**19.** (*b*)	**20.** (*b*)
21. (*a*)	**22.** (*c*)	**23.** (*d*)	**24.** (*c*)	**25.** (*a*)	**26.** (*a*)	**27.** (*a*)	**28.** (*a*)	**29.** (*b*)	**30.** (*b*)
31. (*d*)	**32.** (*d*)	**33.** (*c*)	**34.** (*b*)	**35.** (*a*)	**36.** (*c*)	**37.** (*c*)	**38.** (*d*)	**39.** (*a*)	**40.** (*b*)
41. (*c*)	**42.** (*a*)	**43.** (*a*)	**44.** (*b*)	**45.** (*d*)	**46.** (*b*)	**47.** (*d*)	**48.** (*c*)	**49.** (*b*)	**50.** (*b*)
51. (*a*)	**52.** (*d*)	**53.** (*c*)	**54.** (*c*)	**55.** (*d*)	**56.** (*b*)	**57.** (*d*)			

Advanced Level Multiconcept Questions

1. (*a, c*)	**2.** (*a, c, d*)	**3.** (*c, d*)	**4.** (*b, c*)	**5.** (*a, b, c*)	**6.** (*a, b, d*)	**7.** (*a, d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*b*)

11. (*a*) **12.** (*b*) **13.** A - p ; B - q ; C - t ; D - s **14.** A – q ; B - p ; C - r ; D - s **15.**[0050 cm]

16. [0006] **17.** [2375] **18.** [0800] **19.** [5] **20.** [0400] **21.** [0073] **22.** [8]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*b*)	**2.** (*b*)	**3.** (*a*)	**4.** (*a*)	**5.** (*b*)	**6.** (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*b*)
11. (*a*)	**12.** (*d*)	**13.** (*c*)	**14.** (*a*)	**15.** (*d*)	**16.** (*b*)	**17.** (*d*)	**18.** (*a*)	**19.** (Bonus)	**20.** (*c*)
21. (*d*)	**22.** (*c*)	**23.** (*b*)	**24.** [750]	**25.** (*b*)	**26.** [101]	**27.** (*d*)	**28.** (*d*)	**29.** (*a*)	**30.** [6]

JEE-ADVANCED PREVIOUS YEAR'S

1. (*a,d*)	**2.** (*c*)	**3.** (*a*)	**4.** (*c*)	**5.** (*d*)	**6.** (*a, d*)	**7.** [3]	**8.** [6]	**9.** (*a,c*)
10. (*a,c,d*)	**11.** (*a,c,d*)	**12.** (*b*)	**13.** [9]	**14.** [4]	**15.** (*a,d*)	**16.** [3.74]	**17.** (*a,b,c*)	**18.** (*a, c*)

t.me/realprayas2022

11 THERMAL PROPERTIES OF MATTER

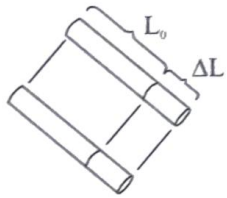
t.me/real prayasians

THERMAL EXPANSION IN SOLID

To varying extents most materials expand when heated and contract when cooled. The increase in any one dimension of a solid is called linear expansion, linear in the sense that the expansion occurs along a line. A rod whose length is L_0 when the temperature is T_0. when the temperature increases to $T_0 + \Delta T$, the length becomes $L_0 + \Delta L$, where ΔT and ΔL are the magnitude of the changes in temperature and length, respectively.

Conversely, when the temperature decreases to $T_0 - \Delta T$, the length decreases to $L_0 - \Delta L$.

For small temperature changes, experiments show that the change in length is directly proportional to the change in temperature ($\Delta L \propto \Delta T$). In addition, the change in length is proportional to the initial length of the rod.



Equation : $\Delta L = \alpha L_0 \Delta T$ expresses the fact that ΔL is proportional to both L_0 and ΔT by using a proportionality constant α, which is called the coefficient of linear expansion.

Unit for the coefficient of linear expansion $= \frac{1}{°C} = (°C)^{-1}$

Variation of time period of pendulum clocks

The time represented by the clock hands of a pendulum clock depends on the number of oscillation performed by pendulum Every time it reaches to its extreme position the second hand of the clock advances by one second, that means second hand moves by two seconds when one oscillation is complete

Let $T = 2\pi\sqrt{\frac{L_0}{g}}$ at temperature θ_0 and $T' = 2\pi\sqrt{\frac{L}{g}}$ at temperature θ.

$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L[1+\alpha\Delta\theta]}{L}} = 1 + \frac{1}{2}\alpha\Delta\theta$$

Therefore change (loss or gain) in time per unit time lapsed is

$$\frac{T'-T}{T} = \frac{1}{2}\alpha\Delta\theta$$

gain or loss in time in duration of 't' in

$\Delta t = \frac{1}{2}\alpha\Delta\theta\, t$, if T is the correct time then

(a) $\theta < \theta_0$, $T' < T$ clock becomes fast and gain time

(b) $\theta > \theta_0$, $T' > T$ clock becomes slow and loose time

Measurement of length by metallic scale:

Case (i)

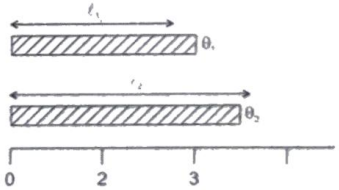
When object is expanded only

$\ell_2 = \ell_1\{1 + \alpha_0(\theta_2 - \theta_1)$

ℓ_1 = actual length of object at θ_1°C = measure length of object at θ_1°C.

ℓ_2 = actual length of object at θ_2°C = measure length of object at θ_2°C.

α_0 = coefficient of linear expansion of object.



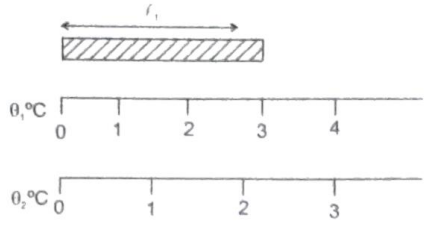
Case (ii)

When only measuring is expanded actual length of object will not change but measured value (MV) decreases.

$MV = \ell_1\{1 - \alpha_S(\theta_2 - \theta_1)\}$

α_S = coefficient of linear expansion measuring instrument.

at θ_1 °C, MV = 3



at θ_2 °C MV = 2.2

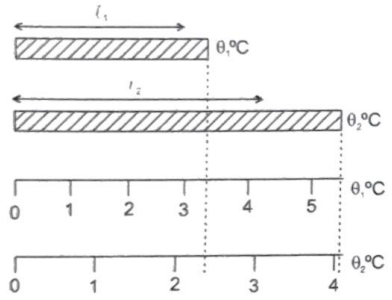
Case (iii)

If both expanded simultaneously

$MV = \{1 + (\alpha_0 - \alpha_s)(\theta_2 - \theta_1)\}$

(i) If $\alpha_0 > \alpha_s$, then measured value is more then the actual value at θ_1°C

(ii) If $\alpha_0 < \alpha_s$, then measured value is less then the actual value at θ_1°C



at θ_1°C MV = 3.4

θ_2°C MV = 4.1

Measured value = calibrated value × {1 + α Δ θ}

where $\alpha = \alpha_0 - \alpha_s$

α_0 = coefficient of linear expansion of object material,

α_s = coefficient of linear expansion of scale material

$\Delta\theta = \theta - \theta_C$

θ = temperature at the time of measurement

θ_C = temperature at the time of calibration.

For scale,

true measurement = scale reading

$[1 + \alpha(\theta - \theta_0)]$

If $\theta > \theta_0$ true measurement > scale reading

$\theta < \theta_0$ true measurement < scale reading

TRAIN YOUR BRAIN

Example 1: A bar measured with a Vernier caliper is found to be 180mm long. The temperature during the measurement is 10°C. The measurement error will be if the scale of the Vernier caliper has been graduated at a temperature of 20°C : ($\alpha = 1.1 \times 10^{-5}$ °C^{-1}. Assume that the length of the bar does not change.)

(*a*) 1.98×10^{-1} mm (*b*) 1.98×10^{-2} mm

(*c*) 1.98×10^{-3} mm (*d*) 1.98×10^{-4} mm

Sol. : True measurement = scale reading $[1 + \alpha(\theta - \theta_0)]$

$= 180\ [1 - 10 \times 1.1 \times 10^{-5}]$

error $= 180 - 180\ [1 - 1.1 \times 10^{-4}] = 1.98 \times 10^{-2}$ mm

Example 2: A brass scale correctly calibrated at 15°C is employed to measure a length at a temperature of 35°C. If the scale gives a reading of 75cm, find the true length. [Linear expansively of brass = 2.0×10^{-5} °C^{-1}]

Sol. Let the distance between two fixed divisions on the scale at 15°C be L_1 and that at 35°C be L_2.

Clearly, $(L_2 - L_1) = \alpha L_1(35 - 15)$

or $L_2 = L_1(1 + 20 \times 2.0 \times 10^{-5})$

$= L_1(1.0004)$

i.e. at 35°C, an actual length of L_2 will be read as L_1 (note) due to the increased separation of the divisions of the scale. In other words, the observed length will be less than the actual length.

Given : $L_1 = 75$ cm

$L_2 = 75\ (1.0004)$ cm $= 75.03$ cm

Example 3: A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct time at 20°C, how fast or slow will it go in 24 hours at 40°C? Coefficient of linear expansion of iron = 1.2×10^{-6}/°C.

Sol. : The time difference occurred in 24 hours (86400 seconds) is given by

$$\Delta t = \frac{1}{2}\alpha\,\Delta\theta\,t$$

$$= \frac{1}{2} \times 1.2 \times 10^{-6} \times 20 \times 86400$$

$= 1.04$ sec. **Ans.**

This is loss of time as θ is greater than θ_0. As the temperature increases, the time period also increases. Thus, the clock goes slow.

Example 4: Estimate the time lost or gained by a pendulum clock at the end of a week when the atmospheric temperature rises to 40°C. The clock is known to give correct time at 15°C and the pendulum is of steel. (Linear expansively of steel is 12×10^{-6} / °C).

Sol. Rise in temperature = 40 − 15 = 25°C

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{\Delta T}{T} = -\frac{1}{2}\frac{\Delta\ell}{\ell}$$

Time lost per second $= \frac{1}{2}\alpha\Delta\theta =$

$$\frac{1}{2} \times (12 \times 10^{-6}/°C) \times (25°C) = 150 \times 10^{-6}\ s/s$$

Therefore, time lost per week (i.e., 7×86400 s)

$= 150 \times 10^{-6} \times 7 \times 86400$ s

$= 90.72$. **Ans.**

Example 5: A glass rod when measured with a zinc scale, both being at 30°C, appears to be of length 100cm. If the scale shows correct reading at 0°C, determine the true length of the glass rod at (a) 30°C and (b) 0°C.

['α' for glass = 8×10^{-6}/°C and for zinc 26×10^{-6}/°C]

Sol. At 30°C, although the reading shown by the zinc scale corresponding to the length of the glass rod is 100cm, but the actual length would be more than 100cm, the reason being the increased separation between the markings, owing to a rise in temperature (from 0°C to 30°C).

Now, an actual (at 0°C) length of 100cm on the zinc scale (or more precisely, two markings or divisions on the scale, separated by a distance of 100cm) would, at a temperature of 30°C, correspond to a length given by:

$l = 100\,(1 + 26 \times 10^{-6} \times 30)$ cm = 100.078 cm

$\therefore$ The true length of the glass rod at 30°C is 100.078 cm, Now, at 0°C, the length of glass rod would be lesser than that at 30°C.

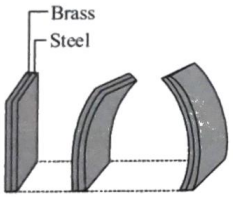
$\therefore$ Using $l_t = l_0\,(1 + \alpha t)$, $l_0 = \dfrac{l_t}{1+\alpha t}$

$\therefore$ The length of the rod at 0°C, will be

$$l_0 = \frac{100.078 \text{ cm}}{(1+8\times10^{-6}\times30)} = 100.054 \text{ cm}$$

THE BIMETALLIC STRIP

A bimetallic strip is made from two thin strips of metal that have different coefficients of linear expansion as figure shows.



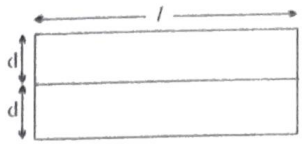
(a) (b) Heated (c) Cooled

(a) A bimetallic strip (b) on heated and (c) on cooled.

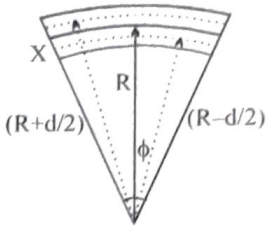
Often brass [$\alpha = 19 \times 10^{-6}$ °C^{-1}] and steel [$\alpha = 12 \times 10^{-6}$ °C^{-1}] are selected. The two pieces are welded or riveted together. When the bimetallic strip is heated, the brass having the larger value of α expands more than the steel. Since, the two metals are bonded together, the bimetallic strips bends into an arc as in part b, with the longer brass piece having a larger radius than the steel piece. When the strip is cooled, the bimetallic strips bends in the opposite direction, as in part c.

Mathematical Solution:

$$\left(R+\frac{d}{2}\right)\phi = L_0\,(1+\alpha_1\Delta\theta)$$



$$\left(R-\frac{d}{2}\right)\phi = L_0\,(1+\alpha_2\Delta\theta)$$



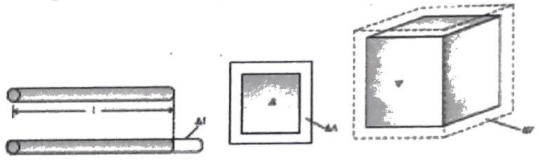
$$\frac{R+\frac{d}{2}}{R-\frac{d}{2}} = \frac{1+\alpha_1\Delta\theta}{1+\alpha_2\Delta\theta}$$

From the above equation we can find the value of R.

The expansion in length is called linear expansion. The expansion in area is called areal expansion.

The expansion in volume is called volume expansion.

If expansion is isotropic than



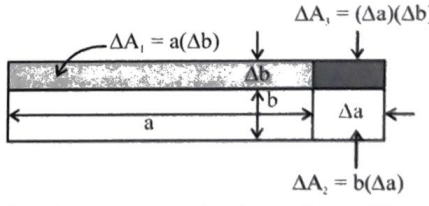
$\dfrac{\Delta l}{l} = \alpha T$ $\qquad$ $\dfrac{\Delta A}{A} = 2\alpha T$ $\qquad$ $\dfrac{\Delta V}{V} = 3\alpha T$

(a) Linear expansion (b) Areal expansion (c) Volume expansion

TRAIN YOUR BRAIN

Example 6: Show that the coefficient of areal expansion $(\Delta A/A)/\Delta T$ of a rectangular sheet of the solid is twice its linear expansion, α_A.

Sol.



Consider a rectangular sheet of the solid material of length a and breadth b in figure. When the temperature increases by ΔT, a increases by $\Delta a = a\alpha_A\Delta T$ and b increases $\Delta b = b\alpha_A\Delta T$. From figure the increase in area.

$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3$

$\Delta A = a\,\Delta b + b\,\Delta a + (\Delta a)\,(\Delta b)$

$= a\,\alpha_A b\,\Delta T + b\,\alpha_A a\Delta T + (\alpha_1)^2\,ab\,(\Delta T)^2$

$= 2\alpha_A\,ab\,\Delta T$

$= 2\alpha_A A\,\Delta T$

Since, $\alpha_A = 10^{-5}\ K^{-1}$ the product $\alpha_A \Delta T$ for fractional temperature is small in comparison with 2 and may be neglected. Hence, $\left(\frac{\Delta A}{A}\right)\frac{1}{\Delta T} = 2\alpha_A$.

VOLUME THERMAL EXPANSION

The volume of a normal material increases as the temperature increases. Most solids and liquids behave in this fashion. By analogy with linear thermal expansion, the change in volume ΔV is proportional to the change in temperature ΔT and to the initial volume V_0, provided the change in temperature is not too large. These two proportionalities can be converted into equation $\Delta V = \gamma V_0 \Delta T$ with the aid of a proportionality constant γ, known as the coefficient of volume expansion. The algebraic form of this equation is similar to that for linear expansion

$$\Delta L = \alpha L_0 \Delta T.$$

The volume V_0 of an object change by an amount ΔV when its temperature changes by an amount ΔT.

$$\Delta V = \gamma V_0 \Delta T$$

Common unit for the coefficient of volume expansion: $°C^{-1}$.

The unit for γ, like that for α, is $°C^{-1}$. Values for γ depend on the nature of the material. The values of γ for liquids are substantially larger than those for solids because liquids typically expand more than solids, given the same initial volumes and temperature expansion is three times greater than the coefficient of linear expansion : $\gamma = 3\alpha$.

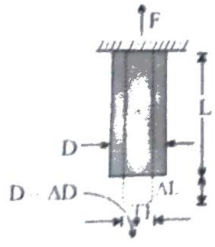
If a cavity exists within a solid object, the volume of the cavity increases when the object expands, just as if the cavity were filled with the surrounding material. The expansion of the cavity is analogous to the expansion of a hole in a sheet of material. Accordingly, the change in volume of a cavity can be found using the relation $\Delta V = \gamma V_0 \Delta T$, where γ is the coefficient of volume expansion of the material that surrounds the cavity.

TRAIN YOUR BRAIN

Example 7: A brass rod of length 1m is fixed to a vertical wall, at one end, with the other end free to expand. When the temperature of the rod is increased by 120°C, the length increases by 2cm. What is the strain?

Sol. After the rod expands, to the new length there are no elastic forces developed internally in it.

So, strain = 0.

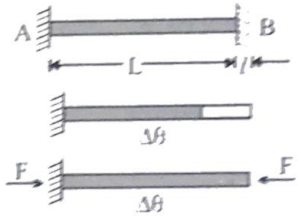


top view

Note: A change in shape, size (i.e. dimensions) does not necessarily imply a strain. For example, if a body is heated to expand, its volume changes as it acquires a new size due to expansion. However, the strain remains zero, as no elastic and no internal elastic forces operate, to bring the body to the original state, no strain exists. When a body is heated, the kinetic energy of molecule increase, owing to an increase in the kinetic energy of the molecules. This results in a shift (increase) in the "equilibrium distance" of molecules and the body acquires a new shape and size, in the expanded form, where by the molecules are in "zero force" state. Hence, there is no strain. However, if the body is restricted to expand during the process of heating, then the molecules become "strained" and even if there is no apparent change in dimensions of the body, there is strain. In such cases, strain is measured as the ratio

In dimension that would have occurred and the change in dimensions that would have occurred, had the body been free to expand or contract, to the original dimension.

When a metal rod is heated or cooled it tends to expand or contract. If it is left free to expand or contract, no temperature stresses will be included. However, if the rod be restricted to change its length then temperature stresses are generated within it. Stress induced due to temperature change can be understood as follows:



Consider a uniform rod AB fixed rigidly between two supports (figure). If L be its length, α the coefficient of linear expansion, then a change in temperature of $\Delta\theta$, would tend to bring a change in its length by $\Delta L = L\alpha\Delta\theta$. Had the rod been free (say one of its ends) its length would have changed by ΔL. Now, let a force be gradually applied so as to restore the natural length. Since the rod, tends to remain in the new state, due to a change in temperature, so when a force F is applied, thermal stress is induced. In equilibrium.

$$\frac{F}{A} = \frac{\Delta l\, Y}{(L \pm l)} \quad [\because \text{stress} = \text{strain} \times Y]$$

Neglecting l in comparison to L,

$$F = \frac{\Delta A Y}{L} = AY\,\alpha\,\Delta\theta$$

Now, if the two ends remain fixed, then this external force is provided from the support.

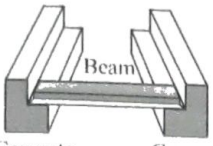
Clearly, strain $= \frac{\Delta l}{L} = \alpha\,\Delta\theta$

TRAIN YOUR BRAIN

Example 8

The stress on a steel beam

A steel beam is used in the road bed of a bridge. The beam is mounted between two concrete supports when the temperature is 22°C, with no room provided for thermal expansion.



What compressional stress must the concrete supports apply to each end of the beam, if they are to keep the beam from expanding when the temperature rises to 42°C?

Sol.: Recall from that the stress (force per unit cross-sectional area of F/A) required to change the length L_0 of an object by an amount ΔL is

$$\text{Stress} = Y\left(\frac{\Delta L}{L_0}\right)$$

where Y is Young's modulus. If the steel beam were free to expand because of the change in temperature, the length would change by $\Delta L = \alpha L_0 \Delta T$. Because the concrete supports do not permit any expansion, they must supply a stress to compress the beam by an amount ΔL. Thus,

$$\text{Stress} = Y\left(\frac{\Delta L}{L_0}\right) = Y\left(\frac{\alpha L_0 \Delta T}{L_0}\right) = Y\alpha\Delta T$$

Young's modulus and the coefficient of linear expansion for steel are $Y = 2.0 \times 10^{11}\ N/m^2$ and $\alpha = 12 \times 10^{-6}\ °C^{-1}$, respectively. The change in temperature from 22°C to 42°C is $\Delta T = 20°C$. The thermal stress is

$$Y\alpha\Delta T = (2.0 \times 10^{11}\ N/m^2)\ [12 \times 10^{-6}\ °C^{-1}]\ \ (20°C) = 4.8 \times 10^7\ N/m^2)$$

Example 9: A rod of length 2m is at a temperature of 20°C. Find the free expansion of the rod, if the temperature is increased to 50°C. Find the temperature stresses produced when the rod is

(i) fully prevented to expand

(ii) permitted to expand by 0.4 mm. ($Y = 2 \times 10^{11}\ Nm^{-2}$; $\alpha = 15 \times 10^{-6}$ per°C)

Sol. Free expansion of the rod $= \alpha\Delta\theta L$

$= (15 \times 10^{-6}/°C) \times (2m) \times (50° - 20°)\ C$

$= 9 \times 10^{-4}\ m = 0.9\ mm$

(i) If the expansion is fully prevented, then strain $= \dfrac{\Delta l}{l}$

$$= \frac{9\times10^{-4}}{2} \Rightarrow 4.5 \times 10^{-4}$$

$\therefore$ stress = strain $\times$ Y $= 4.5 \times 10^{-4} \times 2 \times 10^{11}$ $- 9 \times 10^7\ Nm^{-2}$

(ii) If 0.4 mm expansion is allowed, then length restricted to expand = 0.9 – 0.4 = 0.5 mm

$\therefore$ Strain $= \dfrac{5\times10^{-4}}{2} = 2.5 \times 10^{-4}$

$\therefore$ Temperature stress = strain $\times$ Y $= 2.5 \times 10^{-4} \times 2 \times 10^{11}$ $= 5 \times 10^7\ Nm^{-2}$.

Example 10: Two rods made of different materials are placed between massive walls. The cross section of the rods are A_1 and A_2, their lengths L_1 and L_2 coefficients of linear expansion α_1 and α_2 and the modulus of elasticity of their materials Y_1 and Y_2 respectively. If the rods are heated by t°C. Find the force F with which the rods act on each other.

Sol. Let the first rod expand slightly (say by length δl) and the second rod get compressed by the same amount (since net elongation / compression of the rods is zero.

$\therefore$ Natural increase in length of the first rod (after being heated) when free to expand would have been $\alpha_1 L_1 t$. The expansion allowed is just δl (where $\delta l < \alpha_1 L_1 t$).

$\therefore$ Amount of elongation restricted $= \alpha_1 L_1 t - \delta l$

$\therefore$ Strain $= \dfrac{\text{elongation restricted}}{\text{original length}}$

$$= \frac{\alpha_1 L_1 t - \delta l}{L_1 (1+\alpha_1 t)}$$

Since $\alpha_1 t << 1$

$\therefore$ $1 + \alpha_1 t \simeq 1$

$\therefore$ Strain $= \dfrac{\alpha_1 L_1 t - \delta l}{L_1}$

$\therefore$ Stress = strain $\times$ Y $= \left(\dfrac{\alpha_1 L_1 t - \delta l}{L_1}\right) Y_1$.

or F = stress $\times$ A $= \left(\dfrac{\alpha_1 L_1 t - \delta l}{L_1}\right) Y_1 A_1$ ...(1)

Similarly, $F = \left(\dfrac{\alpha_2 L_2 t - \delta l}{L_2}\right) Y_2 A_2$

or $\delta l = \alpha_1 L_1 t - \dfrac{FL_1}{Y_1 A_1} = \alpha_2 L_2 t - \dfrac{FL_2}{Y_1 A_2}$

or $F = \dfrac{(\alpha_1 L_1 + \alpha_2 L_2)\, t}{\left(\dfrac{L_1}{Y_1 A_1} + \dfrac{L_2}{Y_1 A_2}\right)}$

FLUID EXPANSION

Effect of temperature on density

When a solid or liquid is heated, it expands, with mass remaining constant. Density being the ratio of mass to volume, it decreases. Thus, if V_0 and V_t be the respective volumes of a substance at 0°C and t°C and if the corresponding values of densities be ρ_0 and ρ_t, then the mass m of the substance is given by

$$m = V_0\rho_0 = V_t\rho_t$$

But $V_t = V_0(1 + \gamma t)$,

so $\rho_t = \rho_0(1 + \gamma t)^{-1}$

TRAIN YOUR BRAIN

Example 11: The volume of a glass vessel filled with mercury is 500 cc at 25°C. What volume of mercury will overflow at 45°C?

The coefficients of volume expansion of mercury and glass are 1.8×10^{-4}/°C and 9.0×10^{-6}/°C respectively.

Sol. The volume of mercury overflowing will be the expansion of mercury relative to the glass vessel (i.e. apparent expansion).

Now, since $(\Delta V)_a = (\Delta V)_m - (\Delta V)_g$

Apparent expansion $(\Delta V)_a$ will be

$$(\Delta V)_a = V_m\gamma_m\Delta T - V_g\gamma_g\Delta T$$

$$= 500 \text{ cc } (180 - 9) \times \frac{10^{-6}}{°C} (45 - 25)\ °C$$

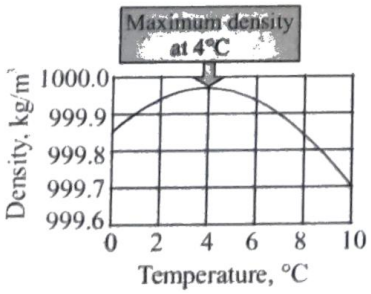
$$= 1.71 \text{ cc}$$

Thus, 1.71 cc of mercury overflows.

ANOMALOUS BEHAVIOUR OF WATER

While most substances expand when heated, a few do not. For instant, it water at 0°C is heated, its volume decreases until the temperature reaches 4°C. Above 4°C water behaves normally, and its volume increases as the temperature increases.

Because a given mass of water has a minimum volume at 4°C, the density (mass per unit volume) of water is greatest at 4°C, as figure shows.



The density of water in the temperature range from 0 to 10°C. Water has a maximum density of 999.973 kg/m^3 at 4°C. (This value for the density is equivalent to the often quoted density of 1.000 grams per millilitre)

Temperature Scales

Defining a temperature scale involves

(i) Choosing the thermometric substance

(ii) The choice of the thermometric property of the substance.

(iii) Choosing the upper fixed point and lower fixed point.

(iv) Choosing the number of divisions between the two fixed points.

Let N be the number of divisions between the fixed upper and fixed lower point. If measure of the thermodynamic property at the upper fixed point be x_N and that at lower fixed point is x_0 then if temperature at the lower fixed point is t_0 and at an unknown temperature t the measure of the thermometric property is x then,

$$t = \left(\frac{x - x_0}{x_N - x_0}\right)N + t_0$$

This equation defines temperature t. Some of the most common thermometric properties are the length of the liquid column (like mercury) in a glass capillary tube. Based on the choice of upper and lower fixed points we define various scales e.g., Celsius scale, Farenheit scale, Requmer scale. The relation between these scales is given by the following equation.

$$\frac{C - 0°}{100° - 0°} = \frac{k - 273}{373 - 273} = \frac{F - 32}{212 - 32} = \frac{R - 0}{4 - 0}$$

TRAIN YOUR BRAIN

Example 12: What is the temperature which has the same numerical value in centigrade scale and Fahrenheit Scale?

Sol. Let x be the required temperature

$$\text{Now } \frac{x - 0}{100 - 0} = \frac{x - 32}{212 - 32}$$

$$\text{or } \frac{x}{100} = \frac{x - 32}{180} \text{ (or) } \frac{x}{5} = \frac{x - 32}{9}$$

$$9x = 5x - 160$$

$$4x = -160$$

$$x = -40$$

$\therefore$ $-40°C$

Example 13: A thermometer has its lower and upper fixed points marked as 10° and 80°. When it reads 38° what is the corresponding temperature on Centigrade scale?

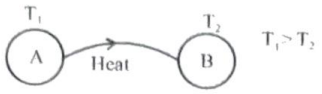
Sol. Let x be the reading on Centigrade scale

$$\frac{x - 0}{100 - 0} = \frac{38 - 10}{80 - 10}$$

$$\frac{x}{100} = \frac{28}{70}, x = \frac{28}{70} \times 100 = \frac{280}{7} = 40°C$$

HEAT

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of receiving body. If should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred. Thus, expressions like "heat in a body" or "heat of body" are meaningless.



When we say that a body is heated it means that its molcules begin to move with greater kinetic energy.

S.I. unit of heat energy is joule (J). Another practicle unit of heat energy is calorie (cal).

MECHANICAL EQUIVALENT OF HEAT

In early days heat was not recongnised as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiment were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write,

$$W = JH$$

Where J is called mechanical equivalent of heat. J is expressed in joule/calories. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

1 calorie : The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5 °C at one atmospheric pressure is 1 calorie.

1 calorie = 4.186 Joule

CALORIMETRY

Specific Heat

Specific heat of substances is equal to heat gain or released by that substance to raise or fall its temperature by 1°C for a unit mass of substance.

When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to :

(a) the mass of the body $\Delta Q \propto m$

(b) rise or fall of temperature of the body $\Delta Q \propto \Delta T$

$\Delta Q \propto m\,\Delta T$

or $\Delta Q = m\,s\,\Delta T$

or $dQ = m\,s\,dT$

or $Q = m\int s\,dT$

where s is a constant and is known as the specific heat of the body $s = \frac{Q}{m\Delta T}$. S.I. unit of s is joule/kg-kelvin and C.G.S unit is cal/gm °C

Specific heat of water : $s = 4180 \approx 4200$ J/kg°C = 1000 cal/kg°C = 1 Kcal/kg°C = 1 cal/gm°C

Specific heat of ice : $s = 2095 \approx 2100$ J/kg°C = 0.5 cal/gm°C

Specific heat of steam : s = 0.46 cal/gm°C

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Example 14: Heat required to increases the temperature of 1 kg water by 20°C

Sol. heat required = $\Delta Q = ms\Delta\theta$

$= 1 \times 20 = 20$ Kcal.

$\therefore$ S = 1 cal/gm°C = 1 Kcal/kg°C

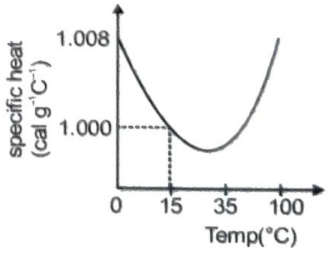
Important Points

(a) We know, $s = \frac{Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature ($\Delta T = 0$), the $s = Q/0 = \infty$. Thus the specific heat of a substance when it melts or boils at constant temprature is infinite.

(b) If the temperature of the substance changes without the transfer of heat ($Q = 0$) then $s = \frac{Q}{m\Delta T} = 0$. Thus when liquid in the thermos flask is shaken, its temperature increases without the transfer of heat and hence the specific heat of liquid in the thermos flask is zero.

(c) To raise the temperature of saturated water vapour, heat (Q) is withdrawn. Hence, specific heat of saturated water vapour is negative. (This is for your information only and not in the course)

(d) The slight variation of specific heat of water with temperature is shown in the graph at 1 atmosphere pressure. Its variation is less than 1% over the interval form 0 to 100°C.



Heat Capacity or Thermal Capacity

Heat capacity of a body is defined as the amount of heat required to raise the temperature of that body by 1°C. If 'm' is the mass and 's' the specific heat of the body, then

Heat capaicty = m s.

Units of heat capacity in : CGS system is, **cal °C^{-1}** ; SI unit is, **JK^{-1}**

Relation between Specific Heat and Water Equivalent

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object

$$ms\,\Delta T = m_w S_w \Delta T \Rightarrow m_w = \frac{ms}{s_w}$$

In calorie $s_w = 1$

$\therefore$ $m_w = ms$

m_w is also represented by W

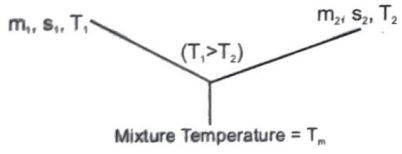
so $W = ms$

Law of Mixture (When phase of substance does not change)

When two substances at different temperatures are mixed together, the exchange of heat continues to take place till their temperature become equal. This temperature is then called final temperature of mixtue. Here,

Heat taken by one substance = Heat given by another substance

$$\Rightarrow m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$$



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Example 15: When a hot liquid is mixed with a cold liquid, the temperature of the mixture –

(*a*) First decrease then becomes constant.

(*b*) First increases then become constant.

(*c*) Continuously increases.

(*d*) Is undefined for some time and then becomes nearly constant.

Sol. (d) Can't say for some time

Example 16: Utensils used for efficient cooking should have-

(*a*) Large heat capacity

(*b*) Small heat capacity

(*c*) Medium heat capacity

(*d*) Any heat capacity

Sol. (b) So that absorb less heat

Example 17: An electric heater of power 1000W raises the temperature of 5 kg of a liquid from 25°C to 31°C in 2 minutes. Heat capacity of the liquid is –

(*a*) 2×10^4 J/°C (*b*) 1×10^4 J/°C

(*c*) 3×10^4 J/°C (*d*) 4×10^4 J/°C

Sol. (a) $1000 \times (2 \times 60) = Q = C\Delta T = C\,(31 - 25)$

$\Rightarrow$ $C = 2 \times 10^4$ J /°C

Example 18: The water equivalent of a 400 g copper calorimeter is (specific heat = 0.1 cal/g°C)

(*a*) 40 g (*b*) 4000 g

(*c*) 200 g (*d*) 4 g

Sol. (a) Water equivalent, $W = ms = 400 \times 0.1 = 40$ gm

Example 19: An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground it loses 25 % energy to surroundings. Then find the temperature rise of the block (Take sp. heat of iron 470 J/kg°C)

Sol. $mS\Delta\theta = \frac{1}{4}mgh \Rightarrow \Delta\theta = \frac{10\times10}{4\times470} = 0.053$

Example 20: The temperature of equal masses of three different liquids A, B, and C are 10°C 15°C and 20°C respectively. The temperatue when A and B are mixed is 13°C and when B and C are mixed, it is 16°C. What will be the temperature when A and C are mixed?

Sol. when A and B are mixed

$mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13)$

$3S_1 = 2S_2$ ...(1)

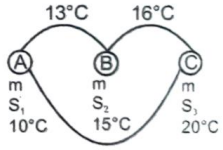
when B and C are mixed

$S_2 \times 1 = S_3 \times 4$ ...(2)

when C and A are mixed

$S_1(\theta - 10) = S_3 \times (20 - \theta)$...(3)

by using equation (1), (2) and (3)



we get $\theta = \frac{140}{11}$°C

Example 21: If three different liquid of different masses, specific heats and temperature are mixed with each other, then what is the temperature mixtrue at thermal equilibrium.

$m_1, s_1, T_1 \rightarrow$ specification for 1st liquid

$m_2, s_2, T_2 \rightarrow$ specification for 2nd liquid

$m_3, s_3, T_3 \rightarrow$ specification for 3rd liquid

Sol. Total heat lost or gain by all substance is equal to zero

$\Delta Q = 0$

$m_1 s_1 (T - T_1) + m_2 s_2 (T - T_2) + m_3 s_3 (T - T_3) = 0$

so $T = \frac{m_1 s_1 T_1 + m_2 s_2 T_2 + m_3 s_3 T_3}{m_1 s_1 + m_2 s_2 + m_3 s_3}$

PHASE CHANGE

Heat required for the change of phase or state,

$Q = mL$, L = latent heat.

(a) Latent heat (L): The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

(b) Latent heat of Fusion (L_f) : The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion.

(c) Latent heat of vaporisation (L_v) : The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization.

If in question latent heat of water are not mentioned and to solve the problem it require to assume that we should consider following values.

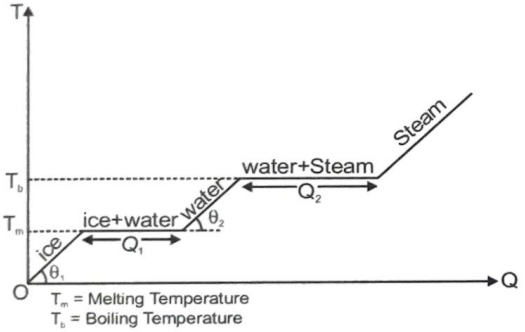
Latent heat of ice:

$L = 80$ cal/gm $= 80$ Kcal/kg $= 4200 \times 80$ J/kg

Latent heat of steam:

$L = 540$ cal/gm $= 540$ Kcal/kg $= 4200 \times 540$ J/kg

The given figure, represents the change of state by different lines



T_m = Melting Temperature
T_b = Boiling Temperature

$\theta_2 > \theta_1$

$Q_2 > Q_1$

Note : If we increases the temperature of liquid (phase) K.E. ↑ & temp. ↑ but at a later time K.E. stop increasing and the phase of the liquid starts changing.

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Example 22: A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on large ice block. The maximum amount of ice that can melt is (Specific heat of copper = 0.39 $Jg^{-1}{}^{\circ}C^{-1}$, latent heat of fusion of water = 335 Jg^{-1})

(a) 1.2 kg (b) 1.455 kg

(c) 1 kg (d) 2.5 kg

Sol. (b) $m_1 L_f = m_c s_c \Delta T_c$

$$\Rightarrow m_1 = \frac{m_c S_C \Delta T_C}{L_f}$$

$$= \frac{2.5 \times 0.39 \times (500 - 0)}{335} = 1.455 \text{ Kg}$$

Example 23: When vapour condenses into liquid -

(a) It absorbs heat

(b) It liberates heat

(c) Its temperature increases

(d) Its temperature decreases

Sol. (b) Phase Change so $\Delta T = 0$

Example 24: Heat required to convert one gram of ice at 0°C into steam at 100°C is (given $L_{steam} = 536$ cal/gm, $L_{ice} = 80$ cal/g)-

(a) 100 calorie (b) 0.01 kilocalorie

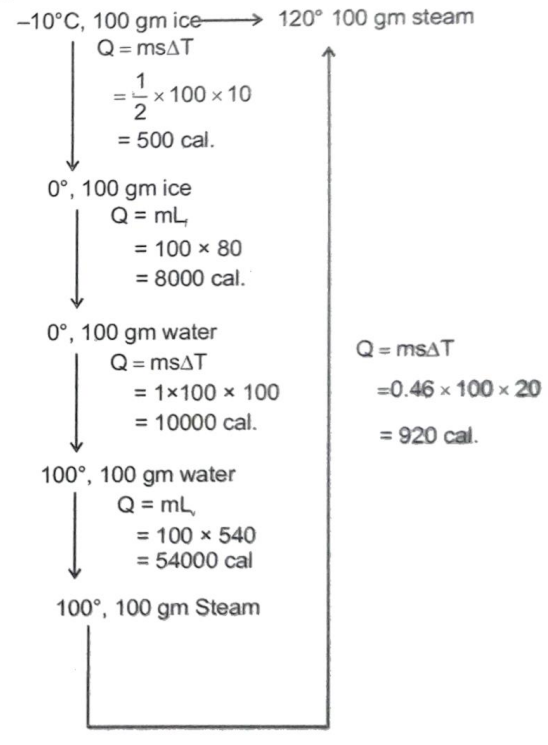
(c) 716 calorie (d) 1 kilocalorie

Sol. (c) $Q = mL_f + mS_w(100) + mL_v$

$= 1[80 + 100 + 536]$ cal $= 716$ cal

Example 25: Find amount of heat released if 100 g ice at – 10°C is converted into 120°C, 100 g steam.

Sol.



$Q_{net} = 73.5$ Kcal.

Example 26: 500 gm of water at 80°C is mixed with 100 gm steam at 120°C. Find out the final mixture.

Sol. 120°C steam ⟶ 100°C steam

Req. heat = 100 × 0.46 × 20 = 920 cal

80°C water ⟶ 100°C water

Req. heat = $500 \times 1 \times 20 = 10$ kcal

100gm steam $\longrightarrow$ 100 gm water at 100°C

Req. heat = $100 \times 540 = 54$ kcal

Total heat = 55 kcal.

Remaining heat = 55 – 10 = 45 kcal

Now we have 600 gm water at 100°C

$\Rightarrow 4500 = m \times 540$

$\Rightarrow m = \frac{250}{3}$ gm

So at last we have $\frac{250}{3}$ gm steam and $\left(600 - \frac{250}{3}\right)$ gm of water

Example 27: A lead ball at 30°C is dropped from a height of 6.2 km. The ball is heated due to the air resistance and it completely melts just before reaching the ground. The molten substance falls slowly on the ground. Calculate the latent heat of fusion of lead. Specific heat capacity of lead = 126 J/kg°C and melting point of lead = 330°C. Assume that any mechanical energy lost is used to heat the ball. Use $g = 10\ m/s^2$.

Sol. The initial gravitational potential energy of the ball

$= mgh$

$= m \times (10\ m/s^2)\ (6.2 \times 10^3\ m)$

$= m \times (6.2 \times 10^4\ m^2/s^2) = m \times (6.2 \times 10^4\ J/kg)$.

All this energy is used to heat the ball as it reaches the ground with a small velocity. Energy required to take the ball from 30°C to 330°C is

$m \times (126\ J/kg°C) \times (300°C)$

$= m \times 37800\ J/kg$

and energy required to melt the ball at 330°C = mL

where L = latent heat of fusion of lead

Thus, $m \times (6.2 \times 10^4\ J/kg) = m \times 37800\ J/kg + mL$

or, $L = \mathbf{24\ kJ/kg.}$

Example 28: How should one kg of water at 10°C be so divided that one part of it when converted into ice at 0°C, would by this change of state provide a quantity of heat that would be sufficient to vaporise the other part?

Sol. Initially 1000 g of water is at 10°C.

Let m gram of it be cooled to ice at 0°C.

Heat released due to this = $(m \times 1 \times 10) + (m \times 80)$

$= 10m + 80m = 90m$ cal.

The heat required by (1000 – m) g of water at 10°C to become steam at 100°C

$= (1000 - m)\ (100 - 10) + (1000 - m)\ 540$ cal

$= (1000 - m)\ (90 + 540)$ cal

$= (1000 - m)\ (630)$ cal

Now, $90\ m = (1000 - m)\ 630$ or, $720\ m = 630 \times 1000$

$m = \frac{630 \times 1000}{720} = 875$ g

Hence **875 g** of water by turning into at 0°C will supply heat to evaporate **125 g** of water.

Example 29: The specific heat of a substance is given by $C = a + bT$, where $a = 1.12\ kJ\ kg^{-1}c^{-1}$ and $b = 0.016\ kJ \cdot kg\ c^{-1}k^{-1}$. The amount of heat required to raise the temperature of 1.2 kg of the material from 280 K to 312 K is –

(*a*) 205 kJ (*b*) 215 kJ

(*c*) 225 kJ (*d*) 235 kJ

Sol. (c) Heat required

$$= Q = \int_{280}^{312} mCdT = \int_{280}^{312} (1.2)(a + bT)dT$$

$$= 1.2\left[aT + \frac{bT^2}{2}\right]_{280}^{312}$$

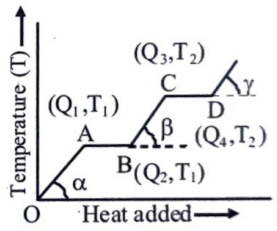
$$= 1.2\left[1.12\ (312 - 280) + \frac{0.016}{2}\{(312)^2 - (280)^2\}\right]$$

$$= 1.2\left[1.12 \times 32 + \frac{0.016}{2} \times (97344 - 78400)\right]$$

$= 1.2\ [35.84 + 0.008 \times 18944]$

$= 224.8704\ KJ = 225KJ$

Example 30: The accompanying graph shows the variation of temperature (T) of one kilogram material with Heat (Q) supplied to it. At O, the substance is in solid state. Which of the following interpretation from the graph is correct –



(*a*) T_2 is the melting point of the solid

(*b*) BC represents the change of state from solid to liquid.

(*c*) $(Q_2 - Q_1)$ represent the latent heat of fusion of the substance.

(*d*) $(Q_3 - Q_1)$ represents the latent heat of vaporisation of the liquid.

Sol. (c) The graph suggests the following processes :

Substance in solid state

(below melting point)

↓

Substance at its melting point

↓

Substance in liquid state at melting point

↓

Substance at its boiling point

↓

Substance at its boiling point in vapour state

↓

Substance at some higher temperature in vapour state

Clearly T_2 is boiling point of substance. BC represents the increase in temperature remaining in same liquid state. $Q_3 - Q_1$ represent heat required to change temperature of substance from its melting point to its boiling point. But $Q_1 - Q_2$ represents heat released when substance changes from liquid state to solid state at its melting point, i.e. latent heat of fusion

HEAT TRANSFER

Heat is energy in transit which flows due to temperature difference; from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three types.

(i) Conduction
(ii) Convection
(iii) Radiation

(a) CONDUCTION

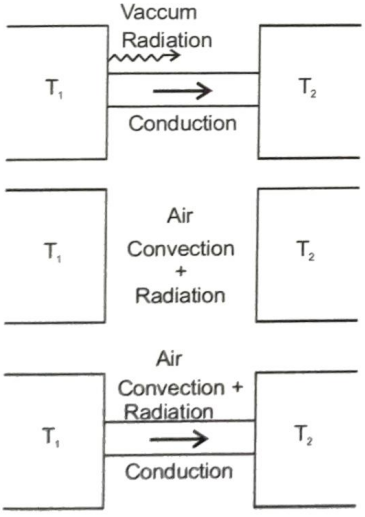
(i) Requires Medium
(ii) Energy is transmitted from one particle to another particle without displacement of particle
(iii)No transfer of particle

(b) CONVECTION

(i) Requires Medium
(ii) Enegy is transfer through movement of the particle of medium.

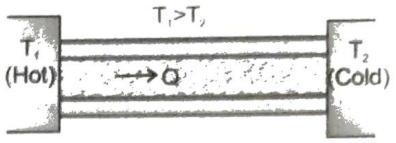
(c) RADIATION

(i) Does not requires any medium
(ii) Enegy is transfer through Electromagnetic waves.



CONDUCTION

Figure shows a rod whose ends are in thermal contact with a hot reservoir at temperature T_1 and a cold reservoir at temperature T_2. The sides of the rod are covered with insulating medium, so the transport of heat is along the rod, not through the sides. The molecules at the hot reservoir have greater vibrational energy. This energy is transferred by collisions to the atoms at the end face of the rod. These atoms transfer energy to their neighbours further along the rod. Such transfer of heat through a substance in which heat is transferred without direct mass transfer is called conduction.



This is much effective mechanism to conduct heat. The free electrons between metals, which move throughout the metal can rapidly carry energy from the hotter to cooler regions, so metals are generally good conductors of heat. The presence of 'free' electrons also causes most metals to be good electrical conductors. A metal rod at 5°C feels colder than a piece of wood at 5°C because heat can flow more easily from your hand into the metal.

Heat transfer occurs only between regions that are at different temperatures, and the rate of heat flow is $\frac{dQ}{dt}$.

This rate is also called the heat current, denoted by H. Experiments show that the heat current is proportional to the cross-section area A of the rod and to the temperature gradient $\frac{dT}{dx}$, which is the rate of change of temperature with distance along the bar. In general

$$H = \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

The negative sign is used to make $\frac{dQ}{dt}$ a positive quantity since $\frac{dT}{dt}$ is negative. The constant k, called the thermal conductivity is a measure of the ability of a material to conduct heat.

A substance with a large thermal conductivity k is a good heat conductor. The value of k depends on the temperature, increasing slightly with increasing temperature, but k can be taken to be practically constant throughout a substance if the temperature difference between its ends is not too great.

A rod of length L and constant cross sectional area A in which a steady state has been reached. In a steady state the temperature at each point is constant in time. Hence.

$$-\frac{dT}{dx} = \frac{T_1 - T_2}{L}$$

Therefore, the heat ΔQ transferred in time Δt is

$$\Delta Q = kA\left(\frac{T_1 - T_2}{L}\right)\Delta t$$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta T}{R}$$

Here, ΔT = temperature difference (TD) and $R = \frac{L}{kA}$

= thermal resistance of the rod.

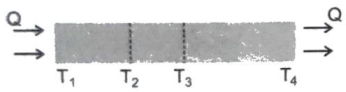
- ***Important Points in conduction***

1. Consider a section ab of a rod as shown in figure. Suppose Q_1 heat enters into the section at 'a' and Q_2 leaves at 'b', then $Q_2 < Q_1$. Part of the energy $Q_1 - Q_2$ is utilized in raising the tempeature of section ab and the remaining is lost to atmosphere through ab. This is called **transient state.**

   If heat is continuously supplied from the left end of the rod, a stage comes when temperature of the section becomes constant. In that case, $Q_1 = Q_2$ if rod is insulated from the surroundings (or loss thorugh ab is zero). This is called the **steady state** condition. Thus, in steady state temperature of different sections of the rod becomes constant (but not same).



transient state



Insulated rod in steady state

T_1 = constant,

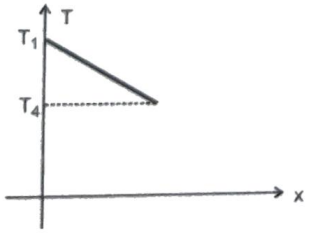
T_2 = constant etc.

and $T_1 > T_2 > T_3 > T_4$

Now, a natural question arises, why the temperature of whole rod not becomes equal when heat is being continuously supplied ?

Here in the rod, there must be a temperature difference in the rod for the heat flow, same as we require a potential difference across a resistance for the current to flow through it.

In steady state, the temperature varies linearly with distance along the rod if it is insluated.



2. Comparing equation number (iii), i.e., heat current

$$H = \frac{dQ}{dt} = \frac{\Delta T}{R}\left(\text{where } R = \frac{L}{kA}\right)$$

with the equation, of current flow through a resistance,

$$i = \frac{dq}{dt} = \frac{\Delta V}{R}\left(\text{where } R = \frac{L}{\sigma A}\right)$$

We find the following similarities in heat flow through a rod and current flow through a resistance.

Heat flow through a conducting rod	Current flow thorugh a resistance
Heat current $H = \frac{dQ}{dt}$ = rate of heat flow	Electric current $i = \frac{dq}{dt}$ = rate of charge flow
$H = \frac{\Delta T}{R}$	$i = \frac{\Delta V}{R}$
$R = \frac{L}{kA}$	$R = \frac{L}{\sigma A}$
k = thermal conductivity	σ = electrical conductivity

From the above table it is evident that flow of heat through rods in series and parallel is analogous to the flow of current through resistances in series and parallel. This analogy is of great importance in solving complicated problems of heat conduction.

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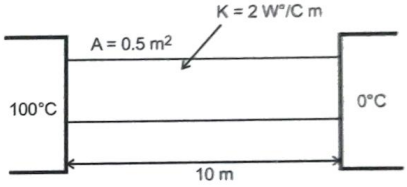
Example 31

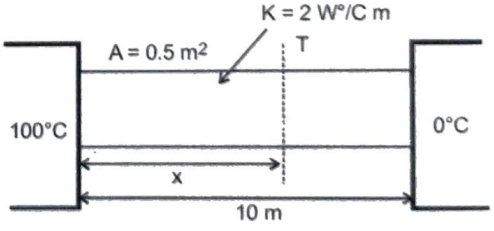


Find out the heat current and temperature at any distance x.

Sol. $R = \frac{10}{2\times 0.5} = 10 = \frac{L}{KA}$

$$i = \frac{100}{10}$$

and temperature at any distance x.



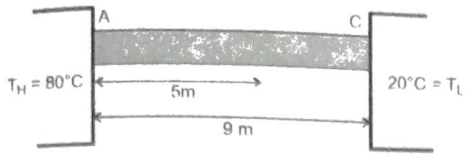
$$\Delta Q = \frac{KA(100-T)}{x} = \frac{KA\Delta T}{\ell}$$

$$\Rightarrow \frac{(100-T)}{x} = \frac{(100-0)}{\ell}$$

$$100\,\ell - T\,\ell = 100\,x$$

$$T = \frac{100(\ell - x)}{\ell}$$

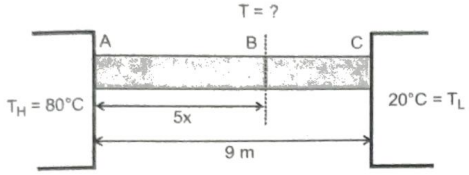
Example 32



Find out the temperature at distance 5 m.

Sol. Heat current is same, so,

$$\frac{T_H - T_L}{\ell} = \frac{T_H - T}{x} \Rightarrow \frac{80-20}{9} = \frac{80-T}{5}$$



$$T = \frac{140}{3}°C$$

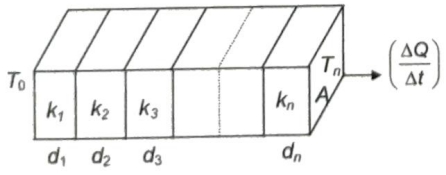
COMBINATION OF LAYERS

Layers in Series

Let us consider a multilayer medium consisted of layers having area of cross-section A each and length $d_1, d_2, \ldots d_n$ as shown. The thermal conductivities of various layers are $K_1, K_2 \ldots, K_n$. If a temperature difference of magnitude $T_0 - T_n$ is maintained between the near and far faces of the multiplayer (where T_0 and T_n are the temperatures of the near and far faces respectively), The rate of heat flow is given by

$$\frac{\Delta Q}{\Delta t} = \frac{A(T_0 - T_n)}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \ldots + \frac{d_n}{k_n}}$$

Which can be obtained directly using the concept of thermal resistance

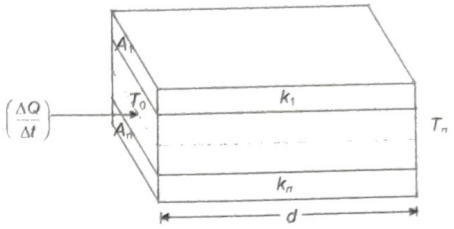


Layers in parallel

Now let us consider a multilayer, this time having same width d but different areas of cross-section $A_1, A_2, \ldots A_n$ as shown. The thermal conductivities of the various layers are $k_1, k_2, \ldots k_n$. If a temperature difference of magnitude $(T_0 - T_n)$ is maintained between the near and far faces of the multiplayer (where T_0 and T_n are the temperatures of near and far faces respectively, the rate of heat flow is given by

$$\frac{\Delta Q}{\Delta t} = \frac{(T_0 - T_n)(k_1A_1 + k_2A_2 + \ldots + k_nA_n)}{d}$$

Which can be obtained directly using the concept of thermal resistance.



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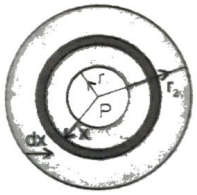
Example 33: Two thin concentric shells made from copper with radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C respectively by keeping a heater of power P at the centre of the two spheres. Find the value of P.

Sol. Heat flowing per second through each cross-section of the sphere = i = heat current

Thermal resistance of the spherical shell of radius x and thickness dx,



$$dR = \frac{dx}{K.4\pi x^2}$$

$$\Rightarrow R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2.K} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

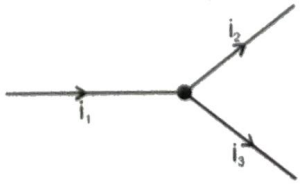
thermal current

$$i = \frac{T_H - T_C}{R} = \frac{4\pi K(T_H - T_C)r_1r_2}{(r_2 - r_1)}$$

Junction Law

Heat current is a Tensor quantity because, it doesn't follow the vector laws but the direction of heat current matter.

According to the Junction law the sum of all the heat current directed towards a point is equal to the sum of all the heat currents directed away from the points.



$$i_1 = i_2 + i_3$$

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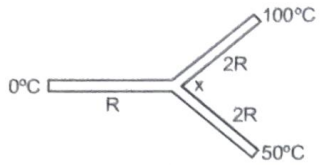
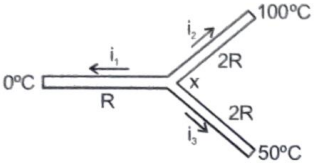
Example 34



Find out the temperature at point x.

Sol. $i_1 + i_2 + i_3 = 0$

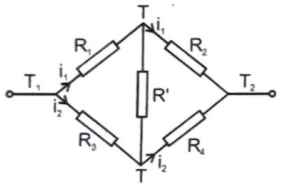
$$i_1 = \frac{(x-0)}{R}, \; i_2 = \frac{(x-100)}{2R}, \; i_3 = \frac{(x-50)}{2R}$$



$4x = 150 \Rightarrow x = 37.5°C$

Wheatstone Bridge

Example 35: Find out the relation between R_1, R_2, R_3 and R_4, so there is no that current in R'.



Sol. $\Delta T = T_1 - T_2$

$T_1 - T = i_1 R_1$ ...(i)

$T_1 - T = i_2 R_3$ ...(ii)

Eq. (i)/(ii)

$$\Rightarrow \frac{i_1 R_1}{i_2 R_3} = 1 \text{ ...(iii)}$$

$T - T_2 = i_1 R_2$ ...(iv)

$T - T_2 = i_2 R_4$ ...(v)

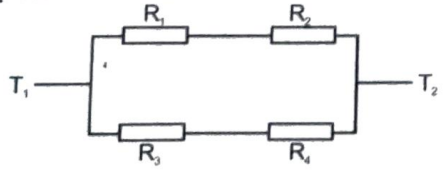
Eq. (4) / eq. (5)

$i_1 R_2 = i_2 R_4$ ...(vi)

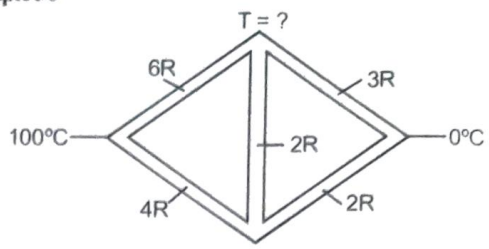
Eq. (iii)/(vi)

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow R_1 R_4 = R_2 R_3$$

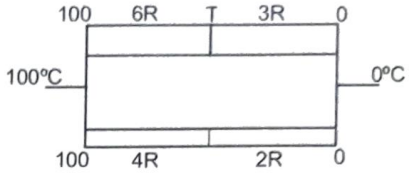
Equivalent circuit of wheatstone bridge can be represented as follow



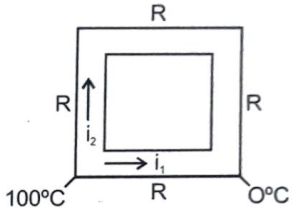
Example 36



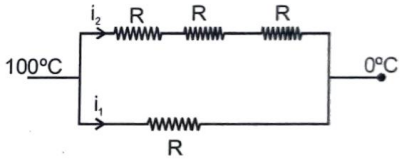
Sol. $\frac{100-T}{6R} = \frac{T-0}{3R} \Rightarrow T = 100/3\,°C$



Example 37: If some rod circuit as shown in figure then find ratio of i_2 to i_1.



Sol. $i_1 : i_2 = \frac{1}{R} : \frac{1}{3R} = 3 : 1$



$$i_1 = \frac{3}{4} \times 100 = 75 \Rightarrow i_2 = \frac{1}{4} \times 100 = 25$$

CONVECTION

When heat is transferred from one point to the other through actual movement of heated particles, the process of heat transfer is called convection. In liquids and gases, some heat may be transported through conduction. But most of the transfer of heat in them occurs through the process of convection.

Can you now see how the following facts can be explained by thermal convection?

(a) Oceans freeze top to down and not bottom to up. (this fact is singularly responsible for presence of aquatic life in temperate and polar waters.)

(b) The temperature in the bottom of deep oceans is invariably 4°C, whether it is winter or summer.

(c) You cannot illuminate the interior of a lift in free fall or an artificial satellite of earth with a candle.

(d) You can Illuminate your room with a candle.

RADIATION

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. These waves are formed due to the superposition of electric and magnetic fields perpendicular to each other and carry energy.

Properties of Radiation

(a) All objects emit radiations simply because their temperature is above absolute zero, and all objects absorb some of the radiation that falls on them from other objects.

(b) Maxwell on the basis of his electromagnetic theory proved that all radiations are electromagnetic waves and their sources are vibrations of charged particles in atoms and molecules.

(c) More radiations are emitted at higher temperature of a body and lesser at lower temperature.

(d) The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this the colour of a body appears to be changing. Radiations from a body at NTP has predominantly infrared waves.

(e) Thermal radiations travels with the speed of light and move in a straight line.

(f) Radiations are electromagnetic waves and can also travel through vacuum.

(g) Similar to light, thermal radiations can be reflected, refracted, diffracted and polarized.

(h) Radiation from a point source obeys inverse square law

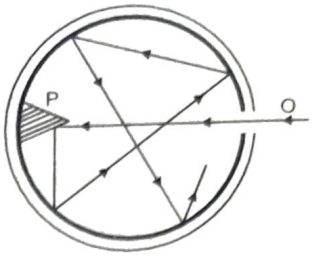
(intensity $\alpha \frac{1}{r^2}$).

Prevost Theory of heat Exchange

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more than what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to the temperature of its surroundings it radiates at the same rate as it absorbs.

Perfectly Black Body and Black Body Radiation (Fery's black body)

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.



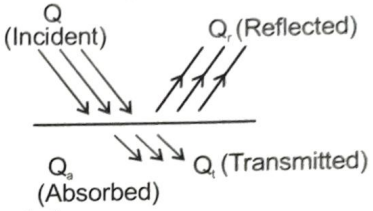
Absorption, Reflection and Emission of Radiations

$$Q = Q_r + Q_t + Q_a$$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

$$1 = r + t + a$$

where r = reflecting power , a = absorptive power



and t = transmission power.

(i) r = 0, t = 0, a = 1, perfect black body

(ii) r = 1, t = 0, a = 0, perfect reflector

(iii) r = 0, t = 1, a = 0, perfect transmitter

Absorptive power

In particular absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body.

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As all the radiations incident on a black body are absorbed, a = 1 for a black body.

Emissive power:

Energy radiated per unit time per unit area along the normal to the area is known as emissive power.

$$E = \frac{Q}{\Delta A\, \Delta t}$$

(Notice that unlike absorptive power, emissive power is not a dimensionless quantity).

Spectral Emissive power (E_λ)

Emissive power per unit wavelength range at wavelength λ is known as spectral emissive power, E_λ. If E is the total emissive power and E_λ is spectral emissive power, they are related as follows,

$$E = \int_0^\infty E_\lambda \, d\lambda \quad \text{and} \quad \frac{dE}{d\lambda} = E_\lambda$$

Emissivity

$$e = \frac{\text{Emissive power of a body at temperature T}}{\text{Emissive power of a black body at same temperature T}}$$

$$= \frac{E}{E_0}$$

Kirchoff's Law

The ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all substances at the same temperature and is equal to the emissive power of a perfectly black body for the same wavelength and temperature.

$$\frac{E(\text{body})}{a(\text{body})} = E \text{ (black body)}$$

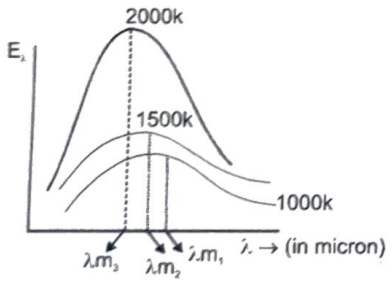
Hence we can conclude that good emitters are also good absorbers.

Nature of Thermal Radiations (Wien's Displacement Law):

From the energy distribution curve of black body radiation, the following conclusions can be drawn :

(a) The higher the temperature of a body, the higher is the area under the curve i.e. more amount of energy is emitted by the body at higher temperature.

(b) The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.



(c) For a given temperature, there is a particular wavelength (λ_m) for which the energy emitted (E_λ) is maximum.

(d) With an increase in the temperature of the black body, the maxima of the curves shift towards shorter wavelengths.

From the study of energy distribution of black body radiation discussed as above, it was established experimentally that the wavelength (λ_m) corresponding to maximum intensity of emission decreases inversely with increase in the temperature of the black body i.e.

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

This is called Wien's displacement law.

Here b = 0.282 cm-K. is the Wien's constant.

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Example 38: Solar radiation is found to have an intensity maximum near the wavelength range of 470 nm. Assuming the surface of sun to be perfectly absorbing (a = 1), calculate the temperature of solar surface.

Sol. Since a =1, sun can be assumed to be emitting as a black body

from Wien's law for a black body

$\lambda_m . T = b$

$$\Rightarrow T = \frac{b}{\lambda_m} = \frac{0.282(\text{cm K})}{(470\times10^{-7}\text{cm})}$$

$\simeq$ 6000 K. **Ans.**

Stefan-Boltzmann's law:

According to this law, the amount of radiation emitted per unit time from an area A of a black body at absolute temperature T is directly proportional to the fourth power of the temperature.

$dQ/dt = \sigma A T^4$

where σ is Stefan's constant $= 5.67 \times 10^{-8}$ W/m^{2}K^4

A body which in not a black body absorbs and hence emits less radiation.

Then, for such a body,

$dQ/dt = e\sigma A T^4$

where e = emissivity (which is equal to absorptive power) which lies between 0 to 1

With the surroundings of temperature T_0, net energy radiated by an area A per unit time.

$\Delta u = u - u_0 = e\sigma A (T^4 - T_0^4)$

TRAIN YOUR BRAIN

Example 39: A block of ice at –10°C is placed in a room which has temperature of 27°C. Then the ice–

(*a*) Does not emit any radiation

(*b*) Emits some radiation but absorbs an equal amount of radiation

(*c*) Absorbs more radiation than it emits

(*d*) Emits more radiation then it absorbs

Sol. (c) Since temp of surrounding is greater than temperature of body

$$\frac{dQ}{dt} = e A\sigma (T_{\text{body}}^4 - T_{\text{surrounding}}^4)$$

Example 40: A thin square steel plate with each side equal to 10 cm is heated by a blacksmith. The rate of radiated energy by the heated plate is 1134 watts. The temperature of the hot steel plate is–

(Stefan's constant $\sigma = 5.67 \times 10^{-8}$ watts $m^{-2} K^{-4}$ emissivity of the plate = 1)

(a) 1000K (b) 1189K

(c) 2000K (d) 2378K

Sol. (b) $\sigma e A T^4 = 1134$

$$\Rightarrow T = \left(\frac{1134}{\sigma e A}\right)^{1/4} = \left[\frac{1134}{5.67\times10^{-8}\times1\times(.1)^2}\right]^{1/4}$$

$T = 1189$ K

Example 41: Two sphere made of same substance have radii 1m and 4m, and temperatures 4000°K and 2000°K respectively. The ratio of power radiated by two spheres is–

(a) 1/2 (b) 1/4

(c) 4 (d) 1

Sol. (d) $$\frac{\sigma e A_1 T_1^4}{\sigma e A_2 T_2^4} = \frac{r_1^2}{r_2^2}\frac{T_1^4}{T_2^4} = \left(\frac{1}{4}\right)^2\left(\frac{4000}{2000}\right)^4 = 1$$

Example 42: The rate of emission of radiation of a body at temperature 27°C is 20 watt. If the temperature of a body is increased to 327°C, the rate of emission of radiation will be –

(a) 20 watt (b) 160 watt

(c) 320 watt (d) 327 watt

Sol. (c) $$\frac{20}{P_1} = \left(\frac{273+27}{327+273}\right)^4$$

$$\Rightarrow P_1 = 20 \times \left(\frac{600}{300}\right)^4 = 320 \text{ Watt}$$

Example 43: A body of emissivity (e = 0.75), surface area of 300 cm^2 and temperature 227°C is kept in a room at temperature 27°C. Calculate the initial value of net power emitted by the body.

Sol. $P = e\sigma A (T^4 - T_0^4)$

$= (0.75)(5.67 \times 10^{-8})(300 \times 10^{-4}) \times \{(500-300)^4\}$

$= 69.4$ Watt.

Example 44: A hot black body emits the energy at the rate of 16 $J m^{-2} s^{-1}$ and its most intense radiation corresponds to 20,000 Å. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000 Å, then find the value of energy radiated in $Jm^{-2} s^{-1}$.

Sol. Wein's displacement law is :

$\lambda_m . T = b$

i.e., $T \propto \frac{1}{\lambda_m}$

Here, λ_m becomes half.

∴ Temperature doubles.

Also $e = \sigma T^4$

$$\Rightarrow \frac{e_1}{e_2} = \left(\frac{T_1}{T_2}\right)^4 \Rightarrow e_2 = \left(\frac{T_2}{T_1}\right)^4 . e_1 = (2)^4 . 16$$

$= 16.16 = 256\ Jm^{-2} s^{-1}$

NEWTON'S LAW OF COOLING

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

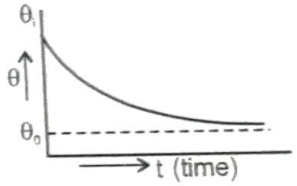
$\frac{d\theta}{dt} \propto (\theta - \theta_0)$, where θ and θ_0 are temperature corresponding to object and surroundings.

From above expression, $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ ...(11.1)

This expression represents Newton's law of cooling. It can be derived directly from stefan's law, when cooling is occur only from radiations.

$$k = \frac{4e\sigma\theta_0^3}{mc} A$$

Now $$\frac{d\theta}{dt} = -k\ [\theta - \theta_0] \Rightarrow \int_{\theta_i}^{\theta_f} \frac{d\theta}{(\theta-\theta_0)} = \int_0^t -k\ dt$$



where θ_i = initial temperature of object and

θ_f = final temperature of object.

$$\Rightarrow \ell n \frac{(\theta_f - \theta_0)}{(\theta_i - \theta_0)} = -kt \Rightarrow (\theta_f - \theta_0) = (\theta_i - \theta_0)\, e^{-kt}$$

$$\Rightarrow \theta_f = \theta_0 + (\theta_i - \theta_0)\, e^{-kt}$$

Limitations of Newton's Law of Cooling

(a) The difference in temperature between the body and surroundings must be small

(b) The loss of heat from the body should be by radiation only.

(c) The temperature of surroundings must remain constant during the cooling of the body.

Approximate Method for Applying Newton's Law of Cooling

Sometime when we need only approximate values from Newton's law, we can assume a constant rate of cooling, which is equal to the rate of cooling corresponding to the average temperature of the body during the interval.

$$\left\langle \frac{d\theta}{dt} \right\rangle = -k(\langle\theta\rangle - \theta_0)$$

$$\frac{\theta_f - \theta_i}{\Delta T} = -k\left(\frac{\theta_i + \theta_f}{2} - \theta_0\right) \quad \ldots (11.2)$$

If θ_i & θ_f be initial and final temperature of the body then,

$$\langle\theta\rangle = \frac{\theta_i + \theta_f}{2}$$

Remember equation (11.1) is only an approximation and equation (11.2) must be used for exact values.

TRAIN YOUR BRAIN

Example 45: A body at temperature 40°C is kept in a surrounding of constant temperature 20°C. It is observed that its temperature falls to 35°C in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30°C.

Sol. $\Delta\theta_f = \Delta\theta_i e^{kt}$ for the interval in which temperature falls from 40 to 35° C.

$$(35 - 20) = (40 - 20) e^{k.10}$$

$$\Rightarrow e^{10k} = \frac{3}{4} \Rightarrow k = \frac{\ell n \frac{4}{3}}{10} = 0.0285$$

for the next interval

$$(30 - 20) = (35 - 20)e^{-kt}$$

$$\Rightarrow e^{kt} = \frac{2}{3} \Rightarrow kt = \ell n \frac{3}{2}$$

$$\Rightarrow \frac{\left(\ell n \frac{4}{3}\right)t}{10} = \ell n \frac{3}{2} \Rightarrow t = 10 \frac{\left(\ell n \frac{3}{2}\right)}{\left(\ell n \frac{4}{3}\right)} \text{ minute}$$

$$= 14.096 \text{ min}$$

Topicwise Questions

THERMAL EXPENSION

1. From a black body, radiation is not :
   (*a*) emitted (*b*) absorbed
   (*c*) reflected (*d*) refracted

2. The energy radiated by a body depends on:
   (*a*) area of body (*b*) nature of surface
   (*c*) mass of body (*d*) temperature of body

3. The rate of cooling of a body by radiation depends on:
   (*a*) area of body
   (*b*) mass of body
   (*c*) specific heat of body
   (*d*) temperature of body and surrounding

4. A rod is fixed between two points at 20°C. The coefficient of linear expansion of material of rod is 1.1×10^{-5}/°C and Young's modulus is 1.2×10^{11} N/m. Find the stress developed in the rod if temperature of rod becomes 10°C
   (*a*) 1.32×10^{7} N/m^2
   (*b*) 1.10×10^{15} N/m^2
   (*c*) 1.32×10^{8} N/m^2
   (*d*) 1.10×10^{6} N/m^2

CALORIMETRY

5. 5 kg of water at 10°C is added to 10 kg of water at 40°C. Neglecting heat capacity of vessel and other losses, the equilibrium temperature will be close to:
   (*a*) 30°C (*b*) 25°C
   (*c*) 35°C (*d*) 33°C

6. The water equivalent of a copper calorimeter 400 g (specific heat = 0.1 cal/g°C)
   (*a*) 40 g (*b*) 4000 g
   (*c*) 200 g (*d*) 4 g

7. There are four objects A, B, C and D. It is observed that A and B are in thermal equilibrium and C and D are also in thermal equilibrium. However, A and C are not in thermal equilibrium. We can conclude that:
   (*a*) B and D are in thermal equilibrium
   (*b*) B and D could be in thermal equilibrium but might not be
   (*c*) B and D cannot be in thermal equilibrium
   (*d*) The zeroth law of thermodynamics does not apply here because there are more than three objects

8. Heat required to convert 1 g of ice at 0°C into steam at 100°C is
   (*a*) 100 cal (*b*) 0.01 cal/°C
   (*c*) 720 cal (*d*) 1 kilocal

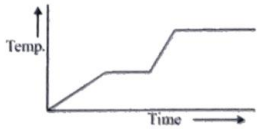
9. A substance of mass m kg requires a power input of P watts to remain in the molten state at its melting point. When the power source is turned off, the substance completely solidifies in t seconds. The latent heat of fusion of the substance is:
   (*a*) $\frac{pt}{\pi M}$ (*b*) $\frac{2pt}{\pi M}$
   (*c*) $\frac{pt}{M}$ (*d*) $\frac{pt}{2M}$

10. One kg of ice at 0°C is mixed with 1 kg of water at 10°C. The resulting temperature will be
    (*a*) between 0°C and 10°C
    (*b*) 0°C
    (*c*) less than 0°C
    (*d*) greater than 0°C

11. The ratio of densities of two bodies is 3 : 4 and ratio of their specific heats is 4 : 3. Then ratio of their heat capacity per unit volume is
    (*a*) 1 : 1 (*b*) 3 : 4
    (*c*) 9 : 16 (*d*) 16 : 9

12. 100 kg solid iron at 730°C is cooled by immersing it into vessel filled by water at 15°C. In the process, final temperature of water is 80°C. How much was mass of water ? (Neglect vaporisation)
    $S_{water} = 4200$ J/kg°C, $S_{iron} = 420$ J/kg°C.
    (*a*) 10 kg (*b*) 100 kg
    (*c*) 500 kg (*d*) 1000 kg

13. Heat is supplied to a certain homogeneous sample of matter at a uniform rate. Its temperature is plotted against time as shown in the figure. Which of the following conclusions can be drawn ?



    (*a*) Its specific heat capacity is greater in the solid state than in the liquid state
    (*b*) Its specific heat capacity is greater in the liquid state than in the solid state
    (*c*) Its latent heat of vaporization is smaller than its latent heat of fusion
    (*d*) None of these

HEAT TRANSFER

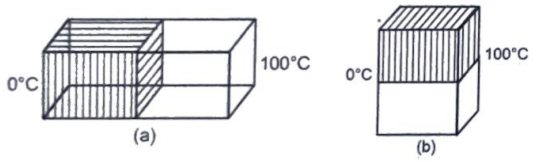
Conduction

14. Water is usually heated by
(*a*) Conduction
(*b*) Convection
(*c*) Radiation
(*d*) All of the above

15. The lengths and radii of two rods made of same material are in the ratios 1 : 2 and 2 : 3 respectively; If the temperature difference between the ends for the two rods be the same, then in the steady state, the amount of heat flowing per second through them will be in the ratio:
(*a*) 1 : 3 (*b*) 4 : 3
(*c*) 8 : 9 (*d*) 3 : 2

16. Which of following qualities suit for a cooking utensil?
(*a*) High specific heat and low thermal conductivity
(*b*) High specific heat and high thermal conductivity
(*c*) Low specific heat and low thermal conductivity
(*d*) Low specific heat and high thermal conductivity

17. Two identical square rods of metal are welded end to end as shown in figure (*a*). Assume that 10 cal of heat flows through the rods in 2 min. Now the rods are welded as shown in figure. (*b*) The time it would take for 10 cal to flow through the rods now, is



(*a*) 0.75 min (*b*) 0.5 min
(*c*) 1.5 min (*d*) 1 min

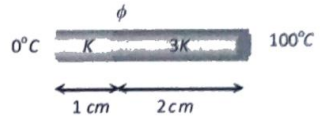
18. Consider a compound slab consisting of two different materials having equal thicknesses and thermal conductivities K and 2K respectively. The equivalent thermal conductivity of the slab is–

(*a*) $\sqrt{2}K$ (*b*) $\frac{1}{3}K$
(*c*) $\frac{3}{2}K$ (*d*) $\frac{2}{3}K$

19. When two ends of a rod wrapped with cotton are maintained at different temperatures and after some time every point of the rod attains a constant temperature, then
(*a*) Conduction of heat at different points of the rod stops because the temperature is not increasing
(*b*) Rod is bad conductor of heat
(*c*) Heat is being radiated from each point of the rod
(*d*) Each point of the rod is giving heat to its neighbour at the same rate at which it is receiving heat

20. Wires A and B have identical lengths and have circular cross-sections. The radius of A is twice the radius of B i.e. $r_A = 2r_B$. For a given temperature difference between the two ends, both wires conduct heat at the same rate. The relation between the thermal conductivities is given by
(*a*) $K_A = 4K_B$ (*b*) $K_A = 2K_B$
(*c*) $K_A = K_B/2$ (*d*) $K_A = K_B/4$

21. Two bars of thermal conductivities K and 3K and lengths 1 cm and 2 cm respectively have equal cross-sectional area, they are joined lengths wise as shown in the figure. If the temperature at the ends of this composite bar is 0°C and 100°C respectively (see figure), then the temperature ϕ of the interface is



(*a*) 50°C (*b*) $\frac{100}{3}$°C
(*c*) 60°C (*d*) $\frac{200}{3}$°C

Radiation Stefan's Law and Wein's Law

22. The velocity of heat radiation in vacuum is
(*a*) Equal to that of light
(*b*) Less than that of light
(*c*) Greater than that of light
(*d*) Equal to that of sound

23. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by
(*a*) Wien's law
(*b*) Rayleigh jeans law
(*c*) Planck's law of radiation
(*d*) Stefan's law of radiation

24. Certain substance emits only the wavelengths λ_1, λ_2, λ_3 and λ_4 when it is at a high temperature. When this substance is at a colder temperature, it will absorb only the following wavelengths
(*a*) λ_1 (*b*) λ_2
(*c*) λ_1 and λ_2 (*d*) λ_1, λ_2, λ_3 and λ_4

25. A black body has maximum wavelength λ_m at temperature 2000 K. Its corresponding wavelength at temperature 3000 K will be

(*a*) $\frac{3}{2}\lambda_m$ (*b*) $\frac{2}{3}\lambda_m$
(*c*) $\frac{4}{9}\lambda_m$ (*d*) $\frac{9}{4}\lambda_m$

26. The amount of radiation emitted by a perfectly black body is proportional to
(a) Temperature on ideal gas scale
(b) Fourth root of temperature on ideal gas scale
(c) Fourth power of temperature on ideal gas scale
(d) Source of temperature on ideal gas scale

27. The rate of radiation of a black body at 0°C is EJ/sec. The rate of radiation of this black body at 273°C will be
(a) 16 E (b) 8 E
(c) 4 E (d) E

28. A black body radiates energy at the rate of E W/m² at a high temperature TK. When the temperature is reduced to $\frac{T}{2}$K, the radiant energy will be
(a) $\frac{E}{16}$ (b) $\frac{E}{4}$
(c) 4 E (d) 16 E

29. Two spheres P and Q, of same colour having radii 8 cm and 2 cm are maintained at temperatures 127°C and 527°C respectively. The ratio of energy radiated by P and Q is
(a) 0.054 (b) 0.0034
(c) 1 (d) 2

30. The energy spectrum of a black body exhibits a maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $\frac{3\lambda_0}{4}$. The power radiated by the black body will now increase by a factor of
(a) 256/81 (b) 64/27
(c) 16/9 (d) 4/3

31. If the sun's surface radiates heat at 6.3×10^7 Wm^{-2}. Calculate the temperature of the sun assuming it to be a black body ($\sigma = 5.7 \times 10^{-8}$ $Wm^{-2}K^{-4}$)
(a) 5.8×10^3 K (b) 8.5×10^3 K
(c) 3.5×10^8 K (d) 5.3×10^8 K

32. Two identical objects A and B are at temperatures T_A and T_B respectively. Both objects are placed in a room with perfectly absorbing walls maintained at temperatures T ($T_A < T < T_B$). The objects A and B attain temperature T eventually which one of the following is correct statement
(a) 'A' only emits radiations while B only absorbs them until both attain temperature
(b) A loses more radiations than it absorbs while B absorbs more radiations that it emits until temperature T is attained
(c) Both A and B only absorb radiations until they attain temperature T
(d) Both A and B only emit radiations until they attain temperature T

33. Newton's law of cooling is a special case of
(a) Stefan's law
(b) Kirchhoff's law
(c) Wien's law
(d) Planck's law

34. Equal masses of two liquids are filled in two similar calorimeters. The rate of cooling will
(a) Depend on the nature of the liquids
(b) Depend on the specific heats of liquids
(c) Be same for both the liquids
(d) Depend on the mass of the liquids

35. A body cools from 60° C to 50°C in 10 minutes. If the room temperature is 25°C and assuming Newton's law of cooling to hold good, the temperature of the body at the end of the next 10 minutes will be
(a) 38.5°C (b) 40°C
(c) 42.85°C (d) 45°C

Learning Plus

THERMAL EXPENSION

1. A steel scale is to be prepared such that the millimeter intervals are to be accurate within 6×10^{-5} mm. The maximum temperature variation from the temperature of calibration during the reading of the millimeter marks is ($\alpha = 12 \times 10^{-6}$ k^{-1})
(a) 4.0 °C (b) 4.5 °C
(c) 5.0 °C (d) 5.5 °C

2. A steel rod 25 cm long has a cross-sectional area of 0.8 cm². Force that would be required to stretch this rod by the same amount as the expansion produced by heating it through 10°C is:
(Coefficient of linear expansion of steel is 10^{-5}/°C and Young's modulus of steel is 2×10^{10} N/m².)
(a) 160 N (b) 360 N
(c) 106 N (d) 260 N

3. If I is the moment of inertia of a solid body having α-coefficient of linear expansion then the change in I corresponding to a small change in temperature ΔT is

(a) $\alpha I \Delta T$ (b) $\frac{1}{2}\alpha I \Delta T$

(c) $2\alpha I \Delta T$ (d) $3\alpha I \Delta T$

4. A metallic wire of length L is fixed between two rigid supports. If the wire is cooled through a temperature difference ΔT (Y = young's modulus, ρ = density, α = coefficient of linear expansion) then the frequency of transverse vibration is proportional to :

(a) $\frac{\alpha}{\sqrt{\rho Y}}$ (b) $\sqrt{\frac{Y\alpha}{\rho}}$

(c) $\frac{\sigma}{\sqrt{Y\alpha}}$ (d) $\sqrt{\frac{\sigma\alpha}{Y}}$

5. A rod of length 20 cm is made of metal. It expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal B having the same length expands by 0.045 cm for the same change in temperature, a third rod of the same length is composed of two parts one of metal A and the other of metal B. Thus rod expand by 0.06 cm for the same change in temperature. The portion made of metal A has the length.

(a) 20 cm (b) 10 cm

(c) 15 cm (d) 18 cm

6. A sphere of diameter 7 cm and mass 266.5 gm floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature 35°C. If the density of a liquid at 0°C is 1.527 gm/cc, then neglecting the expansion of the sphere, the coefficient of cubical expansion of the liquid (f) is :

(a) 8.486×10^{-4} per °C

(b) 8.486×10^{-5} per °C

(c) 8.486×10^{-6} per °C

(d) 8.486×10^{-3} per °C

7. A metal ball immersed in Alcohol weights W_1 at 0°C and W_2 at 50°C. The coefficient of cubical expansion of the metal $(\gamma)_m$ is less than that of alcohol $(\gamma)_{Al}$. Assuming that density of metal is large compared to that of alcohol, it can be shown that

(a) $W_1 > W_2$

(b) $W_1 = W_2$

(c) $W_1 < W_2$

(d) any of (a), (b) or (c)

8. The volume thermal expansion coefficient of an ideal gas at constant pressure is (Here T = absolute temperature of gas)

(a) T (b) T^2

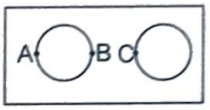
(c) $\frac{1}{T}$ (d) $\frac{1}{T^2}$

9. If two rods of length L and 2L having coefficients of linear expansion α and 2α respectively are connected so that total length becomes 3L, the average coefficient of linear expansion of the composition rod equals:

(a) $\frac{3}{2}\alpha$ (b) $\frac{5}{2}\alpha$

(c) $\frac{5}{3}\alpha$ (d) none of these

10. Two large holes are cut in a metal sheet. If this is heated, distances AB and BC, (as shown)



(a) both will increase

(b) both will decrease

(c) AB increases, BC decreases

(d) AB decreases, BC increases

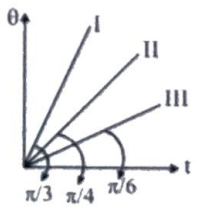
11. Two thermometers x and y have fundamental intervals of 80° and 120°. When immersed in ice, they show the reading of 20° and 30°. If y measures the temperature of a body as 120°, the reading of x is :

(a) 59° (b) 65°

(c) 75° (d) 80°

CALORIMETRY

12. Three bodies A, B and C of masses m, m and $\sqrt{3}$ m respectively are supplied heat at same rate. The change in temperature θ versus time t graph for A, B and C are shown by I, II and III respectively. If their specific heat capacities are S_A, S_B and S_C respectively then which of the following relation is correct? (Initial temperature of body is 0°C)



(a) $S_A > S_B > S_C$ (b) $S_B = S_C < S_A$

(c) $S_A = S_B = S_C$ (d) $S_B = S_C > S_A$

13. 10 gm of ice at 0°C is kept in a calorimeter of water equivalent 10 gm. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat)

(a) 6200 cal (b) 7200 cal

(c) 13600 cal (d) 8200 cal

14. A bullet of mass 70 gm travelling at 250 m/s strikes and gets embedded in ice at 0°C. Assuming that temperature of bullet does not change, find the amount of ice that melts.

(a) $\frac{2}{127}$ kg (b) $\frac{3}{424}$ kg

(c) $\frac{5}{768}$ kg (d) $\frac{7000}{256}$ kg

15. A block of ice with mass m falls into a lake. After impact, a mass of ice m/5 melts. Both the block of ice and the lake have a temperature of 0°C. If L represents the heat of fusion, the minimum distance the ice fell before striking the surface is

(a) $\frac{L}{5g}$ (b) $\frac{5L}{g}$

(c) $\frac{gL}{5m}$ (d) $\frac{mL}{5g}$

16. Water at 0°C, contained in a closed vessel, is abruptly opened in an evacuated chamber. If the latent heats of fusion and vaporization at 0°C are in the ratio $\lambda : 1$, the fraction of water evaporated will be:

(a) $\lambda / 1$ (b) $\lambda / (\lambda + 1)$

(c) $(\lambda - 1) / \lambda$ (d) $(\lambda - 1) / (\lambda + 1)$

17. The density of a material A is 1500 kg/m^3 and that of another mateial B is 2000 kg/m^3. It is found that the heat capacity of 8 volumes of A is equal to heat capacity of 12 volumes of B. The ratio of specific heats of A and B will be

(a) 1 : 2 (b) 3 : 1

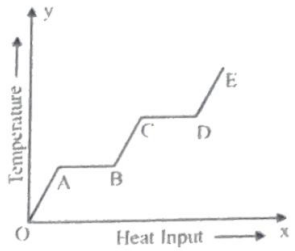
(c) 3 : 2 (d) 2 : 1

18. The specific heat of a solid at low temperature varies according to the relation $c = kT^3$ where k is a constant and T is temperature in kelvin. The heat required to raise the temperature of a mass m of such a solid from T = 0 K to T = 20 K is :

(a) 2×10^4 mk (b) 4×10^4 mk

(c) 8×10^4 mk (d) 16×10^4 mk

19. A solid material is supplied with heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does slope DE represent.



(a) latent heat of liquid

(b) latent heat of vapour

(c) heat capacity of vapour

(d) inverse of heat capacity of vapour

20. 20 gm of water at 30°C is mixed with 5gm of ice at –10°C. Find the final temperature of mixture (in °C) if latent heat of fusion of ice is 80 cal/gm, specific heat of water = 1 cal/gm°C, specific heat of ice = 0.5 cal/gm°C.

(a) 0°C (b) –5°C

(c) 7°C (d) 10°C

HEAT TRANSFER

21. Which of the law can be understood in terms of Stefan's law

(a) Wien's displacement law

(b) Kirchoff's law

(c) Newton's law of cooling

(d) Planck's law

22. A boiler is made of a copper plate 2.4 mm thick with an inside coating of a 0.2 mm thick layer of tin. The surface area exposed to gases at 700°C is 400 cm^2. The amount of steam that could be generated per hour at atmospheric pressure is ($K_{cu} = 0.9$ and $K_{tin} = 0.15$ cal/cm/s/°C and $L_{steam} = 540$ cal/g)

(a) 5000 Kg (b) 1000 kg

(c) 4000 kg (d) 200 kg

23. A pot with a steel bottom 1.2 cm thick rests on a hot stove. The area of the bottom of the pot is 0.150 m^2. The water inside the pot is at 100°C and 0.440 kg vapourise in every 5 minutes. The temperature of the lower surface of the pot, which is in contact with the stove is (Given : $L_v = 2.256 \times 10^6$ J/kg and $K_{steel} = 50.2$ W/m-K)

(a) 105.3 °C (b) 205.3 °C

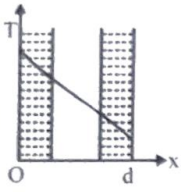
(c) 185.3 °C (d) 115.3 °C

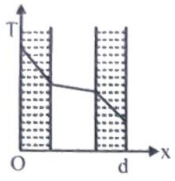
24. Four rods of same material with different radii r and length l are used to connect two reservoirs of heat at different temperatures. Which one will conduct most heat?

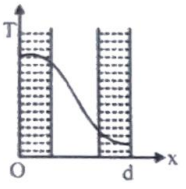
(a) r = 2 cm, l = 0.5m (b) r = 2 cm, l = 2m

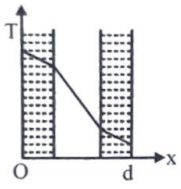
(c) r = 0.5 cm, l = 0.5m (d) r = 1 cm, l = 1 m

25. The wall with a cavity consists of two layers of brick separated by a layer of air. All three layers have the same thickness and the thermal conductivity of the brick is much greater than that of air. The left layer is at a higher temperature than the right layer and steady state condition exists. Which of the following graphs predicts correctly the variation of temperature T with distance d inside the cavity?

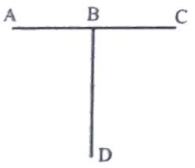
(*a*) 

(*b*) 

(*c*) 

(*d*) 

26. Three conducting rods of same material and cross-section are shown in figure. Temperatures of A, D and C are maintained at 20°C, 90°C and 0°C. The ratio of lengths of BD and BC if there is no heat flow in AB is



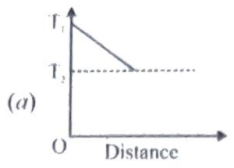
(*a*) 2/7 (*b*) 7/2
(*c*) 9/2 (*d*) 2/9

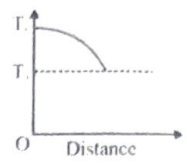
27. Six identical conducting rods are joined as shown in figure. Points A and D are maintained at temperature of 200°C and 20°C respectively. The temperature of junction B will be :

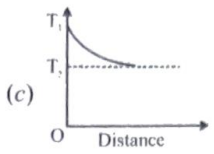


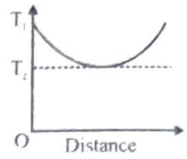
(*a*) 120°C (*b*) 100°C
(*c*) 140°C (*d*) 80°C

28. The ends of a metal bar of constant cross-sectional area are maintained at temperatures T_1 and T_2 which are both higher than the temperature of the surroundings. If the bar is unlagged, which one of the following sketches best represents the variation of temperature with distance along the bar?

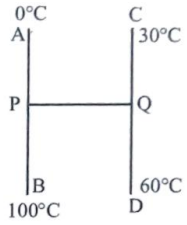
(*a*) 

(*b*) 

(*c*) 

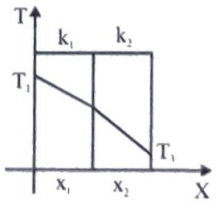
(*d*) 

29. Three identical rods AB, CD and PQ are joined as shown. P an Q are mid points of AB and CD respectively. Ends A, B, C and D are maintained at 0°C, 100°C, 30°C and 60°C respectively. The direction of heat flow in PQ is



(*a*) from P to Q
(*b*) from Q to P
(*c*) heat does not flow in PQ
(*d*) data not sufficient

30. The temperature drop through each layer of two layer furnace wall is shown in figure. Assume that the external temperature T_1 and T_3 are maintained constant and $T_1 > T_3$. If the thickness of the layers x_1 and x_2 are the same, which of the following statements are correct.



(*a*) $k_1 > k_2$
(*b*) $k_1 < k_2$
(*c*) $k_1 = k_2$ but heat flow through material (*a*) is larger then through (*b*)
(*d*) $k_1 = k_2$ but heat flow through material (*a*) is less than that through (*b*)

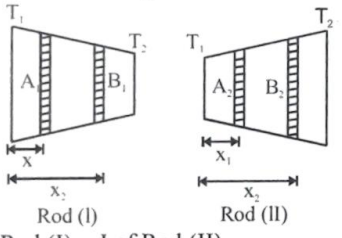
31. Two sheets of thickness d and 3d, are touching each other. The temperature just outside the thinner sheet side is A, and on the side of the thicker sheet is C. The interface temperature is B. A, B and C are in arithmetic progression, then the ratio of thermal conductivity of thinner sheet and thicker sheet is

(a) 1 : 3 (b) 3 : 1
(c) 2 : 3 (d) 1 : 9

32. A cylindrical rod with one end in a steam chamber and the outer end in ice results in melting of 0.1 gm of ice per second. If the rod is replaced by another with half the length and double the radius of the first and if the thermal conductivity of material of second rod is 1/4 that of first, the rate at which ice melts is gm/sec will be

(a) 3.2 (b) 1.6
(c) 0.2 (d) 0.1

33. The two ends of two similar non-uniform rods of length ℓ each and thermal conductivity 'K' are maintained at different but constant temperature. The temperature gradient at any point on the rod is $\frac{\Delta T}{\Delta \ell}$. The heat flow per unit time through the rod is I : **Given** $T_1 > T_2$. Then which of the following is true:



(a) I of Rod (I) = I of Rod (II)
(b) I of Rod (I) > I of Rod (II)
(c) I of Rod (I) < I of Rod (II)
(d) data is insufficient

34. A metallic sphere having radius 0.08 m and mass m = 10kg is heated to a temperature of 227°C and suspended inside a box whose walls are at a temperature of 27°C. The maximum rate at which its temperature will fall is : (**Take** e = 1, Stefan's constant $\sigma = 5.8 \times 10^{-8}\ Wm^{-2}K^{-4}$ and specific heat of the metal s = 90 cal/kg/deg J = 4.2 Joules/Calorie)

(a) .055 °C/sec (b) .066 °C/sec
(c) .044 °C/sec (d) 0.03 °C/sec

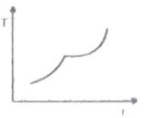
35. A wall has two layers A and B, each made of different material. Both the layers have the same thickness. The thermal conductivity for A is twice that of B. Under steady state, the temperature difference across the whole wall is 36°C. Then the temperature difference across the layer A is

(a) 6°C (b) 12°C
(c) 18°C (d) 24°C

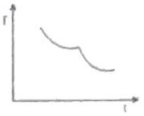
36. Star S_1 emits maximum radiation of wavelength 420 nm and the star S_2 emits maximum radiation of wavelength 560 nm, what is the ratio of the temperature of S_1 and S_2?

(a) 4/3 (b) $(4/3)^{1/4}$
(c) 3/4 (d) $(3/4)^{1/2}$

37. An ice cube at temperature -20°C is kept in a room at temperature 20°C. The variation of temperature of the body with time is given by

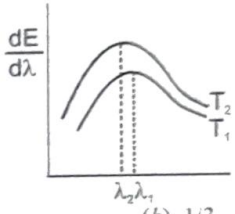
(a) 

(b) 

(c) 

(d) 

38. The spectral emissive power E_λ for a body at temperature T_1 is plotted against the wavelength and area under the curve is found to be A. At a different temperature T_2 the area is found to be 9A Then λ_1/λ_2 =



(a) 3 (b) 1/3
(c) $1/\sqrt{3}$ (d) $\sqrt{3}$

39. Two bodies P and Q have thermal emissivities of ε_P and ε_Q respectively. Surface areas of these bodies are same and the total radiant power is also emitted at the same rate. If temperature of P is θ_P kelvin then temperature of Q i.e. θ_Q is

(a) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{1/4}\theta_P$ (b) $\left(\frac{\varepsilon_P}{\varepsilon_Q}\right)^{1/4}\theta_P$
(c) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{1/4}\times\frac{1}{\theta_P}$ (d) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{4}\theta_P$

40. The rate of emission of radiation of a black body at 273°C is E, then the rate of emission of radiation of this body at 0°C will be

(a) $\frac{E}{16}$ (b) $\frac{E}{4}$
(c) $\frac{E}{8}$ (d) 0

41. A system S receives heat continuously from an electrical heater of power 10 W. The temperature of S becomes constant at 50°C when the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1 °C to 34.9 °C in 1 minute. The heat capacity of S is

(a) 100 J/°C (b) 300 J/°C
(c) 750 J/°C (d) 1500 J/°C

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/STATEMENT/MATCHING

1. An experiment is perfomed to measure the specific heat of copper. A lump of copper is heated in an oven, then dropped into a beaker of water. To calculate the specific heat of copper, the experimenter must know or measure the value of all of the quantities below EXCEPT the
   (*a*) heat capacity of water and beaker
   (*b*) original temperature of the copper and the water
   (*c*) final (equilibrium) temperature of the copper and the water
   (*d*) time taken to achieve equilibrium after the copper isdropped into the water

2. When m, gm of water at 10°C is mixed with m, gm of ice at 0°C, which of the following statements are false?
   (*a*) The temperature of the system will be given by the equation $m \times 80 + m \times 1 \times (T - 0) = m \times 1 \times (10 - T)$
   (*b*) Whole of ice will melt and temperature will be more than 0°C but lesser than 10°C
   (*c*) Whole of ice will melt and temperature will be 0°C
   (*d*) Whole of ice will not melt and temperature will be 0°C

3. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,
   (*a*) the temperature of B is 1934 K
   (*b*) $\lambda_B = 1.5$ μm
   (*c*) the temperature of B is 11604 K
   (*d*) the temperature of B is 2901 K

4. The solar constant is the amount of heat energy received per second per unit area of a perfectly black surface placed at a mean distance of the Earth from the Sun, in the absence of Earth's atmosphere, the surface being held perpendicular to the direction of Sun's rays. Its value is 1388 W/m^2. If the solar constant for the earth is 's'. The surface temperature of the sun is TK. The sun subtends a small angle 'θ' at the earth. Then correct options is/ are:
   (*a*) $s \propto T^2$ (*b*) $s \propto T^4$
   (*c*) $s \propto \theta^2$ (*d*) $s \propto \theta$

5. A force of 200 N is applied at one end of a wire of length 2 m and having area of cross-section 10 cm^2. The other end of the wire is rigidly fixed. If coefficient of linear expansion of the wire $\alpha = 8 \times 10^{-6}$ °C and Young's modulus $Y = 2.2 \times 10^{11}$ N/m^2 and its temperature is increased by 5°C, then the increase in the tension of the wire will be
   (*a*) 4.2 N (*b*) 4.4 N
   (*c*) 2.4 N (*d*) 8.8 NI

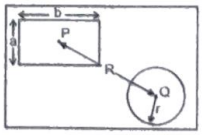
6. In accordance with Kirchhoff's law :
   (*a*) bad absorber is bad emitter
   (*b*) bad absorber is good reflector
   (*c*) bad reflector is good emitter
   (*d*) bad emitter is good absorber

7. A wire of cross-sectional area 3mm^2 is first stretched between two fixed points at a temperature of 20°C. Determine the tension when the temperature falls to 10°C (Coefficient of linear expansion $\alpha = 10^{-5}$ °C^{-1} and $Y = 2 \times 10^{11}$ N/m^2)
   (*a*) 20 N (*b*) 30 N
   (*c*) 60 N (*d*) 120 N

8. The coefficient of linear expansion of brass and steel are α_1 and α_2. If we take a brass rod of length l_1 and steel rod of length l_2 at 0°C, their difference in length $(l_2 - l_1)$ will remain the same at a temperature if
   (*a*) $\alpha_1 l_2 = \alpha_2 l_1$ (*b*) $\alpha_1 l_2^2 = \alpha_2 l_1^2$
   (*c*) $\alpha_1^2 l_1 = \alpha_2^2 l_2$ (*d*) $\alpha_1 l_1 = a_2 l_2$

9. A polished metallic piece and a black painted wooden piece are kept in open in bright sun for a long time:
   (*a*) the wooden piece will absorbs less heat than the metallic piece
   (*b*) the wooden piece will have a lower temperature than the metallic piece
   (*c*) if touched, the metallic piece will feel hotter than the wooden piece
   (*d*) when the two pieces are removed from the open to a cold room, the wooden piece will lose heat at a faster rate than the metallic piece

10. There is a rectangular metal plate in which two cavities in the shape of rectangle and circle are made, as shown with dimensions. P and Q are the centres of these cavities. On heating the plate, which of the following quantities increase ?



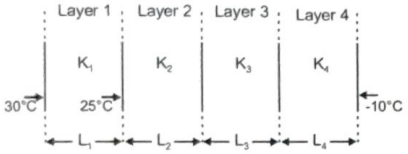
   (*a*) πr^2 (*b*) ab
   (*c*) R (*d*) b

11. Four rods A, B, C, D of same length and material but of different radii r, $r\sqrt{2}$, $r\sqrt{3}$ and 2r respectively are held between two rigid walls. The temperature of all rods is increased by same amount. If the rods do not bend, then
(*a*) the stress in the rods are in the ratio 1 : 2 : 3 : 4
(*b*) the force on the rod exerted by the wall are in the ratio 1 : 2 : 3 : 4
(*c*) the energy stored in the rods due to elasticity are in the ratio 1 : 2 : 3 : 4
(*d*) the strains produced in the rods are in the ratio 1 : 2 : 3 : 4

12. When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%.
(*a*) Percentage rise in the area of a face is 0.4%
(*b*) Percentage rise in the thickness is 0.4%
(*c*) Percentage rise in the volume is 0.6%
(*d*) Coefficient of linear expansion of copper is $0.25 \times 10^{-4}C^{\circ-1}$.

Comprehension – 1 (No. 13 to 15)

Figure shows in cross section a wall consisting of four layers with thermal conductivities $K_1 = 0.06$ W/m°C; $K_3 = 0.04$ W/m°C and $K_4 = 0.10$ W/m°C. The layer thicknesses are $L_1 = 1.5$ cm; $L_3 = 2.8$ cm and $L_4 = 3.5$ cm. The temperature of interfaces is as shown in figure. Energy transfer through the wall is steady.



13. The temperature of the interface between layers 3 and 4 is:
(*a*) – 1°C (*b*) – 3°C
(*c*) 2°C (*d*) 0°C

14. The temperature of the interface between layers 2 and 3 is:
(*a*) 11°C (*b*) 8°C
(*c*) 7.2°C (*d*) 5.4°C

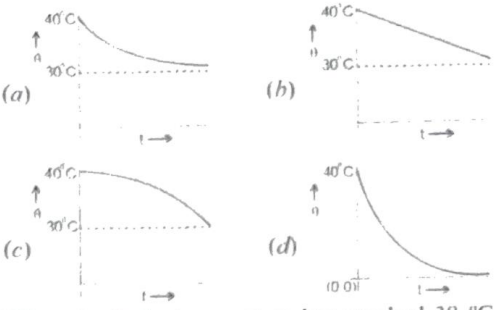
15. If layer thickness L_2 is 1.4 cm, then its thermal conductivity K_2 will have value (in W/mK) :
(*a*) 2×10^{-2} (*b*) 2×10^{-3}
(*c*) 4×10^{-2} (*d*) 4×10^{-3}

Comprehension – 2 (No. 16 to 18)

A body cools in a surrounding of constant temperature 30 °C. Its heat capacity is 2J/°C. Initial temperature of the body is 40°C. Assume Newton's law of cooling is valid. The body cools to 38°C in 10 minutes.

16. In further 10 minutes it will cool from 38°C to :
(*a*) 36°C (*b*) 36.4°C
(*c*) 37°C (*d*) 37.5°C

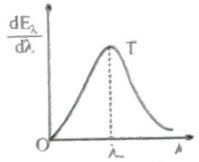
17. The temperature of the body in °C denoted by θ the variation of θ versus time t is best denoted as



18. When the body temperature has reached 38 °C, it is heated again so that it reaches to 40°C in 10 minutes. The total heat required from a heater by the body is :
(*a*) 3.6J (*b*) 0.364J
(*c*) 8 J (*d*) 4 J

Comprehension – 3 (No. 19 to 21)

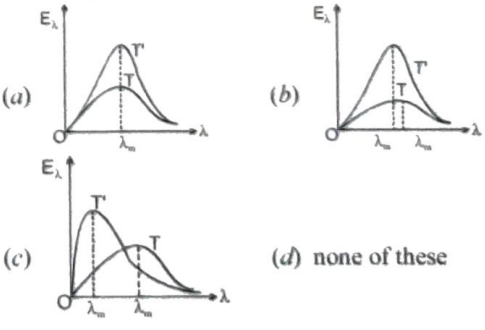
The figure shows a radiant energy spectrum graph for a black body at a temperature T.



19. Choose the correct statement(s)
(*a*) The radiant energy is not equally distributed among all the possible wavelengths
(*b*) For a particular wavelength the spectral intensity is maximum
(*c*) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature
(*d*) None of these

20. If the temperature of the body is raised to a higher temperature T,' then choose the correct statement(s)
(*a*) The intensity of radiation for every wavelength increases
(*b*) The maximum intensity occurs at a shorter wavelength
(*c*) The area under the graph increases
(*d*) The area under the graph is proportional to the fourth power of temperature

21. Identify the graph which correctly represents the spectral intensity versus wavelength graph at two temperatures T' and T(T < T')



(*d*) none of these

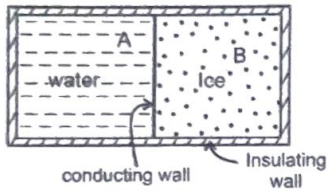
Comprehension – 4 (No. 22 to 24)

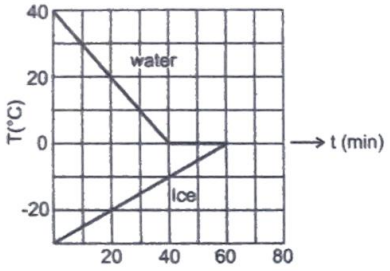
A 0.60 kg sample of water and a sample of ice are placed in two compartments A and B that are separated by a conducting wall, in a thermally insulated container. The rate of heat transfer from the water to the ice though the conducting wall is constant P, until thermal equilibrium is reached. The temperature T of the liquid water and the ice are given in graph as functions of time t. Temperature of the compartments remain homogeneous during whole heat transfer process.

Given specific heat of ice = 2100 J/kg-K

Given specific heat of water = 4200 J/kg-K

Latent heat of fusion of ice = 3.3×10^5 J/kg





22. The value of rate P is

(*a*) 42.0 W (*b*) 36.0 W

(*c*) 21.0 W (*d*) 18.0 W

23. The initial mass of the ice in the container is equal to

(*a*) 0.36 kg (*b*) 1.2 kg

(*c*) 2.4 kg (*d*) 3.6 kg

24. The mass of the ice formed due to conversion from the water till thermal equilibrium is reached, is equal to

(*a*) 0.12 kg (*b*) 0.15 kg

(*c*) 0.25 kg (*d*) 0.40 kg

Comprehension – 5 (No. 25 to 27)

In a container of negligible heat capacity, 200 gm ice at 0°C and 100 gm steam at 100°C are added to 200 gm of water that has temperature 55°C. Assume no heat is lost to the surroundings and the pressure in the container is constant 1.0 atm. (Latent heat of fusion of ice = 80 cal/gm, Latent heat of vaporization of water = 540 cal/gm, Specific heat capacity of ice = 0.5 cal/gm-K, Specific heat capacity of water = 1 cal/gm-K)

25. What is the final temperature of the system ?

(*a*) 48°C (*b*) 72°C

(*c*) 94°C (*d*) 100°C

26. At the final temperature, mass of the total water present in the system, is

(*a*) 472.6 gm (*b*) 483.3 gm

(*c*) 493.6 gm (*d*) 500 gm

27. Amount of the steam left in the system, is equal to

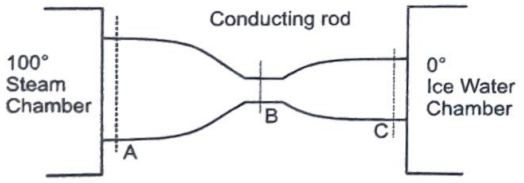
(*a*) 16.7 gm

(*b*) 12.0 gm

(*c*) 8.4 gm

(*d*) 0 gm, as there is no steam left.

28. A copper rod (initially at room temperature 20°C) of non-uniform cross section is placed between a steam chamber at 100°C and ice-water chamber at 0°C.



(*a*) Initially rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be (p) maximum at section A

(*b*) At steady state rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be (q) maximum at section B

(*c*) At steady state temperature gradient $\left|\left(\frac{dT}{dx}\right)\right|$ will be (r) minimum at section A

(*d*) At steady state rate of change of temperature $\left(\frac{dT}{dt}\right)$ at a certain point will be (s) minimum at section B

(t) same for all section

29. In the following question column - I represents some physical quantities & column-II represents their units, match them

Column –I	Column-II
(*a*) Coefficient of linear expansion	(p) Cal/°C
(*b*) Water equivalent	(q) gm
(*c*) heat capacity	(r) $(°C)^{-1}$
(*d*) Specific heat	(s) Cal/g°C

INTEGER TYPE

30. A 30 cm long metal rod expands by 0.0650 cm when its temperature is raised from 0°C to 100°C. A second rod of different metal and of the same length expands by 0.0350 cm for the same rise in temperature. A third composite rod, also 30 cm long, is made-up of pieces of each of the above metals placed end to end and expands by 0.0580 cm when temperature is increased from 0°C to 100°C. Find the length of the longer portion of the composite bar in cm at 0°C.

31. A steel rod at 25°C is welded at both ends and then cooled. By how many °C should the rod be cooled so that it will rupture? Assume that till rupture, Hooke's law is obeyed.
($\alpha_{steel} = 10 \times 10^{-6}/°C$, $Y = 2 \times 10^{11} N/m^2$ and σ_b (breaking stress of steel rod) $= 4 \times 10^8$ N/m^2)

32. In the past, temperature was measured in laboratories using the so called "weight" thermometer, which consists of a hollow platinum sphere filled with mercury and provided with a small hole. An increase in temperature was estimated from the amount of mercury flowing out of the hole. How much temperature (in °C) is increased if 0.0153% of mercury flows out of the hole of such a thermometer, if the completely filllied sphere of thermometer contained mercury at 0°C. The coefficient of volume expansion of platinum is $\gamma_1 = 2.7 \times 10^{-5}/°C$ and of mercury is $\gamma_2 = 1.8 \times 10^{-4}/°C$.

33. A spherical black body of diameter 0.10 m is held at constant temperature. If the power radiated by the body is $5.67\pi \times 10^2$ Js^{-1}, the temperature of the body (in K) is

34. The filament of an incandescent lamp of power 64 W is made of Tungsten. The operation temperature of the lamp is 2000K. Consider the filament a black body and find its radius (in mm).
[**Given :** $\sigma = 6 \times 10^{-8}$ W/m^2 & Length of filament is $\frac{10}{3\pi}$cm]

35. A thermos contains 140 gm coffee at 80°C. To cool it, we drop two 10 gm ice cubes at 0°C into thermos. Ice cubes are at 0°C initially. What is the final temperature of the coffee (in °C), if its specific heat capacity is same as that of water.

36. A liquid A is kept flowing through a tube while it is cooled by water surrounding the tube. The temperature of the incoming liquid A is 60°C and that of the outgoing liquid A is 30°C. The capacity of the cooler tank is 60 litre and the water in it is completely changed in every hour. The initial temperature of the cooling water is 10°C and it is 20°C when it is changed. What is the amount of the liquid (in kg) that goes through the tube in one hour? (Take : Specific heat capacity of water S_{water} = 4200 J/kg-K, $S_{liquid\ A}$ = 140 J/kg-K, density of water = 1000 kg/m^3)

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4s a day if the temperature is 20°C. The temperature at which the clock will show correct time, *and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:*

[*JEE Mains-2016*]

(*a*) $25^0C; \alpha = 1.85 \times 10^{-5}/^0C$
(*b*) $60^0C; \alpha = 1.85 \times 10^{-4}/^0C$
(*c*) $30^0C; \alpha = 1.85 \times 10^{-3}/^0C$
(*d*) $55^0C; \alpha = 1.85 \times 10^{-2}/^0C$

2. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by: (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C.

[*JEE Main-2017*]

(*a*) 1250°C (*b*) 825°C
(*c*) 800°C (*d*) 885° C

3. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$?

[*JEE Main-2019 (January)*]

(*a*) 25 (*b*) 60
(*c*) 40 (*d*) 45

4. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water ? [Specific heat Capacitites of water and metal are, respectively 4200 $Jkg^{-1}K^{-1}$ and 400 $Jkg^{-1}K^{-1}$]

[*JEE Main-2019 (January)*]

(*a*) 15% (*b*) 30%
(*c*) 25% (*d*) 20%

5. When 100g of a liquid A at 100°C is added to 50g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100g of liquid A at 100°C is added to 50g of liquid B at 50°C, will be: [*JEE Main-2019 (January)*]

(*a*) 85°C (*b*) 60°C
(*c*) 80°C (*d*) 70°C

6. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 $J\,kg^{-1}\,K^{-1}$) : [*JEE Main-2019 (January)*]

(*a*) 458 $J\,kg^{-1}\,K^{-1}$ (*b*) 1232 $J\,kg^{-1}\,K^{-1}$
(*c*) 916 $J\,kg^{-1}\,K^{-1}$ (*d*) 654 $J\,kg^{-1}\,K^{-1}$

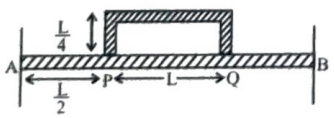
7. Ice at – 20° C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to
(Specific heat of water = 4.2 J/g/°C). Heat of fusion of water at 0°C = 334 J/g)

[*JEE Main-2019 (January)*]

(*a*) 50 g (*b*) 100 g
(*c*) 60 g (*d*) 40 g

8. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to:

[*JEE Main-2019 (January)*]



(*a*) 45° C (*b*) 75° C
(*c*) 60° C (*d*) 35° C

9. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is:

[*JEE Main-2019 (January)*]

(*a*) $\frac{K_1+K_2}{2}$ (*b*) K_1+K_2

(*c*) $\frac{2K_1+3K_2}{5}$ (*d*) $\frac{K_1+3K_2}{4}$

10. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 $W\,K^{-1}m^{-1}$, the energy flux through it in the state is:

[*JEE Main-2019 (January)*]

(*a*) 90 Wm^{-2} (*b*) 120 Wm^{-2}
(*c*) 65 Wm^{-2} (*d*) 200 Wm^{-2}

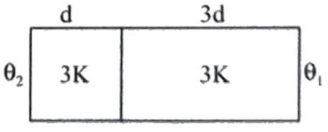
11. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely ? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4384 J/kg K)

[*JEE Main-2019 (April)*]

(*a*) 10^{-3} K (*b*) 10^{-4} K
(*c*) 10^{-1} K (*d*) 10^{-5} K

12. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and '3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is:

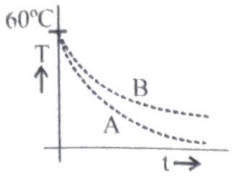
[*JEE Main-2019 (April)*]

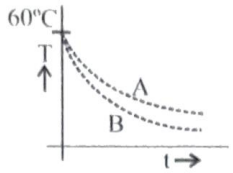


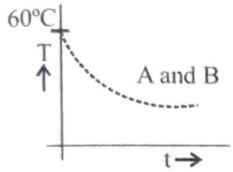
(*a*) $\frac{\theta_2+\theta_1}{2}$ (*b*) $\frac{\theta_1}{10}+\frac{9\theta_2}{10}$

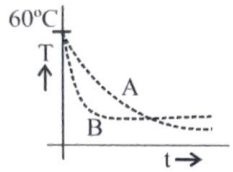
(*c*) $\frac{\theta_1}{3}+\frac{2\theta_2}{3}$ (*d*) $\frac{\theta_1}{6}+\frac{5\theta_2}{6}$

13. Two identical breakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^3 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same) [JEE Main-2019 (April)]

(a) 

(b) 

(c) 

(d) 

14. A thermally insulated vessel contains 150g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to:
(Latent heat of vaporization of water=2.10×10^6 J kg^{-1} and Latent heat of Fusion of water = 3.36×10^5 J kg^{-1}) [JEE Main-2019 (April)]

(a) 130 g (b) 35 g
(c) 20 g (d) 150 g

15. When M_1 gram of ice at –10° C (specific heat = 0.5 cal g^{-1} °C^{-1}) is added to M_2 gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is : [JEE Main-2019 (April)]

(a) $\frac{5M_1}{M_2} - 50$ (b) $\frac{50M_2}{M_1}$

(c) $\frac{50M_2}{M_1} - 5$ (d) $\frac{5M_2}{M_1} - 5$

16. M grams of steam at 100°C is mixed with 200g of ice at its melting point in a thermally insulated container. If the produces liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is_____. [JEE Main-2020 (January)]

17. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in litres) are taken from each container and mixed (assume no loss of heat during the process) [JEE Main-2020 (January)]

C_1	C_2	C_3	T
1ℓ	2ℓ		60°C
	1ℓ	2ℓ	30°C
2ℓ	–	1ℓ	60°C
1ℓ	1ℓ	1ℓ	θ

The value of θ (in °C to the nearest integer) is_____

18. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of 'm' is close to
(Latent heat of water = 540 cal g^{-1}, Specific heat of water = 1 cal g^{-1} °C^{-1}] [JEE Main-2020 (September)]

(a) 2 (b) 3.2
(c) 2.6 (d) 4

19. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? [JEE Main-2020 (September)]

(a) 6 (b) 7
(c) 3 (d) 5

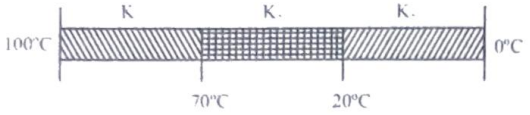
20. The specific heat of water = 4200 J kg^{-1} K^{-1} and the latent heat of ice = 3.4×10^5 J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams) [JEE Main-2020 (September)]

(a) 69.3 (b) 63.8
(c) 64.6 (d) 61.7

21. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to [JEE Main-2020 (September)]

(a) 28°C (b) 35°C
(c) 33°C (d) 31°C

22. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is [JEE Main-2020 (September)]



(a) $K_1 : K_3 = 2 : 3$; $K_2 : K_3 = 2 : 5$
(b) $K_1 < K_2 < K_3$
(c) $K_1 : K_2 = 5 : 2$; $K_1 : K_3 = 3 : 5$
(d) $K_1 > K_2 > K_3$

23. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is: [AIEEE-2012]



(a) $2\pi SY\alpha\Delta T$ (b) $SY\alpha\Delta T$
(c) $\pi SY\alpha\Delta T$ (d) $2SY\alpha\Delta T$

24. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by : [JEE Main-2017]

(a) $\frac{3a}{PK}$ (b) 3 PKa
(c) $\frac{P}{3\alpha K}$ (d) $\frac{P}{\alpha K}$

25. Two rods A and B of identical dimensions are at temperature 30°C. If a heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio fo the coefficients of lienar expansion of A and B is 4 : 3, then the value of T is: [JEE Main-2019 (January)]
(a) 230°C (b) 270°C
(c) 200°C (d) 250°C

26. A rod of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when it temperature rises by ΔT°K. Young's modulus, Y, for this metal is [JEE Main-2019 (January)]

(a) $\frac{F}{A\alpha\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T - 273)}$
(c) $\frac{F}{2A\alpha\Delta T}$ (d) $\frac{2F}{A\alpha\Delta T}$

27. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultanously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equals to : [JEE Main-2019 (April)]
(a) $F/(3\pi r^2YT)$ (b) $3F/(\pi r^2YT)$
(c) $6F/(\pi r^2YT)$ (d) $9F/(\pi r^2YT)$

28. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2m. The value of M is close to : (Coefficient of linear expansion and Young' s modulus of brass are 10^{-5}/°Cand 10^{11} N/m^2, respectively; g = 10 ms^{-2}) [JEE Main-2019 (April)]
(a) 1.5 kg (b) 9 kg
(c) 0.9 kg (d) 0.5 kg

29. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}$ °C^{-1} and $\gamma_{(mercury)} = 1.5 \times 10^{-4}$ °C^{-1}, where γ is the coefficient of volume expansion, then V_m (in cc) is close to _____. [JEE Main-2020 (September)]

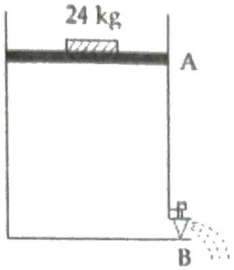
30. A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to
(Given bulk modulus of metal, B = 8×10^{10} Pa)
[JEE Main-2020 (September)]
(a) 0.6 (b) 20
(c) 1.67 (d) 5

31. Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is
[JEE Main-2020 (September)]

(a) $\sqrt[2]{\alpha_1\alpha_2}$ (b) $\frac{4\alpha_1\alpha_2}{\alpha_1+\alpha_2}\frac{L_2L_1}{(L_2+L_1)^2}$
(c) $\frac{\alpha_1+\alpha_2}{2}$ (d) $\frac{\alpha_1L_1+\alpha_2L_2}{L_1+L_2}$

32. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near

the bottom whose cross-section area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v 0.4 ms^{-1}.
The value of v, to the nearest integer, is [Take value of g to be 10 ms^{-2}] [*JEE Main-2021 (March)*]



33. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm^2 by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}$ cm. Then the thickness of oleic acid Layer will be $x \times 10^{-14}$ m. Where x is [*JEE Main-2021 (March)*]

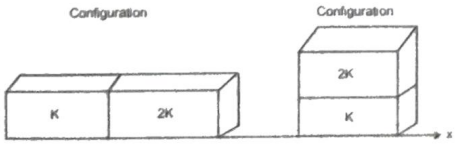
JEE-ADVANCED PREVIOUS YEAR'S

1. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is [*IIT JEE-2012*]

(*a*) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ (*b*) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$

(*c*) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ (*d*) $(97)^{\frac{1}{4}} T$

2. Two rectangular blocks, having indentical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure, One of the blocks has thermal conductivity k and the other 2k. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is : [*JEE -2013*]

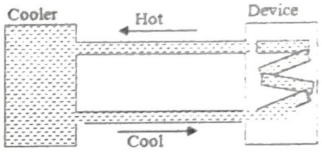


(*a*) 2.0 s (*b*) 3.0 s
(*c*) 4.5 s (*d*) 6.0 s

3. Parallel rays of light of intensity $I = 912$ Wm^{-2} are incident on a spherical balck body kept in surroundings of temperature 300 K. Take Stefan-Biltzmann constant $\sigma = 5.7 \times 10^{-8}$ Wm^{-2} K^{-4} and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to: [*JEE Advanced-2014*]

(*a*) 330 K (*b*) 660 K
(*c*) 990 K (*d*) 1550 K

4. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is [*JEE Advanced -2016*]



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kgm^{-3})

(*a*) 1600 (*b*) 2067
(*c*) 2533 (*d*) 3933

5. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true? [*JEE Advanced -2016*]
   (*a*) The temperature distribution over the filament is uniform
   (*b*) The resistance over small sections of the filament decreases with time
   (*c*) The filament emits more light at higher band of frequencies before it breaks up
   (*d*) The filament consumes less electrical power towards the end of the life of the bulb.

6. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C? [*JEE Advanced-2016*]

7. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of the linear thermal expansion of PQ is $1.2 \times 10^{-5}\ K^{-1}$, the change in length of the wire PQ is: [JEE Advanced -2016]

(a) 0.78 mm (b) 0.90 mm

(c) 1.56 mm (d) 2.34 mm

8. A human body has surface area of approximately $1m^2$. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be $T_0 = 300$ K. For $T_0 = 300$ K, the value of $\sigma T_0^4 = 460 Wm^{-2}$ (where σ is the Stefan-Boltzmann constant.). Which of the following options is/are correct? [JEE Advanced-2017]

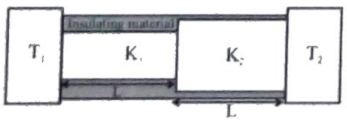
(a) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time.

(b) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(c) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

(d) The amount of energy radiated by the body in 1 second is close to 60 joules

9. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperature $T_1 = 300$ K and $T_2 = 100$ K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady is 200 K, then K_1/K_2 = _____ [JEE Advanced-2018]



10. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as : [JEE Advanced-2019]

$T(t) = T_0(1 + \beta t^{1/4})$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is:

(a) $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$ (b) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

(c) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (d) $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

11. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be _____ °C.

[Neglect the heat exchange with surrounding]

[JEE Advanced-2019]

12. The filament of a light bulb has surface area 64 mm^2. The filament can be considered as a black body at temperature 2500 K emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of 100 m. Assume the pupil of the eyes of the observer to be circular with radius 3 mm. Then (**Take** Stefan-Boltzmann constant = 5.67×10^{-8} $Wm^{-2}\ K^{-4}$, Wien's displacement constant = 2.90×10^{-3} m-K, Planck's constant = 6.63×10^{-34} Js, speed of light in vacuum = $3.00 \times 10^8\ ms^{-1}$)- [JEE Advanced - 2020]

(a) power radiated by the filament is in the range 642 W to 645 W

(b) radiated power entering into one eye of the observer is in the range 3.15×10^{-8} W to 3.25×10^{-8} W

(c) the wavelength corresponding to the maximum intensity of light is 1160 nm

(d) taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range 2.75×10^{11} to 2.85×10^{11}

13. A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm^{-2} and it is absorbed by the water over an effective area of 0.05 m^2. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be _____. (Ignore effect of the container, and take constant for Newton's law of cooling = 0.001 s^{-1}, Heat capacity of water = 4200 J $kg^{-1}\ K^{-1}$)

[JEE Advanced-2020]

ANSWER KEY

Topicwise Questions

1. (*c*, *d*)	**2.** (*a*, *b*, *d*)	**3.** (*a*, *b*, *c*, *d*)	**4.** (*a*)	**5.** (*a*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*b*)
11. (*d*)	**12.** (*b*)	**13.** (*a*)	**14.** (*b*)	**15.** (*c*)	**16.** (*d*)	**17.** (*b*)	**18.** (*c*)	**19.** (*d*)	**20.** (*d*)
21. (*c*)	**22.** (*a*)	**23.** (*c*)	**24.** (*d*)	**25.** (*b*)	**26.** (*c*)	**27.** (*a*)	**28.** (*a*)	**29.** (*c*)	**30.** (*a*)
31. (*a*)	**32.** (*b*)	**33.** (*a*)	**34.** (*b*)	**35.** (*c*)					

Learning Plus

1. (*c*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)	**5.** (*b*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*a*)
11. (*d*)	**12.** (*d*)	**13.** (*d*)	**14.** (*c*)	**15.** (*a*)	**16.** (*b*)	**17.** (*d*)	**18.** (*b*)	**19.** (*d*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** (*a*)	**24.** (*a*)	**25.** (*d*)	**26.** (*b*)	**27.** (*c*)	**28.** (*c*)	**29.** (*a*)	**30.** (*a*)
31. (*a*)	**32.** (*c*)	**33.** (*a*)	**34.** (*b*)	**35.** (*b*)	**36.** (*a*)	**37.** (*b*)	**38.** (*d*)	**39.** (*b*)	**40.** (*a*)
41. (*d*)									

Advanced Level Multiconcept Questions

1. (*d*) **2.** (*a*, *b*, *c*) **3.** (*a*, *b*) **4.** (*b*, *c*) **5.** (*d*) **6.** (*a*, *b*, *c*) **7.** (*c*) **8.** (*d*) **9.** (*c*, *d*)

10. (*a*, *b*, *c*, *d*) **11.** (*b*, *c*) **12.** (*a*, *c*, *d*) **13.** (*b*) **14.** (*a*) **15.** (*a*) **16.** (*b*) **17.** (*a*) **18.** (*c*)

19. (*a*, *b*) **20.** (*a*, *b*, *c*, *d*) **21.** (*b*) **22.** (*a*) **23.** (*c*) **24.** (*b*) **25.** (*d*) **26.** (*b*) **27.** (*a*)

28. (*a*) *p*, *s*, (*b*) *t* (*c*) *q*, *r* (*d*) *t* **29.** (*a*) – (*r*) ; (*b*) – (*q*) ; (*c*) – (*p*) ; (*d*) – (s) **30.** [0023] **31.** [0200] **32.** [0001]

33. [1000] **34.** [0001] **35.** [0060] **36.** [600]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*a*)	**2.** (*d*)	**3.** (*a*)	**4.** (*d*)	**5.** (*c*)	**6.** (*c*)	**7.** (*d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*a*)
11. (*d*)	**12.** (*b*)	**13.** (*a*)	**14.** (*c*)	**15.** (*c*)	**16.** [40]	**17.** [50]	**18.** (*a*)	**19.** (*a*)	**20.** (*d*)
21. (*c*)	**22.** (*a*)	**23.** (*d*)	**24.** (*c*)	**25.** (*a*)	**26.** (*a*)	**27.** (*b*)	**28.** (*b*)	**29.** [20.00]	**30.** (*c*)
31. (*d*)	**32.** [3]	**33.** [25]							

JEE-ADVANCED
PREVIOUS YEAR'S

1. (*c*)	**2.** (*a*)	**3.** (*a*)	**4.** (*b*)	**5.** (*c*, *d*)	**6.** [9]	**7.** (*a*)	**8.** (*b*)	**9.** [4.00]	**10.** (*a*)
11. [270.00]	**12.** (*b*, *c*, *d*)	**13.** [8.33]							

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12 KINETIC THEORY OF GASES AND THERMODYNAMICS

time/real prayasians

KINETIC THEORY OF GASES

SOME DEFINITIONS

m : Mass of the gas molecule
N : Number of molecule in gas
M : Mass of the gas
M = mN

Avogadro Number N_A :
The number of molecules present in 1 gm mole of a gas is defined as Avogadro's Number.

$N_A = 6.02 \times 10^{23}$ per gm. mole

1 gram mole weight M_w :
Mass of one mole gas is known as molecular weight of gas.

Number of moles

$$n = \frac{\text{mass of gas}}{\text{mol wt. of gas}} \quad \text{or} \quad n = \frac{M}{M_w}$$

If n = 1 molten $M = M_w$

$$\Rightarrow M_w = N_A \cdot m \Rightarrow \quad m = \frac{M_w}{N_A}$$

Gram mole:
The quantity of matter in which the number of molecules is equal to the Avogadro's number, is defined as gram mole.

Free path:
The distance covered by the molecules between two successive collisions is called the free path.

Mean free path:
The average distance covered by the molecule between two successive collisions is called the mean free path.

$$\text{i.e. } \lambda = \frac{1}{\sqrt{2}.\pi n d^2}$$

- Mean free path depends on diameter of molecule (d) and it decreases with increasing n.
- At N.T.P. λ for air molecules is 0.01 μm.

Average velocity:
Since gas molecules moves randomly in all possible directions so average velocity of gas molecules is zero.

average displacement of all molecules is zero.

Mean speed:
Mean speed or average speed is the arithmetic mean of the speeds of molecules in a gas at a given temperature.

$$\bar{v} = \frac{v_1 + v_2 + ... + v_N}{N}$$

MEAN SQUARE VELOCITY

Mean square velocity is defined as the average of the square of the velocities of all molecules.

Let $v_1, v_2, ..., v_N$ be the velocities of N molecules of a gas. Then mean square velocity

$$\overline{v^2} = \frac{v_1^2 + v_2^2 + ... + v_N^2}{N}$$

Root mean square velocity (R.M.S. velocity):

The square root of the average of the squares of the velocities of gas molecules is called the R.M.S. velocity.

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + ... + v_N^2}{N}} = \sqrt{\overline{v^2}}$$

IDEAL GAS EQUATION

For n mole $\quad PV = nRT$

For 1 gm. mole $\quad PV = RT$

$PV = N_A kT \quad (N_A . k = R)$

[k is Bolzman constant]

For one gram of gas $PV = \frac{RT}{M_w}$

Values of R, its unit and dimensions :

- R = 2 Calorie/mol/°C,
- R = 8.31 Joule/mol/°C
- $R = 8.31 \times 10^7$ erg/mol/°C

Unit S I : Joule/mol/K.

Dimensions : $ML^2T^{-2}\theta^{-1} mol^{-1}$

ASSUMPTIONS OF THE KINETIC THEORY OF GASES

The kinetic theory of gases is based on the following assumptions:

- All the molecules of a gas are identical as regarding their shape and mass. The molecules of different gases are different.
- The molecules are rigid and perfectly elastic spheres of very small diameters.

- Gas molecules occupy very small space. The actual volume occupied by the molecule is very small compared to the total volume of the gas. Therefore volume of the gas is equal to volume of the vessel.
- The molecules are in a state of random motion, i.e., they are constantly moving with all possible velocities in all possible directions.
- Normally no force acts between the molecules. Hence they move in straight lines with constant speeds.
- The molecules can have all possible velocities lying between zero and infinity.
- The molecules collide with one another and also with the walls of the container and change their direction and speed due to collision. These collisions are perfectly elastic i.e., there is no loss of kinetic energy in these collisions.
- The molecules do not exert any force of attraction or repulsion on each other except during collision. So, the molecules do not posses any potential energy. Their energy is total kinetic.
- The collisions are instantaneous i.e., the time spent by a molecule in a collision is very small as compared to the time elapsed between two consecutive collisions.
- Though the molecules are constantly moving from one place to another, the average number of molecules per unit volume of the gas remains constant.
- The molecules inside the vessel keep on moving continuously in all possible directions, the distribution of molecules in the whole vessel remains uniform.
- The mass of a molecule is negligibly small and the speed is very large, there is no effect of gravity on the motion of the molecules. If this effect were there, the density of the gas would have been greater at the bottom of the vessel.

PRESSURE EXERTED BY AN IDEAL GAS

Pressure P exerted by an ideal gas is

$$P = \frac{1}{3}\frac{mN}{V}\overline{v^2} \quad \text{or} \quad P = \frac{1}{3}\frac{Nm}{V}\cdot v_{rms}^2 \quad \left(v_{rms}^2 = \overline{v^2}\right)$$

$$P = \frac{1}{3}\frac{m}{V}\overline{v^2} \quad \text{or} \quad P = \frac{1}{3}\rho v_{rms}^2$$

$$\left(\frac{Nm}{V} = \rho = \text{density of gas}\right)$$

$$v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}}, \quad v_{rms} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}}$$

$(\because R = N_A k \text{ and } M_w = m \cdot N_A)$

Relation between Pressure and Kinetic Energy:

$$\because \quad P = \frac{1}{3}\frac{Nm}{V}v_{rms}^2 \quad \text{or} \quad PV = \frac{2}{3}N\left(\frac{1}{2}mv_{rms}^2\right)$$

$$\therefore \quad N\left(\frac{1}{2}mv_{rms}^2\right) = \frac{3}{2}PV$$

But $\frac{1}{2}mv_{rms}^2$ = average K.E. of a gas molecule.

$\therefore$ Total K.E. of a gas $E = N\left(\frac{1}{2}mv_{rms}^2\right) = \frac{3}{2}PV$

$\therefore$ K.E. per unit volume of the gas $E = \frac{3}{2}P$

The pressure exerted by a gas is numerically equal to $\frac{2}{3}$rd of the kinetic energy of the molecules present per unit volume of the gas.

DIFFERENT TYPES OF SPEEDS OF GAS MOLECULES

Most probable speed v_{mp}:

It is the speed which maximum number of molecules in a gas have at constant temperature and is:

$$v_{mp} = \sqrt{\frac{2RT}{M_w}} = \left(\sqrt{\frac{2}{3}}\right)v_{rms} = 0.816\,v_{rms}$$

Average speed v_{av}:

It is the arithmetic mean of the speeds of molecules in a gas at a given temperature, i.e,

$$v_{av} = \frac{v_1 + v_2 + v_3 + ... + v_N}{N}$$

and according to kinetic theory of gases

$$v_{av} = \sqrt{\frac{8RT}{\pi M_w}} = \left(\sqrt{\frac{8}{3\pi}}\right)v_{rms} = 0.92\,v_{rms}$$

KINETIC INTERPRETATION OF TEMPERATURE

Mean kinetic energy:

According to kinetic theory of gases

$$PV = \frac{1}{3}mN\overline{v^2} \quad \text{or} \quad PV = nRT$$

$$\text{So} \quad \frac{1}{3}mN\overline{v^2} = nRT \text{ or} \quad \frac{1}{2}m\overline{v^2} = \frac{3}{2}\frac{nRT}{N}$$

$$\text{As} \quad n = \frac{\text{mass}}{\text{mol. wt.}} = \frac{Nm}{M} \quad \text{and} \quad R = N_A k$$

$$\therefore \quad \frac{1}{2}m\overline{v^2} = \frac{3}{2}\frac{Nm}{M}\frac{N_A kT}{N} = \frac{3}{2}kT \quad (\because N_A m = M)$$

i.e.,Translational KE of a molecule

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT \text{ or}$$

Mean kinetic energy per molecule, $E = \frac{3}{2}kT$.

Further, mean kinetic energy per gm mole is given by :

$$E_{mole} = \left[\frac{1}{2}m\overline{v^2}\right]N_A = \frac{3}{2}kTN_A = \frac{3}{2}RT$$

- Average translational KE of a gas molecule depends only on its temperature and is independent of its nature, i.e., molecules of different gases say He, H_2 and O_2, etc, at same temperature will have same translational kinetic energy though their rms speeds are different.
- Mean kinetic energy of molecules is zero at absolute zero if gas continues to remain a gas at that temperature.

KINETIC INTERPRETATION OF PRESSURE

According to kinetic theory of gases,

$$P = \frac{1}{3}\frac{mN}{V}(\overline{v^2}) \text{ with } \overline{v^2} \propto T$$

So the pressure of a gas is controlled by following three independent factors :

- **If volume and temperature of a gas are constant,** $P \propto mN$, i.e., $P \propto$ mass of gas (as mN = mass of gas), i.e., if mass of gas is increased, number of molecules and hence number of collisions per sec increases, i.e., pressure will increase.
- **If mass and temperature of a gas are constant,** $P \propto \frac{1}{V}$, i.e., if volume decreases, number of collisions per sec will increase due to lesser effective distance between the walls resulting in greater pressure.
- **If mass and volume of gas are constant,** $P \propto \overline{v^2} \propto T$, i.e., if temperature increases, the mean square speed of gas molecules will increase and as gas molecules are moving faster, they will collide with the walls more often with greater momentum resulting in greater pressure.

KINETIC INTERPRETATION OF EVAPORATION

Evaporation is a slow process which takes place from the surface of a liquid at all temperatures.

Evaporation depends on :

(a) nature of liquid (boiling point)
(b) temperature of liquid
(c) temperature of surroundings
(d) air currents
(e) relative humidity
(f) pressure

Cooling results due to process of evaporation:

According to kinetic theory of gases at a given temperature in a liquid, molecules are moving in all possible directions with all possible speeds; so at all temperatures high speed molecules can escape from the surface, i.e., evaporation takes place at all temperatures.

As high speed molecules have escaped, the energy and hence the temperature of remaining liquid will decrease, i.e., evaporation produces cooling. Further, greater the area of liquid surface, greater will be the probability of escaping of high speed molecules and so greater will be the rate of evaporation.

MEAN FREE PATH AND KINETIC INTERPRETATION OF BROWNIAN MOTION

- Robert Brown discovered that pollen-grains suspended in water move following zig-zag paths when viewed under a microscope. This phenomenon is called Brownian motion.
- The pollen-grains are hit successively and randomly by moving water molecules, i.e., in matter molecules are not at rest but moving in all possible directions with all possible velocities. The average distance travelled by a gas molecule between two successive collisions is called mean free path.
- According to kinetic theory, it has been shown that the mean free path, $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

  d = the diameter of the molecule

  and n = the number of molecules per unit volume.

  This expression shows that mean free path depends on nature of molecule (d) and with increase in n (i.e., density) it decreases.
- At NTP the mean free path for air molecules is 0.01 μm.

  But as $PV = nRT = nNkT$

  so $n = \frac{N}{V} = \frac{P}{kT}$

  Hence $\lambda = \frac{1}{\sqrt{2}}\frac{kT}{\pi d^2 P}$

DEGREES OF FREEDOM (F)

The term degrees of freedom of a system refers to the possible independent motions a system can have or number of possible independent ways in which a system can have energy.

The independent motions of a system can be translational, rotational or vibrational or any combination of these.

A particle in motion confined to a straight line has only one translational degree of freedom while

If same particle is confined to move in a plane, it will have two translational degrees of freedom.

If the particle is free to move in space, it will have three translational degrees of freedom.

Degree of freedom of gas molecules:

A gas molecule can have following types of energies:

- Translational K.E
- Rotational K.E.
- Vibrational energy (potential + kinetic)

A molecule of a monoatomic gas (like He, Ar, etc.) which consists of a single atom, the translational motion can take place in any direction in space, i.e., it can be resolved along three co-ordinate axes and can have three independent motions and 3 degrees of freedom, all translational.

- A monoatomic molecule can also rotate but due to its small moment of inertia rotational KE is insignificant. Therefore it does not possess rotational degrees of freedom.
- The molecules of a diatomic gas (H_2, O_2) are made of two atoms joined rigidly to one another through a bond.
- This cannot only move bodily, but also rotate about two of three co-ordinate axes. However, its moment of inertia about the axis joining the two atoms is negligible compared to that about the other two axes. Hence, it can have only 2 rotational motions. Thus, a diatomic molecule has 5 degrees of freedom; 3 translational and 2 rotational.
- A non-linear polyatomic molecule (H_2O) can rotate about any of three co-ordinate axes.

  Hence, It has 6 degrees of freedom; 3 translational and 3 rotational.
- At sufficiently high temperatures it has vibrational energy as well providing it two more degrees of freedom (one vibrational kinetic energy and another vibrational potential energy).

  Thus, at high temperatures a diatomic molecule has 7 degrees of freedom, 3 translational, 2 rotational and 2 vibrational. f = 5 (3 translational + 2 rotational) at room temperatures and f = 7 (3 translational + 2 rotational + 2 vibrational) at high temperatures

Degree of freedom of nonlinear polyatomic gas:

A nonlinear polyatomic molecule (such as NH_3) can rotate about any of three co-ordinate axes.

Hence, it has 6 degrees of freedom 3 translational and 3 rotational

At room temperatures a polyatomic gas molecule has vibrational energy greater than that of a diatomic gas.

But at high enough temp. it is also significant. So it has 8 degrees of freedom 3 rotational, 3 translational and 2 vibrational. Thus,

f = 6 (3 translational + 3 rotational) at room temperatures

and f = 8 (3 translational + 3 rotational + 2 vibrational) at high temperatures

Degree of freedom of a solid:

An atom or molecule in a solid has no degrees of freedom for translational and rotational motion.

At high temperatures due to vibration along 3 axes it has $3 \times 2 = 6$ degrees of freedom. f = 6 (all vibrational) at high temperatures

MAXWELL'S LAW OF EQUIPARTITION OF ENERGY

For a dynamical system in thermal equilibrium, the energy of the system is equally distributed amongst the various degrees of freedom and the energy associated with each degree of freedom per molecule is $= \frac{1}{2}kT$.

Translational kinetic energy of a molecule

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

we have $\frac{1}{2}m(v_{rms}^2)_x + \frac{1}{2}m(v_{rms}^2)_y + \frac{1}{2}m(v_{rms}^2)_z = \frac{3}{2}kT$

The molecular motion is a random one and as no direction of motion is a preferred one, the average kinetic energy corresponding to each degree of freedom is the same, i.e.

$$\frac{1}{2}m(v_{rms}^2)_x = \frac{1}{2}m(v_{rms}^2)_y = \frac{1}{2}m(v_{rms}^2)_z$$

$$\Rightarrow \frac{1}{2}m(v_{rms}^2)_x = \frac{1}{2}m(v_{rms}^2)_y = \frac{1}{2}m(v_{rms}^2)_z = \frac{1}{2}kT$$

Mean kinetic energy per molecule per degree of freedom $= \frac{1}{2}kT$

This result was first deduced by Boltzmann

- In each degree of freedom energy of one mole of an ideal gas $= \frac{1}{2}RT$

- If f be the number of degrees of freedom, the internal energy of 1 mole of the gas will be $\frac{f}{2}RT$

  or internal energy of n moles of the gas will be $\frac{n}{2}fRT$

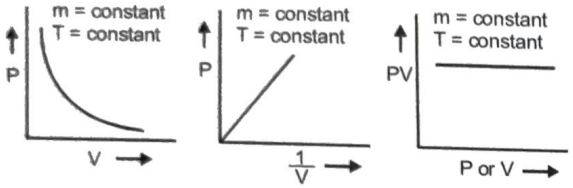
  $U = \frac{n}{2}fRT$

GAS-LAWS

Boyle's law:

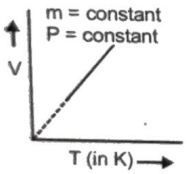
According to Boyle's law for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, i.e., $V \propto \frac{1}{P}$ if mass of gas and T = constant

Graphical forms of the law are as follows:



Charles' law:

According to it for a given mass of an ideal gas at constant pressure,



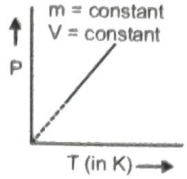
volume of a gas is directly proportional to its absolute temperature,

i.e., $V \propto T$. If m and P = constant

Graphical form of the law is shown in adjoining figure.

Gay-Lussac's law :

According to it for a given mass of an ideal gas at constant volume,



pressure of a gas is directly proportional to its absolute temperature, i.e., $P \propto T$

if m and V = constant

This is shown graphically in adjoining figure.

Avogadro's law:

According to it, at same temperature and pressure, equal volumes of all the gases contain equal number of molecules, i.e., $N_1 = N_2$ if P, V and T are same

Graham's law:

According to it, at constant pressure and temperature, the rate of diffusion of a gas is inversely proportional to the square root of its density.

Rate of diffusion $\propto \frac{1}{\sqrt{\rho}}$ and if P and T = constant

Dalton's law:

According to it, the pressure exerted by a gaseous mixture is equal to the sum of partial pressure of each component present in the mixture, i.e., $P = P_1 + P_2 + ...$

THERMODYNAMICS

INTRODUCTION

Thermodynamics is concerned with the work done by a system and the heat it exchanges with its surroundings. We are concerned only with work done by a system on its surroundings or on the system by the surroundings. We are not concerned with internal work done by one part of a system on another.

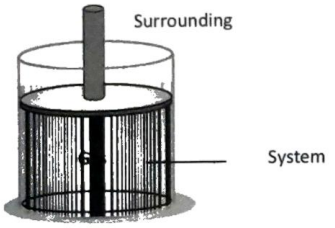
Some Definitions

(1) **Thermodynamic system**

(i) It is a collection of an extremely large number of atoms or molecules

(ii) It is confined within certain boundaries.

(iii) Anything outside the thermodynamic system to which energy or matter is exchanged is called its surroundings.



(iv) Thermodynamic system may be of three types

(a) Open system : It exchange both energy and matter with the surrounding.

(b) Closed system : It exchange only energy (not matter) with the surroundings.

(c) Isolated system : It exchange neither energy nor matter with the surrounding.

(2) **Thermodynamic variables and equation of state :** A thermodynamic system can be described by specifying its pressure, volume, temperature, internal energy and the number of moles. These parameters are called thermodynamic variables. The relation between the thermodynamic variables (P, V, T) of the system is called equation of state.

For n moles of an ideal gas, equation of state is PV = nRT and for 1 mole of an ideal gas it is PV = RT.

(3) **Thermodynamic equilibrium :** In steady state thermodynamic variables are independent of time and the system is said to be in the state of thermodynamic equilibrium. For a system to be in thermodynamic equilibrium, the following conditions must be fulfilled.

(i) Mechanical equilibrium : There is no unbalanced force between the system and its surroundings.

(ii) Thermal equilibrium : There is a uniform temperature in all parts of the system and is same as that of surrounding.

(iii) Chemical equilibrium : There is a uniform chemical composition through out the system and the surrounding.

(4) **Thermodynamic process :** The process of change of state of a system involves change of thermodynamic variables such as pressure P, volume V and temperature T of the system. The process is known as thermodynamic process. Some important processes are

(i) Isothermal process : Temperature remain constant

(ii) Adiabatic process : No transfer of heat

(iii) Isobaric process : Pressure remains constant

(iv) Isochoric process : Volume remains constant

(v) Cyclic and non-cyclic process : In cyclic process Initial and final states are same while in non-cyclic process these states are different.

(5) **Indicator diagram :** Whenever the state of a gas (P, V, T) is changed, we say the gaseous system is undergone a thermodynamic process. The graphical representation of the change in state of a gas by a thermodynamic process is called indicator diagram. Indicator diagram is plotted generally in pressure and volume of gas.

Heat, Internal Energy and Work in Thermodynamics

(1) **Heat (ΔQ) :** It is the energy that is transferred between a system and its environment because of the temperature difference between them.

(i) Heat is a path dependent quantity *e.g.* Heat required to change the temperature of a given gas at a constant pressure is different from that required to change the temperature of same gas through same amount at constant volume.

(ii) For gases when heat is absorbed and temperature changes $\Rightarrow \Delta Q = n C\Delta T$

At constant pressure $(\Delta Q)_P = nC_P\Delta T$

At constant volume $(\Delta Q)_V = nC_V\Delta T$

(2) **Internal energy (U) :** Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

The energy due to molecular motion is called internal kinetic energy U_K and that due to molecular configuration is called internal potential energy U_P *i.e.* Total internal energy $U = U_K + U_P$

(i) For an ideal gas, as there is no molecular attraction $U_P = 0$

i.e. internal energy of an ideal gas is totally kinetic and is given by $U = U_K = \frac{3}{2}nRT$ and change in internal energy $\Delta U = \frac{3}{2}nR\Delta T$

(ii) In case of gases whatever be the process

$$\Delta U = n\frac{f}{2}R\Delta T = nC_V\Delta T = n\frac{R}{(\gamma - 1)}\Delta T$$

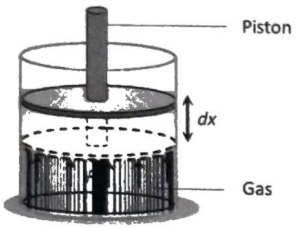
$$= \frac{nR(T_f - T_i)}{\gamma - 1}$$

$$= \frac{nRT_f - nRT_i}{\gamma - 1} = \frac{(P_fV_f - P_iV_i)}{\gamma - 1}$$

(iii) Change in internal energy does not depend on the path of the process. So it is called a point or state function *i.e.* it depends only on the initial and final states of the system, *i.e.* $\Delta U = U_f - U_i$

(3) Work (ΔW) : Suppose a gas is confined in a cylinder that has a movable piston at one end. If P be the pressure of the gas in the cylinder, then force exerted by the gas on the piston of the cylinder

$F = PA$ (A = Area of cross-section of piston)



When the piston is pushed outward an infinitesimal distance dx, the work done by the gas

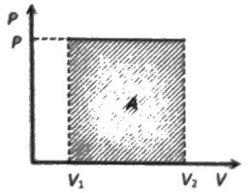
$dW = F.dx = P(A\,dx) = P\,dV$

For a finite change in volume from V_i to V_f

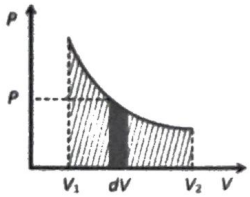
Total amount of work done

$$W = \int_{V_i}^{V_f} P\,dV = P(V_f - V_i)$$

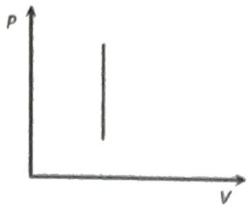
(i) If we draw indicator diagram, the area bounded by PV-graph and volume axis represents the work done



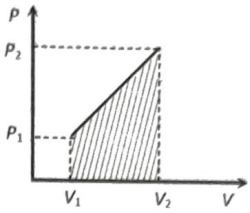
Work = Area = $P(V_2 - V_1)$



$$\text{Work} = \int_{V_1}^{V_2} PdV = P(V_2 - V_1)$$



Work = 0



Work = Area of the shown trapezium

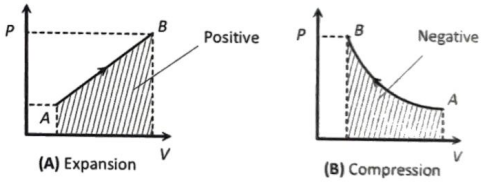
$$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

(ii) From $\Delta W = P\Delta V = P(V_f - V_i)$

If system expands against some external force then $V_f > V_i$

$\Rightarrow \Delta W$ = positive

If system contracts because of external force then $V_f < V_i$



(iii) Like heat, work done is also depends upon initial and final state of the system and path adopted for the process

WORK DONE FOR DIFFERENT-DIFFERENT PROCESS

(1) Isothermal Process:

T = constant [Boyle's law applicable]

PV = constant

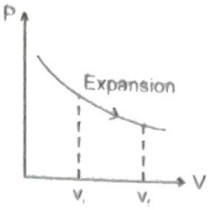


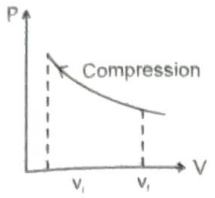
Work done in isothermal process:

$$W = nRT\ \ell n\ \frac{V_f}{V_i}$$

$$\begin{bmatrix}\text{If } V_f > V_i \text{ then W is positive}\\ \text{If } V_f < V_i \text{ then W is negative}\end{bmatrix}$$

$$W = \left[2.303\, n R T \log_{10} \frac{V_f}{V_i}\right]$$





(2) Iso- Choric Process (Isometric Process) :

V = constant

$\Rightarrow$ change in volume is zero

$\Rightarrow \frac{P}{T}$ is constant

$\frac{P}{T}$ = const. (Galussac-law)

Work done in isochoric process :

Since change in volume is zero therefore

$dW = pdV = 0$

(3) Isobaric Process : Pressure remains constant in isobaric process

$\therefore$ P = constant $\Rightarrow \frac{V}{T}$ = constant

Work done in isobaric process :

$\Delta W = P\,\Delta V = P\,(V_{final} - V_{initial}) = nR\,(T_{final} - T_{initial})$

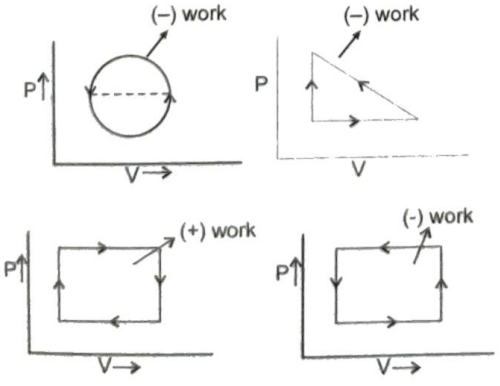
(4) Cyclic Process :

In the cyclic process initial and final states are same therefore initial state = final state

Work done = Area enclosed under P-V diagram.

If the process on P-V curve is clockwise , then net work done is (+ve) and vice-versa.

The graphs shown below explains when work is positive and when it is negative



QUASI-STATIC PROCESS

When we perform a process on a given system, its state is, in general, changed. Suppose the initial state of the system is described by the values P_i, V_i, T_i and the final state by P_f, V_f, T_f. If the process is performed in such a way that at any instant during the process, the system is very nearly in thermodynamic equilibrium, the process is called quasi-static. This means, we can specify the parameters P, V, T uniquely at any instant during such a process.

Actual processes are not quasi-static. To change the pressure of a gas, we can move a piston inside the enclosure. The gas near the piston is acted upon by piston. The pressure of the gas may not be uniform everywhere while the piston is moving. However, we can move the piston very slowly to make the process as close to quasi-static as we wish. Thus, a quasi-static process is an idealised process in which all changes take place infinitely slowly.

REVERSIBLE AND IRREVERSIBLE PROCESS

(1) **Reversible process :** A reversible process is one which can be reversed in such a way that all changes occurring in the direct process are exactly repeated in the opposite order and inverse sense and no change is left in any of the bodies taking part in the process or in the surroundings. For example if heat is absorbed in the direct process, the same amount of heat should be given out in the reverse process, if work is done on the working substance in the direct process then the same amount of work should be done by the working substance in the reverse process. The conditions for reversibility are

(i) There must be complete absence of instantaneous forces such as friction, viscosity, electric resistance etc.

(ii) The direct and reverse processes must take place infinitely slowly.

(iii) The temperature of the system must not differ appreciably from its surroundings.

Some examples of reversible process are

(a) All isothermal and adiabatic changes are reversible if they are performed very slowly.

(b) When a certain amount of heat is absorbed by ice, it melts. If the same amount of heat is removed from it, the water formed in the direct process will be converted into ice.

(c) An extremely slow extension or contraction of a spring without setting up oscillations.

(d) When a perfectly elastic ball falls from some height on a perfectly elastic horizontal plane, the ball rises to the initial height.

(e) If the resistance of a thermocouple is negligible there will be no heat produced due to Joule's heating effect. In such a case heating or cooling is reversible. At a junction where a cooling effect is produced due to Peltier effect when current flows in one direction and equal heating effect is produced when the current is reversed.

(f) Very slow evaporation or condensation. It should be remembered that the conditions mentioned for a reversible process can never be realised in practice. Hence, a reversible process is only an ideal concept. In actual process, there is always loss of heat due to friction, conduction, radiation *etc*.

(2) **Irreversible process :** Any process which is not reversible exactly is an irreversible process. All natural processes such as conduction, radiation, radioactive decay *etc*. are irreversible. All practical processes such as free expansion, Joule-Thomson expansion, electrical heating of a wire are also irreversible. Some examples of irreversible processes are given below

(i) When a steel ball is allowed to fall on an inelastic lead sheet, its kinetic energy changes into heat energy by friction. The heat energy raises the temperature of lead sheet. No reverse transformation of heat energy occurs.

(ii) The sudden and fast stretching of a spring may produce vibrations in it. Now a part of the energy is dissipated. This is the case of irreversible process.

(iii) Sudden expansion or contraction and rapid evaporation or condensation are examples of irreversible processes.

(iv) Heat produced by the passage of an electric current through a resistance is irreversible.

(v) Heat transfer between bodies at different temperatures is also irreversible.

(vi) Joule-Thomson effect is irreversible because on reversing the flow of gas a similar cooling or heating effect is not observed.

FIRST LAW OF THERMODYNAMICS (FLOT)

(1) It is a statement of conservation of energy in thermodynamical process.

(2) According to it heat given to a system (ΔQ) is equal to the sum of increase in its internal energy (ΔU) and the work done (ΔW) by the system against the surroundings.

$\mathbf{\Delta Q = \Delta U + \Delta W}$

(3) It makes no distinction between work and heat as according to it the internal energy (and hence temperature) of a system may be increased either by adding heat to it or doing work on it or both.

(4) ΔQ and ΔW are the path functions but ΔU is the point function.

(5) In the above equation all three quantities ΔQ, ΔU and ΔW must be expressed either in *Joule* or in *calorie*.

(6) The first law introduces the concept of internal energy.

(7) Limitation : First law of thermodynamics does not indicate the direction of heat transfer. It does not tell anything about the conditions, under which heat can be transformed into work and also it does not indicate as to why the whole of heat energy cannot be converted into mechanical work continuously.

Table : Useful sign convention in thermodynamics

Quantity	Sign	Condition
ΔQ	+	When heat is supplied to a system
	−	When heat is drawn from the system
ΔW	+	When work done by the gas (expansion)
	−	When work done on the gas (compression)
ΔU	+	With temperature rise, internal energy increases
	−	With temperature fall, internal energy decreases

THERMODYNAMIC PROCESSES

We now apply the first law of thermodynamics to some simple situations.

(a) Isolated System

Consider first an *isolated system* for which there is no *heat exchange* and *no work is done* on the external environment. In this case $Q = 0$ and $W = 0$, so from the *first law* we conclude

$\Delta U = 0$ or $U =$ constant

The internal energy of an isolated system is constant

(b) Isobaric Process

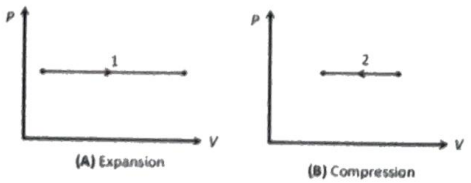
When a thermodynamic system undergoes a physical change in such a way that its pressure remains constant, then the change is known as isobaric process.

(1) **Equation of state :** In this process V and T changes but P remains constant. Hence Charle's law is obeyed in this process.

Hence if pressure remains constant

$$V \propto T \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

(2) **Indicator diagram :** Graph 1 represent isobaric expansion, graph 2 represent isobaric compression.



(i) **In isobaric expansion (Heating)**

Temperature ⟶ increases so ΔU is positive

Volume ⟶ increases so ΔW is positive

Heat ⟶ flows into the system so ΔQ is positive

(ii) **In isobaric compression (Cooling)**

Temperature ⟶ decreases so ΔU is negative

Volume ⟶ decreases so ΔW is negative

Heat ⟶ flows out from the system so ΔQ is negative

(3) **Specific heat :** Specific heat of gas during isobaric process $C_P = \left(\frac{f}{2}+1\right)R$

(4) **Bulk modulus of elasticity :** $K = \frac{\Delta P}{\frac{-\Delta V}{V}} = 0$

[As $\Delta P = 0$]

(5) **Work done in isobaric process**

$$\Delta W = \int_{V_i}^{V_f} P\,dV = P\int_{V_i}^{V_f} dV = P[V_f - V_i]$$

[As P = constant]

$\Rightarrow \Delta W = P(V_f - V_i) = nR[T_f - T_i] = n\,R\Delta T$

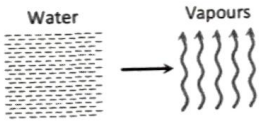
(6) **FLOT in isobaric process :** From $\Delta Q = \Delta U + \Delta W$

$\because \Delta U = nC_v\Delta T = n\frac{R}{(\gamma - 1)}\Delta T$ and $\Delta W = nR\Delta T$

$$\Rightarrow (\Delta Q)_p = n\frac{R}{(\gamma - 1)}\Delta T + nR\Delta T$$

$$= n\left(\frac{\gamma}{\gamma - 1}\right)R\Delta T = nC_P\Delta T$$

(7) **Examples of isobaric process :** All state changes occurs at constant temperature and pressure.



Boiling of water

(i) Water ⟶ vapours

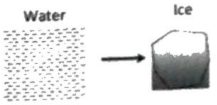
(ii) Temperature ⟶ constant

(iii) Volume ⟶ increases

(iv) A part of heat supplied is used to change volume (expansion) against external pressure and remaining part is used to increase it's potential energy (kinetic energy remains constant)

(v) From FLOT $\Delta Q = \Delta U + \Delta W \Rightarrow mL = \Delta U + P(V_f - V_i)$

Freezing of water



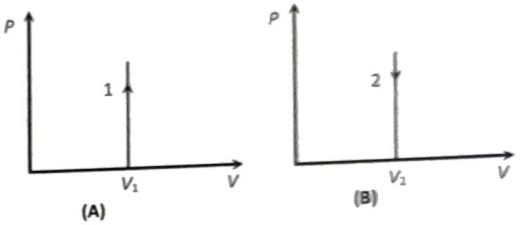
(c) **Isochoric or Isometric Process**

When a thermodynamic process undergoes a physical change in such a way that its volume remains constant, then the change is known as isochoric process.

(1) **Equation of state :** In this process P and T changes but V = constant. Hence Gay-Lussac's law is obeyed in this process *i.e.* $P \propto T$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{constant}$$

(2) **Indicator diagram :** Graph 1 and 2 represent isometric increase in pressure at volume V_1 and isometric decrease in pressure at volume V_2 respectively and slope of indicator diagram $\frac{dP}{dV} = \infty$



(i) Isometric heating

(a) Pressure ⟶ increases

(b) Temperature ⟶ increases

(c) ΔQ ⟶ positive

(d) ΔU ⟶ positive

(ii) Isometric cooling

(a) Pressure ⟶ decreases

(b) Temperature ⟶ decreases

(c) ΔQ ⟶ negative

(d) ΔU ⟶ negative

(3) **Specific heat :** Specific heat of gas during isochoric process $C_V = \frac{f}{2}R$

(4) **Bulk modulus of elasticity :** $K = \frac{\Delta P}{\frac{-\Delta V}{V}} = \frac{\Delta P}{0} = \infty$

(5) **Work done in isochoric process**

$\Delta W = P\Delta V = P[V_f - V_i] = 0$ [As V = constant]

(6) **FLOT in isochoric process :**

From $\Delta Q = \Delta U + \Delta W$

$\because \Delta W = 0$

$$\Rightarrow (\Delta Q)_V = \Delta U = nC_V\Delta T = n\frac{R}{\gamma - 1}\Delta T = \frac{P_fV_f - P_iV_i}{\gamma - 1}$$

(d) Isothermal Process

When a thermodynamic system undergoes a physical change in such a way that its temperature remains constant, then the change is known as isothermal changes.

(1) Essential condition for isothermal process

(i) The walls of the container must be perfectly conducting to allow free exchange of heat between the gas and its surrounding.

(ii) The process of compression or expansion should be so slow so as to provide time for the exchange of heat.

Since these two conditions are not fully realised in practice, therefore, no process is perfectly isothermal.

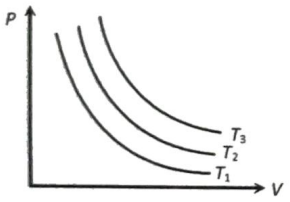
(2) Equation of state : In this process, P and V change but T = constant *i.e.* change in temperature $\Delta T = 0$.

Boyle's law is obeyed *i.e.* PV= constant

$\Rightarrow P_1V_1 = P_2V_2$

(3) Example of isothermal process : Melting of ice (at $0°C$) and boiling of water (at $100°C$) are common example of this process.

(4) Indicator diagram : According to PV = constant, graph between P and V is a part of rectangular hyperbola. The graphs at different temperature are parallel to each other are called isotherms.

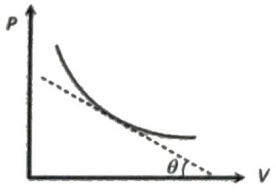


$T_1 < T_2 < T_3$

Two isotherms never intersect

(i) Slope of isothermal curve : By differentiating PV = constant. We get

$P\,dV + V\,dP = 0$



$\Rightarrow PdV = -VdP$

$\Rightarrow \text{Slope} = \tan\theta = \dfrac{dP}{dV} = -\dfrac{P}{V}$

(ii) Area between the isotherm and volume axis represents the work done in isothermal process.

If volume increases ΔW = + (Area under curve) and if volume decreases ΔW = – (Area under curve)

(5) **Specific heat:** Specific heat of gas during isothermal change is infinite.

As $C = \dfrac{Q}{m\Delta T} = \dfrac{Q}{m \times 0} = \infty$ [As $\Delta T = 0$]

(6) **Isothermal elasticity (E_θ):** For this process PV = constant.

$\Rightarrow P\,dV = -V\,dP \Rightarrow P = \dfrac{dP}{-dV/V} = \dfrac{\text{Stress}}{\text{Strain}} = E_\theta$

$\Rightarrow E_\theta = P$ *i.e.* isothermal elasticity is equal to pressure

At N.T.P., E_θ = Atmospheric pressure

$= 1.01 \times 10^5\ N/m^2$

(7) **Work done in isothermal process**

$$W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} \frac{nRT}{V} dV \text{ [As } PV = nRT\text{]}$$

$$W = nRT\log_e\left(\frac{V_f}{V_i}\right) = 2.303\,nRT\log_{10}\left(\frac{V_f}{V_i}\right)$$

or $$W = nRT\log_e\left(\frac{P_i}{P_f}\right) = 2.303\,nRT\log_{10}\left(\frac{P_i}{P_f}\right)$$

(8) **FLOT in isothermal process :**

From $\Delta Q = \Delta U + \Delta W$

$\because \Delta U = 0$ [As $\Delta T = 0$] $\Rightarrow \Delta Q = \Delta W$

i.e. heat supplied in an isothermal change is used to do work against external surrounding.

or if the work is done on the system than equal amount of heat energy will be liberated by the system.

(e) Adiabatic Process

When a thermodynamic system undergoes a change in such a way that no exchange of heat takes place between System and surroundings, the process is known as adiabatic process.

In this process P, V and T changes but $\Delta Q = 0$.

(1) Essential conditions for adiabatic process

(i) There should not be any exchange of heat between the system and its surroundings. All walls of the container and the piston must be perfectly insulating.

(ii) The system should be compressed or allowed to expand suddenly so that there is no time for the exchange of heat between the system and its surroundings.

Since, these two conditions are not fully realised in practice, so no process is perfectly adiabatic.

(2) Some examples of adiabatic process

(i) Sudden compression or expansion of a gas in a container with perfectly non-conducting walls.

(ii) Sudden bursting of the tube of bicycle tyre.

(iii) Propagation of sound waves in air and other gases.

(iv) Expansion of steam in the cylinder of steam engine.

(3) **FLOT in adiabatic process :** From $\Delta Q = \Delta U + \Delta W$

For adiabatic process $\Delta Q = 0 \Rightarrow \Delta U = -\Delta W$

If ΔW = positive then ΔU = negative so temperature decreases *i.e.* adiabatic expansion produce cooling.

If ΔW = negative then ΔU = positive so temperature increases *i.e.* adiabatic compression produce heating.

(4) **Equation of state :** In adiabatic change ideal gases do not obeys Boyle's law but obeys Poisson's law. According to it

PV^{γ} = constant; where $\gamma = \frac{C_P}{C_V}$

(i) For temperature and volume

$TV^{\gamma-1}$ = constant $\Rightarrow T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ or $T \propto V^{1-\gamma}$

(ii) For temperature and pressure $\frac{T^{\gamma}}{P^{\gamma-1}}$ = const.

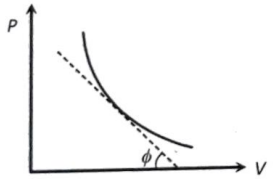
$\Rightarrow T_1^{\gamma}P_1^{1-\gamma} = T_2^{\gamma}P_2^{1-\gamma}$ or $T \propto P^{\frac{\gamma-1}{\gamma}}$ or $P \propto T^{\frac{\gamma}{\gamma-1}}$

Table : Special cases of adiabatic process

Types of gas	$P \propto \frac{1}{V^{\gamma}}$	$P \propto T^{\frac{\gamma}{\gamma-1}}$	$T \propto \frac{1}{V^{\gamma-1}}$
Monoatomic $\gamma = 5/3$	$P \propto \frac{1}{V^{5/3}}$	$P \propto T^{5/2}$	$T \propto \frac{1}{V^{2/3}}$
Diatomic $\gamma = 7/5$	$P \propto \frac{1}{V^{7/5}}$	$P \propto T^{7/2}$	$T \propto \frac{1}{V^{2/5}}$
Polyatomic $\gamma = 4/3$	$P \propto \frac{1}{V^{4/3}}$	$P \propto T^{4}$	$T \propto \frac{1}{V^{1/3}}$

(5) **Indicator diagram**

(i) Curve obtained on PV graph are called adiabatic curve.



(ii) Slope of adiabatic curve : From PV^{γ} = constant

By differentiating, we get

$dP\,V^{\gamma} + P\gamma V^{\gamma-1}\,dV = 0$

$\frac{dP}{dV} = -\gamma\frac{PV^{\gamma-1}}{V^{\gamma}} = -\gamma\left(\frac{P}{V}\right)$

$\therefore$ Slope of adiabatic curve $\tan\phi = -\gamma\left(\frac{P}{V}\right)$

(iii)But we also know that slope of isothermal curve

$\tan\theta = \frac{-P}{V}$

Hence $(\text{Slope})_{Adi} = \gamma \times (\text{Slope})_{Iso}$ or $\frac{(\text{Slope})_{Adi}}{(\text{Slope})_{Iso}} > 1$

(6) **Specific heat :** Specific heat of a gas during adiabatic change is zero As $C = \frac{Q}{m\Delta T} = \frac{0}{m\Delta T} = 0$ [As $Q = 0$]

(7) **Adiabatic elasticity (E_{ϕ}) :** PV^{γ} = constant

Differentiating both sides $d\,PV^{\gamma} + P\gamma V^{\gamma-1}dV = 0$

$\gamma P = \frac{dP}{-dV/V} = \frac{\text{Stress}}{\text{Strain}} = E_{\phi} \Rightarrow E_{\phi} = \gamma P$

i.e. adiabatic elasticity is γ times that of pressure

Also isothermal elasticity $E_{\theta} = P \Rightarrow \frac{E_{\phi}}{E_{\theta}} = \gamma = \frac{C_P}{C_V}$

i.e. the ratio of two elasticity of gases is equal to the ratio of two specific heats.

(8) **Work done in adiabatic process**

$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{K}{V^{\gamma}}dV$

$\Rightarrow W = \frac{[P_iV_i - P_fV_f]}{(\gamma-1)} = \frac{nR(T_i - T_f)}{(\gamma-1)}$

(As $PV^{\gamma} = K$, $P_fV_f = nRT_f$ and $P_iV_i = nRT_i$)

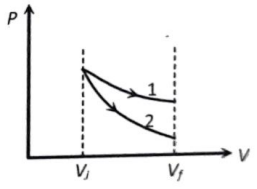
(i) $W \propto$ quantity of gas (either M or n)

(ii) $W \propto$ temperature difference $(T_i - T_f)$

(iii)$W \propto \frac{1}{\gamma-1}$ $\because \gamma_{mono} > \gamma_{di} > \gamma_{tri} \Rightarrow W_{mono} < W_{dia} < W_{tri}$

(9) **Comparison between isothermal and adiabatic indicator diagrams :** Always remember that adiabatic curves are more steeper than isothermal curves

(i) Equal expansion : Graph 1 represent isothermal process and 2 represent adiabatic process



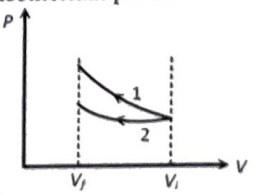
$W_{\text{isothermal}} > W_{\text{adiabatic}}$

$P_{\text{isothermal}} > P_{\text{adiabatic}}$

$T_{\text{isothermal}} > T_{\text{adiabatic}}$

$(\text{Slope})_{\text{Isothermal}} < (\text{Slope})_{\text{adi}}$

(ii) Compression : Graph 1 represent adiabatic process and 2 represent isothermal process



$W_{\text{Adiabatic}} > W_{\text{Isothermal}}$

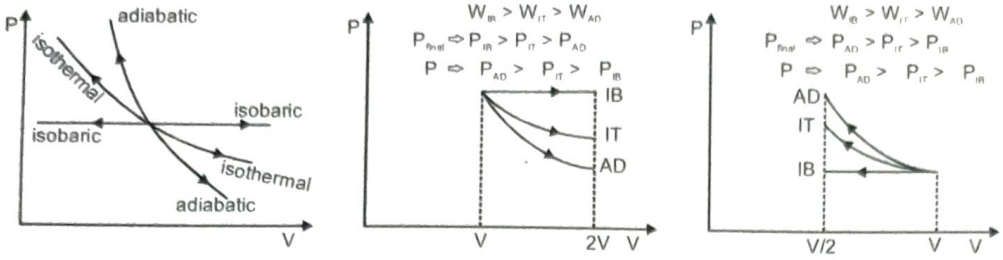
$P_{\text{Adiabatic}} > P_{\text{Isothermal}}$

$T_{\text{Adiabatic}} > T_{\text{Isothermal}}$

$(\text{Slope})_{\text{Isothermal}} < (\text{Slope})_{\text{Adiab}}$

IMPORTANT POINT

When a gas expands its volume increases, then final pressure is less for adiabatic expansion. But, when a gas compresses its volume decreases, then the final pressure is more in case of adiabatic compression.



First Law of Thermodynamics Applied to Different Processes

Process	ΔQ	$=\Delta U$	$+\Delta W$
Cyclic	ΔW	0	Area of the closed curve
Isochoric	ΔU	$nC_V\Delta T$ (n mole of gas)	0
Isothermal	ΔW	0	$nRT \log_e\left[\frac{V_f}{V_i}\right]$
Adiabatic	0	$-\Delta W$	$\frac{nR(T_f - T_i)}{1-\gamma}$
Isobaric	$nC_P\Delta T$	$nC_V\Delta T$	$P(V_f - V_i) = nR(T_f - T_i)$

FREE EXPANSION

Take a thermally insulated bottle with ideal gas at some temperature T_1 and, by means of a pipe with a stopcock, connect this to another insulated bottle which is evacuated.

If we suddenly open the stopcock, the gas will rush from the first bottle into the second until the pressures are equalized.

Experimentally, we find that this process of free expansion does not change the temperature of the gas - when the gas attains equilbrium and stops flowing, the final temperature of both bottles are equal to the iniital temperature T_1.

This process is called a free expansion.



free expansion of a gas

The change in the internal energy of the gas can be calculated by applying the first law of thermodynamics to the free-expansion process.

The process is adiabatic because of the insulation, so **Q = 0**.

No part of the surroundings moves so the system does no work on its surroundings.

For ideal gas

$(\delta W)_{ext.}$ = work done against external atmosphere

= P dV = 0 (because P = 0)

$(\delta W)_{int.}$ = work done against internal molucular forces = 0

$\delta Q = dU + \delta W \Rightarrow 0 = dU + 0$

The internal energy does not change dU = 0

so U and T are constant

The initial and final states of this gas have the same internal energy.

Which implies that the internal energy of an ideal gas does not depend on the volume at all.

The free-expansion process has led us to the following conclusion :

The internal energy U(T) of an ideal gas depends only on the temperature.

For real gas

Free expansion real gases, measurements show that the temperature changes slightly in a free expansion. Which implies that the internal energy of a real gas depends on the volume also.

$\delta Q = 0$ $\quad (\delta W)_{ext.} = 0$ $(\because P = 0)$

$(\delta W)_{int.} \neq 0$(intermolucular forces are present in real gases)

$\delta Q = dU + \delta W \Rightarrow 0 = dU + (\delta W)_{int.} \Rightarrow dU = -(\delta W)_{int.}$

U decreases $\quad$ T decreases

Cyclic process

Cyclic process is that thermodynamic process in which the system returns to its initial stage after undergoing a series of changes.

Non-cyclic process

Non-cyclic process is that process in which the system does not return to its initial stage.

Quasi-static or equilibrium process

Quasi-static is a thermodynamic process which proceeds extremely slowly such that at every instant of time, the temperature and pressure are the same in all parts of the system.

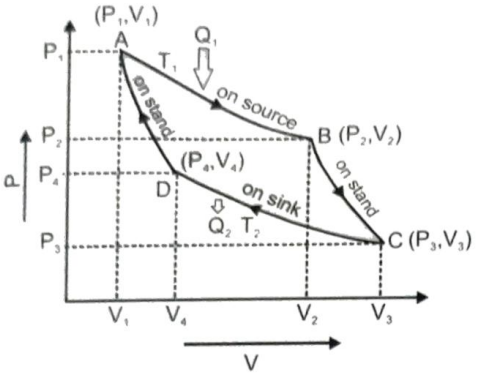
Reversible and Irreversible processes

A reversible process is one in which the changes in heat and work of direct process from initial to a final state are exactly retraced in opposite sense in the reverse process and the system and surroundings are left in their initial states. The reversibility is an ideal concept and can not be realised in practice.

The process which is not reversible is the irreversible process. In nature the processes are irreversible.

CARNOT CYCLE

Carnot devised an ideal engine which is based on a reversible cycle of four operations in succession:



(i) isothermal expansion $A \rightarrow B$
(ii) adiabatic expansion $B \rightarrow C$
(iii) isothermal compression $C \rightarrow D$
(iv) adiabatic compression $D \rightarrow A$

Main parts of Carnot's engine are Source of heat

It is a hot body of very large heat capacity kept at a constant high temperature T_1. Its upper surface is perfectly conducting so that working substance can take heat from it.

Mechanical arrangements and working substance

It is a cylinder whose walls are perfectly non-conducting and its base is perfectly conducting fitted with non conducting piston. This piston move without any friction. Ideal gas enclosed in cylinder as a working substance.

Heat sink

It is a cold body at low temperature T_2. It is a body of large heat capacity and its upper surface is highly conducting so that working substance can reject heat to it.

Stand

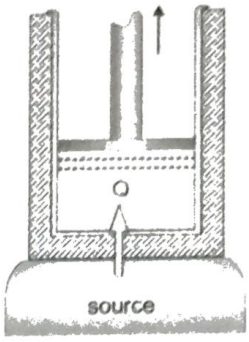
It is made by perfectly insulating material. When cylinder placed on it working substance can expended or compressed adiabaticly.

Working

A set of reversible processes through them working substance is taken back to initial condition to get maximum work from this type of ideal engine.

Processes of Carnot's cycle can be denoted by an indicator diagram.

Isothermal expansion $A \rightarrow B$



Initially the cyclinder is taken to be in thermal equilbrium with the high temperature T_1, this is initial state of working substance denoted by point A (P_1, V_1, T_1).

After that the piston is allowed to move outward slowly. With the movement of the piston. The process is very slow so that it is isothermal.

Heat from reservoir flows through the base of cylinder into the gas so temperature of the gas remains T_1.

Gas expand and receive heat Q_1 from source and gets state $B(P_2, V_2, T_1)$

This heat input Q_1 to the gas from path A to B is utilized for doing work W_1.

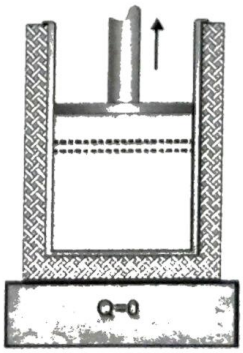
By path A to B the heat input to the gas = the work done against the external pressure.

$$W_1 = Q_1 = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT_1 \ell n \frac{V_2}{V_1}$$

$$\Rightarrow \quad W_1 = nRT_1 \ell n \frac{V_2}{V_1}$$

$$\Rightarrow \quad W_1 = 2.303 nRT_1 \log_{10} \frac{V_2}{V_1}$$

Adiabatic expansion $B \rightarrow C$



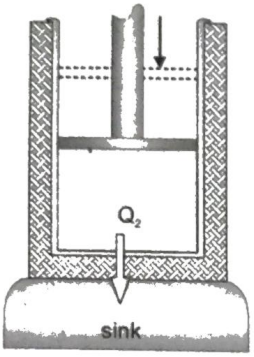
Now cylinder is put in contact with a non-conducting stand and piston is allowed to move outward,because no heat can enter in or leave out so the expansion of gas is adiabatic. The tempeature falls to T_2 and gas describes the adiabatic from B to **point C** (P_3, V_3, T_2)

during this expansion more work is done (W_2) at the expense of the internal energy.

Work done in adiabatic path BC is

$$W_2 = \frac{nR}{\gamma - 1}(T_1 - T_2)$$

Isothermal compression C → D



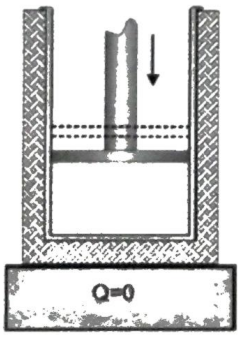
Now the gas cylinder is placed in contact with sink at temperature T_2. The piston is moved slowly inward so that heat produced during compression passes to the sink. The gas is isothermally compressed from C to **point D**. (P_4, V_4, T_2)

The heat rejected Q_2 to the cold reservoir (sink) at T_2 occurs over this path.

Amount of work done on gas W_3 = amount of heat rejected to the sink

$$W_3 = Q_2 = nRT_2 \ell n\frac{V_4}{V_3} \Rightarrow W_3 = Q_2 = -nRT_2 \ell n\frac{V_3}{V_4}$$

Adiabatic compression D → A



The cylinder is removed from the sink and is put in contact with insulating stand now piston moves inward. Heat is not allowed to go out and it increases the internal energy of the system. Now work is done on the gas during adiabatic compression from state D to initial point A (P_1, V_1, T_1).

No heat exchanges occur over the adiabatic path.

Work done on the system W_4

$$= \frac{nR}{\gamma - 1}(T_2 - T_1) = -\frac{nR}{\gamma - 1}(T_1 - T_2)$$

This cycle of operations is called a Carnot cycle.

In first two steps work is done by engine W_1 and W_2 are positive

In last two steps work is done on gas

W_3 and W_4 are negative

The work done in complete cylce W = the area of the closed part of the P–V cycle.

$W = W_1 + W_2 + W_3 + W_4$.

$$\therefore W = nRT_1 \ell n\frac{V_2}{V_1} + \frac{nR}{\gamma - 1}(T_1 - T_2) - nRT_2 \ell n\frac{V_3}{V_4} - \frac{nR}{\gamma - 1}(T_1 - T_2)$$

$$\therefore W = nRT_1 \ell n\frac{V_2}{V_1} - nRT_2 \ell n\frac{V_3}{V_4}$$

B to C and D to A are adiabatic paths

so $T_1V_2^{(\gamma-1)} = T_2V_3^{(\gamma-1)}$ and $T_1V_1^{(\gamma-1)} = T_2V_4^{(\gamma-1)}$

$$\Rightarrow \frac{V_3}{V_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \rho \text{ and } \frac{V_4}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \rho$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Where ρ = adiabatic expansion ratio

$$\therefore \text{ work done in Carnot cycle } W = nR\,(T_1 - T_2)\ \ell n\left(\frac{V_2}{V_1}\right)$$

Efficiency of Carnot Engine, $\eta = \frac{W}{Q_1}$

$$\Rightarrow \eta = \frac{nR(T_1 - T_2)\ell n\frac{V_2}{V_1}}{nRT_1 \ell n\frac{V_2}{V_1}}$$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\Rightarrow \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}\ \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\eta = \frac{T_1 - T_2}{T_1} \times 100\% \Rightarrow \eta = \frac{Q_1 - Q_2}{Q_1} \times 100\%$$

The efficiency for the Carnot engine is the best that can be obtained for any heat engine.

The efficiency of a Carnot engine is never 100% because it is 100% only if temperature of sink $T_2 = 0K$ or $T_1 = \infty$ which is impossible.

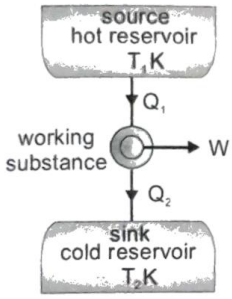
CARNOT THEOREM

No irreversible engine (I) can have efficiency greater than Carnot reversible engine (R) working between same hot and cold reservoirs.

$$\eta_R > \eta_I \Rightarrow 1-\frac{T_2}{T_1} > 1-\frac{Q_2}{Q_1}$$

HEAT ENGINE

Heat engine is a device which converts heat into work.



Three parts of a heat engine:

(i) Source of high temperature reservoir at temperature T_1

(ii) Sink or low temperature reservoir at temperature T_2

(iii) Working substance.

In a cycle of heat engine the working substance extracts heat Q_1 from source, does some work W and rejects remaining heat Q_2 to the sink.

Efficiency of heat engine

$$\eta = \frac{\text{work done (W)}}{\text{heat taken from soruce } (Q_1)}$$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

TYPES OF HEAT ENGINE

(a) External combustion engine

Steam engine is the example of external combustion engine. Its efficiency is 10 to 20% (small).

(b) Internal combustion engine

The examples are petrol engine and diesel engine.

There are four strokes in internal combustion engine.

(i) Intake stroke or Charging stroke

(ii) Compression stroke

(iii) Working (or power) stroke of Expansion stroke

(iv) Exhaust stroke

The useful work is done in third stroke called work stroke or power stroke.

The efficiency of internal combustion engine is 40 to 60%.

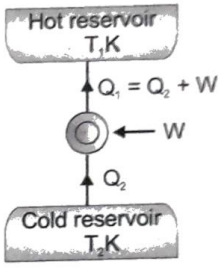
REFRIGERATOR

It is inverse of heat engine. It extracts heat (Q_2) from a cold reservoir, same external work W is done on it and rejects heat (Q_1) to hot reservoir.

The coefficient of performance of a refrigerator.

$$\beta = \frac{\text{heat extracted from cold reservoir}}{\text{work done on refrigerator}}$$

$$= \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$



For Carnot reversible refrigerator $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\therefore \beta = \frac{Q_2}{W} = \frac{1}{\left[\frac{Q_1}{Q_2} - 1\right]} = \frac{1}{\left[\frac{T_1}{T_2} - 1\right]}$$

$$\Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is a generalization of the law of conservation of energy to include heat energy. It tells us that heat and mechanical work are mutually interconvertible.

Second law of thermodynamics tells us in what conditions heat can be converted into useful work.

The following three conditions must be fulfilled to utilize heat for useful work :

1. A device called engine with a working substance is essential.

2. The engine must work in a reversible cyclic process.
3. The engine must operate between two temperatures. It will absorb heat from a hot body (called source), convert a part of into useful work and reject the rest to a cold body (called sink).

There are two conventional statements of second law

(i) Kelvin-Planck Statement

It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert completely into work. In other words, 100% conversion of heat into work is impossible.

(ii) Clausius Statement

It is impossible for a self-acting machine, unaided by any external agency to transfer heat from a cold to hot reservoir. In other words heat can not in itself flow from a colder to a hotter body.

ENTROPY

It is a measure of molecular disorder of a system. Greater is disorder, greater is entropy.

The change in entropy,

$$\Delta S = \frac{\text{heat absorbed}}{\text{absolute temperature}}$$

$$\Delta S = \frac{\Delta Q}{T} \Rightarrow \Delta Q = T\Delta S$$

Mathematical form of second law of thermodynamics $\Delta Q = T\Delta S$

For solids and liquids

The change in entropy during a change of state of mass m of substance $\Delta S = \pm \frac{mL}{T}$

where (+) sign refers to heat absorption and (–) sign for heat evolution.

$$\Delta S = mc \log_e \frac{T_f}{T_i} = 2.3mc \log_{10} \frac{T_f}{T_i}$$

For a perfect gas

The change in entropy for n moles of a gas

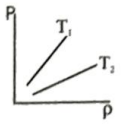
$$\Delta S = n\left[C_v \log_e \frac{T_f}{T_i} + R \log_e \frac{V_f}{V_i}\right]$$

$$= n\left[C_p \log_e \frac{T_f}{T_i} - R \log_e \frac{P_f}{P_i}\right]$$

$$= n\left[C_v \log_e \frac{P_f}{P_i} + C_p \log_e \frac{V_f}{V_i}\right]$$

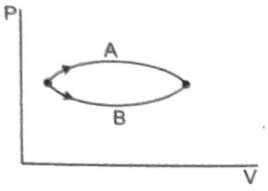
RMS AND KTG

1. The volume of a gas at 20°C is 200 ml. If the temperature is reduced to −20°C at constant pressure, its volume will be
   (*a*) 172.6 ml (*b*) 17.26 ml
   (*c*) 192.7 ml (*d*) 19.27 ml
2. Which of the following quantities is zero on an average for the molecules of an ideal gas in equilibrium ?
   (*a*) kinetic energy (*b*) momentum
   (*c*) density (*d*) speed
3. The mean free path of gas molecules depends on (d = molecular diameter)
   (*a*) d (*b*) d^2
   (*c*) d^{-2} (*d*) d^{-1}
4. A gas behaves more closely as an ideal gas at
   (*a*) low pressure and low temperature
   (*b*) low pressure and high temperature
   (*c*) high pressure and low temperature
   (*d*) high pressure and high temperature
5. Which of the following statements about kinetic theory of gases is wrong
   (*a*) The molecules of a gas are in continuous random motion
   (*b*) The molecules continuously undergo inelastic collisions
   (*c*) The molecules do not interact with each other except during collisions
   (*d*) The collisions amongst the molecules are of short duration
6. Figure shows graphs of pressure vs density for an ideal gas at two temperatures T_1 and T_2.



   (*a*) $T_1 > T_2$
   (*b*) $T_1 = T_2$
   (*c*) $T_1 < T_2$
   (*d*) any of the three is possible
7. Suppose a container is evacuated to leave just one molecule of a gas in it. Let v_a and v_{rms} represent the average speed and the rms speed of the gas.
   (*a*) $v_a > v_{rms}$ (*b*) $v_a < v_{rms}$
   (*c*) $v_a = v_{rms}$ (*d*) v_{rms} is undefined
8. The rms speed of oxygen molecules in a gas is v. If the temperature is doubled and the O_2 molecule dissociate into oxygen atoms, the rms speed will become
   (*a*) v (*b*) $v\sqrt{2}$
   (*c*) 2 v (*d*) 4v
9. Consider a mixture of oxygen and hydrogen kept at room tempertaure. As compared to a hydrogen molecule an oxygen molecule hits the wall
   (*a*) With greater average speed
   (*b*) with smaller average speed
   (*c*) with greater average kinetic energy
   (*d*) with smaller average kinetic energy.
10. Consider the quantity MkT / pV of an ideal gas where M is the mass of the gas. It depends on the
    (*a*) temperature of the gas
    (*b*) volume of the gas
    (*c*) pressure of the gas
    (*d*) nature of the gas
11. The ratio of the mean speed of hydrogen molecules to the mean speed of nitrogen molecules in a sample containing a mixture of the two gases.
    (*a*) $\sqrt{14}$ (*b*) $\sqrt{7}$
    (*c*) $\sqrt{28}$ (*d*) None of these
12. Three closed vessels A, B, and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is V_1, that of the N_2 molecules in vessel B is V_2 the average speed of the O_2 molecules in vessel C will be :
    (*a*) $(V_1 + V_2)/2$ (*b*) V_1
    (*c*) $(V_1 V_2)^{1/2}$ (*d*) $\sqrt{3kT/M}$
13. A flask is filled with 13 gm of an ideal gas at 27°C and its temperature is raised to 52°C. The mass of the gas that has to be released to maintain the temperature of the gas in the flask at 52°C and the pressure remaining the same is
    (*a*) 2.5 g (*b*) 2.0 g
    (*c*) 1.5 g (*d*) 1.0 g
14. Three containers of the same volume contain three different gases. The masses of the molecules are m_1, m_2 and m_3 and the number of molecules in their respective containers are N_1, N_2 and N_3. The gas pressure in the containers are P_1, P_2 and P_3 respectively. All the gases are now mixed and put in one of the containers. The pressure P of mixture will be
    (*a*) $P < (P_1 + P_2 + P_3)$ (*b*) $P = \frac{P_1 + P_2 + P_3}{3}$
    (*c*) $P = P_1 + P_2 + P_3$ (*d*) $P > (P_1 + P_2 + P_3)$

15. Refer to fig Let ΔU_1 and ΔU_2 be the changes in internal energy of the system in the processes A and B then



(*a*) $\Delta U_1 > \Delta U_2$ (*b*) $\Delta U_1 = \Delta U_2$
(*c*) $\Delta U_1 < \Delta U_2$ (*d*) $\Delta U_1 \neq \Delta U_2$

16. Air is pumped into an automobile tube upto a pressure of 200 kPa in the morning when the air temperature is 22°C. During the day, temperature rises to 42°C and the tube expands by 2%. The pressure of the air in the tube at this temperature, will be approximately
(*a*) 212 kPa (*b*) 209 kPa
(*c*) 206 kPa (*d*) 200 kPa

17. To what temperature should the hydrogen at 327°C be cooled at constant pressure, so that the root mean square velocity of its molecules become half of its previous value
(*a*) – 123°C (*b*) 123°C
(*c*) – 100°C (*d*) 0°C

18. The root mean square velocity of a gas molecule of mass m at a given temperature is proportional to
(*a*) m^0 (*b*) m
(*c*) $\sqrt{m}$ (*d*) $\frac{1}{\sqrt{m}}$

19. At room temperature, the r.m.s. speed of the molecules of certain diatomic gas is found to be 1930 m/s. The gas is
(*a*) H_2 (*b*) F_2
(*c*) O_2 (*d*) Cl_2

20. r.m.s. velocity of nitrogen molecules at NTP is
(*a*) 492 m /s (*b*) 517 m/s
(*c*) 546 m/s (*d*) 33 m/s

21. The root mean square speed of the molecules of a gas is
(*a*) Independent of its pressure but directly proportional to its Kelvin temperature
(*b*) Directly proportional to the square roots of both its pressure and its Kelvin temperature
(*c*) Independent of its pressure but directly proportional to the square root of its Kelvin temperature
(*d*) Directly proportional to both its pressure and its Kelvin temperature

22. Let A and B the two gases and given : $\frac{T_A}{M_A} = 4\frac{T_B}{M_B}$; where T is the temperature and M is molecular mass. If C_A and C_B are the r.m.s. speed, then the ratio $\frac{C_A}{C_B}$ will be equal to
(*a*) 2 (*b*) 4
(*c*) 1 (*d*) 0.5

GAS LAW'S

23. To double the volume of a given mass of an ideal gas at 27°C keeping the pressure constant, one must raise the temperature in °C to
(*a*) 54° (*b*) 270°
(*c*) 327° (*d*) 600°

24. A mass of an ideal gas undergoes a reversible isothermal compression. Its molecules will then have compared with initial state, the same
(i) root mean square velocity
(ii) mean mometum
(iii) mean kinetic energy
(*a*) (i), (ii), (iii) correct (*b*) (i), (ii) correct
(*c*) (ii), (iii) correct (*d*) (i) correct

25. Volume of gas become four times if
(*a*) Temperature become four times at constant pressure
(*b*) Temperature become one fourth at constant pressure
(*c*) Temperature becomes two times at constant pressure
(*d*) Temperature becomes half at constant pressure

INTERNAL ENERGY SPECIFIC HEAT OF GAS AND DEGREES OF FREEDOM

26. A monoatomic gas molecule has
(*a*) Three degrees of freedom
(*b*) Four degrees of freedom
(*c*) Five degrees of freedom
(*d*) Six degrees of freedom

27. When an ideal diatomic gas is heated at constant pressure , the fraction of the heat energy supplied which increases the internal energy of the gas is
(*a*) $\frac{2}{5}$ (*b*) $\frac{3}{5}$
(*c*) $\frac{3}{7}$ (*d*) $\frac{5}{7}$

28. Boiling water is changing into steam. Under this condition, the specific heat of water is
(*a*) zero (*b*) one
(*c*) Infinite (*d*) less than one

29. If the degree of freedom of a gas are f, then the ratio of two specific heats C_P/C_V is given by

(a) $\frac{2}{f}+1$ (b) $1-\frac{2}{f}$

(c) $1+\frac{1}{f}$ (d) $1-\frac{1}{f}$

30. Supposing the distance between the atoms of a diatomic gas to be constant, its specific heat at constant volume per mole (gram mole) is

(a) $\frac{5}{2}R$ (b) $\frac{3}{2}R$

(c) R (d) $\frac{7}{2}R$

31. For an ideal gas, the heat capacity at constant pressure is larger than at constant volume because

(a) positive work is done during expansion of the gas by the external pressure

(b) positive work is done during expasion by the gas against external pressure

(c) positive work is done during expansion by the gas against intermolecular forces of attraction

(d) more collisions occur per unit time when volume is kept constant.

32. A gas has:

(a) one specific heat only

(b) two specific heats only

(c) infinite number of specific heats

(d) no specific heat

33. The relation between two specific heats of a gas is

(a) $C_P - C_V = \frac{R}{J}$ (b) $C_V - C_P = \frac{R}{J}$

(c) $C_P - C_V = J$ (d) $C_V - C_P = J$

34. For a solid with a small expansion coefficient,

(a) $C_p - C_v = R$

(b) $C_p - C_v = R$

(c) C_p is slightly greater than C_v

(d) C_p is slightly less than C_v

35. The quantity of heat required to raise one mole through one degree Kelvin for a monoatomic gas at constant volume is

(a) $\frac{3}{2}R$ (b) $\frac{5}{2}R$

(c) $\frac{7}{2}R$ (d) 4R

36. The degrees of freedom of a stationary rigid body about its axis will be

(a) One (b) Two

(c) Three (d) Four

37. A gaseous mixture consists of 16g of helium and 16g of oxygen. The ratio $\frac{C_p}{C_v}$ of the mixture is

(a) 1.4 (b) 1.54

(c) 1.59 (d) 1.62

38. Relationship between P, V, and E for a gas is

(a) $P = \frac{3}{2}EV$ (b) $V = \frac{2}{3}EP$

(c) $PV = \frac{3}{2}E$ (d) $PV = \frac{2}{3}E$

39. The ratio of mean kinetic energy of hydrogen and oxygen at a given temperature is

(a) 1 : 16 (b) 1 : 8

(c) 1 : 4 (d) 1 : 1

40. The mean kinetic energy of a gas at 300 K is 100 J. The mean kinetic energy of the gas at 450 K is equal to

(a) 100 J (b) 3000 J

(c) 450 J (d) 150 J

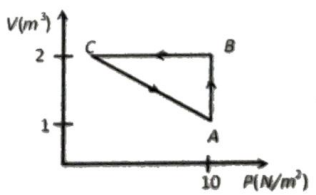
41. Energy of all molecules of a monoatomic gas having a volume V and pressure P is $\frac{3}{2}PV$. The total translational kinetic energy of all molecules of a diatomic gas as the same volume and pressure is

(a) $\frac{1}{2}PV$ (b) $\frac{3}{2}PV$

(c) $\frac{5}{2}PV$ (d) 3 PV

WORK

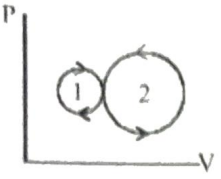
42. An ideal gas is taken through the cycle A → B → C → A, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process C → A is



(a) – 5 J (b) – 10 J

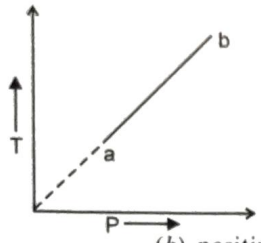
(c) – 15 J (d) – 20 J

43. In the following indicator diagram, the net amount of work done will be



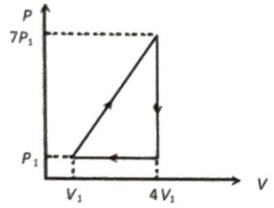
(*a*) Positive (*b*) Negative
(*c*) Zero (*d*) Infinity

44. An ideal gas changes from state a to state b as shown in Fig. What is the work done by the gas in the process?



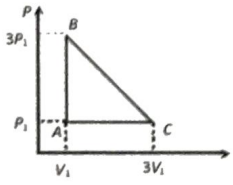
(*a*) zero (*b*) positive
(*c*) negative (*d*) infinite

45. In the cyclic process shown in the figure, the work done by the gas in one cycle is



(*a*) $28\ P_1V_1$ (*b*) $14P_1V_1$
(*c*) $18\ P_1V_1$ (*d*) $9P_1V_1$

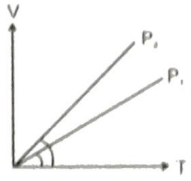
46. An ideal gas is taken around the cycle ABCA as shown in the P-V diagram. The net work done by the gas during the cycle is equal to



(*a*) $12P_1V_1$ (*b*) $6P_1V_1$
(*c*) $3P_1V_1$ (*d*) $2P_1V_2$

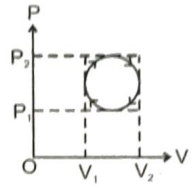
47. A gas is contained in a metallic cylinder fitted with a piston. The piston is suddenly moved in to compress the gas and is maintained at this position. As time passes, after this pressure of the gas in the cylinder
(*a*) increases
(*b*) decreases
(*c*) remains constant
(*d*) increases or decreases depending on the nature of the gas.

48. In the following V-T diagram what is the relation between P_1 and P_2:



(*a*) $P_2 = P_1$ (*b*) $P_2 > P_1$
(*c*) $P_2 < P_1$ (*d*) cannot be predicted

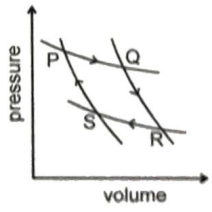
49. In the cyclic process shown on the P – V diagram the magnitude of the work done is:



(*a*) $\pi\left(\frac{P_2 - P_1}{2}\right)^2$ (*b*) $\pi\left(\frac{V_2 - V_1}{2}\right)^2$

(*c*) $\frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$ (*d*) $\pi(P_2V_2 - P_1V_1)$

50. A fixed mass of gas undergoes the cycle of changes represented by PQRSP as shown in Figure. In some of the changes, work is done on the gas and in others, work is done by the gas. In which pair of the changes work is done on the gas?

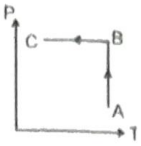


(*a*) PQ and RS (*b*) PQ and QR
(*c*) OR and RS (*d*) RS and SP.

FIRST LAW OF THERMODYNAMICS

51. First law of thermnodynamics is given by
(*a*) $dQ = dU + PdV$
(*b*) $dQ = dU \times PdV$
(*c*) $dQ = (dU + dV)P$
(*d*) $dQ = PdU + dV$

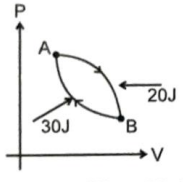
52. Ideal gas is taken through process shown in figure:



(*a*) In process AB, work done by system is positive
(*b*) In process AB, heat is rejected out of the system.
(*c*) In process AB, internal energy increases
(*d*) In process AB internal energy decreases and in process BC internal energy increases.

53. If heat is supplied to an ideal gas in an isothermal process,
(*a*) the internal energy of the gas will increase
(*b*) the gas will do positive work
(*c*) the gas will do negative work
(*d*) the said process is not possible

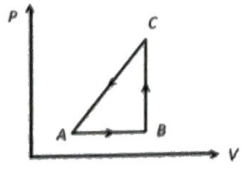
54. In a cyclic process shown in the figure an ideal gas is adiabatically taken from B and A, the work done on the gas during the process B → A is 30 J, when the gas is taken from A → B the heat absorbed by the gas is 20 J. The change in internal energy of the gas in the process A → B is:



(*a*) 20 J (*b*) – 30 J
(*c*) 50 J (*d*) – 10 J

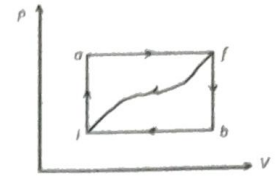
55. A system can be taken from the initial state p_1, V_1 to the final state p_2, V_2 by two different methods. Let ΔQ and ΔW represent the heat given to the system and the work done by the system. Which of the following must be the same in both the methods?
(*a*) ΔQ (*b*) ΔW
(*c*) $\Delta Q + \Delta W$ (*d*) $\Delta Q - \Delta W$

56. The P-V diagram of a system undergoing thermodynamic transformation is shown in figure. The work done on the system in going from A → B → C is 50 J and 20 cal heat is given to the system. The change in internal energy between A and C is



(*a*) 34 J (*b*) 70 J
(*c*) 84 J (*d*) 134 J

57. When a system is taken from state i to a state f along path iaf, Q = 50 J and W = 20 J. Along path ibf, Q = 35 J. If W = – 13 J for the curved return path fi, Q for this path is

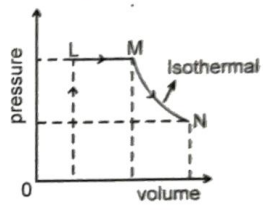


(*a*) 33 J (*b*) 17 J
(*c*) – 7 J (*d*) – 43 J

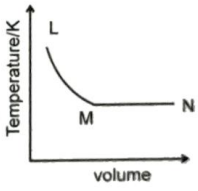
DIFFERENT TYPES OF THERMODYNAMICS PROCESS

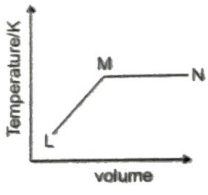
58. For an ideal gas, in an isothermal process
(*a*) Heat content remains constant
(*b*) Heat content and temperature remain constant
(*c*) Temperature remains constant
(*d*) None of these

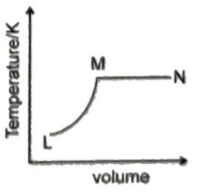
59. A fixed mass of ideal gas undergoes changes of pressure and volume starting at L, as shown in Figure.

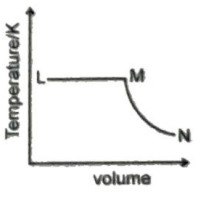


Which of the following is correct:

(*a*) 

(*b*) 

(*c*) 

(*d*) 

60. For free expansion of a gas in an adiabatic container which of the following is true?
(*a*) Q = W = 0 and $\Delta U = 0$
(*b*) Q = 0, W > 0 and $\Delta U = Q$
(*c*) W = 0, Q > 0 and $\Delta U = Q$
(*d*) W = 0, Q < 0 and $\Delta U = 0$

61. The gas law $\frac{PV}{T}$ = constant is true for

(*a*) Isothermal changes only

(*b*) Adiabatic changes only

(*c*) Both isothermal and adiabatic changes

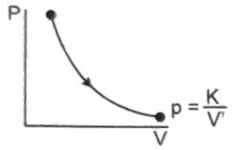
(*d*) Neither isothermal nor adiabatic changes

62. The isothermal Bulk modulus of an ideal gas at pressure P is

(*a*) P (*b*) γP

(*c*) P/2 (*d*) P/γ

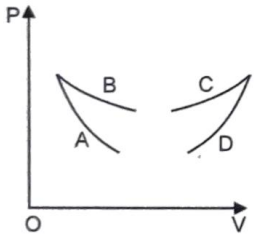
63. The molar heat capacity for the process shown in fig. is



(*a*) $C = C_p$ (*b*) $C = C_v$

(*c*) $C > C_v$ (*d*) $C = 0$

64. Four curves A, B, C and D are drawn in the Fig. for a given amount of gas. The curves which represent adiabatic and isothermal changes are



(*a*) C and D respectively

(*b*) D and C respectively

(*c*) A and B respectively

(*d*) B and A respectively

65. If a cylinder containing a gas at high pressure explodes, the gas undergoes

(*a*) Reversible adiabatic change and fall of temperature

(*b*) Reversible adiabatic change and rise of temperature

(*c*) Irreversible adiabatic change and fall of temperature

(*d*) Irreversible adiabatic change and rise of temperature

66. A given quantity of a gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is:

(*a*) $\frac{2}{3}$P (*b*) P

(*c*) $\frac{3}{2}$P (*d*) 2P

67. An ideal gas is allowed to expand freely against a vacuum in a rigid insulated container. The gas undergoes:

(*a*) an increase in its internal energy

(*b*) a decrease in its internal energy

(*c*) neither an increase nor decrease in temperature or internal energy

(*d*) an increase in temperature

68. The adiabatic bulk modulus of hydrogen gas ($\gamma = 1.4$) at NTP is :

(*a*) 1×10^5 N/m^2 (*b*) 1×10^{-5} N/m^2

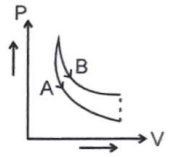
(*c*) 1.4 N/m^2 (*d*) 1.4×10^5 N/m^2

69. An ideal gas at 27°C is compressed adiabatically to $\frac{8}{27}$ of its original volume. If $\gamma = \frac{5}{3}$, then the rise in temperature is

(*a*) 450 K (*b*) 375 K

(*c*) 225 K (*d*) 405 K

70. A and B are two adiabatic curves for two different gases. Then A and B corresponds to:



(*a*) Ar and He respectively

(*b*) He and H_2 respectively

(*c*) O_2 and H_2 respectively

(*d*) H_2 and He respectively

71. Which is the correct statement

(*a*) For an isothermal change PV = constant

(*b*) In an isothermal process the change in internal energy must be equal to the work done

(*c*) For an adiabatic change $\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{\gamma}$, where γ is the ratio of specific heats

(*d*) In an adiabatic process work done must be equal to the heat entering the system

72. The amount of work done in an adiabatic expansion from temperature T to T_1 is

(*a*) $R(T-T_1)$ (*b*) $\frac{R}{\gamma-1}(T-T_1)$

(*c*) RT (*d*) $R(T-T_1)(\gamma-1)$

73. Which relation is correct for isometric process

(*a*) $\Delta Q = \Delta U$ (*b*) $\Delta W = \Delta U$

(*c*) $\Delta Q = \Delta W$ (*d*) None of these

HEAT ENGINES

74. If the door of a refrigerator is kept open then which of the following is true

(*a*) Room is cooled

(*b*) Room is heated

(*c*) Room is either cooled or heated

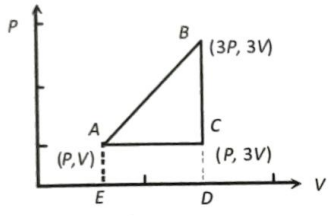
(*d*) Room is neither cooled nor heated

75. An ideal gas heat engine exhausting heat at 77° C is not have a 30% efficiency. It must take heat at

(*a*) 127° C (*b*) 227°

(*c*) 327° C (*d*) 673°C

76. An ideal gas is taken around ABCA as shown in the above P-V diagram. The work done during a cycle is



(*a*) 2PV (*b*) PV

(*c*) 1/2PV (*d*) Zero

77. A Carnot engine works between ice point and steam point. Its efficiency will be -

(*a*) 26.81 % (*b*) 53.36 %

(*c*) 71.23 % (*d*) 85.42 %

78. A Carnot engine operates between 227° C and 27°C. Efficiency of the engine will be

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{5}$

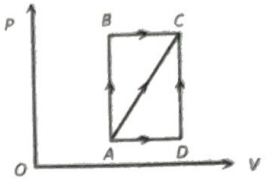
(*c*) $\frac{3}{4}$ (*d*) $\frac{3}{5}$

79. A carnot engine has the same efficiency between 800 K to 500 K and x K to 600 K. The value of x is

(*a*) 1000 K (*b*) 960 K

(*c*) 846K (*d*) 754 K

80. A thermodynamic process is shown in the figure. The pressures and volumes corresponding to some points in the figure are : $P_A = 3 \times 10^4$ Pa. $P_B = 8 \times 10^4$ Pa and $V_A = 2 \times 10^{-3}$ m^3. $V_D = 5 \times 10^{-3}$ m^3. In process AB, 600 J of heat is added to the system and in process BC, 200 J of heat is added to the system. The change in internal energy of the system in process AC would be



(*a*) 560 J (*b*) 800 J

(*c*) 600 J (*d*) 640 J

81. A Cannot engine works between 200°C and 0°C. Another Carnot engine works between 0°C and –200°C. In both cases the working substance absorbs 4 kilocalories of heat from the source. The efficiency of first engine will be -

(*a*) $\frac{100}{473}$ (*b*) $\frac{200}{473}$

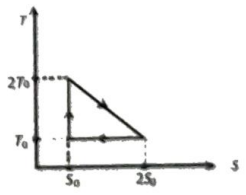
(*c*) $\frac{200}{273}$ (*d*) $\frac{273}{373}$

82. In the above problem, the ratio of efficiencies of two engines will be -

(*a*) 0.18 (*b*) 0.38

(*c*) 0.58 (*d*) 0.78

83. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



(*a*) 1/3

(*b*) 2/3

(*c*) 1/2

(*d*) 1/4

Learning Plus

1. Which of the following is correct for the molecules of a gas in thermal equilibrium?
   (*a*) All have the same speed
   (*b*) All have different speeds which remain constant
   (*c*) They have a certain constant average speed
   (*d*) They do not collide with one another.

2. The temperature at which the r.m.s velocity of oxygen molecules equal that of nitrogen molecules at 100°C is nearly :
   (*a*) 426.3 K (*b*) 456.3 K
   (*c*) 436.3 K (*d*) 446.3 K

3. A gas behaves more closely like an ideal gas at
   (*a*) low pressure and low temperature
   (*b*) low pressure and high temperature
   (*c*) at all pressure and temperature
   (*d*) none of these

4. The average speed of nitrogen molecules in a gas is v. If the temperature is doubled and the N_2 molecule dissociate into nitrogen atoms, then the average speed will be
   (*a*) v (*b*) $v\sqrt{2}$
   (*c*) 2 v (*d*) 4v

5. In a mixture of nitrogen and helium kept at room tempertaure. As compared to a helium molecule nitrogen molecule hits the wall
   (*a*) With greater average speed
   (*b*) with smaller average speed
   (*c*) with greater average kinetic energy
   (*d*) with smaller average kinetic energy.

6. The pressure of an ideal gas is written as $E = \frac{3PV}{2}$.
   Here E stands for
   (*a*) total translational kinetic energy
   (*b*) rotational kinetic energy
   (*c*) average translational kinetic energy
   (*d*) total kinetic energy.

7. The quantities which remains same for all ideal gases at the same temperature is/are ?
   (*a*) the kinetic energy of equal moles of gas
   (*b*) the kinetic energy of equal mass of gas
   (*c*) the number of molecules of equal moles of gas
   (*d*) the number of molecules of equal mass of gas

8. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecule is :
   (*a*) 1 : 1
   (*b*) 1 : 2
   (*c*) 2 : 1
   (*d*) depends on the moments of inertia of the two molecules

9. Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at 1.3×10^5 pascal
   (*a*) 2.4×10^{23} (*b*) 3×10^{23}
   (*c*) 6×10^{23} (*d*) 4.8×10^{23}

10. Keeping the number of moles, volume and pressure the same, which of the following are the same for all ideal gas?
    (*a*) rms speed of a molecule
    (*b*) density
    (*c*) temperature
    (*d*) average of magnitude of momentum.

11. The quantity $\frac{2U}{fkT}$ represents (where U = internal energy of gas f = degree of freedom, k = Boltzmann constant & T = absoulte temprature of gas)
    (*a*) mass of the gas
    (*b*) kinetic energy of the gas
    (*c*) number of moles of the gas
    (*d*) number of molecules in the gas

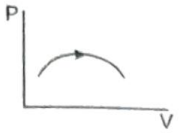
12. A real gas behaves like an ideal gas if its
    (*a*) pressure and temperature are both high
    (*b*) pressure and temperature are both low
    (*c*) pressure is high and temperature is low
    (*d*) pressure is low and temperature is high

13. A gas has:
    (*a*) one specific heat only
    (*b*) two specific heats only
    (*c*) infinite number of specific heats
    (*d*) no specific heat

14. An open and wide glass tube is immersed vertically in mercury in such a way that length 0.05 m extends above mercury level. The open end of the tube is closed and the tube is raised further by 0.43 m. The length of air column above mercury level in the tube will be : Take P_{atm} = 76 cm of mercury
    (*a*) 0.215 m (*b*) 0.2 m
    (*c*) 0.1 m (*d*) 0.4 m

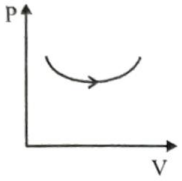
15. A barometer tube, containing mercury, is lowered in a vessel containing mercury until only 50 cm of the tube is above the level of mercury in the vessel. If the atmospheric pressure is 75 cm of mercury, what is the pressure at the top of the tube?
    (*a*) 33.3 kPa (*b*) 66.7 kPa
    (*c*) 3.33 MPa (*d*) 6.67 MPa

16. In figure, P-V curve of an ideal gas is given. During the process, the cumulative work done by the gas



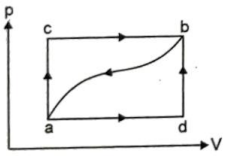
(*a*) continuously increases
(*b*) continuously decreases
(*c*) first increases then decreases
(*d*) first decreases then increases

17. Consider the process on a system shown in fig. During the process, the cumulative work done by the system



(*a*) continuously increase
(*b*) continuously decreases
(*c*) first increases then decreases
(*d*) first decreases then increases

18. When a system is taken from state 'a' to state 'b' along the path 'acb', it is found that a quantity of heat Q = 200 J is absorbed by the system and a work W = 80J is done by it. Along the path 'adb', Q = 144J. The work done along the path 'adb' is



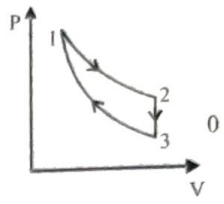
(*a*) 6J (*b*) 12 J
(*c*) 18 J (*d*) 24 J

19. In the above question, if the work done on the system along the curved path 'ba' is 52J, heat abosrbed is
(*a*) – 140 J (*b*) – 172 J
(*c*) 140 J (*d*) 172 J

20. In above question, if U_a = 40J, value of U_b will be
(*a*) – 50 J (*b*) 100 J
(*c*) – 120 J (*d*) 160 J

21. In above question, if U_d = 88 J, heat absorbed for the path 'db' is
(*a*) – 72 J (*b*) 72 J
(*c*) 144 J (*d*) – 144 J

22. Three processes compose a thermodynamics cycle shown in the PV diagram. Process 1 → 2 takes place at constant temperature. Process 2 → 3 takes place at constant volume, and process 3 → 1 is adiabatic. During the complete cycle, the total amount of work done is 10 J. During process 2 → 3, the internal energy decrease by 20J and during process 3 → 1, 20 J of work is done on the system. How much heat is added to the system during process 1 → 2?

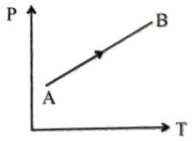


(*a*) 0 (*b*) 10 J
(*c*) 20 J (*d*) 30 J

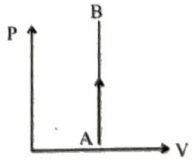
23. A thermodynamic cycle takes in heat energy at a high temperature and rejects energy at a lower temperature. If the amount of energy rejected at the low temperature is 3 times the amount of work done by the cycle, the efficiency of the cycle is
(*a*) 0.25 (*b*) 0.33
(*c*) 0.67 (*d*) 0.9

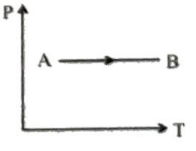
24. When an ideal gas is compressed isothermally then its pressure increases because :
(*a*) its potential energy decreases
(*b*) its kinetic energy increases and molecules move apart
(*c*) its number of collisions per unit area with walls of container increases
(*d*) molecular energy increases

25. The process ΔU = 0, for an ideal gas can be best represented in the form of a graph:

(*a*)


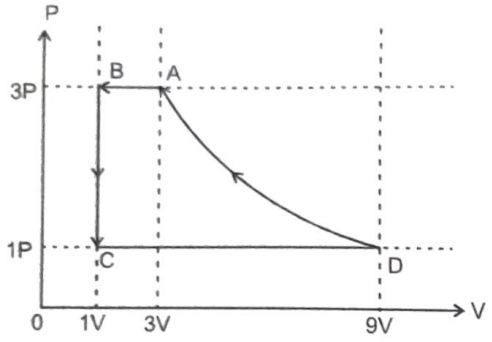
(*b*)


(*c*)


(*d*)


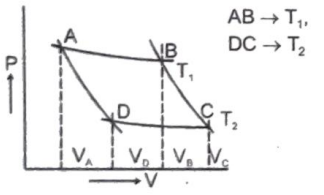
26. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1, the work done in the process is :
(*a*) $\frac{9}{8}RT_1$ (*b*) $\frac{3}{2}RT_1$
(*c*) $\frac{15}{8}RT_1$ (*d*) $\frac{9}{2}RT_1$

27. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



Column-I	Column-II
(*a*) Process A → B	(p) Internal energy decreases
(*b*) Process B → C	(q) Internal energy increases
(*c*) Process C → D	(r) Heat is lost
(*d*) Process D → A	(s) Heat is gained
	(t) Work is done on the gas.

28. In the following P–V diagram of an ideal gas, AB and CD are isothermal where as BC and DA are adiabatic process. The value of V_B/V_C is



(*a*) $= V_A/V_D$ (*b*) $< V_A/V_D$

(*c*) $> V_A/V_D$ (*d*) cannot say

29. Two samples 1 and 2 are initially kept in the same state. The sample 1 is expanded through an isothermal process where as sample 2 through an adiabatic process upto the same final volume. The final temperature in process 1 and 2 are T_1 and T_2 respectively, then

(*a*) $T_1 > T_2$

(*b*) $T_1 = T_2$

(*c*) $T_1 < T_2$

(*d*) The relation between T_1 and T_2 cannot be deduced.

30. Let P_1 and P_2 be the final pressure of the samples 1 and 2 respectively in the previous question then :

(*a*) $P_1 < P_2$

(*b*) $P_1 = P_2$

(*c*) $P_1 > P_2$

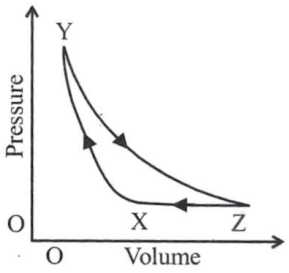
(*d*) The relation between P_1 and P_2 cannot be deduced.

31. In an adiabatic process on a gas with $\gamma = 1.4$, the pressure is increased by 0.5%. The volume decreases by about

(*a*) 0.36% (*b*) 0.5%

(*c*) 0.7% (*d*) 1%

32. In given figure, a fixed mass of an ideal gas undergoes the change represented by XYZX below. Which one of the following sets could describe these of changes?



	X Y	Y Z	Z X
(*a*)	isothermal expansion	adiabatic compression	compression at constant pressure
(*b*)	adiabatic expansion	isothermal compression	pressure reduction constant volume
(*c*)	isothermal compression	adiabatic expansion	compression at constant pressure
(*d*)	adiabatic compression	isothermal expansion	compression at constant pressure

33. The internal energy of an ideal gas is equal to negative of the work done by the system, then

(*a*) The process must be adiabatic

(*b*) The process must be isothermal

(*c*) The process must be isobaric

(*d*) The temperature must decrease

Advanced Level Multiconcept Questions

JEE-ADVANCED
MCQ/STATEMENT/COMPREHENSION/MATCHING

1. Consider a collision between an argon molecule and a nitrogen moecule in a mixture of argon and nitrogen kept at room temperature. Which of the following are possible?
   (*a*) The kinetic energies of both the molecules decrease.
   (*b*) The kinetic energies of both the molecules increase
   (*c*) The kinetic energy of the argon molecule increases and that of the nitrogen molecules decrease.
   (*d*) The kinetic energy of the nitrogen molecules increases and that of the argon molecule decrease.

2. In case of hydrogen and oxygen at N.T.P., which of the following quantities is/are the same?
   (*a*) average momentum per molecule
   (*b*) average kinetic energy per molecule
   (*c*) kinetic energy per unit volume
   (*d*) kinetic energy per unit mass

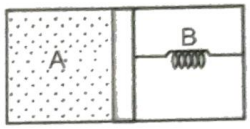
3. Two vessels of the same volume contain the same gas at same temperature. If the pressure in the vessel be in the ratio of 1 : 2, then
   (*a*) the ratio of the average kinetic energy is 1 : 2
   (*b*) the ratio of the root mean square velocity is 1 : 1
   (*c*) the ratio of the average velocity is 1 : 2
   (*d*) the ratio of number of molecules is 1 : 2

4. A mixture of ideal gases 7 kg of nitrogen and 11 kg of CO_2. Then
   (*a*) equivalent molecular weight of the mixture is 36.
   (*b*) equivalent molecular weight of the mixture is 18.
   (*c*) γ for the mixture is 5/2
   (*d*) γ for the mixture is 47/35
   (Take γ for nitrogen and CO_2 as 1.4 and 1.3 respectively)

5. Specific heat of a substance can be:
   (*a*) finite (*b*) infinite
   (*c*) zero (*d*) negative

6. The following sets of values for C_v and C_p of a gas have been reported by different students. The units are cal mole^{-1} K^{-1}. Which of these sets is most reliable?
   (*a*) $C_v = 3, C_p = 5$ (*b*) $C_v = 4, C_p = 6$
   (*c*) $C_v = 3, C_p = 2$ (*d*) $C_v = 3, C_p = 4.2$

7. A thermally insulated chamber of volume $2V_0$ is divided by a frictionless piston of area S & mass m into two equal parts A and B.Part A has an ideal gas at pressure P_0 and temperature T_0 and in part B is vacuum. A massless spring of force constant K is connected with the piston and the wall of the container as shown. Initially the spring is undeformed. The gas in chamber A is allowed to expand. Let in equilibrium the spring is compressed by x_0 . Then :



   (*a*) Pressure of the gas at equilibrium is $\frac{Kx_0}{S}$

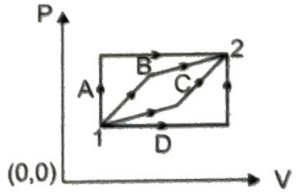
   (*b*) Work done by the gas is $\frac{1}{2}Kx_0^2$

   (*c*) Increase in internal energy of the gas is $\frac{1}{2}Kx_0^2$

   (*d*) Temperature of the gas is decreased

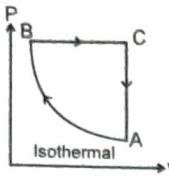
8. A gas kept in a container of finite conductivity is suddenly compressed. The process
   (*a*) must be very nearly adiabatic
   (*b*) must be very nearly isothermal
   (*c*) may be very nearly adiabatic
   (*d*) may be very nearly isothemal

9. An ideal gas is taken from state 1 to state 2 through optional path A, B, C & D as shown in P-V diagram. Let Q, W and U represent the heat supplied, work done & internal energy of tha gas respectively. Then



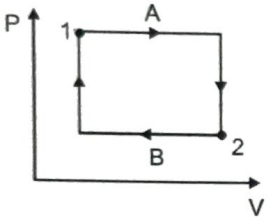
   (*a*) $Q_B - W_B > Q_C - W_C$
   (*b*) $Q_A - Q_D = W_A - W_D$
   (*c*) $W_A < W_B < W_C < W_D$
   (*d*) $Q_A > Q_B > Q_C > Q_D$

10. A cyclic process of an ideal monoatomic gas is shown in figure. The correct statement is (are) :



(*a*) Work done by gas in process AB is more than that of the process BC.
(*b*) net heat energy has been supplied to the system.
(*c*) temperature of the gas is maximum in state B.
(*d*) in process CA, heat energy is rejected out by system.

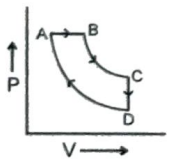
11. In given figure, let ΔU_1 and ΔU_2 be change in internal energy in process A and B respectively. ΔQ and W be the net heat given and net work done by the system in the process A + B, then



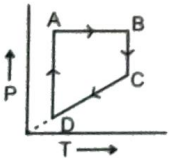
(*a*) $\Delta U_1 + \Delta U_2 = 0$ (*b*) $\Delta U_1 - \Delta U_2 = 0$
(*c*) $\Delta Q - W = 0$ (*d*) $\Delta Q + W = 0$

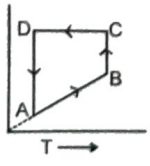
12. An ideal gas of one mole is kept in a rigid container of negligible heat capacity. If 25 J of heat is supplied the gas temperature raises by 2^0C. Then the gas may be
(*a*) helium (*b*) argon
(*c*) oxygen (*d*) carbon dioxide

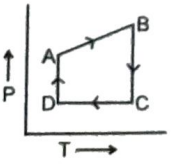
13. A cyclic process ABCD is shown in the P–V diagram. (BC and DA are isothermal)

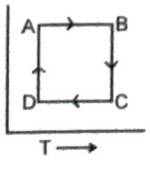


Which of the following curves represents the same process?

(*a*) 

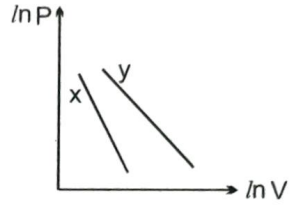
(*b*) 

(*c*) 

(*d*) 

14. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following:
(*a*) Internal energies at A and B are the same
(*b*) Work done by the gas in process AB is $P_0V_0 \ell n\, 4$
(*c*) Pressure at C is $T°/4$
(*d*) Temperature at C is $T°/4$

15. For two different gases X and Y, having degrees of freedom f_1 and f_2 and molar heat capacities at constant vloume C_{V_1} and C_{V_2} respectively, the lnP versus ln V graph is plotted of adiabatic process, as shown



(*a*) $f_1 > f_2$ (*b*) $f_2 > f_1$
(*c*) $C_{V_2} > C_{V_1}$ (*d*) $C_{V_1} > C_{V_2}$

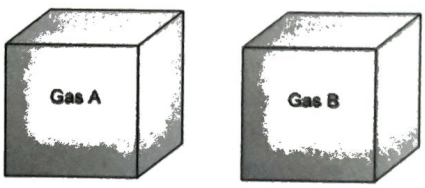
16. A system undergoes a cyclic process in which it absorbs Q_1 heat and gives out Q_2 heat. The efficiency of the process is η and work done is W. Select correct statement:

(*a*) $W = Q_1 - Q_2$ (*b*) $\eta = \dfrac{W}{Q_1}$

(*c*) $\eta = \dfrac{Q_2}{Q_1}$ (*d*) $\eta = 1 - \dfrac{Q_2}{Q_1}$

Comprehension – 1 (17 to 20)

Two closed identical conducting containers are found in the laboratory of an old scientist. For the verification of the gas some experiments are performed on the two boxes and the results are noted.



Experiment 1.

When the two containers are weighed W_A = 225 g, W_B = 160 g and mass of evacuated container W_C = 100 g.

Experiment 2.

When the two containers are given same amount of heat same temperature rise is recorded. The pressure changes found are

ΔP_A = 2.5 atm. ΔP_B = 1.5 atm.

Required data for unknown gas :

Mono (molar mass)	He 4g	Ne 20g	Ar 40 g	Kr 84 g	Xe 131 g	Rd 222 g
Dia (molar mass)	H_2 2g	F_2 19 g	N_2 28g	O_2 32g	Cl_2 71 g	

17. Identify the type of gas filled in container A and B respectively.

(*a*) Mono, Mono (*b*) Dia, Dia

(*c*) Mono, Dia (*d*) Dia, Mono

18. Identify the gas filled in the container A and B.

(*a*) N_2, Ne (*b*) He, H_2

(*c*) O_2, Ar (*d*) Ar, O_2

19. Total number of molecules in 'A' (here N_A = Avagadro number)

(*a*) $\frac{125}{64}N_A$ (*b*) 3.125 N_A

(*c*) $\frac{125}{28}N_A$ (*d*) 31.25 N_A

20. The initial internal energy of the gas in container 'A', If the container were at room temperature 300K initially

(*a*) 1406.25 cal (*b*) 1000 cal

(*c*) 2812.5 cal (*d*) none of these

Comprehension – 2 (21 to 23)

A monatomic ideal gas is filled in a non conducting container. The gas can be compressed by a movable non conducting piston. The gas is compressed slowly to 12.5% of its initial volume.

21. The percentage increase in the temperature of the gas is

(*a*) 400% (*b*) 300%

(*c*) – 87.5% (*d*) 0%

22. The ratio of initial adiabatic bulk modulus of the gas to the final value of adiabatic bulk modulus of the gas is

(*a*) 32 (*b*) 1

(*c*) 1/32 (*d*) 4

23. The ratio of work done by the gas to the change in internal energy of the gas is

(*a*) 1 (*b*) –1

(*c*) ∞ (*d*) 0

24. An ideal monoatomic gas undergoes different types of processes which are described in column-I. Match the corresponding effects in column-II. The letters have usual meaning.

Column-I	Column-II
(*a*) $P = 2V^2$	(p) If volume increases then temperature will also increases.
(*b*) PV^2 = constant	(q) If volume increases then temperature will decreases.
(*c*) $C = C_V + 2R$	(r) For expansion, heat will have to be supplied to the gas.
(*d*) $C = C_V - 2R$	(s) If temperature increases then work done by gas is positive.

25. The figures given below show different processes (relating pressure P and volume V) for a given amount for an ideal gas. W is work done by the gas and ΔQ is heat absorbed by the gas.

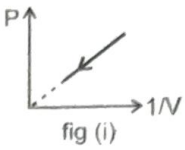


fig (i)

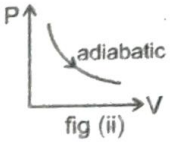


fig (ii)

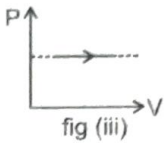


fig (iii)

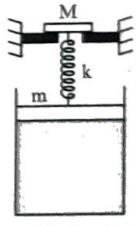


fig (iv)

Column-I	Column-II
(*a*) In Figure (i)	(p) $\Delta Q > 0$.
(*b*) In Figure (ii)	(q) $W < 0$.
(*c*) In Figure (iii)	(r) $\Delta Q < 0$.
(*d*) In Figure (iv) (for complete cycle)	(s) $W > 0$.

INTEGER VALUE BASED

26. 0.01 moles of an ideal diatomic gas is enclosed in an adiabatic cylinder of cross-sectional area $A = 10^{-4}m^2$. In the arrangement shown, a block of mass M = 0.8 kg is placed on a horizontal support, and piston of mass m = 1 kg is suspended from a spring of stiffness constant k = 16 N/m. Initially, the spring is relaxed and the volume of the gas is $V = 1.4 \times 10^{-4}$ m^3.



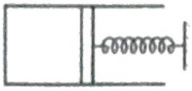
When the gas in the cylinder is heated up the piston starts moving up and the spring gets compressed so that the block *M* is just lifted up. Determine the heat supplied (in Joule).

Take atmospheric pressure $P_0 = 10^5$ Nm^{-2}, g = 10m/s^2.

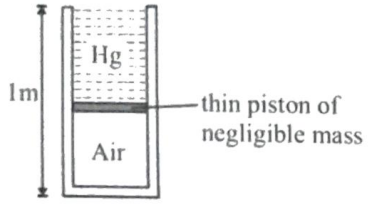
27. A 5000 cm^3 tank contains an ideal gas (M = 40 kg/kmol) at a pressure of 500 kPa and a temperature of 27°C.

What mass of gas (in gm) is in the tank? (Take $R = \frac{25}{3}$ J/mol K).

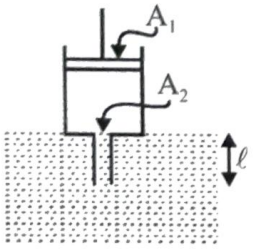
28. An ideal gas is enclosed in a container as shown in figure. The spring constant is 100 N/m, the area of the piston is 1 cm^2 and the length of the gas column is 100 cm. The compression of the spring is 10 cm. There is no friction between the walls and the piston. The whole system is heated to make the temperature 2.4 times the original temperature. Find the distance moved by the piston (in cm) Take atmospheric pressure = 10^5 Pa.



29. The 1 m tall conducting cylinder in figure contains air at a pressure of 76 cm of Hg. A very thin piston of negligible mass is placed at the top of the cylinder to prevent any air from air escaping, then mercury is very slowly poured into the cylinder until no more can be added without the cylinder over flowing. What is the height (in cm) of the compressed air?

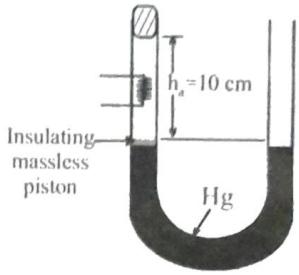


30. How far can I move (in cm) the piston of cross-section A_1, in the device shown in figure, so that 40% of the air enclosed remains. (The atmospheric pressure p_0 = 10 N/cm^2, density of liquid = 5 g/cm^3, ℓ = 20cm, A_1 = 4cm^2, A_2 = 1 cm^2. The temperature is constant. The piston is massless and the initial height of gas in device is 110 cm.

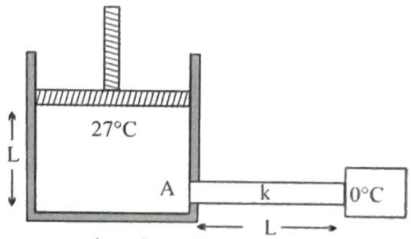


31. In a chemical processing plant, a reaction chamber of fixed volume V_0 is connected to a reservoir chamber of fixed volume $4V_0$ by a passage containing a thermally insulating porous plug. The plug permits the chambers to be at different temperatures. The plug allows gas to pass from either chamber to the other, ensuring that the pressure is the same in both. At one point in the processing, both chambers contain gas at a pressure of 100 kPa and a temperature of 27°C. The reservoir is heated to 127°C while the reaction chamber is maintained at 27°C. What is the pressure in both chambers (in kPa) after this is done?

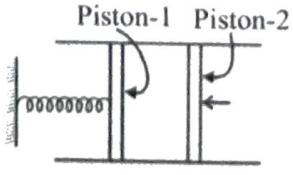
32. A U tube, made of heat-insulating material, is shown in figure. Left limb is closed by a non-conducting cork, the temperature of air in left limb is 300 K. How much heat (in Joule) is required to be given by the coil so that the temperature of air rises to 510 K? The thermal expansion of mercury is negligible. (Take : Area of the tube as 0.1 m^2, P_{atm} = 75 cm of Hg, R = $\frac{25}{3}$ J/mol K, specific gravity of mercury = 13.6). Assume that the initial level of mercury in both limbs is same.



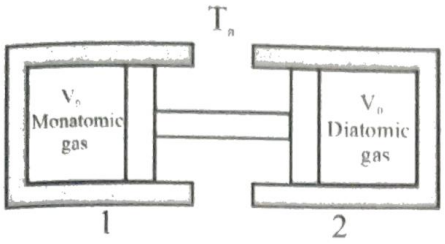
33. 0.5 mole of an ideal gas is kept inside an adiabatic cylinder of length 'L' and cross-sectional area 'A' closed by massless adiabatic piston. The cylinder is attached with a conducting rod of length 'L', cross-sectional area $\frac{1}{900}$ m^2 and thermal conductivity 415.5 W/m-K, whose other end is maintained at 0°C. The piston is moved such that the temperature of the gas remains constant at 27°C. Find the velocity (in mm/sec) of piston when it is at height L/2 from the bottom of cylinder. Rod is well lagged and has negligible heat capacity. R = 8.31 J/mol-K.



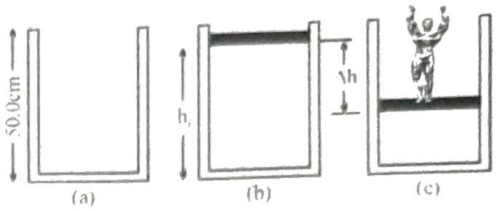
34. A long container has air enclosed inside at room temperature and atmospheric pressure (10^5 pa). It has a volume of 20,000 cc. The area of cross section is 100 cm^2 and force constant of spring is k_{spring} = 1000N/m. We push the right piston isothermally and slowly till it reaches the original position of the left piston which is movable. What is the final length of air column in cm. Assume that spring is initially relaxed.



35. The two conducting cylinder-piston systems shown below are linked. Cylinder 1 is filled with a certain molar quantity of a monatomic ideal gas, and cylinder 2 is filled with an equal molar quantity of a diatomic ideal gas. The entire apparatus is situated inside an oven whose temperature is $T_a = 27°C$. The cylinder volumes have the same initial value $V_0 = 100cc$. When the oven temperature is slowly raised to $T_b = 127°C$. What is the volume change ΔV (in cc) of cylinder 1?



36. A cylinder that has a cross-section area of $100cm^2$ and is 50cm deep is filled with air at 21.0°C and 1.00 atm (figure-a) A 20.0 kg piston is now lowered into the cylinder, compressing the air trapped inside (figure-b). Finally, a 80.0 kg man stands on the piston, further compressing the air, which remains at 21°C (figure-c). To what temperature (in°C) should the gas be heated to raise the piston and the man back to h_i? ($g = 10\ m/s^2$)



37. Three insulated vessels of equal volumes V are connected by thin tubes that can transfer gas but do not transfer heat. Initially all vessels are filled with the same type of gas at a temperature T_0 and pressure P_0. Then the temperature in the first vessel is doubled and the temperature in the second vessel is tripled. The temperature in the third vessel remains unchanged. If initial pressure is 77 cm of Hg. Find the final pressure P' in the system in cm of Hg.

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively):

[*JEE Mains-2016*]

(a) $n=\frac{C_P}{C_V}$

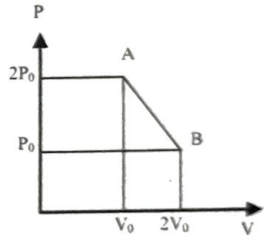
(b) $n=\frac{C-C_P}{C-C_V}$

(c) $n=\frac{C_P-C}{C-C_V}$

(d) $n=\frac{C-C_V}{C-C_P}$

2. 'n' moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature of the gas during the process will be:

[*JEE Mains-2016*]



(a) $\frac{9P_0V_0}{4nR}$ (b) $\frac{3P_0V_0}{2nR}$

(c) $\frac{9P_0V_0}{2nR}$ (d) $\frac{9P_0V_0}{nR}$

3. The temperature of an open room of volume 30 m^3 increases from 17°C t o 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be:

[*JEE Main-2017*]

(a) 2.5×10^{25} (b) -2.5×10^{25}

(c) -1.61×10^{23} (d) 1.38×10^{25}

4. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is : [JEE Main-2017]

(a) $a = 14\,b$ (b) $a = 28\,b$

(c) $a = \frac{1}{14}b$ (d) $a = b$

5. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

[JEE Main-2018]

(a) (a) 195 K (b) –2.7 kJ (b) (a) 189 K (b) –2.7 kJ

(c) (a) 195 K (b) 2.7 kJ (d) (a) 189 K (b) 2.7 kJ

6. In a process, temperature and volume of one mole of an ideal monoatomic gas ar varied according to the relation $VT = K$, where K is a constant. In this procedss the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant)

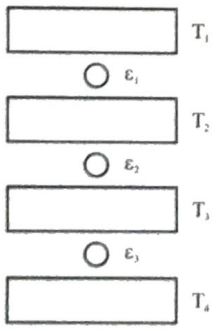
[JEE Main-2019 (January)]

(a) $\frac{1}{2}R\Delta T$ (b) $\frac{1}{2}KR\Delta T$

(c) $\frac{3}{2}R\Delta T$ (d) $\frac{2K}{3}\Delta T$

7. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if:

[JEE Main-2019 (January)]



(a) $T_2 = (T_1T_4)^{1/2}; T_3 = \left(T_1^2T_4\right)^{1/3}$

(b) $T_2 = \left(T_1^2T_4\right)^{1/3}; T_3 = \left(T_1T_2^4\right)^{1/3}$

(c) $T_2 = \left(T_1T_4^2\right)^{1/3}; T_3 = \left(T_1^2T_4\right)^{1/3}$

(d) $T_2 = \left(T_1^3T_4\right)^{1/4}; T_3 = \left(T_1T_4^3\right)^{1/4}$

8. An ideal gas occupies a volume of 2 m^3 at a pressure of 3×10^6 Pa. The energy of the gas is:

[JEE Main-2019 (January)]

(a) 9×10^6 J (b) 6×10^6 J

(c) 10^8 J (d) 3×10^2 J

9. Two kg of a monoatomic gas is at a pressure 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion?

[JEE Main-2019 (January)]

(a) 10^3 J (b) 10^5 J

(c) 10^4 J (d) 10^6 J

10. A charge Q is distributed over three concentric spherical shell of radii a, b, c ($a < b < c$) such that their surface charge densities are equal to one another.
The total potential at a point at distance r from their common centre, where $r < a$, would be:

[JEE Main-2019 (January)]

(a) $\frac{Q}{12\pi\epsilon_0}\frac{ab+bc+ca}{abc}$ (b) $\frac{Q\left(a^2+b^2+c^2\right)}{4\pi\epsilon_0\left(a^3+b^3+c^3\right)}$

(c) $\frac{Q}{4\pi\epsilon_0(a+b+c)}$ (d) $\frac{Q(a+b+c)}{4\pi\epsilon_0\left(a^2+b^2+c^2\right)}$

11. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for the process is TV^x = constant, then x is:

[JEE Main-2019 (January)]

(a) 3/5 (b) 2/5

(c) 2/3 (d) 5/3

12. A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{arg on})}\right]$, is close to:

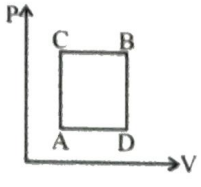
[JEE Main-2019 (January)]

(a) 3.16 (b) 0.32

(c) 0.45 (d) 2.24

13. A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work down by the system is 10J. The heat flow into the system in path ADB is

[JEE Main-2019 (January)]



(a) 40 J (b) 80 J

(c) 100 J (d) 20 J

14. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by has is close to: (Gas constant R = 8.31 J/mol·K)
[JEE Main-2019 (January)]

(a) 581 J (b) 291 J
(c) 146 J (d) 73 J

15. Two Carnot engines A and B are operated in series. The first one, A, receive heat at T_1 (= 600 K) and rejects to a reservoir at temperature T_2. the seconds engine B receives heat rejected by the first engine and, in turns, rejects to a heat reservoir at T_3(= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal:
[JEE Main-2019 (January)]

(a) 600 K (b) 400 K
(c) 300 K (d) 500 K

16. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about:
[Take R = 8.3 J/K mole] [JEE Main-2019 (January)]

(a) 0.9 k J (b) 6 k J
(c) 10 k J (d) 14 k J

17. A vertical closed cylinder is separated into two parts by a fricitonless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1 and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by : (R is universal gas constant and g is the acceleration due to gravity)
[JEE Main-2019 (January)]

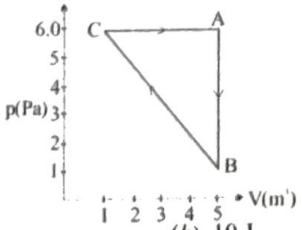
(a) $\frac{RT}{ng}\left[\frac{l_1 - 3l_2}{l_1 l_2}\right]$ (b) $\frac{RT}{g}\left[\frac{2l_1 - l_2}{l_1 l_2}\right]$

(c) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{l}{l_1}\right]$ (d) $\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1 l_2}\right]$

18. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8}s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to
[JEE Main-2019 (January)]

(a) 2×10^{-7} s (b) 4×10^{-8} s
(c) 0.5×10^{-8} s (d) 3×10^{-6} s

19. For the given cyclic process CAB as shown for a gas, the work done is :
[JEE Main-2019 (January)]



(a) 30 J (b) 10 J
(c) 1 J (d) 5 J

20. The specific heats, C_p and C_v of a gas of diatomic molecules, A are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B has the corresponding values 30 and 21. If they are treated as ideal gases, then:
[JEE Main-2019 (April)]

(a) A has one vibrational mode and B has two
(b) Both A and B have a bivrational mode each
(c) A is rigid but B has a vibrational mode
(d) A has a vibrational mode but B has none

21. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is :
[JEE Main-2019 (April)]

(a) $\frac{7}{5}Q$ (b) $\frac{3}{2}Q$

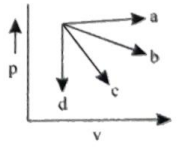
(c) $\frac{5}{3}Q$ (d) $\frac{2}{3}Q$

22. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_o\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_O and V_o are constants. Calculate the change in the temperature of the gas if its volume changes from V_O to $2V_O$.
[JEE Main-2019 (April)]

(a) $\frac{1}{2}\frac{P_o V_o}{R}$ (b) $\frac{3}{4}\frac{P_o V_o}{R}$

(c) $\frac{5}{4}\frac{P_o V_o}{R}$ (d) $\frac{1}{4}\frac{P_o V_o}{R}$

23. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by:
[JEE Main-2019 (April)]



(a) d a c b (b) a d c b
(c) a d b c (d) d a b c

24. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to : [Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg Radius of Earth : 6.4×10^6 m Gravitational acceleration of Earth = 10ms^{-2}] [JEE Main-2019 (April)]

(a) 650 K (b) 3×10^5 K
(c) 10^4 K (d) 800 K

25. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$. m is its mass and k_B is Boltzmann constant, then its temperature will be:

[JEE Main-2019 (April)]

(a) $\frac{m\bar{v}^2}{6k_B}$ (b) $\frac{m\bar{v}^2}{5k_B}$

(c) $\frac{m\bar{v}^2}{3k_B}$ (d) $\frac{m\bar{v}^2}{7k_B}$

26. For a given gas at 1 atm pressure, rms speed of the molecule is 200 m/s at 127° C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be

[JEE Main-2019 (April)]

(a) 80 m/s (b) $100\sqrt{5}$ m/s

(c) $80\sqrt{5}$ m/s (d) 100 m/s

27. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to

[JEE Main-2019 (April)]

(a) $n_0\alpha^{1/4}$ (b) $n_0\alpha^{-3}$

(c) $n_0\alpha^{-3/4}$ (d) $\sqrt{n_0}\alpha^{1/2}$

28. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C. Its efficiency is doubled. The temperatures of the source and the sink are, respectively

[JEE Main-2019 (April)]

(a) 124°C, 62° C (b) 37°C, 99° C

(c) 62°C, 124° C (d) 99°C, 37° C

29. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?

[JEE Main-2019 (April)]

(a) 35 J (b) 40 J

(c) 25 J (d) 30 J

30. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J mol^{-1} K^{-1}]

[JEE Main-2019 (April)]

(a) 748 J (b) 374 J

(c) 350 J (d) 700 J

31. A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm, and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule?

[JEE Main-2019 (April)]

(a) $\sim 10^{11}$ (b) $\sim 10^{13}$

(c) $\sim 10^{10}$ (d) $\sim 10^{12}$

32. n moles of an ideal gas with constant volume heat capcity C_v undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is :

[JEE Main-2019 (April)]

(a) $\frac{4nR}{C_v - nR}$ (b) $\frac{nR}{C_v - nR}$

(c) $\frac{nR}{C_v + nR}$ (d) $\frac{4nR}{C_v + nR}$

33. Two moles of an ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of $\frac{C_P}{C_V}$ for the mixture is :

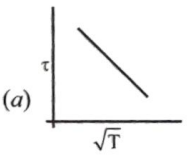
[JEE Main-2020 (January)]

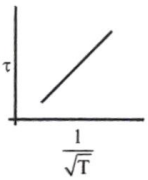
(a) 1.42 (b) 1.47

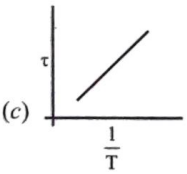
(c) 1.50 (d) 1.45

34. The plot that depicts the behaviour of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is : (Graphs are schematic and not drawn to scale)

[JEE Main-2020 (January)]

(a) 

(b) 

(c) 

(d) 

35. Consider a mixture of n moles of helium gas and 2n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its Cp/Cv value will be :

[JEE Main-2020 (January)]

(a) $\frac{23}{15}$ (b) $\frac{40}{27}$

(c) $\frac{67}{45}$ (d) $\frac{19}{13}$

36. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additonal vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the speciffic heats $\left(C_V^A \text{ and } C_V^B\right)$ of gas A and B, respectively is:

[JEE Main-2020 (January)]

(a) 3 : 5 (b) 5 : 9

(c) 7 : 9 (d) 5 : 7

37. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to : [JEE Main-2020 (January)]

(a) 1.83 (b) 2.3

(c) 3.67 (d) 4.67

38. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true? [JEE Main-2020 (September)]

(a) the mean free path of the molecules decreases.

(b) the mean collision time between the molecules decreases.

(c) the mean free path remains unchanged.

(d) the mean collision time remains unchanged

(a) (c) and (d) (b) (a) and (d)

(c) (b) and (c) (d) (a) and (b)

39. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is [JEE Main-2020 (September)]

(a) 13 (b) 15

(c) 20 (d) 11

40. 

Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is [JEE Main-2020 (September)]

(a) 3RT (b) $\frac{9}{2}RT$

(c) $\frac{3}{2}RT$ (d) $\frac{5}{2}RT$

41. Match the C_p/C_v ratio for ideal gases with different type of molecules [JEE Main-2020 (September)]

Molecule Type	C_p/C_v
(a) Monatomic	(I) 7/5
(b) Diatomic rigid molecules	(II) 9/7
(c) Diatomic non-rigid molecules	(III) 4/3
(d) Triatomic rigid molecules	(IV) 5/3

(a) (a)-(II), (b)-(III), (c)-(I), (d)-(IV)

(b) (a)-(IV), (b)-(II), (c)-(I), (d)-(III)

(c) (a)-(IV), (b)-(I), (c)-(II), (d)-(III)

(d) (a)-(III), (b)-(IV), (c)-(II), (d)-(I)

42. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H$_2$ molecule would be equal to the rms speed of a nitrogen molecule is _____. (Molar mass of N_2 gas 28 g) [JEE Main-2020 (September)]

43. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, g = 980 cm/s^2, density of mercury = 13.6 g/cm^3) [JEE Main-2020 (September)]

(a) 5.8×10^{18} (b) 4.0×10^{15}

(c) 5.8×10^{16} (d) 4.0×10^{14}

44. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T as [JEE Main-2020 (September)]

(a) $\sqrt{T}$ (b) T

(c) $\frac{1}{\sqrt{T}}$ (d) $\frac{1}{T}$

45. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(\frac{C_p}{C_v}\right)$ are given, respectively, by [JEE Main-2020 (September)]

(a) U=5RT and $\gamma = \frac{7}{5}$ (b) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$

(c) U = 5RT and $\gamma = \frac{6}{5}$ (d) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$

46. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is _____. [JEE Main-2020 (September)]

47. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.40$, the work done by air is : ($3^{1.4} = 4.6555$) [Take air to be an ideal gas] [JEE Main-2020 (January)]

(a) 90.5 J (b) 48 J

(c) 60.7 J (d) 100.8 J

48. A carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is..... [JEE Main-2020 (January)]

49. Under an adiabatic process, the volume of an ideal gas gets doubled. consequently the mean collision time between the gas molecules changes from τ_1 to τ_2. If $\frac{C_P}{C_V} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by: [JEE Main-2020 (January)]

(a) $\left(\frac{1}{2}\right)^{\gamma}$ (b) 2

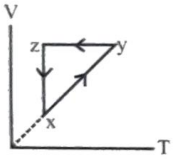
(c) $\frac{1}{2}$ (d) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$

50. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work? [JEE Main-2020 (January)]

(a) $T = \sqrt{T_1 T_2}$ (b) $T = \frac{T_1 + T_2}{2}$

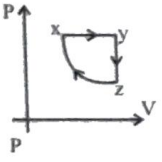
(c) $T = \frac{2T_1T_2}{T_1 + T_2}$ (d) $T = 0$

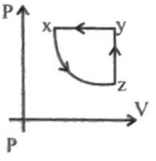
51. A thermodynamic cycle xyzx is shown on a V-T diagram.

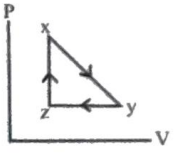


The P-V diagram that best describes this cycle is:(Diagrams as schematic and not to scale)

[JEE Main-2020 (January)]

(a)


(b)


(c)


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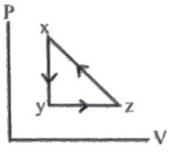
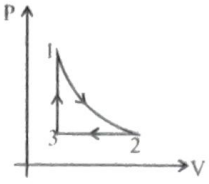


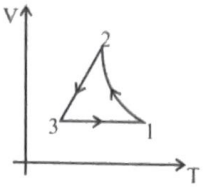
52. A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10J, the amount of heat absorbed from the reservoir at lower temperature is: [JEE Main-2020 (January)]

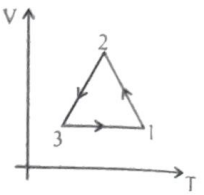
(a) 90 J (b) 1 J

(c) 99 J (d) 100 J

53. Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure ? where, $1 \rightarrow 2$ is adiabatic. (Graphs are schematic and are not to scale) [JEE Main-2020 (January)]



(a)


(b)


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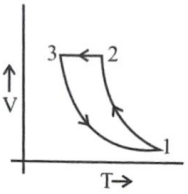
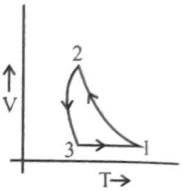


(d)


54. Starting at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically from volume V_1 to $V_2 = V_1/16$. it is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer)_____.

[JEE Main-2020 (January)]

55. A heat engine is involved with exchange of heat of 1915 J, – 40J, + 125 J and –Q J, during one cycle achieving an efficiency of 50.0%. The value of Q is

[JEE Main-2020 (September)]

(a) 980 J (b) 40 J

(c) 400 J (d) 640 J

56. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabatically to $1/10^{th}$ of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is_____.

[JEE Main-2020 (September)]

57. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer ? [JEE Main-2020 (September)]

58. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as [JEE Main-2020 (September)]

(a) Irreversible adiabatic

(b) Reversible adiabatic

(c) Irreversible isothermal

(d) Reversible isothermal

59. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm respectively. If $|\Delta T| = C|\Delta P|$ then value of C in (K/atm) is

[JEE Main-2020 (September)]

60. Match the thermodynamic processes taking place in a system with the correct conditions. In the table : ΔQ is the heat supplied, ΔW is the work done and ΔU is change in internal energy of the system

[JEE Main-2020 (September)]

Process	Condition
(I) Adiabatic	(a) $\Delta W = 0$
(II) Isothermal	(b) $\Delta Q = 0$
(III) Isochoric	(c) $\Delta U \neq 0, \Delta W \neq 0, \Delta Q \neq 0$
(IV) Isobaric	(d) $\Delta U = 0$

(a) (I) - (a), (II) - (d), (III) - (b), (IV) - (c)
(b) (I) - (b), (II) - (d), (III) - (a), (IV) - (c)
(c) (I) - (a), (II) - (b), (III) - (d), (IV) - (d)
(d) (I) - (b), (II) - (a), (III) - (d), (IV) - (c)

61. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to _____.

[JEE Main-2020 (September)]

62. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is:

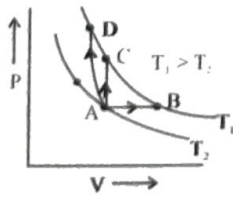
[JEE Main-2020 (September)]

(a) 128 (b) 32

(c) $\frac{1}{32}$ (d) 326

63. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as A → B, A → C and A → D. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the work done as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are

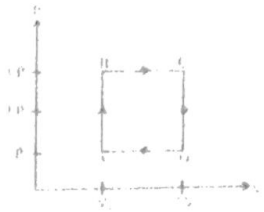
[JEE Main-2020 (September)]



(a) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(b) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$
(c) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$
(d) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$

64. An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to

[JEE Main-2020 (September)]



65. Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of a: b. Then a/b can be expressed as $1/x$ where x is

[JEE Main-2021 (March)]

66. For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be °C. (Round off to the Nearest Integer)

[JEE Main-2021 (March)]

JEE-ADVANCED PREVIOUS YEAR'S

1. A mixture of 2 moles of helium gas (atomic mass = 4 amu), and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(helium)}{v_{rms}(\text{argon})}\right)$ is :

[JEE-2012]

(a) 0.32 (b) 0.45
(c) 2.24 (d) 3.16

2. Two moles of ideal helium gas are in a rubber balloon at 30° C .The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C.The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)

[JEE-2012]

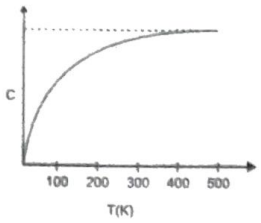
(a) 62J (b) 104 J
(c) 124 J (d) 208 J

3. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is:

[JEE Advanced-2013]

(a) 1 : 4 (b) 1 : 2
(c) 6 : 9 (d) 8 : 9

4. The figure below shows the variation of specific heat capacity (c) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation. *[JEE Advanced-2013]*



(*a*) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.

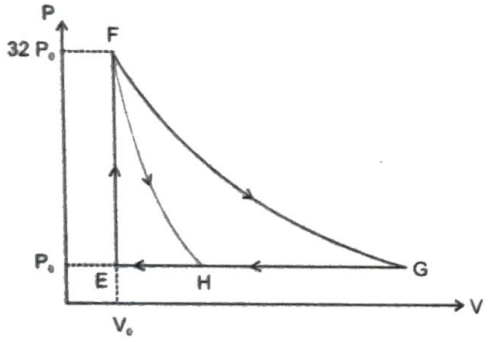
(*b*) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K.

(*c*) there is no change in the rate of heat absorbtion in the range 400–500 K.

(*d*) the rate of heat absorption increases in the range 200–300 K.

5. One mole of a monatomic ideal gas is taken along two cyclic processes E → F → G → E and E → F → H → E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.

[JEE Advanced-2013]



Match the paths in List-I with the magnitudes of the work done in List-II and select the correct answer using the codes given below the lists.

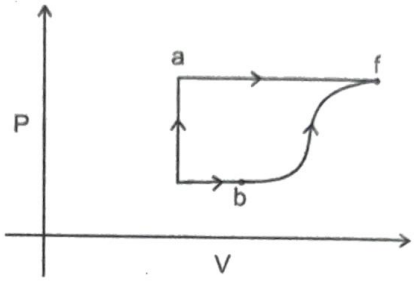
List-I	List-II
P. G → E	1. 60 P_0V_0
Q. G → H	2. 36 P_0V_0
R. F → H	3. 24 P_0V_0
S. F → G	4. 31 P_0V_0

Codes :

	P	Q	R	S
(*a*)	4	3	2	1
(*b*)	4	3	1	2
(*c*)	3	1	2	4
(*d*)	1	3	2	4

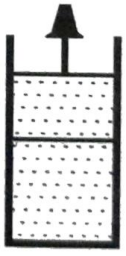
6. A thermodynamic system is taken form an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the fire. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{bf} and Q_{ib} respectively. If the internal energy of the sytem in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf}/Q_{ib} is:

[JEE Advanced-2014]



Paragraph For Questions 7 and 8

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$. *[JEE Advanced-2014]*



7. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(*a*) 550 K (*b*) 525 K

(*c*) 513 K (*d*) 490 K

8. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be:

(*a*) 250 R (*b*) 200 R

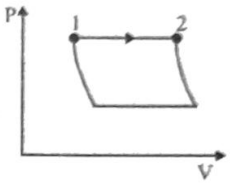
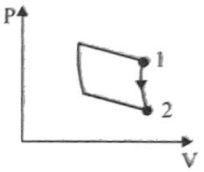
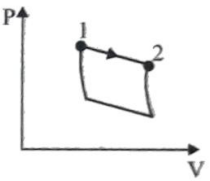
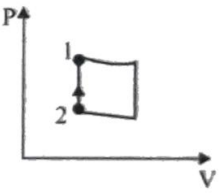
(*c*) 100 R (*d*) –100 R

9. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately. [JEE Advanced-2016]

(a) 112J (b) 294 J (c) 588 J (d) 813 J

Answer Q.10, Q.11 and Q.12 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P - V diagrams in Column-3 of the table. Consider only path from state-1 to state-2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic process. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column-1	Column-2	Column-3
(I) $W_{1\to2} = \frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to2} = -nRT \ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

10. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$? [JEE Advanced-2017]

(a) (II) (iii) (P) (b) (II) (iii) (S)
(c) (III) (iii) (P) (d) (II) (iv) (R)

11. Which one of the following options is the correct combination ? [JEE Advanced-2017]

(a) (II) (iv) (P)
(b) (IV) (ii) (S)
(c) (II) (iv) (R)
(d) (III) (ii) (S)

12. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas ? [JEE Advanced-2017]

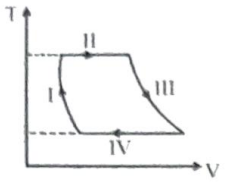
(a) (III) (iv) (R)

(b) (I) (ii) (Q)

(c) (IV) (ii) (R)

(d) (I) (iv) (Q)

13. One mole of a monatomic ideal gas udergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statement below is (are) true ? **[JEE Advanced-2018]**

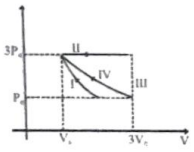


(*a*) Process I is an isochoric process.
(*b*) In process II, gas absorbs heat.
(*c*) In process IV, gas releases heat.
(*d*) Process I and II are not isobaric

14. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J mol^{-1} K^{-1}, the decrease in its internal energy, in Joule, is ..... **[JEE Advanced-2018]**

15. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II

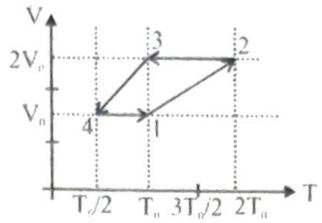
[JEE Advanced-2018]



List-I	List-II
P. In process I	1. Work done by the gas is zero
Q. In process II	2. Temperature of the gas remains unchanged
R. In process III	3. No heat is exchanged between the gas and its surroundings
S. In process IV	4. Work done by the gas is $6P_0V_0$

(*a*) P → 4; Q → 3; R → 1; S → 2
(*b*) P → 1; Q → 3; R → 2; S → 4
(*c*) P → 3; Q → 4; R → 1; S → 2
(*d*) P → 3; Q → 4; R → 2; S → 1

16. One mole of a monoatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/ are: **[JEE Advanced-2019]**
[R is the gas constant]



(*a*) Work done in this thermodynamic cycle (1 → 2 → 3 → 4 → 1) is $|W| = \frac{1}{2} RT_0$

(*b*) The ratio of heat transfer during processes 1 → 2 and 2 → 3 is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(*c*) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(*d*) The ratio of heat transfer during processes 1 → 2 and 3 → 4 is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

17. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0 and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are:

[JEE Advanced-2019]

(Give $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)

(*a*) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

(*b*) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(*c*) The work |W| done during the process is $13RT_0$

(*d*) Adiabatic constant of the gas mixture is 1.6

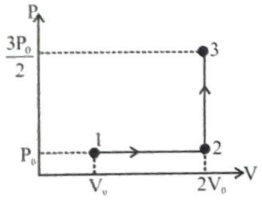
(Direction: 18 and 19) Answer the following by appropriately matching the lists based on the information given in the paragraph.

In a thermodynamics process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by TΔX, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2} R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved and List-II gives some possible values of these quantities.

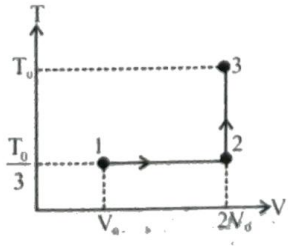
List-I	List-II
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3+\ln 2)$
	(U) $\frac{5}{6}RT_0$

18. If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV - diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is, **[JEE Advanced-2019]**



(*a*) I → Q, II → R, III → P, IV → U
(*b*) I → S, II → R, III → Q, IV → T
(*c*) I → Q, II → R, III → S, IV → U
(*d*) I → Q, II → S, III → R, IV → U

19. If the process on one mole of monatomic ideal gas is an shown is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is **[JEE Advanced-2019]**



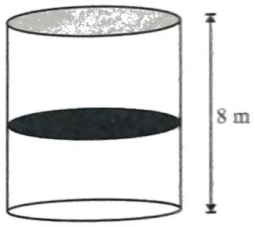
(*a*) I → S, II → T, III → Q, IV → U
(*b*) I → P, II → R, III → T, IV → S
(*c*) I → P, II → T, III → Q, IV → T
(*d*) I → P, II → R, III → T, IV → P

20. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia}, respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f \ln 2$, then f is .....

[JEE (Advanced)-2020]

21. A thermally isolated cylindrical closed vessel of height 8 m is kept vertically. It is divided into two equal parts by a diathermic (perfect thermal conductor) frictionless partition of mass 8.3 kg. Thus the partition is held initially at a distance of 4 m from the top. as shown in the schematic figure below. Each of the two parts of the vessel contains 0.1 mole of an ideal gas at temperature 300 K. The partition is now released and moves without any gas leaking from one part of the vessel to the other. When equilibrium is reached, the distance of the partition from the top (in m) will be _____
(take theacceleration due to gravity =10 ms^{-2} and the universal gas constant = 8.3 J mol^{-1} K^{-1}).

[JEE (Advanced)-2020]



22. A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes (R–a). For a << R the magnitude of the work done in the process is given by $(4\pi p_0 Ra^2)X$, where X is a constant and $\gamma = C_p/C_V = 41/30$. The value of X is _____.

[JEE(Advanced)-2020]

23. A small object is placed at the center of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time t = 0, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2 / t_1) is **[JEE Advanced-2021]**

ANSWER KEY

Topicwise Questions

1. (*a*)	**2.** (*b*)	**3.** (*c*)	**4.** (*b*)	**5.** (*b*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*d*)
11. (*a*)	**12.** (*b*)	**13.** (*d*)	**14.** (*c*)	**15.** (*b*)	**16.** (*b*)	**17.** (*a*)	**18.** (*d*)	**19.** (*a*)	**20.** (*b*)
21. (*c*)	**22.** (*a*)	**23.** (*c*)	**24.** (*a*)	**25.** (*a*)	**26.** (*a*)	**27.** (*d*)	**28.** (*c*)	**29.** (*a*)	**30.** (*a*)
31. (*b*)	**32.** (*c*)	**33.** (*a*)	**34.** (*c*)	**35.** (*a*)	**36.** (*c*)	**37.** (*d*)	**38.** (*d*)	**39.** (*d*)	**40.** (*d*)
41. (*b*)	**42.** (*a*)	**43.** (*b*)	**44.** (*a*)	**45.** (*d*)	**46.** (*d*)	**47.** (*b*)	**48.** (*c*)	**49.** (*c*)	**50.** (*d*)
51. (*a*)	**52.** (*b*)	**53.** (*b*)	**54.** (*b*)	**55.** (*d*)	**56.** (*d*)	**57.** (*b*)	**58.** (*c*)	**59.** (*b*)	**60.** (*a*)
61. (*c*)	**62.** (*a*)	**63.** (*d*)	**64.** (*c*)	**65.** (*c*)	**66.** (*b*)	**67.** (*c*)	**68.** (*d*)	**69.** (*b*)	**70.** (*b*)
71. (*a*)	**72.** (*b*)	**73.** (*a*)	**74.** (*b*)	**75.** (*b*)	**76.** (*a*)	**77.** (*a*)	**78.** (*b*)	**79.** (*b*)	**80.** (*a*)
81. (*b*)	**82.** (*c*)	**83.** (*a*)							

Learning Plus

1. (*c*)	**2.** (*a*)	**3.** (*b*)	**4.** (*c*)	**5.** (*b*)	**6.** (*a*)	**7.** (*c*)	**8.** (*a*)	**9.** (*a*)	**10.** (*c*)
11. (*d*)	**12.** (*d*)	**13.** (*c*)	**14.** (*c*)	**15.** (*a*)	**16.** (*a*)	**17.** (*a*)	**18.** (*d*)	**19.** (*b*)	**20.** (*d*)
21. (*b*)	**22.** (*d*)	**23.** (*a*)	**24.** (*c*)	**25.** (*b*)	**26.** (*a*)	**27.** (*a*) – p,r,t , (*b*) – p,r (*c*) – q,s, (*d*) – r, t			
28. (*a*)	**29.** (*a*)	**30.** (*c*)	**31.** (*a*)	**32.** (*d*)	**33.** (*a*)				

Advanced Level Multiconcept Questions

1. (*c*, *d*) **2.** (*a*, *b*, *c*) **3.** (*b*, *d*) **4.** (*a*, *d*) **5.** (*a*, *b*, *c*, *d*) **6.** (*a*, *b*) **7.** (*a*, *d*) **8.** (*c*, *d*) **9.** (*b*, *d*) **10.** (*b*, *d*)
11. (*a*, *c*) **12.** (*a*, *b*) **13.** (*a*, *b*) **14.** (*a*,*b*, *c*, *d*) **15.** (*b*, *c*) **16.** (*a*, *b*, *d*) **17.** (*c*) **18.** (*d*) **19.** (*b*) **20.** (*c*)
21. (*b*) **22.** (*c*) **23.** (*b*) **24.** (*a*) p, r, s (*b*) q (*c*) p,r,s (*d*) q,r **25.** (*a*) p, s (*b*) s (*c*) p, s (*d*) q, r
26. [0075 J] **27.** [40 gm] **28.** [0020] **29.** [76 cm] **30.** [0075] **31.** [0125] **32.** [2329] **33.** [0005] **34.** [0100]
35. [0000] **36.** [217°] **37.** [0126]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*b*)	**2.** (*a*)	**3.** (*b*)	**4.** (*a*)	**5.** (*b*)	**6.** (*a*)	**7.** (*b*)	**8.** (*a*)	**9.** (*c*)	**10.** (*d*)
11. (*b*)	**12.** (*a*)	**13.** (*a*)	**14.** (*b*)	**15.** (*d*)	**16.** (*c*)	**17.** (*d*)	**18.** (*b*)	**19.** (*b*)	**20.** (*d*)
21. (*a*)	**22.** (*c*)	**23.** (*d*)	**24.** (*c*)	**25.** (*c*)	**26.** (*b*)	**27.** (*c*)	**28.** (*b*)	**29.** (*a*)	**30.** (*a*)
31. (*c*)	**32.** (*c*)	**33.** (*a*)	**34.** (*b*)	**35.** (*d*)	**36.** (*d*)	**37.** (Bonus)	**38.** (*c*)	**39.** (*b*)	**40.** (*a*)
41. (*c*)	**42.** (40.93)	**43.** (*d*)	**44.** (*c*)	**45.** (*d*)	**46.** (05.00)	**47.** (*a*)	**48.** (600)	**49.** (Bonus)	**50.** (*b*)
51. (*a*)	**52.** (*a*)	**53.** (*a*)	**54.** (1819)	**55.** (*a*)	**56.** (46)	**57.** (8791)	**58.** (*a*)	**59.** [150]	**60.** (*b*)
61. (266.67)	**62.** (*a*)	**63.** (None of the option)		**64.** (19)	**65.** (32)	**66.** (113)			

JEE-ADVANCED
PREVIOUS YEAR'S

t.me/realprayaas2

1. (*d*)	**2.** (*d*)	**3.** (*d*)	**4.** (*a*, *b*, *c*, *d*)	**5.** (*a*)	**6.** [2]	**7.** (*d*)	**8.** (*d*)	**9.** (*d*)
10. (*a*)	**11.** (*d*)	**12.** (*d*)	**13.** (*b*, *c*, *d*)	**14.** [900]	**15.** (*c*)	**16.** (*a*, *b*)	**17.** (*a*, *c*, *d*)	**18.** (*c*)
19. (*d*)	**20.** [1.77 to 1.78]	**21.** (6m)	**22.** [2.05]	**23.** [9]				

13 SIMPLE HARMONIC MOTION

t.me/realprayasians

PERIODIC MOTION

Any motion, which repeats itself in equal intervals of time is called **periodic motion**. Often, the displacement of a particle in periodic motion can always be expressed in terms of sine and cosine functions. Since the term harmonic is applied to expressions containing these functions, periodic motion is also called **harmonic motion.**

If a particle in periodic motion moves back and forth (or to and fro) over the same path, then its motion is called **oscillatory or vibratory**. The examples of oscillatory or vibratory motion are:

- the motion of a pendulum
- the motion of a spring fixed at one end, which is stretched or compressed and then released
- the motion of a violin string
- the motion of atoms in molecules or in a solid lattice
- the motion of air molecules as a sound wave passes by

CHARACTERISTICS OF A HARMONIC MOTION

The basic quantities characterizing a periodic motion are the amplitude, period and frequency of vibrations.

Amplitude (A)

The amplitude of oscillations is the maximum displacement of a vibrating body from the position of equilibrium.

Time Period (T)

The time period of oscillations is defined as the time between two successive identical positions passed by the body in the same direction.

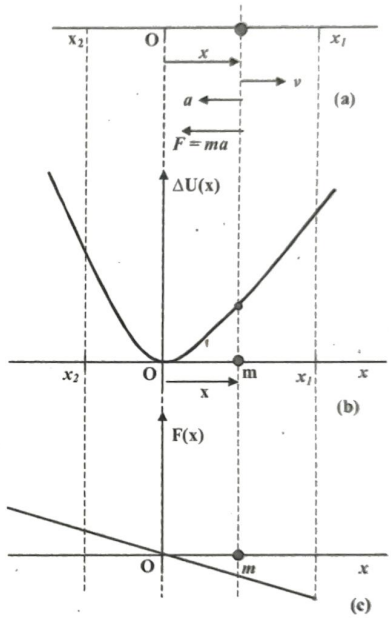
Frequency (f)

The frequency of oscillations is the number of cycles of vibrations of a body completed in one second. The frequency is related to the time period as

$$f = \frac{1}{T}$$

The SI unit of frequency is s^{-1} or Hz (Hertz)

When a body oscillates along a straight line within two fixed limits; its displacement x changes periodically in both magnitude and direction; its velocity v and acceleration a also vary periodically in magnitude and direction, shown in figure. Since F = ma, therefore, the force acting on the body also varies in magnitude and direction with time.



In terms of energy, we can say that a particle executing harmonic motion moves back and forth about a point at which the potential energy is minimum (equilibrium position). The force acting on the body at any position is given by

$$F = -\frac{dU}{dx}$$

SIMPLE HARMONIC MOTION

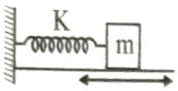
If the restoring force/ torque acting on the body in oscillatory motion is directly proportional to the displacement of body / particle and is always directed towards equilibrium position then the motion is called simple Harmonic Motion (SHM). It is the simplest (easy to analysis) form of oscillatory motion.

When a system is displaced from its equillibrium position, then a force acts on it in such a direction so as to restore the system back to its equilibrium position. This force is called restoring force. It is directed opposite to the displacement.

Types of SHM:

S.H.M. are of two types:

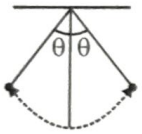
(a) Linear S.H.M. :



When a particle moves to and fro (or up and down) about a fixed point (Called equillibrium position) along a straight line then its motion is called linear simple harmonic motion.

Example : Motion of a block connected to spring

(b) Angular S.H.M. :



When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion

Example : Motion of a bob of a simple pendulum.

NECESSARY CONDITION TO EXECUTE S.H.M.

(a) Motion of particle should be periodic oscillatory

(b) Total mechanical energy of particle should be conserved. KE + PE = constant

(c) **In linear S.H.M.:** the restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle from mean position (x) and directed towards the equilibrium condition

$\therefore F \propto -x \quad \Rightarrow a \propto -x$

(–) ve sign shows that direction of force & acceleration is towards equilibrium position and y is displacement of particle from equilibrium position.

(d) **In angular S.H.M.:** the restoring torque (or acceleration) acting on the particle should always be proportional to the angular displacement of the particle & directed towards the equilibrium condition.

$\therefore \tau \propto -\theta \quad \Rightarrow \alpha \propto -\theta.$

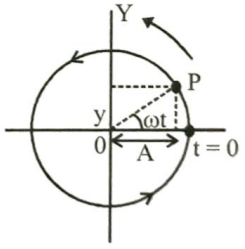
Comparison of Linear & Angular S.H.M. :

S.No.	Linear S.H.M.	Angular S.H.M.
(a)	$F \propto -x$ $F = -kx$ where k is restoring force constant	$\tau \propto -\theta$ $\tau = -C\theta$ Where C is restoring torque constant.
(b)	$a = -\frac{k}{m}x$	$\alpha = -\frac{C}{I}\theta$
(c)	$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$	$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$
(d)	$a = -\omega^2 x$	$\alpha = -\omega^2\theta$
(e)	$\omega^2 = \frac{k}{m}$ $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$	$\omega = \sqrt{\frac{C}{I}} = 2\pi n = \frac{2\pi}{T}$

Geometrical Interpretation of S.H.M.

If a particle is moving with uniform speed along the circumference of a circle, then the straight-line motion of the foot of the perpendicular drawn from the particle on the diameter of the circle is called 'SHM'

Description of S.H.M. Based on Circular Motion



(a) Draw a circle, having radius equal to amplitude (A) of S.H.M.

(b) Suppose particle is executing circular motion with angular velocity ω on the circumference of the circle.

(c) Shadow of particle performs S.H.M. on vertical or horizontal diameter of circle.

(d) By joining centre of circle to particle position, angle θ is determined from horizontal or vertical diameter. After time t, radius vector will turn ωt. Based on these unknown values can be calculated.

(e) As per the position of particle at t = 0 in above diagram, equation of SHM along x-axis can be given as $x = A(\cos\omega t + \phi)$ and along y-axis as $y = A(\sin\omega t + \phi)$

EQUATION OF SHM

The necessary and sufficient condition for SHM is

$F = -kx$

where

k = positive constant for a SHM = Force constant

x = displacement from mean position

$$\text{or } m\frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$$

where $\omega = \sqrt{\frac{k}{m}}$

Its solution is $x = A\sin(\omega t + \phi)$ using a little mathematics as shown below.

Derivation :

$F = -kx \qquad \therefore a = -\frac{k}{m}x$

put $\frac{k}{m} = \omega^2 \qquad \therefore a = -\omega^2 x$

$$\int_0^v v\,dv = -\int_A^x \omega^2 x\,dx$$

$$\Rightarrow \frac{v^2}{2} = -\frac{\omega^2}{2}[x^2 - A^2]$$

Thus $v = \pm\omega\sqrt{A^2 - x^2}$

$$\Rightarrow \frac{dx}{dt} = \pm\omega\sqrt{A^2 - x^2}$$

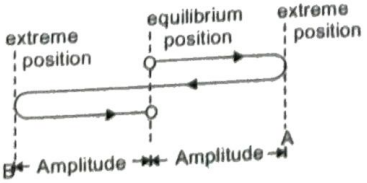
Further integrating $x = A\sin(\omega t + \phi)$ where at $t = 0$; $x = A\sin\phi$

COMPONENTS OF SHM

In the figure shown, path of the particle is on a straight line.

(a) Displacement : It is defined as the distance of the particle from the mean position at that instant. Displacement in SHM at time t is given by $x = A\sin(\omega t + \phi)$

(b) Amplitude (A) : It is the maximum value of displacement of the particle from its equilibrium position



Amplitude = 1/2 (distance between extreme points/ position)

It depends on energy of the system

(c) Angular Frequency (ω) : $\omega = \frac{2\pi}{T} = 2\pi f$ and its units is rad/sec

(d) Frequency (f) : Number of oscillations completed in unit time interval is called frequency of oscillations.

$f = \frac{1}{T} = \frac{\omega}{2\pi}$, its units is sec^{-1} or Hz.

(e) Time period (T) : Smallest time interval after which the oscillatory motion gets repeated is called time period.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

(f) Phase : The physical quantity which represents the state of motion of particle (eg. its position and direction of motion at any instant). Argument of sine function is called phase.

(g) Phase constant (ϕ) : Constant ϕ in equation of SHM is called phase constant or initial phase. It depends on initial position and direction of velocity.

(h) Velocity (v) :

(i) It is defined as the rate of change of the displacement of the particle with respect to time at the given instant.

(ii) Velocity in S.H.M. is given by

$$V = \frac{dx}{dt} = \frac{d}{dt}(A\sin\omega t) \text{ or } V = A\omega\cos\omega t$$

$$V = \pm A\omega\sqrt{1 - \sin^2\omega t} = V = \pm A\omega\sqrt{\left(1 - \frac{x^2}{A^2}\right)}$$

$$[\because x = A\sin\omega t] \quad \boxed{V = \pm\omega\sqrt{(A^2 - x^2)}}$$

Squaring both the sides

$V^2 = \omega^2(A^2 - x^2)$

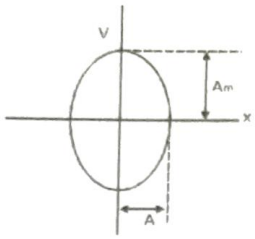
$$\text{or } \frac{V^2}{\omega^2} = A^2 - x^2 \text{ or } \frac{V^2}{\omega^2 A^2} = 1 - \frac{x^2}{A^2}$$

$$\text{or } \boxed{\frac{x^2}{A^2} + \frac{V^2}{A^2\omega^2} = 1}$$

This is equation of ellipse. So curve between displacement and velocity of particle executing S.H.M. is ellipse.

(iii) The graph between velocity and displacement is shown in figure.

If particle oscillates with unit angular frequency ($\omega = 1$) then curve between V and x would be circular.

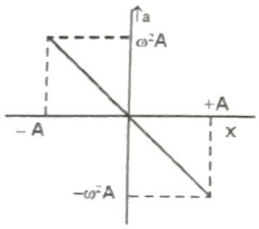


NOTE : (i) The direction of velocity of a particle in S.H.M. is either towards or away from the mean position

(ii) At mean position (x = 0), velocity is maximum (=Aω) and at extreme position (x = ± A), the velocity of particle executing S.H.M. is zero

(i) Acceleration:

(i) It is defined as the rate of change of the velocity of the particle with respect to time at given instant.



(ii) Acceleration in S.H.M. is given by

$$a = \frac{dV}{dt} = \frac{d}{dt}(A\omega\cos\omega t)$$

$$a = -\omega^2 A \sin\omega t$$

$$a = -\omega^2 x$$

(iii) The graph between acceleration and displacement.

NOTE :

(i) The acceleration of a particle executing S.H.M. is always directed towards the mean position.

(ii) The acceleration of the particle executing S.H.M. is maximum at extreme position (= ω^2A) and minimum at mean position (= zero)

TRAIN YOUR BRAIN

Example 1: For a particle performing SHM, equation of motion is given as $\frac{d^2x}{dt^2} + 4x = 0$. Find the time period.

Sol. $\frac{d^2x}{dt^2} = -4x$ $\omega^2 = 4$ $\omega = 2$

Time period : $T = \frac{2\pi}{\omega} = \pi$

Example 2: Amplitude of harmonic oscillator is a, when velocity of particle is half of maximum velocity, then position of particle will be

(a) $\frac{\sqrt{3}}{2}$a (b) $\frac{3}{\sqrt{2}}$a

(c) a/2 (d) a

Sol. $v^2 = \omega^2 (a^2 - x^2)$

But $v = \frac{V_{max}}{2} = \frac{\omega a}{2}$

$\therefore \frac{\omega^2 a^2}{4} = \omega (a^2 - x^2)$

$\Rightarrow a^2 = 4 (a^2 - x^2)$

$x^2 = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

$\Rightarrow x = \pm \frac{\sqrt{3}}{2}a$

GRAPHICAL REPRESENTATION OF DISPLACEMENT, VELOCITY AND ACCELERATION IN SHM

Displacement, $x = A \sin\omega t$

Velocity, $v = A\omega\cos\omega t = A\omega \sin(\omega t + \frac{\pi}{2})$

or $v = \omega\sqrt{A^2 - x^2}$

Acceleration, $a = -\omega^2 A \sin\omega t = \omega^2 A \sin(\omega t + \pi)$

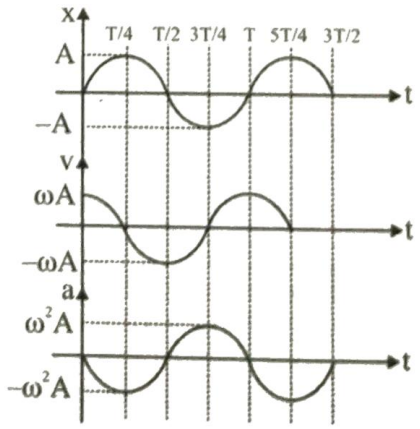
or $a = -\omega^2 x$

Note : $v = \omega\sqrt{A^2 - x^2}$

$a = -\omega^2 x$

These relations are true for any equation of x.

Time, t	0	T/4	T/2	3T/4	T
Displacement, x	0	A	0	–A	0
Velocity, v	Aω	0	–Aω	0	Aω
Acceleration, a	0	$-\omega^2$A	0	ω^2A	0



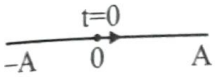
NOTE:

1. All the three quantities displacement, velocity and acceleration vary harmonically with time, having same period.
2. The velocity amplitude is ω times the displacement amplitude ($v_{max} = \omega A$).
3. The acceleration amplitude is ω^2 times the displacement amplitude ($a_{max} = \omega^2 A$).
4. In SHM, the velocity is ahead of displacement by a phase angle of $\frac{\pi}{2}$.
5. In SHM, the acceleration is ahead of velocity by a phase angle of $\frac{\pi}{2}$.

TRAIN YOUR BRAIN

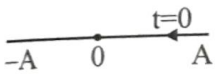
Example 3: A particle starts from mean position and moves towards positive extreme as shown. Find the equation of the SHM. Amplitude of SHM is A.

(a)

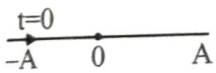


Also write the equation of SHM for the situation shown below.

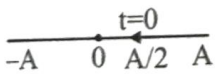
(b)



(c)



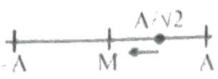
(d)



Ans. **(a)** $x = a\sin\omega t$
(b) $x = A\cos\omega t$;
(c) $x = -A\cos\omega t$;
(d) $x = A\sin(\omega t + \frac{5\pi}{6})$

Example 4: Write equation of S.H.M of ω angular frequency and A amplitude if the particle is situated at $\frac{A}{\sqrt{2}}$ at $t = 0$ and is going towards mean position.

Sol. At t=0 particle was at $\frac{A}{\sqrt{2}}$ and was going towards mean position as shown in the figure(a) below.



Putting $t = 0$ in equation $x = A\sin(\omega t + \phi)$

We get $\frac{A}{\sqrt{2}} = A\sin\phi$

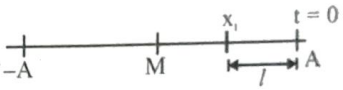
Thus, $\phi = \frac{\pi}{4}, \frac{3\pi}{4}$

but at $t = 0$; $v < 0$ so $\frac{\pi}{4}$ is rejected as $v = \omega A\cos(\omega t + \phi)$

$\therefore x = A\sin(\omega t + \frac{3\pi}{4})$

Example 5: Find distance traveled by a particle of time period T & amplitude A in time T/12 starting from rest.

Sol.



Eqn of SHM $x = A\cos\omega t$

$x_t = A\cos\left(\frac{2\pi}{T}\times\frac{T}{12}\right)$

$x_t = \frac{A\sqrt{3}}{2}$

$l = A - \frac{A\sqrt{3}}{2}$

$= \frac{A(2-\sqrt{3})}{2}$

Example 6: Two particles execute S.H.M. in same straight line, Their amplitude & freqeuency are equal. When their displacement is half of amplitude, they passes each other in opposite direction. Phase difference will be

(*a*) 30° (*b*) 150°
(*c*) 180° (*d*) 120°

Sol. Equation of S.H.M. $y = A\sin(\omega t + \phi)$ when displacement is half of amplitude

$\Rightarrow y = \frac{A}{2}$

$\frac{A}{2} = A\sin(\omega t + \phi)$

$\sin(\omega t + \phi) = 1/2$

$\omega t + \phi = 30^0$ or 150^0

$\Rightarrow \omega t + \phi_1 = 30°$ and $\omega t + \phi_2 = 150°$

$\Rightarrow \phi_2 - \phi_1 = 120°$

Hence correct answer is (*d*)

Example 7: Periodic time of a simple pendulum is 2 second and it can travel to and fro from equilibrium position upto maximum 5cm. At the starting of motion pendulum is at maximum displacement in right side from equilibrium position. Then, the equation of SHM can be

(a) $y = 5 \sin \pi t$ (b) $y = 5 \cos \pi t$

(c) $y = 5 \cos 2\pi t$ (d) $y = 5 \sin 2\pi t$

Sol. Displacement expression for S.H.M. be $y = a \sin(\omega t + \phi)$

Time period of simple pendulum is

$T = 2\pi/\omega = 2 \text{sec}$

$\therefore \omega = \pi$ and $a = 5\text{cm}$

$\therefore y = 5 \sin(\pi t + \phi)$ cm

Now at $t = 0$, displacement $y = 5$ cm

$\therefore 5 = 5 \sin(\pi \times 0 + \phi)$

$\Rightarrow \sin\phi = 1 \Rightarrow \phi = \pi/2$

$\Rightarrow y = 5 \sin(\pi t + \pi/2) \Rightarrow y = 5 \cos \pi t$

Example 8: If equation of displacement of a particle is $y = A \sin \omega t + B \cos \omega t$ then motion of particle is

(a) Simple harmonic motion (b) Linear motion

(c) Uniform circular motion (d) Uniform elliptical motion

Sol. $y = A \sin \omega t + B \cos \omega t$

Differentiate w.r.t. to t,

$$\frac{dy}{dt} = \omega A \cos \omega t - \omega B \sin \omega t$$

Again differentiate w.r.t. to t,

$$\frac{d^2y}{dt^2} = -\omega^2 A \sin \omega t - \omega^2 B \cos \omega t$$

$$\frac{d^2y}{dt^2} = -\omega^2 [A \sin \omega t + B \cos \omega t]$$

$$\Rightarrow \frac{d^2y}{dt^2} + \omega^2 y = 0$$

This is equation for S.H.M

Example 9: The maximum velocity of a harmonic oscillator is α and its maximum acceleration is β. Its time period will be

(a) $\frac{2\pi\alpha}{\beta}$ (b) $\frac{2\pi\beta}{\alpha}$

(c) $2\pi\alpha\beta$ (d) $\frac{\pi\beta}{\alpha}$

Sol. $v_{max} = \omega A = \alpha$ ...(1)

$a_{max} = \omega^2 A = \beta$ ...(2)

Dividing equation (2) by equation (1)

$$\frac{\beta}{\alpha} = \omega \quad \therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi\alpha}{\beta}$$

Example 10: The position of a particle moving along x - axis is given by

$x = 0.08 \sin(12t + 0.3)$ m

where t is in second

(a) What is the amplitude and the period of the motion?

(b) Determine the position, velocity, and acceleration at $t = 0.6$ s.

Sol. (a) On comparing the given equation with the standard equation of SHM, i.e. $y = A \sin(\omega t + \phi)$, we see that the amplitude is $A = 0.08$ m and the angular frequency $\omega = 12$ rad/s. Thus, the period is $T = 2\pi/\omega = 0.524$ s.

(b) The velocity and acceleration at any time are given by

$$v = \frac{dx}{dt} = 0.96 \cos(12t + 0.3) \text{ m/s}$$

$$a = \frac{dv}{dt} = -11.5 \sin(12t + 0.3) \text{ m/s}^2$$

At $t = 0.6$ s, the phase of motion is

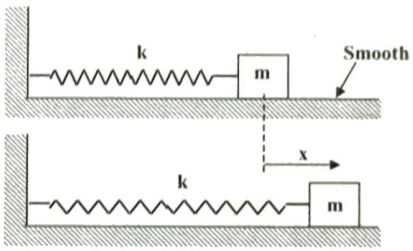
$(12 \times 0.6 + 0.3) = 7.5$ rad.

When this is used in the above expressions, we find $x = 0.075$ m, $v = 0.333$ m/s, and $a = -10.8$ m/s^2.

SPRING MASS SYSTEM

Let us find out the time period of a spring-mass system oscillating on a smooth horizontal surface as shown in the figure.

At the equilibrium position the spring is relaxed. When the block is displaced through x towards right, it experiences a net restoring force $F = -kx$ towards left



The negative sign shows that the restoring force is always opposite to the displacement. That is, when x is positive, F is negative, the force is directed to the left. When x is negative, F is positive, the force is directed to the right. Thus, the force always tends to restore the block to its equilibrium position $x = 0$.

so $F = -kx$

Applying Newton's Second Law,

$$F = m\frac{d^2x}{dt^2} = -kx$$

or $$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Comparing the above equation with standard equation, we get

$$\omega^2 = \frac{k}{m} \qquad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

Note: that the time period is independent of the amplitude. For a given string constant, the period increases with the mass of the block – a more massive block oscillates more slowly. For a given block, the period decreases as k increases. A stiffer spring produces quicker oscillations.

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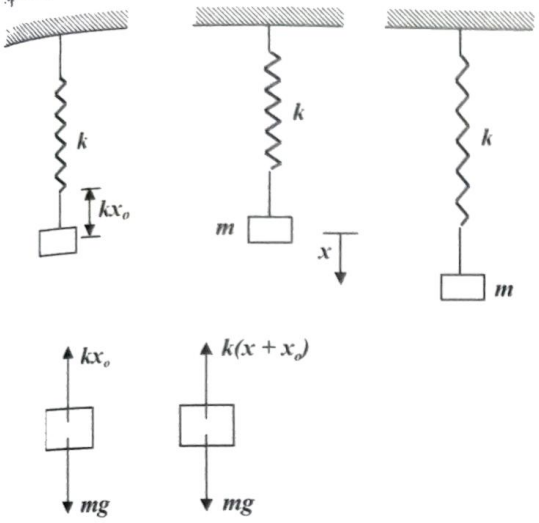
Example 11: Find the period of oscillation of a vertical spring-mass system.



Sol. Let x_o be the deformation in the spring in equilibrium.

Then $kx_o = mg$

When the block is further displaced by x, the net restoring force is given by

$F = -[k(x + x_o) - mg]$

or $F = -kx$

(because $kx_o = mg$)

Using second law of motion,

$$m\frac{d^2x}{dt^2} = -kx$$

or $$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Thus, $$\omega^2 = \frac{k}{m} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

Note that gravity does not influence the time period of the spring-mass system, it merely changes the equilibrium position.

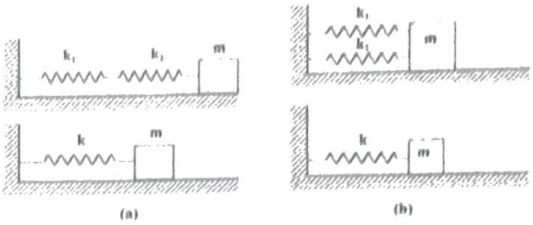
SERIES AND PARALLEL COMBINATIONS OF SPRINGS

When two springs are joined in series, the equivalent stiffness of the combination may be obtained as

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

When two springs are joined in parallel, the equivalent stiffness of the combination is given by

$k = k_1 + k_2$



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Example 12: Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k. When the masses are in equilibrium m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2.

Sol. As m_1 is removed, the mass m_2 will oscillate and so

$$T = 2\pi\sqrt{\frac{m_2}{k}}, \qquad \text{i.e., } \omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m_2}}$$

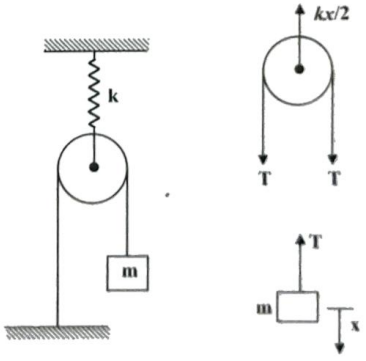
Furthermore, the stretch produced by m_1g will set an amplitude,

i.e. $m_1g = kA$

i.e. $A = (m_1g/k)$

Example 13: For the arrangement shown in the figure, find the period of oscillation.

Sol. Obviously, when the block is displaced down by x, the spring will stretch by $\frac{x}{2}$.



From the free body diagram of the pulley,

$$\frac{kx}{2} = 2T \text{ or } \qquad T = \frac{kx}{4}$$

The net restoring force on the block is T.

Using the Second Law of motion, we get

$$m\frac{d^2x}{dt^2} = -T = -\frac{kx}{4} \quad \text{or} \quad \frac{d^2x}{dt^2} + \left(\frac{k}{4m}\right)x = 0$$

Thus, the period of SHM is given by

$$T = 2\pi\sqrt{\frac{4m}{k}}$$

Note that we have not taken gravity into account as it does not affect the time period.

Example 14: A 2 kg block is attached to a spring for which k = 200 N/m. It is held at an extension of 5 cm and then released at t = 0. Find

(a) the displacement as a function of time

(b) the velocity when x = +A/2;

(c) the acceleration when x = +A/2

Sol. (a) We need to find a, ω, and ϕ in equation. The amplitude is the maximum extension; that is,

A = 0.05 m

We know the angular frequency of the spring-mass system is given by

$$\omega = \sqrt{\frac{k}{m}} = 10 \quad \text{rad/s}$$

To find ϕ we note that at t = 0 we are given x = +A and v = 0.

Thus, from the equation of displacement and velocity, we get

$x = A \sin(\omega t + \phi)$ $\Rightarrow A = A \sin(0 + \phi)$

$v = \omega A(\omega t + \phi)$ $\Rightarrow 0 = 10\, A \cos(0 + \phi)$

Since $\sin\phi = 1$ and $\cos\phi = 0$, it follows that $\phi = \pi/2$ rad. Thus,

$$x = 0.05 \sin\left(10t + \frac{\pi}{2}\right) \text{ m} \quad ....(i)$$

(b) In order to find the velocity we have to find the time t, when x = A/2. Equation (i) yields

$$\frac{1}{2} = \sin\left(10t + \frac{\pi}{2}\right),$$

from which we infer that $(10t + \frac{\pi}{2}) = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

The velocity is given by

$$v = \frac{dx}{dt} = 0.5 \cos\left(10t + \frac{\pi}{2}\right)$$

$$= 0.5 \cos\frac{\pi}{6} \text{ or } 0.5 \cos\frac{5\pi}{6}$$

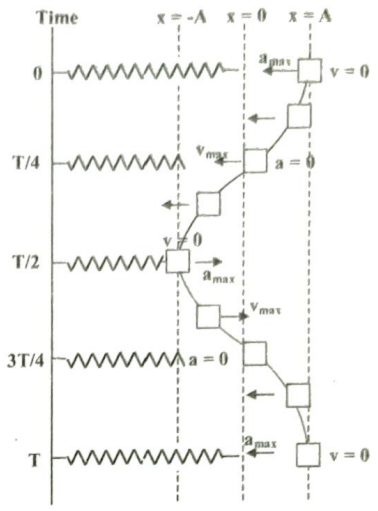
= +0.43 m/s or − 0.43 m/s

At a given position, there are two velocities of equal magnitude but opposite directions.

(c) The acceleration at x = A/2 may be found from the equation,

$$a = -\frac{k}{m}x = -\omega^2 x = -(10 \text{ rad/s})^2(0.05/2 \text{ m})$$

$$= -2.5 \text{ m/s}^2$$



Example 15: A particle of mass m is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass k is pushed slightly against the spring A and released, find the time period of oscillations.

Sol. When the particle of mass m at O is pushed by y in the direction of A, spring A will be compressed by y while B and C will be stretched by y′ = y cos 45°, so the total restoring force on the mass m along OA is given by

$$F = -[F_A + F_{-B}\cos 45° + F_C \cos 45°]$$

i.e. $F = -[ky + 2(ky')\cos 45°]$

or $F = -[ky + 2k(y \cos 45°)\cos 45°]$

i.e., $F = -k'y$ where $k' = 2k$

so $$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

Example 16: The angular displacement of simple pendulum is given by $\theta = 0.1\,\pi \sin\left(2\pi t + \frac{\pi}{6}\right)$ rad

The mass of the bob is 0.4 kg. Calculate

(a) the length of the simple pendulum; and

(b) the velocity of the bob at t = 0.25 s

Sol. (a) We are given $\theta_0 = 0.1\,\pi$ rad, $\phi = \pi/6$ rad, and $\omega = 2\pi$ rad/s. Since $\omega^2 = (g/L)$, we have

$$L = \frac{g}{\omega^2} = \frac{9.8 \text{m/s}^2}{(2 \times 3.14 \text{rad/s})^2} = 0.25 \text{ m}$$

(b) Since $s = L\theta$, the velocity of the bob, $v = \frac{ds}{dt}$, is

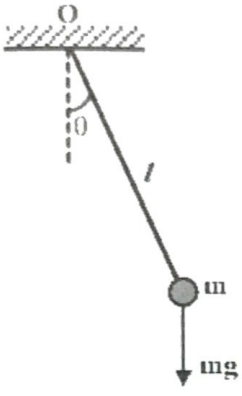
$$v = L\frac{d\theta}{dt} = (0.25\text{ m})(0.1\ \pi)(2\pi)\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$$

$$= -0.123\text{ m/s}$$

PENDULUMS

(i) Simple Pendulum

Ideally, a simple pendulum consists of a point mass suspended at the end of a massless string as shown in the figure.



At the equilibrium position, net torque about the point 'O' is zero, at $\theta = 0$.

When the pendulum is displaced through an angle θ, the restoring torque about the point O is given by

$$\tau_o = -\,mgl\sin\theta$$

For small angles $\sin\theta \approx \theta$, therefore,

$$\tau_o = -mgl\theta$$

Using Newton's Second Law,

$$\tau_o = I_o\alpha = ml^2\frac{d^2\theta}{dt^2}$$

Thus, $$\frac{d^2\theta}{dt^2}+\left(\frac{g}{l}\right)\theta = 0$$

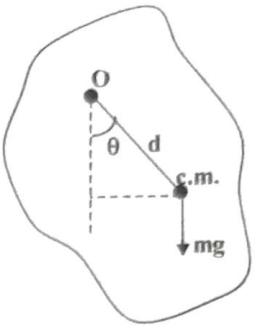
Comparing the above equation with the standard differential equation, we get

$$\omega^2 = \frac{g}{l} \quad \therefore T = 2\pi\sqrt{\frac{l}{g}}$$

The pendulum having a time period equal to two seconds is called a second's pendulum.

(ii) Physical Pendulum

In figure, an extended body is pivoted freely about an axis that does not pass through its center of mass. Such an arrangement forms a physical pendulum that executes simple harmonic motion for small angular displacements. If d is the distance from the pivot to the center of mass, the restoring torque is $-mgd\sin\theta$.



Using Newton's Second law, $\tau = I\alpha$,

we get $-mgd\sin\theta = I\frac{d^2\theta}{dt^2}$

Where I is the moment of inertia about the given axis. If we use the small-angle approximation, $\sin\theta \approx \theta$, then

$$\frac{d^2\theta}{dt^2}+\frac{mgd}{I}\theta = 0$$

Thus, $$\omega = \sqrt{\frac{mgd}{I}}$$

and $$T = 2\pi\sqrt{\frac{I}{mgd}}$$

If the location of the center of mass and d are known, then a measurement of the period allows us to determine the moment of inertia of the body.

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Example 17: A rod of mass M and length L is pivoted about its end O as shown in the figure. Find the period of SHM.

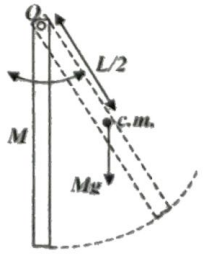
Sol. Restoring torque is

$$\tau_o = -\,Mg\frac{L}{2}\sin\theta$$

Using Newton's Second Law and the small angle approximation, we get

$$I_o\frac{d^2\theta}{dt^2}+Mg\frac{L}{2}\theta = 0$$

Using parallel-axes theorem



$$I_o = I_{cm} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$$

$$\therefore \frac{d^2\theta}{dt^2} + \left(\frac{3}{2}\frac{g}{L}\right)\theta = 0$$

Thus, $$T = 2\pi\sqrt{\frac{2L}{3g}}$$

Example 18: A simple pendulum of length L and mass m has as spring of force constant k connected to it at a distance h below its point of suspension. Find the frequency of vibrations of the system for small values of amplitude.

Sol. As shown in figure, if the pendulum is given a small angular displacement θ, the spring will also stretch by y (= h tan θ). So there restoring torque about O will be due to both force of gravity and elastic force of the spring.

i.e. $\tau = -[mg(L \sin\theta) - k(h \tan\theta)h]$

Now for small θ,

$\tan\theta = \sin\theta = \theta$

so $\tau = -(mgL - kh^2)\theta$

i.e. restoring torque is linear, so motion is angular SHM.

Now as $\tau = I\alpha = mL^2 \frac{d^2\theta}{dt^2}$ (as $I = mL^2$)

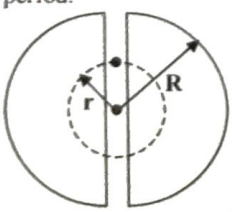
So $$mL^2\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

with $$\omega^2 = \left[\frac{mgL + kh^2}{mL^2}\right]$$

This is the standard equation of angular SHM with frequency $f = (\omega/2\pi)$; so here

$$f = \frac{1}{2\pi}\sqrt{\frac{mgL + kh^2}{mL^2}}$$

Example 19: Suppose a tunnel could be dug through the earth from one side to the other along a diameter, as shown in figure. Show that the motion of the particle is simple harmonic and determine its time period.



Sol. The gravitational force on the particle at a distance r from the centre of the earth arises entirely from that portion of matter of the earth in shells internal to the position of the particle. The external shells exert no force on the particle.

The value of gravity at a distance r is given by

$$g = g_o\frac{r}{R}$$

where $g_o = \frac{GM}{R^2}$ is the gravity on the surface of earth.

The net force on the particle is

$$F = -mg = -mg_o\frac{r}{R}$$

Using Newton's Law,

$$F = m\frac{d^2r}{dt^2} = \frac{-mg_o}{R}r$$

or $$\frac{d^2r}{dt^2} + \left(\frac{g_o}{R}\right)r = 0$$

This is the differential equation of SHM.

The period of oscillation is

$$T = 2\pi\sqrt{\frac{R}{g_o}}$$

Taking $R = 6400$ km $= 6.4 \times 10^6$ m and $g_o = 9.81$ m/s^2, we get

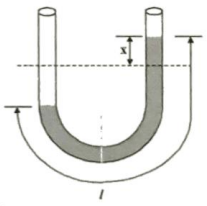
$T = 84.2$ min.

Example 20: An ideal liquid of density ρ occupies length l of an U – tube of uniform cross-section as shown in the figure given below. When the liquid surface is displaced through x the liquid column executes SHM. Find the period of oscillation.

Sol. When the liquid column is displaced through x on the right side, the weight of the unbalanced liquid provides the restoring force.

$F = -[\rho A(2x)]g$

where A is the area of cross-section



Applying Newton's second law

$$F = m\frac{d^2x}{dt^2} = (\rho\ Al)\frac{d^2x}{dt^2}$$

$$\therefore (\rho Al)\frac{d^2x}{dt^2} + \rho\ A(2x)g = 0$$

or $$\frac{d^2x}{dt^2} + \left(\frac{2g}{l}\right)x = 0$$

Time period is $$T = 2\pi\sqrt{\frac{l}{2g}}$$

ENERGY CONSERVATION IN SHM

In a spring-mass system, the instantaneous potential energy and kinetic energy are expressed as

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$$

and $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$

Since $\omega^2 = \frac{k}{m}$, therefore,

$$K = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

The total mechanical energy is given by

$E = K + U$

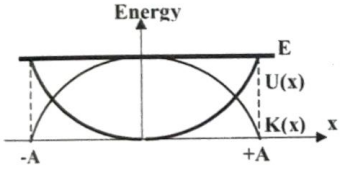
$$E = \frac{1}{2}kA^2\left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right]$$

or $E = \frac{1}{2}kA^2 = \text{constant}$

Thus, the total energy of SHM is constant and proportional to the square of the amplitude.

The variation of K and U as function of x is shown in figure When $x = \pm A$, the kinetic energy is zero and the total energy is equal to the maximum potential energy.

$$E = U_{max} = \frac{1}{2}kA^2$$



There are extreme points or turning points of the SHM. At $x = 0$, $U = 0$ and the energy is purely kinetic,

i.e. $E = K_{max} = \frac{1}{2}m(\omega A)^2$

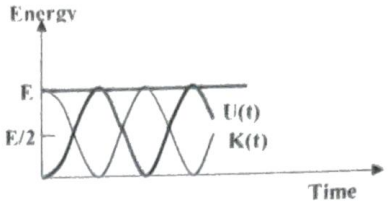
The variation of K and U as function of t is shown in figure. Note that the frequency of variation of the kinetic and potential energy is twice that of the frequency of oscillation.

The instantaneous total mechanical energy of the spring-mass system may be written as

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Differentiating it w.r.t. time, we get

$$\frac{dE}{dt} = \frac{1}{2}m\frac{d}{dt}(v^2) + \frac{1}{2}k\frac{d}{dx}(x^2)$$



or $0 = mv\frac{dv}{dt} + kx\frac{dx}{dt}$

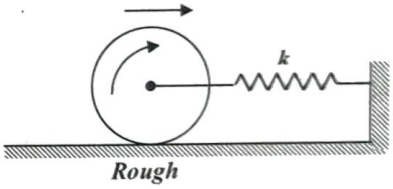
Since $v = \frac{dx}{dt}$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$, therefore,

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

This is the differential equation of SHM.

TRAIN YOUR BRAIN

Example 21: A solid cylinder of mass M and radius R is attached to a spring of stiffness k as shown in the figure. The cylinder can roll without slipping on a rough horizontal surface. Show that the centre of mass of the cylinder executes SHM and determine its time period.



Sol. Consider the situation when the spring is extended by x.

The total energy of the cylinder plus spring system is

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2 = \text{constant}$$

Here, $I = \frac{1}{2}MR^2$ and $\omega = \frac{v}{R}$

$$\therefore \frac{3}{4}Mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Differentiating w.r.t. time, we get

$$\frac{3}{4}Mv^2\frac{dv}{dt} + \frac{1}{2}kx^2\frac{dx}{dt} = 0$$

Noting that $v = \frac{dx}{dt}$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$

We get $\frac{d^2x}{dt^2} + \frac{2k}{3M}x = 0$

$$\therefore T = 2\pi\sqrt{\frac{3M}{2k}}$$

TORSIONAL PENDULUM

In torsional pendulum, an extended object is suspended at the centre by a light torsion wire. A torsion wire is essentially inextensible, but is free to twist about its axis. When the lower end of the wire is rotated by a slight amount the wire applies a restoring torque causing the body to oscillate rotationally



when released. The restoring torque produced is given by

$\tau = -C\theta$ where, C = Torsional constant

or, $I\alpha = -C\theta$

where, I = Moment of inertia about the vertical axis.

or, $\alpha = -\frac{C}{I}\theta$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{I}{C}}$

TRAIN YOUR BRAIN

Example 22: A uniform disc of radius 5.0 cm and mass 200g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20s, find the torsional constant of the wire.

Sol. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2}$$

$$= \frac{(0.200\,kg)\,(5.0\times10^{-2}\,m)^2}{2}$$

$$= 2.5\times10^{-4}\,kg\text{-}m^2$$

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}} \qquad \text{or, } C = \frac{4\pi^2 I}{T^2}$$

$$= \frac{4\pi^2(2.5\times10^{-4}\,kg-m^2)}{(0.20s)^2} = 0.25\,\frac{kg-m^2}{s^2}$$

Example 23: One solid sphere, having mass 1kg & diameter 0.3m is suspended from a wire.

If the twisting couple per unit twist for the wire is 6×10^{-3} N-m/radian, then the time period of small oscillations will be-

(*a*) 0.7 sec (*b*) 7.7 sec

(*c*) 77 sec (*d*) 777 sec

Sol. $T = 2\pi\sqrt{\frac{I}{C}}$ ...(1)

$I = \frac{2}{5}MR^2$ ...(2)

By equation (1) & (2)

$T = 2\pi\sqrt{\frac{2MR^2}{5C}}$ given, M = 1kg

R = 0.15 m

$C = 6\times10^{-3}$ N-m/rad T = ?

$\Rightarrow T = 6.28\sqrt{\frac{0.4\times1\times(0.15)^2}{6\times10^{-3}}}$

$T = 6.28\sqrt{\frac{9\times10^{-3}}{6\times10^{-3}}} = 6.28\times\sqrt{1.5} = 7.7$ sec

SUPERPOSITION OF TWO SHM'S

(a) In same direction and of same frequency

$x_1 = A_1\sin\omega t$

$x_2 = A_2\sin(\omega t+\theta)$, then resultant displacement

$x = x_1 + x_2 = A_1\sin\omega t + A_2\sin(\omega t+\theta) = A\sin(\omega t+\phi)$

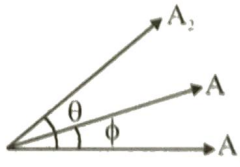
where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$ &

$$\phi = \tan^{-1}\left[\frac{A_2\sin\theta}{A_1 + A_2\cos\theta}\right]$$

If $\theta = 0$, both SHM's are in phase and $A = A_1 + A_2$

If $\theta = \pi$, both SHM's are out of phase and $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHM's of this case can also be found by vector diagram also → **PHASOR DIAGRAM**



1. Amplitude of SHM is taken as length (magnitude) of vector
2. Phase difference between the vectors is taken as the angle between these vectors. The magnitude of resultant of vector's give resultant amplitude of SHM and angle of resultant vector gives phase constant of resultant SHM.

(b) In same direction but are of different frequencies. (special case, if $A_1 = A_2$)

$x_1 = A_1\sin\omega_1 t$ and $x_2 = A_2\sin\omega_2 t$

then resultant displacement $x = x_1 + x_2 = A_1\sin\omega_1 t + A_2\sin\omega_2 t$

(c) In two perpendicular directions.

$x = A\sin\omega t$ and $y = B\sin(\omega t+\phi)$

Case (i) : If $\phi = 0$ or π then $y = \pm(B/A)x$. So path will be straight line & resultant displacement will be

$R = (x^2 + y^2)^{1/2} = (A^2 + B^2)^{1/2}\sin\omega t$

Case (ii) : If $\theta = \frac{\pi}{2}$ then,

$x = A \sin \omega t$

$y = B \sin (\omega t + \pi/2) = B \cos \omega t$

so, resultant will be $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, i.e. equation of an ellipse and if A = B, then superposition will be an equation of circle.

TRAIN YOUR BRAIN

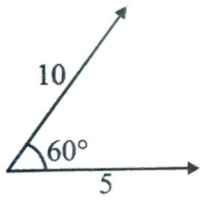
Example 24: $x_1 = 5 \sin (\omega t + 30°)$

$x_2 = 10 \cos (\omega t)$

Find amplitude of resultant SHM.

Sol. $x_1 = 5 \sin (\omega t + 30°)$

$x_2 = 10 \sin \left(\omega t + \frac{\pi}{2}\right)$



Phasor diagram

$$A = \sqrt{5^2 + 10^2 + 2 \times 5 \times 10 \cos 60°}$$

$$= \sqrt{25 + 100 + 50} = \sqrt{175} = 5\sqrt{7}$$

Example 25: $x_1 = 3 \sin \omega t$

$x_2 = 4 \cos \omega t$

Find (i) amplitude of resultant SHM and (ii) equation of the resultant SHM.

Sol. First write all SHM's in terms of sine functions with positive amplitude.

$\therefore x_1 = 3 \sin \omega t$

$x_2 = 4 \sin (\omega t + \pi/2)$

$$A = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \cos \frac{\pi}{2}}$$

$$= \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\tan \phi = \frac{4 \sin \frac{\pi}{2}}{3 + 4 \cos \frac{\pi}{2}} \qquad \phi = 53°$$

Equation $x = 5 \sin (\omega t + 53°)$

Example 26: Equation of two S.H.M.

$x_1 = 5 \sin (2\pi t + \pi/4)$

$x_2 = 5\sqrt{2} (\sin 2\pi t + \cos 2\pi t)$

ratio of amplitude & phase difference will be

(a) 2 : 1, 0 (b) 1 : 2, 0

(c) 1 : 2, $\pi/2$ (d) 2 : 1, $\pi/2$

Sol. $x_1 = 5 \sin (2\pi t + \pi/4)$

$x_2 = 5\sqrt{2} (\sin 2\pi t + \cos 2\pi t)$

$x_2 = 10 (\sin 2\pi t \cos \pi/4 + \cos 2\pi t \sin \pi/4)$

$x_2 = 10 \sin (2\pi t + \pi/4)$

Ratio of amplitude = 5 : 10 = 1 : 2

Phase difference $= (2\pi t + \pi/4) - (2\pi t + \pi/4) = 0$

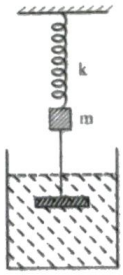
DAMPED SIMPLE HARMONIC MOTION

In the previous section we have considered several examples of oscillatory systems. In each case the amplitude of the oscillation does not depend on time and the system, once set to oscillate, will continue to do so forever. Physical systems are never as ideal as that. Even in the most controlled case there will be dissipative elements.

In each of of these cases the amplitudes of the oscillation will gradually decrease and the motion will be 'damped'.

Most of the time we are interested in the motion of an object through air or other viscous fluids. The motion of the object is then subject to a resistive force called aerodynamic or viscous drag. Experimentally the resistive force is found to be proportional to the velocity of the body for low relative speed with respect to the medium.

A laboratory model for a damped spring-mass system is shown in figure. Where a vertical spring has at its end a mass m which is connected to massless piston which moves through a liquid.



For low speeds it is reasonable to write the frictional force as

$F_f = -bv$ (b in kg/s)

where b is a positive constant known as damping constant. The equation of motion of a harmonic oscillator becomes

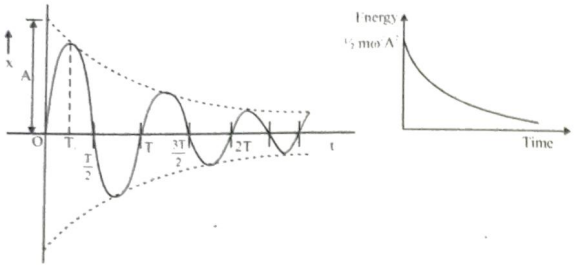
$$m \frac{d^2x}{dt^2} = -kx - b \frac{dx}{dt}$$

On solving

$x_{(t)} = Ae^{-bt/2m}\cos(\omega' t + \phi)$ where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ and

amplitude of oscillation $= Ae^{bt/2m}$

and Total energy $E = \frac{1}{2}KA^2e^{2-bt/m}$



FORCED OSCILLATIONS AND RESONANCE

The oscillations in a system can be indefinitely maintained by supplying energy continuously. In a mechanical system this can be done by subjecting it to an external force which itself has harmonic time dependence.

An intersting phenomenon occurs if the frequency of the external source is equal or nearly equal to the natural frequency of the system. The amplitude of oscillations is found to increase many folds in such cases. This is called **resonance.**

Forced Damped Oscillations

In the presence of resistive forces proportional to velocity and the applied periodic force.

$F = F_0 \sin \omega_d t$ equation becomes

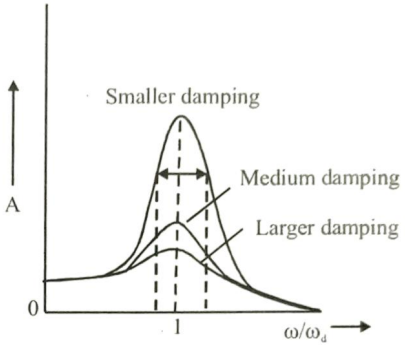
$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = F_0 \sin \omega_d t$$

Thus, $x = A \sin(\omega_d t + \phi)$

where $A = \dfrac{F_0}{m\sqrt{(\omega^2 - \omega_d^2)^2 + \left(\dfrac{b\omega_d}{m}\right)^2}}$

i.e. a function of natural frequency ω and driving frequency ω_d.

The phenomena of increase in amplitude when the driving force is close to the natural frequency of the oscillator is called **resonance**.



Topicwise Questions

KINEMATICS, MECHANICS, ENERGY IN SHM

1. The amplitude and the periodic time of a S.H.M. are 5cm and 6sec respectively. At a distance of 2.5cm away from the mean position, the phase will be
(a) $5\pi/12$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/6$

2. The equation of S.H.M. is $y = a \sin(2\pi nt + \alpha)$, then its phase at time t is
(a) $2\pi nt$ (b) α (c) $2\pi nt + \alpha$ (d) $2\pi t$

3. Which of the following equation does not represent a simple harmonic motion
(a) $y = a \sin \omega t$ (b) $y = a \cos \omega t$
(c) $y = a \sin \omega t + b \cos \omega t$ (d) $y = a \tan \omega t$

4. A particle executing simple harmonic motion along y-axis has its motion described by the equation $y = A \sin(\omega t) + B$. The amplitude of the simple harmonic motion is
(a) A (b) B (c) A + B (d) $\sqrt{A+B}$

5. A simple pendulum performs simple harmonic motion about X = 0 with an amplitude A and time period T. The speed of the pendulum at $X = \frac{A}{2}$ will be
(a) $\frac{\pi A\sqrt{3}}{T}$ (b) $\frac{\pi A}{T}$ (c) $\frac{\pi A\sqrt{3}}{2T}$ (d) $\frac{3\pi^2 A}{T}$

6. A simple harmonic oscillator has a period of 0.01 sec and an amplitude of 0.2 m. The magnitude of the velocity in m sec^{-1} at the centre of oscillation is
(a) 20 π (b) 100 (c) 40 π (d) 100 π

7. A particle executes S.H.M. with a period of 6 second and amplitude of 3 cm. Its maximum speed in cm/sec is
(a) $\pi/2$ (b) π
(c) 2π (d) 3π

8. A particle has simple harmonic motion. The equation of its motion is $x = 5\sin\left(4t - \frac{\pi}{6}\right)$, where x is its displacement. If the displacement of the particle is 3 units, then it velocity is
(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$
(c) 20 (d) 16

9. A particle is performing simple harmonic motion with amplitude A and angular velocity ω. The ratio of maximum velocity to maximum acceleration is
(a) ω (b) $1/\omega$
(c) ω^2 (d) $A\omega$

10. The amplitude of a particle executing SHM is 4 cm. At the mean position the speed of the particle is 16 cm/sec. The distance of the particle from the mean position at which the speed of the particle becomes $8\sqrt{3}$ cm/s will be
(a) $2\sqrt{3}$ cm (b) $\sqrt{3}$ cm
(c) 1 cm (d) 2 cm

11. The instantaneous displacement of a simple pendulum oscillator is given by $x = A\cos\left(\omega t + \frac{\pi}{4}\right)$. Its speed will be maximum at time
(a) $\frac{\pi}{4\omega}$ (b) $\frac{\pi}{2\omega}$ (c) $\frac{\pi}{\omega}$ (d) $\frac{2\pi}{\omega}$

12. A body executing simple harmonic motion has a maximum acceleration equal to 24 metres/sec^2 and maximum velocity equal to 16 metre/sec. The amplitude of the simple harmonic motion is
(a) $\frac{32}{3}$ metres (b) $\frac{3}{32}$ metres
(c) $\frac{1024}{9}$ metres (d) $\frac{64}{9}$ metres

13. A particle moving along the x-axis executes simple harmonic motion, then the force acting on it is given by Where A and K are positive constants
(a) – A Kx (b) A cos (Kx)
(c) A exp (– Kx) (d) A Kx

14. A particle executes simple harmonic motion along a straight line with an amplitude A. The potential energy is maximum when the displacement is
(a) ± A (b) Zero (c) $\pm\frac{A}{2}$ (d) $\pm\frac{A}{\sqrt{2}}$

15. For a particle executing simple harmonic motion, the kinetic energy K is given by $K = K_0 \cos^2 \omega t$. The maximum value of potential energy is
(a) K_0 (b) Zero
(c) $\frac{K_0}{2}$ (d) Not obtainable

16. The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement X from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is
(*a*) $X^2\omega^2 / (a^2 - X^2\omega^2)$ (*b*) $X^2 / (a^2 - X^2)$
(*c*) $(a^2 - X^2\omega^2) / X^2\omega^2$ (*d*) $(a^2 - X^2) / X^2$

17. Consider the following statements. The total energy of a particle executing simple harmonic motion depends on its
(*A*) Amplitude
(*B*) Period
(*C*) Displacement from mean position
(*a*) (*A*) and (*B*) are correct
(*b*) (*B*) and (*C*) are correct
(*c*) (*A*) and (*C*) are correct
(*d*) (*A*), (*B*) and (*C*) are correct

18. A particle of mass m is hanging vertically by an ideal spring of force constant K. If the mass is made to oscillate vertically, total energy of system is
(*a*) Maximum at extreme position
(*b*) Maximum at mean position
(*c*) Minimum at mean position
(*d*) Same at all position

19. A tunnel has been dug through the centre of the earth and a ball is released in it. It will reach the other end of the tunnel after
(*a*) 84.6 minutes
(*b*) 42.3 minutes
(*c*) 1 day
(*d*) Will not reach the other end

20. The kinetic energy of a particle executing S.H.M. is 16 J when it is in its mean position. If the amplitude of oscillations is 25 cm and the mass of the particle is 5.12 kg, the time period of its oscillation is

(*a*) $\frac{\pi}{5}$ sec (*b*) 2π sec (*c*) 20π sec (*d*) 5π sec

21. A particle in SHM is described by the displacement equation $x(t) = A\cos(\omega t + \theta)$. If the initial $(t = 0)$ position of the particle is 1 cm and its initial velocity is π cm/s, what is its amplitude? The angular frequency of the particle is πs^{-1}

(*a*) 1 cm (*b*) $\sqrt{2}$ cm (*c*) 2 cm (*d*) 2.5 cm

22. The displacement x (in metre) of a particle in, simple harmonic motion is related to time t (in seconds) as

$$x = 0.01\cos\left(\pi t + \frac{\pi}{4}\right)$$

The frequency of the motion will be

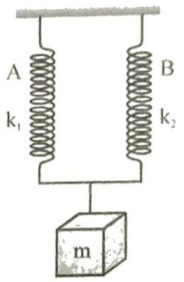
(*a*) 0.5 Hz (*b*) 1.0 Hz (*c*) $\frac{\pi}{2}$ Hz (*d*) π Hz

Finding time period, Spring block system

23. Two bodies M and N of equal masses are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude M to that of N is

(*a*) $\frac{k_1}{k_2}$ (*b*) $\sqrt{\frac{k_1}{k_2}}$ (*c*) $\frac{k_2}{k_1}$ (*d*) $\sqrt{\frac{k_2}{k_1}}$

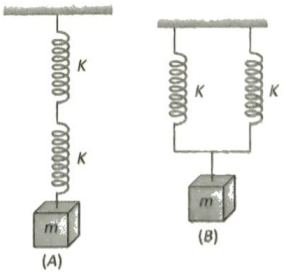
24. A mass m is suspended by means of two coiled spring which have the same length in unstretched condition as in figure. Their force constant are k_1 and k_2 respectively. When set into vertical vibrations, the period will be (The rod from which the mass is hanging remains horizontal)



(*a*) $2\pi\sqrt{\left(\frac{m}{k_1k_2}\right)}$ (*b*) $2\pi\sqrt{m\left(\frac{k_1}{k_2}\right)}$

(*c*) $2\pi\sqrt{\left(\frac{m}{k_1 - k_2}\right)}$ (*d*) $2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

25. Two identical spring of constant K are connected in series and parallel as shown in figure. A mass m is suspended from them. The ratio of their frequencies of vertical oscillations will be



(*a*) 2 : 1 (*b*) 1 : 1 (*c*) 1 : 2 (*d*) 4 : 1

26. A weightless spring which has a force constant oscillates with frequency n when a mass m is suspended from it. The spring is cut into two equal halves and a mass 2m is suspended from one half. The frequency of oscillation will now become

(*a*) n (*b*) 2n (*c*) $n/\sqrt{2}$ (*d*) $n(2)^{1/2}$

27. A block is placed on a frictionless horizontal table. The mass of the block is m and springs are attached on either side with force constants K_1 and K_2. If the block is displaced a little and left to oscillate, then the angular frequency of oscillation will be

(*a*) $\left(\frac{K_1+K_2}{m}\right)^{1/2}$ (*b*) $\left[\frac{K_1K_2}{m(K_1+K_2)}\right]^{1/2}$

(*c*) $\left[\frac{K_1K_2}{(K_1-K_2)m}\right]^{1/2}$ (*d*) $\left[\frac{K_1^2+K_2^2}{(K_1+K_2)m}\right]^{1/2}$

28. The force constants of two springs are K_1 and K_2. Both are stretched till their elastic energies are equal. If the stretching forces are F_1 and F_2, then $F_1 : F_2$ is

(*a*) $K_1 : K_2$ (*b*) $K_2 : K_1$

(*c*) $\sqrt{K_1} : \sqrt{K_2}$ (*d*) $K_1^2 : K_2^2$

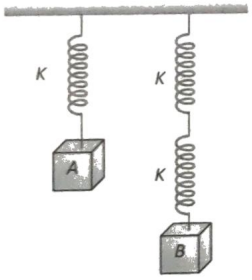
29. One-forth length of a spring of force constant K is cut away. The force constant of the remaining spring will be

(*a*) $\frac{3}{4}K$ (*b*) $\frac{4}{3}K$ (*c*) K (*d*) 4 K

30. Infinite springs with force constant k, 2k, 4k and 8k.... respectively are connected in series. The effective force constant of the spring will be

(*a*) 2k (*b*) k (*c*) k/2 (*d*) 4k

31. The springs shown are identical. When A = 4 kg , the elongation of spring is 1 cm. If B = 6 kg, the elongation produced by it is



(*a*) 4 cm (*b*) 3 cm (*c*) 2 cm (*d*) 1 cm

32. Two masses m_1 and m_2 are suspended together by a massless spring of constant K. When the masses are in equilibrium, m_1 is removed without disturbing the system. The amplitude of oscillations is

(*a*) $\frac{m_1g}{K}$

(*b*) $\frac{m_2g}{K}$

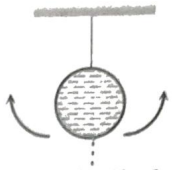
(*c*) $\frac{(m_1+m_2)g}{K}$

(*d*) $\frac{(m_1-m_2)g}{K}$



Simple and compound pendulum, Angular SHM

33. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will



(*a*) Remains unchanged (*b*) Increase
(*c*) Decrease (*d*) Become erratic

34. The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (If it is a second's pendulum on earth)

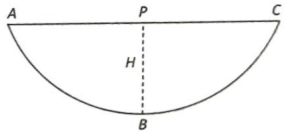
(*a*) $\frac{1}{\sqrt{2}}$ sec (*b*) $2\sqrt{2}$ sec

(*c*) 2 sec (*d*) $\frac{1}{2}$ sec

35. The length of the second pendulum on the surface of earth is 1 m. The length of seconds pendulum on the surface of moon, where g is 1/6th value of g on the surface of earth, is

(*a*) 1 / 6 m (*b*) 6 m (*c*) 1 / 36 m (*d*) 36 m

36. A simple pendulum with a bob of mass 'm' oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is 'g', then the velocity of the bob as it passes through B is



(*a*) mgH (*b*) $\sqrt{2gH}$ (*c*) 2gH (*d*) Zero

37. The time period of a simple pendulum in a lift descending with constant acceleration g is

(*a*) $T = 2\pi\sqrt{\frac{l}{g}}$ (*b*) $T = 2\pi\sqrt{\frac{l}{2g}}$

(*c*) Zero (*d*) not define

38. To show that a simple pendulum executes simple harmonic motion, it is necessary to assume that

(*a*) Length of the pendulum is small
(*b*) Mass of the pendulum is small
(*c*) Amplitude of oscillation is small
(*d*) Acceleration due to gravity is small

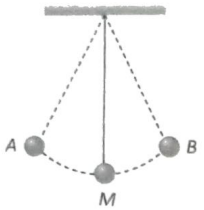
39. The ratio of frequencies of two pendulums are 2 : 3, then their length are in ratio

(*a*) $\sqrt{\frac{2}{3}}$ (*b*) $\sqrt{\frac{3}{2}}$ (*c*) $\frac{4}{9}$ (*d*) $\frac{9}{4}$

40. Two pendulums begin to swing simultaneously. If the ratio of the frequency of oscillations of the two is 7 : 8, then the ratio of lengths of the two pendulums will be

(*a*) 7 : 8 (*b*) 8 : 7 (*c*) 49 : 64 (*d*) 64 : 49

41. What is the velocity of the bob of a simple pendulum at its mean position, if it is able to rise to vertical height of 10cm ($g = 9.8\ m/s^2$)



(*a*) 2.2 m/s (*b*) 1.8 m/s
(*c*) 1.4 m/s (*d*) 0.6 m/s

Superposition of SHM

42. The S.H.M. of a particle is given by the equation $y = 3 \sin \omega t + 4 \cos \omega t$ .The amplitude is

(*a*) 7 (*b*) 1
(*c*) 5 (*d*) 12

43. The resultant of two rectangular simple harmonic motions of the same frequency and unequal amplitudes but differing in phase by $\frac{\pi}{2}$ is

(*a*) Simple harmonic (*b*) Circular
(*c*) Elliptical (*d*) Parabolic

44. In case of a forced vibration, the resonance wave becomes very sharp when the

(*a*) Restoring force is small
(*b*) Applied periodic force is small
(*c*) Quality factor is small
(*d*) Damping force is small

Learning Plus

1. According to a scientists, he applied a force $F = -cx^{1/3}$ on a particle and the particle is performing SHM. No other force acted on the particle. He refuses to tell whether c is a constant or not. Assume that he had worked only with positive x then :

(*a*) as x increases c also increases
(*b*) as x increases c decreases
(*c*) as x increases c remains constant
(*d*) the motion cannot be SHM

2. The displacement of a particle in simple harmonic motion in one time period is

(*a*) A (*b*) 2A (*c*) 4A (*d*) zero

3. The distance moved by a particle in simple harmonic motion in one time period is

(*a*) A (*b*) 2A (*c*) 4A (*d*) zero

4. A particle is executing S.H.M. of amplitude 'a' and time period = 4 second. Then the time taken by it to move from the extreme position to half the amplitude is

(*a*) 1 sec (*b*) $\frac{1}{3}$ sec

(*c*) $\frac{2}{3}$ sec (*d*) $\frac{4}{3}$ sec

5. Two SHM's are represented by $y = a \sin(\omega t - kx)$ and $y = b \cos(\omega t - kx)$. The phase difference between the two is:

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{4}$ (*c*) $\frac{\pi}{6}$ (*d*) $\frac{3\pi}{4}$

6. How long after the beginning of motion is the displacement of a harmonically oscillating particle equal to one half its amplitude if the period is 24s and particle starts from rest.

(*a*) 12s (*b*) 2s
(*c*) 4s (*d*) 6s

7. A particle is made to under go simple harmonic motion. Find its average acceleration in one time period.

(*a*) $\omega^2 A$ (*b*) $\frac{\omega^2 A}{2}$

(*c*) $\frac{\omega^2 A}{\sqrt{2}}$ (*d*) zero

8. The displacement of a body executing SHM is given by $x = A \sin(2\pi t + \pi/3)$. The first time from $t = 0$ when the velocity is maximum is

(*a*) 0.33 sec (*b*) 0.16 sec
(*c*) 0.25 sec (*d*) 0.5 sec

9. A simple harmonic motion having an amplitude A and time period T is represented by the equation :

$y = 5 \sin\pi(t + 4)$ m Then the values of A (in m) and T (in sec) are :

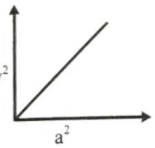
(*a*) A = 5; T = 2 (*b*) A = 10 ; T = 1
(*c*) A = 5 ; T = 1 (*d*) A = 10 ; T = 2

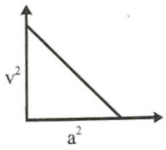
10. Two particles are in SHM on same straight line with amplitude A and 2A and with same angular frequency ω. It is observed that when first particle is at a distance $A/\sqrt{2}$ from origin and going toward mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles.
(*a*) 45° (*b*) 90° (*c*) 135° (*d*) 180°

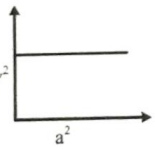
11. Equation of SHM is $x = 10 \sin 10\pi t$. Find the distance between the two points where speed is 50π cm/sec. x is in cm and t is in seconds.
(*a*) 10 cm (*b*) 20 cm (*c*) 17.32 cm (*d*) 8.66 cm.

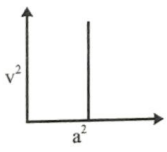
12. Two particles execute S.H.M. of same amplitude and frequency along the same straight line from same mean position. They cross one another without collision, when going in opposite directions, each time their displacement is half of their amplitude. The phase-difference between them is
(*a*) 0° (*b*) 120° (*c*) 180° (*d*) 135°

13. A mass M is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity v is

(*a*) 

(*b*) 

(*c*) 

(*d*) 

14. The K.E. and P.E of a particle executing SHM with amplitude A will be equal when its displacement is

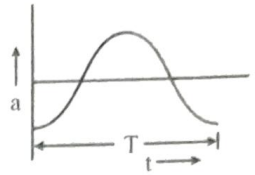
(*a*) $\sqrt{2}A$ (*b*) $\frac{A}{2}$

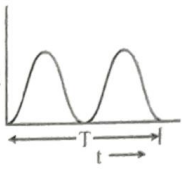
(*c*) $\frac{A}{\sqrt{2}}$ (*d*) $\sqrt{\frac{2}{3}}A$

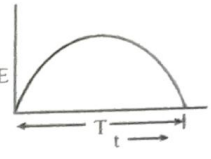
15. The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2 seconds. After another 2 seconds it returns to B. The time period of oscillation is (in seconds)
(*a*) 2 (*b*) 8
(*c*) 6 (*d*) 4

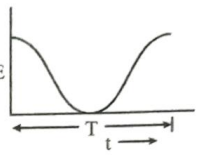
16. The maximum displacement of a particle executing S.H.M. is 1 cm and the maximum acceleration is $(1.57)^2$cm per sec^2. Then the time period is
(*a*) 0.25 sec (*b*) 4.00 sec
(*c*) 1.57 sec (*d*) $(1.57)^2$ sec

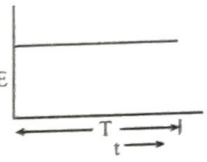
17. Acceleration a versus time t graph of a body in SHM is given by a curve shown below. T is the time period. Then corresponding graph between kinetic energy KE and time *t* is correctly represented by



(*a*) 

(*b*) 

(*c*) 

(*d*) 

18. A particle of mass m oscillating with amplitude A and angular frequency ω. its average energy in one time period is ?

(*a*) $\frac{1}{2}m\omega^2A^2$ (*b*) $\frac{1}{4}m\omega^2A^2$

(*c*) $m\omega^2A^2$ (*d*) zero

19. In SHM particle oscillates with frequency υ then find the frequency of oscillation of its kinetic energy.
(*a*) υ (*b*) $\upsilon/2$ (*c*) 2υ (*d*) zero

20. A body is executing simple harmonic motion. At a displacement *x*, its potential energy is E_1 and at a displacement *y*, its potential energy is E_2. The potential energy E at a displacement $(x + y)$ is

(*a*) $E_1 + E_2$ (*b*) $\sqrt{E_1^2 + E_2^2}$

(*c*) $E_1 + E_2 + 2\sqrt{E_1 E_2}$ (*d*) $\sqrt{E_1 E_2}$

21. A particle performing SHM is found at its equilibrium at $t = 1$ sec. and it is found to have a speed of 0.25 m/s at $t = 2$ sec. If the period of oscillation is 6 sec. Calculate amplitude of oscillation

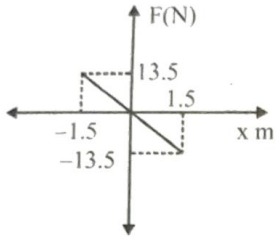
(*a*) $\frac{3}{2\pi}$ m (*b*) $\frac{3}{4\pi}$ m

(*c*) $\frac{6}{\pi}$ m (*d*) $\frac{3}{8\pi}$

22. Two particles execute SHM of same amplitude of 20 cm with same period along the same line about the same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radians is

(*a*) $\frac{2\pi}{3}$ (*b*) $\frac{\pi}{2}$ (*c*) $\frac{\pi}{3}$ (*d*) $\frac{\pi}{4}$

23. A particle of mass 1 kg is undergoing S.H.M., for which graph between force and displacement (from mean position) as shown. Its time period, in seconds, is.



(*a*) $\pi/3$ (*b*) $2\pi/3$ (*c*) $\pi/6$ (*d*) $3/\pi$

24. A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance a/2 from the extreme position is

(*a*) a/T (*b*) 2a/T

(*c*) 3a/T (*d*) a/2T

25. A mass of 1 kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half:

(*a*) 1 kg (*b*) 2 kg

(*c*) 3 kg (*d*) 4 kg

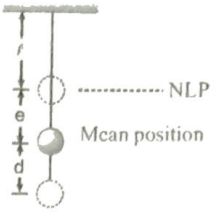
26. A toy car of mass m is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is k and surface is frictionless. The car is displaced from mean position by x cm and released. At the mean position the ribbons are undeformed. Vibration period is



(*a*) $2\pi\sqrt{\frac{m(2k)}{k^2}}$ (*b*) $\frac{1}{2\pi}\sqrt{\frac{m(2k)}{k^2}}$

(*c*) $2\pi\sqrt{\frac{m}{k}}$ (*d*) $2\pi\sqrt{\frac{m}{k+k}}$

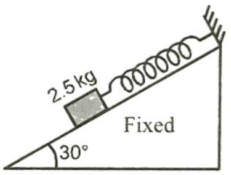
27. An elastic string of length ℓ supports a heavy particle of mass m and the system is in equilibrium with elongation produced being e as shown in figure. The particle is now pulled down below the equilibrium position through a distance d (>e) and released. The angular frequency and amplitude for SHM is



(*a*) $\sqrt{\frac{g}{e}}$, d (*b*) $\sqrt{\frac{g}{\ell}}$, 2e

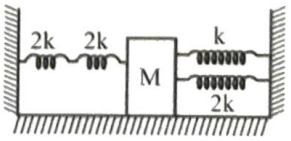
(*c*) $\sqrt{\frac{g}{d+e}}$, d (*d*) $\sqrt{\frac{g}{e}}$, 2d

28. A smooth inclined plane having angle of inclination 30° with horizontal has a mass 2.5 kg held by a spring which is fixed at the upper end as shown in figure. If the mass is taken 2.5 cm up along the surface of the inclined plane, the tension in the spring reduces to zero. If the mass is then released, the angular frequency of oscillation in radian per second is



(*a*) 0.707 (*b*) 7.07 (*c*) 1.414 (*d*) 14.14

29. Four massless springs whose force constants are 2k, 2k, k and 2k respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of the system.



(*a*) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (*b*) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

(*c*) $\frac{1}{2\pi}\sqrt{\frac{k}{7M}}$ (*d*) $\frac{1}{2\pi}\sqrt{\frac{7k}{M}}$

30. The potential energy of a simple harmonic oscillator of mass 2 kg in its mean position is 5 J. If its total energy is 9J and its amplitude is 0.01 m, its time period would be

(a) $\frac{\pi}{10}$ sec (b) $\frac{\pi}{20}$ sec (c) $\frac{\pi}{50}$ sec (d) $\frac{\pi}{100}$ sec

31. A spring mass system performs S.H.M. If the mass is doubled keeping amplitude same, then the total energy of S.H.M. will become :

(a) double (b) half

(c) unchanged (d) 4 times

32. A pendulum clock that keeps correct time on the earth is taken to the moon. It will run

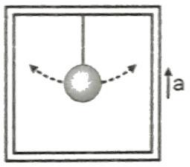
(a) at correct rate (b) 6 times faster

(c) $\sqrt{6}$ times faster $\sqrt{6}$ (d) times slower

33. Two pendulums begin to swing simultaneously. The first pendulum makes 9 full oscillations when the other makes 7. Find the ratio of length of the two pendulums.

(a) $\frac{49}{81}$ (b) $\frac{7}{9}$ (c) $\frac{50}{81}$ (d) $\frac{1}{2}$

34. A scientist measures the time period of a simple pendulum as T in a lift at rest. If the lift moves up with acceleration as one fourth of the acceleration of gravity, the new time period is



(a) $\frac{T}{4}$ (b) 4 T (c) $\frac{2}{\sqrt{5}}$ T (d) $\frac{\sqrt{5}}{2}$ T

35. A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitude is decreased by 5%?

(a) 6 % (b) 3 % (c) 1.5 % (d) 0 %

36. A wall clock uses simple pendulum. It is accurate at earth surface. If it is taken to high altitude:

(a) it will run fast

(b) its length should be increased to keep it accurate.

(c) its length should be decreased to keep it accurate.

(d) Even if length is changed it cannot be accurate.

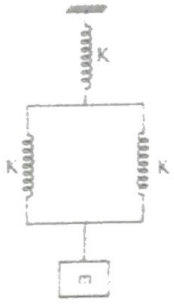
37. A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically above the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then maintain the orientation. If the sphere is then released, its time period of the oscillation will be :

(Assume restoring Torque is proportional to angle turned)

(a) π second (b) $\sqrt{2}\pi$ second

(c) 2π second (d) 4π second

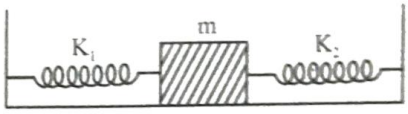
38. A body of mass 'm' hangs from three springs, each of spring constant 'k' as shown in the figure. If the mass is slightly displaced and let go, the system will oscillate with time period



(a) $2\pi\sqrt{\frac{m}{3k}}$ (b) $2\pi\sqrt{\frac{3m}{2k}}$

(c) $2\pi\sqrt{\frac{2m}{3k}}$ (d) $2\pi\sqrt{\frac{3k}{m}}$

39. A block of mass m is connected between two springs (constants K_1 and K_2) as shown in the figure and is made to oscillate, the frequency of oscillation of the system shall be-



(a) $\frac{1}{2\pi}\left(\frac{m}{K_1+K_2}\right)^{1/2}$ (b) $\frac{1}{2\pi}\left(\frac{K_1K_2}{(K_1+K_2)m}\right)^{1/2}$

(c) $\frac{1}{2\pi}\left(\frac{K_1+K_2}{m}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{(K_1+K_2)m}{K_1K_2}\right)^{1/2}$

40. Two pendulums have time periods T and 5T 4. They start SHM at the same time from the mean position. After how many oscillations of the smaller pendulum they will be again in the same phase

(a) 5 (b) 4 (c) 11 (d) 9

41. A ring is suspended at a point on its rim and it behaves as a second's pendulum when it oscillates such that its centre move in its own plane. The radius of the ring would be ($g = \pi^2$)

(a) 0.5 m (b) 1.0 m (c) 0.67 m (d) 1.5 m

42. When two mutually perpendicular simple harmonic motions of same frequency, amplitude and phase are superimposed

(a) the resulting motion is uniform circular motion.

(b) the resulting motion is a linear simple harmonic motion along a straight line inclined equally to the straight lines of motion of component ones.

(c) the resulting motion is an elliptical motion, symmetrical about the lines of motion of the components.

(d) the two S.H.M. will cancel each other.

43. The position of a particle in motion is given by $y = C\sin\omega t + D\cos\omega t$ w.r.t. origin. Then motion of the particle is:

(*a*) SHM with amplitude C+D

(*b*) SHM with amplitude $\sqrt{C^2 + D^2}$

(*c*) SHM with amplitude $\frac{(C+D)}{2}$

(*d*) not SHM

44. The position vector of a particle moving in x-y plane is given by $\vec{r} = (A\sin\omega t)\hat{i} + (A\cos\omega t)\hat{j}$ then motion of the particle is :

(*a*) SHM

(*b*) on a circle

(*c*) on a straight line

(*d*) with constant acceleration

45. The displacement of a particle executing periodic motion is given by $y = 4\cos^2(0.5t)\sin(1000\,t)$. The given expression is composed by minimum

(*a*) four SHMs (*b*) three SHMs

(*c*) one SHM (*d*) None of these

46. The amplitude of the vibrating particle due to superposition of two **SHMs**.

$y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin\omega t$ is :

(*a*) 1 (*b*) $\sqrt{2}$ (*c*) $\sqrt{3}$ (*d*) 2

Advanced Level Multiconcept Questions

JEE-ADVANCED COMPREHENSION/STATEMENT/MATCHING/ MCQ

1. A particle moves on the X-axis according to the equation $x = x_0 \sin^2 \omega t$. The motion is simple harmonic

(*a*) with amplitude $x_0/2$ (*b*) with amplitude $2x_0$

(*c*) with time period $\frac{2\pi}{\omega}$ (*d*) with time period $\frac{\pi}{\omega}$

2. The potential energy of a particle of mass 0.1 kg, moving along x-axis, is given by $U = 5x(x - 4)$ J where x is in metres. It can be concluded that

(*a*) the particle is acted upon by a constant force

(*b*) the speed of the particle is maximum at x = 2 m

(*c*) the particle executes simple harmonic motion

(*d*) the period of oscillation of the particle is $\pi/5$ s

3. If *y*, *v* and *a* represent displacement, velocity and acceleration at any instant for a particle executing SHM, which of the following statements are true?

(*a*) v and y may have same direction

(*b*) v and a may have same direction

(*c*) a and y may have same direction

(*d*) a and v may have opposite direction

4. The equation of motion for an oscillating particle is given by $x = 3\sin(4\pi t) - 4\cos(4\pi t)$, where x is in mm and t is in second

(*a*) The motion is simple harmonic

(*b*) The period of oscillation is 0.5 s

(*c*) The amplitude of oscillation is 5 μm

(*d*) The particle starts its motion from the equilibrium

5. Which of the following functions represent SHM?

(*a*) $\sin 2\omega t$

(*b*) $\sin^2 \omega t$

(*c*) $\sin \omega t + 2\cos \omega t$

(*d*) $\sin \omega t + \cos 2\omega t$

6. The displacement of a particle varies according to the relation $x = 3\sin 100t + 8\cos^2 50t$. Which of the following is/are correct about this motion.

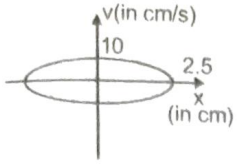
(*a*) the motion of the particle is not S.H.M.

(*b*) the amplitude of the S.H.M. of the particle is 5 units

(*c*) the amplitude of the resultant S.H.M. is $\sqrt{73}$ units

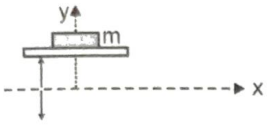
(*d*) the maximum displacement of the particle from the origin is 9 units.

7. The figure shows a graph between velocity and displacement (from mean position) of a particle performing SHM



(*a*) the time period of the particle is 1.57 s
(*b*) the maximum acceleration will be $40 cm/s^2$
(*c*) the velocity of particle is $2\sqrt{21}$ cm/s when it is at a distance 1 cm from the mean position.
(*d*) none of these

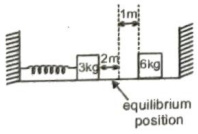
8. As shown in figure a horizontal platform with a mass m placed on it is executing SHM along y-axis. If the amplitude of oscillation is 2.5 cm, the minimum period of the motion for the mass not to be detached from the platform is :
($g = 10 m/sec^2$)



(*a*) $\frac{10}{\pi}$ s (*b*) $\frac{\pi}{10}$ s
(*c*) $\frac{\pi}{\sqrt{10}}$ s (*d*) $\frac{1}{\sqrt{10}}$ s

9. If a SHM is given by $y = (\sin \omega t + \cos \omega t)$ m, which of the following statements are true?
(*a*) The amplitude is 1m
(*b*) The amplitude is $(\sqrt{2})$ m
(*c*) Time is considered from y = 1 m
(*d*) Time is considered from y = 0 m

10. Two blocks of masses 3 kg and 6 kg rest on a horizontal smooth surface. The 3 kg block is attached to a spring with a force constant $k = 900 Nm^{-1}$ which is compressed 2 m from beyond the equilibrium position. The 6 kg mass is at rest at 1m from mean position. 3 kg mass strikes the 6 kg mass and the two stick together.



(*a*) velocity of the combined masses immediately after the collision is $10 ms^{-1}$
(*b*) velocity of the combined masses immediately after thecollision is $5 ms^{-1}$
(*c*) amplitude of the resulting oscillation is $\sqrt{2}$ m
(*d*) amplitude of the resulting oscillation is $\sqrt{5}/2$ m

Comprehension – 01 (11 to 13)

A 2kg block hangs without vibrating at the bottom end of a spring with a force constant of 400 N/m. The top end of the spring is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of $5 m/s^2$ when the acceleration suddenly ceases at time t = 0 and the car moves upward with constant speed. ($g = 10 m s^{-2}$)

11. What is the angular frequency of oscillation of the block after the acceleration ceases?
(*a*) $10\sqrt{2}$ rad/s (*b*) 20 rad/s
(*c*) $20\sqrt{2}$ rad/s (*d*) 32 rad/s

12. The amplitude of the oscillations is
(*a*) 7.5 cm (*b*) 5 cm
(*c*) 2.5 cm (*d*) 1 cm

13. The initial phase angle observed by a rider in the elevator, taking upward direction to be positive and positive extreme position to have $\pi/2$ phase constant, is equal to
(*a*) zero (*b*) $\pi/2$ rad (*c*) π rad (*d*) $3\pi/2$ rad

Comprehension – 02 (14 to 16)

Equations of displacements during three different SHMs of a particle can be written as $x_1 = A_1 \cos\omega_1 t$, $x_2 = A_2 \cos(\omega_2 t + \delta_2)$ and $y = A_3 \cos(\omega_3 t + \delta_3)$ here x_1 and x_2 are the displacement of the particle along x-axis during two different SHMs. y is the displacement of the particle during SHM along y-axis. x and y axis are orthogonal to each other.

Consider the super position of two SHMs along x-axis. The resultant displacement is $x = x_1 + x_2$ and resultant amplitude is $A = [A_1^2 + A_2^2 + 2A_1A_2\cos(\omega_1 - \omega_2)t]^{1/2}$ if $\omega_1 \neq \omega_2$ and $\delta_2 = 0$.

Super position of two SHMs in perpendicular direction may result in SHM along a straight line or motion along a circle or an ellipse in clockwise or counter clockwise direction.

14. In superposition of two SHMs along x-axis identify the correct options :
(*a*) If $A_1 = A_2$ and $\delta_2 = \pi$ the particle is always at rest
(*b*) If $\delta_2 = 0$, $A_1 = A_2$ and $\omega_1 \neq \omega_2$ then the resultant motion is a harmonic motion with angular frequency $\omega = \frac{\omega_1 + \omega_2}{2}$.
(*c*) If $A_1 = A_2$, $\omega_1 \neq \omega_2$, $\delta_2 = 0$ and $A_1 = A_2$ then the particle is at origin at time 't' = $\frac{3\pi}{\omega_1 - \omega_2}$
(*d*) If $A_1 = A_2$, $\omega_1 = \omega_2$ and $\delta_2 = 0$ then the amplitude of resultant SHM is $2A_1$

15. In superposition of two SHMs along mutually perpendicular directions. Identify the correct options.
(*a*) If $\omega_1 = \omega_3$ and $\delta_3 = 0$ then the resulting motion is SHM with amplitude $\sqrt{A_1^2 + A_3^2}$ along line $y = \frac{A_3}{A_1}x$
(*b*) If $\omega_1 = \omega_3$, $A_1 = A_3$ and $\delta_3 = \frac{3\pi}{2}$ then the resulting motion is a circular motion in counter clockwise direction.
(*c*) If $\omega_1 = \omega_2$, $A_1 \neq A_3$ and $\delta_3 = \frac{\pi}{2}$ then the resulting motion is on elliptical path in clockwise direction
(*d*) If $\omega_1 = \omega_3$ and $\delta_3 = \pi$ then the resulting motion is on elliptical path in counter clockwise direction

16. In super position of two SHMs of same frequency along x-axis the initial phase angle of resultant SHM is given as

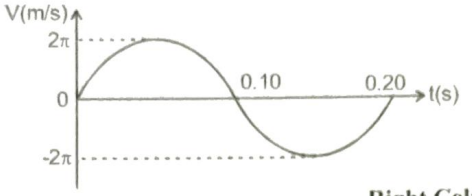
(*a*) $\tan^{-1}\dfrac{A_1\sin\delta_2}{A_2+A_1\cos\delta_2}$

(*b*) $\tan^{-1}\dfrac{A_2\sin\delta_2}{A_1+A_2\cos\delta_2}$

(*c*) $\tan^{-1}\dfrac{A_2\cos\delta_2}{A_1+A_1\sin\delta_2}$

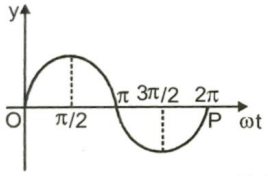
(*d*) $\tan^{-1}\dfrac{A_2\cos\delta_2}{A_1+A_2\sin\delta_2}$

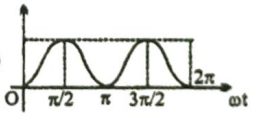
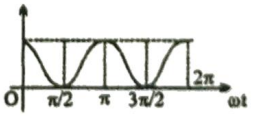
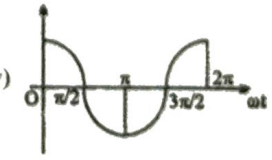
17. A simple harmonic oscillator consists of a block attached to a spring with k = 200 N/m. The block slides on a frictionless horizontal surface, with equilibrium point x = 0. A graph of the block's velocity v as a function of time t is shown. Correctly match the required information in the left column with the values given in the right column. (use $\pi^2 = 10$)



Left Column	**Right Column**
(*a*) The block's mass in kg	(*p*) –0.20
(*b*) The block's displacement at t = 0 in metres	(*q*) –200
(*c*) The block's acceleration at t = 0.10 s in m/s^2	(*r*) 0.20
(*d*) The block's maximum kinetic energy in Joule	(*s*) 4.0

18. The graph plotted between phase angle (ϕ) and displacement of a particle from equilibrium position (y) is a sinusoidal curve as shown below. Then the best matching is



Column-A	**Column-B**
(A) K. E. versus phase angle curve	(i) 
(B) P.E. versus phase angle curve	(ii) 
(C) T.e. versus phase angle curve	(iii) 
(D) Velocity versus phase angle curve	(iv) 

(*a*) (A) - (i), (B) - (ii), (C) - (iii) & (D) - (iv)
(*b*) (A) - (ii), (B) - (i), (C) - (iii) & (D) - (iv)
(*c*) (A) - (ii), (B) - (i), (C) - (iv) & (D) - (iii)
(*d*) (A) - (ii), (B) - (iii), (C) - (iv) & (D) - (i)

NUMERICAL VALUE BASED

19. A 50 g block is attached to a vertical spring whose stiffness constant is 4 N/m. The block is released at the position where the spring is unextended. What is the maximum extension of the spring in meter.

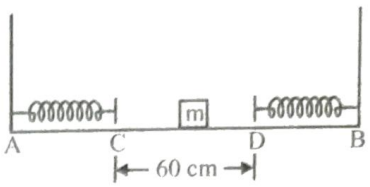
20. A block is resting on a piston which is moving vertically with a SHM of period 1.0 s. At what amplitude **(in m)** of motion will the block and piston separate?

21. In an oscillating block-spring system the spring constant is 2.45 N/m, the amplitude is 16 cm, and the maximum speed is 56 cm/s. What is the mass of the block in kg?

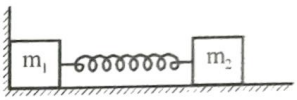
22. If the time period of simple harmonic motion of a uniform rod shown here is π sec. What is its length (in cm)?



23. Two light springs of force constants k_1 and k_2 and a very small block of mass m are in one line AB on smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the springs is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation (in sec) of the block to nearest integer. (k_1 = 1.8 N/m, k_2 = 3.2 N/m, m = 200g) (Take $\pi = \frac{22}{7}$)



24. On a corner on a smooth horizontal surface, we kept 2 masses m_1 and m_2 attached to a spring as shown. The spring has a spring constant of 150 N/m. Now m_2 is suddenly given a velocity to the left. It is seen that spring first comes to natural length at $\frac{\pi}{5\sqrt{2}}$ sec. It next comes to natural length at $\frac{\pi}{10\sqrt{2}}$ sec. What is mass m_1 (in kg).



JEE Mains & Advanced Past Years Questions

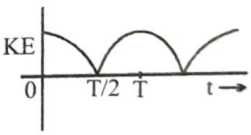
JEE-MAIN
PREVIOUS YEAR'S

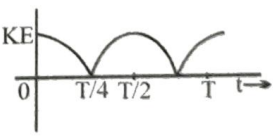
1. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is: **[JEE Mains-2016]**

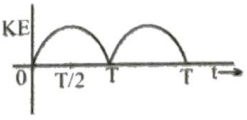
(a) $\frac{A}{3}\sqrt{41}$ (b) 3A

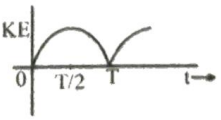
(c) $A\sqrt{3}$ (d) $\frac{7A}{3}$

2. A particle is executing simple harmonic motion with a time period T. AT time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like **[JEE Main-2017]**

(a)


(b)


(c)


(d)


3. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1})

[JEE Main-2018]

(a) 7.1 N/m (b) 2.2 N/m (c) 5.5 N/m (d) 6.4 N/m

4. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by: [JEE Main-2019 (January)]

(a) 10^{-3} rad/s (b) 1 rad/s

(c) 10^{-1} rad/s (d) 10^{-5} rad/s

5. A simple haromic motion is represented by:

$$y = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right) \text{cm}$$

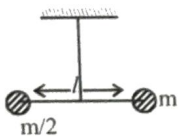
The amplitude and time period of the motion are:

[JEE Main-2019 (January)]

(a) 10 cm, $\frac{2}{3}$s (b) 10 cm, $\frac{3}{2}$s

(c) 5 cm, $\frac{3}{2}$s (d) 5 cm, $\frac{2}{3}$s

6. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length ℓ. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rodmass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be [JEE Main-2019 (January)]



(a) $\frac{3k\theta_0^2}{\ell}$ (b) $\frac{2k\theta_0^2}{\ell}$ (c) $\frac{k\theta_0^2}{\ell}$ (d) $\frac{k\theta_0^2}{2\ell}$

7. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2 then:

[JEE Main-2019 (January)]

(a) $K_2 = 2K_1$ (b) $K_2 = \frac{K_1}{2}$ (c) $K_2 = \frac{K_1}{4}$ (d) $K_2 = K_1$

8. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy o the particle at t = 210 s will be : [JEE Main-2019 (January)]

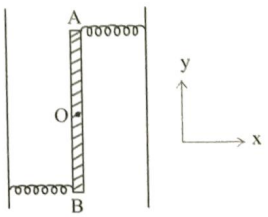
(a) 1/9 (b) 1 (c) 2 (d) 3

9. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The oscillation of the same pendulum on the planet would be: [JEE Main-2019 (January)]

(a) $\frac{\sqrt{3}}{2}$s (b) $\frac{2}{\sqrt{3}}$s (c) $\frac{3}{2}$s (d) $2\sqrt{3}$s

10. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is:

[JEE Main-2019 (January)]



(a) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

11. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis about x = 0. When its potential energy (PE) equals kinetic energy (KE), the position of the particle will be [JEE Main-2019 (January)]

(a) $\frac{A}{2}$ (b) $\frac{A}{2\sqrt{2}}$ (c) $\frac{A}{\sqrt{2}}$ (d) A

12. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to: [JEE Main-2019 (January)]

(a) 0.77 (b) 0.57 (c) 0.37 (d) 0.17

13. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity is SI units is equal to that of its acceleration. Then, its periodic time in seconds is: [JEE Main-2019 (January)]

(a) $\frac{4\pi}{3}$ (b) $\frac{3}{8}\pi$

(c) $\frac{8\pi}{3}$ (d) $\frac{7}{3}\pi$

14. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to:

[JEE Main-2019 (April)]

(a) 100 s (b) 20 s (c) 10 s (d) 50 s

15. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is:

[JEE Main-2019 (April)]

(a) $4T\sqrt{\frac{1}{15}}$ (b) $2T\sqrt{\frac{1}{10}}$

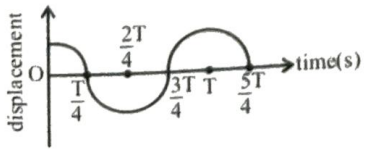
(c) $4T\sqrt{\frac{1}{14}}$ (d) $2T\sqrt{\frac{1}{14}}$

16. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}\cos(10\pi t + \phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to:

[JEE Main-2019 (April)]

(a) 13s (b) 7s (c) 27s (d) 4s

17. The displacement time graph of a particle executing S.H.M is given in figure (sketch is schematic and not to scale) [JEE Main-2020 (September)]



Which of the following statements is/are true for this motion?

(A) The force is zero at $t = \frac{3T}{4}$

(B) The acceleration is maximum at $t = T$

(C) The speed is maximum at $t = \frac{T}{4}$

(D) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(a) (B), (C) and (D) (b) (A) and (D)
(c) (A), (B) and (C) (d) (A), (B) and (D)

18. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is

[JEE Main-2020 (September)]

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 1 (d) $\frac{1}{2}$

19. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. The ratio $\frac{T_1}{T_2}$ will be

[JEE Main-2020 (September)]

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{2}{3}$ (d) $\frac{3}{\sqrt{2}}$

20. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0\sin^2\omega t$, where 'y' is measured from the lower end of unstretched spring. Then ω is

[JEE Main-2020 (September)]

(a) $\sqrt{\frac{5g}{y_0}}$ (b) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$

(c) $\sqrt{\frac{g}{2y_0}}$ (d) $\sqrt{\frac{g}{y_0}}$

21. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is : [JEE Main-2021 (March)]

(a) $\cos(\omega t)+\cos(2\omega t)+\cos(3\omega t)$

(b) $\sin^2(\omega t)$

(c) $\sin(\omega t)+\cos(\omega t)$

(d) $3\cos\left(\frac{\pi}{4}-2\omega t\right)$

22. The time period of a simple pendulum is given by $T=2\pi\sqrt{\frac{l}{g}}$. The measured value of the length of pendulum is 10 cm known to a 1 mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1 s resolution. The percentage accuracy in the determination of 'g' using this pendulum is 'x'. The value of 'x' to be nearest integer is: [JEE Main-2021 (March)]

(a) 2% (b) 3%

(c) 5% (d) 4%

23. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take ln 2 = 0.693) [JEE Main-2021 (March)]

(a) $0.69\times10^{2}\ kgs^{-1}$ (b) $3.3\times10^{2}\ kgs^{-1}$

(c) $1.16\times10^{-2}\ kgs^{-1}$ (d) $5.7\times10^{-3}\ kgs^{-1}$

24. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is [JEE Main-2021 (March)]

(a) $\frac{K_1}{K_2}$ (b) $\sqrt{\frac{K_1}{K_2}}$

(c) $\frac{K_2}{K_1}$ (d) $\sqrt{\frac{K_2}{K_1}}$

25. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500 g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to drop to half of its initial value? (ln 2 = 0.693) [JEE Main-2021 (March)]

(a) 17.32s (b) 34.65s

(c) 0.034s (d) 15.01s

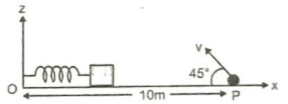
26. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g 2, the time period of pendulum will be: [JEE Main-2021 (March)]

(a) $\sqrt{3}T$ (b) $\sqrt{\frac{2}{3}}T$

(c) $\frac{T}{\sqrt{3}}$ (d) $\sqrt{\frac{3}{2}}T$

JEE-ADVANCED
PREVIOUS YEAR'S

1. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega=\frac{\pi}{3}\ rad\ s$. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 cm from O. If the pebble hits the block at t = 1s, the value of v is (take g = 10 m/s² [IIT JEE-2012]



(a) $\sqrt{50}m/s$ (b) $\sqrt{51}m/s$

(c) $\sqrt{52}m/s$ (d) $\sqrt{53}m/s$

2. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision: [JEE Advanced -2013]

(a) the speed of the particle when it returns to its equilibrium position is u_0.

(b) the time at which the particle passes through the equilibrium position for the first time is $t=\pi\sqrt{\frac{m}{k}}$.

(c) the time at which the maximum compression of the spring occurs is $t=\frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

(d) the time at which the particle passes througout the equilibrium position for the second time is $t=\frac{5\pi}{3}\sqrt{\frac{m}{k}}$.

3. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m ($< M$) is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M? **[JEE Advanced-2016]**

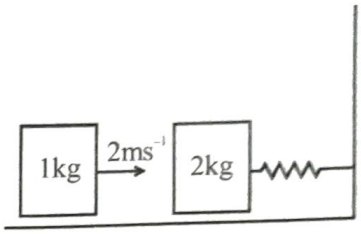
(*a*) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged.

(*b*) The final time period of oscillation in both the cases is same

(*c*) The total energy decreases in both the cases

(*d*) The instantaneous speed at x_0 of the combined masses decreases in both the cases

4. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N m^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _____. **[JEE Advanced-2018]**



ANSWER KEY

Topicwise Questions

1. (*d*)	**2.** (*c*)	**3.** (*d*)	**4.** (*a*)	**5.** (*a*)	**6.** (*c*)	**7.** (*b*)	**8.** (*d*)	**9.** (*b*)	**10.** (*d*)
11. (*a*)	**12.** (*a*)	**13.** (*a*)	**14.** (*a*)	**15.** (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*d*)	**19.** (*b*)	**20.** (*a*)
21. (*b*)	**22.** (*a*)	**23.** (*d*)	**24.** (*d*)	**25.** (*c*)	**26.** (*a*)	**27.** (*a*)	**28.** (*c*)	**29.** (*b*)	**30.** (*c*)
31. (*b*)	**32.** (*a*)	**33.** (*b*)	**34.** (*b*)	**35.** (*a*)	**36.** (*b*)	**37.** (*d*)	**38.** (*c*)	**39.** (*d*)	**40.** (*d*)
41. (*c*)	**42.** (*c*)	**43.** (*c*)	**44.** (*d*)						

Learning Plus

1. (*a*)	**2.** (*d*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)	**6.** (*c*)	**7.** (*d*)	**8.** (*a*)	**9.** (*a*)	**10.** (*c*)
11. (*c*)	**12.** (*b*)	**13.** (*b*)	**14.** (*c*)	**15.** (*b*)	**16.** (*b*)	**17.** (*a*)	**18.** (*a*)	**19.** (*c*)	**20.** (*c*)
21. (*a*)	**22.** (*c*)	**23.** (*b*)	**24.** (*c*)	**25.** (*c*)	**26.** (*c*)	**27.** (*a*)	**28.** (*d*)	**29.** (*b*)	**30.** (*d*)
31. (*c*)	**32.** (*d*)	**33.** (*a*)	**34.** (*c*)	**35.** (*d*)	**36.** (*c*)	**37.** (*d*)	**38.** (*b*)	**39.** (*c*)	**40.** (*a*)
41. (*a*)	**42.** (*b*)	**43.** (*b*)	**44.** (*b*)	**45.** (*b*)	**46.** (*c*)				

Advanced Level Multiconcept Questions

1. (*a*, *d*) **2.** (*b*, *c*, *d*) **3.** (*a*, *b*, *d*) **4.** (*a*, *b*, *c*) **5.** (*a*, *b*, *c*) **6.** (*b*, *d*) **7.** (*a*, *b*, *c*) **8.** (*b*, *d*) **9.** (*b*, *c*) **10.** (*a*, *c*)

11. (*a*) **12.** (*c*) **13.** (*d*) **14.** (*b*, *c*, *d*) **15.** (*a*, *b*, *c*) **16.** (*b*) **17.** (*a*) *r*, (*b*) *p* , (*c*) *q* , (*d*) *s* **18.** (*b*)

19. [0.25 m] **20.** [0.248 m] **21.** [0.2 kg] **22.** [0375] **23.** [0003] **24.** [0001]

JEE Mains & Advanced Past Years Questions

JEE-MAIN

1. (*d*)	**2.** (*b*)	**3.** (*a*)	**4.** (*a*)	**5.** (*a*)	**6.** (*c*)	**7.** (*a*)			
8. (No match) (Correct answer is 1/3)				**9.** (*d*)	**10.** (*c*)	**11.** (*c*)	**12.** (*c*)	**13.** (*c*)	**14.** (*b*)
15. (*a*)	**16.** (*b*)	**17.** (*c*)	**18.** (*a*)	**19.** (*b*)	**20.** (*c*)	**21.** (*d*)	**22.** (*b*)	**23.** (*c*)	**24.** (*d*)
25. (*b*)	**26.** (*b*)								

JEE-ADVANCED

1. (*a*) **2.** (*a*, *d*) **3.** (*a*, *b*, *d*) **4.** [2.09]

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14 WAVES

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INTRODUCTION

A wave is a disturbance which travels or propagates and transports energy and momentum without the transport of matter. The ripples on a pond, the sound we hear, visible light, radio and TV signals are a few examples of waves. Sound, light and radio waves provide us with an effective means of transmitting and receiving energy and information.

TYPES OF WAVES

A wave is disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

The ripples on a water surface, the sound we hear, visible light, radio and TV signals are a few examples of waves.

There are two types of wave

(i) **Mechanical Waves:** Require material medium (elasticity and inertia) for their propagation. These waves are also called elastic waves, water waves and sound waves are example of mechanical waves. They are of two types : Transverse and longitudital. Comparison between the two is given there:

Transverse wave : velocity of wave and velocity of particle are perpendicular to each other.

Longitudinal wave : velocity of wave and velocity of particle are in same direction.

(ii) **Electromagnetic or non-mechanical Waves:** Do not require any material medium for their propagation, such as light and TV signals. Important points regarding these waves are:

(a) Elasticity or inertia do not affect their propagation.

(b) They are always transverse in nature.

WAVE FUNCTION

The disturbance created by a wave is represented by **wave function**. For a string, the wave function is a (Vector) displacement; where as for sound waves, it is (scalar) pressure or density fluctuation. In the case of light or radio waves, the wave function is either an electric or magnetic field vector.

(a) Mathematical Representation of Wave Function

$y = f(x + vt)$

In general, the wave motion in one dimension is given by

$y = f(x \mp vt)$

(b) The Wave Equation

A travelling wave satisfies a differential equation, called the linear wave equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{dt^2}$$

Any function of space and time which satisfies above differential equation is a travelling wave.

Functions such as $y = A \sin \omega t$ or $y = A \sin kx$ do not satisfy above equation, hence do not represent waves.

On the other hand; functions such as

$A \sin (kx - \omega t)$, $A \sin kx \sin \omega t$, $[A \sin (kx - \omega t) + B \cos (kx + \omega t)]$, $\sqrt{(ax + bt)}$, $(ax - bt)^2$, $Ae^{-B(x-vt)^2}$ or $A \cos^2 (kx - \omega t)$ satisfy the wave equation, and hence these are wave functions.

Note: that for a function to be wave function, the three quantities x, t and v must appear in the linear combinations $(x - vt)$ or $(x + vt)$. Thus, $(x - vt)^2$ is acceptable but $(x^2 - v^2t^2)$ is not. (because it is not linear)

Negative sign between t and x implies that the wave is travelling along positive x-axis and vice-versa.

TRAIN YOUR BRAIN

Example 1: The wave function of a pulse is given by $y = \frac{3}{2 + (x - 4t)^2}$, where y is in metres and t is seconds. Determine the wave velocity of the pulse and indicate the direction of propagation of the wave.

Sol. On comparing the given expression with

$y = f(x - vt)$

we get the velocity of the wave as

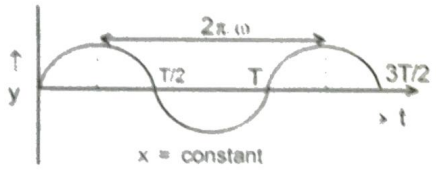
$v = 4$ m/s

Since there occurs negative sign between x and t in the given expression, the wave propagates along the +ve x-axis.

VARIOUS TERMS RELATED TO WAVE MOTION

(1) Amplitude (A): The maximum displacement of a vibrating particle of the medium from the mean position. 'A' shows amplitude in $y = A \sin \omega t$

(2) Periodic time (T): Time taken to complete one oscillation and denoted by T.



$T = \frac{1}{n}$, n is called frequency

(3) Wave Frequency (n): Number of vibrations made per second by the particles and is denoted by n.

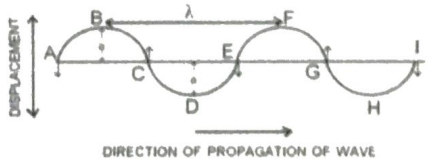
$n = 1/T$ unit : Hz

(4) Angular Frequency: $\omega = 2\pi n$

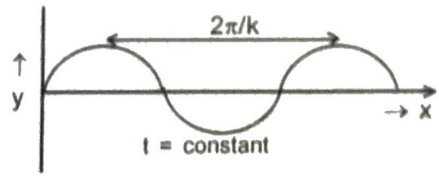
unit : rad/sec

(5) Wave Length (λ): The distance between two consecutive particle in the same phase or the distance travelled by the wave in one periodic time and denoted by λ

$\because$ Wave Velocity $V = n\lambda$



Other Relation :



Position of different particles at some instant in a string

$V = n\lambda = \frac{\lambda}{T} = \frac{\omega}{2\pi}$ $\lambda = \frac{\omega}{k}$

where k is defined as wave number.

(6) Intensity of Wave: In medium, propagation energy perpendicular to per unit area per second is called **intensity of wave.**

$I = 2\pi^2 n^2 A^2 \rho v$

Where ρ = density of the medium, v = Wave velocity, A = amplitude, n = Wave frequency

Unit : W m^{-2} If v & ρ is constant, then $I \propto$ (frequency)^2 & $I \propto$ (Amplitude)^2

(7) Energy Density : Energy per unit volume is called energy density.

E = Energy density $= \frac{\text{Energy Intensity}}{\text{Energy Velocity}}$

$= \frac{2\pi^2 n^2 a^2 \rho v}{v}$, $E = 2\pi^2 n^2 a^2 \rho$

(8) Wave Pressure : When a wave incidents on surface then it produces pressure on it, it is called **wave pressure.**

With Ref. to Point O	Motion in (+) ve x direction		Motion in (−) ve x direction	
Far away from point O	O →	$y = A \sin \omega(t - x/v)$	O ←	$y = A \sin \omega(t + x/v)$
Towards point O	O ←	$y = A \sin \omega(t + x/v)$	O →	$y = A \sin \omega(t - x/v)$

POINT O IS CONSIDERED AS REFERENCE POINT

Speed of Wave Motion

(1) Speed of non-mechanical i.e., electromagnetic wave in vaccum is $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

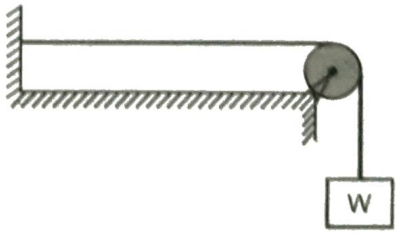
where μ_0 = absolute permeability and ε_0 = absolute permittivity of vacuum or air

(2) Speed of mechanical waves:

(a) Transverse wave in a stretched string

T = tension in the string

μ = mass per unit length



D = diameter of pipe

ρ = density

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{stress}{density}} = \frac{2}{D}\sqrt{\frac{T}{\pi\rho}}$ $\boxed{\mu = Ad = \frac{\pi D^2}{4}\rho}$

(b) Transverse wave in a solid

(i) In a long bar $v = \sqrt{\frac{Y}{\rho}}$ where Y = Young's modulus,

ρ = Density of material

(ii) In an extended solid $v = \sqrt{\frac{Y + \frac{4\eta}{3}}{\rho}}$

where η = modulus of rigidity,

ρ = Density of material

Some Important Points:

Now $v = \nu\lambda$

ν = frequency of wave, which is a constant.

λ = wavelength

$\Rightarrow \lambda \propto v$

As a wave changes medium its speed and wavelength changes but frequency remains same.

Relation between phase difference and path difference

$$\frac{\Delta\varphi}{\Delta x} = \frac{2\pi}{\lambda}$$

$\Delta\phi$ = Phase difference

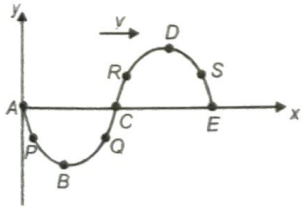
Δx = Path difference

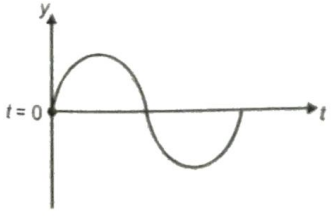
ANALYSIS OF HARMONIC WAVES

1. For a transverse wave $y = A \sin(\omega t - kx)$

at t = 0, photograph of the wave is

For the particle at x = 0





$y = A \sin(-kx)$

$y = A \sin\omega t$

Particles at A, P, S, E are moving upwards

This particle is moving upward at t = 0

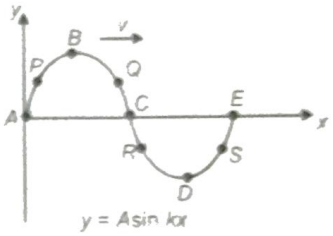
Particles at Q, C, R are moving downwards

Particles at B and D are at rest

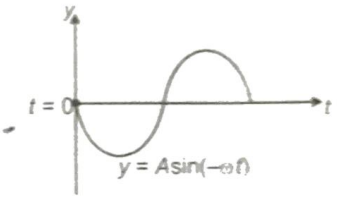
2. For wave $y = A \sin(kx - \omega t)$

at t = 0, photograph of the wave is

For the particle at x = 0



$y = A \sin kx$



$y = A \sin(-\omega t)$

Particles at A, P, S, E are moving downwards. This particle is moving downward at t = 0

Particles at Q, C, R are moving upwards

Particles at B and D are at rest

Some Important Points:

1. Particle velocity, $V_{p_x} = -V\left(\frac{\partial y}{\partial x}\right)$, where V = wave velocity and $\frac{\partial y}{\partial x}$ = slope of the wave
2. In general, points with positive slope $\left(\frac{\partial y}{\partial x}\right)$ move downward.
3. The points with negative slope move upward.
4. The points with maximum slope (A, C, E) have maximum velocity.
5. The points with zero slope are at rest.

INTERFERENCE & PRINCIPLE OF SUPERPOSITION OF WAVES

Principle of Superposition of Waves:

(1) Superposition principle states that the net disturbance at a given place and time caused by a number of waves in the same space is the vector sum of the disturbances which would have been produced by each wave independent of the other.

(2) Resultant displacement

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \text{----------}$$

(3) Value of resultant displacement depends on

(a) Amplitude of waves

(b) Phase difference & path difference of waves

(c) Frequency of waves

(d) Direction of wave motion

Phenomenon observed due to superposition depends on above points (a), (b), (c), (d).

INTERFERENCE

When waves of equal frequency & nearly equal amplitude is superimposed, interference occurs

(i) Mathematical Interference :

At a time t, at point x two waves of equal frequency

$y_1 = a_1\sin(\omega_1 t - k_1 x + \phi_1)$ &

$y_2 = a_2\sin(\omega_2 t - k_2 x + \phi_2)$ is super-imposed, then

Amplitude A & Intensity I of Resultant wave :

$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\Delta\phi$

& $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$

$\Delta\phi = (\omega_2 - \omega_1)t - (k_2 - k_1)x + \phi_2 - \phi_1$

Intensity of resultant wave changes periodically from minimum to maximum & maximum to minimum from one point to another point

(ii) Constructive Interference:

Where, phase difference $\Delta\phi = 2\pi n$

path difference $\Delta x = n\lambda$,

$I_{max} \propto (a_1 + a_2)^2$

(iii) Destructive Interference :

Where phase difference $\Delta\phi = (2n - 1)\pi$

path difference $\Delta x = \left(n - \frac{1}{2}\right)\lambda$

$I_{min} \propto (a_1 - a_2)^2$

TRAIN YOUR BRAIN

Example 2: Two waves of the same frequency but of amplitude in the ratio 1 : 3 are superposed. What is the ratio of maximum to minimum intensity.

Sol. Let the two amplitudes be a and 3a.

Maximum amplitude = a + 3a = 4a

$I_{max} = (4a)^2 = 16\,a^2$

Minimum amplitude = 3a – a = 2a

$I_{min} = (2a)^2 = 4a^2$

$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{16a^2}{4a^2} = \frac{4}{1}$

Example 3: Two waves of the same frequency and amplitudes 2a and 3a are super-imposed on each other.

(i) For what values of the phase difference ϕ the amplitude A of the resultant wave will be maximum and for what values minimum?

(ii) Calculate the ratio of maximum and minimum intensities of the resultant wave.

Sol. (i) The intensity is maximum for $\phi = 0, 2\pi, 4\pi$ .....

The intensity is minimum for $\phi = \pi, 3\pi, 5\pi$, ...

(ii) The maximum intensity is

$I_{max} \propto (a_1 + a_2)^2 = 25a^2$

and minimum intensity is $I_{min} \propto (a_1 - a_2)^2 = a^2$

$\because \frac{I_{max}}{I_{min}} = 25$

Example 4: Consider interference between waves from two sources of intensities I and 4I. Find the intensities at points where the phase difference is (i) $\pi/2$ (ii) π

Sol. Resultant intensity

$I_R = I + 4I + 4I\cos\phi$

(i) $\phi = \pi/2$, $I_R = 5I$

$\therefore I_R = 5I + 4I\cos(\pi/2) = 5I$

(ii) $\phi = \pi$, $I_R = I$

$I_R = 5I + 4I\cos\pi = I$

Example 5: Two sources of intensities I and 4I are used in interference experiment. Find the intensity at points where the waves from the sources super-impose with a phase difference of (a) zero (b) $\pi/2$ (c) π.

Sol. Resultant intensity is

(a) $I_R = I_1 + I_2 + 2\sqrt{(I_1 I_2)}\cos\phi$

$= I + 4I + 4I\cos 0° = 9I$

(b) $I_R = I + 4I + 4I\cos(\pi/2) = 5I$

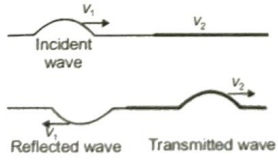
(c) $I_R = I + 4I + 4I\cos\pi = I$

REFLECTION OF WAVES

If a wave travelling in a medium of high velocity gets reflected from the surface of a medium of low velocity, it suffers a phase change π.

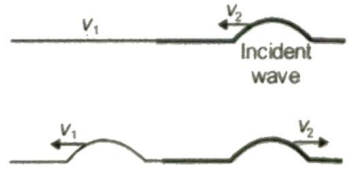
At the interface of a rarer and denser medium.

(a) Wave is moving from rarer to denser medium.



$v_2 < v_1$

(b) Wave is moving from denser to rarer medium



$v_2 < v_1$

The transmitted wave is always in phase with incident wave.

Reflection from fixed end:



Reflection from free end:



STANDING WAVES

Let two waves of equal amplitude are travelling in opposite direction along x-axis. The wave equation of the two waves can be given as

$y_1 = A \sin(\omega t - kx)$ ...(1)

[Wave travelling in +x direction]

and $y_2 = A \sin(\omega t + kx)$ ...(2)

[Wave travelling in - x direction]

When the two waves superpose on medium particles, the resultant displacement of the medium particles can be given as

$y = y_1 + y_2$

or $y = A \sin(\omega t - kx) + A \sin(\omega t + kx)$

or $y = A [\sin\omega t \cos kx - \cos \omega t \sin kx + \sin \omega t \cos kx + \cos \omega t \sin kx]$

or $y = 2A \cos kx \sin \omega t$ ...(3)

Equation (3) can be rewritten as

$y = R \sin \omega t$ ...(4)

Where $R = 2 A \cos kx$ ...(5)

Here equation (4) is an equation of SHM. It implies that after superposition of the two waves the medium particles executes SHM with same frequency ω and amplitude R which is given by equation (5) Here we can see that the oscillation amplitude of medium particles depends on x i.e. the position of medium particles. Thus on superposition of two coherent waves travelling in opposite direction the resulting interference pattern, we call stationary waves, the oscillation amplitude of the medium particle at different positions is different.

At some point of medium the resultant amplitude is maximum which are given as

R is maximum when $\cos kx = \pm 1$

or $\frac{2\pi}{\lambda} x = N\pi \; [N \in I]$

or $x = \frac{N\lambda}{2}$ or $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ .....

and the maximum value of R is given as

$R_{max} = \pm 2 A$ ...(6)

Thus in the medium at position $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$, ...

the waves interfere constructively and the amplitude of oscillations becomes 2A. Similarly at some points of the medium, the waves interfere destructively, the oscillation amplitude become minimum i.e. zero in this case. These are the points where R is minimum, when

$\cos kx = 0$

or $\frac{2\pi x}{\lambda} = (2N + 1)\frac{\pi}{2}$

or $x = (2N + 1)\frac{\lambda}{4}$ $[N \in I]$ or $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.....

and the minimum value of R is given as

$R_{min} = 0$ ...(7)

Thus in the medium at position $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .....

the waves interfere destructively and the amplitude of oscillation becomes zero. These points always remain at rest. Figure (a) shows the oscillation amplitude of different medium particles in a stationary waves.

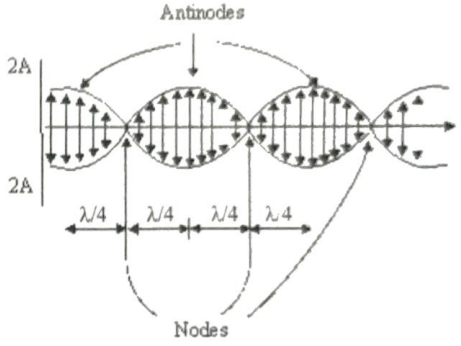


figure (a)

In figure (a) we can see that the medium particles at which constructive interference takes place are called antinodes of stationary wave and the points of destructive interference are called nodes of stationary waves which always remain at rest.

Figure (b) explain the movement of medium particles with time in the region where stationary waves are formed. Let us assume that at an instant t = 0 all the medium particles are at their extreme positions as shown in figure (b – 1). Here points ABCD are the nodes of stationary waves where medium particles remains at rest. All other starts moving towards their mean positions and t = T / 4 all particles cross their mean position as shown in figure (b - 3), you can see in the figure that the particles at nodes are not moving. Now the medium crosses their mean position and starts moving on other side of mean position toward the other extreme position. At time t = T/2, all the particles reach their other extreme position as shown in figure (b - 5) and at time t = 3T/4 again all these particles cross their mean position in opposite direction as shown in figure (b - 7).

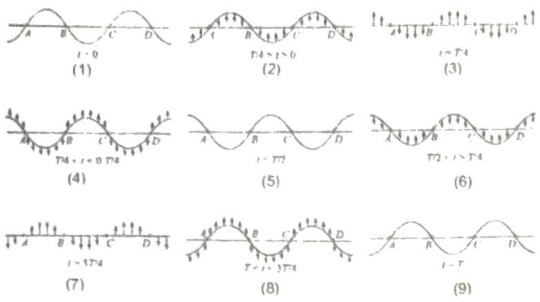


figure (b)

Based on the above analysis of one complete oscillations of the medium particles, we can make some interference for a stationary waves. These are :

(i) In oscillations of stationary wave in a region, some points are always at rest (nodes) and some oscillates with maximum amplitudes (antinodes). All other medium particles oscillate with amplitudes less then those of antinodes.

(ii) All medium particles between two successive nodes oscillate in same phase and all medium particles on one side of a node oscillate in opposite phase with those on the other side of the same node.

(iii) In the region of a stationary wave during one complete oscillation all the medium particles come in the form of a straight line twice.

(iv) If the component wave amplitudes are equal, then in the region where stationary wave is formed, no net flow of energy takes place, only redistribution of energy takes place in the medium.

S.No.	Travelling waves	Stationary waves
1	These waves advance in a medium with a definite velocity	These waves remain stationary between two boundaries in the medium.
2	In these waves, all particles of the medium oscillate with same frequency and amplitude.	In these waves, all particles except nodes oscillate with same frequency but different amplitudes. Amplitude is zero at nodes and maximum at antinodes.
3	At any instant phase of vibration varies continuosly from one particle to the other i.e., phase difference between π two particles can have any value between 0 and 2	At any instant the phase of all particles between two successive nodes is the same, but phase of particles on one side of a node is opposite to the phase of particles on the other side of the node, i.e, phase difference between any two particles can be either 0 or π
4	In these wave, at no instant all the particles of the medium pass through their mean positions simultaneously.	In these waves all particles of the medium pass through their mean position simultaneously twice in each time period.
5	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.

(a) Different Equation for a Stationary Wave

Consider two equal amplitude waves travelling in opposite direction as

$y_1 = A \sin(\omega t - kx)$ ...(1)

and $y_2 = A \sin(\omega t + kx)$ ...(2)

The result of superposition of these two waves is

$y = 2A \cos kx \sin \omega t$ ...(3)

Which is the equation of stationary wave where $2A \cos kx$ represents the amplitude of medium particle situated at position x and $\sin \omega t$ is the time sinusoidal factor. This equation (3) can be written in several ways depending on initial phase differences in the component waves given by equation (1) can (2). If the superposing waves are having an initial phase difference π, then the component waves can be expressed as

$y_1 = A \sin(\omega t - kx)$ ...(4)

$y_2 = -A \sin(\omega t - kx)$ ...(5)

Superposition of the above two waves will result

$y = 2A \sin kx \cos \omega t$ ...(6)

Equation (6) is also an equation of stationary wave but here amplitude of different medium particles in the region of interference is given by

$R = 2A \sin kx$ ...(7)

Similarly the possible equations of a stationary wave can be written as

$y = A_0 \sin kx \cos(\omega t + \phi)$ ...(8)

$y = A_0 \cos kx \sin(\omega t + \phi)$ ...(9)

$y = A_0 \sin kx \sin(\omega t + \phi)$ ...(10)

$y = A_0 \cos kx \cos(\omega t + \phi)$ ...(11)

Here A_0 is the amplitude of antinodes. In a pure stationary wave it is given as

$A_0 = 2A$

Where A is the amplitude of component waves. If we carefully look at equation (8) to (11), we can see that in equation (8) and (10), the particle amplitude is given by

$R = A_0 \sin kx$ ...(12)

Here at $x = 0$, there is nodes as $R = 0$ and in equation (9) and (11) the particle amplitude is given as

$R = A_0 \cos kx$ ...(13)

Here at $x = 0$, there is an antinode as $R = A_0$. Thus we can state that in a given system of co-ordinates when origin of system is at a node we use either equation (8) or (10) for analytical representation of a stationary wave and we use equation (9) or (11) for the same when an antinode is located at the origin of system.

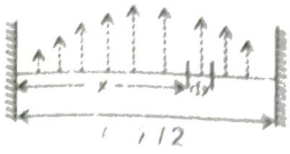
(b) Energy of standing wave in one loop

(i) When all the particles of one loop are at extreme position then total energy in the loop is in the form of potential energy only when the particles reaches

its mean position the total potential energy gets into kinetic energy of the particles, so we can say total energy of the loop remains constant.

Total kinetic energy at mean position is equal to total energy of the loop because potential energy at mean position is zero.

Small kinetic energy of the particle which is in element dx is



$$d(KE) = \frac{1}{2}dmv^2$$

$dm = \mu\, dx$

Velocity of particle at mean position

$= 2A \sin kx\, \omega$

then $d(KE) = \frac{1}{2}\mu dx \cdot 4A^2\omega^2 \sin^2 kx$

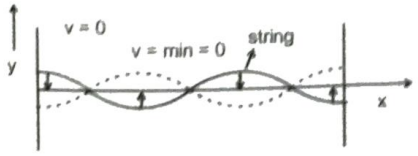
$\Rightarrow d(KE) = 2A^2\omega^2 \mu \cdot \sin^2 kx\, dx$

$$\int d(K.E) = 2A^2\omega^2\mu \int_0^{\lambda/2} \sin^2 kx dx$$

$$\text{Total K.E} = A^2\omega^2\mu \int_0^{\lambda/2} (1 - \cos 2kx)dx$$

$$= A^2\omega^2\mu\left[x - \frac{\sin 2kx}{2k}\right]_0^{\lambda/2} = \frac{1}{2}\lambda A^2\omega^2\mu$$

(ii) As in stationary waves nodes are permanently at rest, so no energy can be transmitted across them, i.e., energy of one region (segment) is confined in that region. However, this energy oscillates between elastic potential energy and kinetic energy of the particles of the medium. When all the particles are at their extreme positions KE is minimum while elastic PE is maximum (as shown in figure A), and when all the particles (simultaneously) pass through their mean position KE will be maximum while elastic PE minimum (Figure B). The total energy confined in a segment (elastic PE + KE), always remains the same.



Elastic PE = max = E
Kinetic energy = min = 0
(A)



Elastic PE = min = 0
Kinetic energy = max = E
(B)

STATIONARY WAVES IN STRINGS

(a) When both end of string is fixed :

A string of length L is stretched between two points. When the string is set into vibrations, a transverse progressive wave begins to travel along the string. It is reflected at the other fixed end. The incident and the reflected waves interfere to produce a stationary transverse wave in which the **ends are always nodes, if both ends of string are fixed.**

Fundamental Mode (0th overtone)

(a) In the simplest form, the string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and frequency of vibration is known as the fundamental frequency or first harmonic.



Since the distance between consecutive nodes is $\frac{\lambda}{2}$

$\therefore L = \frac{\lambda}{2}$

$\therefore \lambda = 2L$

If f_1 is the fundamental frequency of vibration, then the velocity of transverse waves is given as,

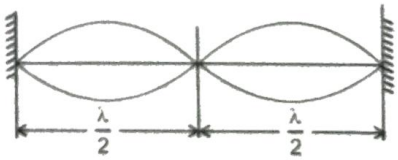
$$v = \lambda f_1 \text{ or } f_1 = \frac{v}{2L} \quad ...(i)$$

First Overtone

(b) The same string under the same conditions may also vibrate in two loops, such that the centre is also the node.

$\therefore L = \frac{2\lambda}{2}$

$\therefore \lambda = L$



If f_2 is frequency of vibrations

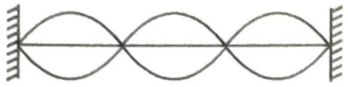
$$\therefore \quad f_2 = \frac{v}{\lambda} = \frac{v}{L}$$

$$\therefore \quad f_2 = \frac{v}{L} \qquad ...(ii)$$

The frequency f_2 is known as second harmonic or first overtone.

Second Overtone

(c) The same string under the same conditions may also vibrate in three segments.



$$\therefore \quad L = \frac{3\lambda}{2}$$

$$\therefore \quad \lambda_3 = \frac{2}{3}L$$

If f_3 is the frequency in this mode of vibration, then,

$$f_3 = \frac{3v}{2L} \qquad ...(iii)$$

The frequency f_3 is known as third harmonic or second overtone.

Thus a stretched string vibrates with frequencies, which are integral multiples of the fundamental frequencies. These frequencies are known as harmonics.

The velocity of transverse wave in stretched string is given as $v = \sqrt{\frac{T}{\mu}}$. Where T = tension in the string.

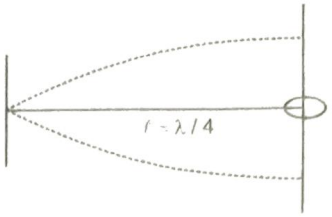
μ = linear density or mass per unit length of string. If the string fixed at two ends, vibrates in its fundamental mode, then

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad ...(17)$$

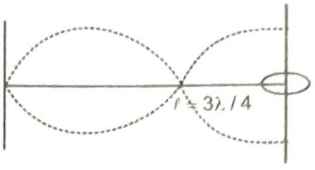
In general $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$ n^{th} harmonic $(n-1)^{th}$ overtone

In general, any integral multiple of the fundamental frequency is an allowed frequency. These higher frequencies are called overtones. Thus, $f_1 = 2f_0$ is the first overtone, $f_2 = 3f_0$ is the second overtone etc. An integral multiple of a frequency is called its harmonic. Thus, for a string fixed at both the ends, all the overtones are harmonics of the fundamental frequency and all the harmonics of the fundamental frequency are overtones.

(b) When one end of the string is fixed and other is free : free end acts as antinode

1. 

fundamental or Ist harmonic

2. 

IIIrd harmonic or Ist overtone

In general : $f = \frac{(2n+1)}{4\ell}\sqrt{\frac{T}{\mu}}$ ((2n + 1)th harmonic, nth overtone)

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Example 6: Standing waves are produced by superposition of two waves

$y_1 = 0.05 \sin(3\pi t - 2x)$ and

$y_2 = 0.05 \sin(3\pi t + 2x)$

Where x and y are measured in metre and t in second. Find the amplitude of particle at x = 0.5m

[cos 57.3 = 0.54]

(*a*) 0.54 m (*b*) 5.4 m

(*c*) 54 m (*d*) 0.054 m

Sol. (d) $A = 2a \cos Kx$

$= 2 \times 0.5 \cos 2 \times 0.5$

$= 0.1 \cos(1)$

$= 0.1 \cos\left(\frac{180^\circ}{\pi}\right)$

$= 0.1 \cos\left(\frac{180^\circ}{3.14}\right) = 0.1 \cos(57.3^\circ)$

$= 0.1 \times 0.54 = 0.054$

Example 7: A standing wave is produced on a string clamped at one end and free at the other. The length of the string -

(*a*) Must be an integral multiple of $\frac{\lambda}{4}$

(*b*) Must be an integral multiple of $\frac{\lambda}{2}$

(c) Must be an integral multiple of λ

(d) May be an integral multiple of $\frac{\lambda}{2}$

Sol. (a) The fixed end is a node and free end is antinode. So, as distance between any two consecutive nodes is $\lambda/4$,. Hence, length of string is $\frac{n\lambda}{4}$

Example 8: The equation of a standing wave produced on a string fixed at both the ends is -

$$y = (0.4)\sin[(0.314\ cm^{-1})x]\cos[(600\ \pi\ s^{-1})t]$$

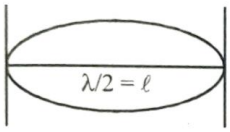
What would be the smallest length of the string?

(a) 20 cm (b) 30 cm

(c) 10 cm (d) 5 cm

Sol. (c)

Both ends of strings are fixed, so both ends are node. So it looks as



So, possible smallest length

$$\ell = \frac{\lambda}{2}$$

$$k = \frac{2\pi}{\lambda}\ ;\ \lambda = \frac{2\pi}{k}$$

From equation,

$$\lambda = \frac{2\pi}{0.314} = \frac{2\times 3.14}{0.314} = 20\ cm$$

$$\ell = \frac{\lambda}{2} = \frac{20}{2} = 10\ cm$$

Example 9: A stretched string vibrates with a frequency of 30 Hz in its fundamental mode when the support are 60 cm apart the velocity of transverse waves in the string is -

(a) 9 m/s (b) 18 m/s

(c) 36 m/s (d) 72 m/s

Sol. (c) Given n = 30 Hz, ℓ = 60 cm

velocity of wave $v = n\lambda$

$\lambda = 2\lambda = 2 \times 60 = 120$ cm

v = 30 Hz = 120 cm

$= 30\ Hz \times \frac{120}{100} m = 36\ m$

Example 10: In Melde's experiment it was found that the string vibrate in 3 loops when 8 gm were placed in the pan. What mass must be placed in the pan to make the string vibrate in 5 loops. (Neglect the mass of string)

(a) 2.98 gm (b) 2.88 gm

(c) 2.78 gm (d) 2.68 gm

Sol. (b) $F_0 = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{P}{2\ell}\sqrt{\frac{mg}{\mu}}$

All are constant except P and m

So, $P\sqrt{m}$ = constant

$$P_1\sqrt{m_1} = P_2\sqrt{m_2}$$

$$3\sqrt{8} = 5\sqrt{m}$$

$m = 0.36 \times 8 = 2.88$ gm

Example 11: A standing wave is produced on a string clamped at one end free at the other. The length of the string–

(a) must be an integral multiple of $\lambda/4$

(b) must be an integral multiple of $\lambda/2$

(c) must be an integral multiple of λ

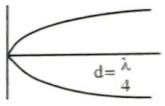
(d) may be an integral multiple of $\lambda/2$

Sol. (a)

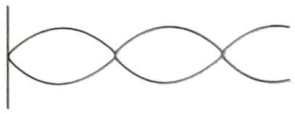
Clamp end of string $\Rightarrow$ node point

Open end of string $\Rightarrow$ antinode point

So, in minimum position it looks as

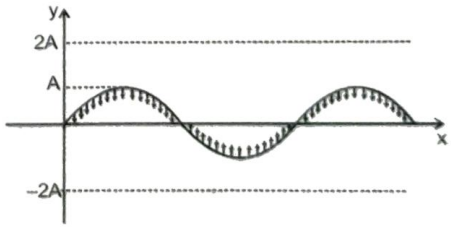


OR



So, it should be $\ell = \frac{\lambda n}{4}$, integer multiple of $\frac{\lambda}{4}$

Example 12: Figure shows the standing waves pattern in a string at t = 0. Find out the equation of the standing wave where the amplitude of antinode is 2A.



Sol. Let we assume the equation of standing waves

is $= A' \sin(\omega t + \phi)$

where $A' = 2A \sin(kx + \theta)$

$\because$ $x = 0$ is node $\Rightarrow A' = 0$, at $x = 0$

$2A \sin\theta = 0$

$\Rightarrow \theta = 0$

at $t = 0$ Particle at is at $y = A$ and going towards mean position.

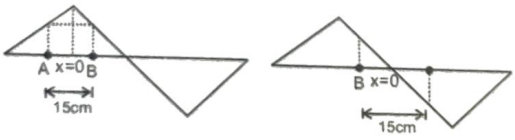
$$\Rightarrow \phi = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$$

so eq. of standing waves is

$$y = 2A\sin kx \sin\left(\omega t + \frac{5\pi}{6}\right)$$

Example 13: A string 120 cm in length sustains standing wave with the points of the string at which the displacement amplitude is equal to 3.5 mm being separated by 15.0cm. The maximum displacement amplitude is X.95 mm then find out the value of X.

Sol. In this problem two cases are possible :



Case - I is that A and B have the same displacement amplitude and **case - 2** is that C and D have the same amplitude which is 3.5 mm. In case 1, if $x = 0$ is taken at antinode then

$A = a \cos kx$

In case -2, if $x = 0$ is taken at node, then

$A = a \sin kx$

But since nothing is given in the question.

Hence from both the cases, result should be same. This is possible only when

$a \cos kx = a \sin kx$

$$\text{or } kx = \frac{\pi}{4} \text{ or } a = \frac{A}{\cos kx} = \frac{3.5}{\cos \pi/4} = 4.95\text{mm}$$

Example 14: A sonometer wire under tension T is in unison with a tuning fork of frequency 250 Hz. If now the sonometer wire is replaced by another hollow wire of the same diameter, the number of loops shall-

(*a*) Decrease

(*b*) Increase

(*c*) Remain the same

(*d*) Incomplete information

Sol. (a) $$\nu = \frac{p}{2\ell}\sqrt{\frac{F}{\mu}} \text{ or } \nu = \frac{p}{2\ell}\sqrt{\frac{F}{\pi r^2 \rho}}$$

$$p = \nu(2\ell)\frac{\sqrt{\pi r^2 \rho}}{\sqrt{F}} \quad \text{or} \quad p \propto \sqrt{\rho}$$

As density of hollow wire is less than that of sonometer wire, so number of loops shall decrease.

Example 15: When a stretched string of length L is vibrating in a particular mode, the distance between two nodes on the string is ℓ. The sound produces in this mode of vibration constitutes the nth overtone on the fundamental frequency of the string

(*a*) $L = (n + 1)\ell$ (*b*) $L = (n - 1)\ell$

(*c*) $L = n\ell$ (*d*) $L = (n + \frac{1}{2})\ell$

Sol. (a) Here, ℓ = distance between the nodes

$$\ell = \frac{n\lambda}{2}$$

Here, no. of loops

= No. of overtones $+1 = n + 1$

So, length of string $= L = (n + 1)\ell$

Example 16: A wire under tension vibrates with a frequency of 450 per second. What would be the fundamental frequency if the wire were half as long, twice as thick and under one-fourth tension–

(*a*) 225 cps (*b*) 190 cps

(*c*) 247 cps (*d*) 174 cps

Sol. (a) We have, $\nu = \frac{1}{2\ell}\sqrt{\frac{F}{\pi r^2 \rho}}$

$$\text{So, } 450 = \frac{1}{2\ell_1}\sqrt{\frac{T_1}{\pi r_1^2 \rho}}$$

$$\nu_2 = \frac{1}{2\ell_2}\sqrt{\frac{T_2}{\pi r_2^2 \rho}}$$

$$\frac{450}{\nu_2} = \frac{\ell_2}{\ell_1}\sqrt{\frac{T_1}{T_2} \times \frac{\pi r_2^2 \rho}{\pi r_1^2 \rho}}$$

$$\frac{450}{\nu_2} = \frac{1}{2}\sqrt{4 \times 4}$$

$$\nu_2 = \frac{450 \times 2}{4} = 225 \text{ cps}$$

Example 17: Two similar wires on a sonometer are tuned to unison. One wire is 25 cm long and is stretched by 100 gram-weight. The length of the other wire which is stretched by 400 gram-weight is-

(*a*) 6.25 cm (*b*) 12.5 cm

(*c*) 50 cm (*d*) 100 cm.

Sol. (c) Here, in first case, $\ell = 0.25$ m, m = 100 g wt = 0.1 kg

So $\nu = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$$\nu_A = \frac{1}{2\ell_A}\sqrt{\frac{T_A}{\mu}} = \frac{1}{2(0.25)}\sqrt{\frac{0.1g}{\mu}}$$

$$\nu_B = \frac{1}{2\ell_B}\sqrt{\frac{T_B}{\mu}} = \frac{1}{2\ell_B}\sqrt{\frac{0.4g}{\mu}}$$

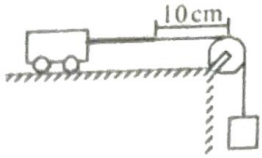
$$\therefore \frac{\nu_A}{\nu_B} = \frac{2\ell_B}{0.5}\sqrt{\frac{1}{4}}$$

$\nu_A = \nu_B = \nu$ = frequency of tuning fork

So $1 = \frac{2\ell_B}{0.5(2)}$

$\Rightarrow \ell_B = 0.5$ m = 50 cm

Example 18: A heavy string is tied at one end to a movable support and to a light thread at the other end as shown in figure. The thread goes over a fixed pulley and supports a weight to produce a tension. The lowest frequency with which the heavy string resonates is 120 Hz. If the movable support is pushed to the right by 10 cm so that the joint is placed on the pulley, then the minimum frequency at which the heavy string can resonate is–



(a) 120 Hz (b) 60 Hz

(c) 240 Hz (d) 480 Hz

Sol. (c) Initially the condition at the two ends of rod are different (one is node (fixed) other is antinode (free). So behaviour is just like a closed pipe closed at one end.

Hence fundamental or lowest frequency is $\frac{V}{4L}$ where L is length of heavy string, so

$$\frac{V}{4L} = 120 \qquad ...(1)$$

Now, in second case, conditions at the two ends of rod become similar (both ends are nodes (fixed). So behaviour is just like a open pipe open at both ends.

So minimum frequency = $\frac{V}{2L} = 2\frac{V}{4L} = 2 \times 120$

= 240 Hz

EXPERIMENT OF MELDE'S

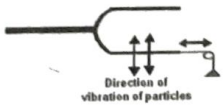
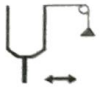
(1) This is experimental show of transverse stationary wave.

(2) Unknown frequency of tuning fork can be determined by this experiment.

(3) Laws of transverse vibration in stretched string can be proved by this experiment

(4) One stretched string is tied to arm of tuning fork. Now it is vibrated.

There are two arrangements:

S.No.	Transverse arrangement	Longitudinal arrangement
1.		
2.	Oscillation of tuning fork arms is perpendicular to string	Oscillation of tuning fork arms is parallel to string length
3.	To complete one oscillation tuning fork & string both take same time. It means frequency is same for both	String complete one oscillation in same time in which tuning fork complete two Oscillation frequency tuning fork = 2 x frequency of string
4.	If P are number of loops in string then $L = P\lambda/2$; $n = \frac{P}{2L}\sqrt{\frac{T}{m}}$ $P\sqrt{T}$ = constant	If P are number of loops in string $L = P\lambda/2$; $n = \frac{P}{2L}\sqrt{\frac{T}{m}}$ Frequency of tuning fork : $n = \frac{2P}{2L}\sqrt{\frac{T}{m}}$, $P\sqrt{T}$ = constant
	If L, m and frequency remains constant	If L, m, and frequency remain constant.

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Example 19: The mathematical form of Meldes law is -

(a) $p\sqrt{T}$ = constant (b) pT = constant

(c) $\frac{P}{\sqrt{T}}$ = constant (d) $\frac{P}{T}$ = constant

Sol. From Melde's law if I and n are constant then $p\sqrt{T}$ = constant.

SOUND WAVES

They are mechanical and longitudinal waves. They propagate in form of compressions and rarefactions. Particle displacements can be represented by wave function

$S = A \sin(\omega t - kx)$

As particles oscillate, pressure variation takes place according to the wave function.

$\Delta P = \Delta P_0 \cos(\omega t - kx)$, ΔP_0 = maximum pressure variation

Note : Points where displacement is maximum, pressure variation is zero i.e. there is a phase difference of 90° between pressure wave and displacement wave.

Characteristic of Sound:

Loudness :

Sensation of sound produced in human ear due to amplitude. It depends upon intensity, density of medium, presence of surrounding bodies,

(a) Intensity of sound wave :

$$I = \frac{P_0^2}{2\rho V}$$

P_0 = Pressure amplitude

ρ = density of medium

V = speed of sound in medium

(b) Intensity Level or (Sound Level) (β) :

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right) \text{ (dB)}$$

$$\left[\begin{array}{l} I_0 = \text{minimum intensity of audible sound} = 10^{-12}\,W/m^2 \\ I = \text{measured intensity} \end{array}\right]$$

Sound level range for audible sound [0 dB to 120 dB]

$\beta_2 - \beta_1 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$, Unit of sound level β is decibel (dB)

Quality: Sensation produced in human ear due to shape of wave. Quality is that characteristic of sound by which we can differentiate between the sound of same pitch and loudness coming from different sources

Pitch:

Sensation produced in human ear due to frequency

(a) Pitch is the characteristics of sound that depends on frequency.

(b) Smaller the frequency smaller the pitch, higher the frequency higher the pitch.

(c) Humming of mosquito has high pitch (high frequency) but low intensity (low loudness) while the roar of a lion has high intensity (loudness) but low pitch

VELOCITY OF SOUND IN A MEDIUM

Medium is necessary for sound propagation. These are longitudinal. In a medium, velocity of sound wave & mechanical wave depends on property of medium. It does not depends on amplitude & nature of wave.

WAVE VELOCITY IS AFFECTED BY FOLLOWING PROPERTIES OF MEDIUM

(i) Elasticity (ii) Inertia

Law of Newton:

In a medium, velocity of sound is given by

E = Elasticity coefficient of medium

d = Density of medium.

$$v = \sqrt{\frac{E}{d}}$$

(A) Velocity of Sound in Solid : for solid E = Y (elasticity coefficient of young)

$$\therefore v = \sqrt{\frac{Y}{d}}$$

(B) Velocity in Liquid : for liquid E = B (elasticity coefficient of volume),

$$v = \sqrt{\frac{B}{d}}$$

(C) Velocity of sound in gas:

(I) Newton's Law : According to it, in a gaseous medium compression & rarefaction are formed slowly during wave propagation. So medium has sufficient time for energy transfer. Thus, temperature of medium remains constant. Sound wave motion is isothermal phenomenon.

$$E = -\frac{dP}{dv/v} \text{ for isothermal } E = P$$

$$\therefore v = \sqrt{\frac{P}{d}} = \sqrt{\frac{RT}{M}}$$

Laplace's Correction:
According to it, compression & rarefactions are formed so fast that sufficient time is not available to energy transfer. Thus, sound wave motion in gaseous medium is adiabatic process.

For adiabatic process:

$$E = -\frac{dP}{dv/v} = \gamma P$$

$$\therefore v = \sqrt{\frac{\gamma P}{d}} = \sqrt{\frac{\gamma RT}{M}}$$

FACTORS AFFECTING VELOCITY OF SOUND IN GASEOUS MEDIUM

(A) Effect of pressure at constant temperature : No effect, because density increases on pressure increasing.

$$v = \sqrt{\frac{\gamma RT}{M}}$$

(B) Effect of temperature : On increasing temperature, velocity of sound increases.

$$v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{T}$$

where T = Temperature in Kelvin.

$\Rightarrow v \propto \sqrt{273+t}$,
where t = temperature in °C

For a given gas $\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{273+t_1}{273+t_2}}$,

t_1 & t_2 are in °C
If velocity of sound at 0^0C in gas is V_0 then at t°C
$v_t = v_0 + 0.61\ t$ m/sec,
where v_0 = 331 m/sec.

Important Points

(1) At t°C, on increasing temperature velocity of sound increases by 0.61 m/sec

(2) At same temperature velocity in different gases

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1 M_2}{\gamma_2 M_1}}$$

(C) Effect of Humidity: On increasing humidity, velocity of sound increases because density decreases with humidity increase. Dry air has more density than humid air.
$\therefore$ Velocity of sound in dry air is less than in humid air.

(D) Effect of amplitude & frequency: No effect. In a medium, all sound waves which falls in sonic range, travels with equal velocity. Resultant velocity of wave remain same due to superposition of these waves.
(Sonic velocity = 20Hz - 20000 Hz)

INTERFERENCE OF SOUND WAVES

When two coherent sound waves superpose in a certain region interference takes place. For waves to be coherent, the phase difference between the two waves should not change with time. This requires that the wavelength (frequency) of the two waves must be the same. Further more to have a better contrast between the maximum and minimum intensities, the two waves producing interference and should propagate in the same direction.
Consider two waves,

$y_1 = a_1 \sin(\omega t + kx)$
$y_2 = a_2 \sin(\omega t + kx + \phi)$
where ϕ is the phase difference between the two waves.

This phase difference may arise because of the path difference [$\phi = 2\pi$ (path diff)/λ] or may be some initial phase difference or may be both. Then the resultant wave obtained from their superposition is written as,

$y = y_1 + y_2$
$= a \sin(\omega t + kx + \theta)$
where the resultant amplitude a is given by,

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$$

The phase θ of the resultant wave is not of any importance in our discussions.
The intensity of the resultant wave is,
$I = Ka^2$,
or $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$
The intensity will be maximum or minimum depending on the value of ϕ.

Constructive interference: The constructive interference takes place when the phase difference between the two waves is,
$\phi = 0, 2\pi, 4\pi, 6\pi..... = 2n\pi$,
Then $\cos\phi = +1$
and the resultant wave amplitude and intensity are maximum
$a_{max} = a_1 + a_2$,
$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$
$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Destructive interference : The destrucitve interference takes place when the phase difference between the two waves is,
$\phi = 0, 3\pi, 5\pi,..... = (2n-1)\pi$,
Where n = 1, 2, 3,.....
Then the resultant wave amplitude and intensity are minimum,
$a_{min} = a_1 - a_2$
$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$
$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

Important Points

(1) If the two interfering waves have the same amplitudes, then

$I_{max} = 4I_0$, $a_{max} = 2a_0$.

$I_{min} = 0$, $a_{min} = 0$

where a_0 is the amplitude and I_0 is the intensity of one wave.

(2) The condition on path difference for observing constructive or destructive interference is,

path diff = $n\lambda$ for constructive

path diff = $\left(n - \frac{1}{2}\right)\lambda$ for destructive

PHASE DIFFERENCE & PATH DIFFERENCE

If the shape of wave does not change as the wave propagates in a medium, with increase in t, x will also increase in such a way that

$$\omega t - kx = \text{constant}$$

The argument of harmonic function ($\omega t - kx$) is called phase of the wave and is constant if the shape of wave remains unchanged. Further if we consider two points at position x_1 and x_2 on a wave at a given instant then,

$\phi_1 = \omega t - kx_1$ and $\phi_2 = \omega t - kx_2$,

so $\phi_2 \sim \phi_1 = k(x_2 - x_1)$

$$\text{i.e., } \Delta\phi = \frac{2\pi}{\lambda}\Delta x \left[\text{as } k = \frac{2\pi}{\lambda}\right]$$

From this it is clear that if $\Delta x = \lambda$, $\Delta\phi = 2\pi$, i.e., a path difference λ corresponds to a phase change of (2π) rad.

STATIONARY WAVES IN SOUND

(1) Two progressive wave of equal amplitude & frequency, travelling in opposite direction are superimposed. Due to superposition one new waves form which do not travel in either direction, this wave is called stationary wave.

(2) Energy do not propagate in medium.

(3) These waves extend & contract relative to its mean position.

CHARACTERISTICS OF STATIONARY WAVES

(1) Certain point in the bounded medium, situated at equal distances; are always in the position of rest, i.e., their displacement remain zero. These points are called nodes. If the stationary waves are longitudinal then at these nodes the change in pressure and density is maximum as compared to other points.

(2) The displacement of the midpoints between the nodes is always maximum as compared to other points. These points are called antinodes. In the longitudinal stationary waves there is no change in pressure & density at these points.

(3) All points between two successive nodes vibrate in the same phase. They reach simultaneously their positions of maximum displacement and pass simultaneously through their mean position.

(4) Except nodes, all points of the medium vibrate but the amplitude of vibration is different from one point to the other. It is zero at the nodes and maximum at the antinodes.

(5) The distance between two consecutive nodes, or between two consecutive antinodes is $\lambda/2$. The distance between a node and its neighbouring antinode is $\lambda/4$.

(6) At any instant, the phase of vibration of the points on one side of a node is opposite from the phase of vibration of the points on the other side.

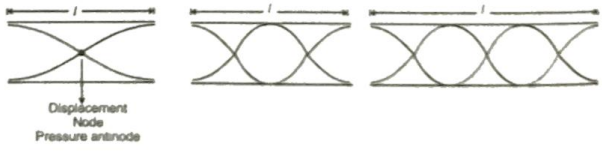
(7) All points of the medium pass through their mean positions simultaneously twice in each period, that is the stationary wave takes the form of a straight line twice.

(8) In longitudinal stationary waves, the nodes are found alternately in the state of maximum compression and maximum rarefaction twice in each period.

VIBRATIONS OF AIR COLUMNS IN PIPES

(1) Organ Pipe: In this case, longitudinal stationary waves are formed

(a) Open organ Pipe:

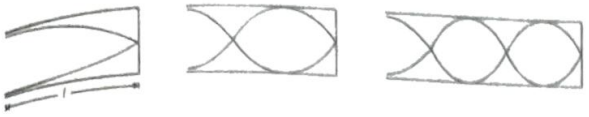


$l = \frac{\lambda}{2}$ or $\lambda = 2l$ $\quad l = \lambda$ $\quad l = \frac{3\lambda}{2}$ or $\lambda = \frac{2l}{3}$

$v_1 = \frac{V}{\lambda} = \frac{V}{2l}$ $\quad v = \frac{V}{l} = 2v_1$ $\quad v_0 = \frac{3V}{2l} = 3v_1$

1st harmonic or Fundamental mode	2nd harmonic 1st overtone	3rd harmonic 2nd overtone

(2) Closed organ pipe :



$l = \frac{\lambda}{4} \Rightarrow \lambda = 4l$ $\quad l = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4l}{3}$ $\quad l = \frac{5\lambda}{4}$

$\lambda = \frac{4l}{5}$

$\nu = \frac{V}{4l}$ $\quad \nu = \frac{3V}{4l}$ $\quad \nu = \frac{5V}{4l}$

Fundamental mode, I[st] harmonic | I[st] overtone, 3rd harmonic | 2nd overtone, 5th harmonic

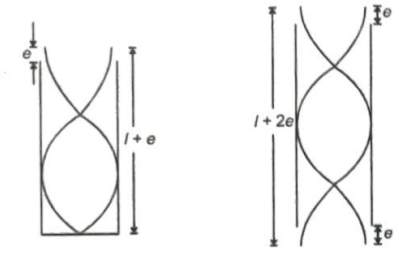
Pipe length 'l'	Fundamental Mode	Ist Overtone	(n – 1)[th] overtone	Ratio of Successive frequency
Open	$\nu = \frac{V}{2l}$ I[st] Harmonic	$\nu = \frac{V}{l}$ 2nd Harmonic	$\nu = n\frac{V}{2l}$ n^{th} Harmonic	1 : 2 : 3 : 4
Closed	$\nu = \frac{V}{4l}$ I[st] Harmonic	$\nu = \frac{3V}{4l}$ 3rd Harmonic	$\nu = (2n-1)\frac{V}{4l}$ $(2n-1)^{th}$ Harmonic	1 : 3 : 5 : 7

Note : Even numbered (i.e., 2nd, 4th .....) harmonics do not exist in close organ pipe.

End correction:

As the antinodes are formed slightly out side the open end.
e = 0.6r = end correction. Thus, we have,

For closed organ pipe For open organ pipe



$\nu = \frac{(2n-1)V}{4(l+e)}$ $\quad \nu = \frac{nV}{2(l+2e)}$

BEATS

It is the phenomenon of periodic variation in intensity at a particular position on account of superposition of wave of nearly equal frequencies. When two waves of same amplitude and nearly equal frequencies ν_1 and ν_2 superimpose on each other.

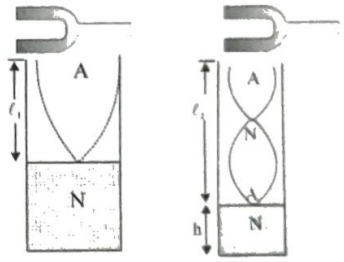
(1) The amplitude at a given position varies with frequency $\left|\frac{\nu_1 - \nu_2}{2}\right|$.

(2) The intensity at a given position varies with frequency $|\nu_1 - \nu_2|$. This frequency of variation of intensity is called beat frequency.

(3) Frequency of the resulting wave is $(\nu_1 + \nu_2)/2$.

Note : To observe the phenomenon of beats, beat frequency should be less than 10 Hz.

TO FIND VELOCITY OF SOUND RESONANCE TUBE METHOD



(a) (b)

A resonance column or a resonance tube is merely a pipe closed at one end with a convenient provision for varying the length of the column. A simple form is shown in figure, where the portion of the pipe above water level acts as the resonance tube. The surface of water inside the pipe serves as a closed end of the air column.

A tuning fork of known frequency is struck with a rubber or cork hammer and is held near the mouth of the tube. The water level is gradually adjusted till for a particular position the air column resonates.

For resonance, the natural frequency of the air column must be equal to that of the tuning fork (n_t). If ℓ_1 is the length of the resonating column, then if end corrections are ignored,

$\ell_1 = \frac{\lambda}{4}$

and $n = \frac{V}{4\ell_1} = n_t$.

Thus a knowledge of n_t and ℓ_1 determines the velocity of sound.

$V = n_t\lambda$.

$= n_t(4\ell_1)$

However, if end correction is not neglected, then the true length that appears in the formula will be,

$$L = \ell_1 + e = \frac{\lambda}{4}$$

So that unless e is known, V can not be determined. To eliminate e, the experiment is repeated for a greater length. This time resonance occurs for the mode shown in fig (b) i.e., when n_t = frequency of the first overtone. Let now the length is ℓ_2 then,

$$L' = \ell_2 + e = \frac{3\lambda}{4}$$

From the relations (ignoring subscript t on n_t.)

$$n = \frac{V}{4L} \text{ and } \quad n = 3\frac{V}{4L}$$

we expect $L = \frac{L'}{3}$

or $\ell_1 + e = \frac{\ell_2 + e}{3}$

This gives,

$$e = \frac{(\ell_2 - 3\ell_1)}{2}$$

and, velocity of sound is determined from the formula

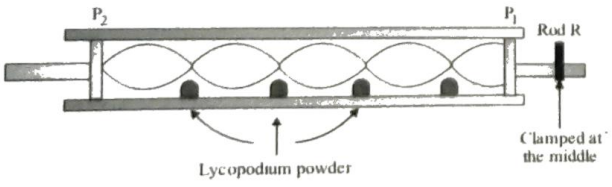
$$n = \frac{V}{2(\ell_2 - \ell_1)}$$

The wavelength of the sound in the air column is,

$\lambda = 2(\ell_2 - \ell_1)$

KUNDT'S TUBE METHOD

The apparatus consists of a long glass tube about 5cm in diameter, fixed horizontally. A metal rod R clamped firmly at the centre is mounted so that its one end carrying a light disc P_1 (of cork or card board) projects some distance into the glass tube. The other, end of the glass tube is closed with a movable piston P_2. Any desired length of the air or gas can be enclosed in between the two discs P_1 and P_2. A small amount of dry lycopodium powder or cork dust is spread along base of the entire length of the tube.



The free end of the metal rod R is rubbed (stroked) along the length with resined cloth. The rod begins to vibrate longitudinaly and emits a very high pitched shrill note. These vibrations are impressed upon the air column in the tube through disc P_1. Let disc P_2 is so adjusted, that the stationary waves are formed in the air (gas) column in the tube. At antinodes powder is set into oscillations vigorously while it remains uneffected at nodes. Heaps of powder are formed at nodes.

(a) Theory of Kundt's tube: Let n is the frequency of vibration of the rod then, this is also the frequency of sound wave in the air column in the tube.

For rod : $\frac{\lambda_{rod}}{2} = \ell_{rod}$

For air : $\frac{\lambda_{air}}{2} = \ell_{air}$

Where ℓ_{air} is the distance between two heaps of powder in the tube (i.e. distance between two nodes). If v_{air} and v_{rod} are velocity of sound waves in the air and rod respectively, then

$$n = \frac{V_{air}}{\lambda_{air}} = \frac{V_{rod}}{\lambda_{rod}}$$

Therefore, $\frac{V_{air}}{V_{rod}} = \frac{\lambda_{air}}{\lambda_{rod}}$

$$= \frac{\ell_{air}}{\ell_{rod}}$$

Thus knowledge of v_{rod}, determines v_{air}.

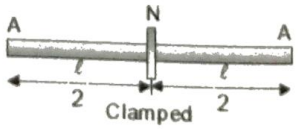
Kundt's tube may be used for

(i) Comparison of velocities of sound in different gases
(ii) Comparison of velocities of sound in different solids.
(iii) Comparison of velocities of sound in a solid and in a gas
(iv) Comparison of density of two gases
(v) Determination of γ of a gas
(vi) Determination of velocity of sound in a liquid

LONGITUDINAL VIBRATIONS OF ROD

(A) Rod free at both ends:

A rod clamped at the middle and free at both ends, vibrate longitudinally with the node in middle and antinodes at the free end. The first two possible modes are shown in figure. The fundamental mode (lowest frequency note) is shown in figure.(a)



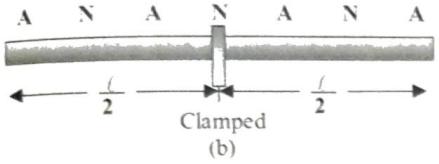
(a)

(a) $\frac{2\lambda_1}{4} = \ell \Rightarrow \lambda_1 = 2\ell$

Its frequency is, $n_1 = \frac{V}{\lambda_1} = \frac{V}{2\ell}$

(b) $\frac{6\lambda_2}{4} = \ell \Rightarrow \lambda_2 = 2\ell / 3$

The next mode has a frequency of,
(See in fig (b))



(b)

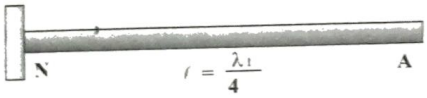
$$n_2 = \frac{V}{\lambda_2} = \left(\frac{V}{2\ell/3}\right)$$

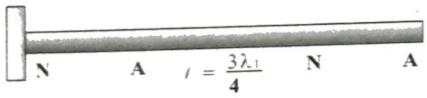
$n_2 = 3n_1$

(B) Rod clamped at one end :

Fundamental mode frequency

$$n_1 = \frac{V}{\lambda_1} = \frac{V}{4\ell}$$





First overtone frequency

$$n_2 = \frac{V}{\lambda_2} = 3\frac{V}{4\ell}$$

$n_2 = 3n_1$

Thus again the overtone frequencies are 3,5,7,.....times that of the fundamental.

DOPPLER'S EFFECT

If a wave source and a receiver are moving relative to each other, the frequency observed by the receiver (f) is different from the actual source frequency (f_0) given by,

$$f = f_0\left(\frac{v \pm v_0}{v \mp v_s}\right)$$

Where v = speed of sound, v_0 = speed of observer, v_s = speed of source

Various cases:

1. Source at rest, observer moves :

(a) Observer moves away from source,

$$f = f_0\left(\frac{v - v_0}{v}\right)$$

(b) Observer moves towards source.

$$f = f_0\left(\frac{v + v_0}{v}\right)$$

2. Observer at rest, source moves :

(a) Source moves towards observer, $f = f_0\left(\frac{v}{v - v_s}\right)$

(b) Source moves away from observer.

$$f = f_0\left(\frac{v}{v + v_s}\right)$$

3. Both move :

(a) Both approaching each other

$$f = f_0\left(\frac{v + v_0}{v - v_S}\right)$$



(b) Source following the observer

$$f = f_0\left(\frac{v - v_0}{v - v_S}\right)$$



(c) Observer following the source

$$f = \left(\frac{v + v_0}{v + v_S}\right) f_0$$



(d) Both moving away from each other

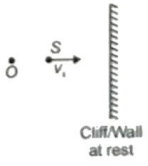
$$f = \left(\frac{v - v_0}{v + v_S}\right) f_0$$



Application of beats and Doppler Effect :

Case - I :

f = frequency of source
v_s = velocity of source
f' = frequency of direct sound
v = velocity of sound
f'' = frequency of reflected sound



$f' = \left(\frac{v}{v + v_s}\right) f$ (source moving away)

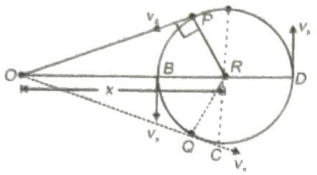
$f'' = \left(\frac{v}{v - v_s}\right) f$ (source moving towards)

Beat frequency =

$f'' - f' = \left(\frac{v}{v - v_s} - \frac{v}{v + v_s}\right) f = \left(\frac{v \times 2v_s}{v^2 - v_s^2}\right) f$

Case - II:

A source of frequency 'f ' is revolving in a circle of radius R with speed v_s. An observer is standing at a distance x from the centre.



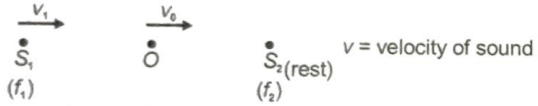
At B and D, observed frequency is 'f ' . At 'P' frequency is maximum as OP ⊥ PR, i.e.,

$$f' = \left(\frac{v}{v - v_s}\right) f \quad OP = \sqrt{OR^2 - PR^2} = \sqrt{x^2 - R^2}$$

At Q frequency is minimum as OQ ⊥ QR, i.e.,

$$f'' = \left(\frac{v}{v + v_s}\right) f$$

Case - III :



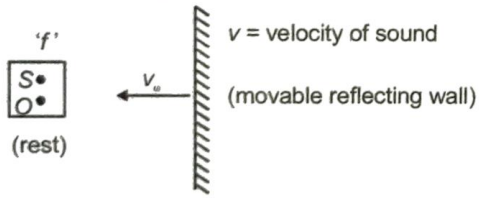
$$f_1' = \left(\frac{v - v_0}{v - v_1}\right) f_1$$

$$f_2' = \left(\frac{v + v_0}{v}\right) f_2 \quad \text{beat frequency} = f_1' - f_2'$$

Case - IV :

Direct frequency = f

$$\text{Reflected frequency} = \left(\frac{v + v_\omega}{v - v_\omega}\right) f$$



$$\text{Beat frequency} = -f\left(\frac{v + v_\omega}{v - v_\omega} - 1\right) = f\left(\frac{2v_\omega}{v - v_\omega}\right)$$

DOPPLER'S EFFECT IN LIGHT

The velocity of light in free space is independent of the motion of source or observer and it is a universal constant given as $c = 3 \times 10^8$ m/s. Thus the doppler's effect in light depends only upon the relative motion of the light source and the observer and it does not matter which one is moving.

(i) When a light source and an observer are approaching each other with a velocity v, then the apparent frequency of light will be

$$\nu' = \nu$$

and $\nu' = \nu\,(1 - v/c)$ if $v << c$

or $\Delta\nu = -\nu\, v/c$

Case (a) When the light source is going away from the earth then

$\Delta\nu =$ ν

$\nu' < \nu$ and $\Delta\lambda =$ λ or $\lambda' > \lambda$ i.e. λ is increased or spectral line will shift towards the red end of the spectrum. This is known as red shift.

Case (b) When the light source is coming nearer to earth.

$\Delta\nu = \nu$ or $\nu' > \nu$

and $\Delta\lambda = -\lambda$ or $\lambda' < \lambda$

⇒ wavelength appears to be decreasing i.e. the spectral line in electromagnetic spectrum gets displaced towards violet end, hence it is known as violet shift.

APPLICATION OF DOPPLER EFFECT

(i) To determine the width of spectral lines (ΔW).
(ii) To determine the velocity of stars and galaxies.
(iii) To determine the frequency of a tuning fork.
(iv) To determine the speed of rotation of sun.
(v) In the discovery of Saturn rings.
(vi) In the discovery of twin stars.
(vii) In E.C.G.
(viii) In very high frequency unidirectional radio ranging (V.O.R.)
(ix) In tracking of artificial satellites.
(x) In RADAR
(xi) In SONAR.

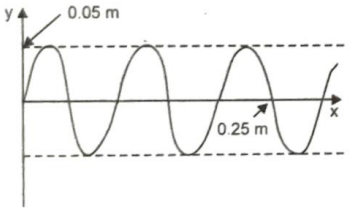
Topicwise Questions

STRING WAVES
WAVE EQUATION

1. The property of a medium necessary for wave propagation is -
(*a*) Its inertia
(*b*) Its elasticity
(*c*) Its low resistance
(*d*) All of the above

2. A sine wave has amplitude A and wavelength λ. If V is the wave velocity and v be the maximum particle velocity, then
(*a*) V = v if $A = \frac{\lambda}{2\pi}$
(*b*) V = v if $A = 2\pi\lambda$
(*c*) V = v if $A = \pi\lambda$
(*d*) V can never be equal to v.

3. If the speed of the wave shown in the figure is 330 m/s in the given medium, then the equation of the wave propagating in the positive x-direction will be - (all quantities are in MKS units)



(*a*) $y = 0.05 \sin 2\pi (4000t - 12.5x)$
(*b*) $y = 0.05 \sin 2\pi (4000t - 122.5x)$
(*c*) $y = 0.05 \sin 2\pi (3300t - 10x)$
(*d*) $y = 0.05 \sin 2\pi (3300x - 10t)$

4. The distance between two consecutive crests in a wave train produced in a string is 5 cm. If 2 complete waves pass through any point per second, the velocity of the wave is
(*a*) 10 cm/sec (*b*) 2.5 cm/sec
(*c*) 5 cm/sec (*d*) 15 cm/sec

5. The phase difference between two points separated by 1m in a wave of frequency 120 Hz is 90°. The wave velocity is
(*a*) 180 m/s (*b*) 240 m/s
(*c*) 480 m/s (*d*) 720 m/s

PROGRASSIVE

6. A plane wave is represented by
$x = 1.2\sin(314t + 12.56y)$
Where x and y are distances measured along in x and y direction in meters and t is time in seconds. This wave has
(*a*) A wavelength of 0.25 m and travels in + ve x direction
(*b*) A wavelength of 0.25 m and travels in + ve y direction
(*c*) A wavelength of 0.5 m and travels in - ve y direction
(*d*) A wavelength of 0.5 m and travels in - ve x direction

7. Wave equations of two particles are given by $y_1 = a\sin(\omega t - kx)$, $y_2 = a\sin(kx + \omega t)$, then
(*a*) They are moving in opposite direction
(*b*) Phase between them is 90°
(*c*) Phase between them is 180°
(*d*) Phase between them is 0°

8. If the wave equation $y = 0.08\sin\frac{2\pi}{\lambda}(200t - x)$ then the velocity of the wave will be
(*a*) $400\sqrt{2}$ (*b*) $200\sqrt{2}$
(*c*) 400 (*d*) 200

VELOCITY OF WAVE

9. A progressive wave is represented by $y = 0.1 \sin \frac{8\pi}{7}\left(0.1t - \frac{x}{20}\right)$. Where all the observations are in MKS system. The wave velocity will be -
(*a*) 2m/sec (*b*) 15 m/sec
(*c*) 20 m/sec (*d*) 40 m/sec

10. The equation of plane progressive wave propagating in positive X-direction is $y = a \sin 2\pi\left[t - \frac{x}{\lambda}\right]$. If the maximum particle velocity is three time the wave velocity then the wavelength of wave is
(*a*) $\frac{\pi a}{3}$ (*b*) πa
(*c*) $\frac{\pi a}{2}$ (*d*) $\frac{2\pi a}{3}$

11. When a sound wave of frequency 300Hz passes through a medium, the maximum displacement of a particle of the medium is 0.1 cm. The maximum velocity of the particle is equal to :
(*a*) 60 π cm/sec (*b*) 30 π cm/sec
(*c*) 30 cm/sec (*d*) 60 cm /sec

INTERFERENCE OF WAVE

12. The amplitude of a wave represented by displacement equation : $y = \frac{1}{\sqrt{a}} \sin \omega t \pm \frac{1}{\sqrt{b}} \cos \omega t$ will be -

(a) $\frac{a+b}{ab}$ (b) $\frac{\sqrt{a}+\sqrt{b}}{ab}$

(c) $\frac{\sqrt{a} \pm \sqrt{b}}{ab}$ (d) $\sqrt{\frac{a+b}{ab}}$

13. The resultant amplitude, when two waves of same frequency but with amplitudes a_1 and a_2 superimpose at phase difference of $\frac{\pi}{2}$, will be -

(a) $a_1 + a_2$ (b) $a_1 - a_2$

(c) $\sqrt{a_1^2 + a_2^2}$ (d) $a_1^2 + a_2^2$

14. The ratio of sound intensities of two waves of the same frequency is 1 : 16. Then the ratio of the amplitudes will be -

(a) 1 : 2 (b) 1 : 4

(c) 1 : 8 (d) 1 : 16

15. If two progressive waves of amplitude A_1 and A_2 superpose, while moving in the same direction, then the amplitude of the resultant wave will be -

(a) $A_1 + A_2$

(b) $\sqrt{A_1^2 + A_2^2}$

(c) Between $A_1 + A_2$ and $A_1 - A_2$

(d) $A_1 - A_2$

STANDING WAVE

16. A stationary wave is represented by -

(a) $Y = 2A \cos Kx \sin \omega t$.

(b) $Y = 2A \sin K(x - Vt) \sin \omega t$

(c) $Y = 2A \cos Kx \cos(\omega t - Kx)$

(d) $Y = 2A \cos K(X - Vt) \cos \omega t$

17. The following equations represent progressive waves – $z_1 = A\cos(kx - \omega t)$, $z_2 = A\cos(ky + \omega t)$ and $z_3 = A\cos(kz - \omega t)$. Which two waves can produce stationary waves?

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) none of these

18. The phase difference between the two particles situated on both the sides of a node is -

(a) 0° (b) 90°

(c) 180° (d) 360°

19. Four wires of same length and same material, whose diameters are in the ratio 4 : 3 : 2 :1, are clamped in such a way that each wire produces note of fundamental frequency double that of the preceding wire. If the tension in the first wire is 2 Kg-wt, then tension in the second wire will be -

(a) 4.5 (b) 8

(c) 9 (d) 16

20. In Melde's experiment, 8 loops are formed with a tension of a 0.75 N. If the tension is increased to four times then the number of loops produced will be -

(a) 4 (b) 8

(c) 2 (d) 12

21. A 1 cm long string vibrates with fundamental frequency of 256 Hz. If the length is reduced to $\frac{1}{4}$ cm keeping the tension unaltered, the new fundamental frequency will be

(a) 64 (b) 256

(c) 512 (d) 1024

22. A string is producing transverse vibration whose equation is $y = 0.021 \sin(x + 30t)$. Where x and y are in metres and t is in t seconds. If the linear density of the string is 1.3×10^{-4} kg/m, then the tension in the string in N will be

(a) 10 (b) 0.5

(c) 1 (d) 0.117

23. If vibrations of a string are to be increased by a factor of two, then tension in the string must be made

(a) Half (b) Twice

(c) Four times (d) Eight times

24. In order to double the frequency of the fundamental note emitted by a stretched string, the length is reduced to $\frac{3}{4}$th of the original length and the tension is changed. The factor by which the tension is to be changed, is

(a) $\frac{3}{8}$ (b) $\frac{2}{3}$

(c) $\frac{8}{9}$ (d) $\frac{9}{4}$

SOUND WAVES

SOUND PROPAGATION AND PRESSURE WAVE

25. A stone is dropped into a lake from a tower 500 metre high. The sound of the splash will be heard by the man approximately after

(a) 11.5 seconds (b) 21 seconds

(c) 10 seconds (d) 14 seconds

26. When sound waves travel from air to water, which of the following remains constant

(a) Velocity (b) Frequency

(c) Wavelength (d) All of the above

27. If velocity of sound in a gas is 360 m/s and the distance between a compression and the nearest rarefaction is 1m, then the frequency of sound is

(a) 90 Hz (b) 180 Hz

(c) 360 Hz (d) 720 Hz

28. The frequency of a running fork is 384 per second and velocity of sound in air is 352 m/s. How far the sound has traversed while fork completes 36 vibration
(a) 3 m (b) 13 m
(c) 23 m (d) 33 m

29. Sound travels in rocks in the form of
(a) Longitudinal elastic waves only
(b) Transverse elastic waves only
(c) Both longitudinal and transverse elastic waves
(d) Non-elastic waves

30. The waves in which the particles of the medium vibrate in a direction perpendicular to the direction of wave motion is known as
(a) Transverse wave (b) Longitudinal waves
(c) Propagated waves (d) None of these

31. If amplitude of waves at distance r from a point source is A, the amplitude at a distance 2r will be
(a) 2A (b) A
(c) A/2 (d) A/4

32. As a wave propagates choose **incorrect** statement :
(a) The wave intensity remains constant for a plane wave
(b) The wave intensity decreases as the inverse of the distance from the source for a spherical wave
(c) The wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(d) Total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times

33. With the propagation of a longitudinal wave through a material medium, the quantities transmitted in the propagation direction are
(a) Energy, momentum and mass
(b) Energy
(c) Energy and mass
(d) Energy and linear momentum

VELOCITY OF SOUND WAVE, ENERGY AND INTENSITY

34. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is
(a) $\sqrt{2/7}$ (b) $\sqrt{1/7}$
(c) $\sqrt{3}/5$ (d) $\sqrt{6}/5$

35. At what temperature velocity of sound is double than that of at 0°C
(a) 819 K (b) 819°C
(c) 600°C (d) 600 K

36. At which temperature the speed of sound in hydrogen will be same as that of speed of sound in oxygen at 100°C
(a) – 148°C (b) 212.5°C
(c) – 317.5°C (d) – 249.7°C

37. When the temperature of an ideal gas is increased by 600 K, the velocity of sound in the gas becomes $\sqrt{3}$ times the initial velocity in it. The initial temperature of the gas is
(a) 73°C (b) 27°C
(c) 127°C (d) 327°C

38. The frequency of a sound wave is n and its velocity is v. If the frequency is increased to 4n, the velocity of the wave will be
(a) v (b) 2v
(c) 4v (d) v/4

39. Quality of a musical note depends on
(a) Harmonics present
(b) Amplitude of the wave
(c) Fundamental frequency
(d) Velocity of sound in the medium

40. The loudness and pitch of a sound depends on
(a) Intensity and velocity
(b) Frequency and velocity
(c) Intensity and frequency
(d) Frequency and number of harmonics

41. If separation between screen and point source is increased by 2% what would be the effect on the intensity
(a) Increases by 4% (b) Increases by 2%
(c) Decreases by 2% (d) Decreases by 4%

42. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at distances of 2m and 3m respectively from the source. The ratio of the intensities of the waves at Pand Q is
(a) 9 : 4 (b) 2 : 3
(c) 3 : 2 (d) 4 : 9

INTERFERENCE AND REFLECTION OF SOUND WAVE

43. When two sound waves with a phase difference of $\pi/2$, and each having amplitude A and frequency ω, are superimposed on each other, then the maximum amplitude and frequency of resultant wave is
(a) $\frac{A}{\sqrt{2}} : \frac{\omega}{2}$ (b) $\frac{A}{\sqrt{2}} : \omega$
(c) $\sqrt{2}A : \frac{\omega}{2}$ (d) $\sqrt{2}A : \omega$

44. If the phase difference between the two wave is 2π during superposition, then the resultant amplitude is
(a) Maximum
(b) Minimum
(c) Maximum or minimum
(d) None of these

45. Two waves of same frequency and intensity superimpose with each other in opposite phases, then after superposition the
(a) Intensity increases by 4 times
(b) Intensity increases by two times
(c) Frequency increases by 4 times
(d) None of these

VIBRATION IN AIR COLUMN AND BEAT

46. The frequencies of two sound sources are 256 Hz and 260 Hz. At t = 0, the intensity of sound is maximum. Then the phase difference at the time t = 1/16 sec will be

(*a*) Zero (*b*) π

(*c*) $\pi/2$ (*d*) $\pi/4$

47. The beats are produced by two sound sources of same amplitude and of nearly equal frequencies. The maximum intensity of beats will be ..... that of one source

(*a*) Same (*b*) Double

(*c*) Four times (*d*) Eight times

48. A tuning fork whose frequency as given by manufacturer is 512 Hz is being tested with an accurate oscillator. It is found that the fork produces a beat of 2 Hz when oscillator reads 514 Hz but produces a beat of 6 Hz when oscillator reads 510 Hz. The actual frequency of fork is

(*a*) 508 Hz (*b*) 512 Hz

(*c*) 516 Hz (*d*) 518 Hz

49. When temperature increases, the frequency of a tuning fork

(*a*) Increases

(*b*) Decreases

(*c*) Remains same

(*d*) Increases or decreases depending on the material

50. For the stationary wave $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\,\pi t)$, the distance between a node and the next antinode is

(*a*) 7.5 (*b*) 15

(*c*) 22.5 (*d*) 30

51. At a certain instant a stationary transverse wave is found to have maximum kinetic energy. The appearance of string at that instant is

(*a*) Sinusoidal shape with amplitude A/3

(*b*) Sinusoidal shape with amplitude A/2

(*c*) Sinusoidal shape with amplitude A

(*d*) Straight line

52. A standing wave having 3 nodes and 2 antinodes is formed between two atoms having a distance 1.21 Å between them. The wavelength of the standing wave is

(*a*) 1.21 Å (*b*) 2.42 Å

(*c*) 6.05 Å (*d*) 3.63 Å

53. At nodes in stationary waves

(*a*) Change in pressure and density are maximum

(*b*) Change in pressure and density are minimum

(*c*) Strain is zero

(*d*) Energy is minimum

54. If the velocity of sound in air is 350 m/s. Then the fundamental frequency of an open organ pipe of length 50 cm, will be

(*a*) 350 Hz (*b*) 175 Hz

(*c*) 900 Hz (*d*) 750 Hz

55. An organ pipe P_1 closed at one end vibrating in its first overtone and another pipe P_2 open at both ends vibrating in its third overtone are in resonance with a given tuning fork. The ratio of lengths of P_1 and P_2 is

(*a*) 1 : 2 (*b*) 1 : 3

(*c*) 3 : 8 (*d*) 3 : 4

56. Two closed pipe produce 10 beats per second when emitting their fundamental nodes. If their length are in ratio of 25 : 26. Then their fundamental frequency in Hz, are

(*a*) 270, 280 (*b*) 260, 270

(*c*) 260, 250 (*d*) 260, 280

57. Fundamental frequency of pipe is 100 Hz and other two frequencies are 300 Hz and 500 Hz then

(*a*) Pipe is open at both the ends

(*b*) Pipe is closed at both the ends

(*c*) One end open and another end is closed

(*d*) None of these

DOPPLER'S EFFECT

58. Doppler shift in frequency does not depend upon

(*a*) The frequency of the wave produced

(*b*) The velocity of the source

(*c*) The velocity of the observer

(*d*) Distance from the source to the listener

59. A source of sound of frequency 450 cycles/sec is moving towards a stationary observer with 34 m/sec speed. If the speed of sound is 340 m/sec, then the apparent frequency will be

(*a*) 410 cycles/sec (*b*) 500 cycles/sec

(*c*) 550 cycles/sec (*d*) 450 cycles/sec

60. The speed of sound in air at a given temperature is 350 m/s. An engine blows whistle at a frequency of 1200 cps. It is approaching the observer with velocity 50 m/s. The apparent frequency in cps heard by the observer will be

(*a*) 600 (*b*) 1050

(*c*) 1400 (*d*) 2400

61. A man sitting in a moving train hears the whistle of the engine. The frequency of the whistle is 600 Hz

(*a*) The apparent frequency as heard by him is smaller than 600 Hz

(*b*) The apparent frequency is larger than 600 Hz

(*c*) The frequency as heard by him is 600 Hz

(*d*) None of these

62. What should be the velocity of a sound source moving towards a stationary observer so that apparent frequency is double the actual frequency (Velocity of sound is v)

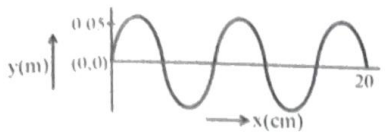
(*a*) v (*b*) 2v

(*c*) $\frac{v}{2}$ (*d*) $\frac{v}{4}$

Learning Plus

STRING WAVES

1. For the wave shown in figure, the equation for the wave, travelling along + x axis with velocity 350 ms $^{-1}$ when its position at t = 0 is as shown



(*a*) 0.05 sin ($\frac{314}{4}$ x – 27475 t)

(*b*) 0.05 sin ($\frac{379}{5}$ x – 27000 t)

(*c*) 1 sin ($\frac{314}{4}$ x – 27500 t)

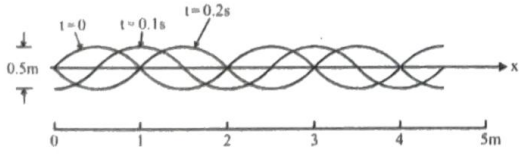
(*d*) 0.05 sin ($\frac{289}{5}$ x + 25700 t)

2. A transverse wave is described by the equation $Y = Y_0 \sin 2\pi (ft - x/\lambda)$. The maximum particle velocity is equal to four times the wave velocity if

(*a*) $\lambda = \pi Y_0/4$ (*b*) $\lambda = \pi Y_0/2$

(*c*) $\lambda = \pi Y_0$ (*d*) $\lambda = 2\pi Y_0$

3. Three consecutive flash photographs of a travelling wave on a string are reproduced in the figure here. The following observations are made. Mark the one which is correct. (Mass per unit length of the string = 3 g/cm.)



(*a*) displacement amplitude of the wave is 0.25 m, wavelength is 1 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.

(*b*) displacement amplitude of the wave is 2.0 m, wavelength is 2 m, wave speed is 0.4 m/s and the frequency of the driving force is 0.7/s.

(*c*) displacement amplitude of the wave is 0.25 m, wavelength is 2 m, wave speed is 5 m/s and the frequency of the driving force is 2.5 /s.

(*d*) displacement amplitude of the wave is 0.5 m, wavelength is 2 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.

4. The plane wave represented by an equation of the form y = f(x vt) implies the propagation along the positive x-axis without change of shape with constant velocity v:

(*a*) $\frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial x}\right)$ (*b*) $\frac{\partial y}{\partial t} = -v\left(\frac{\partial^2 y}{\partial x^2}\right)$

(*c*) $\frac{\partial^2 y}{\partial t^2} = -v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$ (*d*) $\frac{\partial^2 y}{\partial t^2} = v^2 /\left(\frac{\partial^2 y}{\partial x^2}\right)$

5. The displacement of a particle in a medium due to a wave travelling in the x-direction through the medium is given by $y = A \sin(\alpha t - \beta x)$, where t = time, and α and β are constants. Choose the **incorrect** statement :

(*a*) the frequency of the wave is α

(*b*) the frequency of the wave is $\alpha/2\pi$

(*c*) the wavelength is $2\pi/\beta$

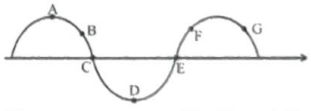
(*d*) the velocity of the wave is α/β

6. The velocity of a wave propagating along a stretched string is 10m/s and its frequency is 100 Hz. The phase difference between the particles situated at a distance of 2.5 cm on the string will be -

(*a*) $\pi/8$ (*b*) $\pi/4$

(*c*) $3\pi/8$ (*d*) $\pi/2$

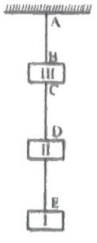
7. The following figure depicts a wave travelling in a medium. Which pair of particles are in phase.



(*a*) A and D (*b*) B and F

(*c*) C and E (*d*) B and G

8. Three blocks I, II, & III having mass of 1.6 kg, 1.6 kg and 3.2 kg respectively are connected as shown in the figure. The linear mass density of the wire AB, CD and DE are 10 g/m, 8 g/m and 10 g/m respectively. The speed of a transverse wave pulse produced in AB, CD and DE are : ($g = 10$ m/sec^2)



(*a*) 80 m/s, $20\sqrt{10}$ m/s, 40 m/s

(*b*) $20\sqrt{10}$ m/s, 80 m/s, 40 m/s

(*c*) $20\sqrt{10}$ m/s in all

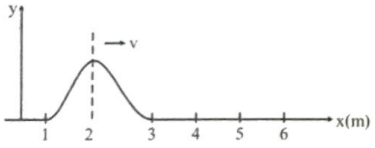
(*d*) 80 m/s in all

9. A copper wire is held at the two ends by rigid supports. At 30°C the wire is just taut, with negligible tension. The speed of transverse waves in this wire at 10°C is : ($\alpha = 1.7 \times 10^{-5}$/°C, $Y = 1.3 \times 10^{11}$ N/m^2, $d = 9 \times 10^3$ kg/m^3).

(*a*) 80 m/sec (*b*) 90 m/sec

(*c*) 100 m/sec (*d*) 70 m/sec

10. Wave pulse on a string shown in figure is moving to the right without changing shape. Consider two particles at positions $x_1 = 1.5$ m and $x_2 = 2.5$ m. Their transverse velocities at the moment shown in figure are along directions :



(*a*) positive y–axis and positive y–axis respectively

(*b*) negative y–axis and positive y–axis respectively

(*c*) positive y–axis and negative y–axis respectively

(*d*) negative y–axis and negative y–axis respectively

11. The displacement produced by a simple harmonic wave is :

$y = \frac{10}{\pi}\sin\left(2000\pi t - \frac{\pi x}{17}\right)$ cm. The time period and maximum velocity of the particle will be respectively

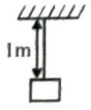
(*a*) 10^{-3} second and 200 m/s

(*b*) 10^{-2} second and 2000 m/s

(*c*) 10^{-3} second and 330 m/s

(*d*) 10^{-4} second and 20 m/s

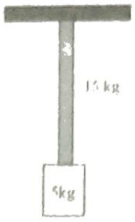
12. A block of mass 1 kg is hanging vertically from a string of length 1 m and Mass/length =0.001 kg/m. A small pulse is generated at its lower end. The Pulse reaches the top end in approximately.



(*a*) 0.2 sec (*b*) 0.1 sec

(*c*) 0.02 sec (*d*) 0.01 sec

13. A uniform rope of length 10 m and mass 15 kg hangs vertically from a rigid support. A block of mass 5 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.08 m is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope will be-



(*a*) 0.08 m (*b*) 0.04 m

(*c*) 0.16 m (*d*) 0 m

14. A wire of 10^{-2} kg m^{-1} passes over a frictionless light pulley fixed on the top of a frictionless inclined plane, which makes an angle of 30^0 with the horizontal. Masses m and M are tied at two ends of wire such that m rests on the plane and M hangs freely vertically downwards. The entire system is in equilibrium and a transverse wave propagates along the wire with a velocity of $100\,ms^{-1}$.

(*a*) M = 5 kg (*b*) $\frac{m}{M} = \frac{1}{4}$ kg

(*c*) m=20 kg (*d*) $\frac{m}{M} = 4$

15. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a average power of 90 W and the string is under a tension of 100 N, then the highest frequency at which the source can operate is (take $\pi^2 = 10$):

(*a*) 45 Hz (*b*) 50 Hz

(*c*) 30 Hz (*d*) 62 Hz

16. The rate of transfer of energy in a wave depends

(*a*) directly on the square of the wave amplitude and square of the wave frequency

(*b*) directly on the square of the wave amplitude and square root of the wave frequency

(*c*) directly on the wave frequency and square of the wave amplitude

(*d*) directly on the wave amplitude and square of the wave frequency

17. For a wave displacement amplitude is 10^{-8} m, density of air 1.3 kg m^{-3}, velocity in air 340 ms^{-1} and frequency is 2000 Hz. The intensity of wave is -

(*a*) 5.3×10^{-4} Wm^{-2} (*b*) 5.3×10^{-6} Wm^{-2}

(*c*) 3.5×10^{-8} Wm^{-2} (*d*) 3.5×10^{-6} Wm^{-2}

18. When two waves of the same amplitude and frequency but having a phase difference of ϕ, travelling with the same speed in the same direction (positive x), interfere, then
(*a*) their resultant amplitude will be twice that of a single wave but the frequency will be same
(*b*) their resultant amplitude and frequency will both be twice that of a single wave
(*c*) their resultant amplitude will depend on the phase angle while the frequency will be the same
(*d*) the frequency and amplitude of the resultant wave will depend upon the phase angle.

19. Two wave pulses travel in opposite directions on a string and approach each other. The shape of the one pulse in inverted with respect to the other.
(*a*) the pulses will collide with each other and vanish after collision.
(*b*) the pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.
(*c*) the pulses will pass through each other but their shapes will be modified
(*d*) the pulses will pass through each other without any change in their shape.

20. A wave pulse, travelling on a two piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has speed v and the transmitted wave v′,
(*a*) $v' > v$
(*b*) $v' = v$
(*c*) $v' < v$
(*d*) nothing can be said about the relation of v and v′.

21. A wave pulse, travelling on a two piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has wavelength λ and the transmitted wave λ'.
(*a*) $\lambda' > \lambda$
(*b*) $\lambda' = \lambda$
(*c*) $\lambda' < \lambda$
(*d*) nothing can be said about the relation of λ and λ'.

22. A stretched sonometer wire resonates at a frequency of 350 Hz and at the next higher frequency of 420 Hz. The fundamental frequency of this wire is
(*a*) 350 Hz (*b*) 5 Hz
(*c*) 70 Hz (*d*) 170 Hz

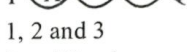
23. Two stretched wires A and B of the same lengths vibrate independently. If the radius, density and tension of wire A are respectively twice those of wire B, then the fundamental frequency of vibration of A relative to that of B is
(*a*) 1 : 1 (*b*) 1 : 2
(*c*) 1 : 4 (*d*) 1 : 8

24. A steel wire of mass 4.0 g and length 80 cm is fixed at the two ends. The tension in the wire is 50 N. The wavelength of the fourth harmonic of the fundamental will be :
(*a*) 80 cm (*b*) 60 cm
(*c*) 40 cm (*d*) 20 cm

25. A string 1m long is drawn by a 300Hz vibrator attached to its end. The string vibrates in 3 segments. The speed of transverse waves in the string is equal to
(*a*) 100 m/s (*b*) 200 m/s
(*c*) 300 m/s (*d*) 400 m/s

26. In a stationary wave represented by $y = a \sin \omega t \cos kx$, amplitude of the component progressive wave is :
(*a*) $\frac{a}{2}$ (*b*) a
(*c*) 2a (*d*) None of these

27. Figure, shows a stationary wave between two fixed points P and Which point(s) of 1, 2 and 3 are in phase with the point x?



(*a*) 1, 2 and 3 (*b*) 1 and 2 only
(*c*) 2 and 3 only (*d*) 3 only

SOUND WAVES

28. A piece of cork is floating on water in a small tank. The cork oscillates up and down vertically when small ripples pass over the surface of water. The velocity of the ripples being 0.21 ms^{-1}, wave length 15 mm and amplitude 5 mm, the maximum velocity of the piece of cork is

$(\pi = \frac{22}{7})$

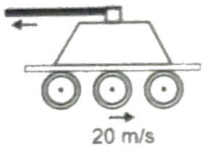


(*a*) 0.44 ms^{-1} (*b*) 0.24 ms^{-1}
(*c*) 2.4 ms^{-1} (*d*) 4.4 ms^{-1}

29. The frequency of a man's voice is 300 Hz and its wavelength is 1 metre. If the wavelength of a child's voice is 1.5 m, then the frequency of the child's voice is:
(*a*) 200 Hz (*b*) 150 Hz
(*c*) 400 Hz (*d*) 350 Hz.

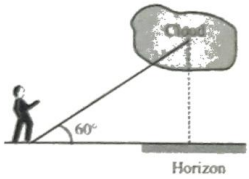
30. A machine gun is mounted on an armored car moving with a speed of 20 ms^{-1} as shown in figure. The gun can point against the direction of motion of car. The muzzle speed of bullet is equal to speed of sound in air i.e.,

340 ms^{-1}. The time difference between bullet actually reaching and sound of firing reaching at a target 544 m away from car at the instant of firing is:



(a) 1.2 s (b) 0.1 s
(c) 1 s (d) 10 s

31. The elevation of a cloud is 60°above the horizontal. A thunder is heard 8 s after the observation of lighting. The speed of sound is 330 ms^{-1}. The vertical height of cloud from ground is:



(a) 2826 m (b) 2682 m
(c) 2286 m (d) 2068 m

32. The ratio of speed of sound in neon to that in water vapours at any temperature (when molecular weight of neon is 2.02×10^{-2} kg mol^{-1} and for water vapours is 1.8×10^{-2} kg mol^{-1})
(a) 1.06 (b) 1.60
(c) 6.10 (d) 15.2

33. A Firecracker exploding on the surface of a lake is heard as two sounds a time interval t apart by a man on a boat close to water surface. Sound travels with a speed u in water and a sped v in air. The distance from the exploding firecracker to the boat is

(a) $\frac{uvt}{u+v}$ (b) $\frac{t(u+v)}{uv}$

(c) $\frac{t(u-v)}{uv}$ (d) $\frac{uvt}{u-v}$

34. Under similar conditions of temperature and pressure, In which of the following gases the velocity of sound will be largest.
(a) H_2 (b) N_2
(c) He (d) CO_2

35. How many times more intense is 90 dB sound than 40 dB sound?
(a) 5 (b) 50
(c) 500 (d) 10^5

36. Sound waves from a tuning fork F reach a point P by two separate routes FAP and FBP (when FBP is greater than FAP by 12 cm there is silence at P). If the difference is 24 cm the sound becomes maximum at P but at 36 cm there is silence again and so on. If velocity of sound in air is 330 ms^{-1}, the least frequency of tuning fork is :
(a) 1537 Hz (b) 1735 Hz
(c) 1400 Hz (d) 1375 Hz

37. The sound intensity is 0.008 $W m^{-2}$ at a distance of 10 m from an isotropic point source of sound. The power of the source is approximately :
(a) 2.5 watt (b) 0.8 watt
(c) 8 watt (d) 10 watt

38. The ratio of intensities between two coherent sound sources is 4:1 The difference of loudness in dB between maximum and minimum intensities when they interfere in space is
(a) 10 log 2 (b) 20 log 3
(c) 10 log 3 (d) 20 log 2

39. In Quincke's tube a detector detects minimum intensity. Now one of the tube is displaced by 5 cm. During displacement detector detects maximum intensity 10 times, then finally a minimum intensity (when displacement is complete). The wavelength of sound is:
(a) 10/9 cm (b) 1 cm
(c) 1/2 cm (d) 5/9 cm

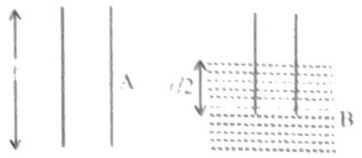
40. A cylindrical tube, open at one end and closed at the other, is in acoustic unison with an external source of frequency held at the open end of the tube, in its fundamental note. Then
(a) the displacement wave from the source gets reflected with a phase change of π at the closed end
(b) the pressure wave from the source get reflected without a phase change at the closed end
(c) the wave reflected from the closed end again gets reflected at the open end
(d) All of the above

41. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particle has maximum amplitude of vibration is (velocity of sound in air is 330 m/s)
(a) 0.125 m (b) 0.5 m
(c) 0.25 m (d) 2 m

42. If λ_1, λ_2, λ_3 are the wavelengths of the waves giving resonance in the fundamental, first and second overtone modes respectively in a open organ pipe, then the ratio of the wavelengths $\lambda_1 : \lambda_2 : \lambda_3$, is :
(a) 1 : 2 : 3 (b) 1 : 3 : 5
(c) 1 : 1/2 : 1/3 (d) 1 : 1/3 : 1/5

43. The maximum variation of pressure in an open organ pipe of length l vibrating in fundamental mode is at.
(a) ends (b) middle of pipe
(c) $\frac{l}{4}$ from centre (d) $\frac{3l}{8}$ from centre

44. Two identical tubes A and B are kept in air and water respectively as shown. If the fundamental frequency of A is f_0, then the fundamental frequency of B is



(a) $\frac{f_0}{4}$ (b) $\frac{f_0}{2}$
(c) f_0 (d) $2f_0$

45. An open organ pipe of length L vibrates in its fundamental mode. The pressure variation is maximum
(a) at the two ends
(b) at the middle of the pipe
(c) at distance L/4 inside the ends
(d) at distance L/8 inside the ends

46. An open organ pipe of length L vibrates in second harmonic mode. The pressure vibration is maximum
(a) At the two ends
(b) At a distance L/4 from either end inside the tube
(c) At the mid-point of the tube
(d) None of these

47. The second overtone of an open pipe A and a closed pipe B have the same frequencies at a given temperature. Both pipes contain air. The ratio of fundamental frequency of A to the fundamental frequency of B is:
(a) 3 : 5 (b) 5 : 3
(c) 5: 6 (d) 6 : 5

48. A resonance tube is resonated with tuning fork of frequency 256 Hz. If the length of first and second resonating air columns are 32 cm and 100 cm, then end correction will be
(a) 1 cm (b) 2 cm
(c) 4 cm (d) 6 cm

49. A closed organ pipe and an open pipe of same length produce 4 beats when they are set into vibrations simultaneously. If the length of each of them were twice their initial lengths, the number of beats produced will be [Assume same mode of vibration in both cases]
(a) 2 (b) 4
(c) 1 (d) 8

50. A pipe's lower end is immersed in water such that the length of air column from the top open end has a certain length 25 cm. The speed of sound in air is 350 m/s. The air column is found to resonate with a tuning fork of frequency 1750 Hz. By what minimum distance should the pipe be raised in order to make the air column resonate again with the same tuning fork?
(a) 7 cm (b) 5 cm
(c) 35 cm (d) 10 cm

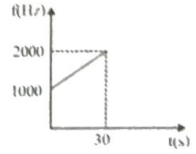
51. A closed organ pipe has length 'l'. The air in it is vibrating in 3rd overtone with maximum displacement amplitude 'a'. The displacement amplitude at distance $l/7$ from closed end of the pipe is :
(a) 0 (b) a
(c) a/2 (d) none of these

52. A tuning fork of frequency 280 Hz produces 10 beats per sec when sounded with a vibrating sonometer string. When the tension in the string increases slightly, it produces 11 beats per sec. The original frequency of the vibrating sonometer string is:
(a) 269 Hz (b) 291 Hz
(c) 270 Hz (d) 290 Hz

53. The speed of sound in a gas, in which two waves of wavelength 1.0 m and 1.02 m produce 6 beats per second, is approximately :
(a) 350 m/s (b) 300 m/s
(c) 380 m/s (d) 410 m/s

54. A source S of frequency f_0 and an observer O, moving with speeds v_1 and v_2 respectively, are moving away from each other. When they are separated by distance a (t = 0), a pulse is emitted by the source. This pulse is received by O at time t_1 then t_1, is equal to
(a) $\frac{a}{v_s + v_2}$ (b) $\frac{a}{v_1 + v_s}$
(c) $\frac{a}{v_s - v_2}$ (d) $\frac{a}{v_1 + v_2 + v_s}$

55. A detector is released from rest over a source of sound of frequency $f_0 = 10^3$ Hz. The frequency observed by the detector at time t is plotted in the graph. The speed of sound in air is ($g = 10\ m/s^2$)



(a) 330 m/s (b) 350 m/s
(c) 300 m/s (d) 310 m/s

56. A source of sound S having frequency f. Wind is blowing from source to observer O with velocity u. If speed of sound with respect to air is C, the wavelength of sound detected by O is :
(a) $\frac{C + u}{f}$ (b) $\frac{C - u}{f}$
(c) $\frac{C(C + u)}{(C - u)f}$ (d) $\frac{C}{f}$

Advanced Level Multiconcept Questions

JEE-ADVANCED
MCQ/COMPREHENSION/MATCHING

1. The particle displacement in a wave is given by
$y = 0.2 \times 10^{-3} \cos(500\,t - 0.025\,x)$
where the distances are measured in metres and time in seconds. Now
(*a*) wave velocity is 2×10^{4} ms^{-1}
(*b*) particle velocity is 2×10^{4} ms^{-1}
(*c*) initial phase difference is $\frac{\pi}{2}$
(*d*) wavelength of the wave is (80π) m

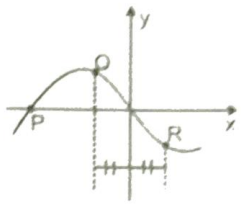
2. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is v/10, where v is the speed of propagation of the wave. if $a = 10^{-3}$ m and v = 10 m/s, then λ and f are given by
(*a*) $\lambda = 2\pi \times 10^{-2}$ m
(*b*) $\lambda = 10^{-3}$ m
(*c*) $f = 10^{3}/(2\pi)$ Hz
(*d*) $f = 10^{4}$ Hz

3. A wave equation which gives the displacement along the Y direction is given by $Y = 10^{-4} \sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave
(*a*) travelling with a velocity of 30 m/s in the negative x direction.
(*b*) of wavelength π metre
(*c*) of frequency $30/\pi$ hertz
(*d*) of amplitude 10^{-4} metre travelling along the negative x direction.

4. A sinusoidal progressive wave is generated in a string. It's equation is given by $y = (2\text{ mm}) \sin(2\pi x - 100\,\pi t + \pi/3)$. The time when particle at x = 4 m first passes through mean position, will be
(*a*) $\frac{1}{150}$ sec (*b*) $\frac{1}{12}$ sec
(*c*) $\frac{1}{300}$ sec (*d*) $\frac{1}{100}$ sec

5. A wave equation is given as $y = \cos(500t - 70x)$, where y is in mm, x in m and t is in sec.
(*a*) the wave must be a transverse propagating wave.
(*b*) The speed of the wave is 50/7 m/s
(*c*) The frequency of oscillations 1000π Hz
(*d*) Two closest points which are in same phase have separation $20\pi/7$ cm.

6. At a certain moment, the photograph of a string on which a harmonic wave is travelling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string.



(*a*) v_p is upwards (*b*) $v_Q = -v_R$
(*c*) $|v_P| > |v_Q| = |v_R|$ (*d*) $v_Q = v_R$

7. One end of a string of length L is tied to the ceiling of a lift accelerating upwards with an acceleration 2g. The other end of the string is free. The linear mass density of the string varies linearly from 0 to λ, from bottom to top.
(*a*) The velocity of the wave in the string will be 0.
(*b*) The acceleration of the wave on the string will be 3g/4 every where.
(*c*) The time taken by a pulse to reach from bottom to top will be $\sqrt{8L/3g}$.
(*d*) The time taken by a pulse to reach from bottom to top will be $\sqrt{4L/3g}$.

8. Two waves of equal frequency f and velocity v travel in opposite directions along the same path. The waves have amplitudes A and 3A. Then :
(*a*) the amplitude of the resulting wave varies with position between maxima of amplitude 4A and minima of zero amplitude
(*b*) the distance between a maxima and adjacent minima of amplitude is V/2f
(*c*) at point on the path the average displacement is zero
(*d*) the position of a maxima or minima of amplitude does not change with time

9. The vibrations of a string of length 600 cm fixed at both ends are represented by the equation
$$y = 4 \sin\left(\pi \frac{x}{15}\right) \cos(96\,\pi\, t)$$
where x and y are in cm and t in seconds.
(*a*) The maximum displacement of a point x = 5 cm is $2\sqrt{3}$ cm.
(*b*) The nodes located along the string are at 15n cm where integer n varies from 0 to 40.

(c) The velocity of the particle at x = 7.5 cm at t = 0.25 sec is 0

(d) The equations of the component waves whose superposition gives the above wave are 2 sin 2π $\left(\frac{x}{30}+48t\right)$, 2 sin2π $\left(\frac{x}{30}-48t\right)$.

10. The length, tension, diameter and density of a wire B are double than the corresponding quantities for another stretched wire A. Then.

(a) Fundamental frequency of B is $\frac{1}{2\sqrt{2}}$ times that of A.

(b) The velocity of wave in B is $\frac{1}{\sqrt{2}}$ times that of velocity in A.

(c) The fundamental frequency of A is equal to the third overtone of B.

(d) The velocity of wave in B is half that of velocity in A.

11. A clamped string is oscillating in nth harmonic, then

(a) total energy of oscillations will be n^2 times that of fundamental frequency

(b) total energy of oscillations will be $(n-1)^2$ times that of fundamental frequency

(c) average kinetic energy of the string over a complete oscillations is half of that of the total energy of the string.

(d) none of these.

12. Which one of the following statements is incorrect for stable interference to occur between two waves?

(a) The waves must have the same wave length

(b) The waves must have a constant phase difference

(c) The waves must be transverse only

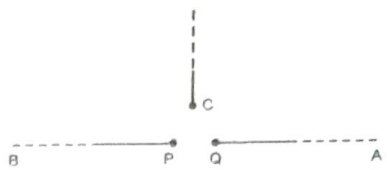
(d) The waves must have equal amplitudes.

13. Two sound waves move in the same direction in the same medium. The pressure amplitude of the waves are equal but the wavelength of the first wave is double that of the second. Let the average power transmitted across a cross section by the two wave be P_1 and P_2 and their displacement amplitudes are s_1 and s_2 then

(a) $P_1/P_2 = 1$ (b) $P_1/P_2 = 2$

(c) $s_1/s_2 = 1/2$ (d) $s_1/s_2 = 2/1$

14. Two monochromatic sources of electromagnetic wave, P and Q emit waves of wavelength λ = 20 m and separated by 5m as shown. A,B and C are three points where interference of these waves is observed. If phase of a wave generated by P is ahead of wave generated by Q by π/2 then (given intensity of both waves is I):



(a) phase difference of these waves at B is 180^0

(b) intensities at A,B and C are in the ratio 2 : 0 : 1 respectively.

(c) intensities at A,B and C are in the ratio 1 : 2 : 0 respectively.

(d) phase difference at A is 0^0.

15. The effect of making a hole exactly at (1/3rd) of the length of the pipe from its closed end is such that :

(a) its fundamental frequency is an octave higher than the open pipe of same length

(b) its fundamental frequency is thrice of that before making a hole

(c) the fundamental frequency is 3/2 time of that before making a hole

(d) the fundamental alone is changed while the harmonics expressed as ratio of fundamentals remain the same

16. In a resonance tube experiment, a closed organ pipe of length 120 cm is used. Initially it is completely filled with water. It is vibrated with tuning fork of frequency 340 Hz. To achieve resonance the water level is lowered then (given v_{air} = 340 m/sec., neglect end correction):

(a) minimum length of water column to have the resonance is 45 cm.

(b) the distance between two successive nodes is 50 cm.

(c) the maximum length of water column to create the resonance is 95 cm.

(d) the distance between two successive nodes is 25 cm.

17. Two identical straight wires are stretched so as to produce 6 beats/sec. when vibrating simultaneously. When changing the tension slightly in one of them the beat frequency remains unchanged. Denoting by T_1, T_2, the higher & the lower initial tensions in the strings, then it could be said that while making the above changes in tension:

(a) T_2 was decreased

(b) T_2 was increased

(c) T_1 was increased

(d) T_1 was decreased

18. At the closed end of an organ pipe :

(a) the displacement is zero

(b) the displacement amplitude is maximum

(c) the pressure amplitude is zero

(d) the pressure amplitude is maximum

19. A girl stops singing a pure note. She is surprised to hear an echo of higher frequency, i.e., a higher musical pitch. Then:

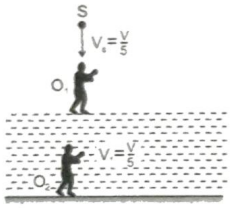
(a) there could be some warm air between the girl and the reflecting surface

(b) there could be two identical fixed reflecting surfaces, one half a wavelength of the sound wave away from the other

(c) the girl could be moving towards a fixed reflector

(d) the reflector could be moving towards the girl

20. In the figure shown an observer O_1 floats (static) on water surface with ears in air while another observer O_2 is moving upwards with constant velocity $V_1 = V/5$ in water. The source moves down with constant velocity $V_s = V/5$ and emits sound of frequency 'f'. The velocity of sound in air is V and that in water is 4V. For the situation shown in figure:



(a) The wavelength of the sound received by O_1 is $\frac{4V}{5f}$

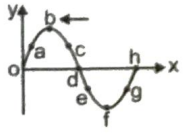
(b) The wavelength of the sound received by O_1 is V/f

(c) The frequency of the sound received by O_2 is $\frac{21f}{16}$

(d) The wavelength of the sound received by O_2 is $\frac{16V}{5f}$

Comprehension – 1 (21 to 24)

The figure represents the instantaneous picture of a transverse harmonic wave traveling along the negative x-axis. Choose the correct alternative(s) related to the movement of the nine points shown in the figure.



21. The points moving upward is/are

(a) a (b) c

(c) f (d) g

22. The points moving downwards is are

(a) e (b) b

(c) d (d) h

23. The stationary points is are

(a) e (b) b

(c) f (d) h

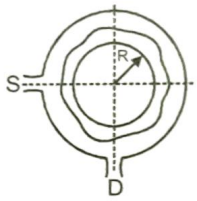
24. The points moving with maximum speed is/are

(a) b (b) c

(c) d (d) h

Comprehension – 2 (25 to 29)

A narrow tube is bent in the form of a circle of radius R, as shown in the figure. Two small holes S and D are made in the tube at the positions right angle to each other. A source placed at S generated a wave of intensity I_0 which is equally divided into two parts : One part travels along the longer path, while the other travels along the shorter path. Both the part waves meet at the point D where a detector is placed



25. If a maxima is formed at the detector then, the magnitude of wavelength λ of the wave produced is given by

(a) πR (b) $\frac{\pi R}{2}$

(c) $\frac{\pi R}{4}$ (d) $\frac{2\pi R}{3}$

26. If the minima is formed at the detector then, the magnitude of wavelength λ of the wave produced is given by

(a) $2\pi R$ (b) $\frac{3\pi R}{2}$

(c) $\frac{2\pi R}{3}$ (d) $\frac{2\pi R}{5}$

27. The maximum intensity produced at D is given by

(a) $4I_0$ (b) $2I_0$

(c) I_0 (d) $3I_0$

28. The maximum value of λ to produce a maxima at D is given by

(a) πR (b) $2\pi R$

(c) $\frac{\pi R}{2}$ (d) $\frac{3\pi R}{2}$

29. The maximum value of λ to produce a minima at D is given by

(a) πR (b) $2\pi R$

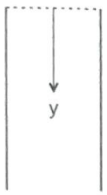
(c) $\frac{\pi R}{2}$ (d) $\frac{3\pi R}{2}$

Comprehension – 3 (30 to 33)

In an organ pipe (may be closed or open) of 99 cm length standing wave is setup, whose equation is given by longitudinal displacement

$$\xi = (0.1 \text{ mm}) \cos \frac{2\pi}{80} (y + 1 \text{ cm}) \cos 2\pi(400)\, t$$

where y is measured from the top of the tube in centimetres and t in second. Here 1 cm is the end correction.



30. The upper end and the lower end of the tube are respectively:

(a) open – closed (b) closed – open

(c) open – open (d) closed – closed

31. The air column is vibrating in

(a) First overtone (b) Second overtone

(c) Third harmonic (d) Fundamental mode

32. Equation of the standing wave in terms of excess pressure is – (Bulk modulus of air B = 5×10^5 N/m^2)

(a) $P_{ex} = (125\ \pi \text{N/m}^2) \sin \frac{2\pi}{80}(y + 1 \text{ cm}) \cos 2\pi(400t)$

(b) $P_{ex} = (125\ \pi \text{N/m}^2) \cos \frac{2\pi}{80}(y + 1 \text{ cm}) \sin 2\pi(400t)$

(c) $P_{ex} = (225\ \pi \text{N/m}^2) \sin \frac{2\pi}{80}(y + 1 \text{ cm}) \cos 2\pi(200t)$

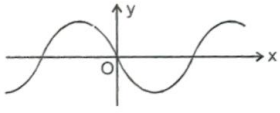
(d) $P_{ex} = (225\ \pi \text{N/m}^2) \cos \frac{2\pi}{80}(y + 1 \text{ cm}) \sin 2\pi(200t)$

33. Assume end correction approximately equals to (0.3) × (diameter of tube), estimate the approximate number of moles of air present inside the tube (Assume tube is at NTP, and at NTP, 22.4 litre contains 1 mole)

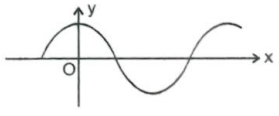
(a) $\frac{10\pi}{36 \times 22.4}$ (b) $\frac{10\pi}{18 \times 22.4}$

(c) $\frac{10\pi}{72 \times 22.4}$ (d) $\frac{10\pi}{60 \times 22.4}$

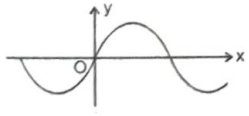
34. For four sine waves, moving on a string along positive x direction, displacement-distance curves (y-x curves) are shown at time t = 0. In the right column, expressions for y as function of distance x and time t for sinusoidal waves are given. All terms in the equations have general meaning. Correctly match y-x curves with corresponding equations.

(a) 

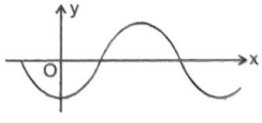
(P) y = A cos (ωt – kx)

(b) 

(Q) y = – A cos (kx – ωt)

(c) 

(R) y = A sin (ωt – kx)

(d) 

(S) y = A sin (kx – ωt)

35. Match the statements in column-I with the statements in column-II.

Column-I	Column-II
(a) A tight string is fixed at both ends and sustaining standing wave in odd harmonic	(p) At the middle, antinode is formed
(b) A tight string is fixed at one end and free at the other end	(q) At the middle, node is formed in even harmonic
(c) A tight string is fixed at both ends and vibrating in four loops	(r) the frequency of vibration is 300% more than its fundamental frequency

(*d*) A tight string is fixed at one end and free at the other end, vibrating in 2nd overtone

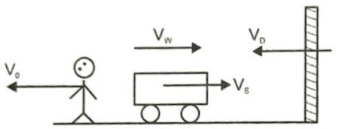
(s) Phase difference between any two particles will be either π or zero.

(t) The frequency of vibration is 400%more than fundamental frequency.

36. Match the Column :

Column-I	**Column-II**
(*a*) $y = 4 \sin (5x - 4t) + 3 \cos (4t - 5x + \pi/6)$ performing SHM	(p) Particles at every position are
(*b*) $y = 10 \cos\left(t - \frac{x}{330}\right) \sin (100)\left(t - \frac{x}{330}\right)$	(q) Equation of travelling wave
(*c*) $y = 10 \sin (2\pi x - 120t) + 10 \cos (120t + 2\pi x)$	(r) Equation of standing wave
(*d*) $y = 10 \sin (2\pi x - 120\ t) + 8 \cos (118t - 59/30\pi x)$	(s) Equation of Beats

37. S, O & W represent source of sound (of frequency f), observer & wall respectively. V_O, V_S, V_D, V are velocity of observer, source, wall & sound (in still air) respectively. V_W is the velocity of wind. They are moving as shown. then match the following : where $f_r = \frac{V + V_w + V_D}{V + V_w - V_S} f$



Column-I	**Column-II**
(*a*) The wavelength of the waves coming towards the observer from source.	(p) $(V - V_W - V_D)/f_r$
(*b*) The wavelength of the waves incident on the wall.	(q) $(V - V_W - V_O)f_r/(V - V_W - V_D)$
(*c*) The wavelength of the waves coming towards observer from the wall.	(r) $(V - V_W + V_S)/f$
(*d*) Frequency of the waves (as detected by O) coming from wall after reflection.	(s) $(V + V_W - V_S)/f$

NUMERICAL VALUE BASED

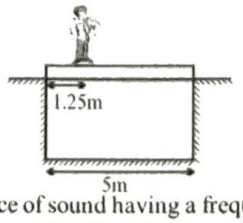
38. A telephone cord is 4.00m long and has a mass of 0.20kg A transverse wave pulse is produced by plucking one end of the that cord. The pulse makes four complete trips forwards and backwards along the cord in 0.800s. What is the tension (in N) in the cord.

39. The nth harmonic on a taut string has a frequency of 720 Hz. The next higher harmonic has a frequency of 840 Hz. What is the value of n?

40. Imagine a very thin uniform wire of length ℓ metre and total mass 1 gram. Fix one end of the wire in the ceiling, hang a large mass of 1 kg from the other end, and then pluck the wire $\frac{\ell}{4}$ m from the top. Assume that as the wire vibrates the large mass is not moving. The lowest frequency heard by us is 100 Hz. What is the length of the wire (in metre). (You may ignore the effect of gravity on the mass of the string)

41. A motorcycle's horn is putting out a 3015 Hz sound as it moves eastward towards a stationary observer. A train sounds its horn at 855 Hz as it heads westward away from that same observer. If the observer hears the train as 850 Hz and the motorcycle's horn as 3060 Hz, what is the speed of the motorcycle relative to the train (in m/s) ? Use 340 m/s as the speed of sound in air.

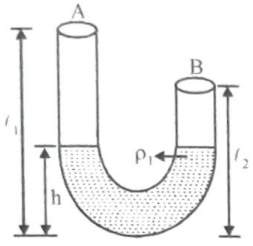
42. A wooden flexible plank is placed over a pit which is 5m wide. Student begins to jump up and down at the center. Such that she jumps at a frequency of 2Hz. The plank forms a standing wave pattern with minimum possible loops. Now student moves at 1.25 m from edge of the pit, and jumps at this position such that this point has maximum amplitude. What is minimum frequency of her jumping?



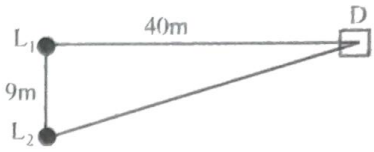
43. A point source of sound having a frequency of 165 Hz is located at the origin. Find the phase difference between points (3,4,0) and (8,6,0) at any given time. The velocity of sound in the medium is 330 m/s. If answer is $n\pi$, fill n in OMR sheet.

44. A point source of sound is located somewhere along the x-axis. Experiments show that the same wave front simultaneously reaches listeners at x = −8 m and x = +2.0 m. A third listener is positioned along the positive y-axis. What is her y-coordinate (in m) if the same wave front reaches her at the same instant as it does the first two listeners?

45. A U-tube having uniform cross-section but unequal arm length $l_1 = 100$ cm and $l_2 = 50$ cm has same liquid of density ρ_1 filled in it upto a height h = 30 cm as shown in figure. Another liquid of density $\rho_2 = \rho_1/2$ is poured in arm A. Both liquids are immiscible. What length of the second liquid (in cm) should be poured in A so that second overtone of A is in unison with fundamental tone of B. (Neglect end correction)



46. Two loudspeakers are, driven by a common oscillator and amplifier, are arranged as shown. The frequency of the oscillator is gradually increased from zero and the detector at D records a series of maxima and minima. If speed of sound is 330 m/s, then what is the frequency in Hz at which the first time a maxima is heard.



47. Straight line AB connects two point sources that are 5.00 m apart, emit 165 Hz sound waves of the same amplitude, and emit exactly in same phase. What is the shortest distance (in cm) between the midpoint of AB and a point on AB where the interfering waves cause maximum oscillation of the air molecules ?
(Take : velocity of sound = 330 m/s)

JEE Mains & Advanced Past Years Questions

JEE MAIN PREVIOUS YEAR'S

String

1. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is: **[JEE Main-2016]**

(a) $2\pi\sqrt{2}$ s (b) 2s

(c) $2\sqrt{2}$ s (d) $\sqrt{2}$ s

2. Equation of travelling wave on a stretched string of linear density 5 g/m is y = 0.03 sin (450 t − 9x) where distance and time are measured in SI united. The tension in the string is: **[JEE Main-2019(January)]**

(a) 10 N (b) 7.5 N

(c) 12.5 N (d) 5 N

3. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100Hz. The separation between successive nodes on the string is close to: **[JEE Main-2019 (January)]**

(a) 10.0 cm (b) 33.3 cm

(c) 16.6 cm (d) 20.0 cm

4. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1 f_2 is: **[JEE Main-2019 (January)]**

(a) 18/17 (b) 19/18

(c) 20/19 (d) 20/21

5. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3}\sin(50t + 2x)$, where x and y are in metre and t is in seconds. Which of the following is a correct statement about the wave? **[JEE Main-2019 (January)]**

(a) The wave is propagating along the negative x-axis with speed 25 ms^{-1}.

(b) The wave is propagating along the positive x-axis with speed 100 ms^{-1}.

(c) The wave is propagating along the positive x-axis with speed 25 ms^{-1}.

(d) The wave is propagating along the negative x-axis with speed 100 ms^{-1}.

6. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is: **[JEE Main-2019(April)]**

(a) 320 m/s, 120 Hz (b) 180 m/s, 80 Hz

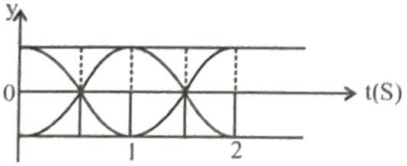
(c) 180 m/s, 120 Hz (d) 320 m/s, 80 Hz

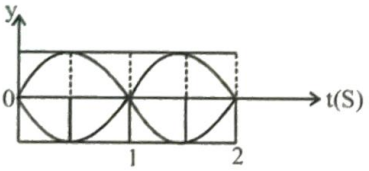
7. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is: [JEE Main-2019 (April)]

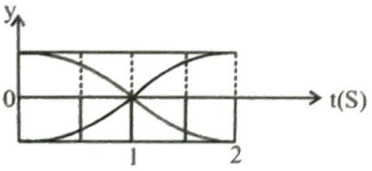


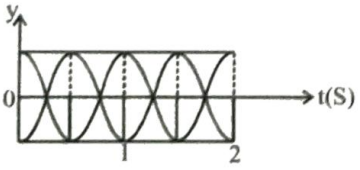
(a) 4 : 9 (b) 3 : 5
(c) 1 : 4 (d) 1 : 2

8. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is: [JEE Main-2019 (April)]

(a)


(b)


(c)


(d)


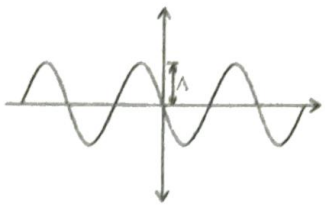
9. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x)\cos(200\pi t)$. The length of the string is : (All quantities are in SI units.)

[JEE Main-2019 (April)]

(a) 20 m (b) 80 m
(c) 60 m (d) 40 m

10. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure:

[JEE Main-2019 (April)]



For this wave, the phase ϕ is :

(a) 0 (b) $-\frac{\pi}{2}$

(c) π (d) $\frac{\pi}{2}$

11. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4N. When the tension is changed to T, the velocity changed to v/2. The value of T is close to : [JEE Main-2020 (January)]

(a) 5.15×10^3 N (b) 10.2×10^2 N
(c) 2.50×10^4 N (d) 30.5×10^4 Ns

12. A wire of length L and mass per unit length 6.0×10^{-3} kgm^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are 420 Hz and 490 Hz. Then L in metres is:

[JEE Main-2020 (January)]

(a) 8.1 m (b) 2.1 m
(c) 1.1 m (d) 5.1 m

13. Two identical strings X and Z made of same material have tension T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_X / T_Z is [JEE Main-2020 (September)]

(a) 1.5 (b) 2.25
(c) 0.44 (d) 1.25

14. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope?

[JEE Main-2020 (September)]

(a) 6 (b) 3
(c) 12 (d) 9

15. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are

[JEE Main-2020 (September)]

(a) 1, 2, 3, ... (b) 1, 3, 5, ...

(c) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}$, ... (d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$, ...

SOUND

16. A pipe open at both ends, has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now: [JEE Main-2016]

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$

(c) $2f$ (d) f

17. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer?
(speed of light = 3×10^8 ms^{-1}) [JEE Main -2017]

(a) 17.3 GHz (b) 15.3 GHz

(c) 10.1 GHz (d) 12.1 GHz

18. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? [JEE Main-2018]

(a) 2.5 kHz (b) 10 kHz

(c) 7.5 kHz (d) 5 kHz

19. A musician using an open flute of length 50 cm produces second harmonic sound waves.
A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to: [JEE Main-2019 (January)]

(a) 660 Hz (b) 753 Hz

(c) 500 Hz (d) 333 Hz

20. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be: (Assume that the highest frequency a person can hear is 20,000 Hz) [JEE Main-2019 (January)]

(a) 6 (b) 4

(c) 7 (d) 5

21. A person standing on an open ground hears the sound of a Jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If υ is the speed of sound, speed of the plane is: [JEE Main-2019 (January)]

(a) $\frac{\sqrt{3}}{2}\upsilon$ (b) $\frac{2\upsilon}{\sqrt{3}}$

(c) υ (d) $\frac{\upsilon}{2}$

22. A resonance tube is old and has jagged end. It is till used in the laboratory to determine velocity of sound in air. A tunign fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to: [JEE Main-2019 (January)]

(a) 322 ms^{-1} (b) 341 ms^{-1}

(c) 335 ms^{-1} (d) 328 ms^{-1}

23. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B? (speed of sound in air = 340 ms^{-1}): [JEE Main-2019 (April)]

(a) 2250 Hz (b) 2060 Hz

(c) 2150 Hz (d) 2300 Hz

24. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ? (Take velocity of sound in air is 350 m/s) [JEE Main-2019 (April)]

(a) 857 Hz (b) 807 Hz

(c) 750 Hz (d) 1143 Hz

25. The pressure wave, $P = 0.01 \sin [1000t - 3x]$ Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day, when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336ms^{-1}. Approximate value of T is : [JEE Main-2019 (April)]

(a) 15° C (b) 12° C

(c) 4° C (d) 11° C

26. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1 = 30$ cm and $l_2 = 70$ cm. Then v is equal to: [JEE Main-2019 (April)]

(a) 332 ms^{-1} (b) 379 ms^{-1}

(c) 384 ms^{-1} (d) 338 ms^{-1}

27. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed v m/s and he hears 6 beats/s. The velocity of sound is 330 m/s. Then, v equals

[JEE Main-2019 (April)]

(a) 2.5 m/s (b) 15 m/s

(c) 5.5 m/s (d) 10.5 m/s

28. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? (Given reference intensity of sound as 10^{-12} W/m^2)

[JEE Main-2019 (April)]

(a) 40 cm (b) 31 cm

(c) 80 cm (d) 20 cm

29. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to: (Speed of sound in water = 1500 ms^{-1})

[JEE Main-2019 (April)]

(a) 496 Hz (b) 502 Hz

(c) 507 Hz (d) 504 Hz

30. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in m/s: (Given speed of sound = 300 m/s)

[JEE Main-2019 (April)]

(a) 16, 14 (b) 12, 18

(c) 12, 16 (d) 8, 18

31. A stationary observer receives sound from two identical turning forks, one of which approaches and the other one recedes with the same speed (much less that the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each returning fork is ν_0 = 1400 Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to:

[JEE Main-2020 (January)]

(a) $\frac{1}{4}$ m/s (b) $\frac{1}{2}$ m/s

(c) 1 m/s (d) $\frac{1}{8}$ m/s

32. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is_____Hz.

[JEE Main-2020 (January)]

33. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1}

[JEE Main-2020 (September)]

(a) 61 kmh^{-1} (b) 91 kmh^{-1}

(c) 81 kmh^{-1} (d) 71 kmh^{-1}

34. A driver in a car approaching a vertical wall notices that the frequency of his car horn has changed from 440 Hz to 490 Hz when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is

[JEE Main-2020 (September)]

(a) 36 km/hr (b) 54 km/hr

(c) 24 km/hr (d) 18 km/hr

35. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is

[JEE Main-2020 (September)]

(a) 2200 Hz (b) 3300 Hz

(c) 2100 Hz (d) 550 Hz

36. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10$ Pa, $v \sim 300$ m/s, $\rho \sim 1$ kg/m^3 and $f \sim 1000$ Hz, then s will be of the order of (take the multiplicative constant to be 1)

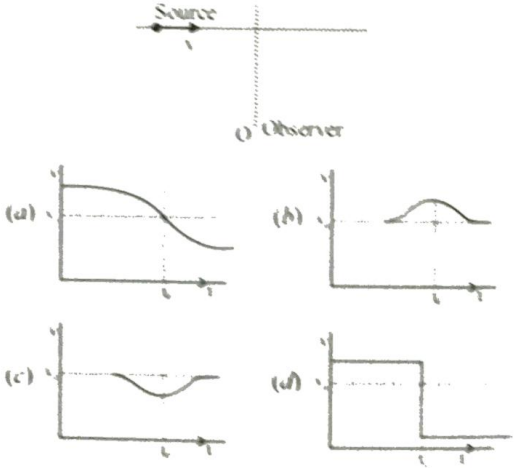
[JEE Main-2020 (September)]

(a) $\frac{3}{100}$ mm (b) 10 mm

(c) 1 mm (d) $\frac{1}{10}$ mm

37. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency ν_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by (t_0 represents the instant when the distance between the source and observer is minimum)

[JEE Main-2020 (September)]



18. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively.

The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is (Round off to the Nearest Integer)

[JEE Main-2021 (March)]

JEE-ADVANCED PREVIOUS YEAR'S

String Waves

1. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01\ m) \sin[(62.8\ m^{-1})x] \cos[(628\ s^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are):

[JEE Advanced-2013]

(a) The number of nodes is 5.
(b) The length of the string is 0.25 m.
(c) The maximum displacement of the midpoint of the string its equilibrium position is 0.01 m.
(d) The fundamental frequency is 100 Hz.

2. One end of a taut string of length 3m along the x-axis is fixed at x = 0. The speed of the waves in the string is 100m/s. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are)

[JEE Advanced-2014]

(a) $y(t) = A \sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(b) $y(t) = A \sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(c) $y(t) = A \sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(d) $y(t) = A \sin\frac{5\pi x}{2}\cos 250\pi t$

3. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is are correct:

[JEE Advanced-2017]



(a) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint at the midpoint of rope
(b) The velocity of any pulse along the rope is independent of its frequency and wavelength
(c) The wavelength of Pulse 1 becomes longer when it reaches point A
(d) The time $T_{OA} = T_{AO}$

Answer the following by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

[JEE Advanced-2019]

List-I	List-II
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) $\frac{1}{2}$
(III) String-3 (3μ)	(R) $\frac{1}{\sqrt{2}}$
(IV) String- 4 (4μ)	(S) $\frac{1}{\sqrt{3}}$
	(T) $\frac{3}{16}$
	(U) $\frac{1}{16}$

4. If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

(a) I → P, II → R, III → S, IV → Q
(b) I → P, II → Q, III → T, IV → S
(c) I → Q, II → S, III → R, IV → P
(d) I → Q, II → P, III → R, IV → T

5. The length of the string 1,2,3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1,2,3 and 4 are vibrated at their 1st, 3rd, 5th and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be

[JEE Advanced-2019]

(a) I → P, II → Q, III → T, IV → U
(b) I → T, II → Q, III → R, IV → U
(c) I → P, II → Q, III → R, IV → T
(d) I → P, II → R, III → T, IV → U

6. Column-I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column-II describing the nature and wavelength of the standing waves.

Column-I **Column-II**

(*a*) Pipe closed at one end



(p) Longitudinal waves

(*b*) Pipe open at both ends



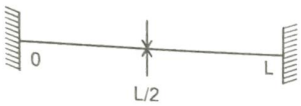
(q) Transverse waves

(*c*) Stretched wire clamped at both ends



(r) $\lambda_f = L$

(*d*) Stretched wire clamped at both ends and at mid-point



(s) $\lambda_f = 2L$

7. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe. *[IIT JEE-2012]*

(*a*) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.

(*b*) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.

(*c*) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.

(*d*) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.

8. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4cm . The distance frequency of the tuning for k is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the metre scale coincides with the top and of the Resonance column. When first resonance occurs, the reading of the water level in the column is *[IIT JEE-2012]*

(*a*) 14.0 (*b*) 15.2

(*c*) 16.4 (*d*) 17.6

9. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are): *[JEE Advanced-2013]*

(*a*) If the wind blows from the observer to the source, $f_2 > f_1$.

(*b*) If the wind blows from the source to the observer, $f_2 > f_1$.

(*c*) If the wind blows from the observer to the source, $f_2 < f_1$.

(*d*) If the wind blows from the source to the observer, $f_2 < f_1$.

10. A student is performing an experiment using a resonance column and a tuning fork of frequency $244 s^{-1}$. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(Useful information) :

$\sqrt{167RT} = 640 j^{1/2}$ mole $^{1/2}$;

$\sqrt{140RT} = 590 j^{1/2}$ mole $^{1/2}$. The molar masses M in grams are given in the options. Take the value of $\sqrt{\frac{10}{M}}$ for each gas as given there.) *[JEE Advanced-2014]*

(*a*) Neon (M = 20, $\sqrt{\frac{10}{20}} = \frac{7}{10}$)

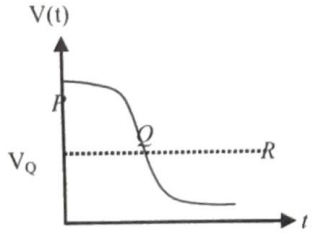
(*b*) Nitrogen (M = 28, $\sqrt{\frac{10}{28}} = \frac{3}{5}$)

(*c*) Oxygen (M = 32, $\sqrt{\frac{10}{32}} = \frac{9}{16}$)

(*d*) Argon (M = 36, $\sqrt{\frac{10}{36}} = \frac{17}{32}$)

11. Two loudspeakers *M* and *N* are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point *P*, 1800 m away from the midpoint Q of the line *MN* and moves towards Q constantly at 60 km/hr along the perpendicular bisector of *MN*. It crosses Q and eventually reaches a point *R*, 1800 m away from Let v(t) represent the beat frequency measured by a person sitting in the car at time *t*. Let v_P, v_Q and v_R be the beat frequencies measured at locations *P*, *Q*, and *R*, respectively. The speed of sound in air is 330 m s^{-1}. Which of the following statement(s) is (are) true regarding the sound heard by the person?

[*JEE Advanced-2016*]



(*a*) The rate of change in beat frequency is maximum when the car passes through Q

(*b*) The plot below represents schematically the variation of beat frequency with time

(*c*) $v_P + v_R = 2v_Q$

(*d*) The plot below represents schematically the variation of beat frequency with time

12. A stationary source emits sound of frequency f_0 = 492 Hz. The sound is reflected by a large car approaching the source with a speed of 2ms $^{-1}$. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz ? (Given that the speed of sound in air is 330 ms $^{-1}$ and the car reflects the sound at the frequency it has received). **[*JEE Advanced-2017*]**

13. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms $^{-1}$ and the man behind walks at a speed 2.0 ms $^{-1}$. A third man is standing at a height 12m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms $^{-1}$. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is _____.

[*JEE Advanced-2018*]

14. In an experment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two susuccessive resonances are heard at air column of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true? **[*JEE Advanced-2018*]**

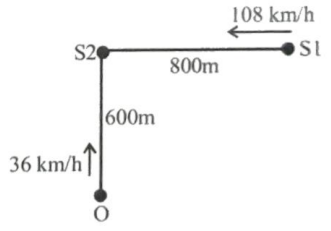
(*a*) The speed of sound determined from this experiment is 332 ms $^{-1}$

(*b*) The end correction in this experiment is 0.9 cm

(*c*) The wavelength of the sound wave is 66.4 cm

(*d*) The resonance at 50.7 cm corresponds to the fundamantal harmonic

15. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is _____.
[Speed of the sound = 330 m/s] **[*JEE Advanced-2019*]**



16. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms $^{-1}$ in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms $^{-1}$, the smallest value of the percentage change required in the length of the pipe is ..... **[*JEE Advanced-2020*]**

17. A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_s, will the sound reaching the pipe lead to a resonance? **[*JEE Advanced-2021*]**

(*a*) $u = 0.8v$ and $f_s = f_0$ (*b*) $u = 0.8v$ and $f_s = 2f_0$

(*c*) $u = 0.8v$ and $f_s = 0.5f_0$ (*d*) $u = 0.5v$ and $f_s = 1.5f_0$

ANSWER KEY

Topicwise Questions

1. (*d*)	**2.** (*a*)	**3.** (*c*)	**4.** (*a*)	**5.** (*c*)	**6.** (*c*)	**7.** (*a*)	**8.** (*d*)	**9.** (*a*)	**10.** (*d*)
11. (*a*)	**12.** (*d*)	**13.** (*c*)	**14.** (*b*)	**15.** (*c*)	**16.** (*a*)	**17.** (*d*)	**18.** (*c*)	**19.** (*a*)	**20.** (*a*)
21. (*d*)	**22.** (*d*)	**23.** (*c*)	**24.** (*d*)	**25.** (*a*)	**26.** (*b*)	**27.** (*b*)	**28.** (*d*)	**29.** (*c*)	**30.** (*a*)
31. (*c*)	**32.** (*b*)	**33.** (*d*)	**34.** (*c*)	**35.** (*b*)	**36.** (*d*)	**37.** (*b*)	**38.** (*a*)	**39.** (*a*)	**40.** (*c*)
41. (*d*)	**42.** (*a*)	**43.** (*d*)	**44.** (*a*)	**45.** (*d*)	**46.** (*c*)	**47.** (*c*)	**48.** (*c*)	**49.** (*b*)	**50.** (*a*)
51. (*d*)	**52.** (*a*)	**53.** (*a*)	**54.** (*a*)	**55.** (*c*)	**56.** (*c*)	**57.** (*c*)	**58.** (*d*)	**59.** (*b*)	**60.** (*c*)
61. (*c*)	**62.** (*c*)								

Learning Plus

1. (*a*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*d*)	**7.** (*d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*b*)
11. (*a*)	**12.** (*d*)	**13.** (*c*)	**14.** (*c*)	**15.** (*c*)	**16.** (*a*)	**17.** (*d*)	**18.** (*c*)	**19.** (*d*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** (*b*)	**24.** (*c*)	**25.** (*b*)	**26.** (*a*)	**27.** (*c*)	**28.** (*a*)	**29.** (*a*)	**30.** (*b*)
31. (*c*)	**32.** (*a*)	**33.** (*d*)	**34.** (*a*)	**35.** (*d*)	**36.** (*d*)	**37.** (*d*)	**38.** (*b*)	**39.** (*b*)	**40.** (*d*)
41. (*a*)	**42.** (*c*)	**43.** (*b*)	**44.** (*c*)	**45.** (*b*)	**46.** (*b*)	**47.** (*b*)	**48.** (*b*)	**49.** (*a*)	**50.** (*d*)
51. (*b*)	**52.** (*d*)	**53.** (*b*)	**54.** (*a*)	**55.** (*c*)	**56.** (*a*)				

Advanced Level Multiconcept Questions

1. (*a*, *d*) **2.** (*a*, *c*) **3.** (*a*, *b*, *c*, *d*) **4.** (*c*) **5.** (*a*, *b*, *d*) **6.** (*c*, *d*) **7.** (*b*, *c*) **8.** (*c*, *d*) **9.** (*a*, *b*, *c*, *d*)
10. (*c*, *d*) **11.** (*a*, *c*) **12.** (*c*, *d*) **13.** (*a*, *d*) **14.** (*a*, *b*, *d*) **15.** (*b*, *d*) **16.** (*a*, *b*, *c*) **17.** (*b*, *d*) **18.** (*a*, *d*) **19.** (*c*, *d*)
20. (*a*, *c*, *d*) **21.** (*a*, *d*) **22.** (*c*) **23.** (*b*, *c*) **24.** (*c*, *d*) **25.** (*a*, *b*, *c*) **26.** (*a*, *c*, *d*) **27.** (*b*) **28.** (*a*) **29.** (*b*)
30. (*a*) **31.** (*b*) **32.** (*a*) **33.** (*a*) **34.** (*a*) R (*b*) P (*c*) S (*d*) Q **35.** (*a*) p,q,s (*b*) s (*c*) q,r,s (*d*) s,t
36. (*a*) p, q (*b*) q, s (*c*) r, (*d*) s,q **37.** (*a*) → (r); (*b*) → (s); (*c*) → (p); (*d*) → (q) **38.** [80] **39.** [6] **40.** [1]
41. [7] **42.** [4] **43.** [5] **44.** [4] **45.** [60] **46.** [0330] **47.** [0050]

JEE Mains & Advanced Past Years Questions

JEE-MAIN

1. (*c*)	**2.** (*c*)	**3.** (*d*)	**4.** (*b*)	**5.** (*c*)	**6.** (*d*)	**7.** (*d*)	**8.** (*d*)	**9.** (*b*)	**10.** (*c*)
11. (*a*)	**12.** (*b*)	**13.** (*b*)	**14.** (*c*)	**15.** (*c*)	**16.** (*d*)	**17.** (*a*)	**18.** (*d*)	**19.** (*a*)	**20.** (*a*)
21. (*d*)	**22.** (*d*)	**23.** (*a*)	**24.** (*c*)	**25.** (*c*)	**26.** (*c*)	**27.** (*a*)	**28.** (*c*)	**29.** (*b*)	**30.** (*b*)
31. (*a*)	**32.** [106]	**33.** (*b*)	**34.** (*b*)	**35.** (*a*)	**36.** (*a*)	**37.** (*a*)	**38.** [4]		

JEE-ADVANCED

1. (*b*, *c*) **2.** (*a*, *c*, *d*) **3.** (*b*, *d*) **4.** (*a*) **5.** (*a*) **6.** (*a*) → p,t , (*b*)→ p,s, (*c*) → q,s, (*d*) → q, r **7.** (*b*, *d*)
8. (*b*) **9.** (*a*, *b*) **10.** (*d*) **11.** (*a*, *b*, *c*) **12.** [6] **13.** [5.00 Hz] **14.** (*a*, *c*) **15.** [8.12 to 8.13]
16. (0.62 to 0.63) **17.** (*a*, *d*)

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PREFACE

In this era of cut throat competition among **JEE** aspirants, only few are lucky enough to make it through. During the preparation, a student needs the most relevant study material to cover all the topics with proper planning to crack **JEE**.

We know there is abundance of study materials floating in the market but what We felt missing, is a self-driven and well-planned Study material. We have tried to bring clarity and Quality in the students by introducing this **Arise for JEE**

The book covers all the important concepts and theories with a rich amount of practice questions as well. This book is designed for the perfect guidance you need.

The study material is consistent with the syllabus offered in NCERT Books as well and has been curated by industry experts under the careful observation of **Physics Wallah Team**.

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1 Electrostatics

ELECTROSTATICS

The branch of physics which deals with properties of charges at rest is called **electrostatics**.

ELECTRIC CHARGE

Charge is scalar physical quantity associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the concept of net charge. A negatively charged body has excess of electrons while a positively charged body has lost some of its electrons.

Properties of Electric Charge

(1) Charges interact with each other *i.e.,* they exert force on each other. Like point charges do not like (repel) each other while unlike point charges like each other (attract).

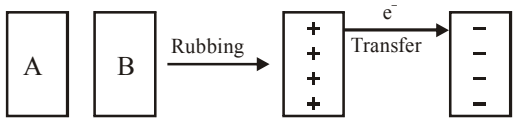
(2) **Charge is of two kind** : Positive and negative.

(3) **Total charge of an isolated system is conserved** (Conservation of charge).

(4) **Charge is quantised:** Charge is integral multiple of electronic charge $Q = Ne$, where $e = 1.6 \times 10^{-19}$.

(5) **Charge can be transferred:** Charge can be transferred from one body to other. This occurs due to transfer of electrons from one body to other. One of the common example of transfer of charge is charging by friction.

Frictional Electricity: When two bodies are rubbed with each other, they are found to attract each other. This is so because, on rubbing, transfer of electrons takes place from one body to other. One of them acquires a positive charge and other acquires a negative charge.



(6) **Charge is invariant :** Charge of a particle is independent of its speed.

(7) Charge cannot exist without mass, while mass can exist without charge. e.g. Neutron, Neutrino, Antineutrino. All are Neutral particles having mass.

SI Unit : coulomb.

[1 coulomb = 1 ampere × 1 second]

C.G.S. : Statcoulomb or frankline

1 coulomb = 3×10^9 stat coulomb

1 coulomb = 3×10^9 esu of charge = $\frac{1}{10}$ emu of charge

[esu = electrostatic unit] [emu = electro magnetic unit]

1 emu of charge = 1 frankline.

Practical Units : 1amp × hr = 3600 coulomb and 1 faraday = 96500 coulomb (charge on 1 mole electrons)

METHODS OF CHARGING

Friction: In friction when two bodies are rubbed together, electrons are transferred from one body to the other. This makes one body become positively charged while the other become negatively charged, e.g., when a glass rod is rubbed with silk, the rod becomes positively charged while the silk is negatively charged. Clouds are also charged by friction. Charging by friction is in accordance with conservation of charge. The positive and negative charges appear simultaneously in equal amounts due to transfer of electrons from one body to the other.

Charging by friction is based on difference in work function (ϕ) of the bodies we are rubbing. We cannot generate charge by rubbing two bodies made up of same material.

Induction: If a charged body is brought near a neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. One side of the neutral body become positively charged while the other side negative.

IMPORTANT POINTS

- Inducing body neither gains nor loses charge.
- The nature of induced charge is always opposite to that of inducing charge.
- Induced charge can be lesser or equal to inducing charge (but never greater) and its maximum value is $q' = -q\left[1 - \frac{1}{K}\right]$

  [only when electric field is uniform]

  q = the inducing charge

  K = the dielectric constant of the material of the uncharged body
- For metals, $K = \infty$ So $q' = -q$

  i.e., in metals induced charge may be equal or less than the inducing charge.

Conduction: When an insulated conductor is brought in contact with a charged body and it gets the same charge as the charged body then conduction takes place. Conduction is only possible in conductors and not in insulators.

COULOMB'S LAW

The force of attraction or repulsion between two stationary point charges is directly proportional to the product of charges and inversely proportional to the square of distance between them. This force acts along the line joining the centre of two charges.

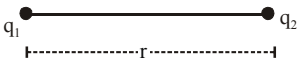
If q_1 and q_2 are charges

r = the distance between them

F = the force acting between them

Then $F \propto q_1 q_2$ & $F \propto \frac{1}{r^2}$ $\therefore F \propto \frac{q_1 q_2}{r^2}$

$$F = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r^2}$$



$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2/\text{C}^2$$

[in electrostatic unit (esu) constant of proportionality = 1]

$\varepsilon_0 = 8.85 \times 10^{-12}$ C²/Nm² = permittivity of free space or vacuum

This law is valid only for stationary point charges and cannot be applied for moving charges.

EFFECT OF MEDIUM

The dielectric constant of a medium is the ratio of the electrostatic force between two charges separated by a given distance in air to electrostatic force between same two charges separated by same distance in that medium.

$$F_{air} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \text{ and } \qquad F_{medium} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}$$

$$\Rightarrow \frac{F_{medium}}{F_{air}} = \frac{1}{\varepsilon_r} = \frac{1}{K}$$

ε_r or K = dielectric constant or relative permittivity or specific inductive capacity of medium.

Permittivity: Permittivity is a measure of the ability of the medium surrounding electric charges to allow electric lines of force to pass through it. It determines the forces between the charges.

Relative Permittivity: The relative permittivity or the dielectric constant (ε_r or K) of a medium is defined as the ratio of the permittivity ε of the medium to the permittivity ε_0 of free space

i.e. ε_r or $K = \frac{\varepsilon}{\varepsilon_0}$

Dimension of permittivity (ε) [k] = [tr] = $M^oL^oT^o$: $[M^{-1} L^{-3} T^4 A^2]$

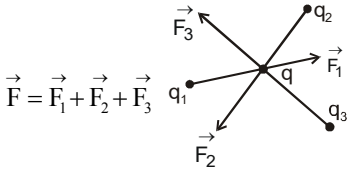
The dielectric constants of different mediums

Medium	Vacuum	Air	Water	Mica
e_r	1	1.00059	80	6

Teflon	Glass	PVC	Metal
2	5-10	4.5	∞

PRINCIPLE OF SUPERPOSITION

The resultant force acting on a charge due to a group of charges is equal to the vector sum of individual forces.



$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

TRAIN YOUR BRAIN

Example 1: Two equal point charges (10^{-3}C)are placed 1 cm apart in medium of dielectric constant K = 5

(*a*) Find the interaction force between the point charges.

(*b*) Net force on any of the charge.

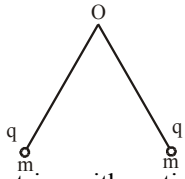
Sol. (*a*) Interaction force between point charges

$$F = \frac{1}{4\pi\in_0}\frac{q_1 q_2}{r^2} = 9\times10^9\frac{\left(10^{-3}\right)^2}{\left(10^{-2}\right)^2} = 9\times10^7\ \text{N}$$

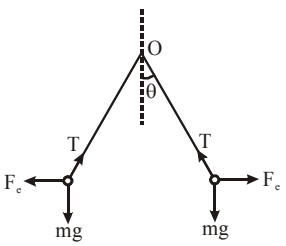
(*b*) Net force

$$F' = \frac{1}{4\pi\in_0 k}\frac{q_1 q_2}{r^2} = \frac{9\times10^9}{5}\frac{\left(10^{-3}\right)^2}{\left(10^{-2}\right)^2} = 18\times10^6\ \text{N}$$

Example 2: Two small balls each of mass m and charge q on each of them are suspended through two light insulating string of length l from a point. Find the expression for angle θ made by any of the string with vertical when under static equilibrium.



Sol. Let angle of any string with vertical be θ as shown



for horizontal direction

$$F_e = \frac{1}{4\pi\in_0}\frac{q^2}{(2\ell\sin\theta)^2} = T\sin\theta \qquad \text{...(i)}$$

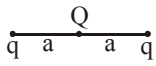
for vertical direction

$T\cos\theta = mg$ ...(ii)

Dividing (i) by (ii)

$\tan\theta = \frac{F_e}{mg}$

Example 3: For system to be in equilibrium Q = ?



Sol. $\frac{Kq^2}{(2a)^2} = \frac{K|Q|q}{a^2}$

$Q = -\frac{q}{4}$

ELECTRIC FIELD

The space around an electric charge in which its influence can be experienced is known as electric field.

ELECTRIC FIELD INTENSITY $\vec{E}$

The electric field intensity at a point in an electric field is defined as the force experienced by a unit positive point charge called test charge supposed to be placed at that point. The test charge does not affect the source charge or charge distribution producing the field.

- If a test charge q_0 at a point P in an electric field experiences a force $\vec{F}$

  Electric field $\vec{E} = \frac{\vec{F}}{q_0}$ or $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

  $\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

- If field is produced by a charge distribution, then by 'principle of superposition' field is :

  $\vec{E} = \vec{E}_1 + \vec{E}_2 + ... = \sum_{i=1}^{n} \vec{E}_i$ with $\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^3}\vec{r}_i$

  While for continuous charge distribution (treating small charge element as a point charge)

  $\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^3}\vec{r}$

IMPORTANT POINTS

SI Unit : N/C or V/m

Dimensions : $[M\,L\,T^{-3}\,A^{-1}]$

- In free space Electric field $E_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

  In a medium of permittivity ε field $E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$

  So $\frac{E}{E_0} = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{K}$ or $E = \frac{E_0}{K}$ $[\text{as } \varepsilon = \varepsilon_0 K]$

  In presence of a dielectric, electric field decreases and becomes $\frac{1}{K}$ times of its value in free space.

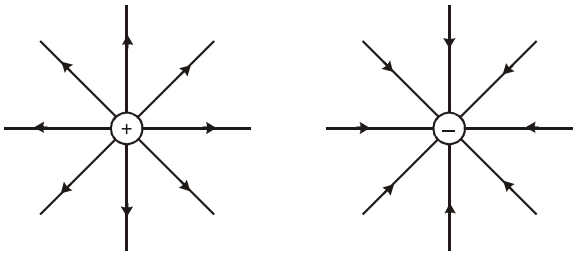
- The electrostatic field i.e. field produced by charge at rest is conservative.

Electric Field due to Point Charge

$E = \frac{kq}{r^2}$

$\vec{E} = \frac{kq}{r^2}\hat{r}$

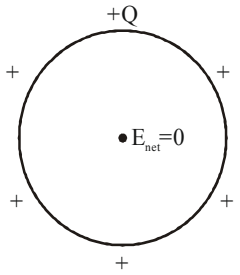
$\vec{E} = \frac{kq}{r^3}\vec{r}$



$\vec{r}$ = position vector of test point with respect to source charge

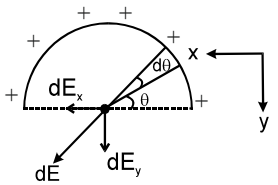
$\vec{r} = \vec{r}_{\text{test point}} - \vec{r}_{\text{source charge}}$

- Similarly electric field due to a uniformly charged ring at the centre of ring :



Electric field intensity at the centre of uniformly charged semicircular ring.

Let λ = linear charge density.



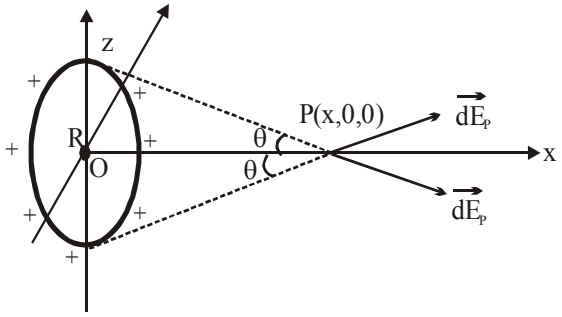
The arc is a collection of large number. of point charges. Consider a part of ring as an element of length Rdθ which subtends an angle dθ at centre of ring and it lies between θ and θ + dθ

$$\vec{dE} = dE_x\,\hat{i} + dE_y\,\hat{j}$$

$$E_x = \int dE_x = 0 \text{ (due to symmetry)}$$

$$E_y = \int dE_y = \int_0^{\pi} dE\sin\theta = \frac{K\lambda}{R}\int_0^{\pi}\sin\theta.d\theta = \frac{2K\lambda}{R}$$

Electric field due to a Charged ring at a point on its axis: Let us consider a uniformly charged ring as shown in figure. We wish to calculate Electric field at a point on its axis.



Consider an element on the ring, then electric field due to this element is given by

$$|\vec{dE_P}| = \frac{k.dQ}{x^2+R^2}$$

From symmetry of charge distribution, it is clear that component perpendicular to x-axis will cancel out.

$$E_P = \int dE_P\cos\theta$$

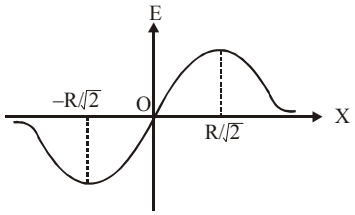
$$\text{or } E_P = \int \frac{k.dQ}{(x^2+R^2)}\cdot\frac{x}{\sqrt{x^2+R^2}}$$

$$= \frac{kx}{(x^2+R^2)^{3/2}}\int dQ$$

$$\text{or } E_P = \frac{kQx}{(x^2+R^2)^{3/2}}$$

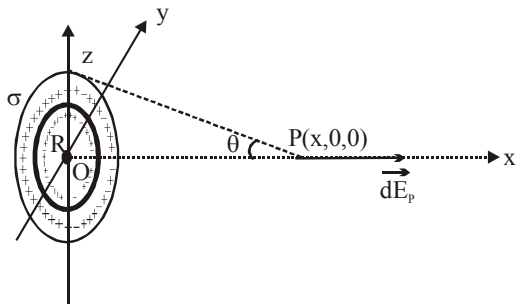
The graphical variation of electric field along x-axis is as shown below:

$$\frac{dE_P}{dx} = 0 \Rightarrow x = \pm\frac{R}{\sqrt{2}}$$



Electric Field due to Charge Disc

Let us consider a uniformly charged disc of radius 'R' and Charged density σ as shown in figure. We wish to calculate Electric field at a point on its axis.



Let us consider an elementary ring of radius 'r' and thickness 'dr', then

dQ = charge on elementary ring

$= (2\pi r)(dr)\sigma = 2\pi\sigma r\,dr$

dE_P = Electric field due to elementary ring

$$= \frac{k.dQx}{(x^2+r^2)^{3/2}}$$

$$\therefore\ \vec{E}_P = \int d\vec{E}_P$$

$$\text{or } E_P = 2\pi k\sigma x\int_0^R \frac{rdr}{(x^2+r^2)^{3/2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

This result is easy to remember in following form

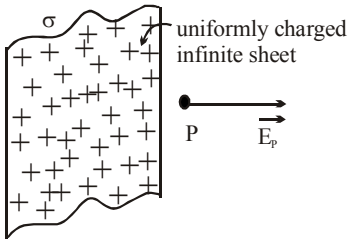
$$E = \frac{\sigma}{2\varepsilon_0}(1-\cos\alpha)$$

where α = semicone angle subtended by disc at point 'P'.

Case : If $R\to\infty$, then $\alpha\to\frac{\pi}{2}$

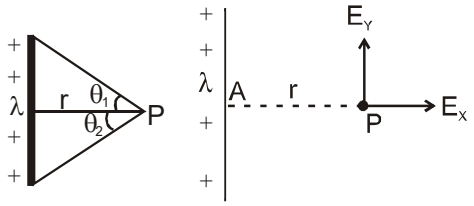
Disc becomes infinite sheet i.e., an infinite sheet can be looked upon as disc of infinite radius.

$$E_{\text{infinite sheet}} = \frac{\sigma}{2\varepsilon_0}$$



Electric Field due to Uniformly Charged Wire

(*i*) Line charge of finite length : Derivation of expression for intensity of electric field at a point due to line charge of finite size of uniform linear charge density λ. The perpendicular distance of the point from the line charge is r and lines joining ends of line charge distribution make angle θ_1 and θ_2 with the perpendicular line.



$$E_x = \frac{K\lambda}{r}[\sin\theta_1 + \sin\theta_2] \quad .....(1)$$

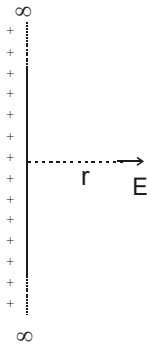
$$E_y = \frac{K\lambda}{r}[\cos\theta_2 - \cos\theta_1]$$

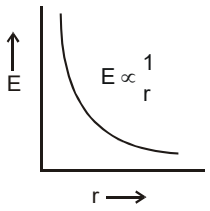
Net electric field at the point

$$E_{net} = \sqrt{E_x^2 + E_y^2}$$

(*ii*) We can derive a result for infinitely long line charge: In above eq. (1) & (2) if we put $q_1 = q_2 = 90^\circ$ we can get required result.

$$E_{net} = E_x = \frac{2K\lambda}{r}$$

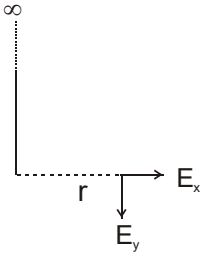




(*iii*) For Semi- infinite wire

$\theta_1 = 90^\circ$ and $\theta_2 = 0^\circ$ so

$$E_x = \frac{K\lambda}{r}, E_y = \frac{K\lambda}{r}$$

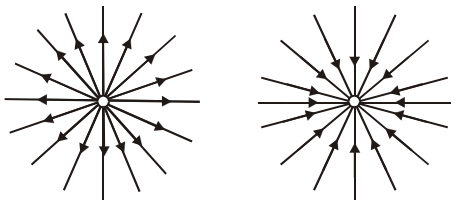


ELECTRIC LINES OF FORCE

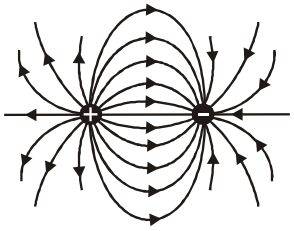
The idea of lines of force was introduced by Michael Faraday. A line of force is an imaginary curve, the tangent to which at a point gives the direction of electric field intensity at that point and the number of lines of force per unit area normal to the surface surrounding that point gives the magnitude of electric field intensity at that point.

IMPORTANT POINTS

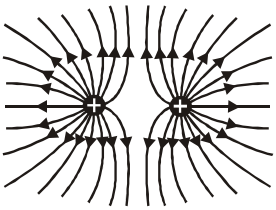
- Electric lines of force usually start or diverge out from positive charge and end or converge on negative charge.



- The number of lines originating or terminating on a charge is proportional to the magnitude of charge.
- Lines of force never cross each other because if they cross then electric field intensity at that point will have two directions which is not possible.
- In electrostatics the electric lines of force can never be closed loops, as a line can never start and end on the same charge. If a line of force is a closed curve, work done round a closed path will not be zero and electric field will not remain conservative.
- Lines of force have tendency to contract longitudinally like a stretched elastic string producing attraction between opposite charges and repel each other laterally resulting in, repulsion between similar charges and 'edge-effect' (curving of lines of force near the edges of a charged conductor)

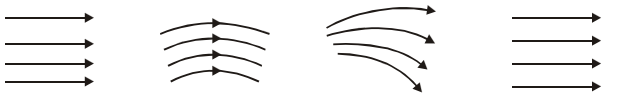


electric lines of force for a dipole



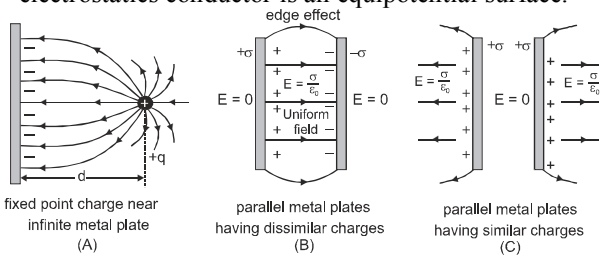
electric lines of force for a system of two positive charges

- If the lines of force are equidistant straight lines the field is uniform and if lines of force are not equidistant or straight lines or both, the field will be non-uniform. The first three represent non-uniform field while last shows uniform field.



magnitude is not constant | direction is not constant | both magnitude and direction not constant | both magnitude and direction constant

- Electric lines of force end or start normally on the surface of a conductor. If a line of force is not normal to the surface of a conductor, electric intensity will have a component along the surface of the conductor and hence conductor will not remain equipotential which is not possible as in electrostatics conductor is an equipotential surface.



fixed point charge near infinite metal plate (A) | parallel metal plates having dissimilar charges (B) | parallel metal plates having similar charges (C)

- If in a region of space, there is no electric field there will be no lines of force. This is why inside a conductor or at a neutral point where resultant intensity is zero there is no lines of force.
- The number of lines of force per unit normal area at a point represents magnitude of electric field intensity. The crowded lines represent strong field while distant lines show a weak field.
- The tangent to the line of force at a point in an electric field gives the direction of intensity. It gives direction of force and hence acceleration which a positive charge will experience there (and not the direction of motion). A positive point charge free to move may or may not follow the line of force. It will follow the line of force if it is a straight line (as direction of velocity and acceleration will be same) and will not follow the line if it is curved as the direction of motion will be different from that of acceleration. The particle will not move in the direction of motion or acceleration (line of force) but other than these which will vary with time as $\vec{v} = \vec{u} + \vec{a}t$

SOME SPECIAL CASES

- Intensity of the electric field inside a charged spherical conductor is zero, since charge resides on the outer surface of the conductor.
- The intensity of the electric field outside the charged spherical conductor is:

  $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

  r > R (R = radius of spherical conductor)
- Intensity of the electric field at some point on the axis of uniformly charged ring $E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+R^2)^{3/2}}$
- Intensity of electric field near an infinite rod of charge $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ = the linear charge density and r is the distance from the axis of rod.
- Intensity of the electric field near a non-conducting infinite sheet of charge $E = \frac{\sigma}{2\varepsilon_0}$,

  where σ = surface charge density
- Intensity of electric feld near a conducting infinite sheet = $\frac{\sigma}{\varepsilon_0}$
- Intensity of the electric field in between the plates of a charged parallel plate capacitor $E = \frac{\sigma}{\varepsilon_0}$

ELECTRIC-FLUX

Electric flux through an elementary area 'ds' is defined as the scalar product of area and field, i.e.,

$$d\phi_E = \vec{E}\cdot\vec{ds} = Eds\cos\theta \qquad \text{i.e., } \phi_E = \int \vec{E}\cdot\vec{ds}$$

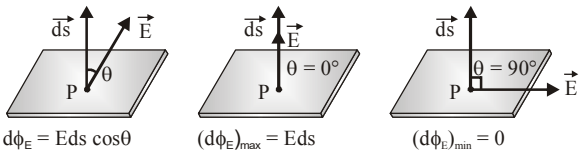
It represents the total lines of force passing through the given area. Here area is treated as a vector.

The direction of area vector is given by direction of normal to the surface.

IMPORTANT POINTS

- It is a scalar physical quantity.
- **SI Units :** volt × m

  dimensions : $[ML^3T^{-3}A^{-1}]$



$d\phi_E = Eds\cos\theta$ | $(d\phi_E)_{max} = Eds$ | $(d\phi_E)_{min} = 0$

- It will be maximum when cos θ is max = 1, i.e., θ = 0°, i.e., electric field is normal to the area with $(d\phi_E)_{max} = E\,ds$
- It will be minimum when cos θ is min = 0, i.e., θ = 90°, i.e. field is parallel to the area with $(d\phi_E)_{min} = 0$
- For a closed body outward flux is taken as positive while inward flux is taken as negative.

GAUSS'S LAW

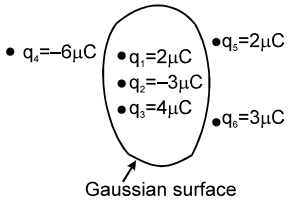
It relates the total flux of an electric field through a closed surface to the net charge enclosed by that surface.

According to it, the total flux linked with a closed surface = $\frac{1}{\varepsilon_0}$ times the charge enclosed by the closed surface.

Mathematically $\oint_s E.ds = \frac{q}{\varepsilon_0}$

TRAIN YOUR BRAIN

Example 4: Find out flux through the given gaussian surface.



Sol. $\phi = \frac{Q_{in}}{\varepsilon_0} = \frac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0} = \frac{3\times10^{-6}}{\varepsilon_0}$ Nm²/C

Example 5: If a point charge q is placed at the centre of a cube then find out flux through any one surface of cube.

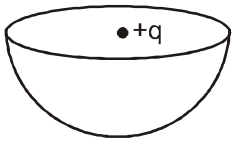
Sol. Flux through 6 surfaces = $\frac{q}{\varepsilon_0}$.

Since all the surfaces are symmetrical

so, flux through one surfaces = $\frac{1}{6}\ \frac{q}{\varepsilon_0}$

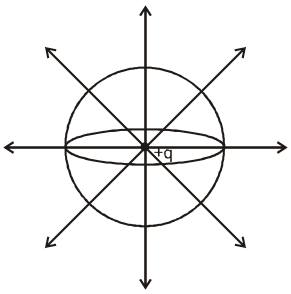
Flux through open surfaces using Gauss's Theorem :

Example 6: A point charge +q is placed at the centre of curvature of a hemisphere. Find flux through the hemispherical surface.



Sol. Lets put an upper half hemisphere.

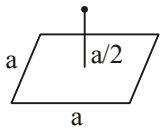
Now flux passing through the entire sphere = $\frac{q}{\varepsilon_0}$



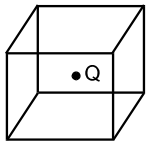
As the charge q is symmetrical to the upper half and lower half hemispheres , so half-half flux will emit from both the surfaces.

Flux cutting lower half surface = $\frac{q}{2\varepsilon_0}$ Flux cutting upper half surface = $\frac{q}{2\varepsilon_0}$

Example 7: A charge Q is placed at a distance a/2 above the centre of a horizontal square surface of edge a as shown in figure. Find the flux of the electric field through the square surface.



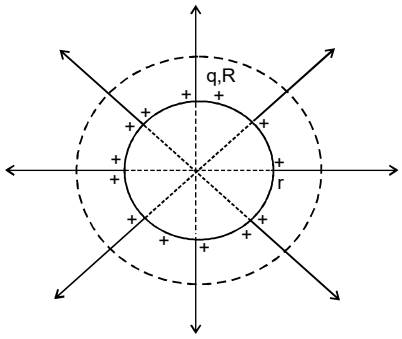
Sol. We can consider imaginary faces of cube such that the charge lies at the centre of the cube. Due to symmetry we can say that flux through the given area (which is one face of cube) $\phi = \frac{Q}{6\varepsilon_0}$



APPLICATIONS OF GAUSS LAW

Finding E due to a Spherical Shell:

Electric field outside the Sphere: Since, electric field due to a shell will be radially outwards. So lets choose a spherical Gaussian surface Applying Gauss`s theorem for this spherical Gaussian surface,



$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{q}{\varepsilon_o}$

↓

$\int |\vec{E}||\vec{ds}| \cos 0$ (because the $\vec{E}$ is normal to the surface)

↓

$E\int ds$ (because value of E is constant at the surface)

↓

$E(4\pi r^2)$

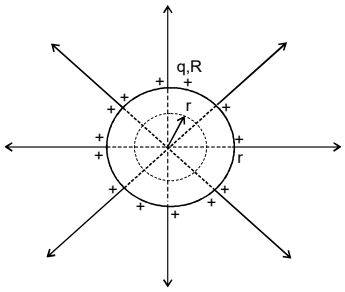
($\int ds$ = total area of the spherical surface = $4\pi r^2$)

$\Rightarrow E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0}$

$\Rightarrow E_{out} = \frac{q}{4\pi\varepsilon_o r^2}$

Electric field inside a spherical shell: Lets choose a spherical gaussian surface inside the shell.

Applying Gauss`s theorem for this surface



$$\frac{q}{4\pi\varepsilon_o r^2} = \phi_{net} = \frac{q_{in}}{\varepsilon_0} = 0$$

$$\downarrow$$

$$\int |\vec{E}||\vec{ds}| \cos 0$$

$$\downarrow$$

$$E \int ds$$

$$\downarrow$$

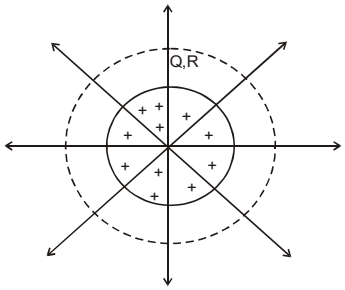
$$E\,(4\pi r^2)$$

$$\Rightarrow E\,(4\pi r^2) = 0$$

$$\Rightarrow E_{in} = 0$$

Electric field due to solid sphere (having uniformly distributed charge Q and radius R) :

Electric field outside the sphere: Direction of electric field is radially outwards, so we will choose a spherical gaussian surface Applying Gauss`s theorem



$$\int \vec{E}\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$$

$$\downarrow$$

$$\int |\vec{E}||\vec{ds}| \cos 0°$$

$$\downarrow$$

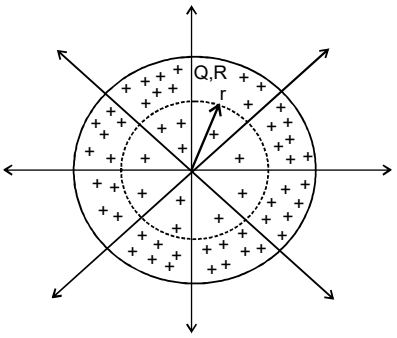
$$E \int ds$$

$$\downarrow$$

$$E\,(4\pi r^2)$$

$$\Rightarrow E\,(4\pi r^2) = \frac{Q}{\varepsilon_o} \qquad \Rightarrow E_{out} = \frac{Q}{4\pi\varepsilon_o r^2}$$

Electric field inside a solid sphere



For this choose a spherical gaussian surface inside the solid sphere Applying gauss`s theorem for this surface

$$\int \vec{E}\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\frac{q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3}{\varepsilon_o} = \frac{qr^3}{\varepsilon_o R^3}$$

$$\downarrow$$

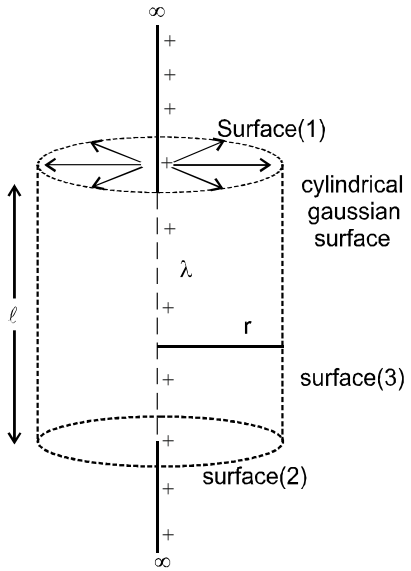
$$\int E\,ds$$

$$\downarrow$$

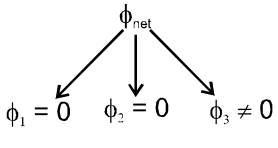
$$E(4\pi r^2) \qquad \Rightarrow E(4\pi r^2) = \frac{qr^3}{\varepsilon_o R^3}$$

$$E = \frac{q\ r}{4\pi\varepsilon_o R^3} \quad \Rightarrow E_{in} = \frac{kQ}{R^3}\ r$$

Electric field due to infinite line charge (having uniformly distributed charge of charge density λ) :



Electric field due to infinite wire is radial so we will choose cylindrical Gaussian surface as shown is figure.



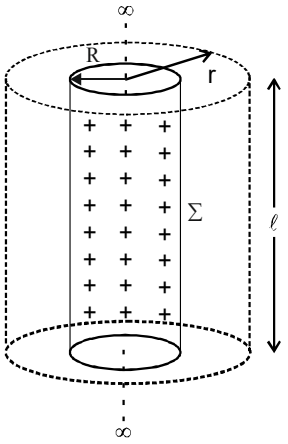
$$\phi_3 = \frac{q_{in}}{\varepsilon_o} = \frac{\lambda \ell}{\varepsilon_o}$$

$$\Rightarrow E\int ds = E(2\pi r\ell)$$

$$= \frac{\lambda \ell}{\varepsilon_o}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_o r} = \frac{2k\lambda}{r}$$

Electric field due to infinitely long charged tube (having uniform surface charge density σ and radius R) :



(*i*) E outside the tube: lets choose a cylindrical gaussian surface

$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\sigma 2\pi R\ell}{\varepsilon_o}$$

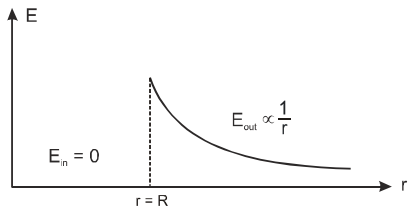
$$\Rightarrow E_{out} \times 2\pi r\ell = \frac{\sigma 2\pi R\ell}{\varepsilon_o}$$

$$\Rightarrow E = \frac{\sigma R}{r\,\varepsilon_0}$$

(*ii*) E inside the tube: lets choose a cylindrical gaussian surface inside the tube.

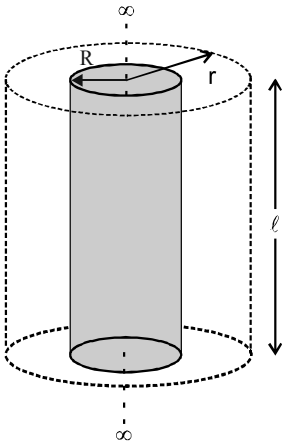
$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = 0$$

So, $E_{in} = 0$



E due to infinitely long solid cylinder of radius R (having uniform volume charge density ρ) :

(*i*) E at outside point :-



Lets choose a cylindrical gaussian surface.

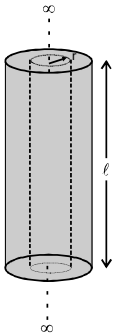
Applying Gauss`s Theorem

$$E \times 2\pi r\ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi R^2 \ell}{\varepsilon_o}$$

$$E_{out} = \frac{\rho R^2}{2r\,\varepsilon_0}$$

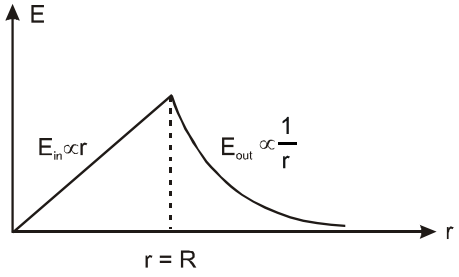
(*ii*) E at inside point: lets choose a cylindrical gaussian surface inside the solid cylinder.

Applying gauss`s theorem



$$E \times 2\pi r\ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi r^2 \ell}{\varepsilon_o}$$

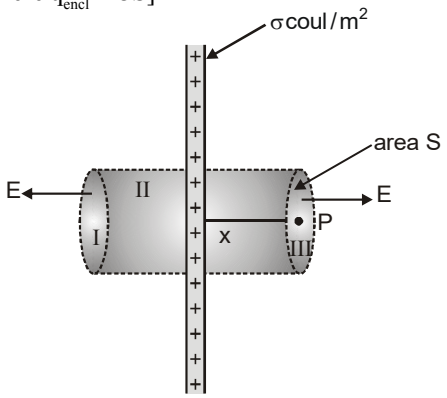
$$E_{in} = \frac{\rho r}{2\varepsilon_0}$$



Electric field Strength due to a Non-conducting Uniformly Charged Sheet: To find the electric field strength at a point P infront of the charged sheet we consider a cylindrical Gaussian surface as shown in figure of face area S. If we apply Gauss law for this surface, we have

$$\oint \vec{E}.\vec{dS} = \frac{q_{encl}}{\epsilon_0} \text{ or } \int_I \vec{E}.\vec{dS} + \int_{II} \vec{E}.\vec{dS} + \int_{III} \vec{E}.\vec{dS} = \frac{\sigma S}{\epsilon_0}$$

[As here $q_{encl} = \sigma S$]



In this case $\int_{II} \vec{E}.\vec{dS} = 0$

as the lateral surface of cylinder is parallel to the direction of electric field strength, no flux is coming out from the lateral surface, hence we have

$$\int_I EdS + \int_{III} EdS = \frac{\sigma S}{\epsilon_0}$$

or $2ES = \frac{\sigma S}{\epsilon_0}$

[As electric field is uniform on both sides]

or $E = \frac{\sigma}{2\epsilon_0}$

ELECTROSTATIC POTENTIAL ENERGY

(*a*) **Electrostatic Potential Energy:** Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Before proceeding further, we should keep in mind the following points, which are useful in understanding potential energy in electric fields.

(*i*) Doing work implies supply of energy

(*ii*) Energy can neither be transferred nor be transformed into any other form without doing work

(*iii*) Kinetic energy implies utilization of energy where as potential energy implies storage of energy

(*iv*) Whenever work is done on a system of bodies, the supplied energy to the system is either used in form of KE of its particles or it will be stored in the system in some form, increasing the potential energy of system.

(*v*) When all particles of a system are separated far apart by infinite distance there will be no interaction between them. This state we take as reference of zero potential energy.

Now potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles.

- **Electrostatic potential energy is defined in two ways.**

(*i*) Interaction energy of charged particles of a system.

(*ii*) Self energy of a charged object (will be discussed later)

(*b*) Electrostatic Interaction Energy: Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to a given configuration.

When some charged particles are at infinite separation, their potential energy is taken as zero as there is no interaction between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system hence final potential energy of system will be positive. If the force between the particles is attractive work will be done by the system and final potential energy of system will be negative.

Let us take some illustrations to understand this concept in detail.

(*c*) Interaction Energy of a System of Two Charged Particles:

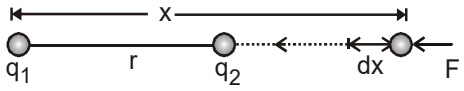


Figure shows two +ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be given as work done in bringing q_2 from infinity to the given separation from q_1. It can be calculated as

$$W = \int_{\infty}^{r} \vec{F}.\vec{dx} = -\int_{\infty}^{r} \frac{Kq_1q_2}{x^2} dx$$

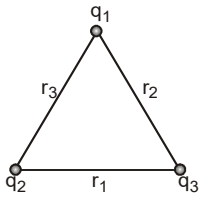
[–ve sign shows that x is decreasing]

$$W = \frac{Kq_1q_2}{r} = U \qquad \text{[Interaction energy]}$$

If the two charges here are of opposite sign, the potential energy will be negative as

$$U = -\frac{Kq_1q_2}{r}$$

Potential Energy for a System of Charged Particles:



When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energy of all the pairs of particles. For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure. The total interaction energy of this system can be given as

$$U = \frac{Kq_1q_2}{r_3} + \frac{Kq_1q_3}{r_2} + \frac{Kq_2q_3}{r_1}$$

Derivation for a System of Point Charges

(*i*) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebric sum of all the works.

Let W_1 = work done in bringing first charge

W_2 = work done in bringing second charge against force due to 1st charge

W_3 = work done in bringing third charge against force due to 1st and 2nd charge.

PE = $W_1 + W_2 + W_3 +$ ..... (This will contain $\frac{n(n-1)}{2} = {}^nC_2$ terms)

(*ii*) Method of calculation (to be used in problems)

U = sum of the interaction energies of the charges.

$$= (U_{12} + U_{13} + ...+ U_{1n}) + (U_{23} + U_{24} + ...+U_{2n}) + (U_{34} + U_{35} + ...U_{3n}) .....$$

(*iii*) Method of calculation useful for symmetrical point charge systems.

Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges.

$$= (U_{12} + U_{13} + ...+ U_{1n})$$

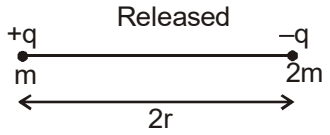
U_2 = PE of second charges due to all other charges.

$$= (U_{21} + U_{23} + ...+U_{2n})$$

U = PE of the system = $\frac{U_1 + U_2 + ...}{2}$

TRAIN YOUR BRAIN

Example 8: Find out speed of particles when separation between them is r.



Sol. By Energy conservation:

$$0 - \frac{kq_1q_2}{2r} = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2 - \frac{kq_1q_2}{r} \qquad ...(1)$$

By Momentum conservation

(as E.F is action-reaction pair)

$$mv_1 = 2mv_2 \Rightarrow v_2 = \frac{v_1}{2} \qquad ...(2)$$

Solving (1) & (2), we get, $v_2 = q\sqrt{\frac{k}{6mr}}$

$$\& \quad v_1 = 2q\sqrt{\frac{k}{6mr}}$$

Example 9: A proton moves from a large distance with a speed u m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons in terms of mass of proton m and its charge e.

Sol. As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach both will move with same velocity. So if v is the common velocity of each particle at closest approach, then by 'conservation of momentum' of the two protons system.

$$mu = mv + mv \qquad \text{i.e., } v = \frac{1}{2}u$$

And by conservation of energy

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

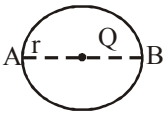
$$\Rightarrow \frac{1}{2}mu^2 - m\left(\frac{u}{2}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad [\text{as } v = \frac{u}{2}]$$

$$\Rightarrow \frac{1}{4}mu^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow r = \frac{e^2}{\pi m\varepsilon_0 u^2}$$

Example 10: A charge Q is placed at the centre of a circle of a radius 'r'. What will be the work done in taking a charge q from A to diametrically opposite point B ?

Sol. Potential energy of q at A



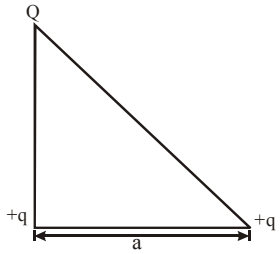
$$= U_A = \frac{1}{4\pi \in_0}\frac{Qq}{r}$$

PE of q at B

$$= U_B = \frac{1}{4\pi \in_0}\frac{Qq}{r}$$

$\therefore$ Work done $= U_B - U_A = 0$

Example 11: Three charges Q, +q and +q are placed at the vertices of a right angled isosceles triangle as shown in the figure. The net electrostatic energy of the configuration is zero., if Q is equal to:



Sol. Net electrostatic energy

$$U = \frac{kQq}{a} + \frac{kQq}{\sqrt{2}a} + \frac{kqq}{a}$$

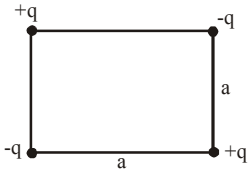
For $U = 0$; $\frac{Qq}{a} + \frac{Qq}{\sqrt{2}\,a} + \frac{qq}{a} = 0$

$$\Rightarrow \quad Qq\left(\frac{1}{a} + \frac{1}{\sqrt{2}\,a}\right) = \frac{-q^2}{a}$$

$$\Rightarrow \quad Q\left(\frac{\sqrt{2}+1}{\sqrt{2}\,a}\right) = \frac{-q}{a}$$

$$\Rightarrow \quad Q = -q\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right) \qquad = -q\left(\frac{2}{2+\sqrt{2}}\right)$$

Example 12: What is work done by the electrostatic field when we put the four charges together, as shown in the figure. Each side of the square has a length a. Initially charges were at infinity.



Sol. $U_i = 0$

[Where charges are separated by infinite distance]

$$U_f = \frac{1}{4\pi\varepsilon_0}\left(\frac{-4q^2}{a} + \frac{q^2}{\sqrt{2}a} + \frac{(-q)^2}{\sqrt{2}a}\right)$$

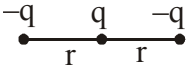
[for 6 pairs of charges]

Work done by field is

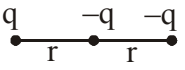
$$W = -\Delta U = U_i = U_f = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}(4-\sqrt{2})$$

Example 13: Two (– ve) charge, each of magnitude q are situated at 2 r distance apart. A (+ ve) charge q is lying at the centre between them. The potential energy of the system is U_1. If the two nearest charges are mutually interchanged and the potential energy becomes U_2, then $\frac{U_1}{U_2}$ will be:

Sol.



$$U_1 = \frac{-kq^2}{r} + \frac{-kq^2}{r} + \frac{kq^2}{2r} = \frac{-3kq^2}{2r}$$

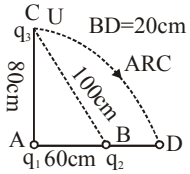


$$U_2 = \frac{-kq^2}{r} + \frac{kq^2}{r} - \frac{kq^2}{2r} = \frac{-kq^2}{2r}$$

So $\frac{U_1}{U_2} = 3$

Example 14: For the given figure charge $q_1 = 2 \times 10^{-8}$C and $q_2 = -0.4 \times 10^{-8}$C. A charge q_3 of 0.2×10^{-8}C is taken along arc of circle from C to D then potential energy of charge q_3 is decreased by ____ percent.

Sol. EPE of q_3 at $C = q_3\left(\frac{kq_1}{0.8} + \frac{kq_2}{1}\right)$



EPE of q_3 at $D = q_3\left(\frac{kq_1}{0.8} + \frac{kq_2}{0.2}\right)$

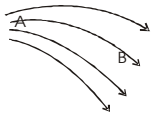
or EPE at C and D are 3.78×10^{-7} J and 0.9×10^{-7} J.

$$\% \text{ decrement} = \frac{2.88}{3.78} \times 100 = 76\%$$

(decreases, 76%)

Example 15: Some electric lines of force are shown in figure, for point A and B

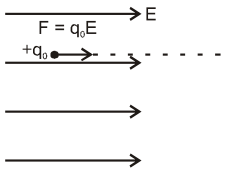
(*a*) $E_A > E_B$
(*b*) $E_B > E_A$
(*c*) $V_A > V_B$
(*d*) $V_B > B_A$



Sol. lines are more dense at A so $E_A > E_B$ In the direction of Electric field, potential decreases so $V_A > V_B$

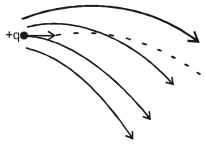
Example 16: If a charge is released in electric field, will it follow lines of force?

Sol. **Case I :** If lines of force are parallel (in uniform electric field) :



In this type of field, if a charge is released, force on it will be q_oE and its direction will be along $\vec{E}$ .So the charge will move in a straight line , along the lines of force.

Case II : If lines of force are curved (in non-uniform electric field):



The charge will not follow lines of force

ELECTRIC POTENTIAL

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field, electric potential is defined as the interaction energy of a unit positive charge.

If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as

$$V = \frac{U}{q_0} \text{ joule/coulomb}$$

As potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces."

Properties:

(*i*) Potential is a scalar quantity, its value may be positive, negative or zero.

(*ii*) S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{coulomb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.

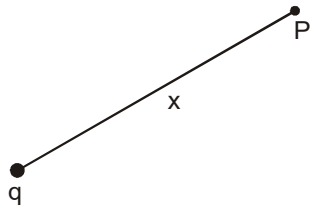
(*iii*) Electric potential at a point is also equal to the negative of the work done by the electric field in taking the point charge from reference point (i.e. infinity) to that point.

(*iv*) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinity. (taking $V_\infty = 0$)

(*v*) Potential decreases in the direction of electric field.

Electric Potential due to a Point Charge

We know the region surrounding a charge is electric field. Thus we can also define electric potential in the surrounding of a point charge. The potential at a point P at a distance x from the charge q can be given as



$$V_p = \frac{U}{q_0}$$

Where U is the potential energy of charge q_0, if placed at point P, which can be given as

$$U = \frac{Kqq_0}{x}$$

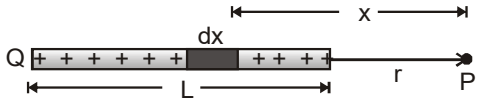
Thus potential at point P is

$$V_P = \frac{Kq}{x}$$

The above result is valid only for electric potential in the surrounding of a point charge. If we wish to find electric potential in the surrounding of a charged extended body, we first find the potential due to an elemental charge dq on body by using the above result and then integrate the expression for the whole body.

Electric Potential due to a Charge Rod

Figure shows a charged rod of length L, uniformly charged with a charge Q. Due to this we will find electric potential at a point P at a distance r from one end of the rod shown in figure shown.



For this we consider an element of width dx at a distance x from the point P. Charge on this element is

$$dq = \frac{Q}{L}dx$$

The potential dV due to this element at point P can be given by using the result of a point charge as

$$dV = \frac{Kdq}{x} = \frac{KQ}{Lx}dx$$

Net electric potential at point P can be given as

$$V = \int dV = \int_{r}^{r+L} \frac{KQ}{Lx}dx$$

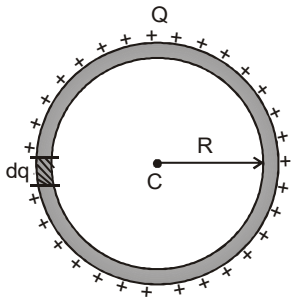
$$= \frac{KQ}{L}[\ell n\, x]_r^{r+L} = \frac{KQ}{L}\ell n\left(\frac{r+L}{r}\right)$$

Electric Potential due to a Charged Ring

Case I : At its centre

To find potential at the centre C of the ring, we first find potential dV at centre due to an elemental charge dq on ring which is given as

$$dV = \frac{Kdq}{R}$$



Total potential at C is

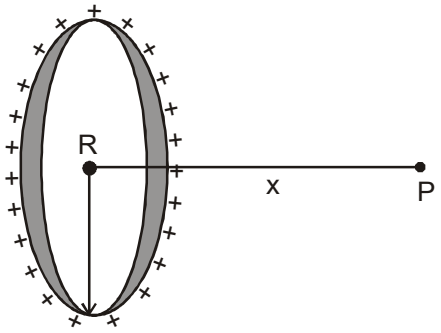
$$V = \int dV$$

$$= \int \frac{Kdq}{R} = \frac{KQ}{R}$$

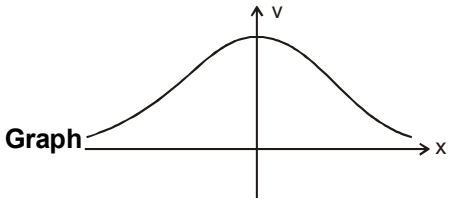
As all dq's of the ring are situated at same distance R from the ring centre C, simply the potential due to all is added as being a scalar quantity, we can directly say that the electric potential at ring centre is $\frac{KQ}{R}$. Here we can also state that even if charge Q is non-uniformly distributed on ring, the electric potential at C will remain same.

Case II : At a Point on Axis of Ring

If we wish to find the electric potential at a point P on the axis of ring as shown, we can directly state the result as here also all points of ring are at same distance $\sqrt{x^2+R^2}$ from the point P, thus the potential at P can be given as



$$V_P = \frac{KQ}{\sqrt{R^2+x^2}}$$



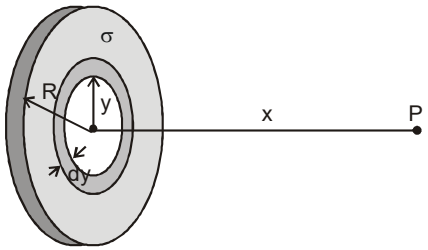
Electric Potential due to a Uniformly Charged Disc

Figure shows a uniformly charged disc of radius R with surface charge density σ. To find electric potential at point P we consider an elemental ring of radius y and width dy, charge on this elemental ring is

$$dq = \sigma . 2\pi y\, dy$$

Due to this ring, the electric potential at point P can be given as

$$dV = \frac{Kdq}{\sqrt{x^2+y^2}} = \frac{K.\sigma.2\pi y\, dy}{\sqrt{x^2+y^2}}$$



Net electric potential at point P due to whole disc can be given as

$$V = \int dV = \int_0^R \frac{\sigma}{2\epsilon_0}\cdot\frac{ydy}{\sqrt{x^2+y^2}}$$

$$= \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R$$

$$V_P = \frac{\sigma}{2\epsilon_0}[\sqrt{x^2+R^2}-x]$$

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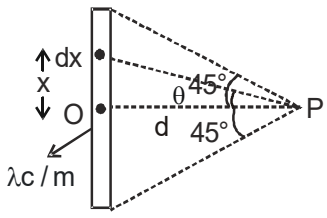
Example 17: Consider the following rod & find the potential due to it at P



Sol. OP = d,

$x = d\tan\theta,\ dx = d\sec^2\theta\, d\theta$

$$dV = \frac{k\lambda dx}{d\sec\theta}$$

$$\Rightarrow \int dV = \int_{-\pi/4}^{+\pi/4} \frac{kd\sec^2\theta d\theta\lambda}{d\sec\theta}$$

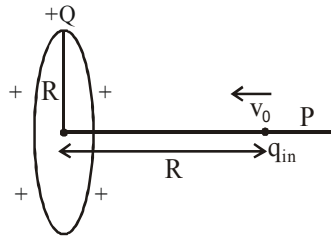
$$V = k\lambda \int_{-\pi/4}^{\pi/4} \sec\theta\, d\theta$$

$$V = k\lambda\, [ln(\sec\theta + \tan\theta)]_{-\pi/4}^{\pi/4}$$

$$V = k\lambda\, [ln(\sqrt{2}+1)] - k\lambda\, [ln(\sqrt{2}-1)]$$

$$V = k\lambda\, ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) = k\lambda\, ln(\sqrt{2}+1)^2$$

Example 18: Find min velocity v_0 such that particle crosses the ring.



Sol. Potential at P = $\frac{kQ}{\sqrt{2}R}$

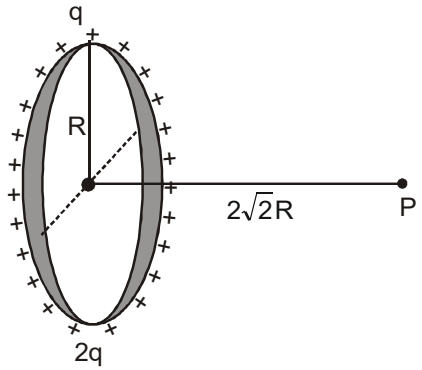
Applying energy conservation

$$\frac{1}{2}mv_0^2 + \frac{kQq}{\sqrt{2}R} = 0 + \frac{kqQ}{R}$$

$$\Rightarrow v_0 = \sqrt{\frac{2kQq}{mR}\left(1 - \frac{1}{\sqrt{2}}\right)}$$

Example 19: A ring of radius R is having two charges q and 2q distributed on its two half parts. Find the electric potential at a point on its axis at a distance $2\sqrt{2}R$ from its centre.

Sol.



Distance of P from periphery of ring is

$$\sqrt{R^2 + (2\sqrt{2}R)^2} = 3R$$

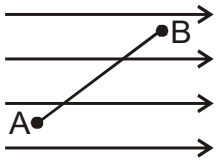
Electric potential = Potential due to upper half + Potential due to lower half

$$V_P = \frac{Kq}{3R} + \frac{2Kq}{3R}$$

$$= \frac{3Kq}{3R} = \frac{Kq}{R}$$

RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL

For Uniform Electric Field



Potential difference between two points A and B

$$V_B - V_A = -\vec{E}.\vec{AB}$$

Non Uniform Electric Field

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$= -\left[\hat{i}\frac{\partial}{\partial x}V + \hat{j}\frac{\partial}{\partial y}V + \hat{k}\frac{\partial}{\partial z}V\right]$$

$$= -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V$$

$$= -\nabla V = -\text{grad}\, V$$

Where $\frac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

$\frac{\partial V}{\partial y}$ = derivative of V with respect to y (keeping z and x constant)

$\frac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

If electric potential and electric field depends only on one coordinate, say r:

$$\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$$

where $\hat{r}$ is a unit vector along increasing r.

$$\int dV = -\int \vec{E}.\vec{dr}$$

$$\Rightarrow V_B - V_A = -\int_{r_A}^{r_B} \vec{E}.\vec{dr}$$

$\vec{dr}$ is along the increasing direction of r.
The potential of a point

$$V = -\int_{\infty}^{r} \vec{E}.\vec{dr}$$

Area under E - x curve gives negative of change in potential.
Negative of slope of V - x curve gives the electric field at that point.

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Example 20:

$V = x^2 + y$, Find $\vec{E}$.

Sol. $\frac{\partial V}{\partial x} = 2x$, $\frac{\partial V}{\partial y} = 1$

and $\frac{\partial V}{\partial z} = 0$

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) = -(2x\hat{i} + \hat{j})$$

Electric field is nonuniform.

Example 21: The electric potential in a region is represented as $V = 2x + 3y - z$. Obtain expression for the electric field strength.

Sol. We know, $\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$

Here, $\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}[2x + 3y - z] = 2$

$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}[2x + 3y - z] = 3$

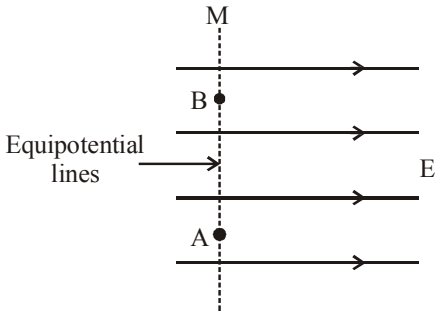
$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}[2x + 3y - z] = -1$

$\therefore \vec{E} = -(2\hat{i} + 3\hat{j} - \hat{k})$

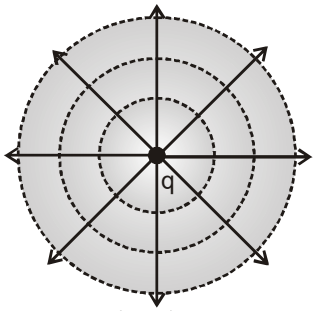
EQUIPOTENTIAL SURFACES

As shown in figure if a charge is shifted from a point A to B on a surface. M which is perpendicular to the direction of electric field, the work done in shifting will obviously, be zero as electric force is normal to the direction of displacement.

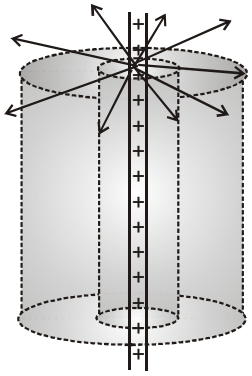
As no work is done in moving from A to B, we can say that A and B are at same potential or we can say that all the points of surface M are at same potential or here we call surface M as equipotential surface.



Following figures show equipotential surfaces in the surrounding of point charge and a long charged wire.



Point charge
Spherical equipotential surfaces



Line charge
Cylindrical equipotential surfaces

Every surface in electric field in which at every point direction of electric field is normal to the surface can be regarded as equipotential surface.

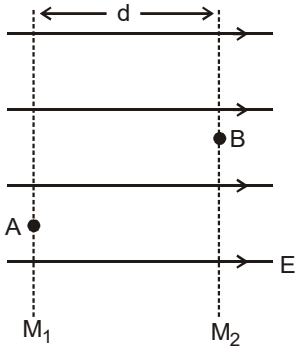
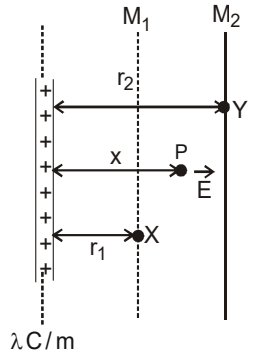


Figure shows two equipotential surfaces in a uniform electric field E. If we wish to find the potential difference between two points A and B shown in figure, we simply find the potential difference between the two equipotential surfaces on which the points lie, given as

$$V_A - V_B = Ed$$

Electric Potential Difference due to Infinte Wire

Figure shows a line charge with linear charge density λ C/m. Here we wish to find potential difference between two points X and Y which lie on equipotential surfaces M_1 & M_2. To find the potential difference between these surfaces, we consider a point P at a distance x from wire as shown. The electric field at point P is



$$E = \frac{2K\lambda}{x}$$

Now the potential difference between surface M_1 and M_2 can be given as

$$V_x - V_y = \int_{r_1}^{r_2} E dx = \int_{r_1}^{r_2} \frac{2K\lambda}{x} dx$$

$$V_X - V_Y = 2K\lambda \ell n\left(\frac{r_2}{r_1}\right)$$

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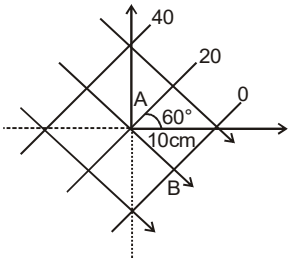
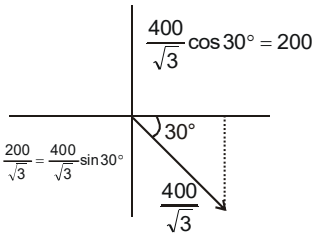
Example 22:



Write down the Electric field in vector form ?

Sol. $v_A - v_B = E \times 0.1 \times \cos 30°$

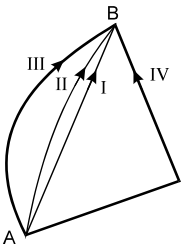
$$20 = E \times 0.1 \times \frac{\sqrt{3}}{2}$$



$$\Rightarrow E = \frac{400}{\sqrt{3}}$$

$$E.F. = 200\hat{i} - \frac{200}{\sqrt{3}}\hat{j} N/C$$

Example 23: In the following fig. Along which path the work done will be maximum in carrying a charge from A to B in the presence of another charge



Sol. Same for each of the paths [Because the work done doesn't depend upon the path]

Example 24: Electric potential for a point (x , y , z) is given by $V = 4x^2$ volt . Electric field at point (1 , 0 , 2) is -

Sol. $E = -\frac{dV}{dx} = -8x$

E at (1, 0 ,2)= – 8 V/m

∴ Magnitude of E

= 8V/m direction along –x axis.

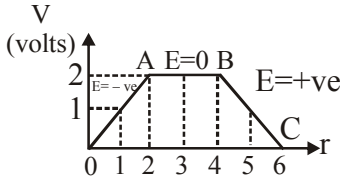
Example 25: Potential in the x-y plane is given as $V = 5(x^2 + xy)$ volts. Find the electric field at the point (1, –2).

Sol. $E_x = -\frac{\partial V}{\partial x} = -(10x + 5y) = -10 + 10 = 0, E_y = -\frac{\partial V}{\partial y} = -5x$

$= -5$ V/m

$\therefore \vec{E} = (-5\hat{j})V/m$

Example 26: Graph shows variation of potential v/s distance. Calculate electric field at various points.

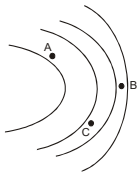


$$E = \frac{-dv}{dr} = -\text{slope} = -\tan\theta$$

Sol.

distance	E(V/m)
r = 1	–1
r = 1.5	–1
r = 2.5	0
r = 5	1

Example 27: Some equipotential surfaces are shown in fig. What is the correct order of electric field intensity ?

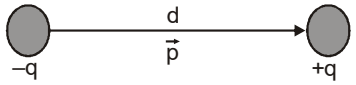


Sol. $E_B > E_C > E_A$, because potential gradient at B is maximum.

ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges. It is given as.

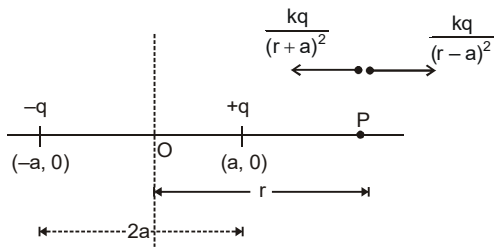
$$\vec{p} = q\,\vec{d}$$



Dipole moment is a vector quantity and convensionally its direction is given from negative pole to positive pole.

Electric Field Due to a Dipole

(1) At an axial point: Figure shows an electric dipole placed on x-axis at origin



Here we wish to find the electric field at point P having coordinates (r, 0) (where r >> 2a). Due to positive charge of dipole electric field at P is in outward direction & due to negative charge it is in inward direction.

$$E_{net} \text{ at } P = \frac{kq}{(r-a)^2} - \frac{kq}{(r+a)^2} = \frac{4kqar}{(r^2-a^2)^2}$$

As $\vec{P} = 2aq$

$$\therefore E_{net} \text{ at } P = \frac{2kpr}{(r^2-a^2)^2}$$

As r >> 2a

$\therefore$ we can neglect a w.r.t. r

$$E_{net} \text{ at } P = \frac{2kp}{r^3}$$

As we can observe that for axial point direction of field is in direction of dipole moment

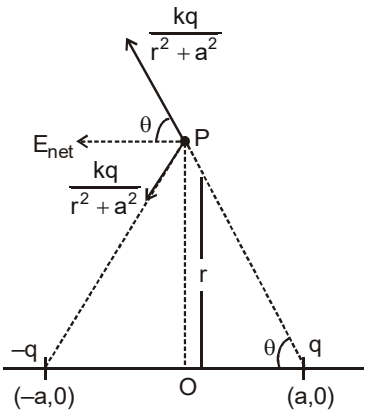
$$\therefore \text{ Vectorially, } \vec{E} = \frac{2k\vec{p}}{r^3}$$

(2) At an equatorial point: Again we consider the dipole placed along the x-axis & we wish to find, electric field at point P which is situated equatorially at a distance r (where r >> 2a) from origin.

Vertical component of the electric field vectors cancel out each other.

$$\therefore E_{net} \text{ at } P = 2E\cos\theta \quad \left[\text{where } E = \frac{kq}{r^2+a^2}\right]$$

$$E_{net} \text{ at } P = \frac{2kq}{r^2+a^2}\cdot\frac{a}{\sqrt{r^2+a^2}} \quad \left[\because \cos\theta = \frac{a}{\sqrt{r^2+a^2}}\right]$$



$$E_{net} = \frac{2kqa}{(r^2+a^2)^{3/2}}$$

$$= \frac{kp}{(r^2+a^2)^{3/2}} \quad (\text{As } p = 2aq)$$

As we have already stated that r > > 2a

$$\therefore E_{net} \text{ at } P = \frac{kp}{r^3}$$

We can observe that the direction of dipole moment & electric field due to dipole at P are in opposite direction.

$\therefore$ Vectorially

$$\vec{E} = \frac{-k\,\vec{P}}{r^3}$$

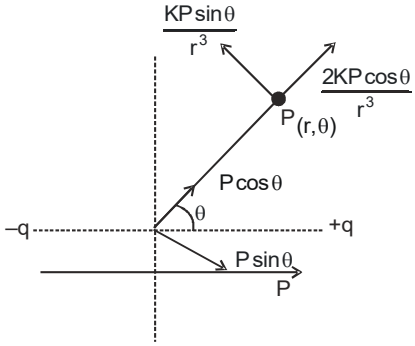
Electric Field at a General Point due to a Dipole

Figure shows a electric dipole placed on x-axis at origin & we wish to find out the electric field at point P with coordinate (r, θ)

$$E_{net} = \sqrt{\left(\frac{2KP\cos\theta}{r^3}\right)^2 + \left(\frac{KP\sin\theta}{r^3}\right)^2}$$

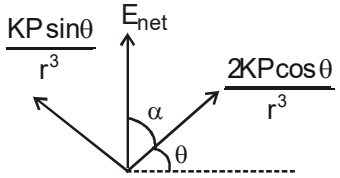
$$= \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\alpha = \frac{\frac{kp\sin\theta}{r^3}}{\frac{2kp\cos\theta}{r^3}}$$



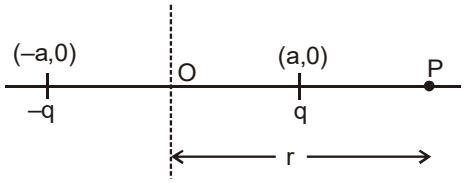
$$\tan\alpha = \frac{\tan\theta}{2}$$

$$\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$$



Electric Potential due to a Dipole

(1) At an axial point



(r >> 2a)

We wish to find out potential at P due to dipole (with p = 2aq)

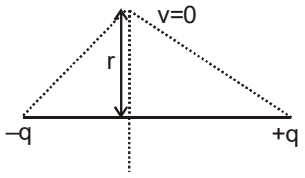
$$V_{net} = \frac{kq}{(r-a)} - \frac{kq}{(r+a)}$$

$$V_{net} = \frac{2akq}{(r^2 - a^2)}$$

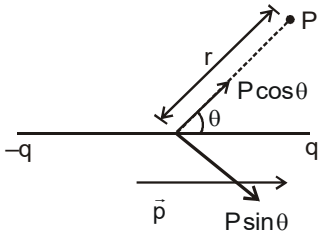
$$V_{net} = \frac{kp}{r^2} \text{ (As } P = 2aq)$$

$$V = \frac{k\vec{p}.\vec{r}}{r^3}$$

(2) At a point on perpendicular bisector: At an equatorial point, electric potential due to dipole is always zero because potential due to +ve charge is cancelled by –ve charge.



(3) Potential due to dipole at a general Point



Potential at P due to dipole = $\frac{kp\cos\theta}{r^2}$

Basic Torque Concept

$\vec{\tau} = \vec{r} \times \vec{F}$

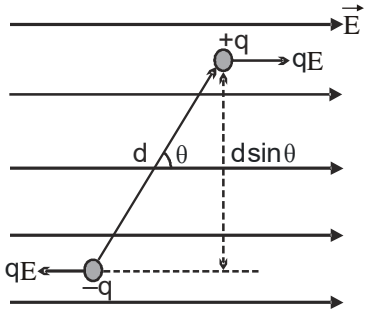
⇒ If the net translational force on the body is zero then the torque of the forces may or may not be zero but net torque of the forces about each point of universe is same.

⇒ If we have to prove that a body is in equilibrium then first we will prove $\vec{F}_{net}$ is equal to zero & after that we will show τ_{net} about any point is equal to zero.

⇒ If the body is free to rotate then it will rotate about the axis passing through centre of mass & parallel to torque vector direction & if the body is hinged then it will rotate about hinged axis.

DIPOLE IN UNIFORM ELECTRIC FIELD

Figure shows a dipole of dipole moment p placed at an angle θ to the direction of electric field. Here the charges of dipole experience forces qε in opposite direction as shown.



thus we can state that when a dipole is placed in a uniform electric field, net forces on the dipole is zero. But as equal and opposite forces act with a separation in their line of action, they produce a couple which tend to align the dipole along the direction of electric field. The torque due to this couple can be given as

τ = Force × separation between lines of action of forces
= qE × d sin θ
= pE sin θ

or vectorially we can write the torque on dipole is

$\vec{\tau} = \vec{p} \times \vec{E}$

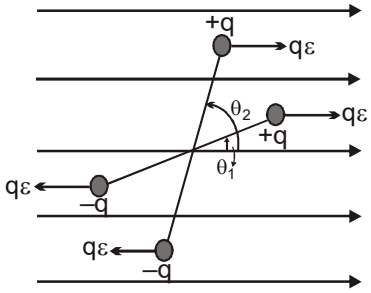
- **Potential Energy of a Dipole in Uniform Electric Field:** When a dipole is kept in an electric field at an angle q, the torque on it due to electric field is

$\tau = p\varepsilon \sin\theta$

In the figure shown, the torque is in clockwise direction. If we rotate the dipole in anticlockwise direction from an angle θ_1 and θ_2 slowly, we have to apply an anticlockwise equal torque, then the work done in process will be given as

$$W = \int dW = \int \tau d\theta$$

$$= \int_{\theta_1}^{\theta_2} p\varepsilon \sin\theta d\theta$$



$= p\varepsilon[-\cos\theta]_{\theta_1}^{\theta_2}$

$= p\varepsilon(\cos\theta_1 - \cos\theta_2)$

$W_{\text{external foce}} = -W_{\text{electricforce}}$

$\therefore\ W_{\text{electric field}} = -pE(\cos\theta_1 - \cos\theta_2)$

As we know that for conservative forces

W.D. $= -\Delta U$

$\Delta U = pE(\cos\theta_1 - \cos\theta_2)$

$U_{\theta_2} - U_{\theta_1} = (-pE\cos\theta_2 - pE\cos\theta_1)$

$\therefore$ We can generalise that

$U_\theta = -pE\cos\theta$

In vector notation we can write potential energy of dipole in electric field is

$U = -\vec{p}.\vec{E}$ [where potential energy at $\theta = 90°$ is zero]

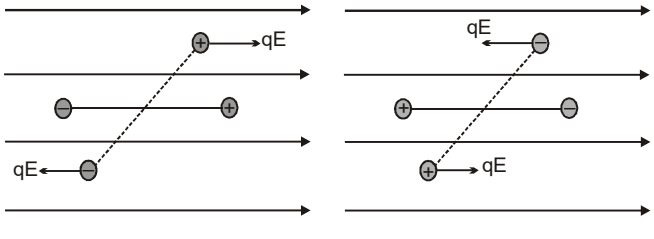
- **Stable and Unstable equilibrium of a Dipole in Electric Field :** We've discussed that when a dipole in an electric field E, the potential energy of dipole can be given as

$U = -pE\cos\theta$

We also know that the net torque on a dipole in electric field can be given as

$\tau = pE\sin\theta$

It shows that net torque on dipole in electric field is zero in two situations when $\theta = 0°$ and $\theta = 180°$ as shown in figure



(a) (b)

We can see that when $\theta = 0$ as shown in figure(a) when torque on dipole is zero, the dipole is in equilibrium. We can verify that here equilibrium is stable. If we slightly tilt the dipole from its equilibrium position in anticlockwise direction as shown by dotted position, The dipole experiences a clockwise torque which tends to rotate the dipole back to its equilibrium position. This shows that at $\theta = 0$, dipole is in stable equilibrium. We can also find the potential energy of dipole at $\theta = 0$, it can be given as

$U = -pE$ (minimum)

Here at $\theta = 0$, potential energy of dipole in electric field is minimum which favours the position of stable equilibrium. Similarly when $\theta = 180°$, net torque on dipole is zero and potential energy of dipole in this state is given as

$U = pE$ (maximum)

Thus at $\theta = 180°$, dipole is in unstable equilibrium. This can also be shown by figure(b). From equilibrium position if dipole is slightly displaced in anticlockwise direction, we can see that torque on dipole also acts in anticlockise direction away from equilirbium position. Thus here dipole is in unstable equilibrium.

- **Angular SHM of Dipole:** When a dipole is suspended in a uniform electric field, it will align itself parallel to the field. Now if it is given a small angular displacement θ about its equilibrium, the (restoring) couple will be

$\tau = -pE\sin\theta$

or $\tau = -pE\,\theta$ [as $\sin\theta \approx \theta$, for small θ]

or $I\dfrac{d^2\theta}{dt^2} = -pE\theta$

or $\dfrac{d^2\theta}{dt^2} = -\dfrac{pE}{I}\theta$

or $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$

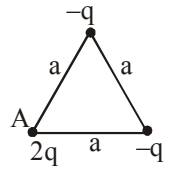
where $\omega^2 = \dfrac{pE}{I}$

This is standard equation of angular simple harmonic motion with time-period $T\left(=\dfrac{2\pi}{\omega}\right)$. So the dipole will execute angular SHM with time-period

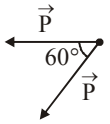
$T = 2\pi\sqrt{\dfrac{I}{pE}}$

TRAIN YOUR BRAIN

Example 28: What is the dipole moment of the system shown in figure



Sol. There are two dipoles of $\vec{P} = q(a)$



so $P_{net} = \sqrt{3}p = \sqrt{3}qa$

Example 29: An electron and a proton are placed at distance of 1Å. What will be dipole moment of so formed dipole ?

Sol. $p = qd = 1.6 \times 10^{-19} \times 1 \times 10^{-10}$
$= 1.6 \times 10^{-29}$ coulomb metre

Example 30: A dipole of dipole moment P lies in a uniform electric field E such that dipole direction is along field. If dipole is rotated through 180º such that dipole direction becomes opposite to the field direction. Find the work done by the electrostatic field.

Sol. $U_i = -\vec{P}.\vec{E} = -PE\cos\theta = -PE$

$U_f = -P.E.\cos(180°) = PE$

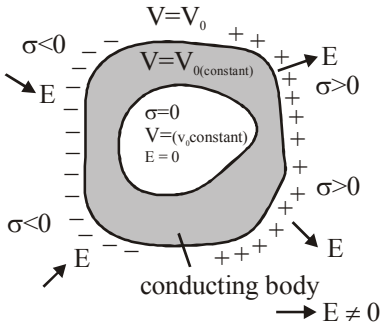
work done by the field = $-\Delta U = U_i - U_f = -2PE$

CONDUCTORS

- **Inside a conductor, electrostatic field is zero :** In the previous chapter, we have already discussed that "when there is no electric current inside or on the surface of a conductor, the electric field inside the conductor is everywhere zero".
- **At the surface of a charged conductor, electrostatic field must be normal to the surface at every point :** If the field $\vec{E}$ is not normal to the surface, it will have a nonzero component along the surface. Hence the free charge on the surface will move due to electrostatic force on it. But free charge on the surface in electrostatics remains at rest. So the electrostatic field at the surface of a charged conductor must be normal to the surface.
- **The interior of a conductor can have no excess charge in the static situation:**
- **Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface :** Since the field E = 0 inside the conductor and its has no nonzero component along the surface, so the potential gradient $\frac{dV}{dr} = 0$ because $-\frac{dV}{dr}$ is equal to the component of the field along dr. If we take dr along the surface or inside the surface, then $-\frac{dV}{dr} = 0$
  $\Rightarrow$ V is constant
- **Electric field at the surface of a charged conductor :** $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ where σ is the surface charge density which may be positive or negative and $\hat{n}$ is the unit vector normal to the surface in the outward direction.
- **Electrostatic Shielding :** In an electrostatic situation, if a conductor contains a cavity and if no charge is present inside the cavity, then there can be no net charge anywhere on the surface of the cavity. This is known as electrostatic shielding. Remember the following facts.

(*i*) There is no net charge anywhere on the surface of the cavity. i.e., $\sigma = 0$

(*ii*) The potential inside (or on the surface of) the cavity = The potential inside the material of the conductor = The potential on the surface of the conductor = V_0 (constant)



(*iii*) The electric field in (or on the surface of) the cavity = The electric field inside material of the conductor = zero

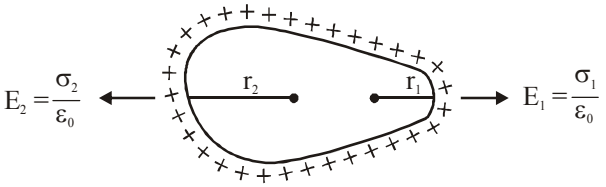
But the electric field on the surface of a charged conductor is not zero.

[Figure shows an uncharged conductor in an external electric field E. The free electrons in the conductor distribute themselves on the surfaces as shown in the figure so that the net electric field inside the conductor becomes zero and the net field at the surface is perpendicular to the surface.]

(*iv*) Except for spherical surfaces, the charge is not distributed uniformly on the surface of a conductor. At the sharp poitns or edges, the surface charge density (σ) is very high and hence the electric field $\left(E = \frac{\sigma}{\varepsilon_0}\right)$ becomes very strong. The air around such sharp points may become ionised, producing the corona discharge in which the charge jumps from th conductor to the air because of the dielectric breakdown of the air.

Figure shows a charged conductor with nonuniform distribution of charge. σ_1 and σ_2 are the surface charge densities at the portions where radii of curvature are r_1 and r_2 then

$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}}$$

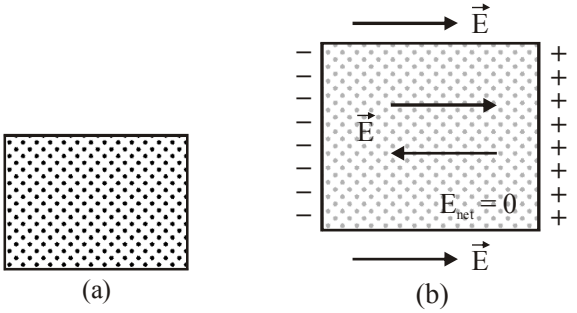


(*a*) The corona discharge from the pointed end of the conductor at the top of a building protect the building from lightning strikes.

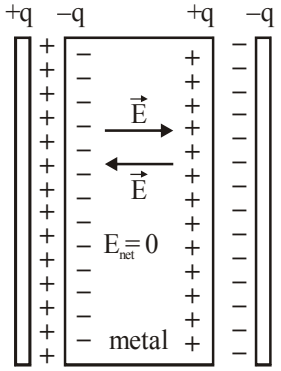
(*b*) It is electrostatic shielding that protects a person from lightning strikes if he is in a car.

(*c*) Figure (a) shows an uncharged conducting plate placed in a region without electric field.

Figure (b) shows an uncharged conducting plate placed in a region having electric field E. Due to electric force on the free electrons in the conductor, they redistribute themselves till the electric field produced due to the redistribution neutralizes the original field. Hence the net field inside the conductor becomes zero.



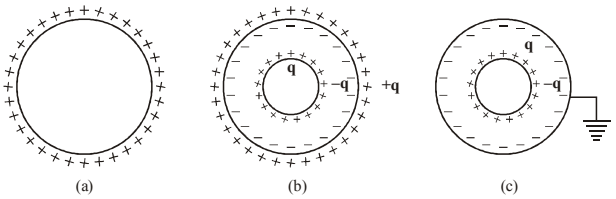
(*d*) Figure shows a metallic slab between two charged plate. The field E due to the charged plate is directed towards right and the field E due to the induced charge in the slab is directed towards left, and hence the net field inside the slab becomes zero.



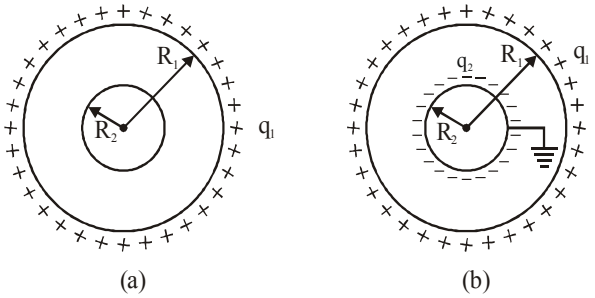
(*e*) Figure (a) shows a conducting shell with charge q. In figure (b), the conducting shell with charge q has been surrounded by another larger conducting shell which is uncharged. Charges – q and + q are induced on the its inner and outer surfaces but net charge in the outer shell is still zero.

In the figure (c), the outer shell of figure (b) is earthed. The free charge on the outer surface goes to the earth but the inner charge remains bounded to the charge on the inner shell so that the potential of the outer shell connected to the earth becomes zero.

[Remember that an earthed conductor is always at zero potential due to flow of charge either from the earth to the conductor or from the conductor to the earth]



(*f*) Figure (a), shows two concentric conducting shells. Some charge q_1 is given to the outer shell. No charge is developed on the inner shell.

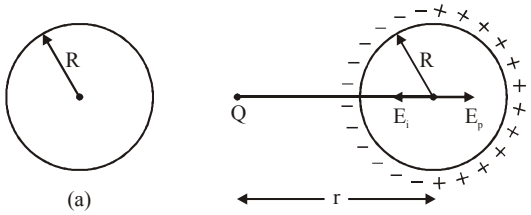


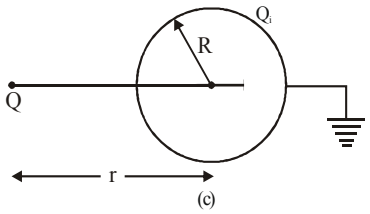
In the figure (b), the inner shell is earthed and hence some charge q_2 is developed on it so that its potential becomes zero.

On the surface of the inner shell, the net potential

is $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{R_2}+\frac{q_1}{R_1}\right)=0$ or, $\boxed{q_2 = -q_1\frac{R_2}{R_1}}$

(*g*) Figure (a) is an uncharged metallic solid sphere of radius R.





In figure (b), a positive point charge Q has been placed at a distance r from the centre of the sphere. The point charge Q exerts force on the electrons in the sphere and hence the free electrons redistribute themselves so that the left half is negatively charged and the right half is positively charged. The charge distribution is **non-uniform**.

At the centre of the sphere, the field due to the point charge Q is $E_P = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, toward right.

Since the field inside the conductor is zero, the field due the induced charge on the sphere is equal in magnitude of E_P but opposite in direction.

i.e., $E_i = \frac{Q}{4\pi\varepsilon_0 r^2}$

or,

The potential at any point of the sphere = potential at its centre = $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

In the figure (c), the sphere is earthed, so V = 0 hence

$\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r}+\frac{Q_i}{R}\right)=0$ or, $Q_i = -QR/r$

Topicwise Questions

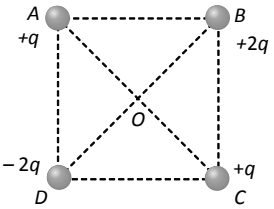
Coulomb's Force

1. When 10^{14} electrons are removed from a neutral metal sphere, the charge on the sphere becomes
(*a*) 16 μC (*b*) –16 μC
(*c*) 32 μC (*d*) –32 μC

2. Number of electrons in one coulomb of charge will be $(1e = 1.6 \times 10^{-9})C$
(*a*) 5.46×10^{29} (*b*) 6.25×10^{18}
(*c*) 1.6×10^{19} (*d*) 9×10^{11}

3. The ratio of the forces between two small spheres with constant charge (a) in air (b) in a medium of dielectric constant K is:
(*a*) 1 : K (*b*) K : 1
(*c*) 1 : K^2 (*d*) K^2 : 1

4. Four charges are arranged at the corners of a square ABCD, as shown in the adjoining figure. The force on the charge kept at the centre O is



(*a*) Zero
(*b*) Along the diagonal AC
(*c*) Along the diagonal BD
(*d*) Perpendicular to side AB

5. A total charge Q is broken in two parts Q_1 and Q_2 and they are placed at a distance R from each other. The maximum force of repulsion between them will occur, when

(*a*) $Q_2 = \frac{Q}{R}, Q_1 = Q - \frac{Q}{R}$ (*b*) $Q_2 = \frac{Q}{4}, Q_1 = Q - \frac{2Q}{3}$

(*c*) $Q_2 = \frac{Q}{4}, Q_1 = \frac{3Q}{4}$ (*d*) $Q_1 = \frac{Q}{2}, Q_2 = \frac{Q}{2}$

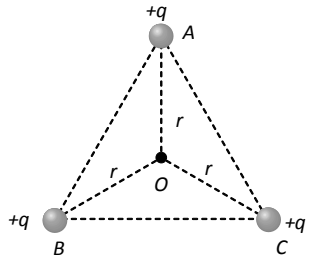
Electric Field and Field Lines

6. A charge q is placed at the centre of the line joining two equal charges The system of the three charges will be in equilibrium, if q is equal to

(*a*) $-\frac{Q}{2}$ (*b*) $-\frac{Q}{4}$

(*c*) $+\frac{Q}{4}$ (*d*) $+\frac{Q}{2}$

7. ABC is an equilateral triangle. Charges +q are placed at each corner. The electric intensity at O will be



(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ (*b*) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

(*c*) Zero (*d*) $\frac{1}{4\pi\varepsilon_0}\frac{3q}{r^2}$

8. The magnitude of electric field intensity E is such that, an electron placed in it would experience an electrical force equal to its weight is given by

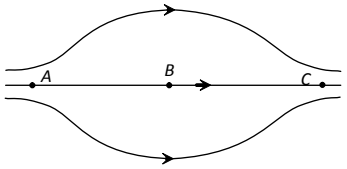
(*a*) mge (*b*) $\frac{mg}{e}$

(*c*) $\frac{e}{mg}$ (*d*) $\frac{e^2}{m^2}g$

9. The distance between the two charges 25 μC and 36 μC is 11 cm At what point on the line joining the two, the intensity will be zero
(*a*) At a distance of 5 cm from 25 μC
(*b*) At a distance of 5 cm from 36 μC
(*c*) At a distance of 10 cm from 25 μC
(*d*) At a distance of 11 cm from 36 μC

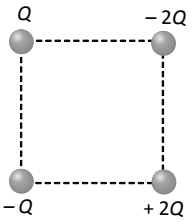
10. The intensity of the electric field required to keep a water drop of radius 10^{-5} cm just suspended in air when charged with one electron is approximately
(g = 10 Newton/ kg, $e = 1.6 \times 10^{-19}$ coulomb)
(*a*) 260 volt/cm
(*b*) 260 Newton/coulomb
(*c*) 130 volt/cm
(*d*) 130 Newton/coulomb

11. The figure shows some of the electric field lines corresponding to an electric field. The figure suggests



(*a*) $E_A > E_B > E_C$ (*b*) $E_A = E_B = E_C$
(*c*) $E_A = E_C > E_B$ (*d*) $E_A = E_C < E_B$

12. Four charges are placed on corners of a square as shown in figure having side of 5 cm . If Q is one microcoulomb, then electric field intensity at centre will be



(*a*) 1.02×10^7 N/C upwards
(*b*) 2.04×10^7 N/C downwards
(*c*) 2.04×10^7 N/C upwards
(*d*) 1.02×10^7 N/C downwards

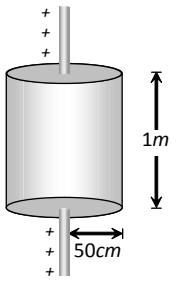
Gauss Law and Application

13. A cylinder of radius R and length L is placed in a uniform electric field E parallel to the cylinder axis. The total flux for the surface of the cylinder is given by
(*a*) $2\pi R^2 E$ (*b*) $\pi R^2 / E$
(*c*) $(\pi R^2 - \pi R) / E$ (*d*) Zero

14. Electric field at a point varies as r^0 for
(*a*) An electric dipole
(*b*) A point charge
(*c*) A plane infinite sheet of charge
(*d*) A line charge of infinite length

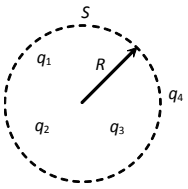
15. A cube of side l is placed in a uniform field E, where $E = E\hat{i}$. The net electric flux through the cube is
(*a*) Zero (*b*) $l^2 E$
(*c*) $4l^2 E$ (*d*) $6l^2 E$

16. Electric charge is uniformly distributed along a long straight wire of radius 1mm. The charge per cm length of the wire is Q coulomb. Another cylindrical surface of radius 50 cm and length 1m symmetrically encloses the wire as shown in the figure. The total electric flux passing through the cylindrical surface is



(*a*) $\frac{Q}{\varepsilon_0}$ (*b*) $\frac{100Q}{\varepsilon_0}$
(*c*) $\frac{10Q}{(\pi\varepsilon_0)}$ (*d*) $\frac{100Q}{(\pi\varepsilon_0)}$

17. q_1, q_2, q_3 and q_4 are point charges located at points as shown in the figure and S is a spherical Gaussian surface of radius R. Which of the following is true according to the Gauss's law?



(*a*) $\oint_S (\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4).d\vec{A} = \frac{q_1 + q_2 + q_3}{2\varepsilon_0}$

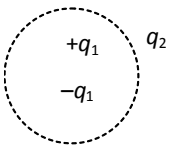
(*b*) $\oint_S (\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4).d\vec{A} = \frac{(q_1 + q_2 + q_3)}{\varepsilon_0}$

(*c*) $\oint_S (\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4).d\vec{A} = \frac{(q_1 + q_2 + q_3 + q_4)}{\varepsilon_0}$

(*d*) None of the above

18. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2 the electric charge inside the surface will be
(*a*) $(\phi_1 + \phi_2)\varepsilon_0$ (*b*) $(\phi_2 - \phi_1)\varepsilon_0$
(*c*) $(\phi_1 + \phi_2)/\varepsilon_0$ (*d*) $(\phi_2 - \phi_1)/\varepsilon_0$

19. Consider the charge configuration and spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface the electric field will be due to

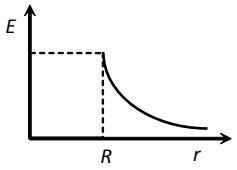


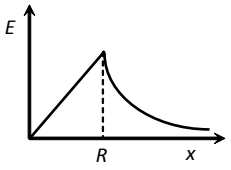
(*a*) q_2
(*b*) Only the positive charges
(*c*) All the charges
(*d*) $+q_1$ and $-q_1$

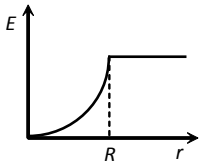
20. An electric dipole is put in north-south direction in a sphere filled with pure water. Which statement is correct
(*a*) Electric flux is coming towards sphere
(*b*) Electric flux is coming out of sphere
(*c*) Electric flux entering into sphere and leaving the sphere are same
(*d*) Water does not permit electric flux to enter into sphere

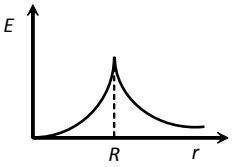
21. Two infinite plane parallel conducting sheets separated by a distance d have equal and opposite uniform charge densities σ. Electric field at a point between the sheets is
(*a*) Zero
(*b*) $\frac{\sigma}{\varepsilon_0}$
(*c*) $\frac{\sigma}{2\varepsilon_0}$
(*d*) Depends upon the location of the point

22. Which of the following graphs shows the variation of electric field E due to a hollow spherical conductor of radius R as a function of distance from the centre of the sphere

(*a*) 

(*b*) 

(*c*) 

(*d*) 

23. A conducting sphere of radius R = 20 cm is given a charge Q = 16 μC. What is $\vec{E}$ at centre

(*a*) 3.6×10^6 N/C (*b*) 1.8×10^6 N/C

(*c*) Zero (*d*) 0.9×10^6 N/C

24. At a point 20 cm from the centre of a uniformly charged dielectric sphere of radius 10 cm, the electric field is 100 V/m. The electric field at 3 cm from the centre of the sphere will be

(*a*) 150 V/m (*b*) 125 V/m

(*c*) 120 V/m (*d*) Zero

ELECTROSTATIC POTENTIAL

Electrostatic Potential and Potential Energy

25. A hollow metal sphere of radius 5 cm is charged so that the potential on its surface is 10 V. The potential at the centre of the sphere is

(*a*) 0 V

(*b*) 10 V

(*c*) Same as at point 5 cm away from the surface

(*d*) Same as at point 25 cm away from the surface

26. If a unit positive charge is taken from one point to another over an equipotential surface, then

(*a*) Work is done on the charge

(*b*) Work is done by the charge

(*c*) Work done is constant

(*d*) No work is done

27. Charges of $+\frac{10}{3}\times 10^{-9}$ C are placed at each of the four corners of a square of side 8 cm. The potential at the intersection of the diagonals is

(*a*) $150\sqrt{2}$ volt (*b*) $1500\sqrt{2}$ volt

(*c*) $900\sqrt{2}$ volt (*d*) 900 volt

28. If E is the electric field intensity of an electrostatic field, then the electrostatic energy density is proportional to

(*a*) E (*b*) E^2

(*c*) $1/E^2$ (*d*) E^3

29. Four equal charges Q are placed at the four corners of a square of each side 'a'. Work done in removing a charge –Q from its centre to infinity is

(*a*) 0 (*b*) $\frac{\sqrt{2}Q^2}{4\pi\varepsilon_0 a}$

(*c*) $\frac{\sqrt{2}Q^2}{\pi\varepsilon_0 a}$ (*d*) $\frac{Q^2}{2\pi\varepsilon_0 a}$

30. Two charge +q and –q are situated at a certain distance. At the point exactly midway between them

(*a*) Electric field and potential both are zero

(*b*) Electric field is zero but potential is not zero

(*c*) Electric field is not zero but potential is zero

(*d*) Neither electric field nor potential is zero

31. In Millikan's oil drop experiment an oil drop carrying a charge Q is held stationary by a potential difference 2400 V between the plates. To keep a drop of half the radius stationary the potential difference had to be made 600 V. What is the charge on the second drop

(*a*) $\frac{Q}{4}$ (*b*) $\frac{Q}{2}$

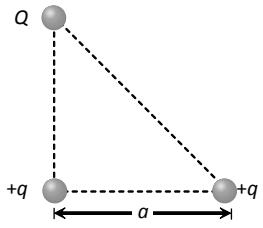
(*c*) Q (*d*) $\frac{3Q}{2}$

32. Two point charges 100 μC and 5 μC are placed at points A and B respectively with AB = 40 cm. The work done by external force in displacing the charge 5μC from B to C, where BC = 30 cm, angle $ABC = \frac{\pi}{2}$ and $\frac{1}{4\pi\varepsilon_0} = 9\times 10^9\ \text{Nm}^2/\text{C}^2$

(*a*) 9 J (*b*) $\frac{81}{20}$ J

(*c*) $\frac{9}{25}$ J (*d*) $-\frac{9}{4}$ J

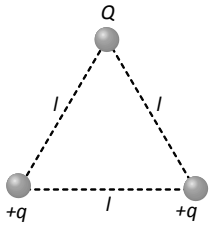
33. Three charges Q, +q and +q are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to



(*a*) $\frac{-q}{1+\sqrt{2}}$ (*b*) $\frac{-2q}{2+\sqrt{2}}$

(*c*) –2q (*d*) +q

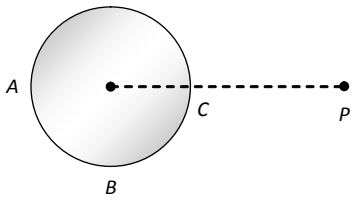
34. Three charges Q, (+q) and (+q) are placed at the vertices of an equilateral triangle of side l as shown in the figure. If the net electrostatic energy of the system is zero, then Q is equal to



(*a*) $\left(-\frac{q}{2}\right)$ (*b*) (–q)

(*c*) (+q) (*d*) Zero

35. A hollow conducting sphere is placed in an electric field produced by a point charge placed at P as shown in figure. Let V_A, V_B, V_C be the potentials at points A, B and C on sphere Then



(*a*) $V_C > V_B$ (*b*) $V_B > V_C$

(*c*) $V_A > V_B$ (*d*) $V_A = V_C$

36. If an insulated non-conducting sphere of radius R has charge density ρ. The electric field at a distance r from the centre of sphere (r < R) will be

(*a*) $\frac{\rho R}{3\varepsilon_0}$ (*b*) $\frac{\rho r}{\varepsilon_0}$

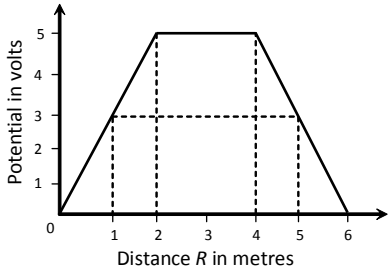
(*c*) $\frac{\rho r}{3\varepsilon_0}$ (*d*) $\frac{3\rho R}{\varepsilon_0}$

37. A hollow metallic sphere of radius R is given a charge Then the potential at the centre is

(*a*) Zero (*b*) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$

(*c*) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{R}$ (*d*) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}$

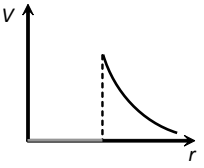
38. The variation of potential with distance R from a fixed point is as shown below. The electric field at R = 5 m is

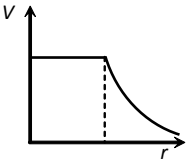


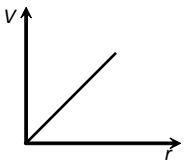
(*a*) 2.5 volt/m (*b*) –2.5 volt/m

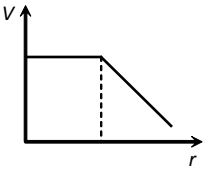
(*c*) 2/5 volt/m (*d*) –2/5 volt/m

39. In a hollow spherical shell potential (V) changes with respect to distance (r) from centre

(*a*) 

(*b*) 

(*c*) 

(*d*) 

Relation Between Field and Potential

40. The electric potential V at any point O (x, y, z all in metres) in space is given by $V = 4x^2$ volt. The electric field at the point (1m, 0, 2m) in volt/metreis

(*a*) 8 along negative X-axis

(*b*) 8 along positive X-axis

(*c*) 16 along negative X-axis

(*d*) 16 along positive Z-axis

41. A uniform electric field having a magnitude E_0 and direction along the positive X-axis exists. If the potential V is zero at x = 0, then its value at X = + x will be

(*a*) $V_x = +xE_0$ (*b*) $V_x = -xE_0$

(*c*) $V_x = +x^2E_0$ (*d*) $V_x = -x^2E_0$

42. Two plates are 2 cm apart, a potential difference of 10 volt is applied between them, the electric field between the plates is

(*a*) 20 N/C (*b*) 500 N/C

(*c*) 5 N/C (*d*) 250 N/C

43. There is an electric field E in X-direction. If the work done on moving a charge 0.2 C through a distance of 2 m along a line making an angle 60° with the X-axis is 4.0, what is the value of E

(*a*) $\sqrt{3}$ N/C (*b*) 4 N/C

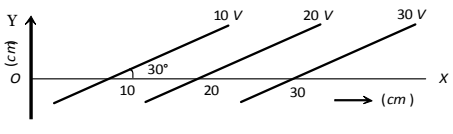
(*c*) 5 N/C (*d*) None of these

44. A charge of 5 C experiences a force of 5000 N when it is kept in a uniform electric field. What is the potential difference between two points separated by a distance of 1 cm along the electric field line.

(*a*) 10 V (*b*) 250 V

(*c*) 1000 V (*d*) 2500 V

45. Equipotential surfaces are shown in figure. Then the electric field strength will be



(*a*) 100 Vm^{-1} along X-axis

(*b*) 100 Vm^{-1} along Y-axis

(*c*) 200 Vm^{-1} at an angle 120° with X-axis

(*d*) 50 Vm^{-1} at an angle 120° with X-axis

Electric Dipole

46. An electric dipole when placed in a uniform electric field E will have minimum potential energy, if the positive direction of dipole moment makes the following angle with E

(*a*) π (*b*) $\pi/2$

(*c*) Zero (*d*) $3\pi/2$

47. An electric dipole of moment $\vec{p}$ is placed normal to the lines of force of electric intensity $\vec{E}$, then the work done in deflecting it through an angle of 180° is

(*a*) pE (*b*) +2pE

(*c*) –2pE (*d*) Zero

48. The electric field due to a dipole at a distance r on its axis is

(*a*) Directly proportional to r^3

(*b*) Inversely proportional to r^3

(*c*) Directly proportional to r^2

(*d*) Inversely proportional to r^2

49. The torque acting on a dipole of moment $\vec{P}$ in an electric field $\vec{E}$ is

(*a*) $\vec{P}\cdot\vec{E}$ (*b*) $\vec{P}\times\vec{E}$

(*c*) Zero (*d*) $\vec{E}\times\vec{P}$

50. Electric potential at an equatorial point of a small dipole with dipole moment P(r, distance from the dipole) is

(*a*) Zero (*b*) $\frac{P}{4\pi\varepsilon_0 r^2}$

(*c*) $\frac{P}{4\pi\varepsilon_0 r^3}$ (*d*) $\frac{2P}{4\pi\varepsilon_0 r^3}$

51. The value of electric potential at any point due to any electric dipole is

(*a*) $k\cdot\frac{\vec{p}\times\vec{r}}{r^2}$ (*b*) $k\cdot\frac{\vec{p}\times\vec{r}}{r^3}$

(*c*) $k\cdot\frac{\vec{p}\cdot\vec{r}}{r^2}$ (*d*) $k\cdot\frac{\vec{p}\cdot\vec{r}}{r^3}$

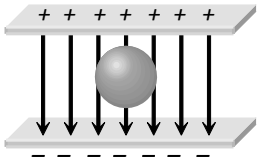
52. Two charges $+3.2\times10^{-19}$ C and -3.2×10^{-9} C kept 2.4 Å apart forms a dipole. If it is kept in uniform electric field of intensity 4×10^5 volt/m then what will be its electrical energy in stable equilibrium

(*a*) $+3\times10^{-23}$ J (*b*) -3×10^{-23} J

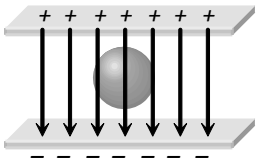
(*c*) -6×10^{-23} J (*d*) -2×10^{-23} J

Conductors and Self-energy and Electrostatic Potential

53. An uncharged sphere of metal is placed in between two charged plates as shown. The lines of force look like



A



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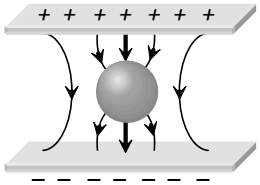
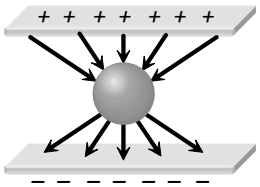


C



D

(*a*) A (*b*) B

(*c*) C (*d*) D

54. Conduction electrons are almost uniformly distributed within a conducting plate. When placed in an electrostatic field $\vec{E}$, the electric field within the plate

(*a*) Is zero

(*b*) Depends upon E

(*c*) Depends upon $\vec{E}$

(*d*) Depends upon the atomic number of the conducting element

Learning Plus

1. One quantum of charge should be at least equal to the charge in coloumb:
(*a*) 1.6×10^{-17} C. (*b*) 1.6×10^{-19} C.
(*c*) 1.6×10^{-10} C. (*d*) 4.8×10^{-10} C.

2. An electron at rest has a charge of 1.6×10^{-19} C. It starts moving with a velocity v = c/2, where c is the speed of light, then the new charge on it is -

(*a*) 1.6×10^{-19} Coulomb

(*b*) $1.6 \times 10^{-19} \sqrt{1-\left(\frac{1}{2}\right)^2}$ Coulomb

(*c*) $1.6 \times 10^{-19} \sqrt{\left(\frac{2}{1}\right)^2 - 1}$ Coulomb

(*d*) $\frac{1.6 \times 10^{-19}}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$ Coulomb

3. Which one of the following statement regarding electrostatics is wrong ?
(*a*) Charge is quantized
(*b*) Charge is conserved
(*c*) There is an electric field near an isolated charge at rest
(*d*) A stationary charge produces both electric and magnetic fields

4. When the distance between two charged particle is halved, the force between them becomes -
(*a*) One fourth (*b*) One half
(*c*) Double (*d*) Four times

5. Five balls, numbered 1 to 5, are suspended using separate threads. Pairs (1, 2), (2, 4), (4, 1) show electrostatic attraction, while pairs (2, 3) and (4, 5) show repulsion. Therefore ball 1:
(*a*) Must be positively charged
(*b*) Must be negatively charged
(*c*) May be neutral
(*d*) Must be made of metal

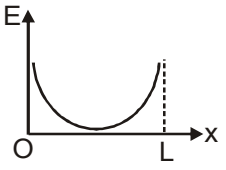
6. Two small balls having equal positive charge Q (Coulomb) on each are suspended by two insulating strings of equal length 'L' metre, from a hook fixed to a stand. The whole set up is taken in a satellite in to space where there is no gravity (state of weight lessness). Then the angle (θ) between the two strings is-
(*a*) 0º (*b*) 90º
(*c*) 180º (*d*) $0º < \theta < 180º$

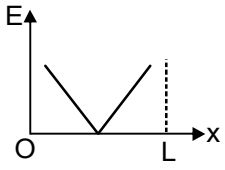
7. Two charges 4q and q are placed 30 cm. apart. At what point the value of electric field will be zero
(*a*) 10 cm. away from q and between the charge
(*b*) 20 cm. away from q and between the charge
(*c*) 10 cm. away from q and out side the line joining the charge.
(*d*) 10 cm. away from 4q and out side the line joining them.

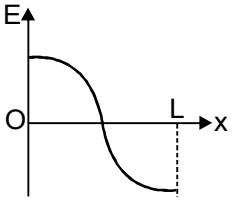
8. A total charge of 20 μC is divided into two parts and placed at some distance apart. If the charges experience maximum coulombian repulsion, the charges should be:
(*a*) 5 μC, 15 μC (*b*) 10 μC, 10 μC
(*c*) 12 μC, 8 μC (*d*) $\frac{40}{3}\mu C, \frac{20}{3}\mu C$

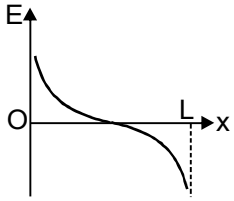
9. Four equal but like charges are placed at four corners of a square. The electric field intensity at the center of the square due to any one charge is E, then the resultant electric field intensity at centre of square will be:
(*a*) Zero (*b*) 4E
(*c*) E (*d*) 1/2E

10. Two identical point charges are placed at a separation of *l*.P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E. E is plotted against x for values of x from close to zero to slightly less than *l*. Which of the following best represents the resulting curve ?

(*a*) 

(*b*) 

(*c*) 

(*d*) 

11. A charged particle of charge q and mass m is released from rest in an uniform electric field E. Neglecting the effect of gravity, the kinetic energy of the charged particle after time 't' seconds is

(*a*) $\frac{Eqm}{t}$ (*b*) $\frac{E^2q^2t^2}{2m}$

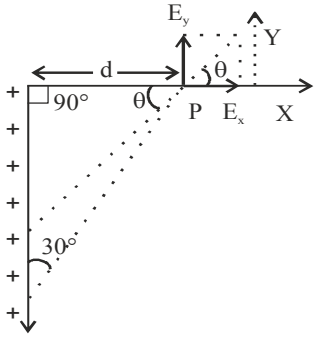
(*c*) $\frac{2E^2t^2}{mq}$ (*d*) $\frac{Eq^2m}{2t^2}$

12. The maximum electric field intensity on the axis of a uniformly charged ring of charge q and radius R will be:

(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{q}{3\sqrt{3}R^2}$ (*b*) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3R^2}$

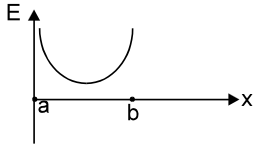
(*c*) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}R^2}$ (*d*) $\frac{1}{4\pi\varepsilon_0}\,i\frac{3q}{2\sqrt{3}R^2}$

13. The direction (θ) of $\vec{E}$ at point P due to uniformly charged finite rod will be -



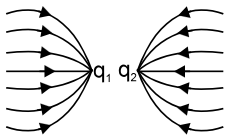
(*a*) at angle 30° from x-axis
(*b*) 45° from x-axis
(*c*) 60° from x-axis
(*d*) None of these

14. Two point charges a & b, whose magnitudes are same are positioned at a certain distance from each other with a at origin. Graph is drawn between electric field strength at points between a & b and distance x from a. E is taken positive if it is along the line joining from a to b. From the graph, it can be decided that



(*a*) a is positive, b is negative
(*b*) a and b both are positive
(*c*) a and b both are negative
(*d*) a is negative, b is positive

15. The given figure gives electric lines of force due to two charges q_1 and q_2. What are the signs of the two charges?



(*a*) Both are negative
(*b*) Both are positive
(*c*) q_1 is positive but q_2 is negative
(*d*) q_1 is negative but q_2 is positive

16. If electric field is uniform, then the electric lines of forces are:

(*a*) Divergent (*b*) Convergent
(*c*) Circular (*d*) Parallel

17. Electric flux through a surface of area 100 m^2 lying in the xy plane is (in V-m) if $\vec{E} = \hat{i} + \sqrt{2}\,\hat{j} + \sqrt{3}\,\hat{k}$

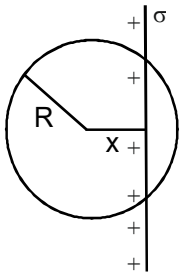
(*a*) 100 (*b*) 141.4
(*c*) 173.2 (*d*) 200

18. A cone of radius (R) and length (L) is placed in a uniform electrical field (E) parallel to the axis of the cone. the total flux for the surface of the cone is given by -

(*a*) $2\pi R^2E$ (*b*) πR^2E

(*c*) $\frac{2\pi R^2}{E}$ (*d*) zero

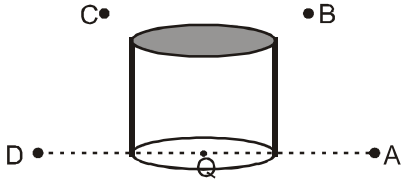
19. An infinite, uniformly charged sheet with surface charge density σ cuts through a spherical Gaussian surface of radius R at a distance x from its center, as shown in the figure. The electric flux Φ through the Gaussian surface is



(*a*) $\frac{\pi R^2\sigma}{\varepsilon_0}$ (*b*) $\frac{2\pi(R^2-x^2)\sigma}{\varepsilon_0}$

(*c*) $\frac{\pi(R-x)^2\sigma}{\varepsilon_0}$ (*d*) $\frac{\pi(R^2-x^2)\sigma}{\varepsilon_0}$

20. Figure shows a charge Q placed at the centre of open face of a cylinder as shown in figure. A second charge q is placed at one of the positions A, B, C and D, out of which positions A and D are lying on a straight line parallel to open face of cylinder. In which position(s) of this second charge, the flux of the electric field through the cylinder remains unchanged?



(*a*) A and D (*b*) B
(*c*) C (*d*) B and C

21. A flat circular fixed disc has a charge +Q uniformly distributed on the disc. A charge +q is thrown with kinetic energy K, towards the disc along its axis. The charge q :

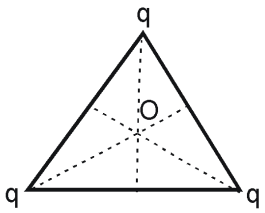
(*a*) may hit the disc at the centre

(*b*) may return back along its path after touching the disc

(*c*) may return back along its path without touching the disc

(*d*) any of the above three situations is possible depending on the magnitude of K

22. Three equal charges are placed at the three corners of an isosceles triangle as shown in the figure. The statement which is true for electric potential V and the field intensity E at the centre of the triangle is-



(*a*) $V=0, E=0$ (*b*) $V=0, E\neq 0$

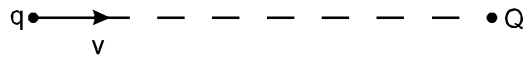
(*c*) $V\neq 0, E=0$ (*d*) $V\neq 0, E\neq 0$

23. A particle A has charge +q and particle B has charge + 4q with each of them having the same mass m. When allowed to fall from rest through same electrical potential difference, the ratio of their speed $v_A : v_B$ will be:

(*a*) 2 : 1 (*b*) 1 : 2

(*c*) 4 : 1 (*d*) 1 : 4

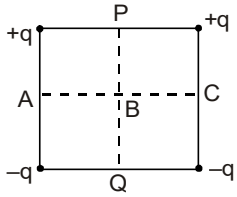
24. A charged particle 'q' is shot from a large distance with speed v towards a fixed charged particle It approaches Q upto a closest distance r and then returns. If q were given a speed '2v', the closest distance of approach would be :



(*a*) r (*b*) 2r

(*c*) $\frac{r}{2}$ (*d*) $\frac{r}{4}$

25. Figure represents a square carrying charges +q, +q, –q, –q at its four corners as shown. Then the potential will be zero at points :



(*a*) A, B, C, P and Q (*b*) A, B and C

(*c*) A, P, C and Q (*d*) P, B and Q

26. The kinetic energy which an electron acquires when accelerated (from rest) through a potential difference of 1 volt is called :

(*a*) 1 joule (*b*) 1 electron volt

(*c*) 1 erg (*d*) 1 watt

27. A particle of charge Q and mass m travels through a potential difference V from rest. The final momentum of the particle is :

(*a*) $\frac{mV}{Q}$ (*b*) $2Q\sqrt{mV}$

(*c*) $\sqrt{2mQV}$ (*d*) $\sqrt{\frac{2QV}{m}}$

28. If a charge is shifted from a high potential region to low potential region, the electrical potential energy:

(*a*) Increases

(*b*) Decreases

(*c*) May increase or decrease

(*d*) Remains constant

29. A particle of mass 2 g and charge 1 μC is held at rest on a frictionless horizontal surface at a distance of 1 m from a fixed charge of 1 mC. If the particle is released it will be repelled. The speed of the particle when it is at distance of 10 m from the fixed charge is:

(*a*) 100 m/s (*b*) 90 m/s

(*c*) 60 m/s (*d*) 45 m/s

30. When the separation between two charges is decreased, the electric potential energy of the charges

(*a*) increases

(*b*) decreases

(*c*) may increase or decrease

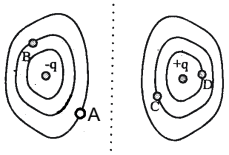
(*d*) remains the same

31. Two identical particles of mass m carry a charge Q each. Initially one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards first particle from a large distance with speed v. The closest distance of approach will be

(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{mv}$ (*b*) $\frac{1}{4\pi\varepsilon_0}\frac{4Q^2}{mv^2}$

(*c*) $\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{mv^2}$ (*d*) $\frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{mv^2}$

32. Figure shows equi-potential surfaces for a two charges system. At which of the labelled point will an electron have the highest potential energy?



(*a*) Point A (*b*) Point B

(*c*) Point C (*d*) Point D

33. In a regular polygon of n sides, each corner is at a distance r from the centre. Identical charges are placed at (n – 1) corners. At the centre, the intensity is E and the potential is V. The ratio V/E has magnitude.

(*a*) r n (*b*) r (n – 1)

(*c*) (n–1)/r (*d*) r(n–1)/n

34. A point positive charge of Q' units is moved round another point positive charge of Q units in circular path. If the radius of the circle is r, then what is the work done on the charge Q' in making one complete revolution i -

(*a*) $\frac{Q}{4\pi \in_0 r}$ (*b*) $\frac{QQ'}{4\pi \in_0 r}$

(*c*) $\frac{Q'}{4\pi \in_0 r}$ (*d*) 0

35. When a negative charge is released and moves in electric field, it moves toward a position of

(*a*) lower electric potential and lower potential energy

(*b*) lower electric potential and higher potential energy

(*c*) higher electric potential and lower potential energy

(*d*) higher electric potential and higher potential energy

36. Two identical thin rings, each of radius R meter are coaxially placed at distance R meter apart. If Q_1 and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is

(*a*) zero

(*b*) $q(Q_1 - Q_2)(\sqrt{2} - 1)/(\sqrt{2}.4\pi\varepsilon_0 R)$

(*c*) $q\sqrt{2}(Q_1 + Q_2)/4\pi\varepsilon_0 R$

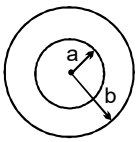
(*d*) $q(Q_1 - Q_2)(\sqrt{2} + 1)/(\sqrt{2}.4\pi\varepsilon_0 R)$

37. Two similar conducting spherical shells having charges 40 μC and –20μC are some distance apart. Now they are touched and kept at same distance. The ratio of the initial to the final force between them is :

(*a*) 8 : 1 (*b*) 4 : 1

(*c*) 1 : 8 (*d*) 1 : 1

38. If the electric potential of the inner metal sphere is 10 volt & that of the outer shell is 5 volt, then the potential at the centre will be:



(*a*) 10 volt (*b*) 5 volt

(*c*) 15 volt (*d*) 0

39. When charge of 3 coulomb is placed in a uniform electric field , it experiences a force of 3000 newton, within this field, potential difference between two points separated by a distance of 1 cm is-

(*a*) 10 Volt (*b*) 90 Volt

(*c*) 1000 Volt (*d*) 3000 Volt.

40. An equipotential surface and a line of force :

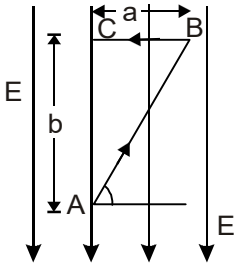
(*a*) never intersect each other

(*b*) intersect at 45°

(*c*) intersect at 60°

(*d*) intersect at 90°

41. The potential difference between points A and B in the given uniform electric field is:



(*a*) Ea (*b*) $E\sqrt{(a^2 + b^2)}$

(*c*) Eb (*d*) $(Eb/\sqrt{2})$

42. An equipotential surface is that surface

(*a*) On which each and every point has the same potential

(*b*) Which has negative potential

(*c*) Which has positive potential

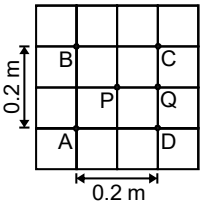
(*d*) Which has zero potential

43. The equation of an equipotential line in an electric field is y = 2x, then the electric field strength vector at (1, 2) may be

(*a*) $4\hat{i} + 3\hat{j}$ (*b*) $4\hat{i} + 8\hat{j}$

(*c*) $8\hat{i} + 4\hat{j}$ (*d*) $-8\hat{i} + 4\hat{j}$

44. A, B, C, D, P and Q are points in a uniform electric field. The potentials at these points are V(*a*) =2 volt. V(P) = V(*b*) = V(*d*) = 5 volt. V(*c*) = 8 volt. The electric field at P is



(*a*) 10 Vm^{-1} along PQ

(*b*) $15\sqrt{2}$ Vm^{-1} along PA

(*c*) 5 Vm^{-1} along PC

(*d*) 5 Vm^{-1} along PA

45. The electric potential V as a function of distance x (in metre) is given by

$V = (5x^2 + 10x - 9)$ volt.

The value of electric field at x = 1 m would be :

(*a*) – 20 volt/m (*b*) 6 volt/m

(*c*) 11 volt/m (*d*) –23 volt/m

46. The electric field in a region is directed outward and is proportional to the distance r from the origin. Taking the electric potential at the origin to be zero, the electric potential at a distance r :

(*a*) increases as one goes away from the origin.

(*b*) is proportional to r^2

(*c*) is proportional to r

(*d*) is uniform in the region

47. An infinite nonconducting sheet of charge has a surface charge density of 10^{-7} C/m^2. The separation between two equipotential surfaces near the sheet whose potential differ by 5V is

(*a*) 0.88 cm (*b*) 0.88 mm

(*c*) 0.88 m (*d*) 5×10^{-7} m

48. If an electric dipole is kept in a uniform electric field, Then it mey experience

(*a*) a force (*b*) a couple and move

(*c*) a couple and rotates (*d*) a force and moves.

49. An electric dipole consists of two opposite charges each of magnitude 1×10^{-6} C separated by a distance 2cm. The dipole is placed in an external field of 10×10^5N/C. The maximum torque on the dipole is -

(*a*) 0.2×10^{-3} N-m (*b*) 1.0×10^{-3} N-m

(*c*) 2×10^{-2} N-m (*d*) 4×10^{-3} N-m

50. The region surrounding a stationary electric dipole has-

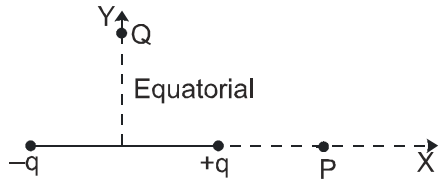
(*a*) electric field only

(*b*) magnetic field only

(*c*) both electric and magnetic fields

(*d*) neither electric nor magnetic field

51. Due to an electric dipole shown in fig., the electric field intensity is parallel to dipole axis :



(*a*) at P only (*b*) at Q only

(*c*) both at P and at Q (*d*) neither at P nor at Q

52. Two opposite and equal charges of magnitude 4×10^{-8} coulomb each when placed 2×10^{-2} cm apart form a dipole. If this dipole is placed in an external electric field of 4×10^8 N/C, the value of maximum torque and the work required in rotating it through 180º from its initial orientation which is along electric field will be : (Assume rotation of dipole about an axis passing through centre of the dipole):

(*a*) 64×10^{-4} N-m and 44×10^{-4} J

(*b*) 32×10^{-4} N-m and 32×10^{-4} J

(*c*) 64×10^{-4} N-m and 32×10^{-4} J

(*d*) 32×10^{-4} N-m and 64×10^{-4} J

53. A neutral spherical metallic object A is placed near a finite metal plate B carrying a positive charge. The electric force on the object will be :

(*a*) away from the plate B

(*b*) towards the plate B

(*c*) parallel to the plate B

(*d*) zero

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/ NUMERICAL

1. Two equal negative charges –q each are fixed at the points (0, a) and (0, -a) on the y-axis .A positive charge Q is released from rest at the point (2a, 0) on the x-axis. The charge Q will :

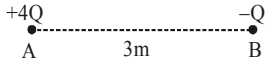
(*a*) Execute simple harmonic motion about the origin

(*b*) At origin velocity of particle is maximum.

(*c*) Move to infinity

(*d*) Execute oscillatory but not simple harmonic motion.

2. Two fixed charges 4Q (positive) and Q (negative) are located at A and B, the distance AB being 3 m.



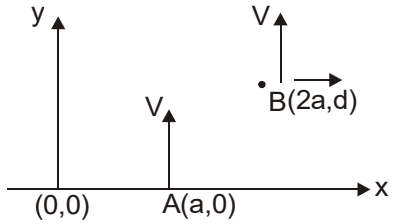
(*a*) The point P where the resultant field due to both is zero is on AB outside AB.

(*b*) The point P where the resultant field due to both is zero is on AB inside AB.

(*c*) If a positive charge is placed at P and displaced slightly along AB it will execute oscillations.

(*d*) If a negative charge is placed at P and displaced slightly along AB it will execute oscillation.

3. A uniform electric field of strength E exists in a region. An electron (charge –e, mass m) enters a point A with velocity $V\hat{j}$. It moves through the electric field & exits at point B.Then:



(*a*) $\vec{E} = -\frac{2\,amv^2}{ed^2}\hat{i}$.

(*b*) Rate of work done by the electric field at B is $\frac{4\,ma^2v^3}{d^3}$.

(*c*) Rate of work by the electric field at A is zero.

(*d*) Velocity at B is $\frac{2av}{d}\hat{i} + v\hat{j}$.

4. An oil drop has a charge - 96×10^{-19} C and mass 1.6×10^{-15} gm. When allowed to fall, due to air resistance force it attains a constant velocity. Then if a uniform electric field is to be applied vertically to make the oil drop ascend up with the same constant speed, which of the following are correct. ($g = 10\ ms^{-2}$)

(**Assume** that the magnitude of resistance force is same in both the cases)

(*a*) The electric field is directed upward

(*b*) The electric field is directed downward

(*c*) The intensity of electric field is $\frac{1}{3} \times 10^2\ NC^{-1}$

(*d*) The intensity of electric field is $\frac{1}{6} \times 10^5\ NC^{-1}$

5. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre.

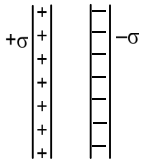
(*a*) increases as r increases, for $r \le R$

(*b*) decreases as r increases, for $0 < r < \infty$.

(*c*) decreases as r increases, for $R < r < \infty$.

(*d*) is discontinuous at r = R

6. Two infinite sheets of uniform charge density $+\sigma$ and $-\sigma$ are parallel to each other as shown in the figure. Electric field at the



(*a*) points to the left or to the right of the sheets is zero

(*b*) midpoint between the sheets is zero

(*c*) midpoint of the sheets is σ/ε_0 and is directed towards right

(*d*) midpoint of the sheets is $2\sigma/\varepsilon_0$ and is directed towards right

7. Charges Q_1 and Q_2 lies inside and outside respectively of a closed surface S. Let E be the field at any point on S and ϕ be the flux of E over S.

(*a*) If Q_1 changes, both E and ϕ will change.

(*b*) If Q_2 changes, E will change but ϕ will not change.

(*c*) If $Q_1 = 0$ and $q_2 \neq 0$ then $E \neq 0$ but $\phi = 0$.

(*d*) If $Q_1 \neq 0$ and $Q_2 = 0$ then $E = 0$ but $\phi \neq 0$.

8. An electric field converges at the origin whose magnitude is given by the expression E = 100rN/C, where r is the distance measured from the origin.

(*a*) total charge contained in any spherical volume with its centre at origin in negative.

(*b*) total charge contained at any spherical volume, irrespective of the location of its centre, is negative.

(*c*) total charge contained in a spherical volume of radius 3 cm with its centre at origin has magnitude 3×10^{-13}C.

(*d*) total charge contained in a spherical volume of radius 3 cm with its centre at origin has magnitude 3×10^{-9} C.

9. Which of the following quantities depend on the choice of zero potential or zero potential energy ?

(*a*) Potential at a particular point

(*b*) Change in potential energy of a two-charge system

(*c*) Potential energy of a two - charge system

(*d*) Potential difference between two points

10. At distance of 5cm and 10cm outwards from the surface of a uniformly charged solid sphere, the potentials are 100V and 75V respectively. Then :

(*a*) Potential at its surface is 150V.

(*b*) The charge on the sphere is $(5/3) \times 10^{-9}$C.

(*c*) The electric field on the surface is 1500 V/m.

(*d*) The electric potential at its centre is 225V.

11. The electric potential decreases uniformly from 180 V to 20 V as one moves on the X-axis from x = – 2 cm to x = + 2 cm. The electric field at the origin :

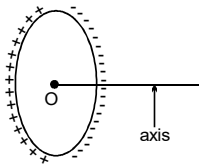
(*a*) must be equal to 40V/cm.

(*b*) may be equal to 40V/cm.

(*c*) may be greater than 40V/cm.

(*d*) may be less than 40V/cm.

12. The figure shows a nonconducting ring which has positive and negative charge non uniformly distributed on it such that the total charge is zero. Which of the following statements is **false**?



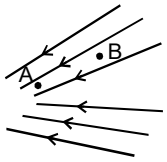
(*a*) The potential at all the points on the axis will be zero.

(*b*) The electric field at all the points on the axis will be zero.

(*c*) The direction of electric field at all points on the axis will be along the axis.

(*d*) If the ring is placed inside a uniform external electric field then net torque and force acting on the ring would be zero.

13. Which of the following is true for the figure showing electric lines of force ? (E is electrical field, V is potential)



(*a*) $E_A > E_B$ (*b*) $E_B > E_A$

(*c*) $V_A > V_B$ (*d*) $V_B > V_A$

14. Potential at a point A is 3 volt and at a point B is 7 volt, an electron is moving towards A from B.

(*a*) It must have some K.E. at B to reach A

(*b*) It need not have any K.E. at B to reach A

(*c*) to reach A it must have K.E. more than or equal to 4eV at B.

(*d*) when it will reach A, it will have K.E. more then or at least equal to 4eV if it was released from rest at B.

15. An electric dipole is placed at the centre of a sphere. Mark the correct answer

(*a*) the flux of the electric field through the sphere is zero

(*b*) the electric field is zero at every point of the sphere.

(*c*) the electric potential is zero everywhere on the sphere.

(*d*) the electric potential is zero on a circle on the surface.

16. An electric dipole is kept in the electric field produced by a point charge.

(*a*) dipole will experience a force.

(*b*) dipole will experience a torque.

(*c*) it is possible to find a path (not closed) in the field on which work required to move the dipole is zero.

(*d*) dipole can be in stable equilibrium.

17. An electric dipole moment $\vec{p} = (2.0\hat{i} + 3.0\hat{j})\mu C$. m is placed in a uniform electric field

$\vec{E} = (3.0\hat{i} + 2.0\hat{k}) \times 10^5 \, N C^{-1}$.

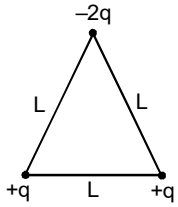
(*a*) The torque that $\vec{E}$ exerts on $\vec{p}$ is $(0.6\hat{i} - 0.4\,\hat{j} - 0.9\,\hat{k})$ Nm

(*b*) The potential energy of the dipole is –0.6 J.

(*c*) The potential energy of the dipole is 0.6 J

(*d*) If the dipole is rotated in the electric field, the maximum potential energy of the dipole is 1.3 J.

18. Three points charges are placed at the corners of an equilateral triangle of side L as shown in the figure.



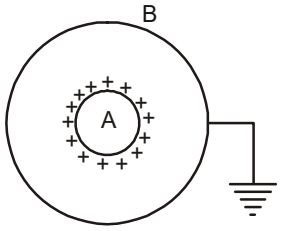
(*a*) The potential at the centroid of the triangle is zero.

(*b*) The electric field at the centroid of the triangle is zero.

(*c*) The dipole moment of the system is $\sqrt{2}\,qL$

(*d*) The dipole moment of the system is $\sqrt{3}\,qL$

19. A and B are two conducting concentric spherical shells. A is given a charge Q while B is uncharged. If now B is earthed as shown in figure.Then:



(*a*) The charge appearing on inner surface of B is -Q

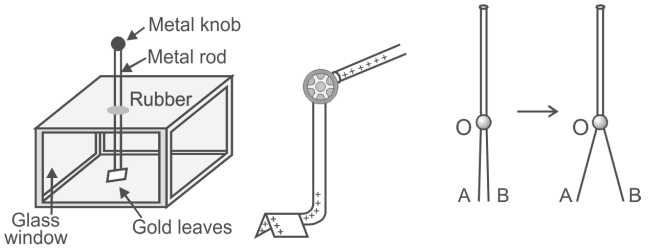
(*b*) The field inside and outside A is zero.

(*c*) The field between A and B is not zero.

(*d*) The charge appearing on outer surface of B is zero.

Comprehension – 1 (No. 20 to 22)

A leaf electroscope is a simple apparatus to detect any charge on a body. It consists of two metal leaves OA and OB, free to rotate about O. Initially both are very slightly separated. When a charged object is touched to the metal knob at the top of the conducting rod, charge flows from knob to the leaves through the conducting rod. As the leaves are now charged similarly, they start repelling each other and get separated, (deflected by certain angle).



The angle of deflection in static equilibrium is an indicator of the amount of charge on the charged body.

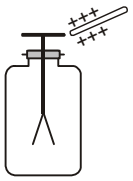
20. When a + 20 C rod is touched to the knob, the deflection of leaves was 5°, and when an identical rod of – 40 C is touched, the deflection was found to be 9°. If an identical rod of +30 C is touched, then the deflection may be :

(*a*) 0 (*b*) 2°

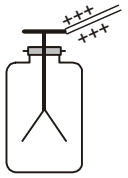
(*c*) 7° (*d*) 11°

21. If we perform these steps one by one.

(*i*) A positively charged rod is brought closer to initially uncharged knob (*a*)



(*ii*) Then the positively charged rod is touched to the knob (*b*)



(*iii*) Now the +vely charged rod is removed, and a negatively charged rod of same magnitude is brought closer at same distance (*c*)



In which case, the leaves will converge (come closer), as compared to the previous state ?

(*a*) (*i*)
(*b*) (*i*) and (*iii*)
(*c*) only (*iii*)
(*d*) In all cases, the leaves will diverge

22. In an electroscope, both leaves are hinged at the top point O. Each leaf has mass m, length ℓ and gets charge q. Assuming the charge to be concentrated at ends A and B only, the small angle of deviation (θ) between the leaves in static equilibrium, is equal to :

(*a*) $\left(\frac{4kq^2}{\ell^2 mg}\right)^{1/3}$ (*b*) $\left(\frac{kq^2}{\ell^2 mg}\right)^{1/3}$

(*c*) $\left(\frac{2kq^2}{\ell^2 mg}\right)^{1/2}$ (*d*) $\left(\frac{64kq^2}{\ell^2 mg}\right)^{1/3}$

Comprehension–2 (No. 23 to 25)

A charged particle is suspended at the centre of two thin concentric spherical charged shells, made of non conducting material. Figure A shows cross section of the arrangement. Figure B gives the net flux ϕ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere.

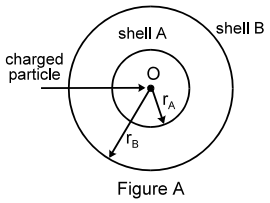


Figure A

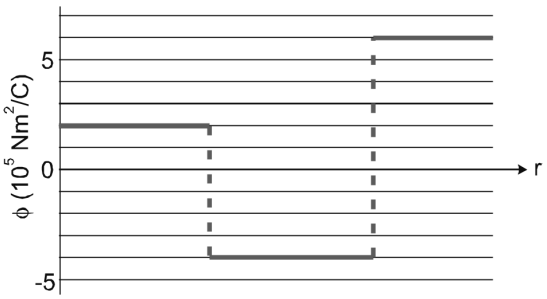


Figure B

23. What is the charge on the central particle ?

(*a*) 0.2 μC (*b*) 2 μC
(*c*) 1.77 μC (*d*) 3.4 μC

24. What is the charge on shell A ?

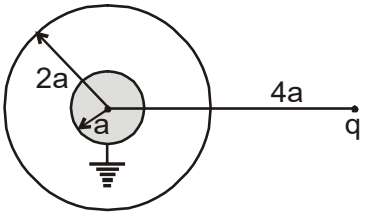
(*a*) 5.31×10^{-6} C (*b*) -5.31×10^{-6} C
(*c*) -3.54×10^{-6} C (*d*) -1.77×10^{-6} C

25. In which range of the values of r is the electric field zero?

(*a*) 0 to r_A
(*b*) r_A to r_B
(*c*) For $r > r_B$
(*d*) For no range of r, electric field is zero.

Comprehension–3 (No. 26 to 28)

A solid conducting sphere of radius 'a' is surrounded by a thin uncharged concentric conducting shell of radius 2a. A point charge q is placed at a distance 4a from common centre of conducting sphere and shell. The inner sphere is then grounded.



26. The charge on solid sphere is :

(*a*) $-\frac{q}{2}$ (*b*) $-\frac{q}{4}$

(*c*) $-\frac{q}{8}$ (*d*) $-\frac{q}{16}$

27. Pick up the correct statement.

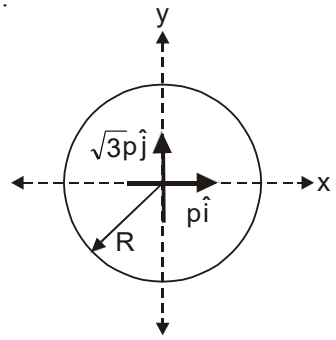
(*a*) Charge on surface of inner sphere is non-uniformly distributed.
(*b*) Charge on inner surface of outer shell is non-uniformly distributed.
(*c*) Charge on outer surface of outer shell is non-uniformly distributed.
(*d*) All the above statements are false.

28. The potential of outer shell is.

(*a*) $\frac{q}{32\pi\varepsilon_o a}$ (*b*) $\frac{q}{16\pi\varepsilon_o a}$

(*c*) $\frac{q}{8\pi\varepsilon_o a}$ (*d*) $\frac{q}{4\pi\varepsilon_o a}$

29. Column I gives a situation in which two dipoles of dipole moment $p\hat{i}$ and $\sqrt{3}\,p\hat{j}$ are placed at origin. A circle of radius R with centre at origin is drawn as shown in figure. Column II gives coordinates of certain positions on the circle. Match the statements in Column I with the statements in Column II .



Column I

(*a*) The coordinate(s) of point on circle where potential is maximum :

(*b*) The coordinate(s) of point on circle where potential is zero :

(*c*) The coordinate(s) of point on circle where magnitude of electric field intensity is $\frac{1}{4\pi\varepsilon_0}\frac{4p}{R^3}$:

(*d*) The coordinate(s) of point on circle where magnitude of electric field intensity is $\frac{1}{4\pi\varepsilon_0}\frac{2p}{R^3}$:

Column II

(p) $\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$

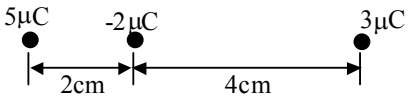
(q) $\left(-\frac{R}{2}, -\frac{\sqrt{3}R}{2}\right)$

(r) $\left(-\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$

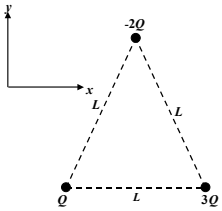
(s) $\left(\frac{\sqrt{3}R}{2}, -\frac{R}{2}\right)$

NUMERICAL VALUE BASED

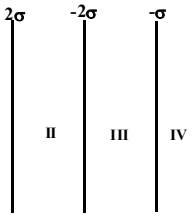
30. Three point charges lie on a straight line, as shown in the figure. Find the magnitude of resultant force exerted on the $-2\mu C$ charge in newton.



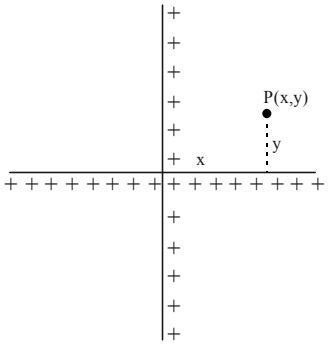
31. Three point charges are held on the corners of an equilateral triangle as shown in the figure.
Take Q = 2μC and L = 3 cm. What is the magnitude of resultant force exerted on the charge +3Q in N.



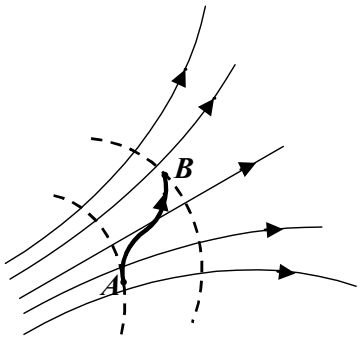
32. The field on either side of an infinite sheet of charge of density of σ C/m^2 is $E = \sigma/2\varepsilon_0$. Electric field in region II is given by $\frac{x\sigma}{\varepsilon_0}$ then x is



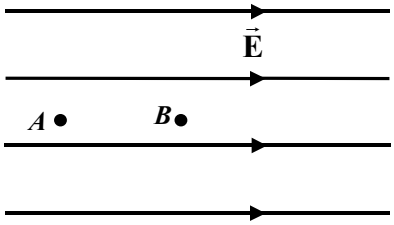
33. Two infinite lines of charge with equal linear charge densities λ C/m are placed along the x and y axes, as in figure. If angle made by resultant electric field with horizontal at pont P is θ then tanθ will be equal to (given y = 2x)



34. Figure shows two equipotential (dashed) surfaces such that $V_A = -5V$ and $V_B = -15$ V. The external work needed to move a -2 μC charge at constant speed from A to B along the indicated path in Joule is given by $x \times 10^{-5}$ then x is -



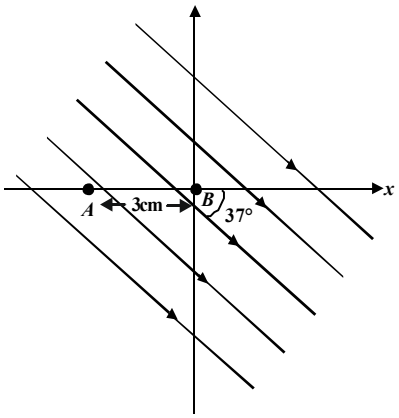
35. In the shown fig. points A and B are 4 cm apart along the lines of a uniform field $\vec{E} = 600\,\hat{i}$ V/m. Find the change in potential $|V_B - V_A|$ in volt.



36. Four point charges of 0.6 μC, 2.2 μC, -3.6 μC and +4.8 μC are placed at the corners of a square of side 10 cm. What is the work done to bring a charge of -5 μC from infinity to the centre of the square in Joule (Assume speed of -5μC charge is kept constant).

37. An electric field is given $\vec{E} = 2x\,\hat{i} - 3y^2\,\hat{j}$ N/C. Find the change in potential in volt from the position $\vec{r}_A = \hat{i} - 2\hat{j}$ m to $\vec{r}_B = 2\hat{i} + \hat{j} + 3\hat{k}$ m.

38. A uniform electric field of 400 V/m is directed at 37° below the x–axis, as shown in figure. Find the changes in potential $V_B - V_A$ in volt.

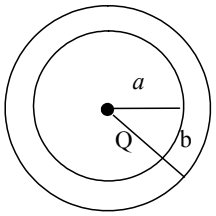


JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge The value of A such that the electric field in the region between the spheres will be constant, is:

[*JEE Main - 2016*]



(*a*) $\frac{Q}{2\pi a^2}$ (*b*) $\frac{Q}{2\pi(b^2 - a^2)}$

(*c*) $\frac{2Q}{\pi(a^2 - b^2)}$ (*d*) $\frac{2Q}{\pi a^2}$

2. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = -\tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\,\hat{j}$ It experiences torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is :

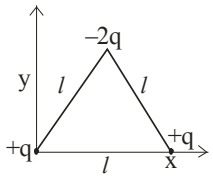
[*JEE Main - 2017*]

(*a*) 60° (*b*) 90°

(*c*) 30° (*d*) 45°

3. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle, as shown in the figure:

[*JEE Main - 2019 (January)*]



(*a*) $\sqrt{3}q\ell\frac{\hat{j} - \hat{i}}{\sqrt{2}}$ (*b*) $(q\ell)\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

(*c*) $2q\ell\hat{j}$ (*d*) $-\sqrt{3}q\ell\hat{j}$

4. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is:

[*JEE Main - 2019 (January)*]

(*a*) $\frac{R}{\sqrt{5}}$ (*b*) $\frac{R}{\sqrt{2}}$

(*c*) R (*d*) $R\sqrt{2}$

5. Three charges +Q, q, +Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by +Q, placed at x = 0, is zero, then value of q is **[*JEE Main - 2019 (January)*]**

(*a*) –Q/4 (*b*) +Q/2

(*c*) +Q/4 (*d*) –Q/2

6. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r)=\frac{A}{r^2}e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution the radius R is:

[*JEE Main - 2019 (January)*]

(*a*) $a\log\left(1-\frac{Q}{2\pi aA}\right)$ (*b*) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(*c*) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (*d*) $\frac{a}{2}\log\left(1-\frac{1}{2\pi aA}\right)$

7. Two point charge $q_1\left(\sqrt{10}\mu C\right)$ and q2 $(-25\mu C)$ are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is, $\left[\text{take}\frac{1}{4\pi\varepsilon_0}=9\times10^9\,Nm^2C^{-2}\right]$

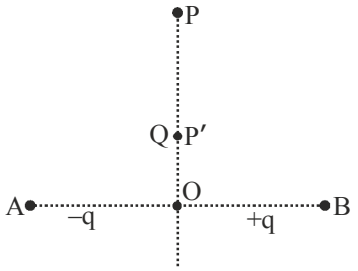
[*JEE Main - 2019 (January)*]

(*a*) $\left(63\hat{i}-27\hat{j}\right)\times10^2$ (*b*) $\left(-63\hat{i}+27\hat{j}\right)\times10^2$

(*c*) $\left(81\hat{i}-81\hat{j}\right)\times10^2$ (*d*) $\left(-81\hat{i}+81\hat{j}\right)\times10^2$

8. Charges – q and + q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. the charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P' such that $OP'=\left(\frac{y}{3}\right)$, the force on Q will be close to $\left(\frac{y}{2}>>2a\right)$

[*JEE Main - 2019 (January)*]



(*a*) 3F (*b*) $\frac{F}{3}$

(*c*) 9F (*d*) 27F

9. Four point charges –q, +q, +q and –q are placed on y-axis at y = –2d, y = –d, y = +d and y = +2d, respectively. The magnitude of the electric field E at a point on the x-axis at x = D, with D>> d, will behave as:-

[*JEE Main-2019 (April)*]

(*a*) $E\propto\frac{1}{D}$ (*b*) $E\propto\frac{1}{D^3}$

(*c*) $E\propto\frac{1}{D^2}$ (*d*) $E\propto\frac{1}{D^4}$

10. The bob of a simple pendulum has mass 2g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000V/m. At equilibrium, the angle that the pendulum makes with the vertical is (Take g = 10 m/s^s)

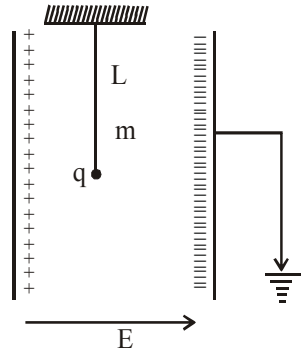
[*JEE Main-2019 (April)*]

(*a*) $\tan^{-1}(5.0)$ (*b*) $\tan^{-1}(2.0)$

(*c*) $\tan^{-1}(0.5)$ (*d*) $\tan^{-1}(0.2)$

11. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by :

[*JEE Main-2019 (April)*]



(*a*) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$ (*b*) $2\pi\sqrt{\frac{L}{\left(g+\frac{qE}{m}\right)}}$

(*c*) $2\pi\sqrt{\frac{L}{\left(g-\frac{qE}{m}\right)}}$ (*d*) $2\pi\sqrt{\frac{L}{\sqrt{g^2-\frac{q^2E^2}{m^2}}}}$

12. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is : **[*JEE Main-2019 (April)*]**

(*a*) $\sqrt{\frac{qE}{2md}}$ (*b*) $2\sqrt{\frac{qE}{md}}$

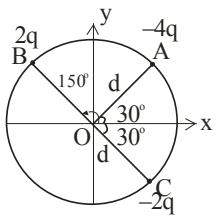
(*c*) $\sqrt{\frac{2qE}{md}}$ (*d*) $\sqrt{\frac{qE}{md}}$

13. Let a total charge 2Q be distributed in a sphere of radius R, with the charge density given by ρ(r) = kr, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then : **[*JEE Main-2019 (April)*]**

(*a*) $a = \frac{3R}{2^{1/4}}$ (*b*) $a = R/\sqrt{3}$

(*c*) $a = 8^{-1/4} R$ (*d*) $a = 2^{-1/4} R$

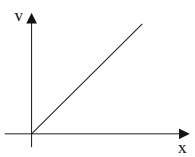
14. Three charged particles A, B and C with charges –4q, 2q and –2q are present on the circumference of a circle of radius d. The charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is: **[*JEE Main-2020 (January)*]**

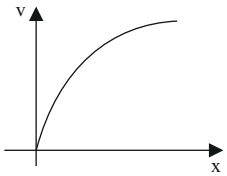


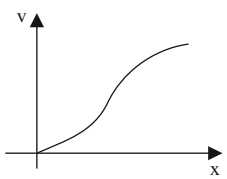
(*a*) $\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$ (*b*) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

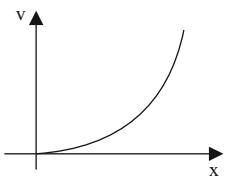
(*c*) $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (*d*) $\frac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$

15. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale) **[*JEE Main-2020 (January)*]**

(*a*)


(*b*)


(*c*)


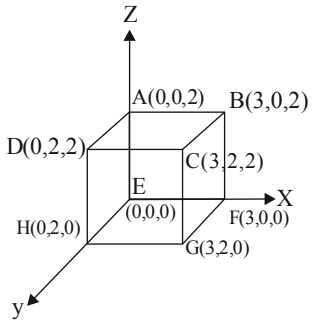
(*d*)


16. An electric dipole of moment $\vec{p} = (-\hat{i} - 3\hat{j} + 2\hat{k})\ 10^{-29}$ C.m. is at the origin (0,0,0). The electric field due to this dipole at $\vec{r} = (+\hat{i} + 3\hat{j} + 5\hat{k})$ (note that $\vec{r}.\vec{p} = 0$) is parallel to: **[*JEE Main-2020 (January)*]**

(*a*) $\left(+\hat{i} + 3\hat{j} - 2\hat{k}\right)$ (*b*) $\left(-\hat{i} + 3\hat{j} - 2\hat{k}\right)$

(*c*) $\left(+\hat{i} - 3\hat{j} - 2\hat{k}\right)$ (*d*) $\left(-\hat{i} - 3\hat{j} + 2\hat{k}\right)$

17. An electric field $\vec{E} = 4x\hat{i} - (y^2 + 1)\hat{j}$ N/C passes through the box shown in figure. The flux of the electric field through surface ABCD and BCGF and marked as ϕ_I and ϕ_{II} respectively. The difference between $(\phi_I - \phi_{II})$ is (in Nm^2/C) **[*JEE Main-2020 (January)*]**



18. In finding the electric field using gauss law the formula $|\vec{E}| = \frac{q_{enc}}{\varepsilon_0 |A|}$ is applicable. In the formula ε_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charges enclosed by the Gaussian surface. This equation can be used in which of the following situation? **[*JEE Main-2020 (January)*]**

(*a*) Only when the Gaussian surface in an equipotential surface

(*b*) Only when $|\vec{E}|$ = constant on the surface.

(*c*) Only when the Gaussian surface is an equipotential surface and $|\vec{E}|$ is constant of the surface.

(*d*) For any choice of Gaussian surface

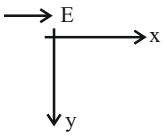
19. Consider two charged metallic spheres S_1 and S_2 of radii R_1 and R_2, respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1/E_2 = R_1/R_2$. Then the ratio V_1(on S_1)/V_2 (on S_2) of the electrostatic potentials on each sphere is **[*JEE Main-2020 (January)*]**

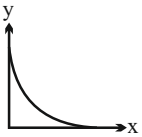
(*a*) $\left(\frac{R_1}{R_2}\right)^3$ (*b*) $\frac{R_1}{R_2}$

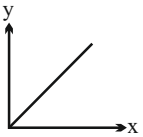
(*c*) $\left(\frac{R_2}{R_1}\right)$ (*d*) $\left(\frac{R_1}{R_2}\right)^2$

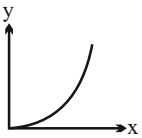
20. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale).

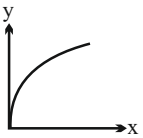
[*JEE Main-2020 (September)*]



(*a*) 

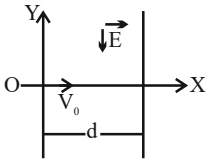
(*b*) 

(*c*) 

(*d*) 

21. A charged particle (mass m and charge q) moves along X-axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is

[*JEE Main-2020 (September)*]



(*a*) $y = \frac{qEd}{mV_0^2}(x-d)$ (*b*) $y = \frac{qEd^2}{mV_0^2}x$

(*c*) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2}-x\right)$ (*d*) $y = \frac{qEd}{mV_0^2}x$

22. A particle of charge q and mass m is subjected to an electric field $E = E_0(1-ax^2)$ in the x-direction, where a and E_0 are constants. Initially the particle was at rest at x = 0. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is

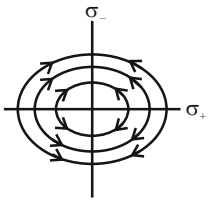
[*JEE Main-2020 (September)*]

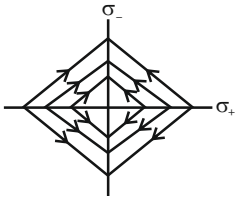
(*a*) $\sqrt{\frac{3}{a}}$ (*b*) $\sqrt{\frac{2}{a}}$

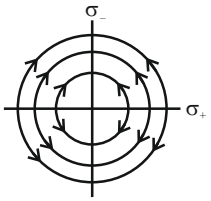
(*c*) $\sqrt{\frac{1}{a}}$ (*d*) a

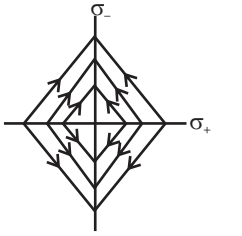
23. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system?

[*JEE Main-2020 (September)*]

(*a*) 

(*b*) 

(*c*) 

(*d*) 

24. Consider the force F on a charge 'q' due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statements is true for F, if 'q' is placed at distance r from the centre of the shell?

[*JEE Main-2020 (September)*]

(*a*) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for all r

(*b*) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2} > F > 0$ For $r < R$

(*c*) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for $r > R$

(*d*) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for $r < R$

25. Two identical electric point dipoles have dipole moments $\vec{p_1} = p\hat{i}$ and $\vec{p_2} = p\hat{i}$ and are held on the x axis at distance 'a' from each other. When released, they move along the x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is 'm', their speed when they are infinitely far apart is

[*JEE Main-2020 (September)*]

(*a*) $\frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$ (*b*) $\frac{P}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$

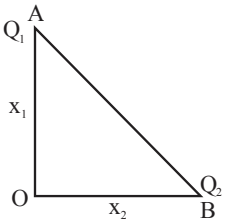
(*c*) $\frac{P}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (*d*) $\frac{P}{a}\sqrt{\frac{3}{2\pi\varepsilon_0 ma}}$

26. Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right)$ W /m The rms electric field, in units of V/m associated with this source is close to the nearest integer is _____.
($\varepsilon_0 = 8.86 \times 10^{-12}\, C^2 Nm^{-2}$; $c = 3 \times 10^8\, ms^{-1}$)

[*JEE Main-2020 (September)*]

27. Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to

[*JEE Main-2020 (September)*]

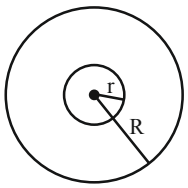


(*a*) $\frac{x_1^3}{x_2^3}$ (*b*) $\frac{x_2^2}{x_1^2}$

(*c*) $\frac{x_1}{x_2}$ (*d*) $\frac{x_2}{x_1}$

28. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R (R > r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is

[*JEE Main-2020 (September)*]



(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$ (*b*) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$

(*c*) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$ (*d*) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$

29. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V(R) – V(4R) is **[*JEE Main-2020 (September)*]**

(*a*) $\frac{3Q_2}{4\pi\varepsilon_0 R}$ (*b*) $\frac{3Q_1}{16\pi\varepsilon_0 R}$

(*c*) $\frac{Q_2}{4\pi\varepsilon_0 R}$ (*d*) $\frac{3Q_1}{4\pi\varepsilon_0 R}$

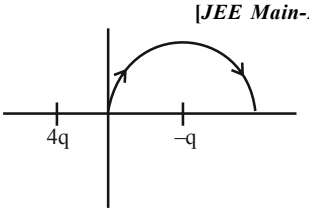
30. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to **[*JEE Main-2020 (September)*]**

(*a*) 2 : 1 (*b*) 1 : 2

(*c*) 10 : 7 (*d*) 5 : 7

31. A two point charges 4q and –q are fixed on the x- axis at x $= -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will

[*JEE Main-2020 (September)*]



(*a*) Decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$ (*b*) Decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$

(*c*) Increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$ (*d*) Increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$

32. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge (+q) each, while 2, 4, 6, 8, 10 have charge (–q) each. The potential V and the electric field E at the centre of the circle are respectively.

(**Take** V = 0 at infinity)

[*JEE Main-2020 (September)*]

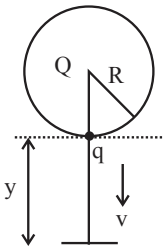
(*a*) V = 0; E = 0

(*b*) $V = \frac{10q}{4\pi\varepsilon_0 R}$; $E = \frac{10q}{4\pi\varepsilon_0 R^2}$

(*c*) $V = \frac{10q}{4\pi\varepsilon_0 R} = E = 0$

(*d*) V = 0; $E = \frac{10q}{4\pi\varepsilon_0 R^2}$

33. A solid sphere of radius R carries a charge Q + q distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then : (assume the remaining portion to be spherical). **[*JEE Main-2020 (September)*]**



(*a*) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(*b*) $v^2 = 2y\left[\frac{qQR}{4\pi\varepsilon_0 (R+y)^3 m} + g\right]$

(*c*) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$

(*d*) $v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

NUMERICAL BASED QUESTIONS

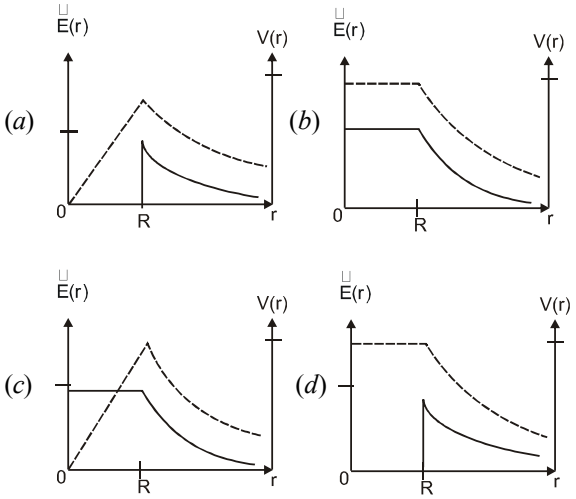
34. An infinite number of point charges, each carrying $1\mu C$ charge, are placed along the y-axis at $y = 1m, 2m$, $4m, 8m$. The total force on a 1 C point charge, placed at the origin, is $x \times 10^3$ N. The value of x_1 to the nearest integer, is . Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

[*JEE Main-2021 (March)*]

JEE-ADVANCED PREVIOUS YEAR'S

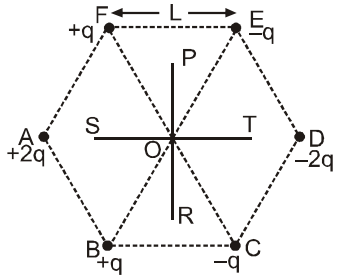
1. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

[*JEE-2012*]



2. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement (s) is (are) correct?

[*JEE-2012*]



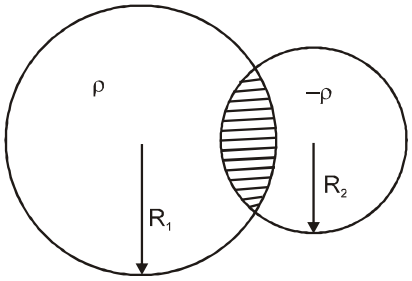
(*a*) the elecric field at O is 6K along OD
(*b*) The potential at O is zero
(*c*) The potential at all points on the line PR is same
(*d*) The potential at all points on the line ST is same.

3. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be ;

(*a*) –4 (*b*) $-\frac{32}{25}$

(*c*) $\frac{32}{25}$ (*d*) 4

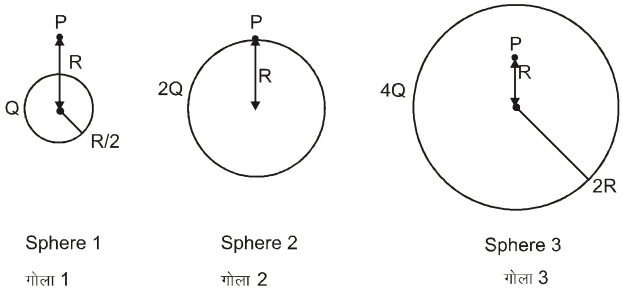
4. Two non–conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities +ρ and –ρ, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region:



(*a*) the electrostatic field is zero
(*b*) the electrostatic potential is constant
(*c*) the electrostatic field is constant in magnitude
(*d*) the electrostatic field has same direction

5. Charges Q, 2Q and 4Q are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii R/2, R and 2R respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1 E_2 and E_3 respectively, then

[*JEE Advanced - 2014*]



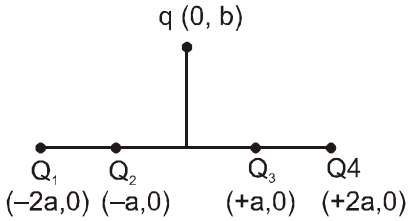
(*a*) $E_1 > E_2 > E_3$ (*b*) $E_3 > E_1 > E_2$
(*c*) $E_2 > E_1 > E_3$ (*d*) $E_3 > E_2 > E_1$

6. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. if $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then **[*JEE Advanced -2014*]**

(*a*) $Q = 4\sigma\pi r_0^2$ (*b*) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(*c*) $E_1(r_0/2) = 2E_2(r_0/2)$ (*d*) $E_2(r_0/2) = 4E_3(r_0/2)$

7. Four charge Q_1, Q_2, Q_3, and Q_4, of same magnitude are fixed along the x axis at x = –2a –a, +a and +2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four options of the signs of these charges are given in List-I . The direction of the forces on the charge q is given in List- II Match List-1 with List-II and select the correct answer using the code given below the lists. **[*JEE Advanced - 2014*]**



List-I	List-II
P. Q_1, Q_2, Q_3, Q_4, all positive	**1.** +x
Q_1, Q_2 positive Q_3, Q_4 negative	**2.** –x
R. Q_1, Q_4 positive Q_2, Q_3 negative	**3.** +y
S. Q_1, Q_3 positive Q_2, Q_4 negative	**4.** –y

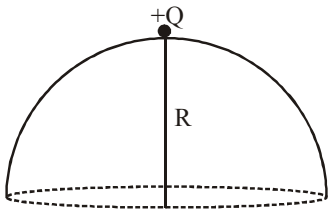
Code :

(*a*) P-3, Q-1, R-4,S-2 (*b*) P-4, Q-2, R-3, S-1

(*c*) P-3, Q-1, R-2,S-4 (*d*) P-4, Q-2, R-1, S-3

8. A point charge +Q is placed just outside an imaginary hemispherical surfce of radius R as shown in the figure. Which of the following statements is/are correct?

[*JEE Advanced - 2017*]



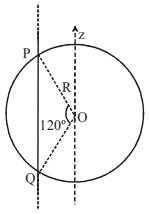
(*a*) Total flux through the curved and the flat surface is $\dfrac{Q}{\varepsilon_0}$

(*b*) The component of the electric field normal to the flat surface is constant over the surface

(*c*) The circumference of the flat surface is an equipotential

(*d*) The electric flux passing through the curved surface of the hemisphere is $-\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$

9. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true? **[*JEE Advanced - 2018*]**



(*a*) The electric flux through the shell is $\sqrt{3}\,R\,\lambda/\varepsilon_0$

(*b*) The z-component of the electric field is zero at all the points on the surface of the shell

(*c*) The electric flux through the shell is $\sqrt{2}\,R\,\lambda/\varepsilon_0$

(*d*) The electric field is normal to the surface of the shell at all points

10. The electric field E is measured at a point P(0,0,d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations betweene E and d. List-II describe different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II. **[*JEE Advanced - 2018*]**

List - I	List - II
P. E is indpendent of d	**1.** A point charge Q at the origion
$E \propto \dfrac{1}{d}$	**2.** A small dipole with point charges Q at (0, 0 , ℓ) and –Q(0, 0, – ℓ). Take $2\ell << d$
R. $E \propto \dfrac{1}{d^2}$	**3.** An infinite line charge coincident with the x-axis, with uniform linear charge density λ.
S. $E \propto \dfrac{1}{d^3}$	**4.** Two infinite wires carrying iniform linear Charge density parallel to the x-axis. The one along (y = 0, z = ℓ) has a charge den sity + λ and the one along (y = 0, z = – ℓ) has a charge density – λ. Take $2\ell << d$
	5. Infinite plane charge coincident with the xy-plane with uniform surface charge density

(*a*) P → 5; Q → 3, 4; R → 1; S → 2

(*b*) P → 5; Q → 3; R → 1, 4; S → 2

(*c*) P → 5; Q → 3; R → 1,2; S → 4

(*d*) P → 4, Q → 2, 3; R → 1; S → 5

11. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2 (\alpha << 1)$ is made on the shell without affecting the rest of the shell. Which one of the following statements is correct?

(*a*) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}$ R from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$ **[*JEE Advanced - 2019*]**

(*b*) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(*c*) The magnitude of electric field at a point, located on a line passing through the hole and shell's center on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

(*d*) The potential at the center of the shell is reduced by $2\alpha V_0$

12. A charged shell of radius R carries a total charge Given Φ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Whichof the following option(s) is/ are correct? [ϵ_0 is the permittivity of free space]

[*JEE Advanced - 2019*]

(*a*) If h > 2R and r > R then $\Phi = \frac{Q}{\epsilon_0}$

(*b*) If h < $\frac{8R}{5}$ and r = $\frac{3R}{5}$ then $\Phi = 0$

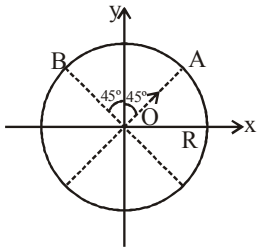
(*c*) If h > 2R and r = $\frac{4R}{5}$ then $\Phi = \frac{Q}{5\epsilon_0}$

(*d*) If h > 2R and r = $\frac{3R}{5}$ then $\Phi = \frac{Q}{5\epsilon_0}$

13. An electric dipole with dipole moment $\frac{P_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are:

[*JEE Advanced - 2019*]

(ε_0 is permittivity of free space, R >> dipole size)



(*a*) $R = \left(\frac{P_0}{4\pi\varepsilon_0 E_0}\right)^{1/3}$

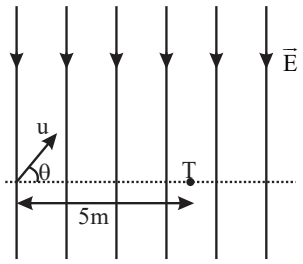
(*b*) The magnitude of total electric field on any two points of the circle will be same

(*c*) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

(*d*) Total electric field at point B is $\vec{E}_B = 0$

14. A uniform electric field, $\vec{E} = -40\sqrt{3}\hat{y}NC^{-1}$ is applied in a region. A charged particle of mass m carrying positive charge q is projected in this region with an initial speed of $2\sqrt{10}\times10^6 ms^{-1}$. This particle is aimed to hit a target T, which is 5 m away from its entry point into the field as shown schematically in the figure. Take $\frac{q}{m} = 10^{10} Ckg^{-1}$.

Then- **[*JEE Advanced - 2020*]**



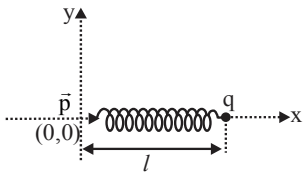
(*a*) the particle will hit T if projected at an angle 45° from the horizontal

(*b*) the particle will hit T if projected either at an angle 30° or 60° from the horizontal

(*c*) time taken by the particle to hit T could be $\sqrt{\frac{5}{6}}$ μs as well as $\sqrt{\frac{5}{2}}$ ms

(*d*) time taken by the particle to hit T $\sqrt{\frac{5}{3}}$ ms

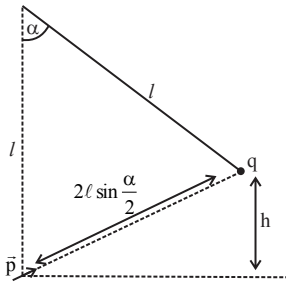
15. One end of a spring of negligible unstretched length and spring constant k is fixed at the origin (0,0). A point particle of mass m carrying a positive charge q is attached at its other end. The entire system is kept on a smooth horizontal surface. When a point dipole $\vec{p}$ pointing towards the charge q is fixed at the origin, the spring gets stretched to a length ℓ and attains a new equilibrium position (see figure below). If the point mass is now displaced slightly by $\Delta\ell << \ell$ from its equilibrium position and released, it is found to oscillate at frequency $\frac{1}{\delta}\sqrt{\frac{k}{m}}$. The value of δ is..... **[*JEE Advanced - 2020*]**



16. A circular disc of radius R carries surface charge density $\sigma(r) = \sigma_0\left(1-\frac{r}{R}\right)$, where σ_0 is a constant and r is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is ϕ_0. Electric flux through another spherical surface of radius $\frac{R}{4}$ and concentric with the disc is ϕ. Then the ratio $\frac{\phi_0}{\phi}$ is ..... **[*JEE Advanced - 2020*]**

17. A point charge q of mass m is suspended vertically by a string of length *l*. A point dipole of dipole moment $\vec{p}$ is now brought towards q from infinity so that the charge moves away. The final equilibrium position of the system including the direction of the dipole, the angles and distances is shown in the figure below. If the work done in bringing the dipole to this position is N×(mgh), where g is the acceleration due to gravity, then the value of N is _____ .

(Note that for three coplanar forces keeping a point mass in equilibrium, $\frac{F}{\sin\theta}$ is the same for all forces, where F is any one of the forces and θ is the angle between the other two forces) **[*JEE(Advanced) - 2020*]**



18. Two identical non-conducting solid spheres of same mass and charge are suspended in air from a common point by two non-conducting, massless strings of same length. At equilibrium, the angle between the strings is α. The spheres are now immersed in a dielectric liquid of density 800 kg m^3 and dielectric constant 21. If the angle between the strings remains the same after the immersion, then **[*JEE Advanced - 2020*]**

(*a*) electric force between the spheres remains unchanged
(*b*) electric force between the spheres reduces
(*c*) mass density of the spheres is 840 kg m^{-3}
(*d*) the tension in the strings holding the spheres remains unchanged

19. Two large circular discs separated by a distance of 0.01 m are connected to a battery via a switch as shown in the figure. Charged oil drops of density 900 kg m^{-3} are released through a tiny hole at the center of the top disc. Once some oil drops achieve terminal velocity, the switch is closed to apply a voltage of 200 V across the discs. As a result, an oil drop of radius 8×10^{-7} m stops moving vertically and floats between the discs. The number of electrons present in this oil drop is _____. (Neglect the buoyancy force, take acceleration due to gravity =10 ms^{-2} and charge on an electron (e) = 1.6×10^{-19} C) **[*JEE Advanced - 2020*]**

ANSWER KEY

Topicwise Questions

1. (*a*) **2.** (*b*) **3.** (*b*) **4.** (*c*) **5.** (*d*) **6.** (*b*) **7.** (*c*) **8.** (*b*) **9.** (*a*) **10.** (*b*)
11. (*c*) **12.** (*a*) **13.** (*d*) **14.** (*c*) **15.** (*a*) **16.** (*b*) **17.** (*b*) **18.** (*b*) **19.** (*c*) **20.** (*c*)
21. (*b*) **22.** (*a*) **23.** (*c*) **24.** (*c*) **25.** (*b*) **26.** (*d*) **27.** (*b*) **28.** (*b*) **29.** (*c*) **30.** (*c*)
31. (*b*) **32.** (*d*) **33.** (*b*) **34.** (*a*) **35.** (*d*) **36.** (*c*) **37.** (*b*) **38.** (*a*) **39.** (*b*) **40.** (*a*)
41. (*b*) **42.** (*b*) **43.** (*d*) **44.** (*a*) **45.** (*c*) **46.** (*c*) **47.** (*d*) **48.** (*b*) **49.** (*b*) **50.** (*a*)
51. (*d*) **52.** (*b*) **53.** (*c*) **54.** (*a*)

Learning Plus

1. (*b*) **2.** (*a*) **3.** (*d*) **4.** (*d*) **5.** (*c*) **6.** (*c*) **7.** (*a*) **8.** (*b*) **9.** (*a*) **10.** (*d*)
11. (*b*) **12.** (*c*) **13.** (*a*) **14.** (*a*) **15.** (*a*) **16.** (*d*) **17.** (*c*) **18.** (*d*) **19.** (*d*) **20.** (*a*)
21. (*d*) **22.** (*c*) **23.** (*b*) **24.** (*d*) **25.** (*b*) **26.** (*b*) **27.** (*c*) **28.** (*c*) **29.** (*b*) **30.** (*c*)
31. (*b*) **32.** (*b*) **33.** (*b*) **34.** (*d*) **35.** (*c*) **36.** (*b*) **37.** (*a*) **38.** (*a*) **39.** (*a*) **40.** (*d*)
41. (*c*) **42.** (*a*) **43.** (*d*) **44.** (*b*) **45.** (*a*) **46.** (*b*) **47.** (*b*) **48.** (*c*) **49.** (*c*) **50.** (*a*)
51. (*c*) **52.** (*d*) **53.** (*b*)

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING

1. (*b*,*d*) **2.** (*a*,*d*) **3.** (*a*,*b*,*c*,*d*) **4.** (*b*,*c*) **5.** (*a*,*c*) **6.** (*a*,*c*) **7.** (*a*,*b*,*c*) **8.** (*a*,*b*,*c*) **9.** (*a*,*c*)
10. (*a*,*b*,*c*,*d*) **11.** (*b*,*c*) **12.** (*b*,*c*,*d*) **13.** (*a*,*d*) **14.** (*a*,*c*) **15.** (*a*,*d*) **16.** (*a*,*c*) **17.** (*a*,*b*,*d*) **18.** (*a*,*d*)
19. (*a*,*c*,*d*) **20.** (*c*) **21.** (*c*) **22.** (*a*) **23.** (*c*) **24.** (*b*) **25.** (*d*) **26.** (*b*) **27.** (*c*)
28. (*a*) **29.** (*a*) p (*b*) r,s (*c*) p,q (*d*) r,s **30.** [0191] **31.** [0208] **32.** [2.5] **33.** [0.5] **34.** [0002]
35. [0024] **36.** [2.55] **37.** [0006] **38.** [–9.6]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*b*) **5.** (*a*) **6.** (*b*) **7.** (*a*) **8.** (*d*) **9.** (*d*) **10.** (*c*)
11. (*a*) **12.** (*c*) **13.** (*c*) **14.** (*a*) **15.** (*b*) **16.** (*a*) **17.** [48] **18.** (*c*) **19.** (*d*) **20.** (*b*)
21. (*c*) **22.** (*a*) **23.** (*d*) **24.** (*c*) **25.** (*a*) **26.** [194.42] **27.** (*c*) **28.** (*a*) **29.** (*b*) **30.** (*a*)
31. (*b*) **32.** (*a*) **33.** (*d*) **34.** (12)

JEE-ADVANCED PREVIOUS YEAR'S

1. (*d*) **2.** (*a*,*b*,*c*) **3.** (*b*,*d*) **4.** (*c*,*d*) **5.** (*c*) **6.** (*c*) **7.** (*a*) **8.** (*c*,*d*) **9.** (*a*,*b*) **10.** (*b*)
11. (*a*) **12.** (*a*,*b*,*d*) **13.** (*a*,*d*) **14.** (*b*,*c*) **15.** (3.14) **16.** (6.40) **17.** (*b*) **18.** (*a*,*c*) **19.** (6)

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

2 Capacitance

CONCEPT OF CAPACITANCE

When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor.

$$Q \propto V \text{ or } Q = CV$$

The constant C is known as the capacity of the conductor.

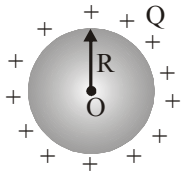
SI Unit :- coulomb/volt or farad (F)

1 farad = 9×10^{11} statfarad (CGS unit).

CAPACITY OF AN ISOLATED SPHERICAL CONDUCTOR

When charge Q is given to a spherical conductor of radius R, then potential at the surface of sphere is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow \frac{Q}{V} = 4\pi\varepsilon_0 R$$



$$C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9}.R$$

If earth is assumed to be a conducting sphere having radius R = 6400 km. It's theoretical capacitance C = 711 μF. But for all particle purpose capacitance of earth is taken infinity and its potential V = 0.

CAPACITY OF VARIOUS CAPACITOR

PARALLEL PLATE CAPACITOR

It consists of two parallel metallic plates (may be circular, rectangular, square) separated by a small distance. If A = Effective overlapping area of each plate.

(i) Electric field between the plates : $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

(ii) Potential difference between the plates : $V = E \times d = \frac{\sigma d}{\varepsilon_0}$

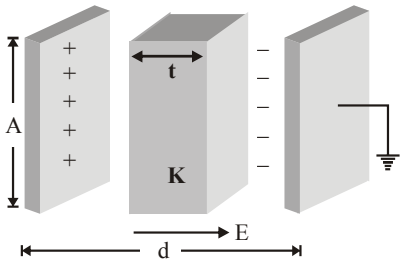
(iii) Capacitance : $C = \frac{\varepsilon_0 A}{d}$

(iv) If a dielectric medium of dielectric constant K is filled completely between the plates then capacitance increases by K times i.e. $C' = \frac{K\varepsilon_0 A}{d} \Rightarrow C' = KC$

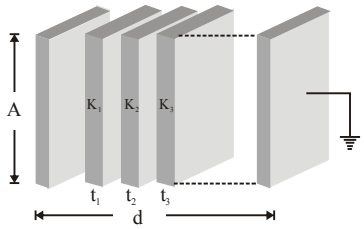
(v) The capacitance of parallel plate capacitor depends on A ($C \propto A$) and d $\left(C \propto \frac{1}{d}\right)$. It does not depend on the charge on the plates or the potential difference between the plates.

(vi) If a dielectric slab is partially filled between the plates

$$\Rightarrow C' = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{K}\right)}$$



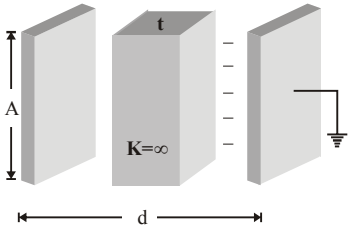
(vii) If a number of dielectric slabs are inserted between the plate as shown.



$$C' = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + ...) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + ...\right)}$$

(viii) When a metallic slab is inserted between the plates

$$C' = \frac{\varepsilon_0 A}{(d - t)}$$



If metallic slab fills the complete space between the plates (i.e. t = d) or both plates are joined through a metallic wire then capacitance becomes infinite.

(ix) Force between the plates of a parallel plate capacitor.

$$|F| = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{CV^2}{2d}$$

(x) Energy density between the plates of a parallel plate capacitor

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

ENERGY STORED IN A CHARGED CONDUCTOR

Let C is capacitance of a conductor. On being connected to a battery, it charges to a potential V from zero potential. If Q is charge on the conductor at that time then

$$Q = CV$$

Let battery supplies small amount of charge dQ to the conductor at constant potential V.

Then small amount of work done by the battery is

$$dW = VdQ = \frac{Q}{C}dQ$$

$$W = \int_0^Q \frac{Q}{C}dQ = \frac{1}{C}\left[\frac{Q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C}$$

This work done is stored by the capacitor in the form of the electric potential energy

$\therefore$ energy stored in the charged conductor

$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV$$

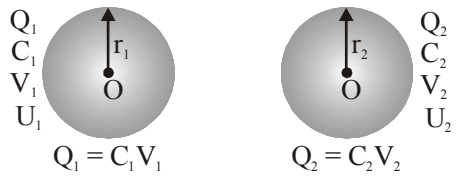
$$\therefore U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

REDISTRIBUTION OF CHARGES AND LOSS OF ENERGY

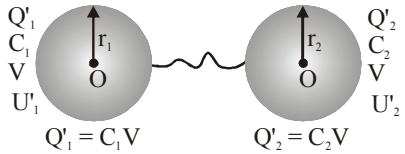
When two charged conductors are joined together through a conducting wire, charge begins to flow from one conductor to another from higher potential to lower potential.

This flow of charge stops when they attain the same potential. Due to flow of charge, loss of energy also takes place in the form of heat through the connecting wire.

Suppose there are two spherical conductors of radii r_1 and r_2, having charge Q_1 and Q_2, potential V_1 and V_2, energies U_1 and U_2 and capacitance C_1 and C_2 respectively.



If these two spheres are connected through a conducting wire, then alteration of charge, potential and energy takes place.



(*a*) **New charge :** According to the conservation of charge

$$Q_1 + Q_2 = Q'_1 + Q'_2 = Q \text{ (say), also } \frac{Q'_1}{Q'_2} = \frac{C_1}{C_2} = \frac{r_1}{r_2}$$

$$\Rightarrow Q'_2 = Q\left[\frac{r_2}{r_1 + r_2}\right] \text{ and similarly } Q'_1 = Q\left[\frac{r_1}{r_1 + r_2}\right]$$

(*b*) **Common potential :** Common potential

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q'_1 + Q'_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

(*c*) **Energy loss :** The loss of energy due to redistribution of charge is given by

$$\Delta U = U_i - U_f = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

TRAIN YOUR BRAIN

Example 1: Can a sphere of radius 1 cm, placed in air be given a charge of 1 coulomb?

Sol. Potential of sphere of 1 cm radius is

$$V = k\frac{q}{r} = 9 \times 10^9 \times \frac{1}{10^{-2}} = 9 \times 10^{11} \text{ volt}$$

This potential is so high that the surrounding air gets ionized, thereby charge leaks to the medium. [air gets ionized at 3×10^6 volt]

Ans. [No]

Example 2: Two conductors having capacities 2μF and 5μF and potentials 2 volts and 10 volt respectively. The ratio of their charges after connecting there by a wire will be:

(*a*) 2 : 5 (*b*) 5 : 2

(*c*) 1 : 5 (*d*) 5 : 1

Sol. $\frac{q_1}{q_2} = \frac{2 \times 10^{-6}}{5 \times 10^{-6}} = \frac{2}{5}$

Ans. [*a*]

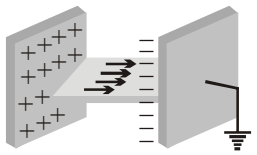
[Charge distributes in the ratio of capacities]

CAPACITOR OR CONDENSER

(*a*) A capacitor is a device that stores electric energy or a capacitor is a pair of two conductors of any shape, which are close to each other and have equal and opposite charge.

(*b*) The capacitance of a capacitor is defined as the magnitude of the charge Q on the positive plate divided

by the magnitude of the potential difference V between the plates i.e., $C = \frac{Q}{V}$



(*c*) A capacitor get's charged when a battery is connected across the plates. Once capacitor get's fully charged, flow of charge carriers stop in the circuit and in this condition potential difference across the plates of capacitor is same as the potential difference across the terminals of battery.

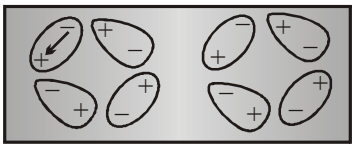
(*d*) net charge on a capacitor is always zero, but when we speak of the charge Q on a capacitor, we are referring to the magnitude of the charge on each plate.

DIELECTRIC

Dielectrics are insulating (non-conducting) materials which transmits electric effect without conducting.

Dielectrics are of Two Types

(*a*) **Polar dielectrics:** A polar molecule has permanent electric dipole moment $(\vec{p})$ in the absence of electric field also. But a polar dielectric has net dipole moment zero in the absence of electric field because polar molecules are randomly oriented as shown in figure.

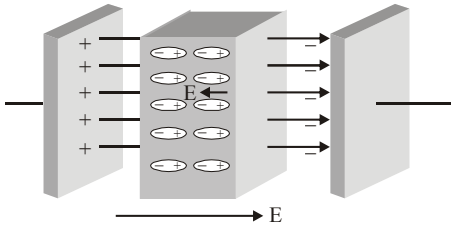


In the presence of electric field polar molecules tends to line up in the direction of electric field, and the substance has finite dipole moment e.g. water, alcohol, CO_2, NH_3, HCl etc. are made of polar atoms/molecules.

(*b*) **Non polar dielectric:** In non-polar molecules, Each molecule has zero dipole moment in its normal state. When electric field is applied, molecules becomes induced electric-dipoles e.g. N_2, O_2, Benzene, Methane etc. are made of non-polar atoms/molecules.

In general, any non-conducting material can be called as a dielectric but broadly non-conducting material having non-polar molecules referred to as dielectric.

(*c*) **Polarization of a dielectric slab:** It is the process of inducing equal and opposite charges on the two faces of the dielectric on the application of electric field.



(i) Electric field between the plates in the presence of dielectric medium is $E' = E - E_i$ where E = Main field, E' = Induced field.

(ii) dielectric constant of dielectric medium is defined as

$$\frac{E}{E'} = \frac{\text{Electric field between the plate with air}}{\text{Electric field beween the plates with medium}} = K$$

(iii) K is also known as relative permittivity (ε_r) of the material

(*d*) **Dielectric breakdown and dielectric strength:** If a very high electric field is created in a dielectric. The dielectric then behaves like a conductor. This phenomenon is known as **dielectric breakdown.**

The maximum value of electric field (or potential gradient) that a dielectric material can tolerate without it's electric breakdown is called it's **dielectric strength.**

S.I. unit of dielectric strength of a material is $\frac{V}{m}$ but practical unit is $\frac{kV}{mm}$

TRAIN YOUR BRAIN

Example 3: The capacitance of a capacitor is C when the distance between the plates is halved, the capacitance will becomes :

(*a*) half (*b*) twice

(*c*) one-fourth (*d*) four-times

Sol. $C \propto \frac{1}{d}$

$$\frac{C_1}{C_2} = \frac{d_2}{d_1} = \frac{d_1/2}{d_1} \Rightarrow \frac{C_1}{C_2} = \frac{1}{2} \Rightarrow C_2 = 2C_1$$ **Ans. [*b*]**

Example 4: If the distance between the plates of a capacitor is made half and the area of plates is doubled, then the capacitance will become:

(*a*) Twice (*b*) Four times

(*c*) Half (*d*) One-Fourth

Sol. $C \propto \frac{1}{d}$ and $C \propto A$

$$\Rightarrow C \propto \frac{A}{d} \qquad \Rightarrow \frac{C_1}{C_2} = \frac{A_1}{A_2}\cdot\frac{d_2}{d_1} = \frac{A_1}{2A_1}\frac{d_1/2}{d_1} = \frac{1}{4}$$

$\Rightarrow C_2 = 4C_1$ **Ans.[*b*]**

Example 5: If 50μF be the capacity of a capacitor in air and 110 μF in oil then the dielectric constant of oil will be:

(*a*) 0.45 (*b*) 0.55

(*c*) 1.10 (*d*) 2.20

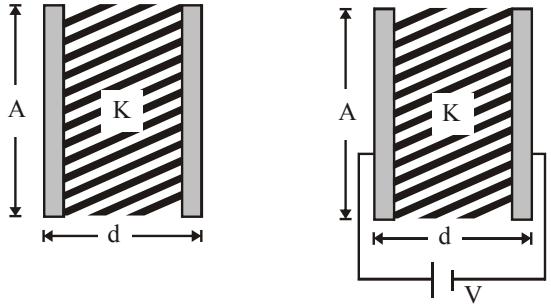
Sol. $C \propto K \Rightarrow \frac{C_1}{C_2} = \frac{K_1}{K_2}$

$$\Rightarrow \frac{50\times10^{-6}}{110\times10^{-6}} = \frac{1}{K_2} \text{ [for air } K_1 = 1]$$

$\Rightarrow K_2 = 2.20$ **Ans. [*d*]**

VARIATION OF DIFFERENT VARIABLE (Q, C, V, E AND U) OF PARALLEL PLATE CAPACITOR WHEN DIELECTRIC IS INTRODUCED

Quantity Battery is RemovedBattery Remains connected



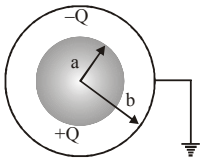
Quantity		
Capacity	C' = KC	C' = KC
Charge	Q' = Q	Q'= KQ
Potential	V' = V/K	V' = V
Electric field	E' = E/K	E' = E
Energy	U' = U/K	U'' = KU

Spherical capacitor :

It consists of two concentric conducting spheres of radii a and b (a < b). Inner sphere is given charge + Q, while outer sphere is earthed

(i) Potential difference : Between the spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

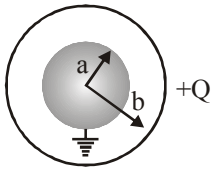


(ii) Capacitance : $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

(iii) If outer sphere is given a charge + Q while inner sphere is earthed

Induced charge on the inner sphere

$Q' = -\frac{a}{b}.Q$ and capacitance of



the system $C' = 4\pi\varepsilon_0.\frac{b^2}{b-a}$

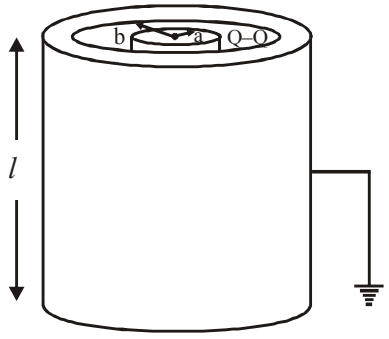
Capacitance is equivalent to the sum of capacitance of spherical capacitor and spherical conductor i.e.

$$4\pi\varepsilon_0.\frac{b^2}{b-a} = 4\pi\varepsilon_0\frac{ab}{b-a} + 4\pi\varepsilon_0 b$$

CYLINDRICAL CAPACITOR

It consists of two co-axial cylinders of radii a and b (a < b), inner cylinder is given charge + Q while outer cylinder is earthed. Common length of the cylinders is *l* then

$$C = \frac{2\pi\varepsilon_0\ell}{\log_e\left(\frac{b}{a}\right)}$$



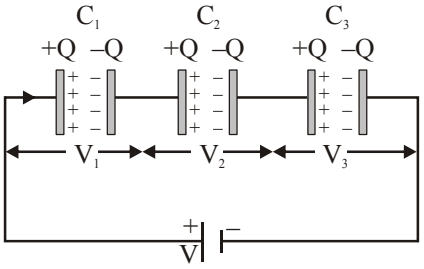
Grouping of capacitors

SERIES GROUPING

(i) Charge on each capacitor remains same and equals to the main charge supplied by the battery but potential difference distributes i.e. $V = V_1 + V_2 + V_3$

(ii) Equivalent capacitance

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



(iii) In series combination potential difference and energy distributes in the reverse ratio of capacitances i.e.

$$V \propto \frac{1}{C} \text{ and } U \propto \frac{1}{C}$$

(iv) If two capacitors having capacitances C_1 and C_2 are connected in series then

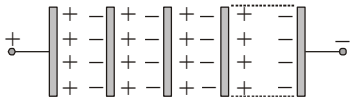
$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{\text{Multiplication}}{\text{Addition}}$$

$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right).V \text{ and } V_2 = \left(\frac{C_1}{C_1 + C_2}\right).V$$

(v) If n identical capacitors each having capacitances C are connected in series with supply voltage V then

Equivalent capacitance $C_{eq} = \frac{C}{n}$ and potential difference across each capacitor $V' = \frac{V}{n}$

(vi) If n identical plates are arranged as shown below, they constitute (n – 1) capacitors in series. If each capacitor has capacitance $\frac{\varepsilon_0 A}{d}$ then $C_{eq} = \frac{\varepsilon_0 A}{(n-1)d}$



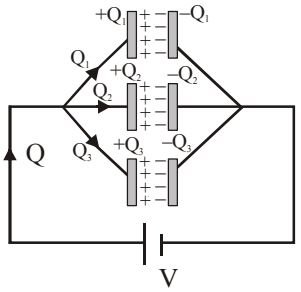
In this situation except two extreme plates each plate is common to adjacent capacitor.

(vii) Here, effective capacitance C_{eq} is even less than the least of the individual capacitance.

PARALLEL GROUPING

(i) Potential difference across each capacitor remains same and equal to the applied potential difference but charge distributes

i.e. $Q = Q_1 + Q_2 + Q_3$



(ii) $C_{eq} = C_1 + C_2 + C_3$

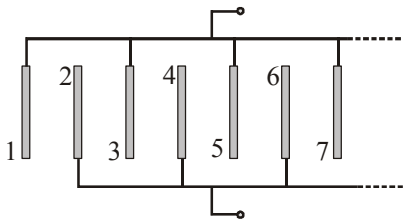
(iii) In parallel combination charge and energy distributes in the ratio of capacitance i.e. $Q \propto C$ and $U \propto C$

(iv) If two capacitors having capacitance C_1 and C_2 respectively are connected in parallel then $C_{eq} = C_1 + C_2$

$$Q_1 = \left(\frac{C_1}{C_1 + C_2}\right).Q \text{ and } Q_2 = \left(\frac{C_2}{C_1 + C_2}\right).Q$$

(v) If n identical capacitors are connected in parallel, then Equivalent capacitance $C_{eq} = nC$ and Charge on each capacitor $Q' = \frac{Q}{n}$

If n identical plates are arranged such that even numbered plates are connected together and odd numbered plates are connected together, then (n – 1) capacitors will be formed and they will be in parallel grouping.



Equivalent capacitance C' = (n – 1) C

where C = capacitance of a capacitor = $\frac{\varepsilon_0 A}{d}$

(vi) This type of combination is used when high capacity is required.

(vii) If C_p is the effective capacity when n identical capacitors are connected in parallel and C_s is their effective capacity when connected in series, then $\frac{C_p}{C_s} = n^2$

TRAIN YOUR BRAIN

Example 6: The capacitor of capacitance 4 μF and 6 μF are connected in series. A potential difference of 500 volts is applied to the outer plates of the two capacitor system. Then the charge on each capacitor is numerically:

(*a*) 6000 C (*b*) 1200 C

(*c*) 1200 μC (*d*) 6000 μC

Sol. $C_R = \frac{C_1 C_2}{C_1 + C_2} = 2.4\mu F$

Charge flown through the circuit

$= 2.4 \times 500 \times 10^{-6} C$

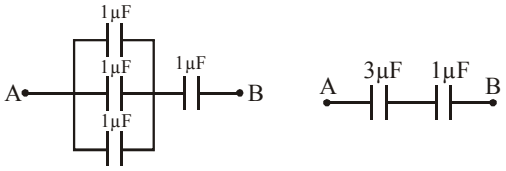
$= 1200\ \mu C$ **Ans. [*c*]**

Example 7: Three capacitors each of capacitance 1μF are connected in parallel. To this combination, a fourth capacitor of capacitance 1μF is connected in series. The resultant capacitance of the system is :

(*a*) 4μF (*b*) 2 μF

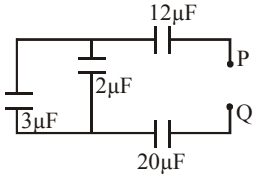
(*c*) 4/3 μF (*d*) 3/4 μF

Sol.



$\frac{1}{C} = \frac{1}{3} + \frac{1}{1} \Rightarrow C = \frac{3}{4}\mu F$ **Ans. [*d*]**

Example 8: In the circuit diagram shown in the adjoining figure, the resultant capacitance between P and Q is:

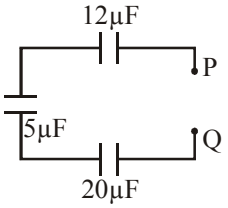


(*a*) 47 μF (*b*) 3 μF

(*c*) 60 μF (*d*) 10 μF

Sol. The given circuit can be drawn as

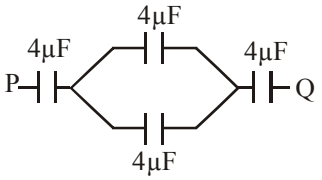
Where C = (3 + 2) μF = 5μF



$$\frac{1}{C_{PQ}} = \frac{1}{5} + \frac{1}{20} + \frac{1}{12} = \frac{20}{60} = \frac{1}{3}$$

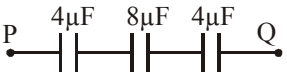
$\Rightarrow C_{PQ} = 3\ \mu F$ **Ans. [*b*]**

Example 9: Four capacitor each capacity 4μF are connected as shown in figure $V_P - V_Q = 15$ volts. The energy stored in the system is :



(*a*) 2400 erg (*b*) 1800 erg

(*c*) 3600 erg (*d*) 5400 erg

Sol. 

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{4}$$

$$\frac{1}{C_{eq}} = \frac{2+1+2}{8} \qquad \Rightarrow C_{eq} = \frac{8}{5}\mu F$$

Energy stored = $\frac{1}{2}\ C_{eq}V^2$

$= \frac{1}{2} \times \frac{8}{5} \times 10^{-6} \times 225$

$= 180 \times 10^{-6}$ joule

$= 180 \times 10^{-6} \times 10^{7}$ erg

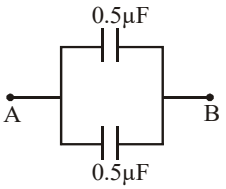
= 1800 erg **Ans. [*b*]**

Example 10: Two capacitors each of 0.5 μF capacitance are connected in parallel and are then charged by 200 volts. D.C. supply. The total energy stored in the system (in joules) is :

(*a*) 0.01 (*b*) 0.02

(*c*) 0.04 (*d*) 0.06

Sol. $C_{eq} = 1\mu F$, $E = \frac{1}{2}\ C_{eq}V^2$



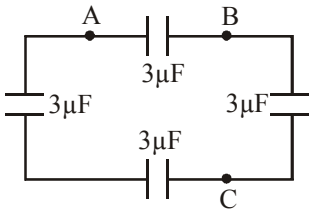
$C_{eq} = C_1 + C_2$

$C_{eq} = 1\mu F$

$\Rightarrow E = \frac{1}{2} \times 1 \times 10^{-6} \times 200 \times 200$

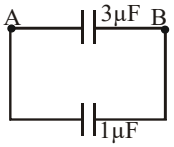
$= 2 \times 10^{-2} = 0.02$ J **Ans. [*b*]**

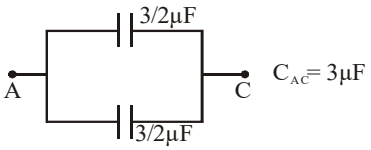
Example 11: Four capacitors of each capacity 3μF are connected as shown in the adjoining figure. The ratio of equivalent capacitance between A and B and between A and C will be :



(*a*) 4 : 3 (*b*) 3 : 4

(*c*) 2 : 3 (*d*) 3 : 2

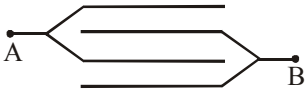
Sol. C_{AB} =4μF 



C_{AC}= 3μF

$\therefore C_{AB} : C_{AC} = 4 : 3$ **Ans. [*a*]**

Example 12: Four plates of the same area of cross-section are joined as shown in the figure. The distance between each plates is d. The equivalent capacity across AB will be :



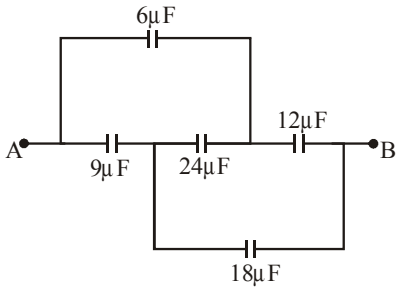
(*a*) $\frac{2\varepsilon_0 A}{d}$ (*b*) $\frac{3\varepsilon_0 A}{d}$

(*c*) $\frac{3\varepsilon_0 A}{2d}$ (*d*) $\frac{\varepsilon_0 A}{d}$

Sol. The arrangement shown in the figure is equivalent to three capacitance in parallel hence resultant capacitance

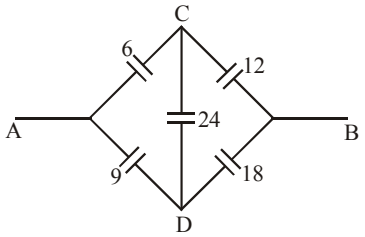
$= \frac{3\varepsilon_0 A}{d}$ **Ans. [*b*]**

Example 13: In the connection shown in the adjoining figure. The equivalent capacity between A and B will be :



(*a*) 8 μF (*b*) 12 μF
(*c*) 20 μF (*d*) 10 μF

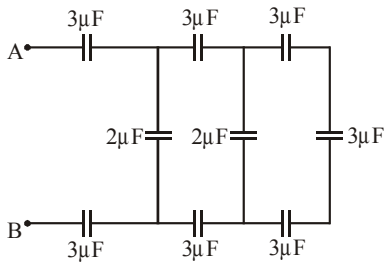
Sol. Given circuit can be drawn as
Here C, D are equipotential points
∴ 24 μF capacitor holds no charge as



$\Delta V = 0$

$\therefore C_{eq} = \frac{12\times6}{12+6} + \frac{9\times18}{9+18} = 10\ \mu F$ **Ans.[*d*]**

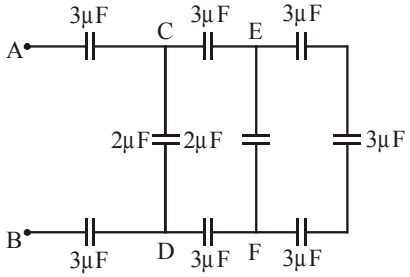
Example 14: The resultant capacitance between A and B in the following figure is equal to :



(*a*) 1 μF (*b*) 3 μF
(*c*) 2 μF (*d*) 1.5 μF

Sol. Total series capacitance across Ef = 3 μF
Capacitance across EF = 2 + 1 = 3 μF
This capacitor is in series with 3 μF capacitance at CD.
Hence total series capacitance

$\frac{1}{C'} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1;$



∴ C' = 1 μF
This capacitance is in parallel with capacitance of 2 μF across CD.
Hence $C_{T(AB)} = 2 + 1 = 3\ \mu F$
This capacitance is in series with 3 μF capacitance across AB. Hence total capacitance across AB.

$\frac{1}{C''} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1;$

∴ C'' = 1 μF **Ans. [*a*]**

Example 15: Two capacitor of capacities 2C and C joined in parallel and charged upto potential V. The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K. The p.d. across the capacitor will now be :

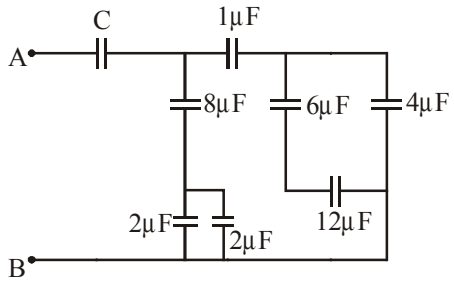
(*a*) $\frac{3V}{K+2}$ (*b*) $\frac{3V}{K}$

(*c*) $\frac{V}{K+2}$ (*d*) $\frac{V}{K}$

Sol. $q_1 = 2\ CV$, $q_2 = CV$
Now condenser of capacity C is filled with dielectric K, therefore $C_2 = KC$
As charge is conserved,
∴ $q_1 + q_2 = (C + 2C)V$

$\Rightarrow V' = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}$ **Ans. [*a*]**

Example 16: In the following circuit, the resultant capacitance between A and B is 1 μF. Then value of C is :



(*a*) $\frac{32}{11}\mu F$ (*b*) $\frac{11}{32}\mu F$

(*c*) $\frac{23}{32}\mu F$ (*d*) $\frac{32}{23}\ \mu F$

Sol. 12μF and 6μF are in series and again are in parallel with 4μF.
Therefore resultant of these three will be

$= \frac{12\times6}{12+6} + 4 = 8\mu F,$

This equivalent system is series with 1μF , its equivalent capacitance

$= \frac{8\times1}{8+1} = \frac{8}{9}$ μF ..... (*a*)

Equivalent of 8μF , 2μF and

$2\mu F = \frac{4\times8}{4+8} = \frac{8}{3}\mu F$ ..... (*b*)

(*a*) and (*b*) are in parallel and are in series with C.

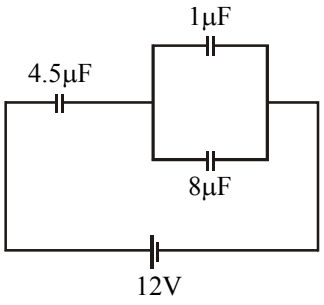
$\therefore \frac{8}{9} + \frac{8}{3} = \frac{32}{9}$ and $C_{eq} = 1$

$= \frac{\frac{32}{9}\times C}{\frac{32}{9}+C}$ $\Rightarrow \frac{32}{9} + C = \frac{32}{9}C$

$\Rightarrow \frac{32}{9}C - C = \frac{32}{9}$

$\Rightarrow C = \frac{32}{9} \times \frac{9}{23} = \frac{32}{23}$ μF **Ans. [*d*]**

Example 17: In the circuit shown in the figure, the potential difference across the 4.5mF capacitor is :



(*a*) $\frac{8}{3}$ volts (*b*) 4 volts

(*c*) 6 volts (*d*) 8 volts

Sol. $\frac{1}{C_{eq}} = \frac{1}{4.5} + \frac{1}{9}$

$\Rightarrow C_{eq} = 3$ μF

The charge through the circuit

$= 3 \times 12 = 36$ μC

$\therefore$ Potential difference across 4.5 μF capacitor

$V = \frac{q}{C} = \frac{36}{4.5} = 8$ volts **Ans. [*d*]**

Example 18: A capacitor 4 mF charged to 50V is connected to another capacitor of 2 mF charged to 100V with plates of like charges connected together. The total energy before and after connection in multiples of (10^{-2} J) is :

(*a*) 1.5 and 1.33 (*b*) 1.33 and 1.5

(*c*) 3.0 and 2.67 (*d*) 2.67 and 3.0

Sol. The total energy before connection

$= \frac{1}{2} \times 4 \times 10^{-6} \times (50)^2 + \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2$

$= 1.5 \times 10^{-2}$ J

When connected in parallel

$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \Rightarrow V = \frac{4(50) + 2(100)}{6}$

$\Rightarrow V = \frac{200}{3}$ Volt(common)

Total energy after connection

$U = \frac{1}{2}\times6\times10^{-6}\times\left(\frac{200}{3}\right)^2$

$U = 1.33 \times 10^{-2}$ Joule **Ans. [*a*]**

Example 19: Two capacitors of 3pF and 6pF are connected in series and a potential difference of 5000V is applied across the combination. They are then disconnected and reconnected in parallel. The potential between the plates is :

(*a*) 2250 V (*b*) 1111 V

(*c*) 2.25×10^6 V (*d*) 1.1×10^6 V

Sol. $\frac{1}{C} = \frac{1}{3} + \frac{1}{6} \Rightarrow C = 2$pF

Total charge $= 2 \times 10^{-12} \times 5000 = 10^{-8}$ C

The new potential when the capacitors are connected in parallel is

$V = \frac{10^{-8}}{(3+6)\times10^{-12}} = 1111$ V **Ans. [*b*]**

Example 20: Three capacitor are connected in series across a 75 volt supply. The voltages across them are 20, 25 and 30 volts respectively and the charge on each is 3×10^{-3} C. Find the capacity of each capacitor and also total capacity of the combination.

Sol. Here

$Q = 3 \times 10^{-3}$ C

$V = 75$ V, $V_1 = 20$ V, $V_2 = 25$ V, $V_3 = 30$ V

Let C_1, C_2 and C_3 be the capacities of the capacitor respectively and C be the capacity of the combination.

since $C = \frac{Q}{V}$ (In series combination, charge is same)

$\therefore C_1 = \frac{3\times10^{-3}}{20} = 1.5 \times 10^{-4}$ F $= 15 \times 10^{-5}$ F

$C_2 = \frac{3\times10^{-3}}{25} = 1.2 \times 10^{-4}$ F $= 12 \times 10^{-5}$ F

$C_3 = \frac{3\times10^{-3}}{30} = 1.0 \times 10^{-4}$ F $= 10 \times 10^{-5}$ F

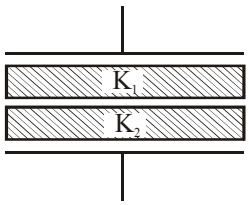
Total capacity of the combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

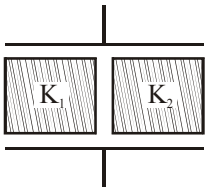
On solving C = 40 μF

Example 21:

(*a*) Two dielectric slabs of dielectric constant K_1 and K_2 have been filled in between the plates of a capacitor as shown in fig. What will be the capacitance in each case.



(*b*) A capacitor if filled with two dielectric of same dimensions but of dielectric constant 2 and 3 respectively.



Sol. (*a*) The arrangement shown in fig is equivalent to two capacitors joined in series. Let their capacitance be C_1 and C_2 respectively. The

$$C_1 = \varepsilon_0 \frac{K_1 A}{d/2} \text{ and } C_2 = \varepsilon_0 \frac{K_2 A}{d/2}$$

$$\text{Now } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2\varepsilon_0 K_1 A} + \frac{d}{2\varepsilon_0 K_2 A}$$

$$= \frac{d}{2\varepsilon_0 A}\left[\frac{K_2 + K_1}{K_1 K_2}\right]$$

$$C = \frac{2\varepsilon_0 A}{d}\left[\frac{K_2 K_1}{K_2 + K_1}\right] \quad ...(b)$$

(*b*) Let A be the area of each plate of the capacitor and d be the distance between the two plates. If the capacitance be C_1 and C_2 respectively, then

$$C_1 = \varepsilon_0 \frac{K_1 A/2}{d} \text{ and } C_2 = \varepsilon_0 \frac{K_2 A/2}{d}$$

Let C be the equivalent capacitance, then C

$$= C_1 + C_2 = \varepsilon_0 \frac{K_1 A}{2d} + \varepsilon_0 \frac{K_2 A}{2d}$$

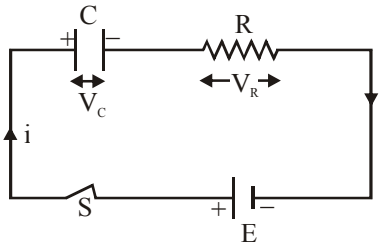
$$C_{equ} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

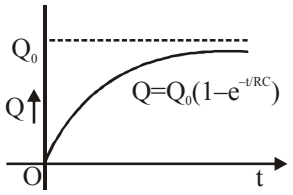
{$\therefore$ Two condensers are in parallel}

CHARGING AND DISCHARGING OF CAPACITOR IN SERIES RC CIRCUIT

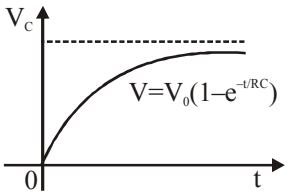
(i) **Charging :** In transient state of charging charge on the capacitor at any instant $Q = CE\left(1 - e^{-t/RC}\right)$ and potential difference across the capacitor at any instant $V_C = E\left(1 - e^{-t/RC}\right)$

(Here Q and V are the instantaneous values of charge and potential difference

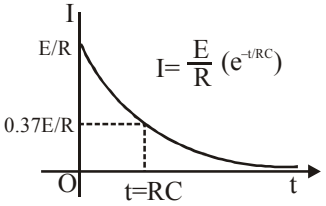




Charge on the capacitor increases with time during charging

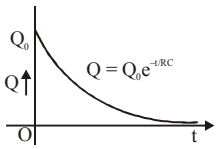


Charging

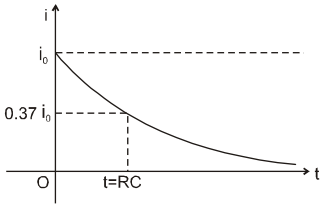


Charging

(ii) **Discharging :** After the completion of charging, if battery is removed capacitor starts discharging. In transient state charge on the capacitor at any instant $Q = CEe^{-t/RC}$ and potential difference cross the capacitor at any instant $V = Ee^{-t/RC}$.



Charge on the capacitor decreases with time during discharging



(iii) **Time constant (τ) :** The quantity RC is called the time constant as it has the dimension of time During charging if $t = \tau = RC$, $Q = Q_0(1 - e^{-1}) = 0.63\ Q_0$ = 63% of Q_0 $\left(\frac{1}{e} = 0.37\right)$ or during discharging it is defined as the time during which charge on a capacitor falls to 0.37 times (37%) of the initial charge on the capacitor.

TRAIN YOUR BRAIN

Example 22: A capacitor is discharged through a 10 MW resistor and it is found that the time constant is 200s. Then, find the value of the capacitance.

Sol. $\tau = RC = 200$, thus

$C = 200 / 10 \times 10^6 = 20\ \mu F$

Example 23: A 2500 mF capacitor is charged through a 1 KW resistor by a 12V d.c. source. What is the voltage across the capacitor after 5 sec ?

Sol. The time constant of the circuit is

$\tau = RC = 10^3 \times 2500 \times 10^{-6} = 2.5$ sec

For charging

$V = V_0 (1 - e^{-t/RC})$

put t = 5sec, and $\tau = 2.5$ sec, then

$V = 12(1 - e^{-2}) = 12(1 - 0.135)$

$= 12 \times 0.865 = 10.38$ volt.

Topicwise Questions

CAPACITANCE

Finding Capacitance, Parallel, Spherical, Cylindrical

1. Eight drops of mercury of equal radii possessing equal charges combine to form a big drop. Then the capacitance of bigger drop compared to each individual small drop is

(*a*) 8 times (*b*) 4 times

(*c*) 2 times (*d*) 32 times

2. The capacity of parallel plate condenser depends on

(*a*) The type of metal used

(*b*) The thickness of plates

(*c*) The potential applied across the plates

(*d*) The separation between the plates

3. Eight small drops, each of radius r and having same charge q are combined to form a big drop. The ratio between the potentials of the bigger drop and the smaller drop is

(*a*) 8 : 1 (*b*) 4 : 1

(*c*) 2 : 1 (*d*) 1 : 8

4. N identical spherical drops charged to the same potential V are combined to form a big drop. The potential of the new drop will be

(*a*) V (*b*) V/N

(*c*) $V \times N$ (*d*) $V \times N^{2/3}$

Equivalent Capacitance (Series & Parallel Combination)

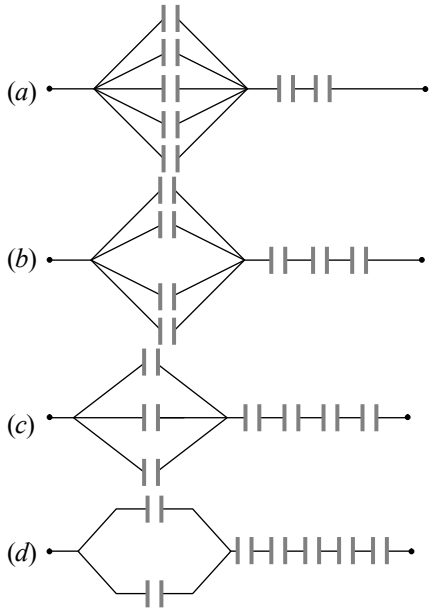
5. The condensers of capacity C_1 and C_2 are connected in parallel, then the equivalent capacitance is

(*a*) $C_1 + C_2$ (*b*) $\frac{C_1 C_2}{C_1 + C_2}$

(*c*) $\frac{C_1}{C_2}$ (*d*) $\frac{C_2}{C_1}$

6. Seven capacitors each of capacity $2\mu F$ are to be so connected to have a total capacity $\frac{10}{11}\mu F$. Which will be the necessary figure as shown

(*a*)

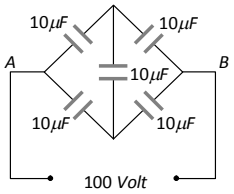


(*b*)

(*c*)

(*d*)

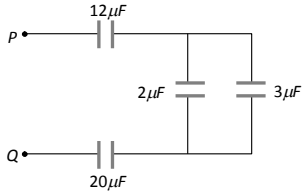
7. Five capacitors of 10 μF capacity each are connected to a d.c. potential of 100 volts as shown in the adjoining figure. The equivalent capacitance between the points A and B will be equal to



(*a*) 40 μF (*b*) 20 μF

(*c*) 30 μF (*d*) 10 μF

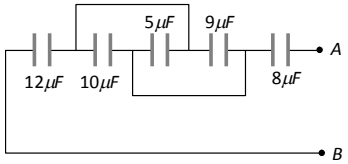
8. In the circuit diagram shown in the adjoining figure, the resultant capacitance between P and Q is



(*a*) 47 μF (*b*) 3 μF

(*c*) 60 μF (*d*) 10 μF

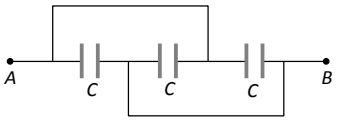
9. The capacities and connection of five capacitors are shown in the adjoining figure. The potential difference between the points A and B is 60 volts. Then the equivalent capacity between A and B and the charge on 5 μF capacitance will be respectively



(*a*) 44 μF; 300 μC (*b*) 16 μF; 150 μC

(*c*) 15 μF; 200 μC (*d*) 4 μF; 50 μC

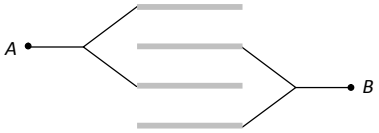
10. Three equal capacitors, each with capacitance C are connected as shown in figure. Then the equivalent capacitance between A and B is



(*a*) C (*b*) 3C

(*c*) $\frac{C}{3}$ (*d*) $\frac{3C}{2}$

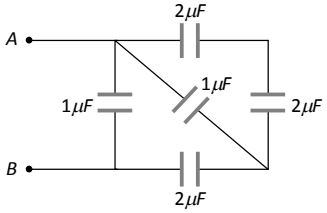
11. Four plates of the same area of cross-section are joined as shown in the figure. The distance between each plate is d. The equivalent capacity across A and B will be



(*a*) $\frac{2\varepsilon_0 A}{d}$ (*b*) $\frac{3\varepsilon_0 A}{d}$

(*c*) $\frac{3\varepsilon_0 A}{2d}$ (*d*) $\frac{\varepsilon_0 A}{d}$

12. The total capacity of the system of capacitors shown in the adjoining figure between the points A and B is



(*a*) 1 μF (*b*) 2 μF

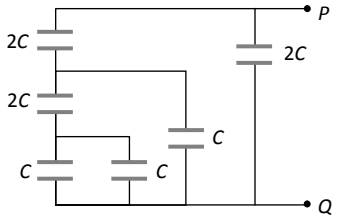
(*c*) 3 μF (*d*) 4 μF

13. Three capacitors each of 6μF are available. The minimum and maximum capacitances which may be obtained are

(*a*) 6 μF, 18 μF (*b*) 3 μF, 12 μF

(*c*) 2 μF, 12 μF (*d*) 2 μF, 18 μF

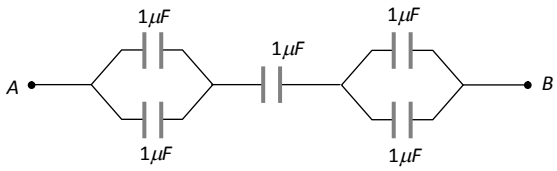
14. The resultant capacitance of given circuit is



(*a*) 3C (*b*) 2C

(*c*) C (*d*) $\frac{C}{3}$

15. The equivalent capacitance between A and B is



(*a*) 2 μF (*b*) 3 μF

(*c*) 5 μF (*d*) 0.5 μF

Circuit Analysis

16. A capacitor having capacitance C is charged to a voltage V. It is then removed and connected in parallel with another identical capacitor which is uncharged. The new charge on each capacitor is now

(*a*) CV (*b*) CV/2

(*c*) 2CV (*d*) CV/4

17. Two capacitors of 3pF and 6pF are connected in series and a potential difference of 5000 V is applied across the combination. They are then disconnected and reconnected in parallel. The potential between the plates is

(*a*) 2250 V (*b*) 2222 V

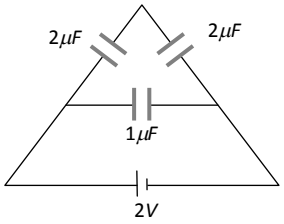
(*c*) 2.25×10^6 V (*d*) 1.1×10^6 V

18. Three capacitors of capacitance 3 μF, 10 μF and 15 μF are connected in series to a voltage source of 100V. The charge on 15 μF is

(*a*) 50 μC (*b*) 100 μC

(*c*) 200 μC (*d*) 280 μC

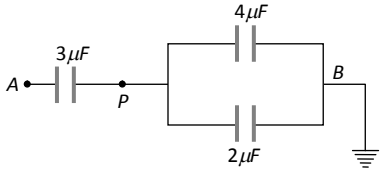
19. The charge on any one of the 2 μF capacitors and 1 μF capacitor will be given respectively (in μC) as



(*a*) 1, 2 (*b*) 2, 1

(*c*) 1, 1 (*d*) 2, 2

20. In the figure a potential of + 1200 V is given to point A and point B is earthed, what is the potential at the point P



(*a*) 100 V (*b*) 200 V

(*c*) 400 V (*d*) 600 V

ENERGY AND HEAT DISSIPATED

21. A condenser has a capacity 2 μF and is charged to a voltage of 50 V. The energy stored is

(*a*) 25×10^5 Joule (*b*) $\frac{1}{2} \times 2 \times 50^2$ J

(*c*) 25×10 erg (*d*) 2500×10^{-6}J

22. A capacitor of capacitance 6μF is charged upto 100 volt. The energy stored in the capacitor is

(*a*) $\frac{1}{2} \times 6 \times 100^2$ Joule (*b*) 0.06 Joule

(*c*) 3×10 Joule (*d*) 0.03 Joule

23. A 40 μF capacitor in a defibrillator is charged to 3000 V. The energy stored in the capacitor is sent through the patient during a pulse of duration 2ms. The power delivered to the patient is

(*a*) 45 kW (*b*) 90 kW

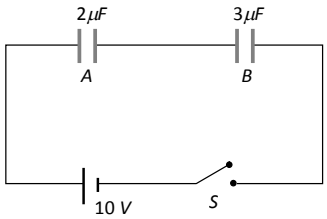
(*c*) 180 kW (*d*) 360 kW

24. The energy stored in a condenser is in the form of

(*a*) Kinetic energy (*b*) Potential energy

(*c*) energy (*d*) Magnetic energy

25. Two capacitors A and B are connected in series with a battery as shown in the figure. When the switch S is closed and the two capacitors get charged fully, then



(*a*) The potential difference across the plates of A is 4V and across the plates of B is 6V

(*b*) The potential difference across the plates of A is 6V and across the plates of B is 4V

(*c*) The ratio of electrical energies stored in A and B is 2 : 3

(*d*) The ratio of charges on A and B is 3 : 2

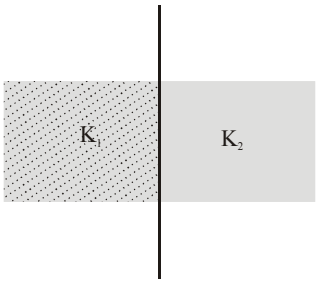
Dielectrics

26. An air filled parallel plate capacitor has capacity C. If distance between plates is doubled and it is immersed in a liquid then capacity becomes twice. Dielectric constant of the liquid is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

27. A parallel plate capacitor with air as medium between the plates has a capacitance of 10 μF. The area of capacitor is divided into two equal halves and filled with two media as shown in the figure having dielectric constant $k_1 = 2$ and $k_2 = 4$. The capacitance of the system will now be

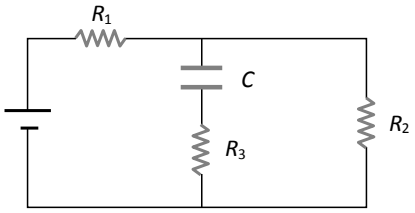


(*a*) 10 μF (*b*) 20 μF

(*c*) 30 μF (*d*) 40 μF

RC Circuit

28. In the circuit here, the steady state voltage across capacitor C is a fraction of the battery e.m.f. The fraction is decided by



(*a*) R_1 only (*b*) R_1 and R_2 only

(*c*) R_1 and R_3 only (*d*) R_1,R_2 and R_3

Learning Plus

1. The radii of two metallic spheres are 5 cm and 10 cm and both carry equal charge of 75μC. If the two spheres are shorted then charge transferred is.

(*a*) 25 μC from smaller to bigger

(*b*) 25 μC from bigger to smaller

(*c*) 50 μC from smaller to bigger

(*d*) 50 μC from bigger to smaller

2. Two isolated charged metallic spheres of radii R_1 and R_2 having charges Q_1 and Q_2 respectively are connected to each other, then there is:

(*a*) No change in the electrical energy of the system

(*b*) An increase in the electrical energy of the system

(*c*) Always a decrease in the electrical energy of the system

(*d*) A decrease in electrical energy of the system until $Q_1 R_2 = Q_2 R_1$

3. No current flows between two charged bodies connected together when they have the same

(*a*) capacitance or Q/V ratio

(*b*) charge

(*c*) resistance

(*d*) potential or Q/C ratio

4. The capacitance of a parallel plate condenser does not depend upon

(*a*) the distance between the plates

(*b*) area of the plates

(*c*) medium between the plates

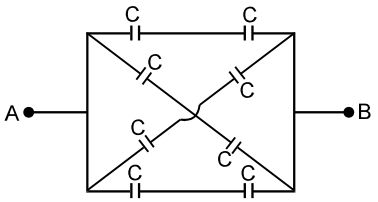
(*d*) metal of the plates

5. Two spherical conductors of capacitance 3.0 μF and 5.0 μF are charged to potentials of 300Volt and 500Volt. The two are connected resulting in redistribution of charges. Then the final potential is -

(*a*) 300 volt (*b*) 500 volt

(*c*) 425 volt (*d*) 400 volt

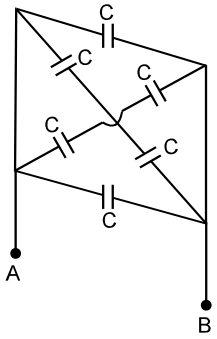
6. In the adjoining circuit, the capacity between the points A and B will be -



(*a*) C (*b*) 2C

(*c*) 3C (*d*) 4C

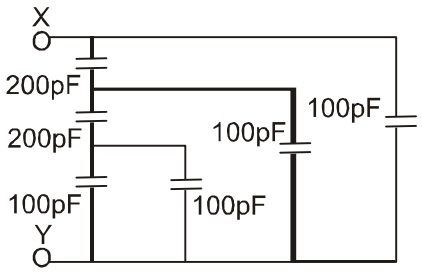
7. The resultant capacity between the points A and B in the adjoining circuit will be -



(*a*) C (*b*) 2C

(*c*) 3C (*d*) 4C

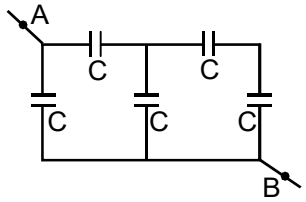
8. The equivalent capacitance between the terminals X and Y in the figure shown will be–



(*a*) 100 pF (*b*) 200 pF

(*c*) 300 pF (*d*) 400 pF

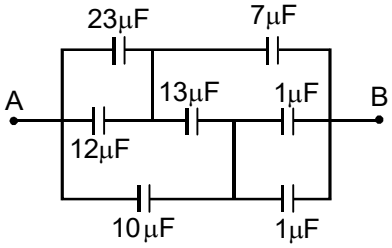
9. What is the equivalent capacitance of the system of capacitors between A & B



(*a*) $\frac{7}{6}$ C (*b*) 1.6 C

(*c*) C (*d*) None

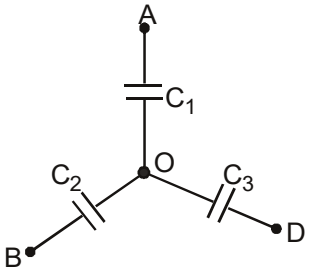
10. Find the equivalent capacitance across A & B



(*a*) $\frac{28}{3}$ μ F (*b*) $\frac{15}{2}$ μ F

(*c*) 15 μ F (*d*) None

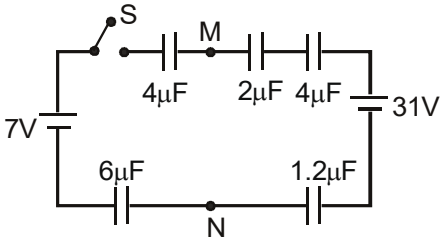
11. Three uncharged capacitors of capacitane C_1 = 1μF, C_2 = 2μF and C_3 = 3μF are connected as shown in figure to one another and to points A, B and D potential ϕ_A = 10V, ϕ_B = 25V and ϕ_D = 20 V, Determine the potential (ϕ_0) at point O.



(*a*) 20 V (*b*) 30 V

(*c*) 40 V (*d*) 10 V

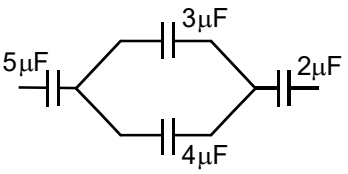
12. Five capacitors are connected as shown in the figure. Initially S is opened and all capacitors are uncharged. When S is closed, steady state is obtained. Then find out potential difference between the points M and N.



(*a*) 14 (*b*) 12

(*c*) 10 (*d*) 15

13. If charge on left plate of the 5 μ F capacitor in the circuit segment shown in the figure is-20 μ C, the charge on the right plate of 3 μ F capacitor is :-



(*a*) +8.57 μ C (*b*) –8.57 μ C

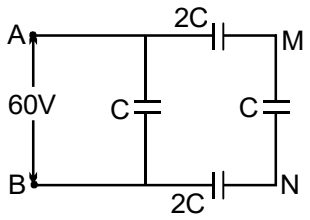
(*c*) +11.42 μ C (*d*) –11.42 μ C

14. A capacitor $C_1 = 4\mu F$ is connected in series with another capacitor $C_2 = 1\mu F$. The combination is connected across a d.c. source of voltage 200V. The ratio of potential across C_1 and C_2 is -

(*a*) 1 : 4 (*b*) 4 : 1

(*c*) 1 : 2 (*d*) 2 : 1

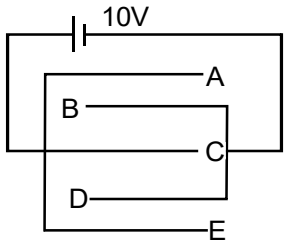
15. In the circuit shown, a potential difference of 60V is applied across AB. The potential difference between the points M and N is



(*a*) 10 V (*b*) 15 V

(*c*) 20 V (*d*) 30 V

16. Five identical capacitor plates are arranged such that they make capacitors each of 2 μ F. The plates are connected to a source of emf 10 V. The charge on plate C is



(*a*) +20 μ C (*b*) +40 μ C

(*c*) +60 μ C (*d*) +80 μ C

17. A capacitor of capacitance 1 μ F withstands the maximum voltage 6 kV while a capacitor of 2 μ F withstands the maximum voltage 4 kV. What maximum voltage will the system of these two capacitor withstands if they are connected in series?

(*a*) 10 kV (*b*) 12 kV

(*c*) 8 kV (*d*) 9 kV

18. Three capacitors 2 μ F, 3 μ F and 5 μ F can withstand voltages to 3 V, 2V and 1V respectively. Their series combination can withstand a maximum voltage equal to

(*a*) 5 Volts (*b*) (31/6) Volts

(*c*) (26/5) Volts (*d*) None

19. A capacitor is connected to a battery. The force of attraction between the plates when the separation between them is halved

(*a*) remains the same (*b*) becomes eight times

(*c*) becomes four times (*d*) becomes two times

20. A_1 is a spherical conductor of radius (r) placed concentrically inside a thin spherical hollow conductor A_2 of radius (R). A_1 is earthed and A_2 is given a charge +Q then the charge on A_1 is-

(*a*) –Q (*b*) Qr/R

(*c*) –rQ/R (*d*) –Q (R–r)/R

21. The energy of a charged capacitor resides in

(*a*) the electric field only

(*b*) the magnetic field only

(*c*) both the electric and magnetic field

(*d*) neither in electric nor magnetic field

22. The work done against electric forces in increasing the potential difference of a condenser from 20V to 40V is W. The work done in increasing its potential difference from 40V to 50V will be (consider capacitance of capacitor remain constant)

(*a*) 4W (*b*) $\frac{3W}{4}$

(*c*) 2W (*d*) $\frac{W}{2}$

23. A conductor of capacitance 0.5μF has been charged to 100volts. It is now connected to uncharged conductor of capacitance 0.2μF. The loss in potential energy is nearly-

(*a*) 7×10^{-4} J (*b*) 3.5×10^{-4} J

(*c*) 14×10^{-4} J (*d*) 7×10^{-3} J

24. A parallel plate condenser of capacity C is connected to a battery and is charged to potential V. Another condenser of capacity 2C is connected to another battery and is charged to potential 2V. The charging batteries are removed and now the condensers are connected in such a way that the positive plate of one is connected to negative plate of another. The final energy of this system is–

(*a*) zero (*b*) $\frac{25CV^2}{6}$

(*c*) $\frac{3CV^2}{2}$ (*d*) $\frac{9CV^2}{2}$

25. A 2 μ F capacitor is charged to a potential = 10 V. Another 4 μ F capacitor is charged to a potential = 20V. The two capacitors are then connected in a single loop, with the positive plate of one connected with negative plate of the other. What heat is evolved in the circuit?

(*a*) 300 μ J (*b*) 600 μ J

(*c*) 900 μ J (*d*) 450 μ J

26. The distance between the plates of a parallel plate condenser is d. If a copper plate of same area but thickness $\frac{d}{2}$ is placed between the plates then the new capacitance will become-

(*a*) half (*b*) double

(*c*) one fourth (*d*) unchanged

27. A parallel plate condenser is connected to a battery of e.m.f. 4 volt. If a plate of dielectric constant 8 is inserted into it, then the potential difference on the condenser will be-

(*a*) 1/2 V (*b*) 2V

(*c*) 4V (*d*) 32V

28. In the above problem if the battery is disconnected before inserting the dielectric, then potential difference will be-

(*a*) 1/2 V (*b*) 2V

(*c*) 4V (*d*) 32V

29. A parallel plate condenser with plate separation d is charged with the help of a battery so that U_0 energy is stored in the system. A plate of dielectric constant K and thickness d is placed between the plates of condenser while battery remains connected. The new energy of the system will be-

(*a*) KU_0 (*b*) K^2U_0

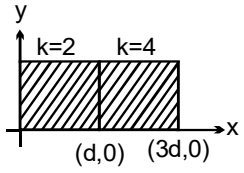
(*c*) $\frac{U_0}{K}$ (*d*) $\frac{U_0}{K^2}$

30. A metallic plate of thickness (t) and face area of one side (*a*) is inserted between the plates of a parallel plate air capacitor with a separation (d) and face are (*a*). Then the equivalent capacitance is :

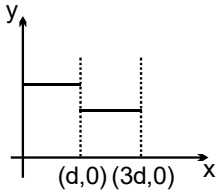
(*a*) $\frac{\epsilon_0 A}{d}$ (*b*) $\frac{\epsilon_0 A}{(d \times t)}$

(*c*) $\frac{\epsilon_0 A}{(d - t)}$ (*d*) $\frac{\epsilon_0 A}{(d + t)}$

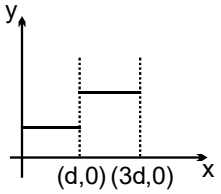
31. A parallel plate capacitor has two layers of dielectric as shown in figure. This capacitor is connected across a battery. The graph which shows the variation of electric field (E) and distance (x) from left plate.



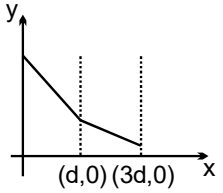
(*a*)



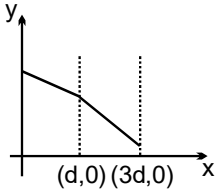
(*b*)



(*c*)



(*d*)



32. A parallel plate isolated condenser consists of two metal plates of area A and separation 'd'. A slab of thickness 't' and dielectric constant K is inserted between the plates with its faces parallel to the plates and having the same surface area as that of the plates. If K = 2, for what value of t/d will the capacitance of the system be 3/2 times that of the condenser with air filling the full space ?

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{3}{2}$ (*d*) 3

33. Hard rubber has a dielectric constant of 2.8 and a dielectric strength of 18×10^8 volts/meter. If it is used as the dielectric material filling the full space in a parallel plate capacitor. What minimum area may the plates of the capacitor have in order that the capacitance be 7.0×10^{-2} μf and that the capacitor be able to withstand a potential difference of 4000 volts.

(*a*) $0.62\,m^{-2}$ (*b*) $0.32\,m^{-2}$

(*c*) $0.42\,m^{-2}$ (*d*) $0.52\,m^{-2}$

34. The area of the plates of a parallel plate capacitor is A and the gap between them is d. The gap is filled with a non homogeneneous dielectric whose dielectric constant varies with the distance 'y' from one plate as: $K = \lambda \sec(\pi y/2d)$, where λ is a dimensionless constant. The capacitance of this capacitor is

(*a*) $\pi\epsilon_0\lambda$ A/2d (*b*) $\pi\epsilon_0\lambda$ A/d

(*c*) $2\pi\epsilon_0\lambda$ A/d (*d*) None

35. A capacitor stores 60 μ C Charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 120 μ C flows through the battery. The dielectric constant of the material inserted is

(*a*) 1 (*b*) 2

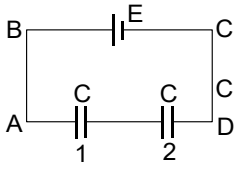
(*c*) 3 (*d*) None

36. Condenser A has a capacity of 15 μ F when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity 1 μ F with air between the plates. Both are charged separately by a battery of 100 V. After charging, both are connected in parallel without the battery and the dielectric material being removed. The common potential now is

(*a*) 400 V (*b*) 800 V

(*c*) 1200 V (*d*) 1600 V

37. In the adjoining figure, capacitor (*a*) and (*b*) have a capacitance 'C' each. When the dielectric of dielectric constant K is inserted between the plates of one of the capacitor, the total charge flowing through battery is



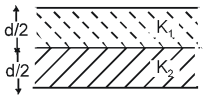
(*a*) $\frac{KCE}{K+1}$ from B to C

(*b*) $\frac{KCE}{K+1}$ from C to B

(*c*) $\frac{(K-1)CE}{2(K+1)}$ from B to C

(*d*) $\frac{(K-1)CE}{2(K+1)}$ from C to B

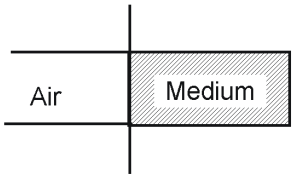
38. Two dielectric slabs of constant K_1 and K_2 have been filled in between the plates of a capacitor as shown below. What will be the capacitance of the capacitor



(*a*) $\frac{2\varepsilon_0 A}{d}(K_1+K_2)$ (*b*) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1+K_2}{K_1\times K_2}\right)$

(*c*) $\frac{d}{2\varepsilon_0 A}$ (*d*) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1\times K_2}{K_1+K_2}\right)$

39. The capacitance of a parallel plate capacitor is 2.5μF. When it is half filled with a dielectric as shown in the figure, Its capacitance becomes 5μF, the dielectric constant of the dielectric is



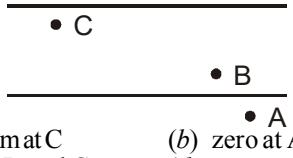
(*a*) 7.5 (*b*) 3.0
(*c*) 0.33 (*d*) 4.0

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/ NUMERICAL

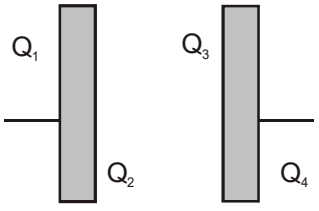
1. When a charged capacitor is connected with an uncharged capacitor, then which of the following is/are correct option/ options.
(*a*) the magnitude of charge on the charged capacitor decreases.
(*b*) a steady state is obtained after which no further flow of charge occurs.
(*c*) the total potential energy stored in the capacitors remains conserved.
(*d*) the charge conservation is always true.

2. For a charged parallel plate capacitor shown in the figure, the force experienced by an alpha particle will be :



(*a*) maximum at C (*b*) zero at A
(*c*) same at B and C (*d*) zero at C

3. In an isolated parallel plate capacitor of capacitance C the four surfaces have charges Q_1, Q_2, Q_3 and Q_4 as shown in the figure. The potential difference between the plates is :



(*a*) $\frac{Q_1+Q_2}{C}$

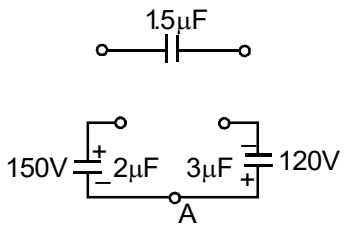
(*b*) $\left|\frac{Q_2}{C}\right|$

(*c*) $\left|\frac{Q_3}{C}\right|$

(*d*) $\frac{1}{C}[(Q_1+Q_2)-(Q_3-Q_4)]$

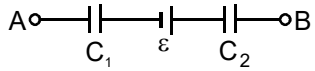
4. Each plate of a parallel plate capacitor has a charge q on it. The capacitor is now connected to a battery. Now,
(*a*) the facing surfaces of the capacitor have equal and opposite charges
(*b*) the two plates of the capacitor have equal and opposite charges
(*c*) the battery supplies equal and opposite charges to the two plates
(*d*) the outer surfaces of the plates have equal charges

5. Two capacitors of 2μF and 3μF are charged to 150 volt and 120 volt respectively. The plates of capacitor are connected as shown in the figure. A discharged capacitor of capacity 1.5 μF falls on the free ends of the wire. Then



(*a*) charge on the 1.5 μF capacitor is 180 μC
(*b*) charge on the 2 μF capacitor is 120 μC
(*c*) charge flows through A from right to left
(*d*) charge flows through A from left to right

6. A circuit shown in the figure consists of a battery of emf 10V and two capacitors C_1 and C_2 of capacitances 1.0μF and 2.0μF respectively. The potential difference $V_A - V_B$ is 5V



(*a*) charge on capacitor C_1 is equal to charge on capacitor C_2

(*b*) Voltage across capacitor C_1 is 5V

(*c*) Voltage across capacitor C_2 is 10 V

(*d*) Energy stored in capacitor C_1 is two times the energy stored in capacitor C_2

7. A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges +Q and –Q respectively. A third plate C, identical to A and B, with charge +Q, is now introduced midway between A and B, parallel to them. Which of the following are correct?

(*a*) The charge on the inner face of B is now $-\frac{3Q}{2}$

(*b*) There is no change in the potential difference between A and B

(*c*) The potential difference between A and C is one-third of the potential difference between B and C

(*d*) The charge on the inner face of A is now Q/2

8. The separation between the plates of a isolated charged parallel plate capacitor is increased. Which of the following quantities will change ?

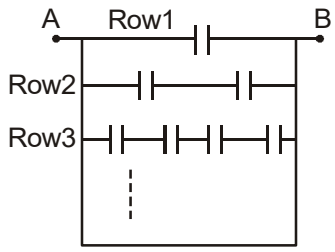
(*a*) charge on the capacitor

(*b*) potential difference across the capacitor

(*c*) energy of the capacitor

(*d*) energy density between the plates

9. Rows of capacitors containing 1, 2, 4, 8, ...∞ capacitors, each of capacitance 2F, are connected in parallel as shown in figure. The potential difference across AB = 10 volt, then :



(*a*) Total capacitance across AB is 4F

(*b*) Charge of each capacitor will be same

(*c*) Charge on the capacitor in the first row is more than on any other capacitor

(*d*) Energy of all the capacitors is 50 J

10. The two plates X and Y of a parallel plate capacitor of capacitance C are given a charge of amount Q each. X is now joined to the positive terminal and Y to the negative terminal of a cell of emf E = Q/C.

(*a*) Charge of amount Q will flow from the negative terminal to the positive terminal of the cell inside it.

(*b*) The total charge on the plate X will be 2Q

(*c*) The total charge on the plate Y will be zero

(*d*) The cell will supply CE^2 amount of energy

11. When two identical capacitors are charged individually to different potentials and connected parallel to each other, after disconnecting them from the source :

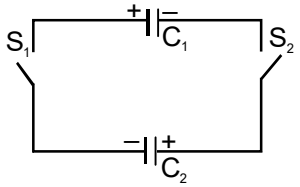
(*a*) net charge on connected plates is less than the sum of initial individual charges.

(*b*) net charge on connected plates equals the sum of initial charges.

(*c*) the net potential difference across them is different from the sum of the individual initial potential differences.

(*d*) the net energy stored in the two capacitors is less than the sum of the initial individual energies.

12. Two capacitors $C_1 = 4\mu F$ and $C_2 = 2\mu F$ are charged to same potential V = 500 Volt, but with opposite polarity as shown in the figure. The switches S_1 and S_2 are closed.



(*a*) The potential difference across the two capacitors are same and is given by 500/3V

(*b*) The potential difference across the two capacitors are same and is given by 1000/3V

(*c*) The ratio of final energy to initial energy of the system is 1/9

(*d*) The ratio of final energy to initial energy of the system is 4/9

13. The terminals of a battery of emf V are connected to the two plates of a parallel plate capacitor. If the space between the plates of the capacitor is filled with an insulator of dielectric constant K, then :

(*a*) the electric field in the space between the plates does not change

(*b*) the capacitance of the capacitor increases

(*c*) the charge stored in the capacitor increases

(*d*) the electrostatic energy stored in the capacitor decreases

14. The plates of a parallel plate capacitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor.

(*a*) the energy stored in the capacitor will become K–times

(*b*) the electric field inside the capacitor will decrease to K-times

(*c*) the force of attraction between the plates will increase to K^2 – times

(*d*) the charge on the capacitor will increase to K-times

15. A parallel plate air capacitor is connected to a battery. The quantities charge, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as :

(*a*) $Q > Q_0$ (*b*) $V > V_0$

(*c*) $E > E_0$ (*d*) $U > U_0$

16. A parallel plate capacitor A is filled with a dielectric whose dielectric constant varies with applied voltage as K = V. An identical capacitor B of capacitance C_0 with air as dielectric is connected to voltage source V_0 = 30 V and then connected to the first capacitor after disconnecting the voltage source. The charge and voltage on capacitor.

(*a*) A are 25 C_0 and 25 V (*b*) A are 25 C_0 and 5 V

(*c*) B are $5c_0$ and 5V (*d*) B are $5C_0$ and 25 V

17. A parallel plate air-core capacitor is connected across a source of constant potential difference. When a dielectric plate is introduced between the two plates then :

(*a*) some charge from the capacitor will flow back into the source

(*b*) some extra charge from the source will flow back into the capacitor

(*c*) the electric field intensity between the two plate does not change

(*d*) the electric field intensity between the two plates will decrease

18. Following operations can be performed on a capacitor X – connect the capacitor to a battery of emf E Y – disconnect the battery Z – reconnect the battery with polarity reversed W – insert a dielectric slab in the capacitor

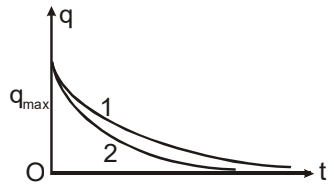
(*a*) In XYZ (perform X, then Y, then Z) the stored electric energy remains unchanged and no thermal energy is developed

(*b*) The charge appearing on the capacitor is greater after the action XWY than after the action XYW.

(*c*) The electric energy stored in the capacitor is greater after the action WXY than after the action XYW.

(*d*) The electric field in the capacitor after the action XW is the same as that after WX

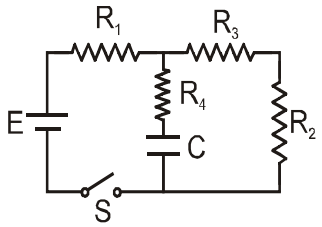
19. The instantaneous charge on capacitor in two discharging RC circuits is plotted with respect to time in figure. Choose the correct statement(s) (where E_1 and E_2 are emfs of two DC sources in two different charging circuits and capacitors are fully charged).



(*a*) $R_1C_1 > R_2C_2$ (*b*) $\frac{R_1}{R_2} < \frac{C_2}{C_1}$

(*c*) $R_1 > R_2$ if $E_1 = E_2$ (*d*) $C_2 > C_1$ if $E_1 = E_2$

20. In the circuit shown in figure the switch S is closed at t = 0.



A long time after closing the switch

(*a*) voltage drop across the capacitor is E

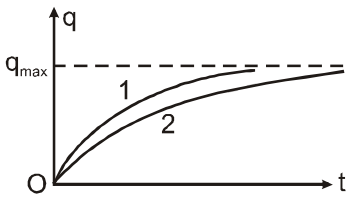
(*b*) current through the battery is $\frac{E}{R_1 + R_2 + R_3}$

(*c*) energy stored in the capacitor is

$$\frac{1}{2}C\left(\frac{(R_2 + R_3)E}{R_1 + R_2 + R_3}\right)^2$$

(*d*) current through the resistance R_4 becomes zero

21. The charge on capacitor in two different RC circuits 1 and 2 are plotted as shown in figure.



Choose the correct statement(s) related to the two circuits.

(*a*) Both the capacitors are charged to the same magnitude of charge

(*b*) The emf's of cells in both the circuits are equal.

(*c*) The emf's of the cells may be different

(*d*) The emf E_1 is more than E_2

22. Capacitor C_1 of the capacitance 1 microfarad and capacitor C_2 of capacitance 2 microfarad are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time t = 0.

(*a*) the current in each of the two discharging circuits is zero at t = 0.

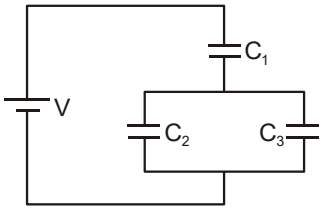
(*b*) the current in the two discharging circuits at t = 0 are equal but non zero.

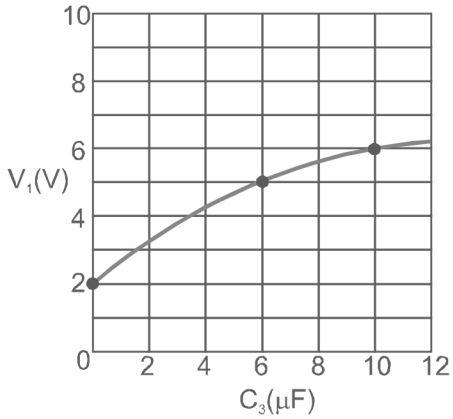
(*c*) the current in the two discharging circuits at t = 0 are unequal

(*d*) capacitor C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge

Comprehension Type Questions – 1 (No. 23 to 25)

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3. Electric potential V_1 approaches on asymptote of 10 V as $C_3 \to \infty$.





23. EMF of the battery is equal to:

(*a*) 10 V (*b*) 12 V

(*c*) 16 V (*d*) 20 V

24. The capacitance of the capacitor C_1 has value:

(*a*) 2 μF (*b*) 6 μF

(*c*) 8 μF (*d*) 12 μF

25. The capacitance of C_2 is equal to:

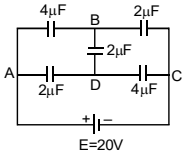
(*a*) 2 μF (*b*) 6 μF

(*c*) 8 μF (*d*) 12 μF

Comprehension Type Questions – 2 (No. 26 to 29)

The figure shows a diagonal symmetric arrangement of capacitors and a battery

26. Identify the correct statements.



(*a*) Both the 4μF capacitors carry equal charges in opposite sense

(*b*) Both the 4μF capacitors carry equal charges in same sense

(*c*) $V_B - V_D > 0$

(*d*) $V_D - V_B > 0$

27. If the potential of C is zero, then

(*a*) $V_A = +20V$

(*b*) $4(V_A - V_B) + 2(V_D - V_B) = 2V_B$

(*c*) $2(V_A - V_D) + 2(V_B - V_D) = 4V_D$

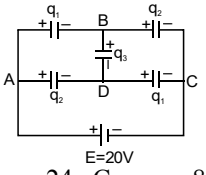
(*d*) $V_A = V_B + V_D$

28. The potential of the point B and D are

(*a*) $V_B = 8V$ (*b*) $V_B = 12V$

(*c*) $V_D = 8V$ (*d*) $V_D = 12V$

29. The value of charge q_1, q_2 and q_3 as shown in the figure are



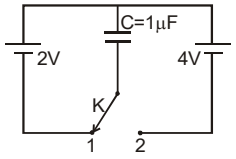
(*a*) $q_1 = 32\ \mu C\ ;\ q_2 = 24\ \mu C\ ;\ q_3 = -8\ \mu C$

(*b*) $q_1 = 48\ \mu C\ ;\ q_2 = 16\ \mu C\ ;\ q_3 = +8\ \mu C$

(*c*) $q_1 = 32\ \mu C\ ;\ q_2 = 24\ \mu C\ ;\ q_3 = +8\ \mu C$

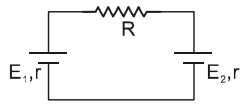
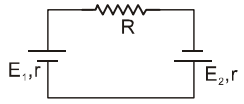
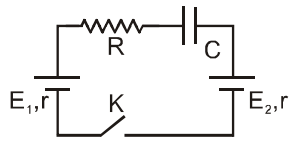
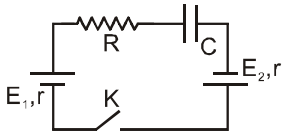
(*d*) $q_1 = 3\ \mu C\ ;\ q_2 = 4\ \mu C\ ;\ q_3 = +2\ \mu C$

30. The circuit involves two ideal cells connected to a 1 μF capacitor via a key K. Initially the key K is in position 1 and the capacitor is charged fully by 2V cell. The key is then pushed to position 2. Column I gives physical quantities involving the circuit after the key is pushed from position 1. Column II gives corresponding results. Match the statements in Column I with the corresponding values in Column II.



Column I	**Column II**
(*a*) The net charge crossing the 4 volt cell in μC is	(p) 2
(*b*) The magnitude of work done by 4 Volt cell in μJ is	(q) 6
(*c*) The gain in potential energy of capacitor in μJ is	(r) 8
(*d*) The net heat produced in circuit in μJ is	(s) 16

31. In each situation of column-I, a circuit involving two non-ideal cells of unequal emf E_1 and E_2 ($E_1 > E_2$) and equal internal resistance r are given. A resistor of resistance R is connected in all four situations and a capacitor of capacitance C is connected in last two situations as shown. Assume battery can supply infinity charge to the circuit (r, $R \neq 0$, $E_1, E_2 \neq 0$). Four statements are given in column-II. Match the situation of column-I with statements in column-II.

Column-I	**Column -II**
(*a*) 	(p) magnitude of potential difference across both cells can never be same.
(*b*) 	(q) cell of lower emf absorbs energy, that is, it gets charged up as long as current flows in circuit
(*c*) The capacitor is initially uncharged. 	(r) potential difference across cell of After the key K is closed lower emf may be zero.
(*d*) The capacitor is initially uncharged. 	(s) current in the circuit can never be zero After the key K is closed. (even after steady state is reached).

NUMERICAL VALUE BASED

32. A parallel plate capacitor contains a mica sheet (thickness 10^{-3} m) and a sheet of fibre (thickness 0.5×10^{-3} m). The dielectric constant of mica is 8 and that of the fibre is 2.5. Assuming that the fibre breaks down when subjected to an electric field of 6.4×10^6 V/m, find the maximum safe voltage in k.V that can be applied to the capacitor.

33. A 4 μF and a 9 μF capacitor are connected in series across a 26 V battery. What is voltage of the battery required to charge a parallel combination of the two capacitors to the same total energy?

34. A 10 μF capacitor is fully charged across a 12 volt battery. The capacitor is then disconnected from the battery and connected across an initially uncharged capacitor, C. The voltage across each capacitor is now 3 volts. What is the unknown capacitance C (in μF)?

35. What is the force of attraction (in μN) between the plates of a flat plate air capacitor of area 17.7 cm^2, if the voltage on the plates is 500 V and the distance between them is 8.85 mm?

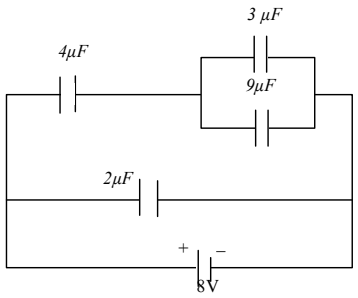
36. 2 conducting objects one with charge of +Q and another with –Q are kept on x-axis at x = –3 and x = + 4 respectively. The electric field on the x-axis is given by $3Q\left(x^2+\frac{4}{3}\right)$. What is the capacitance C of this configuration of objects. Fill $\frac{1}{C}$ (in F^{-1}) in OMR sheet.

JEE Mains & Advanced Past Years Questions

JEE MAIN PREVIOUS YEAR'S

1. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4μF and 9 μF capacitors), at a point distant 30 m from it, would equal:

[*JEE Mains-2016*]



(*a*) 240 N / C (*b*) 360 N / C

(*c*) 420 N / C (*d*) 480 N / C

2. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more t han 300 V.

The minimum number of capacitors required to achieve this is : **[*JEE Main - 2017*]**

(*a*) 24 (*b*) 32

(*c*) 2 (*d*) 16

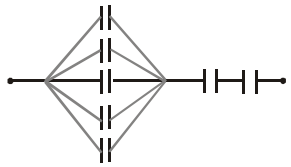
3. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be : **[*JEE Main-2018*]**

(*a*) 0.3 n C (*b*) 2.4 n C

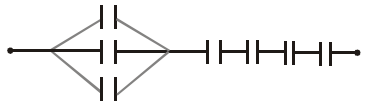
(*c*) 0.9 n C (*d*) 1.2 n C

4. Seven capacitors, each of the capacitance 2μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)$ μF. Which of the combinations, shown in figures below, will achieve the desired value? **[*JEE Main-2019 (January)*]**

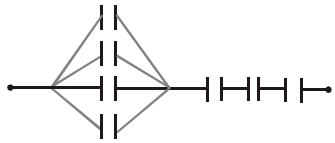
(*a*)



(*b*)



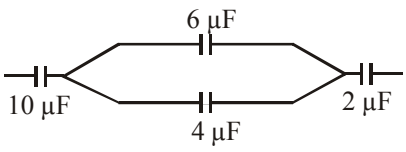
(*c*)



(*d*)



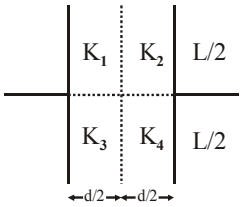
5. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30μC. The charge on the right place of the 6μF capacitor is : **[*JEE Main-2019 (January)*]**



(*a*) –12 μC (*b*) +12 μC

(*c*) –18 μC (*d*) +18 μC

6. A parallel palate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be: **[*JEE Main-2019 (January)*]**



(*a*) $K = \frac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

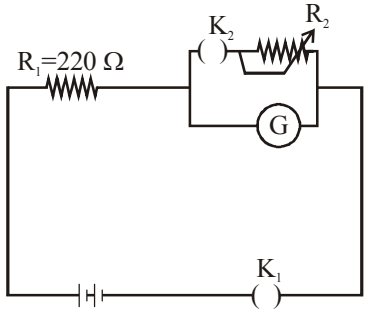
(*b*) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

(*c*) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(*d*) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

7. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5 Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery]:

[*JEE Main-2019 (January)*]



(*a*) 5 Ω (*b*) 22 Ω

(*c*) 25 Ω (*d*) 12 Ω

8. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. the charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is:

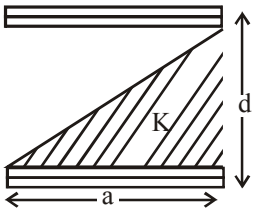
[*JEE Main-2019 (January)*]

(*a*) 692 pJ (*b*) 508 pJ

(*c*) 560 pJ (*d*) 600 pJ

9. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d(d <<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is :

[*JEE Main-2019 (January)*]

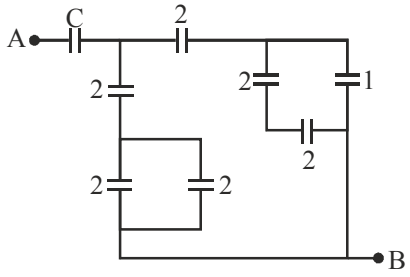


(*a*) $\frac{K\varepsilon_0 a^2}{d(K-1)}\ln K$ (*b*) $\frac{K\varepsilon_0 a^2}{d}\ln K$

(*c*) $\frac{K\varepsilon_0 a^2}{2d(K+1)}\ln K$ (*d*) $\frac{1}{2}\frac{K\varepsilon_0 a^2}{d}$

10. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF.

[*JEE Main-2019 (January)*]



(*a*) $\frac{7}{11}\mu F$ (*b*) $\frac{6}{5}\mu F$

(*c*) 4 μF (*d*) $\frac{7}{10}\mu F$

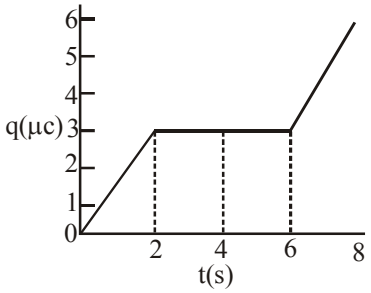
11. A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is:

[*JEE Main-2019 (January)*]

(*a*) 7.85×10^{-10}C (*b*) 6.85×10^{-10}C

(*c*) 8.85×10^{-10}C (*d*) 9.85×10^{-10}C

12. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure:



What is the value of current at t = 4 s ?

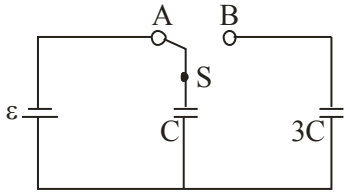
[*JEE Main-2019 (January)*]

(*a*) zero (*b*) 3μA

(*c*) 2μA (*d*) 1.5μA

13. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is:

[*JEE Main-2019 (January)*]



(*a*) $\frac{1}{8}\frac{Q^2}{C}$ (*b*) $\frac{3}{8}\frac{Q^2}{C}$

(*c*) $\frac{5}{8}\frac{Q^2}{C}$ (*d*) $\frac{3}{4}\frac{Q^2}{C}$

14. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :

[*JEE Main-2019 (April)*]

(*a*) $\frac{V}{K+n}$ (*b*) V

(*c*) $\frac{(n+1)V}{(K+n)}$ (*d*) $\frac{nV}{K+n}$

15. Voltage rating of a parallel plate capacitor is 500V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2 . What is the dielectric constant is the capacitance is 15 pF ?

[*JEE Main-2019 (April)*]

(**Given** $\in_0 = 8.86 \times 10^{-12}$ C^2/Nm^2)

(*a*) 3.8 (*b*) 4.5

(*c*) 6.2 (*d*) 8.5

16. A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is :

[*JEE Main-2019 (April)*]

(*a*) 5V (*b*) 2V

(*c*) 3V (*d*) 1V

17. A capacitor with capacitance 5μF is charged to 5μC. If the plates are pulled apart to reduce the capacitance to 2μF, how much work is done ?

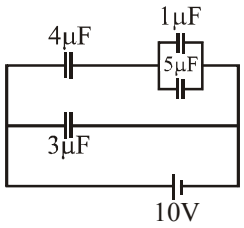
[*JEE Main-2019 (April)*]

(*a*) 3.75×10^{-6} J (*b*) 2.55×10^{-6} J

(*c*) 2.16×10^{-6} J (*d*) 6.25×10^{-6} J

18. In the given circuit, the charge on 4 μFcapacitor will be

[*JEE Main-2019 (April)*]



(*a*) 5.4 μC (*b*) 24 μC

(*c*) 13.4 μC (*d*) 9.6 μC

19. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in figure-I, and the second one is filled as shown in figure-I.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)).

[*JEE Main-2019 (April)*]

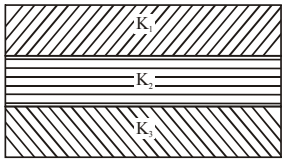


Figure-I

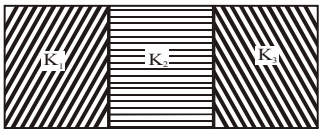


Figure-II

(*a*) $\frac{E_1}{E_2}=\frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

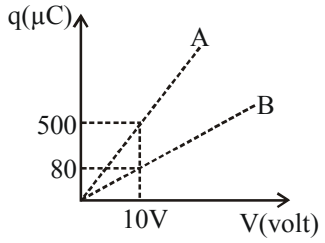
(*b*) $\frac{E_1}{E_2}=\frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(*c*) $\frac{E_1}{E_2}=\frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

(*d*) $\frac{E_1}{E_2}=\frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

20. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are :

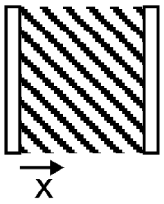
[*JEE Main-2019 (April)*]



(*a*) 50 μF and 30 μF (*b*) 20 μF and 30 μF

(*c*) 60 μF and 40 μF (*d*) 40 μF and 10 μF

21.



A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression:

[*JEE Main-2020 (January)*]

(*a*) $\frac{A\in_0 K}{d}\left(1+\frac{\alpha^2d^2}{2}\right)$ (*b*) $\frac{AK\in_0}{d}\left(1+\frac{\alpha d}{2}\right)$

(*c*) $\frac{A\in_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$ (*d*) $\frac{AK\in_0}{d}(1+\alpha d)$

22. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ).

[*JEE Main-2020 (January)*]

23. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10μF. When these capacitors are individually connected to a voltage source of 1V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be:

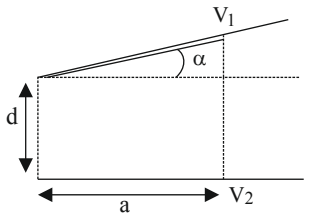
[*JEE Main-2020 (January)*]

(*a*) 4.2 μF (*b*) 8.4 μF

(*c*) 3.2 μF (*d*) 1.6 μF

24. A capacitor is made of two square plates each of side 'a' making a very small angle α between them, as shown in figure. The capacitance will be close to :

[*JEE Main-2020* (*January*)]



(*a*) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$ (*b*) $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{2d}\right)$

(*c*) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$ (*d*) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$

25. A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is

[*JEE Main-2020* (*September*)]

(*a*) 20 μF (*b*) 10 μF

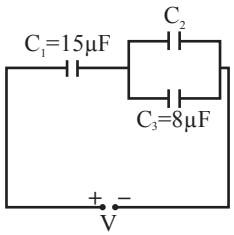
(*c*) 15 μF (*d*) 30 μF

26. A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J then value of X to the nearest integer is _____.

[*JEE Main-2020* (*September*)]

27. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is

[*JEE Main-2020* (*September*)]



(*a*) 650 μC (*b*) 590 μC

(*c*) 160 μC (*d*) 450 μC

28. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12μC and – 3μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively

[*JEE Main-2020* (*September*)]

(*a*) 4.5 μC on both (*b*) +4.5 μC and –4.5 μC

(*c*) 6 μC and 3 μC (*d*) 3 μC and 6 μC

29. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is

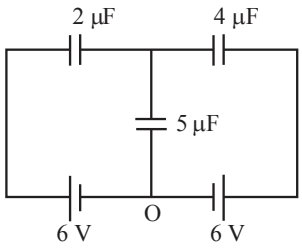
[*JEE Main-2020* (*September*)]

(*a*) $\frac{1}{4}CV_0^2$ (*b*) $\frac{1}{3}CV_0^2$

(*c*) $\frac{1}{6}CV_0^2$ (*d*) $\frac{1}{2}CV_0^2$

30. In the circuit shown, charge on the 5 μF capacitor is

[*JEE Main-2020* (*September*)]



(*a*) 16.36 μC (*b*) 18.00 μC

(*c*) 5.45 μC (*d*) 10.90 μC

31. A parallel plate capacitor has plate of length 'ℓ', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant k = 4 is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored?

[*JEE Main-2020* (*September*)]

(*a*) $\frac{\ell}{4}$ (*b*) $\frac{\ell}{2}$

(*c*) $\frac{2\ell}{3}$ (*d*) $\frac{\ell}{3}$

32. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is

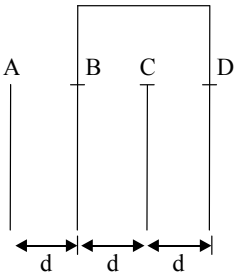
[*JEE Main-2020* (*September*)]

(*a*) $\frac{3}{2}CV^2$ (*b*) $\frac{9}{2}CV^2$

(*c*) Zero (*d*) $\frac{25}{6}$

33. Four identical rectangular plates with length, $l = 2$ cm and breadth, $b = \frac{3}{2}$cm are arranged as shown in figure. The equivalent capacitance between A and C is $\frac{w_0}{d}$. The value of x is . (Round off to the Nearest Integer)

[*JEE Main-2021* (*March*)]

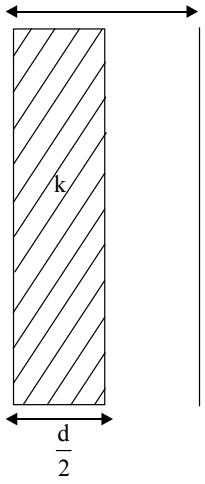


34. A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference V = 12V between its plates. The charging battery is now disconnected and a porcelin plate with k = 7 is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of…….pJ. (Assume no friction)

[*JEE Main-2021* (*March*)]

35. In a parallel plate capacitor set up, the plate area of capacitor is $2m^2$ and the plates are separated by 1m. If the space between the plates are filled with a dielectric material of thickness $0.5\,m$ and area $2m^2$ (see fig.) the capacitance of the set-up will be ε_0. (Dielectric constant of the material = 3.2) (Round off to the Nearest Integer)

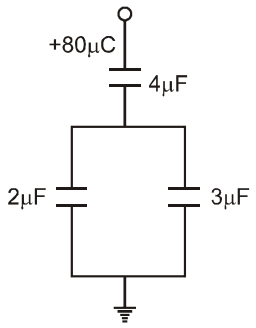
[*JEE Main-2021* (*March*)]



JEE ADVANCED PREVIOUS YEAR'S

1. In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is :

[*JEE Advanced-2012*]

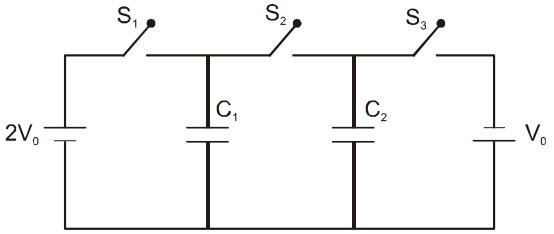


(*a*) +32 μC (*b*) +40 μC

(*c*) +48 μC (*d*) +80 μC

2. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time.

[*JEE Advanced-2013*]



(*a*) the charge on the upper plate of C_1 is $2CV_0$

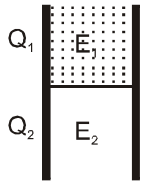
(*b*) the charge on the upper plate of C_1 is CV_0

(*c*) the charge on the upper plate of C_2 is 0

(*d*) the charge on the upper plate of C_2 is $-CV_0$

3. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. Choose the correct option/ options, igonoring edge effects.

[*JEE Advanced-2014*]

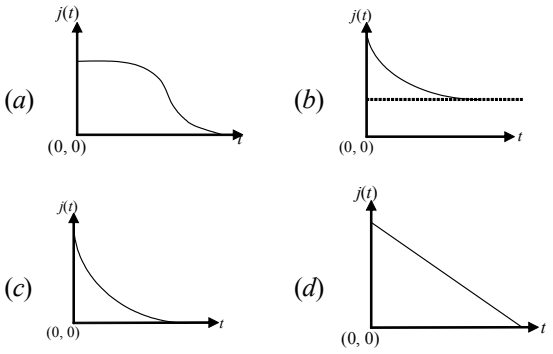


(*a*) $\frac{E_1}{E_2} = 1$ (*b*) $\frac{E_1}{E_2} = \frac{1}{K}$

(*c*) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (*d*) $\frac{C}{C_1} = \frac{2+K}{K}$

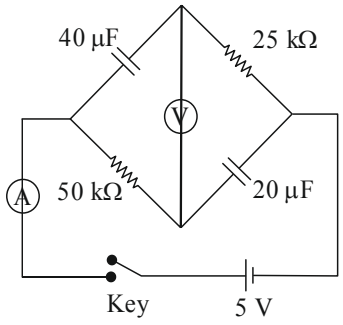
4. *An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R.* At time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?

[*JEE Advanced-2016*]



5. In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement (s) is (are) true?

[*JEE Advanced-2016*]



(*a*) The voltmeter displays – 5V as soon as the key is pressed, and displays + 5 V after a long time

(*b*) The voltmeter will display 0 V at time $t = l\text{n } 2$ seconds

(*c*) The current in the ammeter becomes 1/e of the initial value after 1second

(*d*) The current in the ammeter becomes zero after a long time

Comprehension Type Questions # 1 (No. 8 to 9)

Consider a simple RC circuit as shown in figure 1.

Process 1: In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T > > RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T > > RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in figure 2.

[*JEE Advanced-2017*]

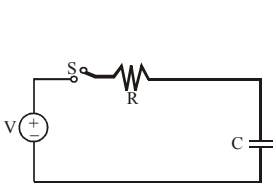


Figure 1

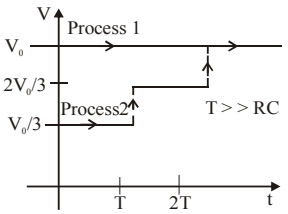


Figure 2

6. In process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by :

(*a*) $E_C = \frac{1}{2}E_D$ (*b*) $E_C = E_D \ln 2$

(*c*) $E_C = 2E_D$ (*d*) $E_C = E_D$

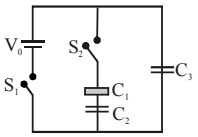
7. In process 2, total energy dissipated across the resistance E_D is :

(*a*) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (*b*) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

(*c*) $E_D = 3CV_0^2$ (*d*) $E_D = \frac{1}{2}CV_0^2$

8. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with. a dielectric material of relative permittivity $\in_r$. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of $\in_r$.

[*JEE Advanced-2018*]



9. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the mth layer is $K_m = K\left(1 + \frac{m}{N}\right)$. For a very large N (> 10^3), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d\ell n 2}\right)$. The value of α will be ___. [$\in_0$ is the permittivity of free space]

[*JEE Advanced-2019*]

ANSWER KEY

Topicwise Questions

1. (*c*) **2.** (*d*) **3.** (*b*) **4.** (*d*) **5.** (*a*) **6.** (*a*) **7.** (*d*) **8.** (*b*) **9.** (*d*) **10.** (*b*)
11. (*b*) **12.** (*b*) **13.** (*d*) **14.** (*a*) **15.** (*d*) **16.** (*b*) **17.** (*b*) **18.** (*c*) **19.** (*d*) **20.** (*c*)
21. (*d*) **22.** (*c*) **23.** (*b*) **24.** (*b*) **25.** (*b*) **26.** (*d*) **27.** (*c*) **28.** (*b*)

Learning Plus

1. (*a*) **2.** (*d*) **3.** (*d*) **4.** (*d*) **5.** (*c*) **6.** (*b*) **7.** (*c*) **8.** (*b*) **9.** (*b*) **10.** (*b*)
11. (*a*) **12.** (*b*) **13.** (*a*) **14.** (*a*) **15.** (*d*) **16.** (*b*) **17.** (*d*) **18.** (*b*) **19.** (*c*) **20.** (*c*)
21. (*a*) **22.** (*b*) **23.** (*a*) **24.** (*c*) **25.** (*b*) **26.** (*b*) **27.** (*c*) **28.** (*a*) **29.** (*a*) **30.** (*c*)
31. (*a*) **32.** (*b*) **33.** (*a*) **34.** (*a*) **35.** (*c*) **36.** (*b*) **37.** (*d*) **38.** (*d*) **39.** (*b*)

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. (*a*,*b*,*d*) **2.** (*b*,*c*) **3.** (*b*,*c*) **4.** (*a*,*c*,*d*) **5.** (*a*,*b*,*c*) **6.** (*a*,*d*) **7.** (*a*,*b*,*c*,*d*) **8.** (*b*,*c*) **9.** (*a*,*c*)
10. (*a*,*b*,*c*,*d*) **11.** (*b*,*c*,*d*) **12.** (*a*,*c*) **13.** (*a*,*b*,*c*) **14.** (*a*,*c*,*d*) **15.** (*a*,*d*) **16.** (*b*,*c*) **17.** (*b*,*c*) **18.** (*b*,*c*,*d*) **19.** (*a*,*c*)
20. (*b*,*c*,*d*) **21.** (*a*,*c*) **22.** (*b*,*d*) **23.** (*a*) **24.** (*c*) **25.** (*a*) **26.** (*b*,*c*) **27.** (*a*,*b*,*c*,*d*) **28.** (*b*,*c*) **29.** (*c*)
30. (*a*) p (*b*) r (*c*) q (*d*) p **31.** (*a*) p,q,s (*b*) p,r,s (*c*) p,q (*d*) p,r **32.** [5.2] **33.** [0012] **34.** [0030] **35.** [0025]
36. [0119]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*c*) **2.** (*b*) **3.** (*d*) **4.** (*b*) **5.** (*d*) **6.** (Bonus) **7.** (*b*) **8.** (*b*) **9.** (*a*) **10.** (*a*)
11. (*c*) **12.** (*a*) **13.** (*b*) **14.** (*c*) **15.** (*d*) **16.** (*d*) **17.** (*a*) **18.** (*b*) **19.** (*a*) **20.** (*d*)
21. (*b*) **22.** (6) **23.** (*d*) **24.** (*a*) **25.** (*c*) **26.** (*d*) **27.** (*b*) **28.** (*c*) **29.** (*c*) **30.** (*a*)
31. (*d*) **32.** (*a*) **33.** (2) **34.** (864) **35.** (*c*)

JEE-ADVANCED
PREVIOUS YEAR'S

1. (*c*) **2.** (*b*,*d*) **3.** (*a*,*d*) **4.** (*b*) **5.** (*a*,*b*,*c*,*d*) **6.** (*d*) **7.** (*b*) **8.** (1.50) **9.** (1.00)

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

3 Current Electricity

ELECTRIC CURRENT

The time rate of flow of charge through any cross-section is called current. If Δq charge passes through a cross-section in time Δt then

Average current $I_{av} = \frac{\Delta q}{\Delta t}$ and

Instantaneous current $I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$

TRAIN YOUR BRAIN

Example 1: Given $q = 2t^2 + 3$

(i) Calculate I at $t = 2$ sec.

(ii) I_{avg} in time interval of $t = 0$ to $t = 2$sec.

Sol. (i) $I = \frac{dq}{dt} = 4t$

$\therefore$ I at 2 sec $= 4 \times 2 = 8$ A

(ii) at $t = 0$, $q = 3$ C, at $t = 2$ sec. $q = 11$ C

$\Delta q = 11 - 3 = 8$ C

$I_{avg} = \frac{\Delta q}{\Delta t} = \frac{8}{2} = 4$ A

- Current is a fundamental quantity.
- Current is a scalar quantity and its
  SI Unit : Ampere
  CGS : biot (Bi) or ab ampere (ab amp.)
  dimension : $M^0 L^0 T^0 A^1$

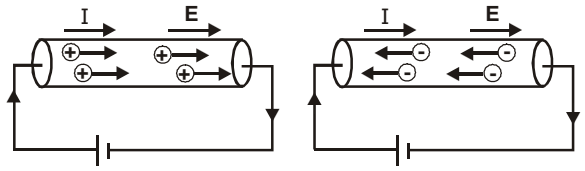
Ampere :

The current through a conductor is said to be one ampere if one coulomb of charge is flowing per second through a cross-section of wire.

$$1 \text{ ampere} = \frac{1 \text{coulomb}}{\text{second}}$$

$$= \frac{(1/10) \text{ emu of charge}}{\text{second}} = \frac{1}{10} \text{ Biot}$$

- The conventional direction of current is the direction of flow of positive charge or applied field. It is opposite to direction of flow of negatively charged electrons.



If n electrons pass through a point in a conductor in time t then current through the conductor is $I = \frac{q}{t} = \frac{ne}{t}$

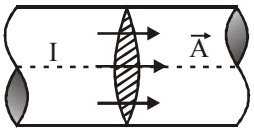
Number of electrons flowing through conductor in t second is $n = \frac{I \times t}{e}$

For 1 ampere of current $n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$ electrons/second

- The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second.
- For a given conductor current does not change with change in its cross-section because current is simply rate of flow of charge.

Current Density:

Current flowing per unit area through any cross-section is called current density.



$J = \frac{I}{A}$

Unit : A/m^2.

It is a vector quantity and directed along the motion of positive charge.

$I = \vec{J}.\vec{A} = JA\cos\theta$ (where θ is the angle between $\vec{J}$ & $\vec{A}$)

Mean free path:

The average distance travelled by a free electron between two consecutive collisions is called as mean free path λ.

$$\text{Mean free path } \lambda = \frac{\text{total distance travelled}}{\text{number of collisions}}$$

Relaxation time :

The time taken by an electron between two successive collisions is called as relaxation time τ. ($\tau \sim 10^{-14}$ s)

Relaxation time $\tau = \dfrac{\text{total time taken}}{\text{number of collisions}}$

thermal speed can be written as $v_T = \dfrac{\lambda}{\tau}$

In absence of applied potential difference electrons have random motion.

The average displacement and average velocity is zero. There is no flow of current due to thermal motion of free electrons in a conductor.

Drift velocity v_d :

The average velocity with which the free electrons are drifted towards the positive end of a conductor under the influence of an external electric field is called drift velocity v_d.

Using $\vec{v} = \vec{u} + \vec{a}\,t$

we have $\vec{v}_d = \dfrac{-e\vec{E}}{m}\tau$ $\quad (v_d \sim 10^{-4}$ m/sec)

The direction of drift velocity for electrons in a metal is opposite to that of applied field E.

Relation between current and drift velocity

Let n be number density of free electrons and A be area of cross-section of conductor.

$$I = neAv_d$$

- The current flowing through a conductor is directly proportional to the drift velocity ($I \propto v_d$)

  The current density

$$J = \frac{I}{A} = nev_d = ne \times \frac{eE}{m}\tau = \frac{ne^2\tau}{m} \times E$$

  so $J \propto E$ or $J = \sigma E$

- Conductivity of conductor depends on temperature and nature of material. $\sigma = \dfrac{ne^2\tau}{m}$
- Vector form of ohm's law, $\vec{J} = \sigma\vec{E}$

The drift velocity depends on nature of metal, τ, applied potential difference, length of conductor.

$$v_d = \frac{eE}{m}\tau = \frac{eV}{mL}\tau$$

v_d is independent of radius or area of cross-section of a conductor.

The rise of temperature causes increase in v_{rms} and hence a decrease in λ and relaxation time τ, causing a decrease in drift velocity.

Mobility of a charge carrier is defined as drift velocity acquired per unit electric field.

$$\text{Mobility } \mu = \frac{v_d}{E} = \frac{e\tau}{m} = \frac{e}{m}\frac{m\sigma}{ne^2} = \frac{\sigma}{ne}$$

SI Unit : $m^2 V^{-1} s^{-1}$

Dimensions : $M^{-1} T^2 A^1$

The mobility depends on applied potential difference, length of conductor, number density of charge carriers, current in conductor, area of cross-section of conductor.

OHM'S LAW AND RESISTANCE

Ohm's Law

If the physical state i.e. temperature, nature of material and dimensions of a conductor remain unchanged then the ratio of potential differene applied across its ends to current flowing through it remains constant.

$V \propto I$ or $V = IR$

$R = \dfrac{V}{I}$ = resistance of conductor

Resistance :

SI Unit : volt/ampere or **ohm (Ω)**

Dimensions : $\left[M^1L^2T^{-3}A^{-2}\right]$

Conductance : The reciprocal of resistance is called conductance. $G = \dfrac{1}{R}$

SI Unit : ohm^{-1} or mho or siemens

Dimensions : $[M^{-1} L^{-2} T^3 A^2]$

The substances which obey ohm's law are called ohmic or linear conductor. The resistance of such conductors is independent of magnitude and polarity of applied potential difference. Here the graph between I and V is a straight line passing through the origin.

- The reciprocal of slope of straight line gives resistance $R = \dfrac{V}{I} = \dfrac{1}{\tan\theta}$ = constant.
- The substances which do not obey ohm's law are called nonohmic or non linear conductors.
- The I–V curve is not a straight line.

  Example: p-n diode, transistors, thermionic valves, rectifiers etc.

Variation of Resistance with Dimensions of Conductor

1. **Variation with length:** $R = \rho\dfrac{l}{A}$

   (a) If a wire is cut to alter its length, then area remains same.

   $\therefore R \propto l$

   (b) If a wire is stretched or folded, area varies but volume remains constant.

   $\Rightarrow R \propto l^2$

   For small percentage changes in length by stretching or folding, then,

   $\dfrac{\Delta R}{R} = \dfrac{2\Delta l}{l}$

2. **Variation with area of cross-section or thickness:**

   (a) If area is increased / decreased but length is kept same.

   $\therefore R \propto \dfrac{1}{A}$ or $R \propto \dfrac{1}{r^2}$ (r = radius / thickness)

   (b) If area is increased / decreased but volume remains same.

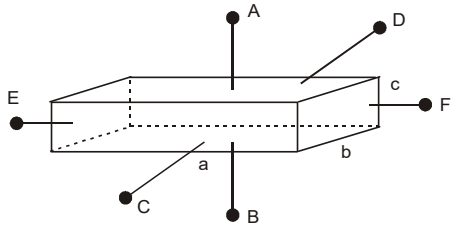
   $R \propto \dfrac{1}{A^2}$ or $R \propto \dfrac{1}{r^4}$

   (c) A resistance wire has a resistance R. Half of this wire is stretched to double its length and half is twisted to double its thickness, then

   $R' = \left(\dfrac{R}{2}\right) \times 4 + \dfrac{R}{2} \times \dfrac{1}{16} = \dfrac{65R}{32}$

TRAIN YOUR BRAIN

Example 2: The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.



Sol. $R = \frac{\rho \ell}{A} = \frac{\text{Resistivity} \times \text{length}}{\text{Area of cross section}}$

where A = Area of cross-section perpendicular to direction of current

l = length along the direction of current.

$R_{AB} = \frac{\rho c}{ab}$, $R_{CD} = \frac{\rho b}{ac}$, $R_{EF} = \frac{\rho a}{bc}$

Example 3: One end of an aluminium wire, whose diameter is 2.5 mm is welded to one end of a copper wire whose diameter is 2.0 mm. The composite wire carries a steady current 6.25 A. The current densities in Al and Cu will be respectively –

(*a*) 127 A/cm^2, 200 A/cm^2
(*b*) 126 A/cm^2, 180 A/cm^2
(*c*) 125 A/cm^2, 160 A/cm^2
(*d*) 125 A/cm^2, 180 A/cm^2

Ans. (a)

Sol. $\because J = I/A = \frac{I}{\pi r^2}$

$\therefore$ For Aℓ, $J = \frac{6.25}{\frac{\pi}{4} \times 2.5^2 \times 10^{-6}}$

$= 127 \times 10^6$ A/m^2 = 127 A/cm^2

and for Cu, $J = \frac{6.25}{\frac{\pi}{4} \times 2.0^2 \times 10^{-6}} = 200$ A/cm^2

Example: A current of 5 amp. is passing through metallic wire of cross–sectional area 4×10^{-6} m^2, the drift speed of electrons will be (in m/sec) (density of electron per unit volume is 5×10^{26})

(*a*) 1.56×10^{-2} (*b*) 1.6×10^{-19}
(*c*) 1.56×10^2 m/sec (*d*) 1.6×10^{19}

Sol.(*a*) We know $I = neAv_d$

$\Rightarrow v_d = \frac{5}{5 \times 10^{26} \times 1.6 \times 10^{-19} \times 4 \times 10^{-6}}$

$= 1.56 \times 10^{-2}$ m/sec

VARIATION OF RESISTANCE WITH TEMPERATURE

- The variation of resistance with temperature is approximately given by the linear relation

  $R_t = R_0(1 + \alpha t)$

  α = the temperature coefficient of resistance

  SI Unit of α : °C^{-1} or K^{-1}

  For many pure metals, $\alpha = \frac{1}{273}$ per °C.

- At temperature t_1, $R_1 = R_0(1 + \alpha t_1)$ ....(1)
  and at temperature t_2, $R_2 = R_0(1 + \alpha t_2)$ .... (2)
  Subtracting eqution (1) from (2), we get

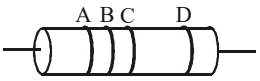
  $\alpha = \frac{R_2 - R_1}{R_0(t_2 - t_1)}$ per °C

- α is positive for conductors.
- α is negative for semiconductors, electrolytes, carbon, mica and Indian rubber.
- α is zero for superconductors (resistance of a superconductor is also zero).

Thermistors

A thermistor is a heat sensitive device made of semiconducting material where resistivity changes rapidly with change of temperature. It has a large temperature coefficient. The temperature coefficient may be positive or negative. These can be used over wide range of temperature.

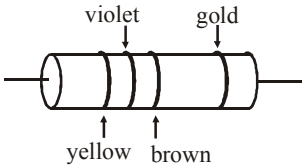
COLOUR CODE FOR CARBON RESISTORS



Colour	Strip A (digit 1)	Strip B (digit 2)	Strip C (Multiplier)	Strip D (Tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Grey	8	8	10^8	
White	9	9	10^9	
Gold	-	-	10^{-1}	±5%
Silver	-	-	10^{-2}	±10%
No Colour	-	-	-	±20%

• Aid to memory BBROY of Great Britain has a Very Good Wife.

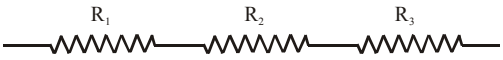
Example 5: What is resistance of following resistor ?



Sol. Number for yellow is 4. Number for violet is 7
Brown colour gives multiplier 10^1, Gold gives a tolerance of $\pm$ 5%
So resistance of resistor is
$47 \times 10^1\,\Omega \pm 5\% = 470 \pm 5\%\,\Omega$

COMBINATION OF RESISTANCES

Series Combination:

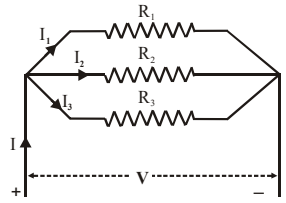


Resistances are said to be connected in series between two points if they provide only a single path between two points.
Resistances are connected in series if same current flows through each resistance when some potential difference is applied across the combination.

- Equivalent resistance $R_s = R_1 + R_2 + R_3$
- The equivalent resistance is equal to sum of individual resistances.
- The equivalent resistance is greater than largest of individual resistance.

The resistances are connected in series:
(a) to increase the resistance and
(b) to divide large potential difference across many resistances.

Parallel Combination :



Resistances are said to be connected in parallel between two points if it is possible to proceed from one point to another along different paths.Resistances are said to be in parallel if potential across each resistance is same and equal to applied potential.
Reciprocal of equivalent resistance

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- The equivalent resistance is smaller than smallest of individual resistance.
- The resistances are connected in parallel to decrease resistance.

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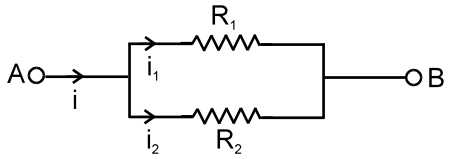
Example 6: When two resistors are in parallel combination then determine i_1 and i_2, if the combination carries a current i ?

Sol. $\because V_1 = V_2$

$\therefore\ i_1R_1 = i_2R_2$

or $\dfrac{i_1}{i_2} = \dfrac{R_2}{R_1}$ ... (i)

$i = i_1 + i_2$ ...(ii)

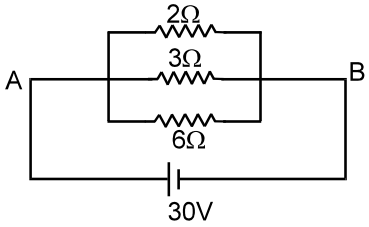


From equation (i) & (ii)

$$i_1 = \frac{R_2 i}{R_1 + R_2}\ ,\ i_2 = \frac{R_1 i}{R_1 + R_2}$$

Remember this law of $i \propto \dfrac{1}{R}$ in the resistors connected in parallel. It can be used in problems.

Example 7: Find current passing through the battery and each resistor.



Sol.Method (I): It is easy to see that potential difference across each resistor is 30 V.

$\therefore$ current is each resistors are $\dfrac{30}{2} = 15$ A, $\dfrac{30}{3} = 10$ A and $\dfrac{30}{6} = 5$ A

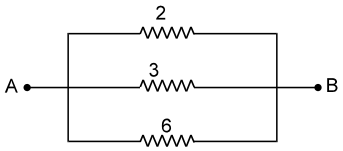
$\therefore$ Current through battery is = 15 + 10 + 5 =30 A.

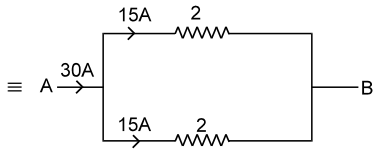
Method (II) : $\dfrac{1}{R_{eq}} = \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{6} = 1\Omega \Rightarrow R_{eq} = 1\,\Omega$

By ohm's law $i = \dfrac{V}{R_{eq}}$

$\Rightarrow\ i = \dfrac{30}{1} = 30$ A

Now distribute this current in the resistors in their inverse ratio.

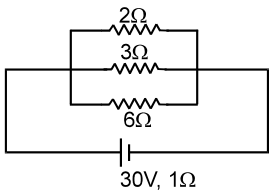




Total current in 3 Ω and 6 Ω is 15 A, it will be divided as 10 A and 5 A.

Note: The method (I) is better. But you will not find such an easy case every where.

Example 8: Find current which is passing through battery.



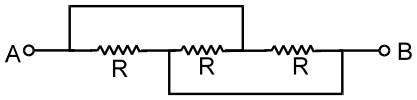
Sol. Here potential difference across each resistor is not 30 V

∵ battery has internal resistance. Here the concept of combination of resistors is useful.

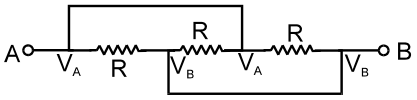
$R_{eq} = 1 + 1 = 2\,\Omega$

$i = \frac{30}{2} = 15\,A.$

Example 9: Find equivalent Resistance



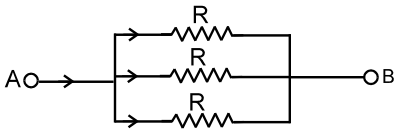
Sol.



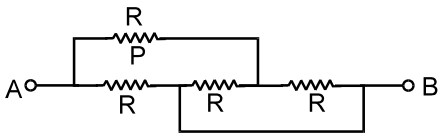
Here all the Resistance are connected between the terminals A and B

Modified circuit is

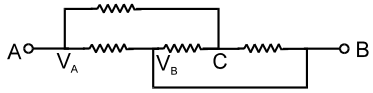
So $R_{eq} = \frac{R}{3}$



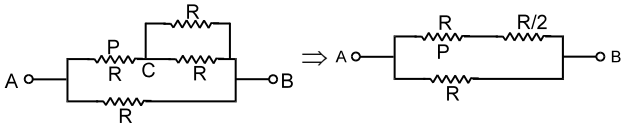
Example 10: Find the current in Resistance P if voltage supply between A and B is V volts



Sol.



Modified circuit

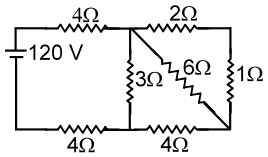


$R_{eq} = \frac{3R}{5}$

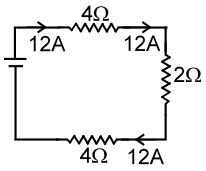
$I = \frac{5V}{3R}$

Current in P $= \frac{R \times \frac{5V}{3R}}{1.5R + R} = \frac{2V}{3R}$

Example 11: Find the current in 2Ω resistance



Sol. 2Ω, 1Ω in series = 3Ω



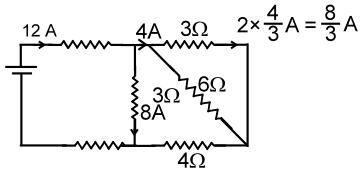
3Ω, 6Ω in parallel $= \frac{18}{9} = 2\Omega$

2Ω, 4Ω in series = 6Ω

6Ω, 3Ω in parallel = 2Ω

$R_{eq} = 4 + 4 + 2 = 10\Omega$

$i = \frac{120}{10} = 12A$



So current in 2Ω Resistance $= \frac{8}{3}$ A

SOME OTHER IMPORTANT POINTS CONCERNING VARIATION OF RESISTANCE

- Resistance of bismuth wire increases when placed in a magnetic field.
- Resistance of carbon granules decreases when pressure is increased.
- Resistance of semiconductor decreases when light falls on it.
- Resistance of an intrinsic semiconductor decreases when doped with a trivalent or pentavalent impurity.
- The specific resistance of alloys increases with a rise in temperature but this increase is much smaller when compared to pure metals.

Electrochemical Cell:

An electrochemical cell is a device which by converting chemical energy into electrical energy maintains the flow of charge in a circuit. It usually consists of two electrode of different materials and an electrolyte. The electrode at higher potential is called anode and the one at lower potential is cathode.

Electromotive Force (EMF):

The emf of a cell is defined as work done by cell in moving a unit positive charge in the whole circuit including the cell once.

SI Unit : Joule/Coulomb or Volt

emf is the maximum potential difference between the two electrodes of the cell when no current is drawn from the cell.

emf is the characteristic property of cell and depends on the nature of electrodes and electrolyte used in cell.

emf is independent of quantity of electrolyte, size of electrodes and distance between the electrodes.

Internal Resistance of Cell:

The opposition offered by the electrolyte of the cell to the flow of electric current through it, is called the internal resistance of the cell.

The internal resistance of cell depends on:

- Distance between electrodes ($r \propto d$) larger is the separation between electrodes more is the length of electrolyte through which ions have to move so more is internal resistance.
- Conductivity or nature of electrolyte $r \propto \frac{1}{\sigma}$
- Concentration of electrolyte $r \propto c$
- Temperature of electrolyte $r \propto \frac{1}{T}$
- Nature and area of electrodes dipped in electrolyte $r \propto \frac{1}{A}$

Terminal Potential Difference:

The potential difference between the two electrodes of a cell in a closed circuit i.e. when current is being drawn from the cell is called terminal potential difference.

(a) When cell is discharging : $V = E - Ir$

(b) When cell is charging : When cells is charging current inside the cell is from anode to cathode.

$V = E + Ir$

(c) When cell is in open circuit : In open circuit terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

In open circuit $R = \infty$

$V = E$

(d) When cell is short circuited : In short circuit $R = 0$

so $$I = \frac{E}{R + r} = \frac{E}{r}$$

and $$V = IR = 0$$

In short circuit current from cell is maximum and terminal potential difference is zero.

The emf of cell is taken to be positive in a circuit if current inside a cell is from negative to positive i.e. during discharging otherwise negative.

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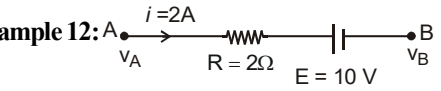
Example 12:

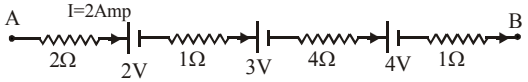


Find $v_A - v_B$

Sol. $v_A - iR - E = v_B$

$v_A - v_B = iR + E = 4 + 10 = 14$ volt

Example 13: Find Potential at A if potential at B is zero.



Sol. $V_A = 2 + 4 + 8 + 3 + 2 + 2 + 4 = 25$ V

CAPACITY OF A BATTERY

Capacity of a battery is equal to product of current in ampere and time in hour for which a cell can operate. It depends on the amount of electrolyte and so on the size of cell.

e.g. capacity 8Ah means we can draw 8A current for one hour or 2A current for 4 hours.

COMBINATIONS OF CELLS

(i) Series Combination

If N sources of *emf* are connected in series, then the equivalent *emf* is given by

$E = E_1 + E_2 + E_3 + \ldots\ldots\ldots + E_N$

And, total internal resistance is given by

$r = r_1 + r_2 + r_3 + \ldots\ldots\ldots\ldots + r_N$

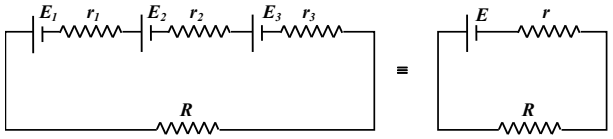


Fig. Series combination of batteries E_1, E_2 and E_3

(ii) Parallel Combination

The *emf* and *internal resistance* of the equivalent battery are given by

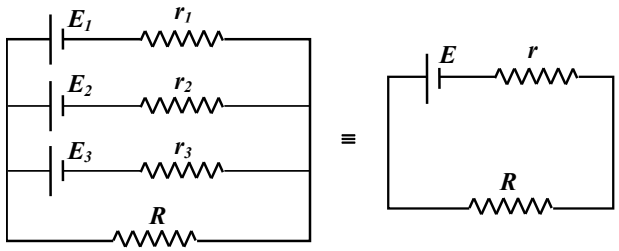


Fig. Parallel Combination of batteries E_1, E_2 and E_3

$$E=\frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}$$

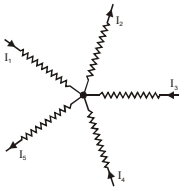
and $$\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$

If 'n' identical cells with emf E and internal resistance 'r', p cells are connected in opposite polarity then,

$E_{net}=(n-2p)E \text{ and } r_{net}=nr$

KIRCHHOFF'S LAW

Kirchhoff's first law or current law or junction rule(KCL)



In any electrical network, the algebraic sum of currents meeting at a junction is always zero i.e. $\Sigma I = 0$

$I_1 - I_2 + I_3 + I_4 - I_5 = 0$

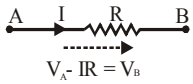
or $I_1 + I_3 + I_4 = I_2 + I_5$

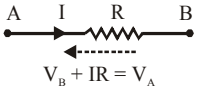
- The sum of currents flowing towards a junction is equal to sum of currents leaving the junction.
- By convention the current directed towards the junction is positive while those directed away from the junction is taken as negative.
- The first law is in accordance with **conservation of charge**.
- The charges do not accumulate at a junction. The total charge entering a junction is equal to total charge leaving the junction.

Kirchhoff's second law or voltage law or loop rule (KVL)

The algebraic sum of rise up and drop of potential along any closed path in a network is zero. i.e. $\Sigma V = 0$

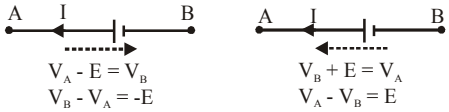
- The second law is in accordance with **conservation of energy**.
- According to second law the electric energy given to the charge by a source of emf is lost in passing through resistance.
- The change in potential in covering a resistance in the direction of current is negative (–IR) while in opposite direction it is positive.





The potential falls along direction of current. The potential fall is taken as negative while potential rise is taken as positive.

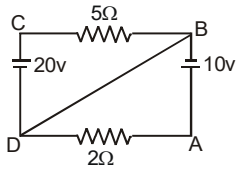
- The emf of source is taken as positive when it is traversed from negative to positive terminal while it is taken as negative when it is traversed from positive to negative irrespective of the direction of current.



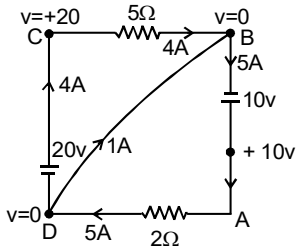
- If there are n loops their will be (n – 1) equations according to loop rule.
- The algebraic sum of products of currents and resistances in a closed loop is equal to sum of emf's applied in the circuit i.e. $\Sigma E = \Sigma IR$

TRAIN YOUR BRAIN

Example 14: Shown in the figure. Find out the current in the wire BD

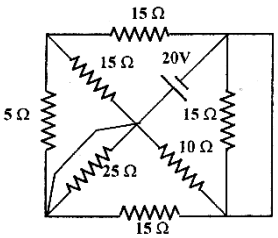


Sol. Let at point D potential = 0 and write the potential of other points then current in wire $AD = \frac{10}{2} = 5A$ from A to D, current in wire $CB = \frac{20}{5} = 4A$ from C to B



$\therefore$ current in wire BD = 1 A from D to B

Example 15: In the shown circuit, the current in 25 Ω and 10Ω resistance will be



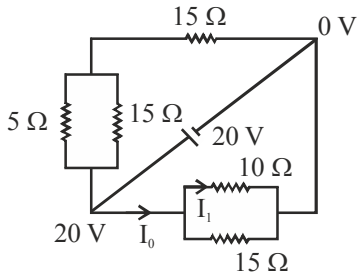
(*a*) 0 A and 1A (*b*) A and 2 A
(*c*) 0 A and 2 A (*d*) none

Ans. (*c*)

Sol.: Here, 15 Ω and 25 Ω is short-circuited.

$\therefore$ Current in 25 Ω is zero.

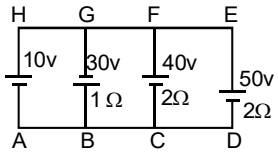
Now, Modified circuit is



$$\therefore\ I_0 = \frac{20}{\left(\frac{10\times15}{25}\right)} = 20\left(\frac{25}{10\times15}\right) = \frac{10}{3}\text{A}$$

$$I_1 = \left(\frac{15}{10+15}\right)I_0 = \frac{15}{25}\times\frac{10}{3} = 2\text{A}$$

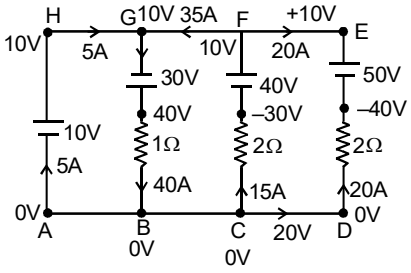
Example 16: Find the current in each wire



Sol. Let potential at point A is 0 volt then potential of other points is shown in figure.

Current in BG $= \frac{40-0}{1} = 40$ A from G to B

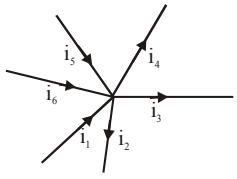
Current in FC $= \frac{0-(-30)}{2} = 15$A from C to K



current in DE $= \frac{0-(-40)}{2} = 20$A from D to E

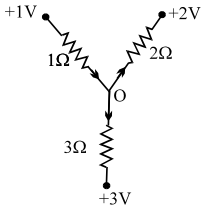
current in wire AH = 40 – 35 = 5 A from A to H

Example 17: Find relation in between current i_1, i_2, i_3, i_4, i_5 and i_6



Sol. $i_1 - i_2 - i_3 - i_4 + i_5 + i_6 = 0$

Example 18: The potential of point O in the steady state circuit shown on the right is



(*a*) 11/12 V (*b*) 18/11 V

(*c*) 16/9 V (*d*) None of these

Ans. (*b*)

Sol.: Let at point 'O', potential is x.

According to KCL,

$I_1 = I_2 + I_3$

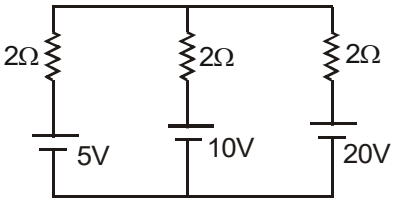
$$\Rightarrow \frac{1-x}{1} = \frac{x-2}{2} + \frac{x-3}{3}$$

$$\Rightarrow 1-x = \frac{3(x-2)+2(x-3)}{6}$$

$$\Rightarrow 6-6x = 5x-12 \Rightarrow 11x = 18$$

$$\Rightarrow x = \frac{18}{11}\text{V}$$

Example 19: Find the current in every branch ?

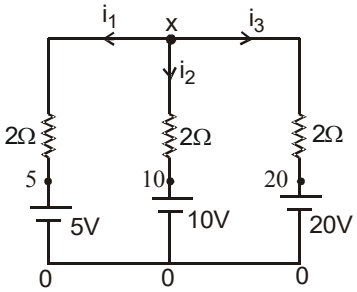


Sol. Let we assume x potential at the top junction & zero potential at lower junction

As from KCL, net current on a junction is 0

$i_1 + i_2 + i_3 = 0$

$$\frac{x-5}{2} + \frac{x-10}{2} + \frac{x-20}{2} = 0$$



$$3x = 35 \Rightarrow x = \frac{35}{3}$$

$$\therefore\ i_1 = \frac{\frac{35}{3}-5}{2} = \frac{10}{3}\text{ A}$$

Similarly, $i_2 = \frac{5}{6}$ A ; $i_3 = -\frac{25}{6}$ A.

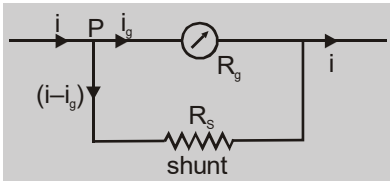
Measuring Devices

Galvanometer : Galvanometer detects the presence of current in the branch where it is connected. An instrument that measures current is called an ammeter, and one that measures potential difference is called a **voltmeter.**

CONVERSION OF GALVANOMETER INTO AMMETER

A galvanometer can be converted into an ammeter by connecting low resistance in parallel to its coil.

- The value of shunt resistance to be connected in parallel to galvanometer coil is given by $R_S = \frac{R_g i_g}{i - i_g}$



Where i = Range of ammeter

i_g = Current required for full scale deflection of galvanometer.

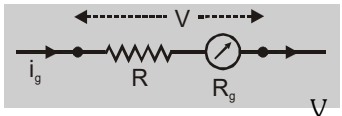
R_g = Resistance of galvanometer coil.

- $R_S = \frac{R_g i_g}{i}$ if $i_g << i$
- On connecting shunt in parallel to galvanometer coil, its equilvalent resistance becomes less than the shunt resistance.

CONVERSION OF GALVANOMETER INTO VOLTMETER

- The galvanometer can be converted into voltmeter by connecting high resistance in series with its coil.
- The high resistance to be connected in series with galvanometer coil is given by -

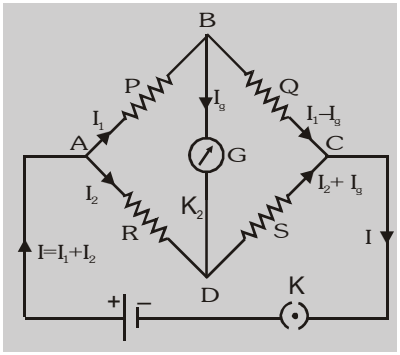
$$R = \frac{V}{i_g} - R_g$$



- If R_g is negligible, then $R = \frac{V}{i_g}$

Wheat Stone Bridge

- The configuration in the adjacent figure is called Wheat



Stone Bridge

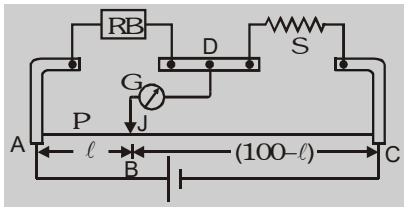
- If current in galvanometer is zero ($I_g = 0$) then bridge is said to be balanced

$V_D = V_B$ or $\frac{P}{Q} = \frac{R}{S}$

- If $\frac{P}{Q} < \frac{R}{S}$ then $V_B > V_D$ and current will flow from B to D.
- If $\frac{P}{Q} > \frac{R}{S}$ then $V_B < V_D$ and current will flow from D to B.

METRE BRIDGE

It is based on principle of wheatstone bridge. It is used to find out unknown resistance of wire. AC is 1 m long uniform wire R.B. is known resistance and S is unknown resistance.



A cell is connected across 1 m long wire and Galvanometer is connected between jockey and midpoint of R & S.

To find out unknown resistance we touch jockey from A to C and find balance condition. Let balance is at B point on wire.

AB = ℓ cm. P = r ℓ

BC = (100 - ℓ) cm.Q = r(100 - ℓ)

r = resistance per unit length on wire.

At balance condition :

$\frac{P}{Q} = \frac{R}{S}$ $\Rightarrow \frac{r\ell}{r(100-\ell)} = \frac{R}{S}$

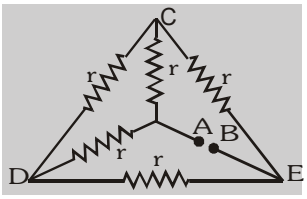
$\Rightarrow S = \frac{(100-\ell)}{\ell} R$

GOLDEN KEY POINT

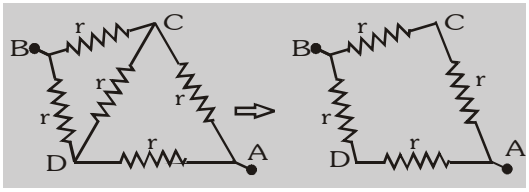
- To increase the range of an ammeter a shunt is connected in parallel with the galvanometer.
- To convert an ammeter of range I ampere and resistance R_g Ω into an ammeter of range nI amp., the value of resistance to be connected in parallel will be $R_g/(n-1)$
- To increase the range of a voltmeter a high resistance is connected in series with it.
- To convert a voltmeter of resistance R_g Ω and range V volt into a voltmeter of range nV volt, the value of resistance to be connected in series will be $(n-1)R_g$.
- Resistance of ideal ammeter is zero & resistance of ideal voltmeter is infinite.

TRAIN YOUR BRAIN

Example 20: In the adjoining network of resistors, each resistance is of r Ω. Find the equivalent resistance between point A and B.



Sol. Given circuit is balanced Wheatstone Bridge

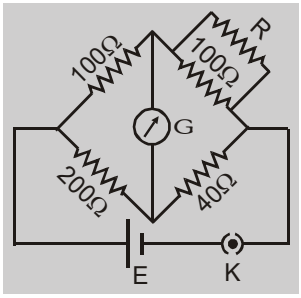


$$\therefore \quad \frac{1}{R_{AB}} = \frac{1}{2r} + \frac{1}{2r} = \frac{1}{r} \quad \Rightarrow \quad R_{AB} = r$$

Example 21: For the following diagram the galvanometer shows zero deflection then what is the value of R ?

Sol. For balanced Wheatstone bridge

$$\frac{100}{\frac{100R}{(100+R)}} = \frac{200}{40} \qquad \Rightarrow \quad \frac{100+R}{R} = 5$$



$$100 + R = 5R \qquad \Rightarrow \quad R = \frac{100}{4} = 25\Omega$$

Example 22: A 100 volt voltmeter whose resistance is 20 kΩ is connected in series to a very high resistance R. When it is joined in a line of 110 volt, it reads 5 volt. What is the magnitude of resistance R ?

Sol. When voltmeter connected in 110 volt line,

Current through the voltmeter

$$I = \frac{110}{(20\times10^3 + R)}$$

The potential difference across the voltmeter

$V = IR_V$

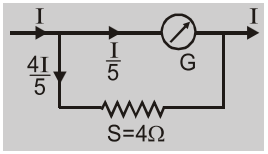
$$\Rightarrow \quad 5 = \frac{110\times20\times10^3}{(20\times10^3+R)}$$

$$\Rightarrow \quad 20\times10^3 + R = 440\times10^3$$

$$\Rightarrow \quad R = 420\times10^3\,\Omega$$

Example 23: When a shunt of 4Ω is attached to a galvanometer, the deflection reduces to $\frac{1}{5}$th. If an additional shunt of 2Ω is attached. What will be the deflection ?

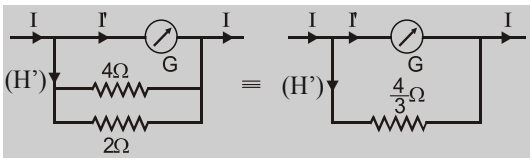
Sol. Initial condition



When shunt of 4Ω used

$$\frac{I}{5}\times G = \frac{4}{5}I\times 4 \quad \Rightarrow \quad G = 16\Omega$$

When additional shunt of 2Ω used



$$I'\times 16 = (I-I')\frac{4}{3}$$

$$12\,I' = I - I'$$

$$I' = \frac{I}{13}$$

$\therefore$ it will reduce to $\frac{I}{13}$ of the initial deflection.

Example 24: A galvanometer having 30 divisions has current sensitivity of 20μA/division. It has a resistances of 25Ω.

(i) How will you convert it into an ammeter measuring upto 1 ampere.

(ii) How will you convert this galvanometer into a voltmeter upto 1 volt.

Sol. The current required for full scale deflection

$I_g = 20\,\mu A\times 30 = 600\,\mu A = 6\times10^{-4}A$

(i) To convert it into ammeter, a shunt is required is parallel with it shunt resistance

$$R'_S = \frac{I_g R_g}{(I-I_g)} = \left(\frac{6\times10^{-4}}{1-6\times10^{-4}}\right)25 = 0.015\Omega$$

(ii) To convert galvanometer into voltmeter, a high resistance in series with it is required.

Series resistance

$$R = \frac{V}{i_g} - R_g = \frac{1}{6\times10^{-4}} - 25$$

$$= 1666.67 - 25 = 1641.67\,\Omega$$

POTENTIOMETER

Necessity of potentiometer

Practically voltameter has a finite resistance. (ideally it should be ∞) in other words it draws some current from the circuit . To overcome this problem potentiometer is used because at the instant of measurement, it draws no current from the circuit.

Working principle of potentiometer

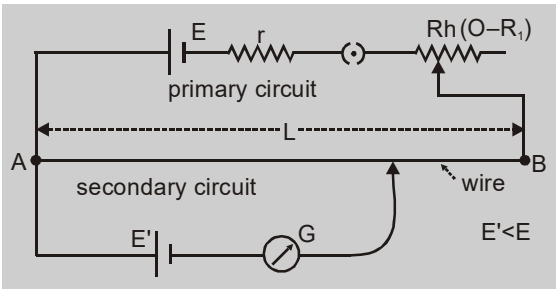
Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over entire length of potentiometer wire. This process is named as zero deflection or null deflection method.

Note :

(i) Potentiometer wire : Made up of alloys of magnin, constantan and eureka.

(ii) Specific properties of these alloys are high specific resistance, negligible temprature co-efficient of resistance (α). Invariability of resistance of potentiometer wire over a long period.

Circuits of potentiometer



Primary circuit contains constant source of voltage rheostat or Resistance Box

Secondary, Unknown or galvanometer circuit

Let ρ = Resistance per unit length of potentiometer wire

Potential gradient (x)

- Potential gradient corresponding to unit length of potentiometer is also called potential gradient.
- The potential difference per unit length of potentiometer wire is equal to potential gradient.

  Let r = 0 and $R_1 = 0$ then $V_{AB} = E$ (max. in the case of ideal cell) then $x = \frac{E}{L}$

  S.I. unit : V/m ; Dimension : $[MLT^{-3}A^{-1}]$
- Always $V_{AB} < E$;

  $x = \frac{V_{AB}}{L}$ Now $V_{AB} = I R_P$

  (R_P = resistance of potentiometer wire)

  So $x = \frac{I R_P}{L} = I\rho$ $\rho = \frac{R_P}{L}$

  current in primary circuit $I = \frac{E}{R_1 + r + R_P}$;

  $x = \frac{E}{R_1 + R_P + r}\left(\frac{R_P}{L}\right)$
- If radius is uniform, than x is uniform over entire length of potentiometer wire.
- If I is constant then, $x \propto \frac{1}{(\text{radius})^2}$
- 'x' directly depends on $\rightarrow \rho, r, \sigma$ etc.

Factor Affecting 'x'

- If V_{AB} = const. and L = const. then for any change x remains unchanged.
- If there is no information about V_{AB} then always take V_{AB} as constant so ($x \propto \frac{1}{L}$)
- If V_{AB} and L are constant :
  For any change like radius of wire and substance of wire (σ) there is no change in x.
- Any change in secondary circuit causes no change in x because x is an element of primary circuit.

Note : $x = \frac{E}{R_P + r + R_1}\left(\frac{R_P}{L}\right)$

x_{max} or x_{min} on the basis of range of rheostat or resistance box (R.B.)

If $R_1 = 0 \Rightarrow x_{max} = \frac{E}{R_P} \times \frac{R_P}{L}$ ($r \simeq 0$)

If $R_1 = R \Rightarrow x_{min} = \frac{E}{R_P + R}\left(\frac{R_P}{L}\right)$

then $\frac{x_{max}}{x_{min}} = \frac{R_P + R}{R_P}$

Standardization and Sensitivity of Potentiometer

Standardization process of evaluating x experimentally

If balanced length for standard cell (emf E) is = ℓ_0 then potential gradient $x = \frac{E}{\ell_0}$

Sensitivity :

(i) x also indicates about sensitivity of potentiometer.

(ii) If $x \downarrow \Rightarrow$ sensitivity $\uparrow$

(iii) To increase sensitivity $\rightarrow R_h \uparrow$ (current in primary ckt should be reduced), $L \uparrow$

(iv) Any change in secondary ckt, no effect on sensitivity.

(v) Balanced length for unknown potential difference $\uparrow \Rightarrow$ sensitivity $\uparrow$

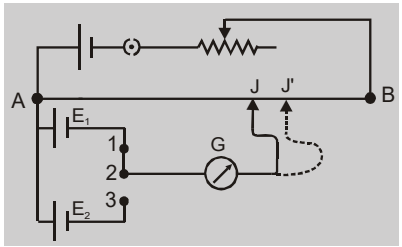
Applications of Potentiometer

(1) To measure potential difference across a resistance.
(2) To find out emf of a cell.
(3) Comparision of two emfs $\frac{E_1}{E_2}$
(4) To find out internal resistance of a primary cell.
(5) Comparision of two resistances.
(6) To find out an unknown resistance which is connected in series with the given resistance.
(7) To find out current in a given circuit.
(8) Calibration of an ammeter or to have a check on reading of (A)
(9) Calibration of a voltmeter or to have a check on reading of (V)
(10) To find out thermocouple emf (e_t) (mV or μV)

Note : (i) For application 3-6 no need of standard cell and no need of value of x.
(ii) For 7, 8, 9, 10 - Always require a standard cell ($E_0 = x\ell_0$)
(iii) For 1 – 9 order of voltage drop (0.1 to 1v)

(*a*) Comparision of emf of two cells

plug only in (1– 2) plug only in (2 – 3)

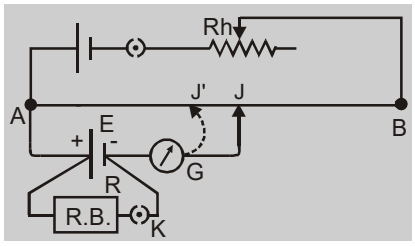


balance length AJ = ℓ_1 and balance length AJ' = ℓ_2
$E_1 = x\ell_1$ and $E_2 = x\ell_2$

$\Rightarrow \frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$

(*b*) Internal resistance of a given primary cell

$E = V + Ir$ $\Rightarrow r = \frac{E - V}{I}$



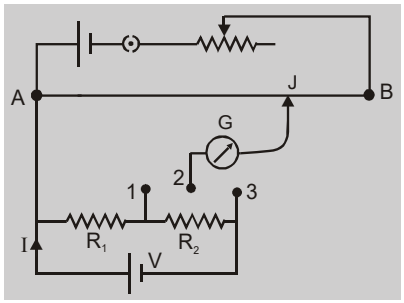
or $r = \left(\frac{E - V}{V}\right)R$

Key K open $E = x\ell_1$ $(AJ = \ell_1)$
Key K closed T.P.D. $V = x\ell_2$ $(AJ' = \ell_2)$

$$\boxed{r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R}$$

(*c*) Comparision of two resistances

Plug only in (1–2) potential difference across R_1 is balanced

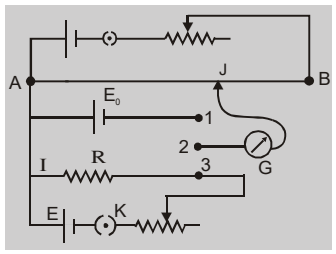


$I R_1 = x\ell_1$
Plug only in (2-3)
potential difference across (R_1+R_2) is balanced
$I(R_1+R_2) = x\ell_2$

$\frac{R_1 + R_2}{R_1} = \frac{\ell_2}{\ell_1}$ $\Rightarrow \frac{R_2}{R_1} = \frac{\ell_2 - \ell_1}{\ell_1}$

(*d*) Measurement of current

Plug only in (1–2)
$E_0 = x\ell_0$
Plug only in (2–3)



$V = IR = x\ell_1$

$I = \frac{\ell_1}{R} \times \frac{E_0}{\ell_0}$

TRAIN YOUR BRAIN

Example 25: There is a definite potential difference between the two ends of a potentiometer. Two cells are connected in such a way that first time they help each other, and second time they oppose each other. They are balanced on the potentiometer wire at 120 cm and 60 cm length respectively. Compare the electromotive force of the cells.

Sol. Suppose the potential gradient along the potentiometer wire = x and the emf's of the two cells are E_1 and E_2.

When the cells help each other, the resultant emf $= (E_1 + E_2)$
$E_1 + E_2 = x \times 120\,cm.$ .....(i)
When the cells oppose each other, the resultant emf $= (E_1 - E_2)$
$E_1 - E_2 = x \times 60\,cm.$ .....(ii)
From equation (i) and (ii)

$\frac{E_1 + E_2}{E_1 - E_2} = \frac{120\,cm}{60\,cm} = \frac{2}{1}$

or $E_1 + E_2 = 2(E_1 - E_2)$

or $3E_2 = E_1$ or $\frac{E_1}{E_2} = \frac{3}{1}$

Topicwise Questions

Microscopic Analysis, Drift Velocity Ohm's Law Conductivity

1. There is a current of 40 ampere in a wire of 10^{-6} m^2 area of cross-section. If the number of free electron per m^3 is 10^{29}, then the drift velocity will be

(a) 1.25×10^3 m/s (b) 2.50×10^{-3} m/s

(c) 25.0×10^{-3} m/s (d) 250×10^{-3} m/s

2. When current flows through a conductor, then the order of drift velocity of electrons will be

(a) 10^{10} m/sec (b) 10^{-2} cm/sec

(c) 10^4 cm/sec (d) 10^{-1} cm/sec

3. Every atom has one free electron in copper. If 1.1 ampere current is flowing in the wire of copper having 1 mm diameter, then the drift velocity (approx.) will be (Density of copper = 9×10^3 kg m^{-3} and atomic weight = 63)

(a) 0.3 mm/sec (b) 0.1 mm/sec

(c) 0.2 mm/sec (d) 0.2 cm/sec

4. Drift velocity v_d varies with the intensity of electric field as per the relation

(a) $v_d \propto E$ (b) $v_d \propto \frac{1}{E}$

(c) v_d = constant (d) $v_d \propto E^2$

5. A certain wire has a resistance R. The resistance of another wire identical with the first except having twice its diameter is

(a) 2R (b) 0.25 R

(c) 4R (d) 0.5 R

6. A metallic block has no potential difference applied across it, then the mean velocity of free electrons is T = absolute temperature of the block)

(a) Proportional to T

(b) Proportional to $\sqrt{T}$

(c) Zero

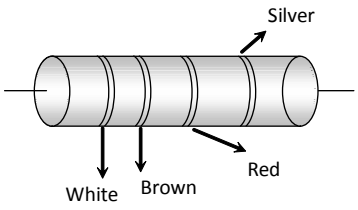
(d) Finite but independent of temperature

7. A uniform wire of resistance R is uniformly compressed along its length, until its radius becomes n times the original radius. Now resistance of the wire becomes

(a) $\frac{R}{n^4}$ (b) $\frac{R}{n^2}$

(c) $\frac{R}{n}$ (d) nR

8. In the figure a carbon resistor has bands of different colours on its body as mentioned in the figure. The value of the resistance is



(a) 2.2 $k\Omega$ (b) 3.3 $k\Omega$

(c) 5.6 $k\Omega$ (d) 9.1 $k\Omega$

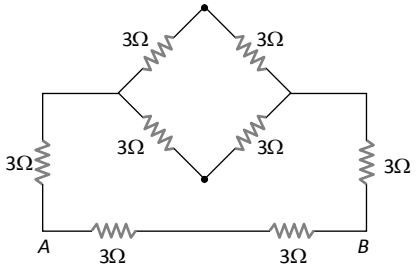
Series and Parallel Combination of Resistance

9. There are 8 equal resistances R. Two are connected in parallel, such four groups are connected in series, the total resistance of the system will be

(a) R/2 (b) 2R

(c) 4R (d) 8R

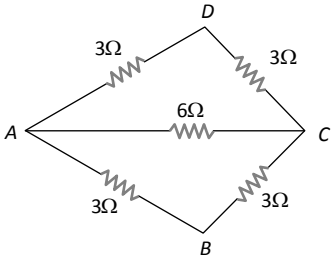
10. Equivalent resistance between A and B will be



(a) 2 Ω (b) 18 Ω

(c) 6 Ω (d) 3.6 Ω

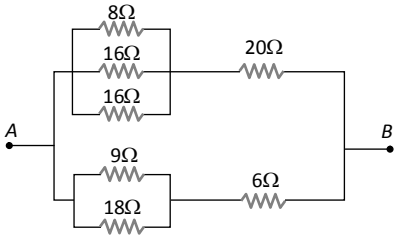
11. The effective resistance between the points A and B in the figure is



(a) 5Ω (b) 2Ω

(c) 3Ω (d) 4Ω

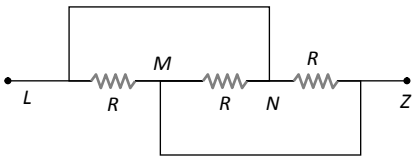
12. The equivalent resistance of the arrangement of resistances shown in adjoining figure between the points A and B is



(a) 6 Ω (b) 8 Ω

(c) 16 Ω (d) 24 Ω

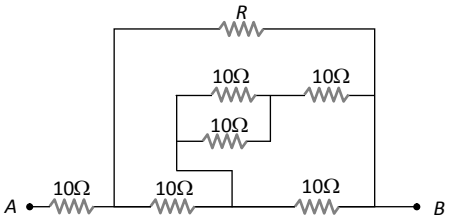
13. Three equal resistances each of value R are joined as shown in the figure. The equivalent resistance between M and N is



(*a*) R (*b*) $2R$

(*c*) $\frac{R}{2}$ (*d*) $\frac{R}{3}$

14. For what value of R the net resistance of the circuit will be $18\ \Omega$



(*a*) 8Ω (*b*) 10Ω

(*c*) 16Ω (*d*) 24Ω

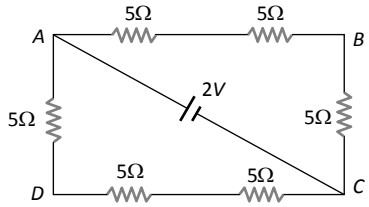
15. Two wires of equal diameters, of resistivities ρ_1 and ρ_2 and lengths l_1 and l_2, respectively, are joined in series. The equivalent resistivity of the combination is

(*a*) $\frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$ (*b*) $\frac{\rho_1 l_2 + \rho_2 l_1}{l_1 - l_2}$

(*c*) $\frac{\rho_1 l_2 + \rho_2 l_1}{l_1 + l_2}$ (*d*) $\frac{\rho_1 l_1 - \rho_2 l_2}{l_1 - l_2}$

Problem Solving using Ohm's Law

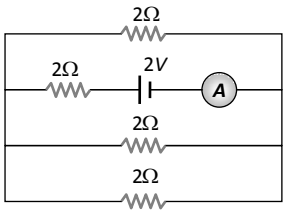
16. The potential difference between points A and B of adjoining figure is



(*a*) $\frac{2}{3}$V (*b*) $\frac{8}{9}$V

(*c*) $\frac{4}{3}$V (*d*) 2V

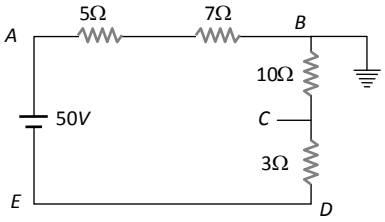
17. The reading of the ammeter as per figure shown is



(*a*) $\frac{1}{8}$A (*b*) $\frac{3}{4}$A

(*c*) $\frac{1}{2}$A (*d*) 2A

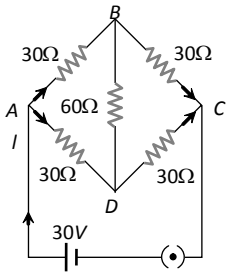
18. In the circuit shown, the point 'B' is earthed. The potential at the point 'A' is



(*a*) 14 V (*b*) 24 V

(*c*) 26 V (*d*) 50 V

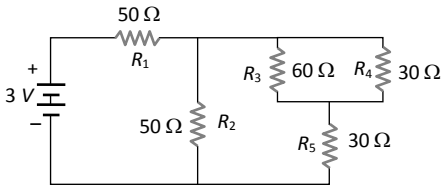
19. The current between B and D in the given figure is



(*a*) 1 amp (*b*) 2 amp

(*c*) Zero (*d*) 0.5 amp

20. In circuit shown below, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3 *volt*. The voltage across the resistance R_4 is

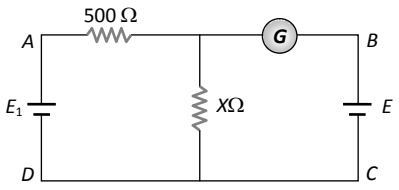


(*a*) 0.4 *V* (*b*) 0.6 *V*

(*c*) 1.2 *V* (*d*) 1.5 *V*

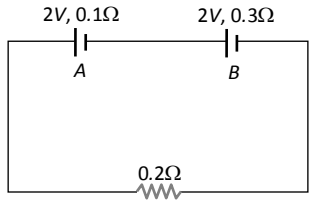
Kirchoff's Law and Equivalent Battery

21. In the adjoining circuit, the battery E_1 has an e.m.f. of 12 volt and zero internal resistance while the battery E has an e.m.f. of 2 volt. If the galvanometer G reads zero, then the value of the resistance X in ohm is



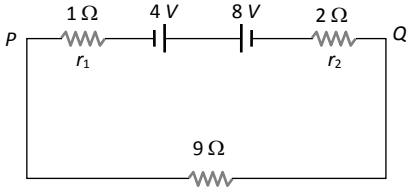
(*a*) 10 (*b*) 100
(*c*) 500 (*d*) 200

22. The internal resistances of two cells shown are 0.1 Ω and 0.3 Ω. If R = 0.2Ω, the potential difference across the cell



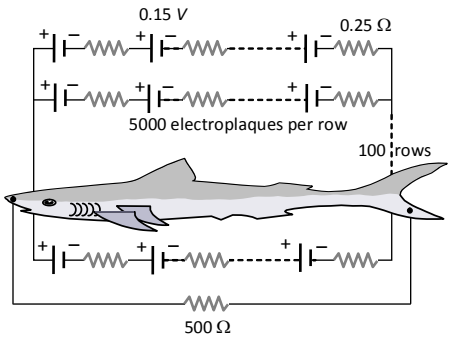
(*a*) *B* will be zero
(*b*) *A* will be zero
(*c*) *A* and *B* will be 2*V*
(*d*) *A* will be > 2V and *B* will be < 2V

23. Two batteries of e.m.f. 4V and 8 V with internal resistances 1 Ω and 2 Ω are connected in a circuit with a resistance of 9 Ω as shown in figure. The current and potential difference between the points P and Q are



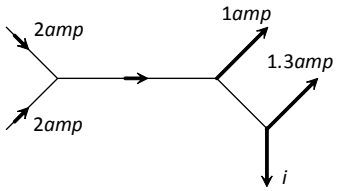
(*a*) $\frac{1}{3}$A and 3V (*b*) $\frac{1}{6}$A and 4V
(*c*) $\frac{1}{9}$A and 9V (*d*) $\frac{1}{2}$A and 12V

24. Eels are able to generate current with biological cells called electroplaques. The electroplaques in an eel are arranged in 100 rows, each row stretching horizontally along the body of the fish containing 5000 electroplaques. The arrangement is suggestively shown below. Each electroplaques has an emf of 0.15 V and internal resistance of 0.25 Ω



The water surrounding the eel completes a circuit between the head and its tail. If the water surrounding it has a resistance of 500 Ω, the current an eel can produce in water is about
(*a*) 1.5A (*b*) 3.0A
(*c*) 15A (*d*) 30A

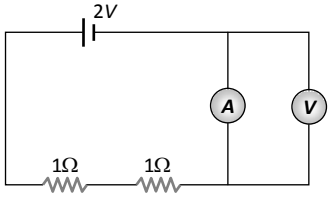
25. The figure below shows currents in a part of electric circuit. The current *i* is



(*a*) 1.7 amp (*b*) 3.7 amp
(*c*) 1.3 amp (*d*) 1 amp.

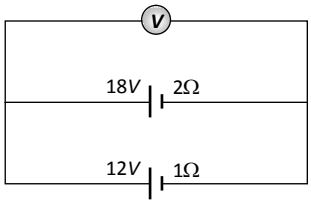
Current Measuring Instrument

26. In the circuit shown, A and V are ideal ammeter and voltmeter respectively. Reading of the voltmeter will be



(*a*) 2V (*b*) 1V
(*c*) 0.5V (*d*) Zero

27. Two batteries, one of emf 18 volts and internal resistance 2Ω and the other of emf 12 volt and internal resistance 1Ω, are connected as shown. The voltmeter V will record a reading of



(*a*) 15 volt (*b*) 30 volt
(*c*) 14 volt (*d*) 18 volt

28. In meter bridge or Wheatstone bridge for measurement of resistance, the known and the unknown resistances are interchanged. The error so removed is)

(*a*) End correction (*b*) Index error

(*c*) Due to temperature effect (*d*) Random error

29. A galvanometer can be converted into an ammeter by connecting

(*a*) Low resistance in series

(*b*) High resistance in parallel

(*c*) Low resistance in parallel

(*d*) High resistance in series

30. A 2 volt battery, a 15Ω resistor and a potentiometer of 100 cm length, all are connected in series. If the resistance of potentiometer wire is 5Ω, then the potential gradient of the potentiometer wire is

(*a*) 0.005 V/cm (*b*) 0.05 V/cm

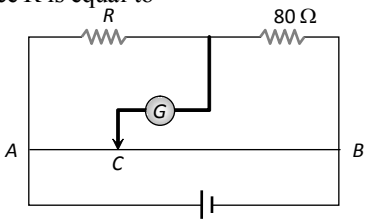
(*c*) 0.02 V/cm (*d*) 0.2 V/cm

31. An ammeter of 5 ohm resistance can read 5 mA. If it is to be used to read 100 volts, how much resistance is to be connected in series

(*a*) 19.9995Ω (*b*) 199.995Ω

(*c*) 1999.95Ω (*d*) 19995Ω

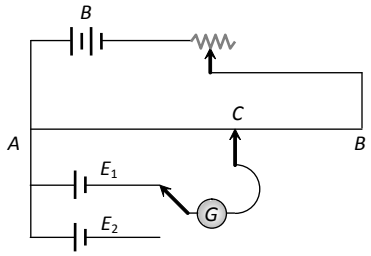
32. AB is a wire of uniform resistance. The galvanometer G shows no current when the length AC = 20cm and CB = 80 cm. The resistance R is equal to



(*a*) 2Ω (*b*) 8Ω

(*c*) 20Ω (*d*) 40Ω

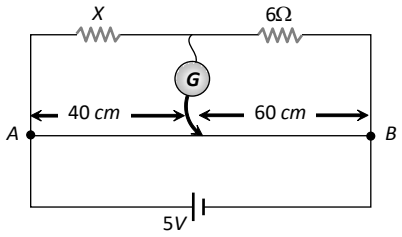
33. The circuit shown here is used to compare the e.m.f. of two cells E_1 and $E_2(E_1 > E_2)$. The null point is at C when the galvanometer is connected to E_1. When the galvanometer is connected to E_2, the null point will be



(*a*) To the left of C (*b*) To the right of C

(*c*) At C itself (*d*) Nowhere on AB

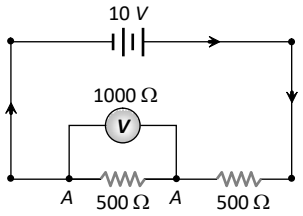
34. In the circuit shown, a meter bridge is in its balanced state. The meter bridge wire has a resistance 0.1 Ω/cm. The value of unknown resistance X and the current drawn from the battery of negligible resistance is



(*a*) 6 Ω, 5 amp (*b*) 10 Ω, 0.1 amp

(*c*) 4 Ω, 1.0 amp (*d*) 12 Ω, 0.5 amp

35. What is the reading of voltmeter in the following figure



(*a*) 3V (*b*) 2V

(*c*) 5V (*d*) 4V

Learning Plus

1. In the presence of an applied electric field ($\vec{E}$) in a metallic conductor.

(*a*) The electrons move in the direction of $\vec{E}$

(*b*) The electrons move in a direction opposite to $\vec{E}$

(*c*) The electrons may move in any direction randomly, but slowly drift in the direction of $\vec{E}$.

(*d*) The electrons move randomly but slowly drift in a direction opposite to $\vec{E}$.

2. A current (I)flows through a uniform wire of diameter (d) when the mean drift velocity is V. The same current will flow through a wire of diameter d/2 made of the same material if the mean drift velocity of the electron is

(*a*) V/4

(*b*) V/2

(*c*) 4V

(*d*) 2V

3. A steady current is passing through a linear conductor of non-uniform cross-section. The current density in the conductor is
(*a*) independent of area of cross-section
(*b*) directly proportional to area of cross-section
(*c*) inversely proportional to area of cross-section
(*d*) inversely proportional to the square root of area of cross-section

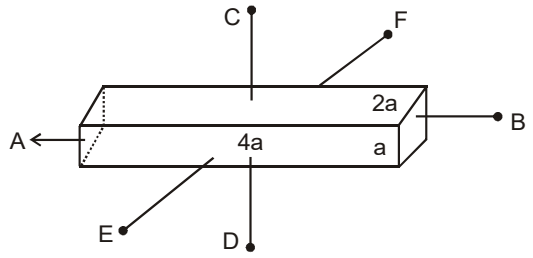
4. The resistance of wire is 20Ω. The wire is stretched to three times its length. Then the resistance will now be
(*a*) 6.67 Ω (*b*) 60 Ω
(*c*) 120 Ω (*d*) 180 Ω

5. In order to increase the resistance of a given wire of uniform cross section to four times its value, a fraction of its length is stretched uniformly till the full length of the wire becomes $\frac{3}{2}$ $I = neAv_d$ times the original length what is the value of this fraction?

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{8}$

(*c*) $\frac{1}{16}$ (*d*) $\frac{1}{6}$

6. A conductor with rectangular cross section has dimensions (a × 2a × 4a)as shown in figure. Resistance across AB is x, across CD is y and across EF is z. Then



(*a*) x = y = z (*b*) x > y > z
(*c*) y > z > x (*d*) x > z > y

7. In a wire of cross-section radius r, free electrons travel with drift velocity v when a current I flows through the wire. What is the current in another wire of half the radius and of the same material when the drift velocity is 2v ?
(*a*) 2I (*b*) I
(*c*) I/2 (*d*) I/4

8. Two coils connected in series have resistances 600 Ω and 300 Ω at 20°C and temperature coefficient of resistivity 0.001 K^{-1} and 0.004 K^{-1} respectively.
The resistance of the combination at temperature 50°C is
(*a*) 426 Ω (*b*) 954 Ω
(*c*) 1806 Ω (*d*) 214 Ω

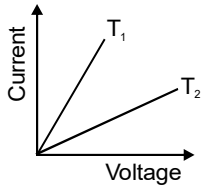
9. Two coils connected in series have resistances 600 Ω and 300 Ω at 20°C and temperature coefficient of resistivity 0.001 K^{-1} and 0.004 K^{-1} respectively.
The effective temperature coefficient of the combination is

(*a*) $\frac{1}{1000}$ degree^{-1} (*b*) $\frac{1}{250}$ degree^{-1}

(*c*) $\frac{1}{500}$ degree^{-1} (*d*) $\frac{3}{1000}$ degree^{-1}

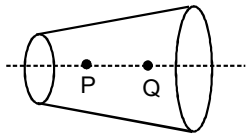
10. Two wires of the same material having radii in has the ratio 1:2, carry currents in the ratio 4 : 1. The ratio of drift velocities of electrons in them is:
(*a*) 1:16 (*b*) 16:1
(*c*) 1:4 (*d*) 4:1

11. The current in a metallic conductor is plotted against voltage at two different temperatures T_1 and T_2. Which is correct



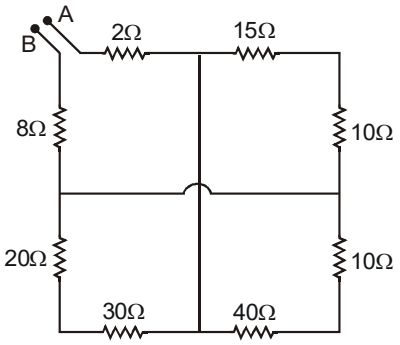
(*a*) $T_1 > T_2$ (*b*) $T_1 < T_2$
(*c*) $T_1 = T_2$ (*d*) none

12. A wire has a non-uniform cross-section as shown in figure. A steady current flows through it. The drift speed of electrons at points P and q is v_p and v_q



(*a*) $v_p = v_q$ (*b*) $v_p < v_q$
(*c*) $v_p > v_q$ (*d*) Data insufficient

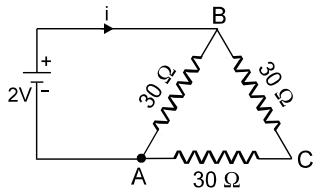
13. The equivalent resistance between points A and B is :



(*a*) $\frac{65}{2}\Omega$ (*b*) $\frac{45}{2}\Omega$

(*c*) $\frac{5}{2}\Omega$ (*d*) $\frac{91}{2}\Omega$

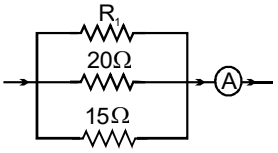
14. The current i in the circuit of figure is -



(*a*) $\frac{1}{45}$amp. (*b*) $\frac{1}{15}$amp.

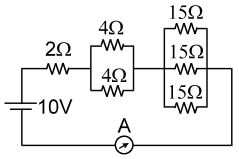
(*c*) $\frac{1}{10}$amp. (*d*) $\frac{1}{5}$amp.

15. In the given circuit the current flowing through the resistance 20 ohms is 0.3 ampere while the ammeter reads 0.8 ampere. What is the value of R_1 ?



(*a*) 30 ohms (*b*) 40 ohms
(*c*) 50 ohms (*d*) 60 ohms

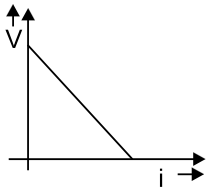
16. The current through the ammeter shown in figure is 1 A. If each of the 4Ω resistor is replaced by 2Ω resistor, the current in circuit will become nearly :

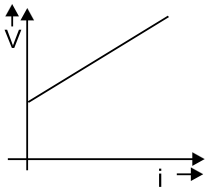


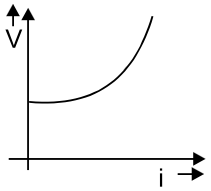
(*a*) $\frac{10}{9}$ A (*b*) $\frac{5}{4}$

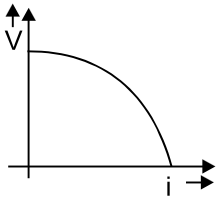
(*c*) $\frac{9}{8}$ A (*d*) $\frac{5}{8}$ A

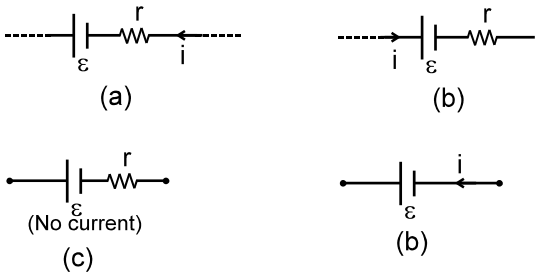
17. If internal resistance of a cell is proportional to current drawn from the cell. Then the best representation of terminal potential difference of a cell with current drawn from cell will be:

(*a*)


(*b*)


(*c*)


(*d*)


18.


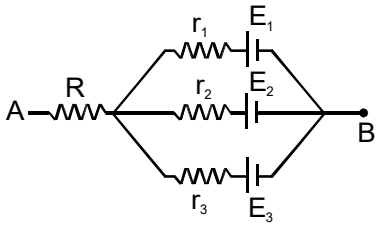
In which of the above cells, the potential difference between the terminals of a cell exceeds its emf.

(*a*) a (*b*) b
(*c*) c (*d*) d

19. A storage battery is connected to a charger for charging with a voltage of 12.5 Volts. The internal resistance of the storage battery is 1 Ω. When the charging current is 0.5 A, the emf of the storage battery is :

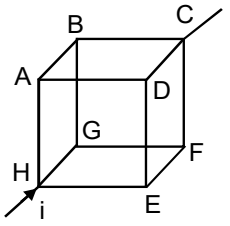
(*a*) 13 Volts (*b*) 12.5 Volts
(*c*) 12 Volts (*d*) 1.5 Volts

20. In the network shown the potential difference between A and B is ($R = r_1 = r_2 = r_3 = 1\ \Omega$, $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V)



(*a*) 1V (*b*) 2V
(*c*) 3V (*d*) 4V

21. In the box shown current i enters at H and leaves at C. If $i_{AB} = \frac{i}{6}, i_{DC} = \frac{2i}{3}, i_{HA} = \frac{i}{2}, i_{GF} = \frac{i}{6}, i_{HE} = \frac{i}{6}$, choose the branch in which current is zero



(*a*) BG (*b*) FC
(*c*) ED (*d*) none

22. If 2 bulbs rated 2.5 W – 110 V and 100 W – 110 V are connected in series to a 220 V supply then

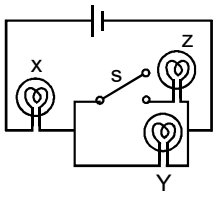
(*a*) 2.5 W bulb will fuse (*b*) 100 W bulb will fuse
(*c*) both will fuse (*d*) both will not fuse

23. Power generated across a uniform wire connected across a supply is H. If the wire is cut into n equal parts and all the parts are connected in parallel across the same supply, the total power generated in the wire is

(*a*) $\frac{H}{n^2}$ (*b*) n^2H

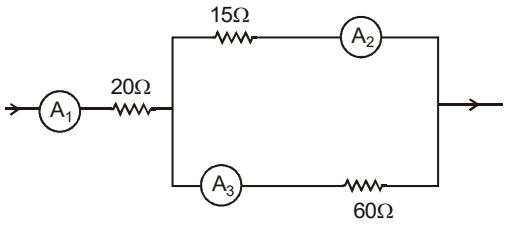
(*c*) nH (*d*) $\frac{H}{n}$

24. If X, Y and Z in figure are identical lamps, which of the following changes to the brightnesses of the lamps occur when switch S is closed ?



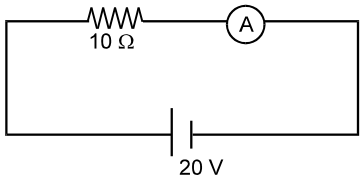
(*a*) X stays the same, Y decreases
(*b*) X increases, Y decreases
(*c*) X increases, Y stays the same
(*d*) X decreases, Y increases

25. If the reading of ammeter A_3 in figure is 0.75 A. Neglecting the resistance of the ammeters, the reading of ammeter A_2 will be :



(*a*) 1.5 A (*b*) 3 A
(*c*) 4.5 A (*d*) 6 A

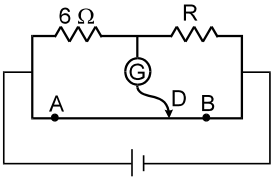
26. The ammeter shown in figure consists of a 480 Ω coil connected in parallel to a 20 Ω shunt. Find the reading of the ammeter.



(*a*) $\frac{50}{73}$ A (*b*) $\frac{40}{53}$ A

(*c*) $\frac{50}{93}$ A (*d*) $\frac{73}{50}$ A

27. The meter-bridge wire AB shown in figure is 50 cm long. When AD = 30 cm, no deflection occurs in the galvanometer. Find R.

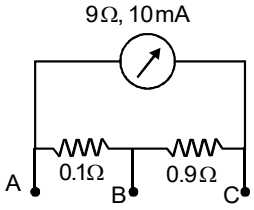


(*a*) 1 Ω (*b*) 2 Ω
(*c*) 3 Ω (*d*) 4 Ω

28. If an ammeter is to be used in place of a voltmeter then we must connect with the ammeter a

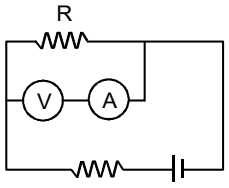
(*a*) Low resistance in parallel
(*b*) High resistance in parallel
(*c*) High resistance in series
(*d*) Low resistance in series

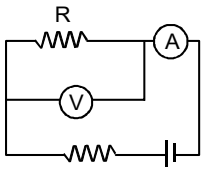
29. A milliammeter of range 10mA and resistance 9Ω is joined in a circuit as shown. The metre gives full-scale deflection for current I when A and B are used as its terminals, i.e., current enters at A and leaves at B (C is left isolated). The value of I is

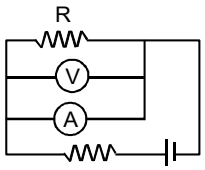


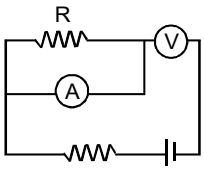
(*a*) 100 mA (*b*) 900 mA
(*c*) 1 A (*d*) 1.1 A

30. Which of the following wiring diagrams could be used to experimentally determine R using ohm's law? Assume an ideal voltmeter and an ideal ammeter.

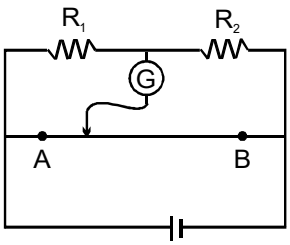
(*a*)


(*b*)


(*c*)


(*d*)


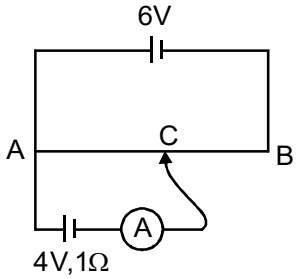
31. In the figure shown for gives values of R_1 and R_2 the balance point for Jockey is at 40 cm from A. When R_2 is shunted by a resistance of 10 Ω, balance shifts to 50 cm. R_1 and R_2 are (AB = 1m)



(*a*) $\frac{10}{3}\Omega, 5\Omega$ (*b*) $20\Omega, 30\Omega$

(*c*) $10\Omega, 15\Omega$ (*d*) $5\Omega, \frac{15}{2}\Omega$

32. A 6V battery of negligible internal resistance is connected across a uniform wire of length 1m. The positive terminal of another battery of emf 4V and internal resistance 1Ω is joined to the point A as shown in figure. The ammeter shows zero deflection when the jockey touches the wire at the point C. Then AC is equal to



(*a*) 2/3 m (*b*) 1/3 m
(*c*) 3/5 m (*d*) 1/2 m

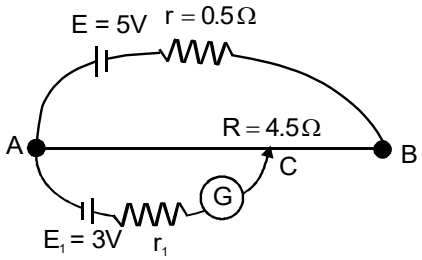
33. A potentiometer wire has length 10 m and resistance 10Ω. It is connected to a battery of EMF 11 volt and internal resistance 1Ω, then the potential gradient in the wire is
(*a*) 10 V/m (*b*) 1 V/m
(*c*) 0.1 V/m (*d*) none

34. The length of a potentiometer wire is *l*. A cell of emf E is balanced at a length *l*/3 from the positive end of the wire. if the length of the wire is increased by *l*/2. At what distance will the same cell give a balance point.

(*a*) $\frac{2l}{3}$ (*b*) $\frac{l}{2}$

(*c*) $\frac{l}{6}$ (*d*) $\frac{4l}{3}$

35. In the given potentiometer circuit length of the wire AB is 3m and resistance is R = 4.5 Ω. The length AC for no deflection in galvanometer is



(*a*) 2m (*b*) 1.8m
(*c*) dependent on r_1 (*d*) none of these

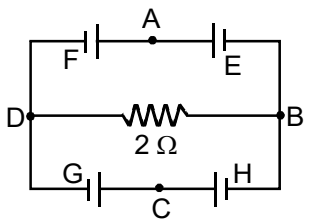
Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/ NUMERICAL

1. A current passes through a wire of non-uniform cross-section. Which of the following quantities are independent of the cross-section?
(*a*) the charge crossing in a given time interval
(*b*) drift speed
(*c*) current density
(*d*) free-electron density.

2. The current density in a wire is 10 A/cm^2 and the electric field in the wire is 5 V/cm. If ρ = resistivity of material, σ = conductivity of the material then (in S.I. units):
(*a*) $\rho = 5 \times 10^{-3}$ (*b*) $\rho = 200$
(*c*) $\sigma = 5 \times 10^{-3}$ (*d*) $\sigma = 200$

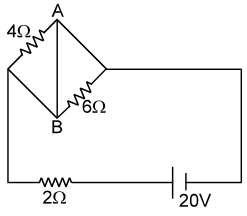
3. In the circuit shown E, F, G and H are cells of e.m.f. 2V, 1V, 3V and 1V respectively and their internal resistances are 2Ω, 1Ω, 3Ω and 1Ω respectively.



(*a*) $V_D - V_B = -2/13$ V
(*b*) $V_D - V_B = 2/13$ V
(*c*) $V_G = 21/13$ V = potential difference across G.
(*d*) $V_H = 19/13$ V = potential difference across H.

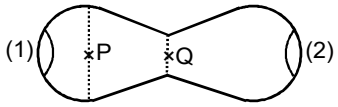
4. A bulb is connected to an ideal battery of emf 10 V so that the resulting current is 10 mA. When the bulb is connected to 220 V mains (ideal), the current is 50 mA. Choose the correct alternative(s)
(*a*) In the first case, the resistance of the bulb is 1kΩ and in second case, it is 4.4 kΩ.
(*b*) It is not possible since ohm's law is not followed
(*c*) The increase in resistance is due to heating of the filament of the bulb when it is connected to 220 V mains
(*d*) None of these

5. In the circuit shown in figure



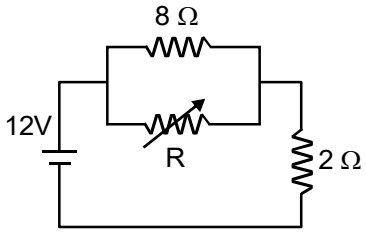
(*a*) power supplied by the battery is 200 watt
(*b*) current flowing in the circuit is 5 A
(*c*) potential difference across 4 Ω resistance is equal to the potential difference across 6Ω resistance
(*d*) current in wire AB is zero

6. A metallic conductor of irregular cross-section is as shown in the figure. A constant potential difference is applied across the ends (*a*)and (*b*). Then :



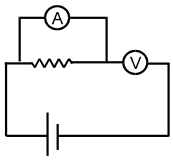
(*a*) the current at the cross-section P equals the current at the cross-section Q
(*b*) the electric field intensity at P is less than that at
(*c*) the rate of heat generated per unit time at Q is greater than that at P
(*d*) the number of electrons crossing per unit area of cross-section at P is less than that at

7. The value of the resistance R in figure is adjusted such that power dissipated in the 2Ω resistor is maximum. Under this condition



(*a*) R = 0
(*b*) R = 8Ω
(*c*) power dissipated in the 2Ω resistor is 72 W
(*d*) power dissipated in the 2Ω resistor is 8 W

8. By mistake, a voltmeter is placed in series and an ammeter in parallel with a resistance in an electric circuit, with a cell in series.



(*a*) The main current in the circuit will be very low and almost all current will flow through the ammeter, if resistance of ammeter is much smaller than the resistance in parallel.
(*b*) If the devices are ideal, a large current will flow through the ammeter and it will be damaged
(*c*) If the devices are ideal, ammeter will read zero current and voltmeter will read the emf of cell
(*d*) The devices may get damaged if emf of the cell is very high and the meters are nonideal.

9. A micro-ammeter has a resistance of 100 Ω and full scale range of 50 μA. It can be used as a voltmeter and an ammeter of a higher range provided a resistance is added to it. Pick the correct range and resistance combination (s):
(*a*) 50 V range with 10 kΩ resistance is series
(*b*) 10 V range with 200 kΩ resistance in series
(*c*) 5 mA range with 1 Ω resistance in parallel
(*d*) 10 mA range with 1 Ω resistance in parallel

10. In a potentiometer arrangement. E_1 is the cell establishing current in primary circuit. E_2 is the cell to be measured. AB is the potentiometer wire and G is a galvanometer. Which of the following are the essential condition for balance to be obtained.
(*a*) The emf of E_1 must be greater than the emf of E_2
(*b*) Either the positive terminals of both E_1 and E_2 or the negative terminals of both E_1 and E_2 must be joined to one end of potentiometer wire.
(*c*) The positive terminals of E_1 and E_2 must be joined to one end of potentiometer wire.
(*d*) The resistance of G must be less than the resistance of AB.

11. Why is high tension wire thick wire. Select the most appropriate option
(*a*) so that more current may flow
(*b*) so that resistance will be less thereby reducing power loss in the transmission line.
(*c*) so that it may bear high tension & therefore sag less.
(*d*) so that when in future population increases, the existing wire may serve the purpose.

12. Consider a bird having effective resistance 10Ω between its feet sitting on this high tension wire. The distance between its feet is 10 cm. The potential difference between the feet of the bird is approximately.
(*a*) 0.1 V (*b*) 1 V
(*c*) 10 V (*d*) 0.05 V

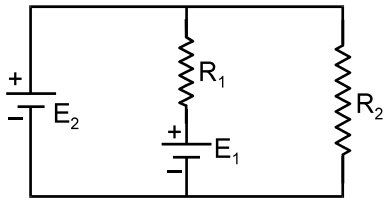
13. In the above question find the current through the bird.
(*a*) 10 A (*b* 1 A
(*c*) 0.01 A (*d*) 0.005 A

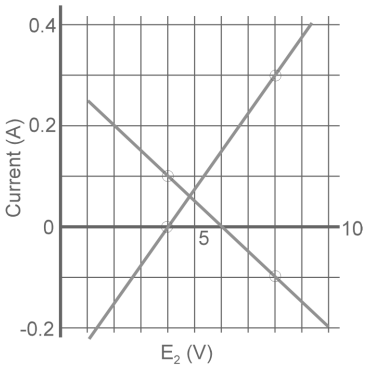
14. In the above question if the potential difference between HT at A and corresponding point on earth wire is 11 kV, find potential difference between point B and corresponding point on earth wire.
(*a*) 1 KV (*b*) 2 KV
(*c*) 9 KV (*d*) 10 KV

15. If the 'Critical Current' for the bird is 0.1 A, find the maximum power at 11 KV that can be transmitted at point A so that the bird may not get shock. Assume that the distance between the feet is 10 cm and the resistance of the bird is 10 Ω.
(*a*) 111 KW (*b*) 11 KW
(*c*) 101 KW (*d*) 110011 KW

Comprehension Type Questions-2 (No. 16 to 18)

In the circuit given below, both batteries are ideal. Emf E_1 of battery 1 has a fixed value, but emf E_2 of battery 2 can be varied between 1.0 V and 10.0 V. The graph gives the currents through the two batteries as a function of E_2, but are not marked as which plot corresponds to which battery. But for both plots, current is assumed to be negative when the direction of the current through the battery is opposite the direction of that battery's emf. (direction of emf is from negative to positive)





16. The value of emf E_1 is
(*a*) 8V (*b*) 6V
(*c*) 4V (*d*) 2V

17. The resistance R_1 has value
(*a*) 10 Ω (*b*) 20 Ω
(*c*) 30 Ω (*d*) 40 Ω

18. The resistance R_2 is equal to :
(*a*) 10 Ω (*b*) 20 Ω
(*c*) 30 Ω (*d*) 40 Ω

19. Match the following :
The following table gives the lengths of four copper rods at the same temperature, their diameters, and the potential differences between their ends.

Rod	Length	Diameter	Potential Difference
1	L	3d	V
2	2L	d	3V
3	3L	2d	2V
4	3L	d	V

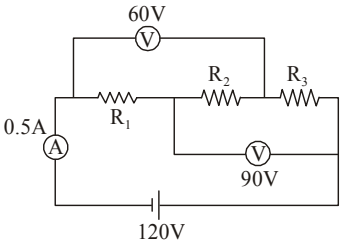
Correctly match the physical quantities mentioned in the left column with the rods as marked.
(*a*) Greatest Drift speed of the electrons. (p) Rod 1
(*b*) Greatest Current (q) Rod 2
(*c*) Greatest rate of thermal energy produced (r) Rod 3
(*d*) Greatest Electric field (s) Rod 4

20. Match the statements in Column I with the current element in Column II.

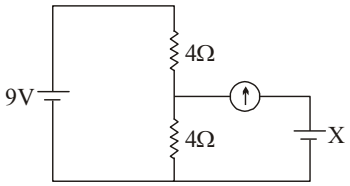
Column - I	Column - II
(*a*) Current always flows from higher potential to lower potential	(p) A Resistor
(*b*) Energy dissipated in an element is always zero	(q) Ideal cell/ Battery
(*c*) Current flow through the element is always zero	(r) Non-Ideal cell/ Battery
(*d*) Potential difference may be zero	(s) Short-circuited resistor

NUMERICAL VALUE BASED

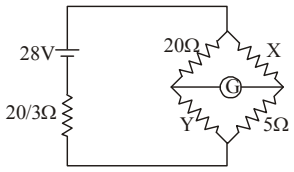
21. In the circuit shown, all voltmeters & ammeter are ideal. The internal resistance of battery is 20Ω. The readings of voltmeter and ammeter are marked in the figure. What is resistance R_2 (in ohm)?



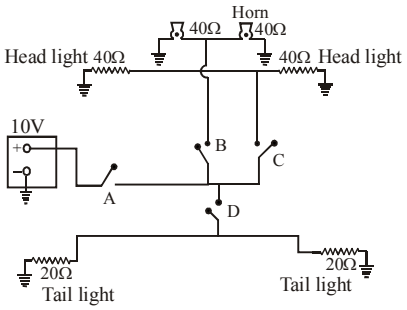
22. If each battery in figure has internal resistance of 1Ω, what should be the e.m.f. of X (in V) for there to be no deflection on the galvanometer?



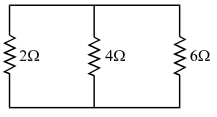
23. In the balanced wheatstone bridge arrangement, the current drawn from the battery is 1.4A. Find resistance X in Ω.



24. Figure shows an automobile circuit. How much power (in watt)is dissipated by the automobile circuit when switches A, B, C and D are all closed.



25. In one of the branches of the circuit shown in figure a 11V battery is to be inserted so that it dissipates minimum power. What will be current (in Ampere) through the 2Ω resistance for this position of the battery?



JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300 – 400 K, is best described by: **[*JEE Main - 2016*]**

(*a*) Linear increase for Cu, linear increase for Si.
(*b*) Linear increase for Cu, exponential increase for Si.
(*c*) Linear increase for Cu, exponential decrease for Si.
(*d*) Linear decrease for Cu, linear decrease for Si.

2. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A , is : **[*JEE Main - 2016*]**

(*a*) 0.01Ω (*b*) 2Ω
(*c*) 0.1 Ω (*d*) 3Ω

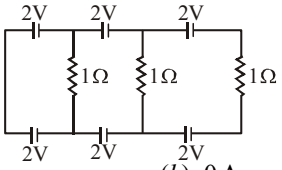
3. Which of the following statements is false ? **[*JEE Main-2017*]**

(*a*) A rheostat can be used as a potential divider
(*b*) Kirchhoff's second law represents energy conservation
(*c*) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.
(*d*) In a balanced wheatstone bridge, if the cell and the galvanometer are exchanged, the null point is disturbed.

4. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0 – 10 V is: **[*JEE Main-2017*]**

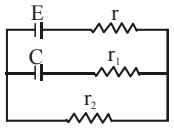
(*a*) $2.535 \times 10^3\ \Omega$ (*b*) $4.005 \times 10^3\ \Omega$
(*c*) $1.985 \times 10^3\ \Omega$ (*d*) $2.045 \times 10^3\ \Omega$

5. In the above circuit the current in each resistance is: **[*JEE Main-2017*]**



(*a*) 0.5 A (*b*) 0 A
(*c*) 1 A (*d*) 0.25 A

6. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be : **[*JEE Main-2017*]**



(*a*) $CE\frac{r_2}{(r+r_2)}$ (*b*) $CE\frac{r_1}{(r_1+r)}$

(*c*) CE (*d*) $CE\frac{r_1}{(r_2+r)}$

7. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **[*JEE Main-2018*]**

(*a*) 1.5 Ω (*b*) 2 Ω
(c) 2.5 Ω (*d*) 1 Ω

8. Two batteries with e.m.f 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between. **[*JEE Main-2018*]**

(*a*) 11.5 V and 11.6 V (*b*) 11.4 V and 11.5 V
(*c*) 11.7 V and 11.8 V (*d*) 11.6 V and 11.7 V

9. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances ? **[*JEE Main-2018*]**

(*a*) 505 Ω (*b*) 550 Ω
(*c*) 910 Ω (*d*) 990 Ω

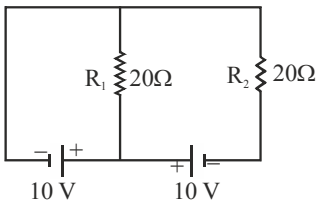
10. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere / division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt is : **[*JEE Main - 2019 (January)*]**

(*a*) 100 Ω (*b*) 120 Ω
(*c*) 80 Ω (*d*) 125 Ω

11. A 2 W carbon resistor is colour coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is: **[*JEE Main - 2019 (January)*]**

(*a*) 20 mA (*b*) 100 mA
(*c*) 0.4 mA (*d*) 63 mA

12. In the given circuit, cells have zero internal resistance. The currents (in Amperes)passing through resistance R_1 and R_2 respectively, are: **[*JEE Main - 2019 (January)*]**



(*a*) 1, 2 (*b*) 2, 2
(*c*) 0.5, 0 (*d*) 0, 1

13. Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then: **[*JEE Main - 2019 (January)*]**

(*a*) P_1=16 W, P_2 = 4 W (*b*) P_1=16 W, P_2 = 9 W
(*c*) P_1 = 9 W, P_2 = 16 W (*d*) P_1 = 4 W, P_2 = 16 W

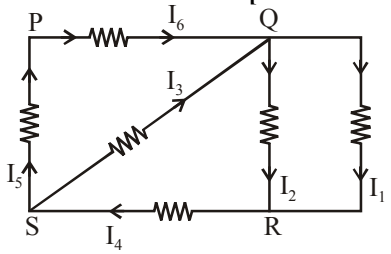
14. A copper wire is stretched to make it 0.5% longer. The percentage change in its electric resistance if its volume remains unchanged is **[*JEE Main - 2019 (January)*]**

(*a*) 2.0% (*b*) 2.5%

(*c*) 1.0% (*d*) 0.5%

15. In the given circuit diagram currents, $I_1 = 0.3$ A, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2, I_3 and I_6, respectively are :

[*JEE Main - 2019 (January)*]

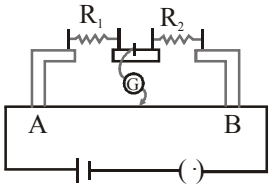


(*a*) 1.1 A, –0.4 A, 0.4 A (*b*) 1.1 A, 0.4 A, 0.4 A

(*c*) 0.4 A, 1.1 A, 0.4 A (*d*) –0.4 A, 0.4 A, 1.1 A

16. In the experimental setup of metre bridge shown in the figure, the null point is obtained at a dsitance of 40cm from A. If a 10Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is :

[*JEE Main - 2019 (January)*]



(*a*) 20Ω (*b*) 40Ω

(*c*) 60Ω (*d*) 30Ω

17. In a Wheatstone bridge (see figure) Resistance P and Q are approximately equal. When R = 400 Ω, the bridge is balanced. On interchanging P and Q, the value of R, for balance is 405 Ω. The value of X is close to

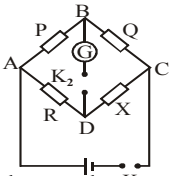
[*JEE Main - 2019 (January)*]

(*a*) 401.5 ohm

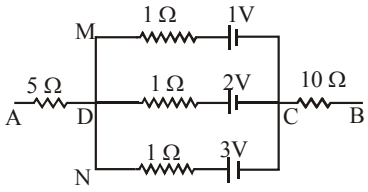
(*b*) 404.5 ohm

(*c*) 403.5 ohm

(*d*) 402.5 ohm



18. In the circuit shown, the potential difference between A and B is : **[*JEE Main - 2019 (January)*]**



(*a*) 1V (*b*) 2V

(*c*) 3V (*d*) 4V

19. Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be :

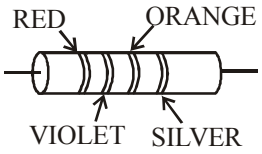
[*JEE Main - 2019 (January)*]

(*a*) 60 W (*b*) 240 W

(*c*) 120 W (*d*) 30 W

20. A resistance is shown in the figure. Its value and tolerance are given respectively by :

[*JEE Main - 2019 (January)*]



(*a*) 270 Ω, 10% (*b*) 27 kΩ, 10%

(*c*) 27 kΩ, 20% (*d*) 270 kΩ, 5%

21. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is $9 \times 10^{28}/m^3$, value of v in mm/s is close to (Take charge of electron to be 1.6×10^{-19} C)

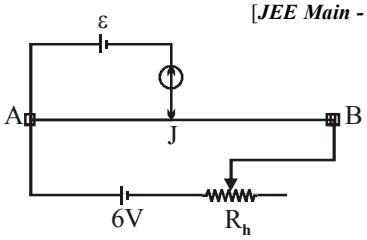
[*JEE Main - 2019 (January)*]

(*a*) 0.02 (*b*) 3

(*c*) 2 (*d*) 0.2

22. The resistance of the meter bridge AB in given figure is 4Ω. With a cell of emf ε = 0.5 V and rheostat resistance R_h = 2 Ω, the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$, the same null point J is found for $R_h = 6\ \Omega$. The emf ε_2 is:

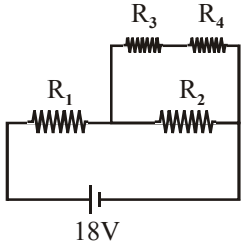
[*JEE Main - 2019 (January)*]



(*a*) 0.4 V (*b*) 0.3 V

(*c*) 0.6 V (*d*) 0.5 V

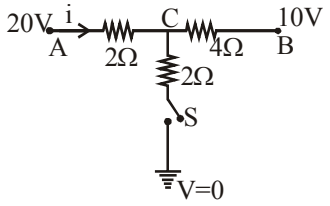
23. In the given circuit, the internal resistance of the 18 V cell is negligible. If $R_1 = 400\Omega$, $R_3 = 100\Omega$ and $R_4 = 500\Omega$ and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be **[*JEE Main - 2019 (January)*]**



(*a*) 300 Ω (*b*) 450 Ω

(*c*) 550 Ω (*d*) 230 Ω

24. When the switch S, in the circuit shown, is closed, then the value of current i will be **[*JEE Main - 2019 (January)*]**

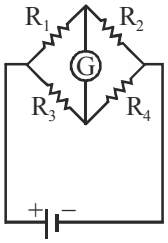


(*a*) 3A (*b*) 5A
(*c*) 4A (*d*) 2A

25. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal voltage supply of 11 V is connected across it is:
[*JEE Main - 2019 (January)*]

(*a*) 11×10^{-5} W (*b*) 11×10^{-3} W
(*c*) 11×10^{-4} W (*d*) 11×10^{5} W

26. The Wheatstone bridge shown in Figure, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80 Ω and 40 Ω, respectively. Assuming that the colour code for the carbon resistors given their accurate values, the colour code for the carbon resistor, used as R_3 would be :
[*JEE Main - 2019 (January)*]

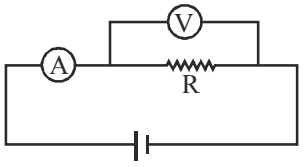


(*a*) Brown, Blue, Brown
(*b*) Brown, Blue, Black
(*c*) Red, Green, Brown
(*d*) Grey, Black, Brown

27. A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer)deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of :
[*JEE Main - 2019 (January)*]

(*a*) 250 ohm (*b*) 200 ohm
(*c*) 6200 ohm (*d*) 6250 ohm

28. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{l}$, where V and l are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is : **[*JEE Main - 2019 (January)*]**

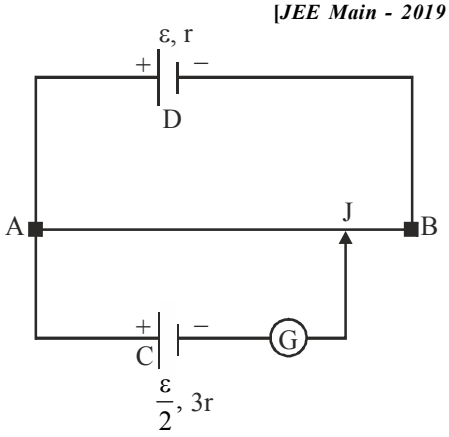


(*a*) 600 Ω (*b*) 570 Ω
(*c*) 35 Ω (*d*) 350 Ω

29. A uniform metallic wire has resistance of 18Ω and is bent into an equilateral triangle, then resistance between any two vertices of the triangle is **[*JEE Main - 2019 (January)*]**

(*a*) 4 Ω (*b*) 8 Ω
(*c*) 12 Ω (*d*) 2 Ω

30. A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf ε/2 and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is:
[*JEE Main - 2019 (January)*]

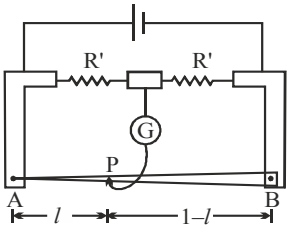


(*a*) $\frac{11}{12}$L (*b*) $\frac{11}{24}$L
(*c*) $\frac{13}{24}$L (*d*) $\frac{5}{12}$L

31. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5 Ω. The value of R, to give a difference of 5 mV across 10 cm of potentiometer wire, is:
[*JEE Main - 2019 (January)*]

(*a*) 490 Ω (*b*) 480 Ω
(*c*) 395 Ω (*d*) 495 Ω

32. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{d\ell}$ of its resistance R with length ℓ is $\frac{dR}{d\ell} \propto \frac{1}{\sqrt{\ell}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP? **[*JEE Main - 2019 (January)*]**



(*a*) 0.2 m (*b*) 0.3 m
(*c*) 0.25 m (*d*) 0.35 m

33. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60º at the centre, the equivalent resistance between these two points will be :

[*JEE Main-2019 (April)*]

(*a*) $\frac{12}{5}\Omega$ (*b*) $\frac{5}{3}\Omega$

(*c*) $\frac{5}{2}\Omega$ (*d*) $\frac{7}{2}\Omega$

34. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A?

[*JEE Main-2019 (April)*]

(*a*) 0.2 ohm (*b*) 0.002 ohm

(*c*) 0.02 ohm (*d*) 0.5 ohm

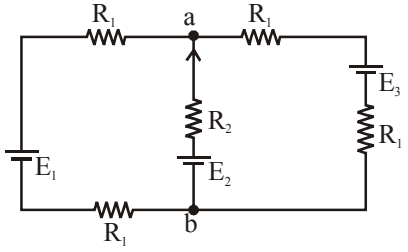
35. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25 fs (femto second), it's approximate resistivity is:-
($m_e = 9.1 \times 10^{-31}$ kg) **[*JEE Main-2019 (April)*]**

(*a*) 10^{-5} Ωm (*b*) 10^{-6} Ωm

(*c*) 10^{-7} Ωm (*d*) 10^{-8} Ωm

36. For the circuit shown, with $R_1 = 1.0\Omega$, $R_2 = 2.0\Omega$, $E_1 = 2$ V and $E_2 = E_3 = 4$ V, the potential difference between the points 'a' and 'b' is approximately (in V):

[*JEE Main-2019 (April)*]



(*a*) 2.7 (*b*) 3.3

(*c*) 2.3 (*d*) 3.7

37. A 200Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be :

[*JEE Main-2019 (April)*]

(*a*) 100 Ω (*b*) 400 Ω

(*c*) 500 Ω (*d*) 300 Ω

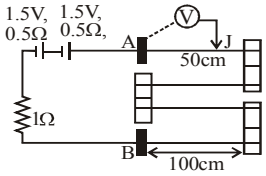
38. Space between two concentric conducting spheres of radii a and b (b > a)is filled with a medium of resistivity ρ. The resistance between the two spheres will be :

[*JEE Main-2019 (April)*]

(*a*) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (*b*) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

(*c*) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ (*d*) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$

39. In the circuit shown, a four wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01 Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be :- **[*JEE Main-2019 (April)*]**



(*a*) 0.20 V (*b*) 0.25 V

(*c*) 0.75 V (*d*) 0.50V

40. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when :-

[*JEE Main - 2019(April)*]

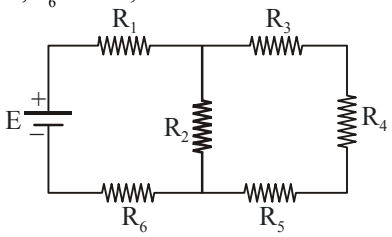
(*a*) R = 1000 r (*b*) R = 0.001 r

(*c*) R = 2r (*d*) R = r

41. In the figure shown, what is the current (in Ampere) drawn from the battery ? You are given :

[*JEE Main-2019 (April)*]

$R_1 = 15\Omega$, $R_2 = 10\Omega$, $R_3 = 20\Omega$, $R_4 = 5\Omega$,
$R_5 = 25\Omega$, $R_6 = 30\,\Omega$, $E = 15$V

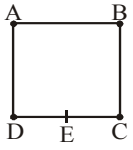


(*a*) $\frac{7}{18}$ (*b*) $\frac{13}{24}$

(*c*) $\frac{9}{32}$ (*d*) $\frac{20}{3}$

42. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is : (E is mid-point of arm CD)

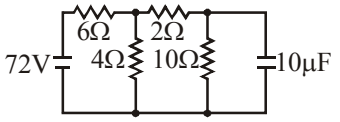
[*JEE Main - 2019(April)*]



(*a*) R (*b*) $\frac{1}{16}$R

(*c*) $\frac{7}{64}$R (*d*) $\frac{3}{4}$R

43. Determine the charge on the capacitor in the following circuit: **[*JEE Main-2019 (April)*]**



(*a*) 2μC (*b*) 60 μC

(*c*) 200 μC (*d*) 10 μC

44. A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to:

[*JEE Main-2019 (April)*]

(*a*) 10V (*b*) 20V
(*c*) 40V (*d*) 15V

45. One kg of water, at 20ºC, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to : [Specific heat of water = 4200 J/kg ºC), Latent heat of water = 2260 kJ/kg]

[*JEE Main-2019 (April)*]

(*a*) 3 minutes (*b*) 22 minutes
(*c*) 10 minutes (*d*) 16 minutes

46. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to I_0 ($I_0 > I_g$) by connecting a shunt resistance R_A to it and (ii)into a voltmeter of range 0 to V(where $V = GI_0$) by connecting a series resistance R_V to it. Then,

[*JEE Main-2019 (April)*]

(*a*) $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

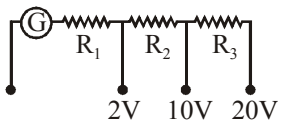
(*b*) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(*c*) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

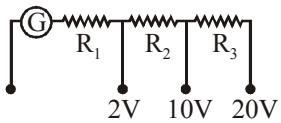
(*d*) $R_A R_V = G^2\left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

47. A galvanometer of resistance 100Ω has 50 divisions on its scale and has sensitivity of 20μA/division. It is to be converted to a voltmeter with three ranges, of 0-2 V, 0-10 V and 0-20 V. The appropriate circuit to do so is

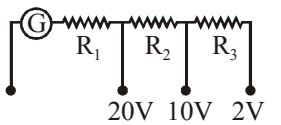
[*JEE Main-2019 (April)*]

(*a*) 

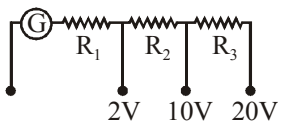
$R_1 = 1900\ \Omega$
$R_2 = 9900\ \Omega$
$R_3 = 19900\ \Omega$

(*b*) 

$R_1 = 2000\ \Omega$
$R_2 = 8000\ \Omega$
$R_3 = 10000\ \Omega$

(*c*) 

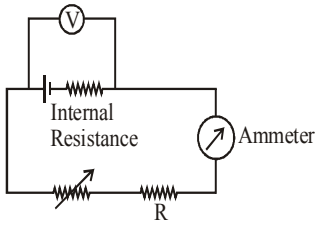
$R_1 = 19900\ \Omega$
$R_2 = 9900\ \Omega$
$R_3 = 1900\ \Omega$

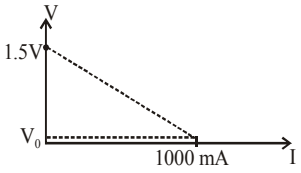
(*d*) 

$R_1 = 1900\ \Omega$
$R_2 = 8000\ \Omega$
$R_3 = 10000\ \Omega$

48. To verify Ohm's law, a student connects the voltmeter across the battery as shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained. If V_0 is almost zero, identify the correct statement

[*JEE Main-2019 (April)*]

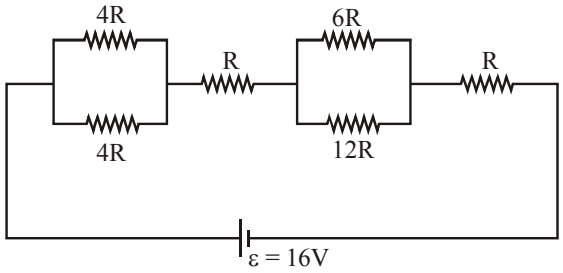




(*a*) The value of the resistance R is 1.5Ω
(*b*) The emf of the battery is 1.5 V and the value of R is 1.5 Ω
(*c*) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω
(*d*) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA.

49. The resistive network shown below is connected to a D.C. source of 16V. The power consumed by the network is 4 Watt. The value of R is :

[*JEE Main-2019 (April)*]



(*a*) 8Ω (*b*) 6Ω
(c) 1Ω (*d*) 16 Ω

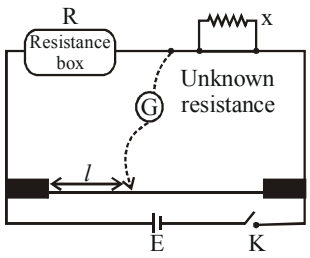
50. A current of 5 A passes through a copper conductor (resistivity = 1.7×10^{-8} Ωm)of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s.

[*JEE Main-2019 (April)*]

(*a*) 1.3 m^2/Vs (*b*) 1.5 m^2/Vs
(*c*) 1.8 m^2/Vs (*d*) 1.0 m^2/Vs

51. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure

[*JEE Main-2019 (April)*]



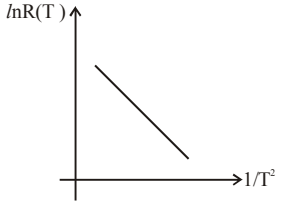
SI.No.	R(Ω)	*l*(cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the reading is inconsistent?

(*a*) 4 (*b*) 1
(*c*) 2 (*d*) 3

52. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line. One may conclude that:

[*JEE Main-2019 (April)*]



(*a*) $R(T) = \frac{R_0}{T^2}$

(*b*) $R(T) = R_0 e^{-T^2/T_0^2}$

(*c*) $R(T) = R_0 e^{-T_0^2/T^2}$

(*d*) $R(T) = R_0 e^{T^2/T_0^2}$

53. A moving coil galvanometer allows a full scale current of 10^{-4}A. A series resistance of 2MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0-10 mA is:

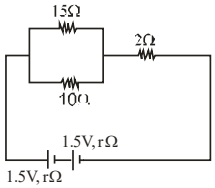
[*JEE Main-2019 (April)*]

(*a*) 200Ω (*b*) 100Ω

(*c*) 10Ω (*d*) 500Ω

54. In the given circuit, an ideal voltmeter connected across the 10Ω resistance reads 2V. The internal resistance r, of each cell is :

[*JEE Main-2019 (April)*]

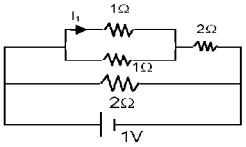


(*a*) 1Ω (*b*) 1.5Ω

(*c*) 0Ω (*d*) 0.5Ω

55. The current I_1 (in A)flowing through 1Ω resistor in the following circuit is

[*JEE Main-2020 (January)*]



(*a*) 0.2 (*b*) 0.4

(*c*) 0.5 (*d*) 0.25

56. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of building will be:

[*JEE Main-2020 (January)*]

(*a*) 20A (*b*) 15A

(*c*) 10A (*d*) 25 A

57. The balancing length for a cell is 560 cm in a potentiometer experiment. when an external resistance of 10Ω is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer then value of N is_____.

[*JEE Main-2020 (January)*]

58. The length a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5V and internal resistance of 20Ω, the null point of it is found to be at 1000 cm. The resistance of whole wire is:

[*JEE Main-2020 (January)*]

(*a*) 60Ω (*b*) 120Ω

(*c*) 100Ω (*d*) 80Ω

59. Four resistances of 15Ω, 12Ω, 4Ω and 10Ω respectively in cyclic order to form Wheatstone's network. The resistance that is to be connected in parallel with the resistance of 10Ω to balance the network is _____Ω.

[*JEE Main-2020 (January)*]

60. The series combination of two batteries, both of the same emf 10V, but different internal resistance of 20Ω and 5Ω, is connected to the parallel combination of two resistors 30Ω and RΩ. The voltage difference across the battery of internal resistance 20Ω is zero, the value of R (in Ω) is

[*JEE Main-2020 (January)*]

61. A galvanometer having a coil resistance 100Ω gives a full scale deflection when a current of 1mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V ?

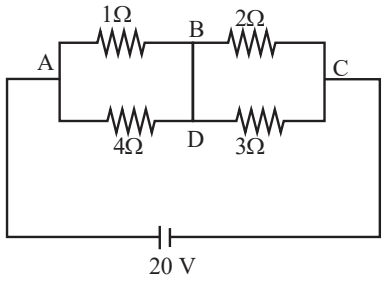
[*JEE Main-2020 (January)*]

(*a*) 8.9kΩ (*b*) 10kΩ

(*c*) 9.9kΩ (*d*) 7.9kΩ

62. In the given circuit diagram, a wire is joining points B and D. The current in this wire is :

[*JEE Main-2020 (January)*]

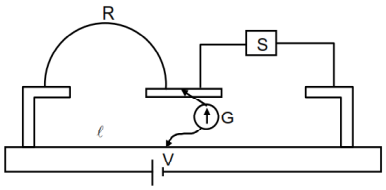


(*a*) zero (*b*) 2A

(*c*) 0.4A (*d*) 4A

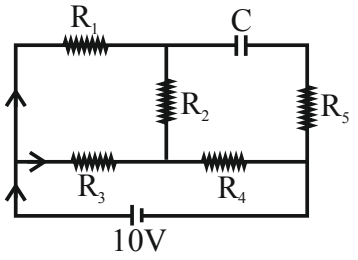
63. In a meter bridge experiment S is a standard resistance. R is a resistance wire. It is found that balancing length is ℓ= 25 cm. If R is replaced by a wire of half - length and half diameter that of R of same material, then the balancing distance ℓ (in cm) will now be

[*JEE Main-2020 (January)*]



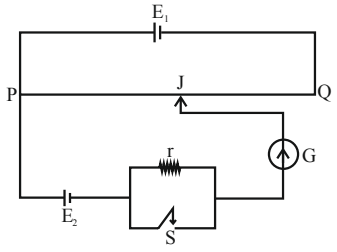
64. An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2 Ω. The potential difference (in V) across the capacitor when it is fully charged is _____.

[*JEE Main-2020 (September)*]



65. A potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from The potential gradient in the potentiometer wire is

[*JEE Main-2020 (September)*]



(*a*) 0.04 V/cm (*b*) 0.01 V/cm
(*c*) 0.02 V/cm (*d*) 0.03 V/cm

66. Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity ρ_C, ρ_T, ρ_M and ρ_A respectively. **[*JEE Main-2020 (September)*]**

(*a*) $\rho_M > \rho_A > \rho_T > \rho_C$ (*b*) $\rho_C > \rho_A > \rho_T$
(*c*) $\rho_A > \rho_M > \rho_C$ (*d*) $\rho_M > \rho_T > \rho_A > \rho_C$

67. Two resistors 400 Ω and 800 Ω are connected in series across a 6 V battery. The potential difference measured by a voltmeter of 10 kΩ across 400 Ω resistor is close to

[*JEE Main-2020 (September)*]

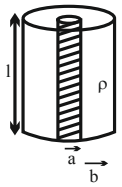
(*a*) 2.05 V (*b*) 1.8 V
(*c*) 2 V (*d*) 1.95 V

68. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed?

[*JEE Main-2020 (September)*]

(*a*) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.
(*b*) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.
(*c*) Multimeter shows a deflection, accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.
(*d*) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.

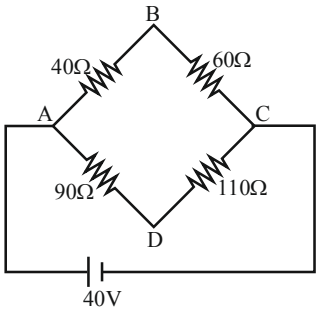
69. Model a torch battery of length *l* to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R, the maximum Joule heating in R will take place for **[*JEE Main-2020 (September)*]**



(*a*) $R = \frac{\rho}{\pi l}\ln\left(\frac{b}{a}\right)$ (*b*) $R = \frac{2\rho}{\pi l}\ln\left(\frac{b}{a}\right)$

(*c*) $R = \frac{\rho}{2\pi l}\ln\left(\frac{b}{a}\right)$ (*d*) $R = \frac{\rho}{2\pi l}\left(\frac{b}{a}\right)$

70.

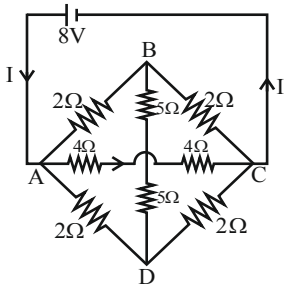


Four resistances 40 Ω, 60 Ω, 90 Ω and 110 Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf 40 V and internal resistance negligible. The potential difference across BD in V is _____.

[*JEE Main-2020 (September)*]

(*a*) 4V (*b*) 2V
(*c*) 6V (*d*) 8V

71. The value of current i_1 flowing from A to C in the circuit diagram is : **[*JEE Main-2020 (September)*]**



(*a*) 1 A (*b*) 4A
(*c*) 5A (*d*) 2A

72. A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated within the internal resistance is

[*JEE Main-2020 (September)*]

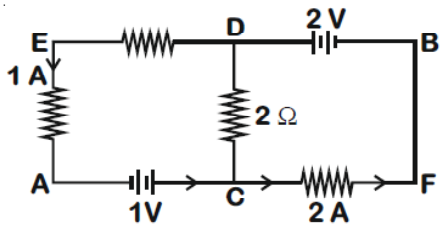
(*a*) 0.10 W (*b*) 0.072 W
(*c*) 0.50 W (*d*) 0.125 W

73. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6 mA it produces a deflection of 2°, its figure of merit is close to **[*JEE Main-2020 (September)*]**

(*a*) 6×10^{-3} A/div. (*b*) 666° A/div.

(*c*) 3×10^{-3} A/div. (*d*) 333° A/div.

74. In the circuit, given in the figure currents in different branches and value of one resistor are shown. Then potential at point B with respect to the point A is **[*JEE Main-2020 (September)*]**



(*a*) +1 V (*b*) –2V

(*c*) +2 V (*d*) –1 V

75. A galvanometer of resistance G is converted into a voltmeter of range 0 – 1 V by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0 – 2 V will be **[*JEE Main-2020 (September)*]**

(*a*) G (*b*) R_1

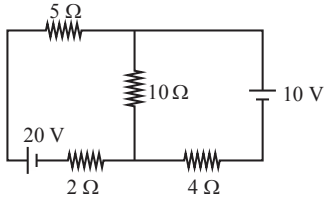
(*c*) $R_1 + G$ (*d*) $R_1 - G$

76. An electrical power line, having a total resistance of 2 Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately **[*JEE Main-2020 (September)*]**

(*a*) 85% (*b*) 96%

(*c*) 72% (*d*) 91%

77.



In the figure shown, the current in the 10 V battery is close to **[*JEE Main-2020 (September)*]**

(*a*) 0.36 A from negative to positive terminal

(*b*) 0.42 A from positive to negative terminal

(*c*) 0.71 A from positive to negative terminal

(*d*) 0.21 A from positive to negative terminal

78. A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected correctly to the resistor. In the circuit **[*JEE Main-2020 (September)*]**

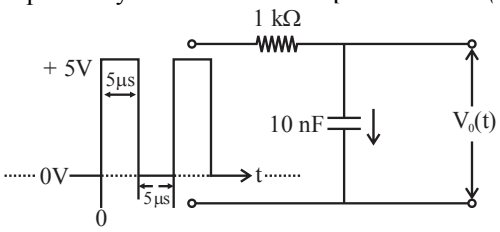
(*a*) Ammeter is always connected in series and voltmeter in parallel

(*b*) Both ammeter and voltmeter must be connected in parallel

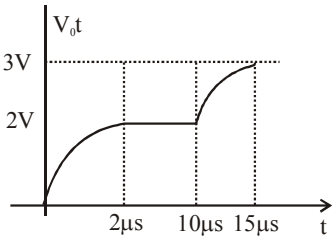
(*c*) Ammeter is always used in parallel and voltmeter is series

(*d*) Both ammeter and voltmeter must be connected in series

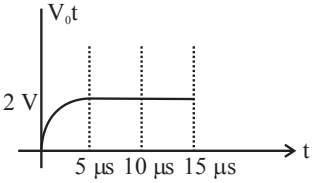
79. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_0(t)$, across the capacitor is correctly depicted by **[*JEE Main-2020 (September)*]**



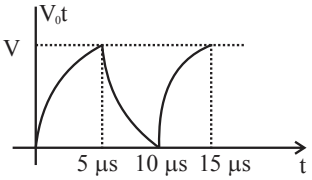
(*a*)



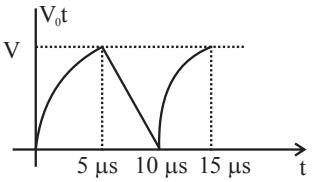
(*b*)



(*c*)



(*d*)



80. The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{j} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0 \times 10^3 \frac{\text{N}}{\text{o}}$. The flux of this field through a rectangular surface area $0.4\,\text{m}^2$ parallel to the $Y-Z$ plane is Nm^2C^{-1}. **[*JEE Main-2021 (March)*]**

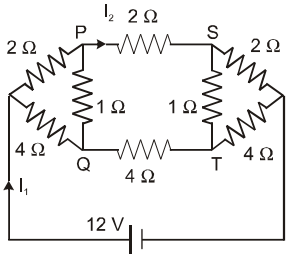
81. The electric field intensity produced by the radiation coming from a $100\,\text{W}$ bulb at a distance of $3\,\text{m}$ is E. The electric field intensity produced by the radiation coming from $60\,\text{W}$ at the same distance is $\sqrt{\frac{x}{5}}\text{E}$. Where the value of $x =$ ---_____. **[*JEE Main-2021 (March)*]**

82. The equivalent resistance of series combination of two resistors is 's'. When they are connected in parallel, the equivalent resistance is ' p '. If $s = np$, then the minimum value for n is ..... (Round off to the Nearest Integer)

[*JEE Main-2021 (March)*]

JEE-ADVANCED PREVIOUS YEAR'S

1. For the resistance network shown in the figure, choose the correct option(s). **[*JEE-2012*]**



(*a*) The current through PQ is zero.

(*b*) $I_1 = 3$ A.

(*c*) The potential at S is less than that at

(*d*) $I_2 = 2$ A.

2. Heater of electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40K ? **[*JEE Advanced-2014*]**

(*a*) 4 if wires are in parallel

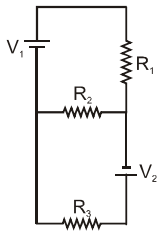
(*b*) 2 if wires are in series

(*c*) 1 if wires are in series

(*d*) 0.5 if wires are in parallel.

3. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if

[*JEE Advanced - 2014*]



(*a*) $V_1 = V_2$ and $R_1 = R_2 = R_3$

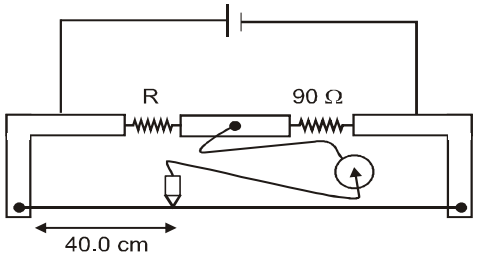
(*b*) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

(*c*) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$

(*d*) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

4. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0-30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0-1.5 A. The value of n is. **[*JEE Advanced - 2014*]**

5. During an experiment with a metre bridge, the galvanometer shows a null point when the jochey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the meter bridge is 1 mm. The unknown resistance is **[*JEE Advanced - 2014*]**



(*a*) $60 \pm 0.15\Omega$ (*b*) $135 \pm 0.56\Omega$

(*c*) $60 \pm 0.25\Omega$ (*d*) $135 \pm 0.23\Omega$

6. Consider two identical galvanometers and two identical resistors with resistance R . If the internal resistance of the galvanometers Rc < R /2, which of the following statement(s)about any one of the galvanomenters is (are) true? **[*JEE Advanced - 2014*]**

(*a*) The maximum voltage range is obtained when all the components are connected in series

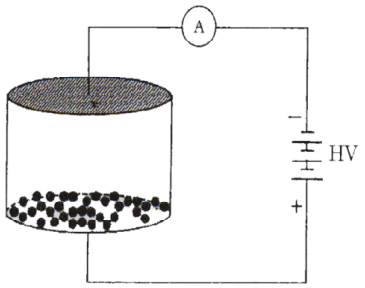
(*b*) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

(*c*) The maximum current range is obtained when all the components are connected in parallel

(*d*) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors

Comprehension Type Questions (No. 7 to 8)

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plates is at $+ V_0$ and the top plate at $- V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



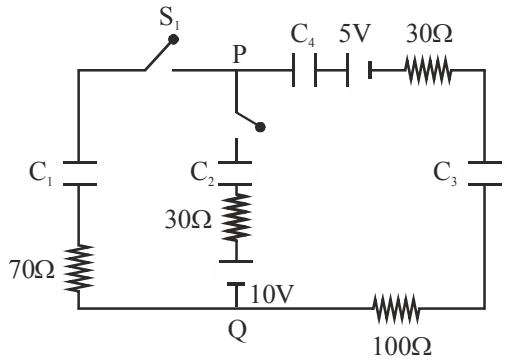
7. Which one of the following statements is correct?
(*a*) The balls will bounce back to the bottom plate carrying the opposite charge they went up with
(*b*) The balls will stick to the top plate and remain there
(*c*) The balls will execute simple harmonic motion between the two plates
(*d*) The balls will bounce back to the bottom plate carrying the same charge they went up with

8. The average current in the steady state registered by the ammeter in the circuit will be
(*a*) proportional to $V_0^{1/2}$
(*b*) zero
(*c*) proportional to V_0^2
(*d*) proportional to the potential V_0

9. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The megnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} N m rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0–1.0A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is .....

[*JEE Advanced - 2018*]

10. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are C_1 = 10 μF, C_2 = 30 μF and $C_3 = C_4 = 80$ μF. **[*JEE Advanced - 2019*]**



Which of the statement(s)is/are correct ?
(*a*) The keys S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).
(*b*) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.
(*c*) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.
(*d*) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C_1 will be 4V.

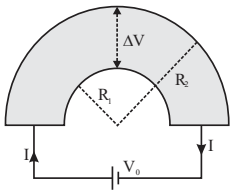
11. Two identical moving coil galvanometer have 10 Ω resistance and full scale deflection at 2 μA current.One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000 Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct ?

[*JEE Advanced - 2019*]

(*a*) The measured value of R will be 978 Ω < R < 982 Ω.
(*b*) The resistance of the Voltmeter will be 100 kΩ.
(*c*) The resistance of the Ammeter will be 0.02 Ω (round off to 2nd decimal place)
(*d*) If the ideal cell is replaced by a cell having internal resistance of 5 Ω then the measured value of R will be more than 1000 Ω.

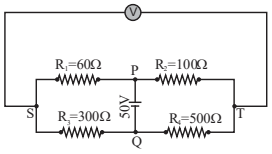
12. Shown in the figure is a semicircular metallic strip that has thickness t and resistivity ρ . Its inner radius is R_1 and outer radius is R_2. If a voltage V_0 is applied between its two ends, a current I flows in it. In addition, it is observed that a transverse voltage ΔV develops between its inner and outer surfaces due to purely kinetic effects of moving electrons (ignore any role of the magnetic field due to the current). Then (figure is schematic and not drawn to scale):

[*JEE Advanced - 2020*]



(*a*) $I = \dfrac{V_0 t}{\pi\rho} \ln\left(\dfrac{R_2}{R_1}\right)$
(*b*) the outer surface is at a higher voltage than the inner surface
(*c*) the outer surface is at a lower voltage than the inner surface
(*d*) $\Delta V \propto I^2$

13. In the balanced condition, the values of the resistances of the four arms of a Wheatstone bridge are shown in the figure below. The resistance R_3 has temperature coefficient 0.0004 °C^{-1}. If the temperature of R_3 is increased by 100 °C, the voltage developed between S and T will be _____ volt. **[*JEE Advanced - 2020*]**



ANSWER KEY

Topicwise Questions

1. (*b*) **2.** (*b*) **3.** (*b*) **4.** (*a*) **5.** (*b*) **6.** (*b*) **7.** (*a*) **8.** (*d*) **9.** (*b*) **10.** (*d*)
11. (*b*) **12.** (*b*) **13.** (*d*) **14.** (*c*) **15.** (*a*) **16.** (*c*) **17.** (*b*) **18.** (*b*) **19.** (*c*) **20.** (*a*)
21. (*b*) **22.** (*a*) **23.** (*a*) **24.** (*a*) **25.** (*a*) **26.** (*d*) **27.** (*c*) **28.** (*a*) **29.** (*c*) **30.** (*a*)
31. (*d*) **32.** (*c*) **33.** (*a*) **34.** (*c*) **35.** (*d*)

Learning Plus

1. (*d*) **2.** (*c*) **3.** (*c*) **4.** (*d*) **5.** (*b*) **6.** (*d*) **7.** (*c*) **8.** (*b*) **9.** (*c*) **10.** (*b*)
11. (*b*) **12.** (*c*) **13.** (*b*) **14.** (*c*) **15.** (*d*) **16.** (*a*) **17.** (*d*) **18.** (*b*) **19.** (*c*) **20.** (*b*)
21. (*b*) **22.** (*a*) **23.** (*b*) **24.** (*b*) **25.** (*b*) **26.** (*a*) **27.** (*d*) **28.** (*c*) **29.** (*c*) **30.** (*b*)
31. (*a*) **32.** (*a*) **33.** (*b*) **34.** (*b*) **35.** (*d*)

ADVANCED LEVEL MULTICONCEPT QUESTIONS

PREVIOUS YEAR'S MCQ/STATEMENT/COMPREHENSION/MATCHIN

1. (*a*,*d*) **2.** (*a*,*d*) **3.** (*a*,*c*,*d*) **4.** (*a*,*c*) **5.** (*a*,*c*) **6.** (*a*,*b*,*c*,*d*) **7.** (*a*,*c*) **8.** (*a*,*c*,*d*) **9.** (*b*,*c*) **10.** (*a*,*b*)
11. (*b*) **12.** (*a*) **13.** (*c*) **14.** (*c*) **15.** (*a*) **16.** (*b*) **17.** (*b*) **18.** (*d*) **19.** (*a*)q, (*b*)p, (*c*)p,(*d*)q
20. (*a*)p, (*b*)q,s, (*c*)s, (*d*)p,r,s **21.** [0080] **22.** [0004] **23.** [25 W] **24.** [0020] **25.** [0001

JEE Mains & Advanced Past Years Questions

JEE-MAINS PREVIOUS YEAR'S

1. (*c*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*b*) **6.** (*a*) **7.** (*a*) **8.** (*a*) **9.** (*b*) **10.** (*c*)
11. (*a*) **12.** (*c*) **13.** (*a*) **14.** (*c*) **15.** (*b*) **16.** (*c*) **17.** (*d*) **18.** (*b*) **19.** (*b*) **20.** (*b*)
21. (*a*) **22.** (*b*) **23.** (*a*) **24.** (*b*) **25.** (*a*) **26.** (*a*) **27.** (*b*) **28.** (*b*) **29.** (*a*) **30.** (*c*)
31. (*c*) **32.** (*c*) **33.** (*b*) **34.** (*a*) **35.** (*d*) **36.** (*b*) **37.** (*c*) **38.** (*a*) **39.** (*b*) **40.** (*d*)
41. (*c*) **42.** (*c*) **43.** (*c*) **44.** (*b*) **45.** (*b*) **46.** (*b*) **47.** (*d*) **48.** (*c*) **49.** (*a*) **50.** (*d*)
51. (*a*) **52.** (*c*) **53.** Bonus **54.** (*d*) **55.** (*a*) **56.** (*a*) **57.** [12] **58.** (*c*) **59.** [10] **60.** [30]
61. (*c*) **62.** (*b*) **63.** [40] **64.** [8] **65.** (*c*) **66.** (*d*) **67.** (*d*) **68.** (*a*) **69.** (*c*) **70.** (*b*)
71. (*a*) **72.** (*a*) **73.** (*c*) **74.** (*a*) **75.** (*c*) **76.** (*b*) **77.** (*d*) **78.** (*a*) **79.** (*a*) **80.** [640]
81. [3] **82.** [4]

JEE-ADVANCED PREVIOUS YEAR'S

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

1. (*a*,*b*,*c*,*d*) **2.** (*b*,*d*) **3.** (*a*,*b*,*d*) **4.** [5] **5.** (*c*) **6.** (*a*,*c*) **7.** (*a*) **8.** (*c*) **9.** [5.55] **10.** (*c*,*d*)
11.(*a*,*c*) **12.** (*a*,*c*,*d*) **13.** (0.26 to 0.27)

4 Moving Charge and Magnetism

MAGNETIC FIELD

In earlier lessons we found it convenient to describe the interaction between charged objects in terms of electric fields. Recall that an electric field surrounding an electric charge. The region of space surrounding a moving charge includes a magnetic field in addition to the electric field. A magnetic field also surrounds a magnetic substance.

In order to describe any type of field, we must define its magnitude, or strength, and its direction.

Magnetic field is the region surrounding a moving charge in which its magnetic effects are perceptible on a moving charge (electric current). Magnetic field intensity is a vector quantity and also known as magnetic induction vector. It is represented by $\vec{B}$.

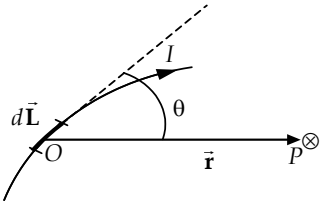
There are two methods of calculating magnetic field at some point. One is **Biot-Savart law,** which gives the magnetic field due to an infinitesimally small current carrying wire at some point and the another is **Ampere law,** which is useful in calculating the magnetic field of a symmetric configuration carrying a steady current.

The unit of magnetic field is weber/m^2 and is known as tesla (T) in the SI system.

BIOT SAVART LAW

Biot-Savart law gives the magnetic induction due to an infinitesimal current element.

- The magnetic field grows weaker as we move farther from its source. In particular, the magnitude of the magnetic field dB is inversely proportional to the square of the distance from the current element $Id\vec{L}$.



- The larger the electric current, the larger is the magnetic field. In particular, the magnitude of the magnetic field $d\vec{B}$ is proportional to the current I.
- The magnitude of the magnetic field $d\vec{B}$ is proportional to sin θ, where θ is the angle between the current element $Id\vec{L}$ and vector $\vec{r}$ that points from the current element to the point in space where $d\vec{B}$ is evaluated. The direction of $d\vec{L}$ is the same as the direction of current at that point.
- The direction of the $\vec{B}$ is not radially away from its source as the gravitational field and the electric field are from their sources. In fact, the direction of $d\vec{B}$ is perpendicular to both $I\,d\vec{L}$ and the vector $\vec{r}$.

These features of field $d\vec{B}$ can be written compactly as

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{Id\vec{L}\times\hat{r}}{r^2} = \left(\frac{\mu_0}{4\pi}\right)\frac{Id\vec{L}\times\vec{r}}{r^3}$$

Here, $(\mu_0/4\pi)$ is a constant of proportionality and $\hat{r}$ is unit vector in the direction of $\vec{r}$.

The constant μ_0 is called the permeability of free space or the permeability constant. Its value is

$\mu_0 = 4\pi\times10^{-7}$ T m/A

The magnetic field $\vec{B}$ is also called **magnetic induction**, or **flux density**.

Its **SI unit** is **tesla** (T) which is equivalent to Wb/m^2.

The dimensions of $\vec{B}$ are $[MT^{-2}A^{-1}]$

The **magnitude** of $d\vec{B}$ can be obtained by

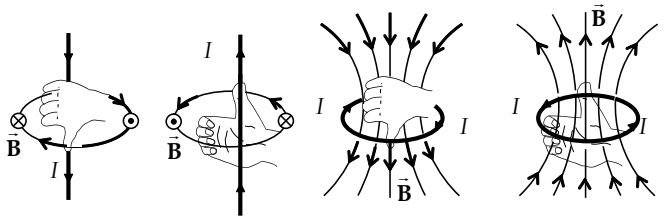
$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,dL\sin\theta}{r^2}$$

The **direction** of $d\vec{B}$ is perpendicular to both $d\vec{L}$ and $\vec{r}$, i.e perpendicular to the plane of the paper. From the definition of the cross product, the field at the point P is directed into the paper. Such a field direction is shown by crossed circle (⊗), according to convention.

The conventional sign for a field coming out of the plane, and perpendicular to it, is a *dot*, namely ◉. One way of remembering the sign convention is to imagine an arrow pointing in the direction of the vector. If the vector points out of the paper, we see the head of the arrow, namely, the ◉. If the vector points into the paper, we see the tail of the arrow, namely, ⊗.

The **direction** of $\vec{B}$ at a point in case of linear and circular current-carrying conductors can be found as follows :

(*a*) **For Linear Conductors**: If the wire is grasped in the palm of the right hand with the stretched thumb pointing in the direction of the current, the fingers curl in the direction of the magnetic field $\vec{B}$ (i.e., the lines of field) [Fig. (A)].



Linear current (*A*) Circular current (*B*)

(*b*) **For Circular Conductors** : If the direction of the current coincides with the direction of the curl of the fingers of the right hand, the stretched thumb points in the direction of the magnetic field $\vec{B}$.

MAGNETIC FIELD FOR SOME IMPORTANT CASES

For a conducting path of finite length, the total magnetic field B at any given point in space is computed by summing the contributions from each small element of the current along its total path. It can be done by integrating

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{Id\vec{L}\times\vec{r}}{r^3}$$

along the current path. In practice one comes across two cases, each requiring different strategy.

- The $d\vec{B}$ contributions from each $Id\vec{L}$ all point in the same direction (or at worst in opposite directions). In this situation one can integrate immediately using the magnitude expression.
- The $d\vec{B}$ contributions from $Id\vec{L}$ sources located on different parts of the conductor point in different directions. In this case, before integration one must resolve the $d\vec{B}$ contributions into components.

Field due to Straight Current-Carrying Conductor

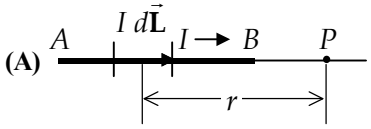
According to Biot-Savart law,

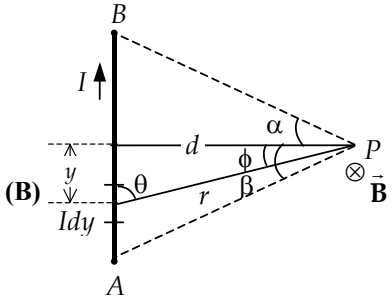
$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{Id\vec{L}\times\vec{r}}{r^3}$$

$$\therefore\ \vec{B} = \left(\frac{\mu_0}{4\pi}\right)\int\frac{Id\vec{L}\times\vec{r}}{r^3}$$

Case (A) : The point *P* is along the length of the wire, as in Fig. (A). Then $d\vec{L}$ and $\vec{r}$ will be either parallel or antiparallel. That is, $\theta = 0$ or π, so $d\vec{L}\times\vec{r} = 0$, hence

$\vec{B} = 0$





Case (B) : The point *P* is at a distance *d* from the wire, as shown in Fig. (*B*). Every current element $I\,d\vec{L}$ of the wire contributes to $\vec{B}$ in the same direction (which is ⊗). Therefore,

$$B = \left(\frac{\mu_0}{4\pi}\right)\int\frac{I\,dL\,r\sin\theta}{r^3} = \left(\frac{\mu_0}{4\pi}\right)\int_A^B\frac{I\,dy\,\sin\theta}{r^2}$$

But $y = d\tan\phi \Rightarrow dy = d\sec^2\phi\,d\phi$,
and $r = d\sec\phi$, $\theta = (90° - \phi)$.

$$\therefore\ B = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{d}\int_{-\beta}^{\alpha}\cos\phi\,d\phi$$

or $$B = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{d}(\sin\alpha + \sin\beta)$$

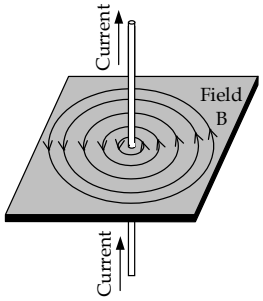
Note: that

(1) For points along the length of the wire (but not on it) the field is always zero.

(2) For points at a perpendicular distance d from the wire, field B varies inversely with distance,

$B \propto \frac{1}{d}$ [and not $\frac{1}{d^2}$].

(3) The field is always perpendicular to the plane containing the wire and the point. So in a plane perpendicular to the wire the lines of force are concentric circles.



(4) If the wire is of **infinite length** and the point P is not near its any end, $\alpha = \beta = (\pi/2)$. Hence,

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{d}[1+1] \quad \text{or} \quad B = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{d}$$

(5) If **the point is near one end of an infinitely long wire**, $\alpha = (\pi/2)$ and $\beta = 0$. Hence,

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{d}[1+0] \quad \text{or} \quad B = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{d}$$

(6) If the wire is of **finite length and the point is on its perpendicular bisector,** $\alpha = \beta$. Hence,

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{d}\sin\alpha \quad \text{with}$$

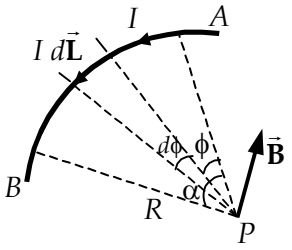
$$\sin\alpha = \frac{L}{\sqrt{(L^2 + 4d^2)}}$$ where L is length of the wire.

(7) If the wire is of **finite length and the point is near its one end,** $\beta = 0$. Hence,

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{d}\sin\alpha \quad \text{with}$$

$$\sin\alpha = \frac{L}{\sqrt{(L^2 + d^2)}}$$

- **Field due to a Circular Current-Carrying Segment at its Centre:** Let AB be a circular segment of radius R. Point P is at its centre. Here,



each element is at the same distance from the centre, i.e., r = R = constant,

(*i*) the angle between element $d\vec{L}$ and $\vec{r}$ is always $\pi/2$, and

(*ii*) the contribution of each element to $\vec{B}$ is in the same direction (i.e., out of the page if the current is anticlockwise and into the page if clockwise).

$$\therefore \vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{L}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\int_A^B\frac{I\,dL}{R^2}\odot$$

But $dL = R\,d\phi$,

$$\therefore B = \frac{\mu_0}{4\pi}\int_0^\alpha \frac{IR\,d\phi}{R^2}$$

or $B = \dfrac{\mu_0 I\alpha}{4\pi R}$

Note: that

(1) The angle α is in radians

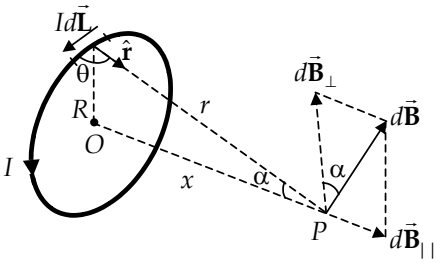
(2) If the loop is semicircular (i.e, $\alpha = \pi$),

$$B = \frac{\mu_0}{4\pi}\frac{\pi I}{R} = \frac{\mu_0 I}{4R}$$

(3) If the loop is a full circle with *N* turns

(i.e., $\alpha = 2\pi N$), $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi NI}{R} = \dfrac{N\mu_0 I}{2R}$

Field at an Axial Point of a Current-Carrying Circular Loop



Here, angle θ between the element $d\vec{L}$ and $\vec{r}$ is $\pi/2$ everywhere and *r* is same for all elements,

$$\therefore \left|d\vec{L}\times\vec{r}\right| = r\,dL\sin\theta = r\,dL$$

$$\therefore dB = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,dL\,r}{r^3} = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,dL}{r^2}$$

The field $d\vec{B}$ has components both along and perpendicular to the axis. However, if we consider the contributions of the current elements that are diametrically opposite, we see that their components normal to the axis $d\vec{B}_\perp$ will cancel.

But, the component of $d\vec{B}$ along the axis is

$$dB_\parallel = dB\sin\alpha = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,dL}{r^2}\left(\frac{R}{r}\right)$$

The total field is given by the integral of this expression over all elements,

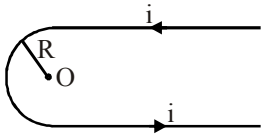
$$\therefore B = B_\parallel = \int dB_\parallel = \left(\frac{\mu_0}{4\pi}\right)\frac{IR}{r^3}\int_0^{2\pi R} dL = \left(\frac{\mu_0}{4\pi}\right)\frac{2I\pi R^2}{r^3}$$

However, as $r^2 = R^2 + x^2$, and if the coil has N turns, we have

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi R^2 IN}{(R^2 + x^2)^{3/2}}$$

TRAIN YOUR BRAIN

Example 1: An electric current is flowing in a very long pin as shown in the figure. The value of magnetic flux density at point O will be

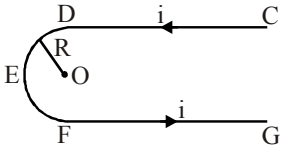


(*a*) $\frac{\mu_0 i}{4\pi R}[\pi+2]$ (*b*) $\frac{\mu_0 i}{4\pi R}[\pi+1]$

(*c*) $\frac{\mu_0 i}{4\pi R}[\pi-2]$ (*d*) $\frac{\mu_0 i}{4\pi R}[\pi-1]$

Sol. $B_O = B_{CD} + B_{DEF} + B_{FG}, B_{CD} = B_{FG} = \left[\frac{\mu_0 i}{4\pi R}\right]$,

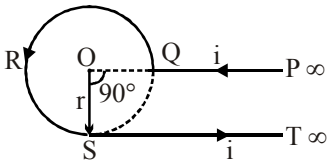
$B_{DEF} = \frac{1}{2}\left[\frac{\mu_0 i}{2R}\right]$



$\therefore B_O = \frac{\mu_0 i}{4\pi R} + \frac{\mu_0 i}{4R} + \frac{\mu_0 i}{4\pi R}$

$= \frac{\mu_0 i}{4R}\left[\frac{1}{\pi}+\frac{1}{\pi}+1\right] = \frac{\mu_0 i}{4R}\left[\frac{2}{\pi}+1\right] = \frac{\mu_0 i}{4\pi R}[2+\pi]$

Example 2: A current i is flowing in a conductor shaped as shown in the figure. The radius of curved part is r and length of straight portions is very large. The value of magnetic field at the centre will be

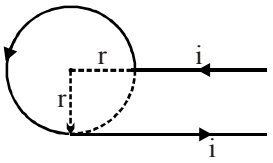


(*a*) $\frac{\mu_0 i}{4\pi r}\left[\frac{3\pi}{2}+1\right]$ (*b*) $\frac{\mu_0 i}{4\pi r}\left[\frac{3\pi}{2}-1\right]$

(*c*) $\frac{\mu_0 i}{4\pi r}\left[\frac{\pi}{2}+1\right]$ (*d*) $\frac{\mu_0 i}{4\pi r}\left[\frac{\pi}{2}-1\right]$

Sol. $B_O = B_{PQ} + B_{QR} + B_{ST}, B_{ST} = \frac{1}{2}\left[\frac{\mu_0 i}{2\pi r}\right]$

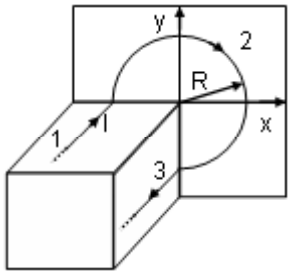
$B_{PQ} = 0, B_{QRS} = \frac{\mu_0 i}{4\pi r^2} \times \frac{3}{4} \times 2\pi r$



$B_O = \frac{\mu_0 i}{4\pi r} + \frac{3}{4}\frac{\mu_0 i}{2r} = \frac{\mu_0 i}{4r}\left[\frac{3}{2}+\frac{1}{\pi}\right]$

$= \frac{\mu_0 i}{4\pi r}\left[\frac{3\pi}{2}+1\right]$

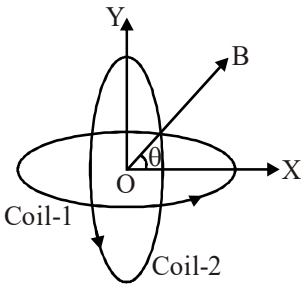
Example 3: Find the magnetic induction at the point O if the wire carrying a current I A has the shape shown in Fig. The radius of the curved part of the wire is R , the linear parts of the wire are very long.



Sol. $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$= \frac{\mu_0 I}{4\pi R}\left(-\hat{j}\right) + \frac{\mu_0 I}{4\pi R}\left(\frac{3\pi}{2}\right)\left(-\hat{k}\right) + \frac{\mu_0 I}{4\pi R}\left(-\hat{i}\right)$

Example 4: For the arrangement of figure the magnetic field at the center O will be



(*a*) $\sqrt{2}\left(\frac{\mu_0 NI}{2a}\right)$ (*b*) $\frac{\mu_0 NI}{2\sqrt{2}a}$

(*c*) $\frac{\mu_0 NI}{2}$ (*d*) $\frac{\mu_0 NI}{2a}$

Sol. The two coils are perpendicular to each other. Coil 1 produce field along X axis and coil 2 produce field along Y axis. Thus the resultant field will be

$B = \sqrt{B_1^2 + B_2^2}$ making an angle θ, $\tan\theta = \frac{B_2}{B_1}$

For identical coils,

$B = \sqrt{2}\left(\frac{\mu_0 NI}{2a}\right)$ and $\theta = 45°$

Example 5: A charge of one Coulomb is placed at one end of a non-conducting rod of length 0.6 m. The rod is rotated in a vertical plane about a horizontal axis passing through the other end of the rod with angular frequency $10^4\ \pi$ rad/s.

(*a*) Find the magnetic field at a point on the axis of rotation at a distance of 0.8 m from the centre of the path.

(*b*) Now half of the charge is removed from one end and placed on the other end. The rod is rotated in a vertical plane about horizontal axis passing through the mid-point of the rod with the same angular frequency. Calculate the magnetic field at a point on the axis at a distance of 0.4 m from the centre of the rod.

Sol. (*a*) As the revolving charge q is equivalent to a current

$$i = qf = q \times \frac{\omega}{2\pi} = 1 \times \frac{10^4\pi}{2\pi} = 5 \times 10^3\ \mathbf{A}$$

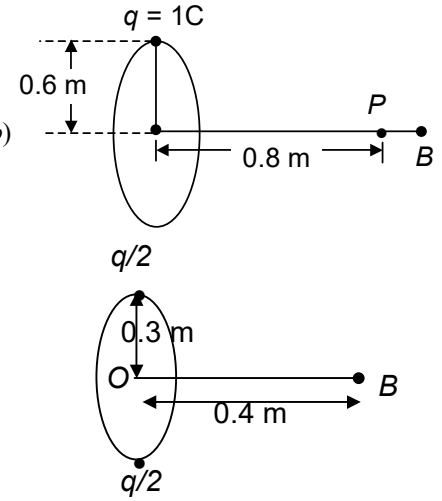
Now $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi I R^2}{(R^2 + x^2)^{3/2}}$

$$\therefore\ B = 10^{-7} \times \frac{2\pi \times 5 \times 10^3 (.6)^2}{[(.6)^2 + (.8)^2]^{3/2}} = \mathbf{1.13 \times 10^{-3}\ T}$$

If half of the charge is placed at the other end and the rod is rotated at the same frequency

$$I' = \left(\frac{q}{2}\right)f + \left(\frac{q}{2}\right)f = qf = I = 5 \times 10^3\ \mathbf{A}$$

(*b*)



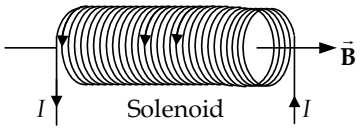
In this case, R′ = 0.3 m and x′ = 0.4 m

$$\therefore\ B' = 10^{-7} \times \frac{2\pi \times 1 \times 5 \times 10^3 (.3)^2}{[(.3)^2 + (.4)^2]^{3/2}} = \mathbf{2300\ \mu T}$$

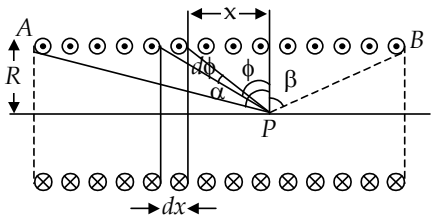
Field at an Axial Point of a Solenoid

If many turns of an insulated wire are wound around a cylinder, the resulting coil is called a **solenoid** [Fig. (A)]. The field at a point on the axis of a solenoid can be obtained by superposition of fields due to large number of identical coils all having their centre on the axis of the solenoid.

(A)



(B)



Considering a coil of width dx at a distance x from the point P on the axis, [Fig. (B)],

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi N I R^2}{(R^2 + x^2)^{3/2}}$$

If n is the number of turns per unit length, N = n dx and as in Fig. (B), x = R tan ϕ, i.e.,
dx = R sec$^2\ \phi\ d\phi$, so

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi(n\ dx)IR^2}{(R^2 + R^2\tan^2\varphi)^{3/2}} = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)\cos\ \varphi\ d\varphi$$

$$\therefore\quad B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)\int_{-\alpha}^{\beta}\cos\varphi\ d\varphi$$

$$\text{or}\quad B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)[\sin\alpha + \sin\beta]$$

$$\Rightarrow B = \frac{\mu_0 nI}{2}\ [\sin\alpha + \sin\beta]$$

Note: that

(1) If the solenoid is of **infinite length** and **the point is well inside** the solenoid,
$\alpha = \beta = (\pi/2)$,

$$B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)[1+1] \quad \text{or} \quad B = \mu_0 nI\ .$$

(2) If the solenoid is of **infinite length** and **the point is near one end,** $\alpha = 0$ and $\beta = (\pi/2)$,

$$B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)[1+0] \quad \text{or} \quad B = \tfrac{1}{2}\,(\mu_0 nI)$$

(3) If the solenoid is of **finite length** and **the point is on the perpendicular bisector of its axis,** $\alpha = \beta$,

$$B = \left(\frac{\mu_0}{4\pi}\right)(4\pi nI)\sin\alpha \quad \text{with}$$

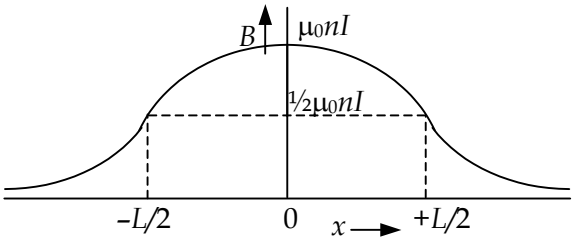
$$\sin\alpha = \frac{L}{\sqrt{L^2 + 4R^2}}\ \text{where L is the length of solenoid.}$$

(4) If the solenoid is of **finite length** and **the point is on its axis but near one end,** $\beta = 0$,

$$B = \left(\frac{\mu_0}{4\pi}\right)(2\pi nI)\sin\alpha \quad \text{with}$$

$$\sin\alpha = \frac{L}{\sqrt{L^2 + R^2}}$$

(5) The field variation with distance along the axis of a solenoid is as shown in figure.



Magnetic Field due to Uniformly Moving Charge

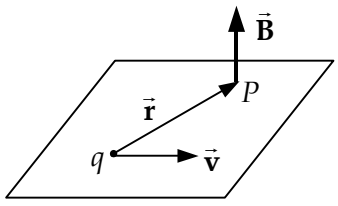
Biot-Savart law gives the magnetic field produced by a current element $I\,d\vec{L}$. However, since I = dq/dt, we can write,

$$I\,d\vec{L} = \frac{dq}{dt}d\vec{L} = dq\frac{d\vec{L}}{dt} = dq\,\vec{v}$$

$$\therefore\ d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,d\vec{L}\times\vec{r}}{r^3} = \left(\frac{\mu_0}{4\pi}\right)dq\frac{\vec{v}\times\vec{r}}{r^3}$$

If a single charge q is moving with a velocity $\vec{v}$, it creates a magnetic field given by

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q[\vec{v}\times\vec{r}]}{r^3}$$



AMPERE'S CIRCUITAL LAW

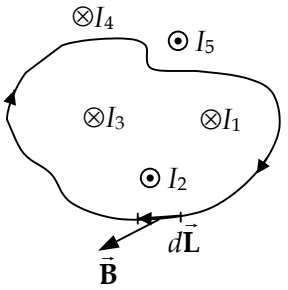
Ampere's circuital law gives another method to calculate the magnetic field due to a given current distribution. It states :

The integral $\oint\vec{B}.d\vec{L}$ of the resultant magnetic field along a closed plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus,

$$\oint\vec{B}.d\vec{L} = \mu_0 I$$

Note: that

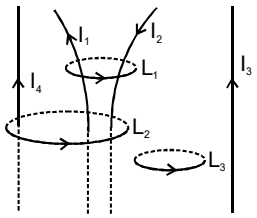
- The magnetic field $\vec{B}$ on the left-hand side is the resultant field due to all the current existing anywhere, including I_4 and I_5.
  (2) On the right hand side, the current I is the algebraic sum of all currents passing through the area. Any current outside the area is not included. (Thus, I_4 and I_5 are not included in I.)



- To find the sign of the enclosed currents, we assign a sense to the curve by putting an arrow on the curve. This becomes the direction of $d\vec{L}$. If you curl the fingers of the right hand along the arrow on the curve, the stretched thumb gives the positive direction of the currents. Thus,
  $I = I_1 + I_3 - I_2$
  Ampere's law is more useful in finding $\vec{B}$ in certain symmetrical conditions.

TRAIN YOUR BRAIN

Example 6: Find the value of $\oint\vec{B}.d\vec{\ell}$ for the loops L_1, L_2, L_3 in the figure shown. The sense of $d\vec{\ell}$ is mentioned in the figure.



Sol. for L_1 $\oint\vec{B}.d\vec{\ell} = \mu_0(I_1 - I_2)$

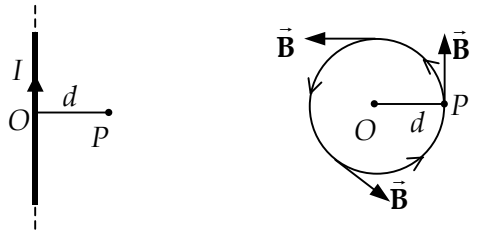
here I_1 is taken positive because magnetic lines of force produced by I_1 is anti clockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $d\vec{\ell}$ is anticlockwise as seen from top.

for L_2 : $\oint\vec{B}.d\vec{\ell} = \mu_0(I_1 - I_2 + I_4)$

for L_3 : $\oint\vec{B}.d\vec{\ell} = 0$

APPLICATIONS OF AMPERE'S LAW

Field due to an Infinitely Long Straight Current-Carrying Conductor: Consider a circular path passing through point P, with centre at O and radius d.



(A) **(B)**

We have arbitrarily put the arrow on the circle in anticlockwise direction. The magnetic field is along the tangent, as shown. Same is the direction of the length-element $d\vec{L}$. By symmetry, the magnitude of the magnetic field should be the same at all these points. The circulation of magnetic field along the circle is

$$\oint \vec{B}.d\vec{L} = \oint B\, dL = B\oint dL = B2\pi d$$

The current crossing the area bounded by the circle is I. Thus, from Ampere's law,

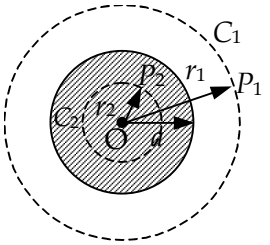
$$B2\pi d = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi d}$$

Field Outside and Inside a Thick Long Straight Cylinder: It follows from symmetry that the lines of $\vec{B}$ are in the form of circles with centre at the cylinder axis. For any point P_1, outside the cylinder,

$$\oint \vec{B}.d\vec{L} = \mu_0 I$$

$$\text{or } B2\pi r_1 = \mu_0 I$$

$$\text{or } B_{out} = \frac{\mu_0 I}{2\pi r_1}$$



Thus, for any external point, the straight current-carrying cylindrical conductor behaves as a straight current-carrying wire.

Consider a point P_2 lying on the circular contour C_2 of radius r_2 inside the cylinder. The current enveloped by this contour is

$$I_2 = I\left(\frac{\pi r_2^2}{\pi a^2}\right) = I\left(\frac{r_2}{a}\right)^2$$

Where a is the radius of the cylinder
From Ampere's Law,

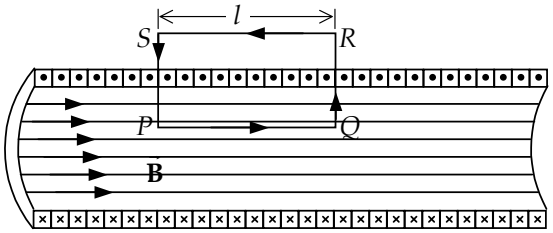
$$\oint \vec{B}.d\vec{L} = \mu_0 I_2$$

$$\text{or } B2\pi r_2 = \mu_0 I\left(\frac{r_2}{a}\right)^2 \qquad \text{or } B_{in} = \frac{\mu_0}{2\pi}\frac{Ir_2}{a^2}$$

Note: that if the cylinder has a shape of a tube, the field outside is same as given above, but the field inside the tube is zero, as no current is enclosed by the circular contour.

Field due to a Solenoid

A wire helically wound on the surface of a long cylinder is called a **linear solenoid.** Let there be n turns per unit length and I be the current flowing. Longer the solenoid, the less is the field outside. The lines of the field $\vec{B}$ inside are directed along the axis.



Consider a rectangular contour PQRS. The line PQ is parallel to the solenoid axis and hence parallel to the magnetic field $\vec{B}$ inside the solenoid. Thus,

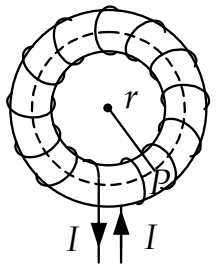
$$\int_P^Q \vec{B}.d\vec{l} = Bl$$

On the remaining three sides, $\vec{B}.d\vec{l}$ is zero everywhere as $\vec{B}$ is either zero (outside the solenoid) or perpendicular to $d\vec{l}$ (inside the solenoid).
Using Ampere's Law,

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{total} \text{ or } Bl = \mu_0 Inl \qquad \text{or} \quad B = \mu_0 nI\,.$$

Field in a Toroid

If a solenoid is bent in a circular shape and the ends are joined, we get a toroid. Alternatively, one can start with a non-conducting ring or a torus and wind a conducting wire closely on it.



Draw a circle through the point P and concentric with the toroid. By symmetry, the field will have equal magnitude at all points of this circle. Also, the field is everywhere tangential to the circle, Thus,

$$\oint \vec{B}d\vec{L} = \int B\, dL = B\int dL = 2\pi rB$$

If the total number of turns is n, the current crossing the area bounded by the circle is nI. Using Ampere's law on this circle,

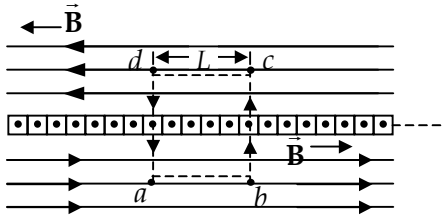
$$\oint \vec{B}.d\vec{L} = \mu_0 nI \quad \text{or} \quad 2\pi rB = \mu_0 nI \quad \text{or} \quad B = \frac{\mu_0 nI}{2\pi r}$$

Note: that the magnetic field varies as 1/r inside a toroid. If the chosen contour passes inside or outside the toroid, it does not envelop any currents, hence $2\pi rB = 0$, i.e., $B = 0$.

Field due to an Infinite Current Sheet

Consider an infinitely large sheet of current, with λ linear current density (i.e, the current per unit length). Imagine a rectangular loop abcda around the sheet as shown. The current enveloped by this contour is

$I = \lambda L$



For this symmetry, the magnetic field is uniform on the two sides of the sheet. Applying Ampere's law,

$$\oint \vec{B}.d\vec{L} = \mu_0 I$$

or

$$\int_a^b \vec{B}.d\vec{L} + \int_b^c \vec{B}.d\vec{L} + \int_c^d \vec{B}.d\vec{L} + \int_d^a \vec{B}.d\vec{L} = \mu_0 \lambda L$$

Here, along the paths $b \to c$ and $d \to a$, $\vec{B}$ is perpendicular to $d\vec{L}$. Hence,

$$\int_b^c \vec{B}.d\vec{L} = \int_d^a \vec{B}.d\vec{L} = 0$$

Further, along the paths $a \to b$ and $c \to d$, $\vec{B}$ is parallel to $d\vec{L}$. Hence,

$$\int_a^b \vec{B}.d\vec{L} = \int_c^d \vec{B}.d\vec{L} = BL$$

$$\therefore 2BL = \mu_0 \lambda L \quad \text{or} \quad B = \frac{\mu_0 \lambda}{2}.$$

MAGNETIC FORCE ON MOVING CHARGES

An electric charge at rest, kept in a magnetic field, experiences no force. Magnetic field acts only on moving charges.

The force exerted by a magnetic field $\vec{B}$ on a charged particle q moving with a velocity $\vec{v}$ is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

The **magnitude** of this force is given as

$F = qvB \sin\theta$

where θ is the angle between velocity vector $\vec{v}$ and the magnetic field vector $\vec{B}$.

The **direction** of the force is same as the cross-product. It is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$, and is given by the **right-hand rule**.

If the vector $\vec{v}$ and $\vec{B}$ are specified in a three-dimensional space in terms of their components along x, y and z-axis as

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

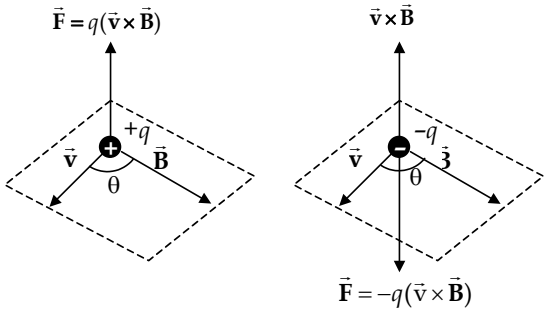
Then the magnetic force $\vec{F}$ is given as

$$\vec{F} = q(\vec{v} \times \vec{B}) = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= q\,\hat{i}\,(v_yB_z - v_zB_y) - q\,\hat{j}\,(v_xB_z - v_zB_x) + q\,\hat{k}\,(v_xB_y - v_yB_x)$$

Note: carefully that

(1) The force on a positive charge is in the direction of $\vec{v} \times \vec{B}$, but the force on a negative charge is opposite to the vector $\vec{v} \times \vec{B}$.



(2) There can be any angle between the vectors $\vec{v}$ and $\vec{B}$. But the resulting force $\vec{F}$ on the charge is always perpendicular to $\vec{v}$ (as well as to $\vec{B}$).

(3) If the motion of the charged particle is collinear with the field, i.e., $\theta = 0°$ or $180°$,
$F = qvB \sin\theta = 0$
Thus, a moving charged particle does not experience any force in a magnetic field if its motion is parallel or antiparallel to the field.

DIFFERENCE BETWEEN MAGNETIC FORCE AND ELECTRIC FORCE

(1) Magnetic force is always perpendicular to the field, whereas electric force is collinear with the field.

(2) Magnetic force is velocity dependent, i.e., acts only when the charged particle is in motion, whereas electric force (qE) is independent of the state of rest or motion of the charged particle.

(3) Magnetic force does no work when the charged particle is displaced (since magnetic force is perpendicular to velocity), whereas electric force does work in displacing the charged particle.

TRAIN YOUR BRAIN

Example 7: A 64-μC charge traveling with velocity $\vec{v} = (4\hat{i} + 2\hat{j})$ m/s enters a region of magnetic field of induction $\vec{B} = (-5\hat{i} + 3\hat{j})$T. Compute the force experienced by the charge.

Sol. Both $\vec{v}$ and $\vec{B}$ lie in the x-y plane. Therefore, the magnetic force $\vec{F}$ must be along the z-axis. Let us compute $\vec{F}$ using determinant method,

$$\vec{F} = q(\vec{v}\times\vec{B}) = (64\times10^{-6})\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 0 \\ -5 & 3 & 0 \end{vmatrix}$$

$$= (64\times10^{-6})\,[\hat{i}(0) - \hat{j}(0) + \hat{k}(12+10)]$$

$$= \mathbf{(1.41\times10^{-3}\,\hat{k})\ N}$$

MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Case I : Straight Line Motion

If a charged particle q is projected into a uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$ which is parallel to the field lines, the force experienced by the charge is zero. Hence it travels in a straight line with uniform velocity.

Case II : Uniform Circular Motion

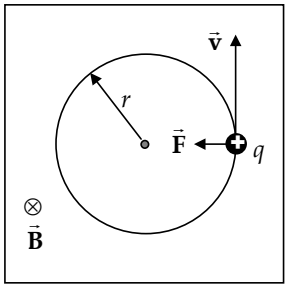
If a charged particle q is projected into a uniform magnetic field $\vec{B}$ with an initial velocity $\vec{v}$ perpendicular to the magnetic field, it gets trapped in a circular path. The force exerted by the field provides the necessary centripetal force,

$$qvB = \frac{mv^2}{r}$$

where r is the radius of the circular path. Thus, we have

$$r = \frac{mv}{qB}$$

The radius of the orbit is directly proportional to the linear momentum of the particle and inversely proportional to the magnitude of magnetic field.



The **time period** of the orbit is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Note: that time period is independent of velocity. All particles with same charge-to-mass ratio (q/m), have the same time period in a given magnetic field.

TRAIN YOUR BRAIN

Example 8: An electron with a kinetic energy of 10^3 eV moves perpendicular to the direction of a uniform magnetic field, $B = 10^{-4}$T.

(*a*) What is the period of its orbit ?

(*b*) What is the radius of the orbit ?

Sol. (*a*) Time period is given by

$$T = \frac{2\pi m}{eB} = \frac{2\pi(9.1\times10^{-31})}{(1.6\times10^{-19})(10^{-4})}$$

or $T = \mathbf{3.6\times10^{-7}s}$

(*b*) Radius of the orbit is given by

$$r = \frac{mv}{eB} = \frac{p}{eB}$$

The momentum of the electron can be obtained from kinetic energy as

$$p = mv = \sqrt{2Km} = \sqrt{2(1.6\times10^{-16})\ (9.1\times10^{-31})}$$

$$= 1.7\times10^{-23}\ \text{kg m/s}$$

$$\therefore\quad r = \frac{(1.7\times10^{-23})}{(1.6\times10^{-19})(10^{-4})} = \mathbf{1.07\ m}$$

Example 9: An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 T [Mass of α-particle = 6.4×10^{-27} kg].

Sol. When an α-particle is accelerated by 10^4 volts, its kinetic energy will be $K = (2e)(10^4\text{ V}) = 2\times10^4$ eV.

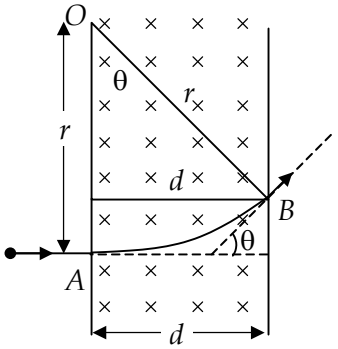
The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$= \frac{(2\times6.4\times10^{-27}\times2\times10^4\times1.6\times10^{-19})^{1/2}}{2\times1.6\times10^{-19}\times0.1}$$

$$= 0.2\text{ m}$$

Now, the angle between tangents at two points will be equal to the angle between *OA* and *OB*. Hence the change in the direction of the particle as it passes the field,



$$\theta = \sin^{-1}\left(\frac{d}{r}\right) = \sin^{-1}\left(\frac{0.1}{0.2}\right) = \mathbf{30°}$$

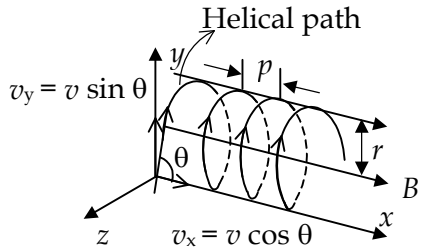
Case III : Motion in Helical Path

When a charged particle is projected into a uniform magnetic field at an angle, it travels in a helical path. The figure shows uniform magnetic field $\vec{B}$ along positive *x*-direction. A charge *q* is projected with velocity $\vec{v}$ making an angle θ with *x*-axis. The two components of the velocity are $v_x = v\cos\theta$ and $v_y = v\sin\theta$. The component v_x is along the field and hence remains unchanged by the field. However, the component v_y is perpendicular to the field, and the particle describes a circle (in a plane perpendicular to the field, i.e., in *y*-*z* plane) of radius,

$$r = \frac{m(v\sin\theta)}{qB}$$

and time period

$$T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi m}{qB}$$



The projection of the helical path onto y-z plane will be a **circle**.

Whereas the projections onto the x-y plane or the x-z plane will be **sinusoidal**. The **pitch** of the helix is defined as the linear distance moved by the charged particle along the magnetic field in one revolution, and is given as

$$p = (v\cos\theta)\,T = \frac{2\pi m}{qB}v\cos\theta$$

TRAIN YOUR BRAIN

Example 10: A beam of protons with a velocity of 4×10^5 m/s enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix.

Sol. Component of the velocity perpendicular to the direction of the field,

$v_\perp = v\sin\theta = 4\times10^5\sin60° = 2\sqrt{3}\times10^5$ m/s

Component of the velocity parallel to the field,

$v_\parallel = v\cos\theta = 2\times10^5$ m/s

The radius of the helix is given by

$$r = \frac{mv_\perp}{qB} = \frac{1.67\times10^{-27}\times2\sqrt{3}\times10^5}{0.3\times1.6\times10^{-19}} = \mathbf{0.012\text{ m}}$$

The time period of revolution is

$$T = \frac{2\pi m}{qB} = \frac{2\times3.14\times1.67\times10^{-27}}{0.3\times1.6\times10^{-19}}$$

$$= 2.185\times10^{-7}\text{ s}$$

Pitch of the helix $= T\times v_\parallel$

$= 2.185\times10^{-7}\times2\times10^5$

$= \mathbf{4.37\times10^{-2}\text{ m}}$

LORENTZ FORCE

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v}\times\vec{B})$ Therefore, net force on it will be

$$\vec{F} = q[\vec{E} + (\vec{v}\times\vec{B})]$$

which is called the **Lorentz-force equation**.

Lorentz equation is universal. It is valid for constant as well as for varying electric and magnetic fields, and for any velocity $\vec{v}$ of the charge q.

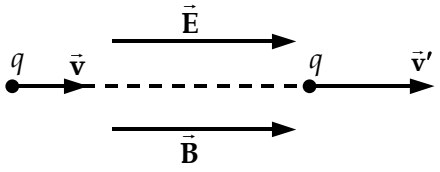
Depending on the directions of $\vec{v}, \vec{E}$ and $\vec{B}$ various situations are possible. The motion of the charged particle in general is quite complex. Here, we consider only two simple cases :

Case I : When $\vec{v}, \vec{E}$ and $\vec{B}$ are Collinear

As the particle is moving parallel or antiparallel to the magnetic field, the magnetic force on it will be zero. The electric force $\vec{F}_e$ will produce an acceleration,

$$\vec{a} = \frac{\vec{F}_e}{m} = \frac{q\vec{E}}{m}$$

The particle follows a straight line path with change in speed. So in this situation speed, velocity, momentum and kinetic energy all will change, without change in direction, of motion as shown in figure.



$\vec{v}, \vec{E}$ and $\vec{B}$ are collinear

Case II : $\vec{v}, \vec{E}$ and $\vec{B}$ are Mutually Perpendicular

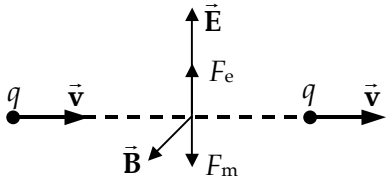
In this situation we consider a specific case when $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0 \qquad \Rightarrow \qquad \vec{a} = (\vec{F}/m) = 0$$

The particle will pass through the field with same velocity. And in this situation, as

$F_e = F_m$ or $qE = qvB$

$\therefore\ v = E/B$



$\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular

FORCE ON A CURRENT-CARRYING CONDUCTOR IN A MAGNETIC FIELD

A conductor has enormous number of free electrons, the negatively charged particles. When current flows through a conductor, these moving electrons experience a magnetic force which is then transmitted to the conductor.

Thus, the basis of a current-carrying conductor experiencing a force when kept in a magnetic field is the Lorentz force on moving charges.

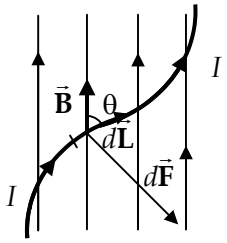
When a current element $Id\vec{L}$ is placed in a magnetic field $\vec{B}$, it experience a force,

$$d\vec{F} = Id\vec{L} \times \vec{B}$$

The **magnitude** of this force is $dF = BI\, dL \sin\theta$

Obviously,

(*i*) The force will be minimum (= 0) when $\sin\theta = 0$, i.e., $\theta = 0°$ or $180°$. That is a current element in a magnetic field does not experience any force if the current in it is collinear with the field.



(*ii*) The force will be maximum when $\sin\theta = 1$, i.e., $\theta = 90°$. That is, the force on a current element in a magnetic field is maximum (= BI dL) when it is perpendicular to the field.

The direction of the force is given by the cross product $Id\vec{L} \times \vec{B}$.

Important Points

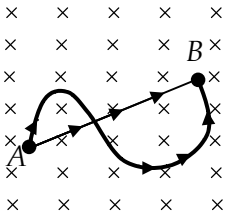
(1) The force $BI\, dL \sin\theta$ is not a function of position r. That is, the magnetic force on a current element is non-central. [A central force is of the form $F = Kf(r)\,\hat{n}_r$]

(2) The force $d\vec{F}$ is always perpendicular to both $\vec{B}$ and $Id\vec{L}$, though $\vec{B}$ and $Id\vec{L}$ may or may not be perpendicular to each other.

(3) If the magnetic field is uniform and the current is constant, both can be taken out from the integral,

$$\vec{F} = \int d\vec{F} = \int I\, d\vec{L} \times \vec{B} = I\left(\oint d\vec{L}\right) \times \vec{B}$$

The integral $\int d\vec{L}$ represents the vector sum of all the length elements from initial point (A) to final point (B). It shows that the force on a curved current carrying wire of any arbitrary shape between points A and B kept in a uniform magnetic field, is same as that on a straight current carrying wire joining these points.



FORCE OF INTERACTION BETWEEN TWO LONG PARALLEL CURRENT-CARRYING WIRES

The net force on a current-carrying conductor due to its own field is zero. However, if there are two long parallel current-carrying wires 1 and 2 as shown, wire-1 will be in the field of wire-2 and vice-versa. So the force on dL_2 length of wire-2 due to field of wire-1,

$$dF_2 = I_2 dL_2 B_1 = I_2 dL_2 \left[\left(\frac{\mu_0}{4\pi}\right)\frac{2I_1}{d}\right] = \left(\frac{\mu_0}{4\pi}\right)\frac{2I_1 I_2}{d} dL_2$$

Therefore, the force per unit length is

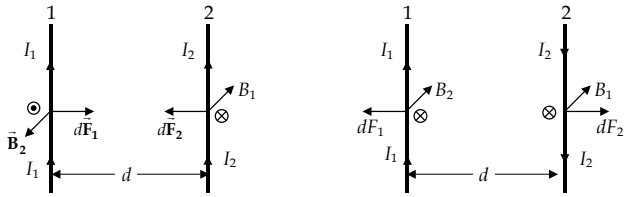
$$\frac{dF_2}{dL_2}=\left(\frac{\mu_0}{4\pi}\right)\frac{2I_1I_2}{d}$$

Same will be true for wire-1 in the field of wire-2,

$$\frac{dF_1}{dL_1}=\left(\frac{\mu_0}{4\pi}\right)\frac{2I_1I_2}{d}$$

The two forces are seen to be equal in magnitude. Therefore, the interaction force per unit length in case of two parallel current-carrying wires separated by a distance d is given by

$$\frac{dF}{dL}=\left(\frac{\mu_0}{4\pi}\right)\frac{2I_1I_2}{d}$$



(A) **(B)**

The direction of the force can be determined in accordance with the rules specified. It is found that the force between the wires is attractive if the currents in them are in the same direction, otherwise repulsive [this is **opposite** to that of what happens between two charges].

SI Unit of Current

The SI unit of current **ampere,** which is one of the base units is defined using the above concept as follows :

One ampere of current is the current which when passing through each of two parallel infinitely long straight conductors placed in free space at a distance of 1 m from each other produces between them a force of 2×10^{-7} newton for one metre of their length.

CURRENT-CARRYING LOOP IN A MAGNETIC FIELD

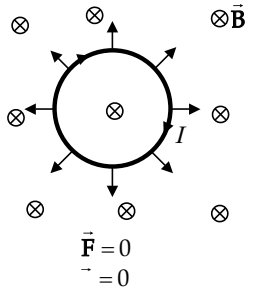
(A) Uniform Magnetic Field: If the current-carrying loop of any arbitrary shape is placed in a uniform magnetic field, the net force is

$$\vec{F}=\oint d\vec{F}=\oint Id\vec{L}\times\vec{B}=I\left\{\oint d\vec{L}\right\}\times\vec{B}=0$$

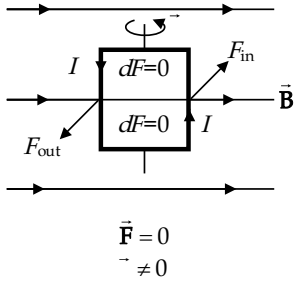
(as the vector sum of $d\vec{L}$ for a closed loop is always zero)

Thus, the net magnetic force on current loop in a uniform magnetic field is always zero.

Note: that different parts of the loop may experience elemental force due to which the loop may be under tension [Fig. (a)] or may experience a torque [Fig. (b)].



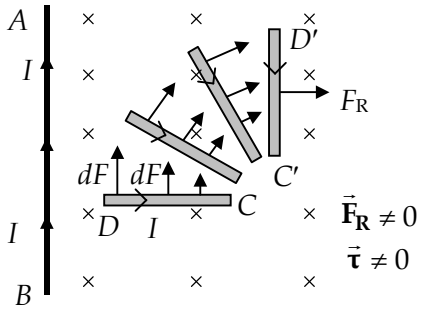
(a) Closed loop under tension



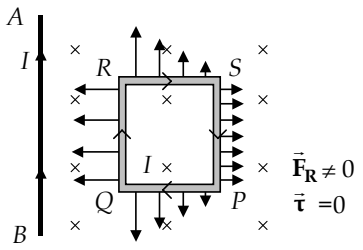
(b) Closed loop experiences a torque

(B) Nonuniform Magnetic Field: When a current-carrying conductor is situated in nonuniform magnetic field field, its different elements will experience different forces. Therefore, the net force F_R is not zero. However, the torque τ acting on a loop may or may not be zero. Consider a current-carrying rod CD placed perpendicular to a long current-carrying conductor AB.

The element of the rod nearer to the conductor AB will experience larger force dF, as shown. Obviously, the rod experience a net force and also a net torque. If free to move, the root CD will translate as well as rotate to a new position C′ D′. In this position, the torque τ becomes zero and the rod experiences a net repulsive force.



Next consider a rectangular loop PQRS placed in the magnetic field produced by a long current-carrying conductor AB. The elements of sides PQ and RS experience different forces, as shown. The forces acting on the side PQ are equal and opposite to the forces acting on side RS. Hence, the loop experiences no net force in vertical direction.



The side QR experiences an attractive force and the side SP experiences a repulsive force. However, as QR is nearer to AB as compared to SP, the attractive force is large than the repulsive force. Hence the loop experience a net attractive force F_R. Note that no torque is acting on the loop.

MAGNETIC DIPOLE MOMENT

Magnetic field lines due to a solenoid are very similar to those of a bar magnet. A bar magnet (or a solenoid) can be thought of as a 'dipole'. Its field lines resemble very close to those of an electric dipole. This connection is made more precise by associating a magnetic moment with a current loop.

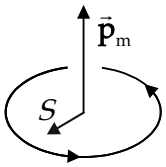
Current Loop: The magnetic moment $\vec{p}_m$ associated with a current loop carrying current I and of area A is defined as

$$\vec{p}_m = IA\hat{n}$$

Where $\hat{n}$ is a unit vector normal to the loop whose direction is connected with the direction of current in the loop through the **right-hand screw rule.** If the current is directed along the four curved fingers of the **right**-hand, $\hat{n}$ (and hence $\vec{p}_m$) is directed along the stretched out thumb.

Note: that

(1) Magnetic moment is a vector, with magnitude, $p_m = IA$
(2) Its SI unit is $A\,m^2$
(3) Its dimensions are $[L^2A]$
(4) Its value depends on the current in the loop and its area. It is independent of the shape of the loop, whether circular or rectangular or any other arbitrary shape.
(5) If the loop (or the coil) contains N number of turns, $p_m = NIA$



CHARGED PARTICLE MOVING IN A CIRCLE

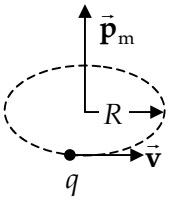
Consider a particle of mass m and having charge q moving with a velocity $\vec{v}$ in a circle of radius R. The associated current is

$$I = \frac{q}{T} = \frac{q\,v}{2\pi R}$$

Area of the circular path is
$A = \pi R^2$

$$\therefore\ \vec{p}_m = IA\hat{n} = \frac{qv}{2\pi R}\pi R^2\hat{n} = \frac{1}{2}qvR\hat{n}$$

But, the **angular momentum** of this particle is



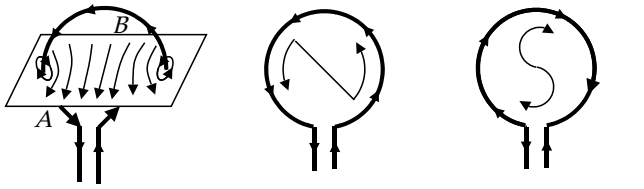
$$\vec{L} = mvR\hat{n}$$

$$\therefore\ \vec{p}_m = \frac{q}{2m}\vec{L}$$

Thus, the magnetic moment of a charged particle moving in a circle is (q/2m) times its angular momentum. Note that if the charge on the particle is negative (e.g., an electron), the $\vec{p}_m$ is directed opposite to $\vec{L}$.

FIELD PATTERN DUE TO CIRCULAR COIL

A circular coil carrying current behaves like a **thin disc magnet**. One face of this magnet is north pole and the other is south pole. Just look at one face of the coil. If the current around the face is in anticlockwise direction, it is a north pole (see figure). On the other hand, if the current is in clockwise direction, then the face is a south pole (see figure.) Note that the anticlockwise direction of current coincides with the arrowheads at the ends of the letter N; and the clockwise direction of current coincides with the arrowheads at the ends of the letter S.



(a) Perspective view (b) View from side A (c) View from side A

MAGNETIC FIELD DUE TO A MAGNETIC DIPOLE

Magnetic field due to a current-carrying coil of radius R, at an axial point at a distance d is given as

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{IR}{(R^2 + d^2)^{3/2}}2\pi R$$

If d >> R, i.e., for very distant axial point,

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{IR}{d^3}2\pi R = \left(\frac{\mu_0}{4\pi}\right)\frac{2I\,\pi R^2}{d^3}$$

or $$B = \frac{\mu_0}{4\pi}\frac{2p_m}{d^3}$$

Compare this, with the expression for the electric field due to an electric dipole,

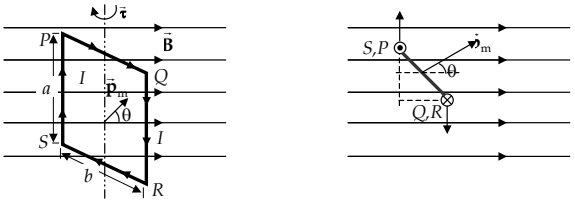
$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2p_e}{d^3}$$

The remarkable similarity of the above two expressions justify associating a magnetic dipole with a small current loop.

TORQUE ON A MAGNETIC DIPOLE

Just as an electric dipole placed in an electric field experiences a torque, similarly a magnetic dipole (a current loop) placed in a magnetic field experiences a torque.

Consider a rectangular loop of sides a and b, carrying a current I. Let it be placed in a uniform magnetic field B. Let the angle between the direction of B and the perpendicular to the plane of the coil be θ.



(A) Side View **(B) Top View**

The element PQ experiences an upward force. The element RS experiences an equal force downward. So, the net force and torque due to these are zero. As shown in Fig. (B), the force on SP is equal and opposite to the force on QR. Each of these forces has magnitude IaB. Since these are not along the same line, a net torque is produced,

τ = force × (force - arm) = IaB × b sin θ

or τ = IabB sin θ = (IS sin θ) B

where A = ab is the area of the loop. The direction of the torque is such as to rotate the loop clockwise. The torque is fully defined by

$$\vec{\tau} = \vec{p}_m \times \vec{B}$$

where p_m = IA is the magnetic moment having a direction perpendicular to the plane of the loop.

The torque on an electric dipole (of dipole moment $\vec{p}_e$) placed in an electric field $\vec{E}$ is given as

$$\vec{\tau} = \vec{p}_e \times \vec{E}$$

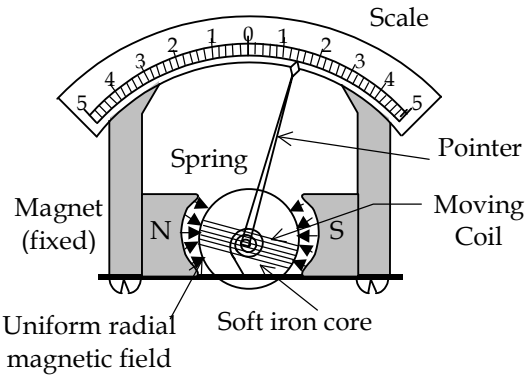
Once again, the similarity of these two expressions confirm that a current loop is a magnetic dipole.

If the loop has *N* turns, the torque produced becomes

τ = NIBA sin θ

MOVING COIL GALVANOMETER

The torque acting on a current loop placed in a magnetic field forms the basis of working of a galvanometer (and also of electric motor).



By placing a cylindrical soft iron core inside the coil, the magnetic field is made uniform and radial. Thus, the angle θ = 90° for any position of the coil. The deflecting torque acting on the coil becomes

τ_d = BINA

The coil rotates, the pointer moves on the scale, the spring winds and produces a restoring torque τ_r. If *k* is the restoring torque per unit twist, and ϕ is the final or steady state deflection (or rotation) of the coil, we have

Deflecting torque τ_d = Restoring torque τ_r

INA = $k\phi$

$$\Rightarrow I = \frac{k}{BIA}\varphi$$

or $I = K_G\phi$

Where $K_G = \frac{k}{BIA}$ is galvanometer constant.

Thus, we see that the current passing through the coil is directly proportional to its deflection.

By suitable modifications, a galvanometer can be converted into an ammeter or a voltmeter.

TRAIN YOUR BRAIN

Example 11: 19 A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 ampere is passed through it and the magnetic field is parallel to its plane.

Sol. The area of the coil is

$$A = \frac{1}{2}L \times L\sin 60° = \frac{1}{2}\times(0.02)^2 \times \frac{\sqrt{3}}{2} = \sqrt{3}\times10^{-4}\ m^2$$

Its magnetic moment,

$$p_m = IS = 0.1\times\sqrt{3}\times10^{-4} = \sqrt{3}\times10^{-5}\ A\,m^2$$

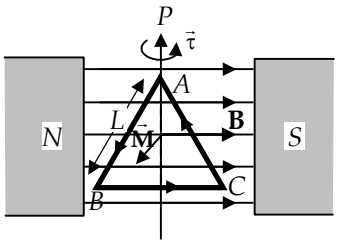
Now the couple on current-carrying coil in a magnetic field is given by

$$\vec{\tau} = \vec{p}_m \times \vec{B}$$

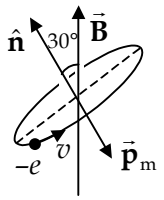
or $\tau = p_m \sin 90° = p_m B$

or $\tau =$

$$(\sqrt{3} \times 10^{-5}) \times 5 \times 10^{-2} = 5\sqrt{3} \times 10^{-7} \text{ N m}$$



Example 12: A electron in the ground state of hydrogen atom is revolving in anticlockwise direction in a circular orbit of radius r. (a) Obtain an expression for the orbital magnetic moment of the electron. (b) The atom is placed in a uniform magnetic induction $\vec{B}$ such that the plane-normal of the electron orbit makes an angle of 30° with the magnetic induction as shown. Find the torque experienced by the orbiting electron.



Sol. (*a*) In case of a charged particle moving in a circle,

$$I = \frac{q}{T} = \frac{-e\ v}{2\pi r}$$

Therefore, the magnetic moment is

$$\vec{p}_m = I\vec{A} = -\frac{e\ v}{2\pi r}\pi r^2 \hat{n} = -\frac{1}{2} evr\ \hat{n}$$

The angular momentum for the circular motion of electron is

$$\vec{L} = mvr\ \hat{n}$$

Thus, the magnetic moment in terms of angular momentum is given as

$$\vec{p}_m = -\frac{1}{2}\frac{e}{m}\vec{L}$$

Now, as from Bohr's postulate,

$$L = mvr = n\left(\frac{h}{2\pi}\right)$$

For ground state, $n = 1$, $L = (h/2\pi)$ So

$$\vec{p}_m = \frac{eh}{4\pi\ m}(-\hat{n})$$

(*b*) In case of a current loop in a magnetic field, the torque is given by

$$\vec{\tau} = \vec{p}_m \times \vec{B}$$

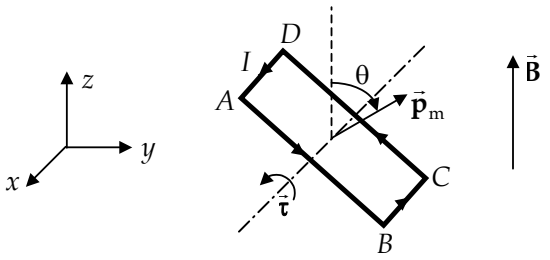
or $\vec{\tau} = \frac{ehB}{8\pi\ m}\hat{k}$, where $\hat{k}$ is a unit vector coming of the page.

POTENTIAL ENERGY OF A CURRENT LOOP

When a bar magnet is freely suspended in Earth's magnetic field, it aligns itself such that its north pole points towards geographical north. Similarly, when a current loop is suspended in a uniform magnetic field, it rotates to attain equilibrium position, in which it lies with its vector magnetic moment $\vec{p}_m$ in the same direction as the field $\vec{B}$.

Consider a rectangular loop ABCD, carrying current I placed in a magnetic field B along z-axis. The loop can rotate about an axis parallel to x-axis, and passing through the mid-points of the sides AB and CD. If the normal to loop makes an angle θ with the z-axis, the torque acting on the loop is

$$\tau = -p_m B \sin\theta$$



The extra minus sign is included here because the torque is in the direction of decreasing θ.

Work done dW in rotating the dipole through a small angle dθ against this torque is

$$dW = \tau\, d\theta = -p_m B \sin\theta\, d\theta$$

Therefore, the work done in rotating the dipole from angular position θ_1 to θ_2,

$$W = -\int_{\theta_2}^{\theta_1} p_m B \sin\theta\, d\theta = p_m B\ [\cos\theta_1 - \cos\theta_2]$$

If the dipole is rotated from field direction, i.e., $\theta_1 = 0$ to any position θ, i.e., θ_2, we have

$$W = p_m B\ [1 - \cos\theta]$$

For a dipole placed in a magnetic field, the potential energy U of the dipole is defined as the work done in rotating the dipole from a direction perpendicular to the field [Fig. (B)] to the given direction, i.e.,

$$U = W_\theta - W_{90°}$$
$$= p_m B\ (1 - \cos\theta) - p_m B$$
$$= -p_m B \cos\theta$$

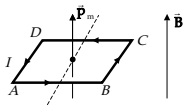
or $U = -\vec{p}_m \cdot \vec{B}$.

This is similar to the potential energy of an electric dipole moment $\vec{p}_e$ placed in an

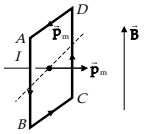
electric field $\vec{E}$,

$U = -\vec{p}_e.\vec{E}$

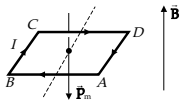
The potential energy of magnetic dipole is minimum $(= -p_mB)$ when $\vec{p}_m$ and $\vec{B}$ are parallel [Fig. (A)]. This is the position of **stable equilibrium**. In this position, $\tau = 0$. When $\vec{p}_m$ and $\vec{B}$ are antiparallel [Fig. (C)], again $\tau = 0$, work done is $2p_mB$ and the potential energy is p_mB. This is the maximum potential energy the current loop can have. It is in **unstable equilibrium**.



(A)



(B)



(C)

$\theta = 0°$ ($\vec{p}_m$ is parallel to $\vec{B}$)	$\theta = 90°$ ($\vec{p}_m$ is $\perp$ to $\vec{B}$)	$\theta = 180°$ ($\vec{p}_m$ is antiparallel to $\vec{B}$)
$\tau = 0$ (min)	$\tau = p_mB$ (max)	$\tau = 0$ (min)
$W = 0$ (min)	$W = p_mB$	$W = 2p_mB$ (max)
$U = -p_mB$ (min)	$U = 0$	$U = p_mB$ (max)
Stable equilibrium	No equilibrium	Unstable equilibrium

Comparison between Electric and Magnetic Dipoles

S.No.	Property	Electric Dipole	Magnetic Dipole
1.	Definition	$\oplus\ q$; $2a$; $p_e = 2qa$; $\ominus\ -q$	$p_m = IA$; S; I
2.	Field at a distant point along axis (end-on position)	$E_{\|\|} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2p_e}{d^3}$	$B_{\|\|} = \left(\frac{\mu_0}{4\pi}\right)\frac{2p_m}{d^3}$
3.	Field at a distant point along perpendicular bisector (broad-side-on position)	$E_\perp = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{p_e}{d^3}$	$B_\perp = \left(\frac{\mu_0}{4\pi}\right)\frac{p_m}{d^3}$
4.	Torque in an external field	$\vec{\tau} = \vec{p}_e \times \vec{E}$	$\vec{\tau} = \vec{p}_m \times \vec{B}$
5.	Work done in rotating the dipole in an external field from its equilibrium position	$W = p_eE\,(1 - \cos\theta)$ $= p_eE - \vec{p}_e.\vec{E}$	$W = p_mB\,(1 - \cos\theta)$ $= p_mB - \vec{p}_m.\vec{B}$
6.	Potential energy in an external field	$U = -\vec{p}_e.\vec{E}$	$U = -\vec{p}_m.\vec{B}$

Topicwise Questions

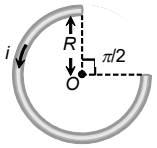
BIOT SAVART LAW

1. The magnetic field $d\vec{B}$ due to a small current element $d\vec{l}$ at a distance $\vec{r}$ and element carrying current i is,

(*a*) $d\vec{B} = \frac{\mu_0}{4\pi} i \left(\frac{d\vec{l} \times \vec{r}}{r} \right)$ (*b*) $d\vec{B} = \frac{\mu_0}{4\pi} i^2 \left(\frac{d\vec{l} \times \vec{r}}{r} \right)$

(*c*) $d\vec{B} = \frac{\mu_0}{4\pi} i^2 \left(\frac{d\vec{l} \times \vec{r}}{r^2} \right)$ (*d*) $d\vec{B} = \frac{\mu_0}{4\pi} i \left(\frac{d\vec{l} \times \vec{r}}{r^3} \right)$

2. A current *i ampere* flows in a circular arc of wire whose radius is *R*, which subtend an angle $3\pi/2$ radian at its centre. The magnetic induction *B* at the centre is



(*a*) $\frac{\mu_0 i}{R}$ (*b*) $\frac{\mu_0 i}{2R}$

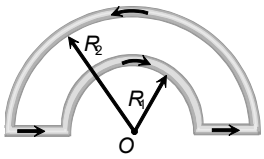
(*c*) $\frac{2\mu_0 i}{R}$ (*d*) $\frac{3\mu_0 i}{8R}$

3. A helium nucleus makes a full rotation in a circle of radius 0.8 *metre* in two seconds. The value of the magnetic field *B* at the centre of the circle will be

(*a*) $\frac{10^{-19}}{\mu_0}$ (*b*) $10^{-19}\mu_0$

(*c*) $2 \times 10^{-10}\mu_0$ (*d*) $\frac{2 \times 10^{-10}}{\mu_0}$

4. The magnetic induction at the centre *O* in the figure shown is



(*a*) $\frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (*b*) $\frac{\mu_0 i}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

(*c*) $\frac{\mu_0 i}{4}(R_1 - R_2)$ (*d*) $\frac{\mu_0 i}{4}(R_1 + R_2)$

5. A battery is connected between two points *A* and *B* on the circumference of a uniform conducting ring of radius *r* and resistance *R*. One of the arcs *AB* of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is

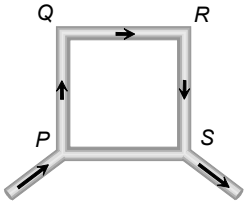
(*a*) Proportional to $2(180° - \theta)$

(*b*) Inversely proportional to *r*

(*c*) Zero, only if $\theta = 180°$

(*d*) Zero for all values of θ

6. *PQRS* is a square loop made of uniform conducting wire the current enters the loop at *P* and leaves at *S*. Then the magnetic field will be



(*a*) Maximum at the centre of the loop

(*b*) Zero at the centre of loop

(*c*) Zero at all points inside the loop

(*d*) Zero at all points outside of the loop

Ampere's law and it's Application

7. If a long hollow copper pipe carries a direct current, the magnetic field associated with the current will be

(*a*) Only inside the pipe

(*b*) Only outside the pipe

(*c*) Neither inside nor outside the pipe

(*d*) Both inside and outside the pipe

8. There are 50 turns of a wire in every *cm* length of a long solenoid. If 4 *ampere* current is flowing in the solenoid, the approximate value of magnetic field along its axis at an internal point and at one end will be respectively

(*a*) 12.6×10^{-3} Weber / m^2, 6.3×10^{-3} Weber / m^2

(*b*) 12.6×10^{-3} Weber / m^2, 25.1×10^{-3} Weber / m^2

(*c*) 25.1×10^{-3} Weber / m^2, 12.6×10^{-3} Weber / m^2

(*d*) 25.1×10^{-5} Weber / m^2, 12.6×10^{-5} Weber / m^2

9. A solenoid is 1.0 *metre* long and it has 4250 turns. If a current of 5.0 *ampere* is flowing through it, what is the magnetic field at its centre

$[\mu_0 = 4\pi \times 10^{-7}$ weber / amp – m]

(*a*) 5.4×10^{-2} weber / m^2

(*b*) 2.7×10^{-2} weber / m^2

(*c*) 1.35×10^{-2} weber / m^2

(*d*) 0.675×10^{-2} weber / m^2

Motion of a Charged Particle in Magnetic Field

10. A uniform electric field and a uniform magnetic field are produced, pointed in the same direction. An electron is projected with its velocity pointing in the same direction

(*a*) The electron will turn to its right

(*b*) The electron will turn to its left

(*c*) The electron velocity will increase in magnitude

(*d*) The electron velocity will decrease in magnitude

11. An electron has mass 9×10^{-31} kg and charge 1.6×10^{-19} C is moving with a velocity of 10^6 m /s, enters a region where magnetic field exists. If it describes a circle of radius 0.10 *m*, the intensity of magnetic field must be

(*a*) 1.8×10^{-4} T (*b*) 5.6×10^{-5} T

(*c*) 14.4×10^{-5} T (*d*) 1.3×10^{-6} T

12. A proton is moving along *Z*-axis in a magnetic field. The magnetic field is along *X*-axis. The proton will experience a force along

(*a*) *X*-axis (*b*) *Y*-axis
(*c*) *Z*-axis (*d*) Negative *Z*-axis

13. A deutron of kinetic energy 50 *keV* is describing a circular orbit of radius 0.5 *metre* in a plane perpendicular to magnetic field $\vec{B}$. The kinetic energy of the proton that describes a circular orbit of radius 0.5 *metre* in the same plane with the same $\vec{B}$ is

(*a*) 25 *keV* (*b*) 50 *keV*
(*c*) 200 *keV* (*d*) 100 *keV*

14. Lorentz force can be calculated by using the formula

(*a*) $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ (*b*) $\vec{F} = q(\vec{E} - \vec{v} \times \vec{B})$

(*c*) $\vec{F} = q(\vec{E} + \vec{v} \cdot \vec{B})$ (*d*) $\vec{F} = q(\vec{E} \times \vec{B} + \vec{v})$

15. The charge on a particle *Y* is double the charge on particle *X*. These two particles *X* and *Y* after being accelerated through the same potential difference enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of *X* to that of *Y* is

(*a*) $\left(\frac{2R_1}{R_2}\right)^2$ (*b*) $\left(\frac{R_1}{2R_2}\right)^2$

(*c*) $\frac{R_1^2}{2R_2^2}$ (*d*) $\frac{2R_1}{R_2}$

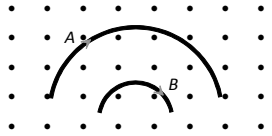
16. A charged particle enters a magnetic field *H* with its initial velocity making an angle of 45º with *H*. The path of the particle will be

(*a*) A straight line (*b*) A circle
(*c*) An ellipse (*d*) A helix

17. An electron enters a region where magnetic (*B*) and electric (*E*) fields are mutually perpendicular to one another, then

(*a*) It will always move in the direction of *B*
(*b*) It will always move in the direction of *E*
(*c*) It always possess circular motion
(*d*) It can go undeflected also

18. Two particles *A* and *B* of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B respectively, and the trajectories are as shown in the figure. Then



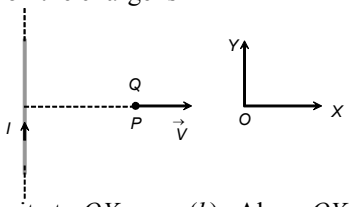
(*a*) $m_A v_A < m_B v_B$
(*b*) $m_A v_A > m_B v_B$
(*c*) $m_A < v_B$ and $v_A < v_B$
(*d*) $m_A = m_B$ and $v_A = v_B$

19. A particle of mass *M* and charge *Q* moving with velocity $\vec{v}$ describes a circular path of radius *R* when subjected to a uniform transverse magnetic field of induction *B*. The work done by the field when the particle completes one full circle is

(*a*) $BQv2\pi R$ (*b*) $\left(\frac{Mv^2}{R}\right)2\pi R$

(*c*) Zero (*d*) $BQ2\pi R$

20. A very long straight wire carries a current *I*. At the instant when a charge +Q at point P has velocity $\vec{V}$, as shown, the force on the charge is



(*a*) Opposite to *OX* (*b*) Along *OX*
(*c*) Opposite to *OY* (*d*) Along *OY*

Magnetic Force on Current Carrying Wire

21. Two long and parallel wires are at a distance of 0.1 *m* and a current of 5 *A* is flowing in each of these wires. The force per unit length due to these wires will be

(*a*) 5×10^{-5} N/m (*b*) 5×10^{-3} N/m
(*c*) 2.5×10^{-5} N/m (*d*) 2.5×10^{-4} N/m

Torque and Potential Energy of Coil in External Magnetic Field

22. A circular coil of radius 4 cm has 50 turns. In this coil a current of 2A is flowing. It is placed in a magnetic field of 0.1 weber/m^2 perpendicular to the palane of coil. The amount of work done in rotating it through 180° from its equilibrium position will be

(*a*) 0.1 J (*b*) 0.2 J
(*c*) 0.4 J (*d*) 0.8 J

23. The radius of a circular loop is r and a current i is flowing in it. The equivalent magnetic moment will be

(*a*) ir (*b*) $2\pi ir$

(*c*) $i\pi r^2$ (*d*) $\frac{1}{r^2}$

24. To make the field radial in a moving coil galvanometer

(*a*) The number of turns in the coil is increased
(*b*) Magnet is taken in the form of horse-shoe
(*c*) Poles are cylindrically cut
(*d*) Coil is wounded on aluminium frame

25. In a moving coil galvanometer, the deflection of the coil θ is related to the electrical current *i* by the relation

(*a*) $i \propto \tan\theta$ (*b*) $i \propto \theta$

(*c*) $i \propto \theta^2$ (*d*) $i \propto \sqrt{\theta}$

26. A small coil of N turns has an effective area A and carries a current I. It is suspended in a horizontal magnetic field $\vec{B}$ such that its plane is perpendicular to $\vec{B}$. The work done in rotating it by 180° about the vertical axis is

(*a*) $NAIB$ (*b*) $2NAIB$

(*c*) $2\pi NAIB$ (*d*) $4\pi NAIB$

27. A triangular loop of side l carries a current I. It is placed in a magnetic field B such that the plane of the loop is in the direction of B. The torque on the loop is

(*a*) Zero (*b*) IBl

(*c*) $\frac{\sqrt{3}}{2} Il^2B^2$ (*d*) $\frac{\sqrt{3}}{4} Il^2B^2$

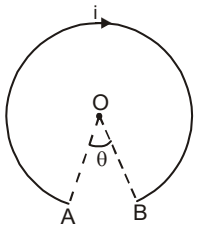
Learning Plus

1. Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the current are in the same direction, the magnetic field at a point midway between the wire is $20\mu T$. If the direction of i_1 is reversed, the field becomes $30\mu T$. The ratio i_1/i_2 is

(*a*) 4 (*b*) 3

(*c*) 5 (*d*) 1

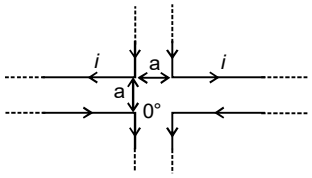
2. A current carrying wire AB of the length $2\pi R$ is turned along a circle, as shown in figure. The magnetic field at the centre O.



(*a*) $\frac{\mu_0 i}{2R}\left(\frac{2\pi-\theta}{2\pi}\right)^2$ (*b*) $\frac{\mu_0 i}{2R}\left(\frac{2\pi-\theta}{2\pi}\right)$

(*c*) $\frac{\mu_0 i}{2R}(2\pi-\theta)$ (*d*) $\frac{\mu_0 i}{2R}(2\pi+\theta)^2$

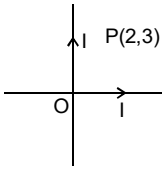
3. Four infinitely long 'L' shaped wires, each carrying a current i have been arranged as shown in the figure. Obtain the magnetic field intensity at the point 'O' equidistant from all the four corners.



(*a*) 1 wb/m^2 (*b*) 0 wb/m^2

(*c*) 2 wb/m^2 (*d*) None of these

4. Two mutually perpendicular insulated long conducting wires carrying equal currents I, intersect at origin. Then the resultant magnetic induction at point P (2m, 3m) will be -



(*a*) $\frac{\mu_0 I}{5a}$ (*b*) $\frac{5\mu_0 I}{2\pi}$

(*c*) $\frac{\mu_0 I}{I2\pi}$ (*d*) 0

5. Two parallel straight long conducting wires, which are placed at a distance r from each other, are carrying equal currents i in opposite directions. The value of magnetic induction at a point situated at a point situated x from one wire in between the wires will be :

(*a*) $\frac{\mu_0 i}{2\pi}\left\{\frac{1}{r-x}-\frac{1}{x}\right\}$ (*b*) $\frac{\mu_0 i}{2\pi}\left\{\frac{1}{r-x}+\frac{1}{x}\right\}$

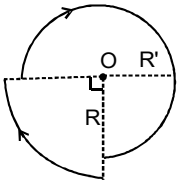
(*c*) $\frac{\mu_0 i}{2\pi(r-x)}$ (*d*) $\frac{\mu_0 i}{2\pi x}$

6. Two circular coils of wire each having a radius of 4 cm and 10 turns have a common axis and are 6 cm apart. If a current of 1 A passess through each coil in the opposite direction find the magnetic induction. At a point on the axis, midway between them.

(*a*) 13×10^{-5} T (*b*) zero

(*c*) 15×10^{-5}T (*d*) none of these

7. A current of i ampere is flowing through each of the bent wires as shown the magnitude and direction of magnetic field at O is

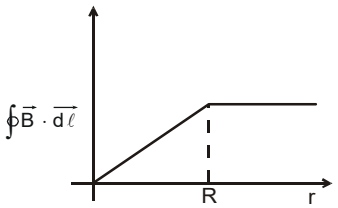


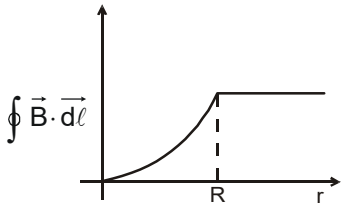
(*a*) $\frac{\mu_0 i}{4}\left(\frac{1}{R}+\frac{2}{R'}\right)$ (*b*) $\frac{\mu_0 i}{4}\left(\frac{1}{R}+\frac{3}{R'}\right)$

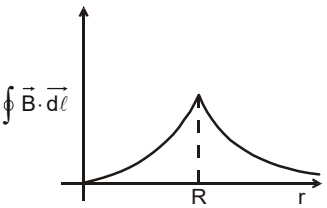
(*c*) $\frac{\mu_0 i}{8}\left(\frac{1}{R}+\frac{3}{2R'}\right)$ (*d*) $\frac{\mu_0 i}{8}\left(\frac{1}{R}+\frac{3}{R'}\right)$

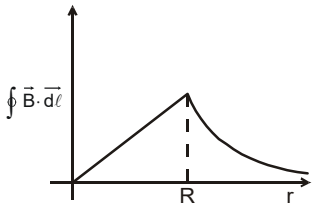
8. A point charge is moving in a circle with constant speed. Consider the magnetic field produced by the charge at a fixed point P (not centre of the circle) on the axis of the circle.
(*a*) it is constant in magnitude only
(*b*) it is constant in direction only
(*c*) it is constant in direction and magnitude both
(*d*) it is not constant in magnitude and direction both.

9. A cylindrical wire of radius R is carrying current i uniformly distributed over its cross-section. If a circular loop of radius 'r' is taken as amperian loop, then the variation value of $\oint \vec{B}\cdot\vec{d\ell}$ over this loop with radius 'r' of loop will be best represented by:

(*a*) 

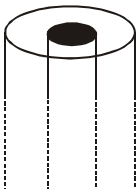
(*b*) 

(*c*) 

(*d*) 

10. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then
(*a*) the magnetic field at all points inside the pipe is the same, but not zero
(*b*) the magnetic field at any point inside the pipe is zero
(*c*) the magnetic field is zero only on the axis of the pipe
(*d*) the magnetic field is different at different points inside the pipe.

11. In a coaxial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero.



(*a*) outside the cable
(*b*) inside the inner conductor
(*c*) inside the outer conductor
(*d*) in between the two conductors

12. Electric current i enters and leaves a square loop made of homogeneous wire of uniform cross-section through diagonally opposite corners. A charge particle q moving along the axis of the square loop. Passes through centre at speed υ. The magnetic force acting on the particle when it passes through the centre has a magnitude

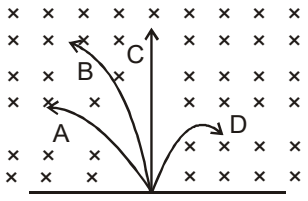
(*a*) $q\upsilon\frac{\mu_0 i}{2a}$ (*b*) $q\upsilon\frac{\mu_0 i}{2\pi a}$

(*c*) $q\upsilon\frac{\mu_0 i}{a}$ (*d*) zero

13. A negative charged particle falling freely under gravity enters a region having uniform horizontal magnetic field pointing towards north. The particle will be deflected towards
(*a*) East (*b*) West
(*c*) North (*d*) South

14. A positively charged particle moves in a region having a uniform magnetic field and uniform electric field in same direction. At some instant, the velocity of the particle is perpendicular to the field direction. The path of the particle will be
(*a*) a straight line
(*b*) a circle
(*c*) a helix with uniform pitch
(*d*) a helix with increasing pitch

15. A neutron, a proton, an electron and an a-particle enters a uniform magnetic field with equal velocities. The field is directed along the inward normal to the plane of the paper. Which of these tracks followed are by a - particle.



(*a*) A (*b*) B
(*c*) C (*d*) D

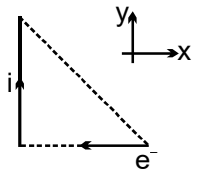
16. Electrons moving with different speeds enter a uniform magnetic field in a direction perpendicular to the field. They will move along circular paths.
(*a*) of same radius
(*b*) with larger radii for the faster electrons.
(*c*) with smaller radii for the faster electrons.
(*d*) either (*b*) or (*c*) depending on the magnitude of the magnetic field

17. A particle having charge of 1 C, mass 1 kg and speed 1 m/s enters a uniform magnetic field, having magnetic induction of 1 T, at an angle $\theta = 30°$ between velocity vector and magnetic induction. The pitch of its helical path is (in meters)

(*a*) $\frac{\sqrt{3}\pi}{2}$ (*b*) $\sqrt{3}\pi$

(*c*) $\frac{\pi}{2}$ (*d*) p

18. The direction of magnetic force on the electron as shown in the diagram is along



(*a*) y-axis (*b*) –y-axis
(*c*) z-axis (*d*) –z-axis

19. An electron is moving along positive x-axis. A uniform electric field exists towards negative y-axis. What should be the direction of magnetic field of suitable magnitude so that net force of electron is zero.
(*a*) positive z-axis (*b*) negative z-axis
(*c*) positive y-axis (*d*) negative y-axis

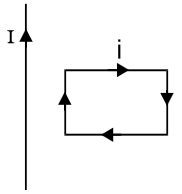
20. Two infinitely long, thin, insulated, straight wires lie in the x-y plane along the x and y-axis respectively. Each wire carries a current I, respectively in the positive x-direction and positive y-direction. The magnetic field will be zero at all points on the straight line:
(*a*) $y = x$ (*b*) $y = -x$
(*c*) $y = x - 1$ (*d*) $y = -x + 1$

21. A straight current carrying conductor is placed in such a way that the current in the conductor flows in the direction out of the plane of the paper. The conductor is placed between two poles of two magnets, as shown. The conductor will exerience a force in the direction towards



(*a*) P (*b*) Q
(*c*) R (*d*) S

22. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and the plane of the loop is same of the left wire. If a steady current I is established in the wire as shown in the (fig) the loop will -



(*a*) Rotate about an axis parallel to the wire
(*b*) Move away from the wire
(*c*) Move towards the wire
(*d*) Remain stationary

23. A bar magnet has a magnetic moment 2.5 JT^{-1} and is placed in a magnetic field of 0.2 T. Work done in turning the magnet from parallel to antiparallel position relative to the field direction.
(*a*) 0.5 J (*b*) 1 J
(*c*) 2.0 J (*d*) Zero

24. A circular loop of area 1 cm^2, carrying a current of 10 A, is placed in a magnetic field of 0.1 T perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is
(*a*) zero (*b*) 10^{-4} N-m
(*c*) 10^{-2} N-m (*d*) 1 N-m

25. The magnetic moment of a circular orbit of radius 'r' carrying a charge 'q' and rotating with velocity v is given by

(*a*) $\frac{qvr}{2\pi}$ (*b*) $\frac{qvr}{2}$

(*c*) $qv\pi r$ (*d*) $qv\pi r^2$

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. A long straight wire carries a current along the x-axis. Consider the points A(0, 1, 0), B (0, 1, 1), C (1, 0, 1) and D (1, 1, 1). Which of the following pairs of points will have magnetic fields of the same magnitude ?
(*a*) A and B (*b*) A and C
(*c*) B and C (*d*) B and D

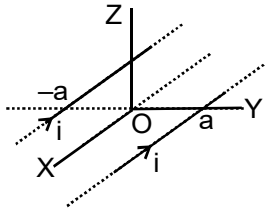
2. In the previous question, if the current is *i* and the magnetic field at D has magnitude B,

(*a*) $B = \frac{\mu_0 i}{2\sqrt{2}\pi}$

(*b*) $B = \frac{\mu_0 i}{2\sqrt{3}\pi}$

(*c*) B is parallel to the x-axis
(*d*) B makes an angle of 45° with the xy plane

3. Two long thin, parallel conductors carrying equal currents in the same direction are fixed parallel to the x-axis, one passing through y = a and the other through y = –a. The resultant magnetic field due to the two conductors at any point is B. Which of the following are correct?



(*a*) B = 0 for all points on the x-axis
(*b*) At all points on the y-axis, excluding the origin, B has only a z-component.
(*c*) At all point of the z-axis, excluding the origin, B has only a y-component.
(*d*) B cannot have an x-component.

4. A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field
(*a*) increases linearly from the axis to the surface
(*b*) is constant inside the tube
(*c*) is zero at the axis
(*d*) is non-zero outside the tube at finite distance from surface

5. Two identical charged particles enter a uniform magnetic field with same speed but at angles 30° and 60° with field. Let a, b and c be the ratio of their time periods, radii and pitches of the helical paths than
(*a*) abc = 1 (*b*) abc > 1
(*c*) abc < 1 (*d*) a = bc

6. Consider the following statements regarding a charged particle in a magnetic field. Which of the statements are true :
(*a*) Starting with zero velocity, it accelerates in a direction perpendicular to the magnetic field.
(*b*) While deflecting in magnetic field its energy gradually increases.
(*c*) Only the component of magnetic field perpendicular to the direction of motion of the charged particle is effective in deflecting it.
(*d*) Direction of deflecting force on the moving charged particle is perpendicular to its velocity.

7. A beam of electrons moving with a momentum p enters a uniform magnetic field of flux density B perpendicular to its motion. Which of the following statement(s) is (are) true?

(*a*) Energy gained is $\frac{p^2}{2m}$

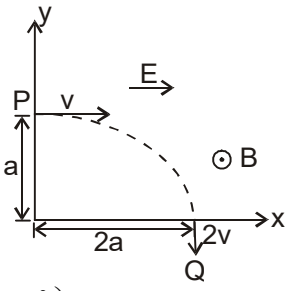
(*b*) Centripetal force on the electron is $Be\frac{m}{p}$

(*c*) Radius of the electron's path is $\frac{p}{Be}$

(*d*) Work done on the electrons by the magnetic field is zero

8. Two ions have equal masses but one is singly-ionized and other is tripply-ionized. They are projected from the same place in a uniform magnetic field with the same velocity perpendicular to the field.
(*a*) Both ions will go along circles of equal radii.
(*b*) The circle described by the single-ionized charge will have a radius tripply that of the other circle
(*c*) The two circles do not touch each other
(*d*) The two circles touch each other

9. A particle of charge +q and mass m moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$ follows a trajectory from P and Q as shown in figure. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$. Which of the following statement(s) is/are correct?



(*a*) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(*b*) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

(*c*) Rate of work done by the electric field at P is zero.
(*d*) Rate of work done by both fields at Q is zero.

10. Let $\vec{E}$ and $\vec{B}$ denote the electric and magnetic fields in a certain region of space. A proton moving with a velocity $\vec{v}$ along a straight line enters the region and is found to pass through it undeflected. Indicate which of the following statements are possible for the observation:

(*a*) $\vec{E} = 0$ and $\vec{B} = 0$

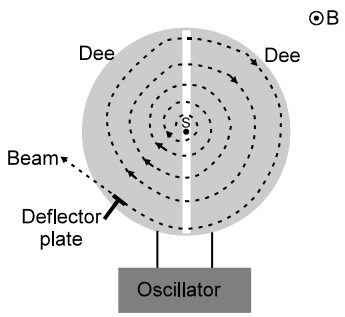
(*b*) $\vec{E} \neq 0$ and $\vec{B} = 0$

(*c*) $\vec{E} \neq 0$, $\vec{B} \neq 0$ and both $\vec{E}$ and $\vec{B}$ are parallel to $\vec{v}$

(*d*) $\vec{E}$ is parallel to $\vec{v}$ but $\vec{B}$ is perpendicular to $\vec{v}$

Comprehension–1 (No. 11 to 15)
(Read the following passage and answer the questions. They have only one correct option)

In the given figure of a cyclotron, showing the particle source S and the dees. A uniform magnetic field is directed up from the plane of the page. Circulating protons spiral outward within the hollow dees, gaining energy every time they cross the gap between the dees.



Suppose that a proton, injected by source S at the centre of the cyclotron in Fig., initially moves toward a negatively charged dee. It will accelerate toward this dee and enter it. Once inside, it is shielded from electric field by the copper walls of the dee; that is the electric field does not enter the dee. The magnetic field, however, is not screened by the (nonmagnetic) copper dee, so the proton moves in circular path whose radius, which depends on its speed, is given by

$$\text{Eq. } r = \frac{mv}{qB} \qquad ...(1)$$

Let us assume that at the instant the proton emerges into the center gap from the first dee, the potential difference between the dees is reversed. Thus, the proton again faces a negatively charged dee and is again accelerated. Thus, the proton again faces a negatively charged dee and is again accelerated. This process continues, the circulating proton always being in step, with the oscillations of the dee potential, until the proton has spiraled out to the edge of the dee system. There a deflector plate sends it out through a portal.

The key to the operation of the cyclotron is that the frequency f at which the proton circulates in the field (and that does not depend on its speed) must be equal to the fixed frequency f_{osc} of the electrical oscillator, or

$$f = f_{osc} \text{ (resonance condition)} \qquad ...(2)$$

This resonance condition says that, if the energy of the circulating proton is to increase, energy must be fed to it at a frequency f_{osc} that is equal to the natural frequency f at which the proton circulates in the magnetic field.

Combining Eq. 1 and 2 allows us to write the resonance condition as

$$qB = 2\pi m f_{osc}. \qquad ...(3)$$

For the proton, q and m are fixed. The oscillator (we assume) is designed to work at a single fixed frequency f_{osc}. We then "tune" the cyclotron by varying B until eq. 3 is satisfied and then many protons circulate through the magnetic field, to emerge as a beam.

11. Ratio of radius of successive semi circular path

(*a*) $\sqrt{1} : \sqrt{2} : \sqrt{3} : \sqrt{4}$ .....

(*b*) $\sqrt{1} : \sqrt{3} : \sqrt{5}$ .....

(*c*) $\sqrt{2} : \sqrt{4} : \sqrt{6}$ .....

(*d*) 1 : 2 : 3 .....

12. Change in kinetic energy of charge particle after every time period is :

(*a*) 2qV (*b*) qV

(*c*) 3qV (*d*) None of these

13. If q/m for a charge particle is 10^6, frequency of applied AC is 10^6 Hz. Then applied magnetic field is:

(*a*) 2π tesla (*b*) π tesla

(*c*) 2 tesla (*d*) can not be defined

14. Distance travelled in each time period are in the ratio of:

(*a*) $\sqrt{1} + \sqrt{3} : \sqrt{5} + \sqrt{7} : \sqrt{9} + \sqrt{11}$

(*b*) $\sqrt{2} + \sqrt{3} : \sqrt{4} + \sqrt{5} : \sqrt{6} + \sqrt{7}$

(*c*) $\sqrt{1} : \sqrt{2} : \sqrt{3}$

(*d*) $\sqrt{2} : \sqrt{3} : \sqrt{4}$

15. For a given charge particle a cyclotron can be "tune" by:

(*a*) changing applied A.C. voltage only

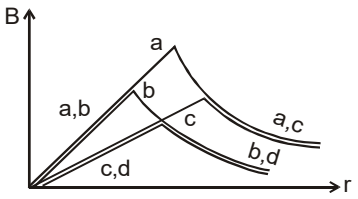
(*b*) changing applied A.C. voltage and magnetic field both

(*c*) changing applied magnetic field only

(*d*) by changing frequency of applied A.C.

Comprehension–2 (No. 16 to18)

Curves in the graph shown give, as functions of radial distance r (from the axis), the magnitude B of the magnetic field (due to individual wire) inside and outside four long wires a, b, c and d, carrying currents that are uniformly distributed across the cross sections of the wires. Overlapping portions of the plots are indicated by double labels. All curves start from the origin.



16. Which wire has the greatest radius?

(*a*) a (*b*) b

(*c*) c (*d*) d

17. Which wire has the greatest magnitude of the magnetic field on the surface?

(*a*) a (*b*) b

(*c*) c (*d*) d

18. The current density in wire a is

(*a*) greater than in wire c.

(*b*) less than in wire c.

(*c*) equl to that in wire c.

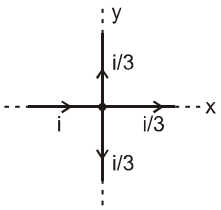
(*d*) not comparable to that of in wire c due to lack of information.

19. Column-II gives four situations in which three (in q,r,s) and four (in p) semi infinite current carrying wires are placed in xy-plane as shown. The magnitude and direction of current is shown in each figure. Column-I gives statements regarding the x and y components of magnetic field at a point P whose coordinates are P (0, 0, d). Match the statements in column-I with the corresponding figures in column-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in OMR.

Column I **Column II**

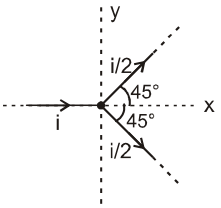
(*a*) The x component of magnetic field at point P is zero in

(p)



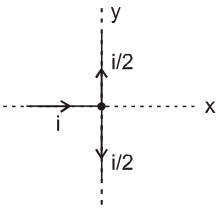
(*b*) The z component of magnetic field at point P is zero in

(q)



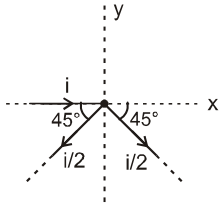
(*c*) The magnitude of magnetic field at point P is $\frac{\mu_0 i}{4\pi d}$ in

(r)



(*d*) The magnitude of magnetic field at point P is less than $\frac{\mu_0 i}{2\pi d}$ in

(s)



20. There are four situations given in column **I** involving a magnetic dipole of dipole moment $\vec{\mu}$ placed in uniform external magnetic field $\vec{B}$. Column **II** gives corresponding results. Match the situtations in column **I** with the corresponding results in column **II**

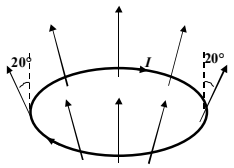
Column - I	**Column - II**
(*a*) Magnetic dipole moment $\vec{\mu}$ is parallel to uniform external magnetic field $\vec{B}$ (angle between both vectors is zero)	(p) force on dipole is zero
(*b*) Magnetic dipole moment $\vec{\mu}$, is perpendicular to uniform external magnetic field $\vec{B}$	(q) torque on dipole is zero
(*c*) Angle between magnetic dipole moment $\vec{\mu}$ and uniform external magnetic field $\vec{B}$ is acute	(r) magnitude of torque is (μB)
(*d*) Angle between magnetic dipole moment $\vec{\mu}$ and uniform external magnetic field $\vec{B}$ is 180°.	(s) potential energy of dipole due to external magnetic field is (μB)

NUMERICAL VALUE BASED

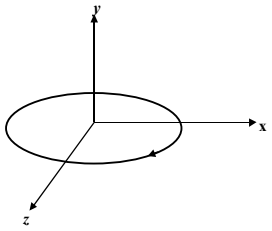
21. A charge $q = -4\ \mu C$ has an instantaneous velocity $\vec{v} = (2 \times 10^6\ \hat{i} - 3 \times 10^6\ \hat{j} + 10^6\ \hat{k})$ m/s in a uniform field $\vec{B} = (2 \times 10^{-2}\ \hat{i} + 5 \times 10^{-2}\ \hat{j} - 3 \times 10^{-2}\ \hat{k})$ T. What is the magnitude of x component of force on the charge in N?

22. An electron accelerated by a potential difference V = 1.0 kV moves in a uniform magnetic field at an angle $\alpha = 30°$ to the vector B whose modulus is B = 29mT . Find the pitch of the helical trajectory of the electron in mm.

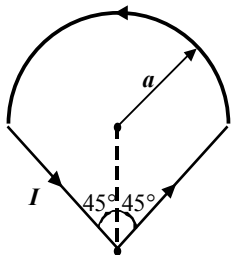
23. A super conducting ring has a radius of 1.4 cm and a mass of 30g. The ring carries a constant *I* and is placed in a 0.5 T magnetic field with field lines that are tilted at a 20° angle, outward from the vertical at every location around the ring. What must the current *I* in Ampere be for the ring to float in the magnetic field?
[Given sin20° = 0.34 ; cos20° = 0.94]



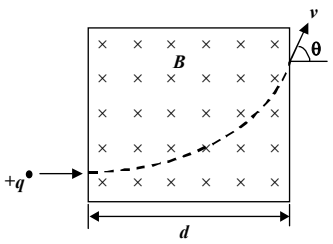
24. A coil of radius 25 cm has 15 turns and lies in the x-z plane. It carries a current of 2A, as in figure. Find magnitude of torque on the coil for $\vec{B} = 0.2\,\hat{i}$ T in N-m.



25. Two perpendicular straight wires join the ends of a semicircular loop of radius *a*, as shown in the figure. If the current is *I*, the resultant field at the centre of the circular section is given by $x \times 10^{-7}\ \frac{I}{a}$ then x is -



26. A proton accelerated by a potential difference *V* = 500 kV flies through a uniform transverse magnetic field with induction *B* = 0.51 T. The field occupies a region of space *d* = 10 cm in thickness. Find the angle θ in degree through which the proton deviates from the initial direction of its motion.



27. The magnetic field existing in a region is given by

$$\vec{B} = B_0\left(1 + \frac{x}{l}\right)\hat{k}$$

A square loop of side *l* and carrying a current I, is placed with its sides parallel to the x and y axes. The magnitude of the net magnetic force experienced by the loop is kBI*l* then k is

28. A current *I* = 10A flows in a ring of radius r_0 = 15 cm made of a very thin wire. The tensile strength of the wire is equal to *T* = 1.5 N. The ring is placed in a magnetic field which is perpendicular to the plane of the ring so that the forces tend to break the ring. Find *B* in Tesla at which the ring is broken.

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. Two identical wires A and B, each of length '*l* ', carry the same current *I*. Wire A is bent into a circle of radius R and wire B is bent to form a square of side ' a '. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is:

[*JEE Main-2016*]

(*a*) $\frac{\pi^2}{8}$ (*b*) $\frac{\pi^2}{16\sqrt{2}}$

(*c*) $\frac{\pi^2}{16}$ (*d*) $\frac{\pi^2}{8\sqrt{2}}$

2. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e , r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is: **[*JEE Main-2018*]**

(*a*) $r_e < r_p = r_\alpha$ (*b*) $r_e < r_p < r_\alpha$

(*c*) $r_e < r_\alpha < r_p$ (*d*) $r_e > r_p = r_\alpha$

3. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2.

The ratio $\frac{B_1}{B_2}$ is: **[*JEE Main-2018*]**

(*a*) $\sqrt{3}$ (*b*) $\sqrt{2}$

(*c*) $\frac{1}{\sqrt{2}}$ (*d*) 2

4. The region between y = 0 and y = d contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velcoity $\vec{v} = v\hat{i}$. If $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is :

[*JEE Main - 2019 (January)*]

(*a*) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$ (*b*) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$

(*c*) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$ (*d*) $\frac{qvB}{m}\left(\frac{\hat{j}+\hat{i}}{\sqrt{2}}\right)$

5. In an experiment, electrons are accelerated, from rest by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied. [Charge of electron = 1.6×10^{-19} C, Mass of electron = 9.1×10^{-31} kg]

[*JEE Main - 2019 (January)*]

(*a*) 7.5×10^{-3}m (*b*) 7.5×10^{-2}m

(*c*) 7.5 m (*d*) 7.5×10^{-4}m

6. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (given charge of electron = 1.6×10^{-19}C)

[*JEE Main - 2019 (January)*]

(*a*) 9.1×10^{-31} kg (*b*) 1.6×10^{-27} kg

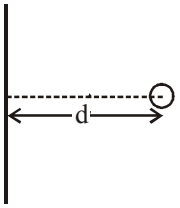
(*c*) 1.6×10^{-19} kg (*d*) 2.0×10^{-24} kg

7. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the centre of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be **[*JEE Main - 2019 (January)*]**

(*a*) N (*b*) $\frac{1}{N}$

(*c*) N^2 (*d*) $\frac{1}{N^2}$

8. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d >> a). If the loop applies a force F on the wire then: **[*JEE Main - 2019 (January)*]**



(*a*) F = 0 (*b*) $F \propto \left(\frac{a}{d}\right)$

(*c*) $F \propto \left(\frac{a^2}{d^3}\right)$ (*d*) $F \propto \left(\frac{a}{d}\right)^2$

9. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then: **[*JEE Main - 2019 (January)*]**

(*a*) $T_h = T_c$ (*b*) $T_h = 2T_C$

(*c*) $T_h = 1.5T_C$ (*d*) $T_h = 0.5T_C$

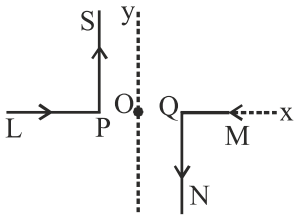
10. An insulating thin rod of length ℓ has a linear charge density $\rho(x) = \rho_0 \frac{x}{\ell}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is:

[*JEE Main - 2019 (January)*]

(*a*) $\pi n\rho_0\ell^3$ (*b*) $\frac{\pi}{3}n\rho_0\ell^3$

(*c*) $\frac{\pi}{4}n\rho_0\ell^3$ (*d*) $n\rho_0\ell^3$

11. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4 cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal current (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}): **[*JEE Main - 2019 (January)*]**



(*a*) 20 A, perpendicular out of the page

(*b*) 40 A, perpendicular out of the page

(*c*) 20 A, perpendicular into the page

(*d*) 40 A, perpendicular into the page

12. A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1:2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be:

[*JEE Main-2019 (January)*]

(*a*) $1:\sqrt{2}$ (*b*) $1:2$

(*c*) $1:3$ (*d*) $1:\sqrt{3}$

13. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N–m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately :-

[*JEE Main-2019 (April)*]

(*a*) 10^{-3} (*b*) 10^{-1}

(*c*) 10^{-4} (*d*) 10^{-2}

14. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is

[*JEE Main-2019 (April)*]

(*a*) $B\pi r^2 IN$ (*b*) $\dfrac{Br^2 I}{\pi N}$

(*c*) Zero (*d*) $\dfrac{B\pi r^2 I}{N}$

15 The magnitude of the magnetic field at the center of an equilateral triangular loop of side l m which is carrying a current of 10 A is : [Take $\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$]

[*JEE Main-2019 (April)*]

(*a*) 18 μT (*b*) 3μT

(*c*) 1 μT (*d*) 9 μT

16. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be :

[*JEE Main-2019 (April)*]

(*a*) $\dfrac{3m}{\pi}$ (*b*) $\dfrac{4m}{\pi}$

(*c*) $\dfrac{2m}{\pi}$ (*d*) $\dfrac{m}{\pi}$

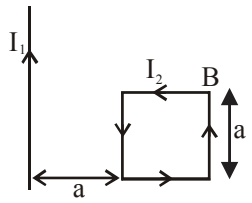
17. Two very long, straight and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure. These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be :

[*JEE Main-2019 (April)*]

(*a*) Zero (*b*) $\dfrac{+\mu_0 I}{\pi d}(\hat{z})$

(*c*) $-\dfrac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$ (*d*) $\dfrac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

18. A rigid square loop of side 'a' and carrying current I_2 is lying on horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to wire will be: **[*JEE Main-2019 (April)*]**



(*a*) Attractive and equal to $\dfrac{\mu_0 I_1 I_2}{3\pi}$

(*b*) Repulsive and equal to $\dfrac{\mu_0 I_1 I_2}{4\pi}$

(*c*) Repulsive and equal to $\dfrac{\mu_0 I_1 I_2}{2\pi}$

(*d*) Zero

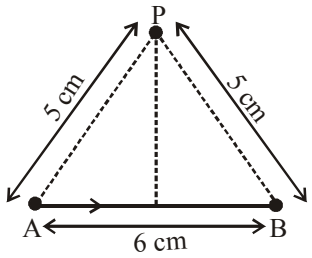
19. A rectangular coil (dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is:

[*JEE Main-2019 (April)*]

(*a*) 0.55 Nm (*b*) 0.27 Nm

(*c*) 0.38 Nm (*d*) 0.42 Nm

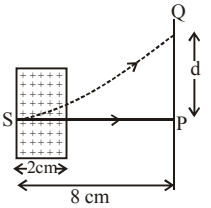
20. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. in figure ($\mu_0 = 4\pi \times 10^{-7}\ N\text{-}A^{-2}$) **[*JEE Main-2019 (April)*]**



(*a*) 3.0×10^{-5} T (*b*) 2.5×10^{-5} T

(*c*) 2.0×10^{-5} T (*d*) 1.5×10^{-5} T

21. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = \left(1.5\times10^{-3}\,T\right)\hat{k}$ at S (See figure). The field extends between x = 0 and x=2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is :(Electron's charge = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31} kg) **[*JEE Main-2019 (April)*]**



(*a*) 12.87 cm (*b*) 1.22 cm

(*c*) 11.65 cm (*d*) 2.25 cm

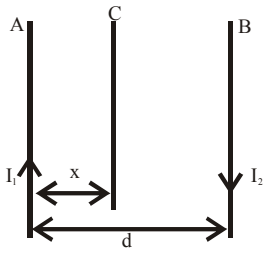
22. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

[*JEE Main-2019 (April)*]

(*a*) $r_e > r_p > r_{He}$ (*b*) $r_e < r_p < r_{He}$

(*c*) $r_e < r_p = r_{He}$ (*d*) $r_e > r_p = r_{He}$

23. Two wires A & B are carrying currents I_1 & I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are: **[*JEE Main-2019 (April)*]**



(*a*) $x = \left(\frac{I_1}{I_1 - I_2}\right)d$ and $x = \frac{I_2}{(I_1 + I_2)}d$

(*b*) $x = \pm\frac{I_1 d}{(I_1 - I_2)}$

(*c*) $x = \left(\frac{I_1}{I_1 + I_2}\right)d$ and $x = \frac{I_2}{(I_1 - I_2)}d$

(*d*) $x = \left(\frac{I_2}{I_1 + I_2}\right)d$ and $x = \left(\frac{I_2}{I_1 - I_2}\right)d$

24. The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(kz + \omega t)$. At t = 0, a positively charged particle is at the point $(x, y, z) = \left(0, 0, \frac{\pi}{k}\right)$. If its instantaneous velocity at (t = 0) is $v_0\hat{k}$, the force acting on it due to the wave is: **[*JEE Main-2020 (January)*]**

(*a*) Zero (*b*) parallel to $\hat{k}$

(*c*) antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (*d*) parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

25. A particle of mass m and charge q has an initial velocity $\vec{v} = v_0\hat{j}$. If an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ act on the particle, its speed will double after a time: **[*JEE Main-2020 (January)*]**

(*a*) $\frac{2mv_0}{qE_0}$ (*b*) $\frac{3mv_0}{qE_0}$

(*c*) $\frac{\sqrt{2}mv_0}{qE_0}$ (*d*) $\frac{\sqrt{3}mv_0}{qE_0}$

26. Proton with kinetic energy of 1 MeV moves from south to north. It gets an acceleration of 10^{12}m/s^2 by an applied magnetic field (west to east). The value of magnetic field. (Rest mass of proton is 1.6×10^{-27} kg)

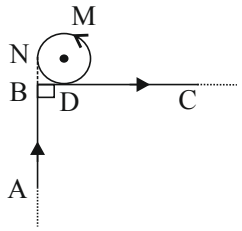
[*JEE Main-2020 (January)*]

(*a*) 7.1 mT (*b*) 71 mT

(*c*) 0.71 mT (*d*) 0.071 mT

27. A very long wire ABDMNDC is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn DMND of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is :

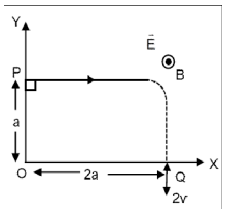
[*JEE Main-2020 (January)*]



(*a*) $\frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$ (*b*) $\frac{\mu_0 I}{2R}$

(*c*) $\frac{\mu_0 I}{2\pi R}\left(\pi - \frac{1}{\sqrt{2}}\right)$ (*d*) $\frac{\mu_0 I}{2\pi R}(\pi + 1)$

28. A charged particle of mass 'm' and charge 'q' moving under the influence of uniform electric field $E\vec{i}$ and a uniform magnetic field $B\hat{k}$ follows a trajectroy from point P to Q as shown in figure. The velocities at P and Q are respectively, $v\vec{i}$ and $-2v\vec{j}$. Then which of the following statements (A,B,C,D) are the correct ? (Trajectroy shown is schematic and not to scale)



(*A*) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(*B*) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

(*C*) Rate of work done by both the fields at Q is zero

(*D*) The difference between the magnitude of angular momentum of the particle at P and Q is 2mav.

[*JEE Main-2020 (January)*]

(*a*) (A), (B), (C) (*b*) (A), (B), (C), (D)

(*c*) (A), (C), (D) (*d*) (B), (C), (D)

29. A long, straight wire of radius a carries a current distributed uniformly over its cross–section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and 2a, respectively from the axis of the wire is:

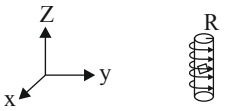
[*JEE Main-2020 (January)*]

(*a*) $\frac{1}{2}$ (*b*) 2

(*c*) $\frac{3}{2}$ (*d*) $\frac{2}{3}$

30. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning)

[*JEE Main-2020 (January)*]



(*a*) $\frac{e\mu_0 nIR}{2m}$ (*b*) $\frac{e\mu_0 nIR}{m}$

(*c*) $\frac{2e\mu_0 nIR}{m}$ (*d*) $\frac{e\mu_0 nIR}{4m}$

31. A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m then :

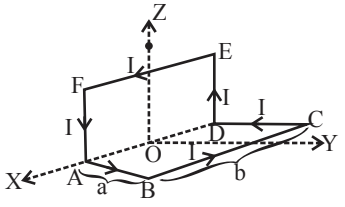
[*JEE Main-2020 (January)*]

(*a*) $T = \sqrt{\frac{\pi m}{IB}}$ (*b*) $T = \sqrt{\frac{2m}{IB}}$

(*c*) $T = \sqrt{\frac{\pi m}{2IB}}$ (*d*) $T = \sqrt{\frac{2\pi m}{IB}}$

32. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangle ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is

[*JEE Main-2020 (September)*]



(*a*) abl, along $\left(\frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}}\right)$

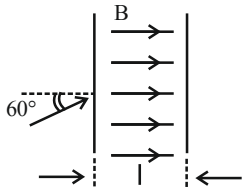
(*b*) $\sqrt{2}$ abl, along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

(*c*) abl, along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

(*d*) $\sqrt{2}$ abl, along $\left(\frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}}\right)$

33. The figure shows a region of length 'l' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity 4×10^5 ms^{-1} making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, 'l' is close to (mass of proton $=1.67 \times 10^{-27}$ kg, charge of the proton = 1.6×10^{-19} C)

[*JEE Main-2020 (September)*]



(*a*) 0.22 m (*b*) 0.11 m

(*c*) 0.88 m (*d*) 0.44 m

34. A beam of protons with speed 4×10^5 enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to (Mass of the proton = 1.67×10^{-27} kg, charge of the proton = 1.69×10^{-19} C)

[*JEE Main-2020 (September)*]

(*a*) 2 cm (*b*) 12 cm

(*c*) 5 cm (*d*) 4 cm

35. A galvanometer coil has 500 turns and each turn has an average area of 3×10^{-4} m^2. If a torque of 1.5 Nm is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is _____.

[*JEE Main-2020 (September)*]

36. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is

[*JEE Main-2020 (September)*]

(*a*) $500\sqrt{3}$ (*b*) $250\sqrt{3}$

(*c*) $50\sqrt{3}$ (*d*) $5\sqrt{3}$

37. A charged particle carrying charge 1 μC is moving with velocity $(2\hat{i}+3\hat{j}+4\hat{k})$ ms^{-1}. If an external magnetic field of $(5\hat{i}+3\hat{j}-6\hat{k})\times10^{-3}$ T exists in the region where the particle is moving then the force on the particle is $\vec{F}\times10^{-9}$ N. The vector $\vec{F}$ is **[*JEE Main-2020 (September)*]**

(*a*) $-3.0\hat{i}+3.2\hat{j}-0.9\hat{k}$

(*b*) $-300\hat{i}+320\hat{j}-90\hat{k}$

(*c*) $-0.30\hat{i}+0.32\hat{j}-0.09\hat{k}$

(*d*) $-30\hat{i}+32\hat{j}-9\hat{k}$

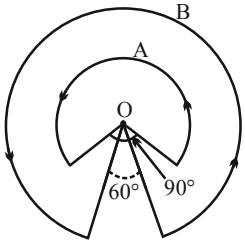
38. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the vertical, it starts rotating, around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be **[*JEE Main-2020 (September)*]**

(*a*) 10 rad s^{-1} (*b*) 10π rad s^{-1}

(*c*) 20 rad s^{-1} (*d*) 20π rad s^{-1}

39. A wire A, bent in the shape of an arc of a circle, carrying a current of 2 A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is

[*JEE Main-2020 (September)*]



(*a*) 2 : 5 (*b*) 6 : 5

(*c*) 4 : 6 (*d*) 6 : 4

40. A square loop of side 2a, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point (0, b, 0), (b > > a). The magnitude of the torque on the loop about z-axis is given by

[*JEE Main-2020 (September)*]

(*a*) $\frac{2\mu_0 I^2a^2}{\pi b}$ (*b*) $\frac{\mu_0 I^2a^2}{2\pi b}$

(*c*) $\frac{\mu_0 I^2a^3}{2\pi b^2}$ (*d*) $\frac{2\mu_0 I^2a^3}{\pi b^2}$

41. An electron is constrained to move along the y-axis with a speed of 0.1 c (c is the speed of light) in the presence of electromagnetic wave, whose electric field is $\vec{E}=30\hat{j}$ $\sin(1.5\times10^7\,t-5\times10^{-2}x)$ V/m. The maximum magnetic force experienced by the electron will be (**Given** c = 3 × 10^8 ms^{-1} and Electron charge = 1.6 × 10^{-19} C)

[*JEE Main-2020 (September)*]

(*a*) 4.8×10^{-19} N (*b*) 2.4×10^{-18} N

(*c*) 3.2×10^{-18} N (*d*) 1.6×10^{-19} N

42. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field $\vec{B}$. The magnetic moment of this moving particle :

[*JEE Main-2020 (September)*]

(*a*) $-\frac{mv^2\vec{B}}{2B^2}$ (*b*) $\frac{mv^2\vec{B}}{2B^2}$

(*c*) $-\frac{mv^2\vec{B}}{2\pi B^2}$ (*d*) $-\frac{mv^2\vec{B}}{B^2}$

43. A particle moving in the xy plane experiences a velocity dependent force $\vec{F}=k(v_y\hat{i}+v_x\hat{j})$, where v_x and v_y are the x and y components of its velocity $\vec{v}$. If $\vec{a}$ is the acceleration of the particle, then which of the following statements is true for the particle?

[*JEE Main-2020 (September)*]

(*a*) Quantity $\vec{v}\cdot\vec{a}$ is constant in time

(*b*) Kinetic energy of particle is constant in time

(*c*) Quantity $\vec{v}\times\vec{a}$ is constant in time

(*d*) $\vec{F}$ arises due to a magnetic field

44. A square loop of side 2a and carrying current I is kept in xz plane with its centre at origin. A long wire carrying the same current I is placed parallel to z-axis and passing through point (0, b, 0), (b >> a). The magnitude of torque on the loop about z-axis will be

[*JEE Main-2020 (September)*]

(*a*) $\frac{\mu_0 I^2a^2}{2\pi b}$ (*b*) $\frac{2\mu_0 I^2a^2b}{\pi(a^2+b^2)}$

(*c*) $\frac{\mu_0 I^2a^2b}{2\pi(a^2+b^2)}$ (*d*) $\frac{2\mu_0 I^2a^2}{\pi b}$

45. A particle of charge q and mass m is moving with a velocity $-v\hat{i}(v\neq0)$ towards a large screen placed in the Y - Z plane at a distance d. If there is a magnetic field $\vec{B}=B_0\hat{k}$, the minimum value of v for which the particle will not hit the screen is

[*JEE Main-2020 (September)*]

(*a*) $\frac{2qdB_0}{m}$ (*b*) $\frac{qdB_0}{3m}$

(*c*) $\frac{qdB_0}{2m}$ (*d*) $\frac{qdB_0}{m}$

46. An electron is moving along + x direction with a velocity of 6×10^6 ms^{-1}. It enters a region of uniform electric field of 300 V/cm pointing along + y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be **[*JEE Main-2020 (September)*]**

(*a*) 5×10^{-3} T, along + z direction

(*b*) 5×10^{-3} T, along – z direction

(*c*) 3×10^{-4} T, along + z direction

(*d*) 3×10^{-4} T, along – z direction

47. Which of the following statements are correct?

[*JEE Main-2021 (March)*]

(A) Electric monopoles do not exist whereas magnetic monopoles exist.

(B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.

(C) Magnetic field lines are completely confined within a toroid.

(D) Magnetic field lines inside a bar magnet are not parallel.

(E) $\chi = -1$ is the condition for a perfect diamagnetic material, where x is its magnetic susceptibility.

Choose the correct answer from the options given below:

(*a*) (A) and (B) only

(*b*) (B) and (C) only

(*c*) (C) and (E) only

(*d*) (B) and (D) only

48. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field $(\vec{E})$ and magnetic field $(\vec{B})$ components.

[*JEE Main-2021 (March)*]

(*a*) $E_{x_1}\, B_z$ or $E_{z_1}\, B_x$ (*b*) $E_{x_1} B_y$ or $E_{y!} B_x$

(*c*) $E_{y,1} B_y$ or $E_{z,} B_z$ (*d*) $E_{y'} B_x$ or $E_{x'} B_y$

49. A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire.

[*JEE Main-2021 (March)*]

(*a*) Loop assumes circular shape with its plane normal to the field.

(*b*) Loop assumes circular shape with its plane parallel to the field.

(*c*) Wire gets stretched to become straight.

(*d*) Shape of the loop remains unchanged.

50. A plane electromagnetic wave of frequency 100MHz is travelling in vacuum along the x-direction. At a particular point in space and time, $\vec{B} = 2.0 \times 10^{-8} \hat{k} T$. (where, $\hat{k}$ is unit vector along z-direction) What is $\vec{E}$ at this point?

[*JEE Main-2021 (March)*]

(*a*) $0.6\hat{j}$ V / m (*b*) $6.0\hat{k}$ V / m

(*c*) $6.0\hat{j}$ V / m (*d*) $0.6\hat{k}$ V / m

JEE-ADVANCED PREVIOUS YEAR'S

1. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with x-axis. Which of the following option(s) is(are) correct for time t > 0 ? **[*IIT JEE-2012*]**

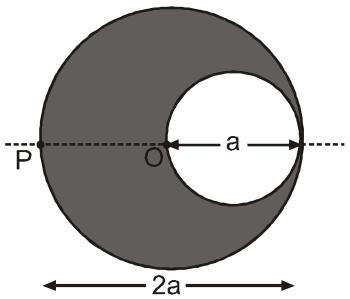
(*a*) If θ = 0°, the charge moves in a circular path in the x-z plane.

(*b*) If θ = 0°, the charge undergoes helical motion with constant pitch along the y-axis.

(*c*) If θ = 10°, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

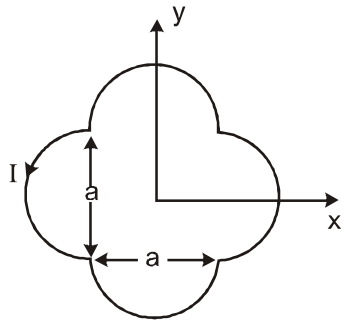
(*d*) If θ = 90°, the charge undergoes linear but accelerated motion along the y-axis.

2. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0$ aJ, then the value of N is: **[*IIT JEE-2012*]**



3. A loop carrying current I lies in the x-y plane as shown in the figure. the unit vector $\hat{k}$ is coming out of the plane of the paper. the magnetic moment of the current loop is:

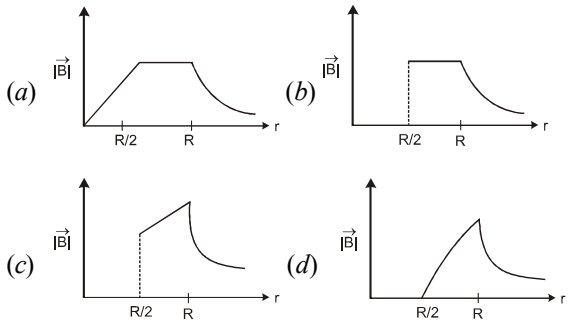
[*IIT JEE-2012*]



(*a*) $a^2 I\hat{k}$ (*b*) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

(*c*) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (*d*) $(2\pi+1)a^2 I\hat{k}$

4. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along is length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by: **[*IIT JEE-2012*]**



5. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}\ ms^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2\left(\sqrt{3}\,\hat{i} + \hat{j}\right)\ ms^{-1}$. The correct statement(s) is (are): **[*JEE Advanced-2013*]**

(*a*) The direction of the magnetic field is –z direction.

(*b*) The direction of the magnetic field is +z direction

(*c*) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.

(*d*) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

6. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are): **[*JEE Advanced -2013*]**

(*a*) In the region 0 < r < R, the magnetic field is non–zero.

(*b*) In the region R < r < 2R, the magnetic field is along the common axis.

(*c*) In the region R < r < 2R, the magnetic field is tangential to the circle of radius r, centered on the axis.

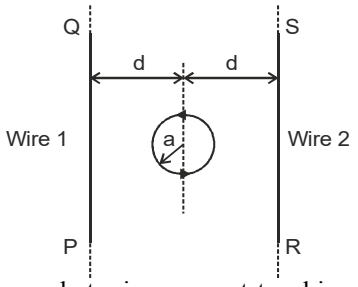
(*d*) In the region r > 2R, the magnetic field is non–zero.

7. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have direction opposite to each other, the radius of curvature of the path is R_2. If $\frac{x_0}{x_1} = 3$, the value of $\frac{R_1}{R_2}$ is. **[*JEE Advanced -2014*]**

Paragraph For Questions 11 to 12

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above.

[*JEE Advanced -2014*]



8. When d ≈ a but wires are not touching the loop, it is found that the net magnetic filed on the axis of the loop is zero at a height h above the loop. In that case **[*JEE Advanced -2014*]**

(*a*) current in wire 1 and wire 2 is the direction PQ and RS, respectively and h ≈ a

(*b*) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ a

(*c*) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ 1.2 a

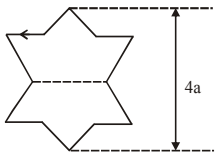
(*d*) current in wire 1 and wire 2 is the direction PQ and RS, resepectively and h ≈ 1.2 a

9. Consider d >> a, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop) **[*JEE Advanced-2014*]**

(*a*) $\frac{\mu_0 I^2 a^2}{d}$ (*b*) $\frac{\mu_0 I^2 a^2}{2d}$

(*c*) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (*d*) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

10. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is: **[*JEE Advanced -2017*]**

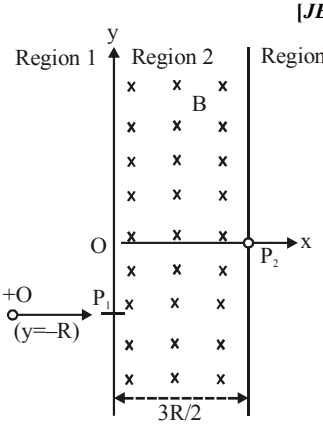


(*a*) $\frac{\mu_o I}{4\pi a} 6\left[\sqrt{3}-1\right]$ (*b*) $\frac{\mu_o I}{4\pi a} 6\left[\sqrt{3}+1\right]$

(*c*) $\frac{\mu_o I}{4\pi a} 3\left[2+\sqrt{3}\right]$ (*d*) $\frac{\mu_o I}{4\pi a} 3\left[\sqrt{3}-1\right]$

11. A uniform magnetic field B exists in the region between x = 0 and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point P_1 (y = – R). Which of the following option(s) is /are correct?

[*JEE Advanced-2017*]



(*a*) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between P_1 and the farthest point from y-axis is p/ $\sqrt{2}$.

(*b*) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

(*c*) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

(*d*) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1.

12. Two infinitely long straight wires lie in the xy-plane along the lines x = ± R. The wire located at x = +R carries a constant current I_1 and the wire located at x = – R carries a constant current I_2. A circular loop of radius R is suspended with its centre (0, 0, $\sqrt{3}R$) and in a plane parallel to the xy-plane. This loop carries a constant I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regardint the magnetic field $\vec{B}$ is (are) true?

[*JEE Advanced - 2018*]

(*a*) If $I_1 = I_2$ then $\vec{B}$ cannot be equal to zero at the origin (0, 0, 0).

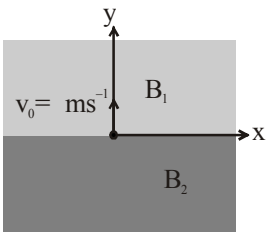
(*b*) If $I_1 > 0$ and $I_2 < 0$. then $\vec{B}$ can be equal to zero at the origin (0, 0, 0).

(*c*) If $I_1 < 0$ and $I_2 > 0$ then $\vec{B}$ can be equal to zero at the origin (0, 0, 0).

(*d*) If $I_1 = I_2$ then the z -component of the magnetic field at the centre of the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$.

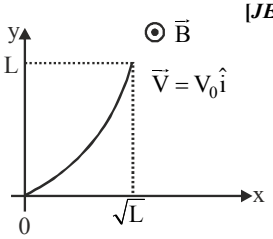
13. In the x-y plane, the region y > 0 has a uniform magnetic field $B_1\hat{k}$ and the region y < 0 has a another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi ms^{-1}$ at t = 0, as shown in the figure. Neglect gravity in this problem. Let t = T be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is_____.

[*JEE Advanced - 2018*]



14. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non -uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and Δϕ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are :

[*JEE Advanced - 2019*]



(*a*) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, y = x initially, of length $\sqrt{2}L$

(*b*) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

(*c*) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for B = 0

(*d*) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for β = 2

ANSWER KEY

Topicwise Questions

1. (*d*) **2.** (*d*) **3.** (*b*) **4.** (*a*) **5.** (*d*) **6.** (*b*) **7.** (*b*) **8.** (*c*) **9.** (*b*) **10.** (*d*)
11. (*b*) **12.** (*b*) **13.** (*d*) **14.** (*a*) **15.** (*c*) **16.** (*d*) **17.** (*d*) **18.** (*b*) **19.** (*c*) **20.** (*d*)
21. (*a*) **22.** (*a*) **23.** (*c*) **24.** (*c*) **25.** (*b*) **26.** (*b*) **27.** (*d*)

Learning Plus

1. (*c*) **2.** (*a*) **3.** (*b*) **4.** (*c*) **5.** (*b*) **6.** (*b*) **7.** (*d*) **8.** (*a*) **9.** (*b*) **10.** (*b*)
11. (*a*) **12.** (*d*) **13.** (*b*) **14.** (*d*) **15.** (*b*) **16.** (*b*) **17.** (*b*) **18.** (*a*) **19.** (*b*) **20.** (*a*)
21. (*b*) **22.** (*c*) **23.** (*b*) **24.** (*a*) **25.** (*b*)

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. (*b*,*d*) **2.** (*a*,*d*) **3.** (*a*,*b*,*c*,*d*) **4.** (*b*,*c*,*d*) **5.** (*a*,*d*) **6.** (*c*,*d*) **7.** (*c*,*d*) **8.** (*b*,*d*) **9.** (*a*,*b*,*d*) **10.** (*a*,*b*,*c*)
11. (*b*) **12.** (*a*) **13.** (*a*) **14.** (*a*) **15.** (*c*) **16.** (*c*) **17.** (*a*) **18.** (*a*)
19. (*a*)-p,q,r; (*b*)-p,q,r,s; (*c*)-r; (*d*)-p,q,r,s **20.** (*a*)-p,q; (*b*)-p,r; (*c*)-p; (*d*) p,q,s **21.** [0.16 N] **22.** [19.3 mm]
23. [19.5 *a*] **24.** [1.18 $\hat{k}$ N-m] **25.** [7.14] **26.** [30°] **27.** [0001] **28.** [0001]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*d*) **2.** (*a*) **3.** (*b*) **4.** (Bonus) **5.** (*d*) **6.** (*d*) **7.** (*d*) **8.** (*d*) **9.** (*a*) **10.** (*c*)
11. (*c*) **12.** (*a*) **13.** (*a*) **14.** (*a*) **15.** (*a*) **16.** (*b*) **17.** (*a*) **18.** (*b*) **19.** (*b*) **20.** (*d*)
21. (*a*) **22.** (*c*) **23.** (*b*) **24.** (*c*) **25.** (*d*) **26.** (*c*) **27.** (*a*) **28.** (*a*) **29.** (*d*) **30.** (*a*)
31. (*d*) **32.** (*b*) **33.** (*d*) **34.** (*d*) **35.** [20] **36.** (*a*) **37.** (*d*) **38.** (Bonus) **39.** (*b*) **40.** (*a*)
41. (*a*) **42.** (*a*) **43.** (*c*) **44.** (*b*) **45.** (*d*) **46.** (*a*) **47.** (*c*) **48.** (*a*) **49.** (*a*) **50.** (*c*)

JEE-ADVANCED
PREVIOUS YEAR'S

1. (*c*,*d*) **2.** [5] **3.** (*b*) **4.** (*d*) **5.** (*a*,*c*) **6.** (*a*,*d*) **7.** (*c*) **8.** (*c*) **9.** (*b*) **10.** (*a*)
11. (*c*,*d*) **12.** (*a*,*b*,*d*) **13.** [2.00] **14.** (*a*,*b*,*d*)

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PREFACE

In this era of cut throat competition among **JEE** aspirants, only few are lucky enough to make it through. During the preparation, a student needs the most relevant study material to cover all the topics with proper planning to crack **JEE**.

We know there is abundance of study materials floating in the market but what We felt missing, is a self-driven and well-planned Study material. We have tried to bring clarity and Quality in the students by introducing this **Arise for JEE**

The book covers all the important concepts and theories with a rich amount of practice questions as well. This book is designed for the perfect guidance you need.

The study material is consistent with the syllabus offered in NCERT Books as well and has been curated by industry experts under the careful observation of **Physics Wallah Team**.

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2. Capacitance **23-37**
3. Current Electricity **38-55**
4. Moving Charge and Magnetism **56-70**

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1 Electrostatics

Topicwise Questions

1. (*a*) $Q = ne = 10^{14} \times 1.6 \times 10^{-19}$
$\Rightarrow Q = 1.6 \times 10^{-5}\,C = 16\,\mu C$
Electrons are removed, so chare will be positive.

2. (*b*) $n = \dfrac{Q}{e} = \dfrac{1}{1.6\times10^{-19}} = 6.25\times10^{18}$

3. (*b*) $F_a = \dfrac{q_1q_2}{4\pi\varepsilon_0 r^2}$, $F_b = \dfrac{q_1q_2}{K4\pi\varepsilon_0 r^2} \Rightarrow F_a : F_b = K : 1$

4. (*c*) We put a unit positive charge at O. Resultant force due to the charge placed at A and C is zero and resultant force due to B and D is towards D along the diagonal BD.

5. (*d*) $Q_1 + Q_2 = Q$ ...(*i*)

and $F = k\dfrac{Q_1Q_2}{r^2}$ ...(*ii*)

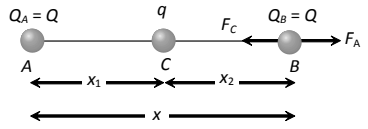
From (i) and (ii)

$$F = \frac{kQ_1(Q-Q_1)}{r^2}$$

For F to be maximum $\dfrac{dF}{dQ_1} = 0 \Rightarrow Q_1 = Q_2 = \dfrac{Q}{2}$

6. (*b*) Suppose in the following figure, equilibrium of charge B is considered. Hence for it's equilibrium $|F_A| = |F_C|$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4x^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2} \Rightarrow q = \frac{-Q}{4}$$



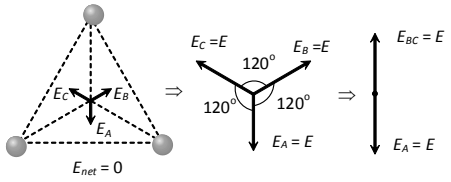
Short Trick: For such type of problem the magnitude of middle charge can be determined if either of the extreme charge is in equilibrium by using the following formula.

If charge A is in equilibrium then $q = -Q_B\left(\dfrac{x_1}{x}\right)^2$

If charge B is in equilibrium then $q = -Q_A\left(\dfrac{x_2}{x}\right)^2$

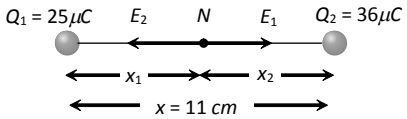
If the whole system is in equilibrium then use either of the above formula.

7. (*c*)



8. (*b*) According to the question, $eE = mg \Rightarrow E = \dfrac{mg}{e}$

9. (*a*) Suppose electric field is zero at point N in the figure then



At N $|E_1| = |E_2|$

which gives $x_1 = \dfrac{x}{\sqrt{\dfrac{Q_2}{Q_1}}+1} = \dfrac{11}{\sqrt{\dfrac{36}{25}}+1} = 5$ cm

10. (*b*) For balance $mg = eE \Rightarrow E = \dfrac{mg}{e}$

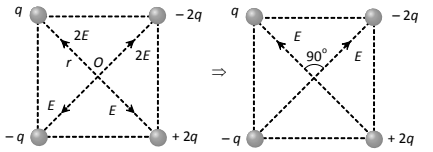
Also $m = \dfrac{4}{3}\pi r^3 d = \dfrac{4}{3}\times\dfrac{22}{7}\times(10^{-7})^3\times1000$ kg

$$\Rightarrow E = \frac{\frac{4}{3}\times\frac{22}{7}\times(10^{-7})^3\times1000\times10}{1.6\times10^{-19}} = 260\ N/C$$

11. (*c*) At A and C, electric lines are equally spaced and dense that's why $E_A = E_C > E_B$

12. (*a*) Side $a = 5 \times 10^{-2}$ m

Half of the diagonal of the square $r = \frac{a}{\sqrt{2}}$

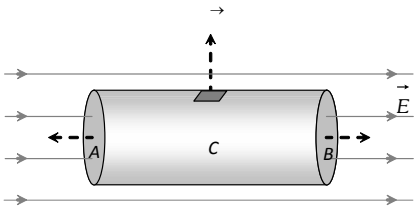


Electric field at centre due to charge q, $E = \frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2}$

Now field at O = $\sqrt{E^2 + E^2} = E\sqrt{2} = \frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2} \cdot \sqrt{2}$

$= \frac{9 \times 10^9 \times 10^{-6} \times \sqrt{2} \times 2}{(5 \times 10^{-2})^2} = 1.02 \times 10^7$ N/C (upward)

13. (*d*) Flux through surface A, $\phi_A = E \times \pi R^2$ and $\phi_B = -E \times \pi R^2$



Flux through curved surface

$C = \int \vec{E}.\vec{ds} = \int Eds\cos 90° = 0$

$\therefore$ Total flux through cylinder = $\phi_A + \phi_B + \phi_C = 0$

14. (*c*) $E = \sigma/(2\varepsilon_0)$

15. (*a*) As there is no charge residing inside the cube, hence net flux is zero.

16. (*b*) Charge enclosed by cylindrical surface (length 100 cm) is $Q_{enc} = 100\,Q$.

By applying Gauss's law $\phi = \frac{1}{\varepsilon_0}(Q_{enc}) = \frac{1}{\varepsilon_0}(100\,Q)$

17. (*b*) By using $\int \vec{E}.\overrightarrow{dA} = \frac{1}{\varepsilon_0}(Q_{enc})$

18. (*b*) $\phi_{net} = \frac{1}{\varepsilon_0} \times Q_{enc} \Rightarrow Q_{enc} = (\phi_2 - \phi_1)\,\varepsilon_0$

19. (*c*) The electric field is due to all charges present whether inside or outside the given surface.

20. (*c*) In electric dipole, the flux coming out from positive charge is equal to the flux coming in at negative charge i.e. total charge on sphere = 0. From Gauss law, total flux passing through the sphere = 0.

21. (*b*) According to Gauss's applications.

22. (*a*) Electric field due to a hollow spherical conductor is governed by following equation E = 0,
for $r < R$ ...(i)

and $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ for $r \geq R$ ...(ii)

i.e. inside the conductor field will be zero and outside the conductor will vary according to $E \propto \frac{1}{r^2}$

23. (*c*) Electric field inside a conductor is always zero.

24. (*c*) Electric field outside of the sphere $E_{out} = \frac{kQ}{r^2}$ ...(i)

Electric field inside the dielectric sphere $E_{in} = \frac{kQx}{R^3}$ ...(ii)

From (i) and (ii),

$E_{in} = E_{out} \times \frac{r^2 x}{R}$

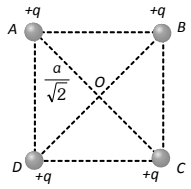
At 3 cm,

$E = 100 \times \frac{3(20)^2}{10^3} = 120$ V/m

25. (*b*) Since potential inside the hollow sphere is same as that on the surface.

26. (*d*) On the equipotential surface, electric field is normal to the charged surface (where potential exists) so that no work will be done.

27. (*b*)



Potential at the centre O, $V = 4 \times \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{a/\sqrt{2}}$

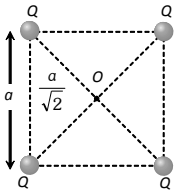
where $Q = \frac{10}{3} \times 10^{-9}$ C and $a = 8$ cm $= 8 \times 10^{-2}$ m

So $V = 5 \times 9 \times 10^9 \times \frac{\frac{10}{3} \times 10^{-9}}{\frac{8 \times 10^{-2}}{\sqrt{2}}} = 1500\sqrt{2}$ volt

28. (*b*) Electrostatic energy density $\frac{dU}{dV} = \frac{1}{2} K\varepsilon_0 E^2$

$\therefore \quad \frac{dU}{dV} \propto E^2$

29. (*c*) Potential at centre O of the square

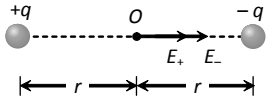


$$V_0 = 4\left(\frac{Q}{4\pi\varepsilon_0\left(a/\sqrt{2}\right)}\right)$$

Work done in shifting (– Q) charge from centre to infinity

$$W = -Q(V_\infty - V_O) = QV_0 = \frac{4\sqrt{2}Q^2}{4\pi\varepsilon_0 a} = \frac{\sqrt{2}Q^2}{\pi\varepsilon_0 a}$$

30. (*c*) At O, $E \neq 0$, $V = 0$



31. (*b*) In balance condition

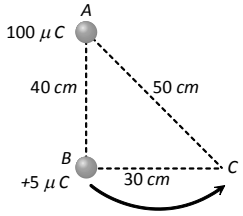
$$\Rightarrow QE = mg \Rightarrow Q\frac{V}{d} = \left(\frac{4}{3}\pi r^3\rho\right)g$$

$$\Rightarrow Q \propto \frac{r^3}{V} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^3 \times \frac{V_2}{V_1}$$

$$\Rightarrow \frac{Q}{Q_2} = \left(\frac{r}{r/2}\right)^3 \times \frac{600}{2400} = 2$$

$$\Rightarrow Q_2 = Q/2$$

32. (*d*) Work done in displacing charge of 5 μC from B to C is $W = 5 \times 10^{-6}(V_C - V_B)$ where



$$V_B = 9\times10^9 \times \frac{100\times10^{-6}}{0.4} = \frac{9}{4}\times10^6\,V$$

and $V_C = 9\times10^9 \times \frac{100\times10^{-6}}{0.5} = \frac{9}{5}\times10^6\,V$

So $W = 5\times10^{-6}\times\left(\frac{9}{5}\times10^6 - \frac{9}{4}\times10^6\right) = -\frac{9}{4}J$

33. (*b*) Net electrostatic energy $U = \frac{kQq}{a} + \frac{kq^2}{a} + \frac{kQq}{a\sqrt{2}} = 0$

$$\Rightarrow \frac{kq}{a}\left(Q + q + \frac{Q}{\sqrt{2}}\right) = 0 \Rightarrow Q = -\frac{2q}{2+\sqrt{2}}$$

34. (*a*) Potential energy of the system

$$U = k\frac{Qq}{l} + \frac{kq^2}{l} + \frac{kqQ}{l} = 0 \Rightarrow \frac{kq}{l}(Q + q + Q) = 0$$

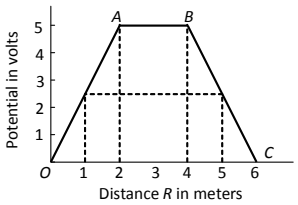
$$\Rightarrow Q = -\frac{q}{2}$$

35. (*d*) Conducting surface behaves as equipotential surface.

36. (*c*) For non-conducting sphere $E_{in} = \frac{k.Qr}{R^3} = \frac{\rho r}{3\varepsilon_0}$

37. (*b*) Potential V any where inside the hollow sphere, including the centre is $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$

38. (*a*) Intensity at 5m is same as at any point between B and C because the slope of BC is same throughout (i.e., electric field between B and C is uniform). Therefore electric field at R = 5m is equal to the slope of line BC hence by $E = \frac{-dV}{dr}$;



$$E = -\frac{(0-5)}{6-4} = 2.5\frac{V}{m}$$

39. (*b*) $V_{inside} = \frac{Q}{4\pi\varepsilon_0 R}$ for $r \le R$ ...(i)

and $V_{out} = \frac{Q}{4\pi\varepsilon_0 r}$ for $r \ge R$ ...(ii)

i.e. potential inside the hollow spherical shell is constant and outside varies according to $V \propto \frac{1}{r}$.

40. (*a*) The electric potential $V(x, y, z) = 4x^2$ volt

Now $\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$

Now $\frac{\partial V}{\partial x} = 8x$, $\frac{\partial V}{\partial y} = 0$ and $\frac{\partial V}{\partial z} = 0$

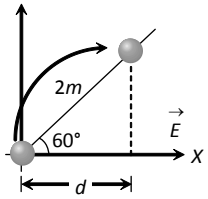
Hence $\vec{E} = -8x\hat{i}$, so at point (1m, 0, 2m)

$\vec{E} = -8\hat{i}$ volt / metre or 8 along negative X-axis.

41. (*b*) $\because\ E = -\frac{dV}{dX} \Rightarrow V_x = -xE_0$

42. (*b*) $E = \frac{V}{d} = \frac{10}{2\times10^{-2}} = 500\ N/C$

43. (*d*)



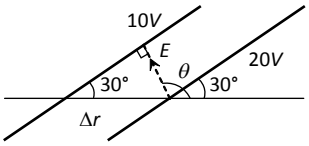
$W = qV = qE.d$

$\Rightarrow 4 = 0.2 \times E \times (2\cos 60^\circ) = 0.2\,E \times (2 \times 0.5)$

$\therefore\ E = \frac{4}{0.2} = 20\ NC^{-1}$

44. (*a*) $F = QE = \frac{QV}{d} \Rightarrow 5000 = \frac{5 \times V}{10^{-2}} \Rightarrow V = 10$ volt

45. (*c*)



Using $dV = -\vec{E}.\vec{dr}$

$\Rightarrow \Delta V = -E.\Delta r \cos\theta$

$\Rightarrow E = \frac{-\Delta V}{\Delta r \cos\theta}$

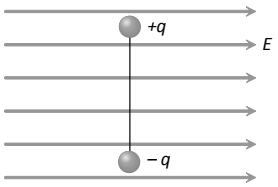
$\Rightarrow E = \frac{-(20-10)}{10 \times 10^{-2} \cos 120°}$

$= \frac{-10}{10 \times 10^{-2}(-\sin 30°)} = \frac{-10^2}{-1/2} = 200$V/m

Direction of E be perpendicular to the equipotential surface i.e. at 120° with x-axis.

46. (*c*) Potential energy = – pE cosθ
When θ = 0. Potential energy = – pE (minimum)

47. (*d*) Work done = $\int_{90}^{270} pE \sin\theta \, d\theta = \left[-pE\cos\theta\right]_{90}^{270} = 0$



48. (*b*) $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3}$

49. (*b*)

50. (*a*)

51. (*d*) Potential due to dipole in general position is given by

$V = \frac{k.p\cos\theta}{r^2} \Rightarrow V = \frac{k.p\cos\theta r}{r^3} = \frac{k\cdot(\vec{p}\,\vec{r})}{r^3}$

52. (*b*) Potential energy of electric dipole
$U = -pE\cos\theta = -(q \times 2l) E\cos\theta$
$U = -(3.2 \times 10^{-19} \times 2.4 \times 10^{-10}) 4 \times 10^5 \cos\theta$
$U = -3 \times 10^{-23}$ (approx.)

53. (*c*) Electric lines of force never intersect the conductor. They are perpendicular and slightly curved near the surface of conductor.

54. (*a*) Electric field inside a conductor is zero in electrostatic equilibrium.

Learning Plus

1. (*b*)

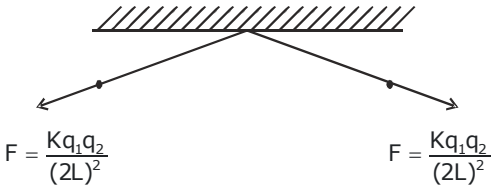
2. (*a*)

3. (*d*)

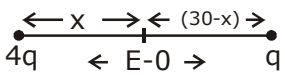
4. (*d*) $F = \frac{Kq_1q_2}{r^2} \Rightarrow F_1 = \frac{Kq_1q_2}{(r/2)^2} = \frac{4.Kq_1q_2}{r^2} = 4F$

5. (*c*) Attraction is possible between a charged and a neutral object.

6. (*c*)



7. (*a*)



$\frac{K(4q)}{x^2} = \frac{Kq}{(30-x)^2}$

x = 20 cm from 4q
10 cm away from q

8. (*b*)



Let the two charges are q & (20 – q) μC

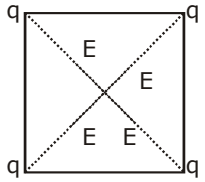
$\therefore F_e = \frac{K(q)(20-q)}{r^2}$

F_e will be max, when $\frac{dF_e}{dq} = 0$

or $\frac{dFe}{dq} = \frac{K}{r^2}(20-2q) = 0$

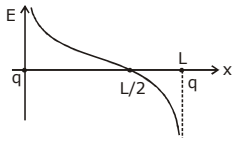
$\Rightarrow \therefore q = 10\,\mu C.$

9. (*a*)



$E_{Net} = 0$

10. (*d*)



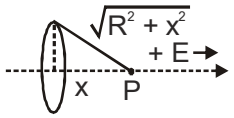
11. (*b*) $a = \frac{qE}{m}$

After time t

$v = \frac{qE}{m}t$

$KE = \frac{1}{2}mv^2 = \frac{E^2q^2t^2}{2m}$

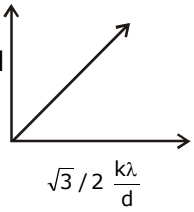
12. (*c*) At point P on axis, $E = \dfrac{kqx}{(R^2 + x^2)^{3/2}}$



For max E, $\dfrac{dE}{dx} = 0 \Rightarrow$ or $x = \dfrac{R}{\sqrt{2}}$

$\therefore$ Putting x in (i) $E_{max} = \dfrac{2kq}{3\sqrt{3}R^2}$

13. (*a*)



$\theta_1 = 0,\ \theta_2 = 60°$

$$E_{\perp} = \frac{k\lambda}{d}[\sin 60° + \sin 0°] = \frac{\sqrt{3}}{2}\frac{k\lambda}{d}$$

$$E_{||} = \frac{k\lambda}{d}[\cos 60° - \cos 0°] = \frac{-k\lambda}{2d}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$$

14. (*a*) a & b can't be both +ve or both – ve otherwise field would have been zero at their mid point.
b can't be positive even, otherwise the field would have been in –ve direction to the right of mid point

15. (*a*) By definition

16. (*d*)

17. (*c*) $\vec{A} = 100\hat{k}$, $\vec{E} = \hat{i} + \sqrt{2}\,\hat{j} + \sqrt{3}\,\hat{k}$

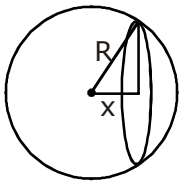
$\phi = \vec{E} \bullet \vec{A}$

$\phi = 100\sqrt{3}$

18. (*d*) Incoming flux = Outgoing flux

19. (*d*) Radius of the cutting

disc = $\sqrt{R^2 - x^2}$



charge on disc

$q = \sigma A$

$q = \sigma\pi(R^2 - x^2)$

Now $\phi = \dfrac{q}{\varepsilon_0} = \dfrac{\sigma\pi(R^2 - x^2)}{\varepsilon_0}$

20. (*a*) If charge is at A or D, its all field lines cut the given surface twice which means that net flux due to this charge remains zero and flux through given surface remains unchanged.

21. (*d*) By M.E. conservation between initial & final point:
$U_i + K_i = U_f + K_f$
$\therefore$ Answer is (4)

22. (*c*)

23. (*b*) $\therefore\ \dfrac{1}{2}mV_A^2 = qV,\ \dfrac{1}{2}mV_B^2 = 4\,qV \Rightarrow \therefore\ \dfrac{V_A^2}{V_B^2} = \dfrac{1}{4}$

$\Rightarrow \dfrac{V_A}{V_B} = \dfrac{1}{2}$

24. (*d*) Comparison can be shown as :

$V \to 2V \Rightarrow k \to 4k \Rightarrow PE_{max} \to 4PE_{max} \Rightarrow r \to \dfrac{r}{4}$

25. (*b*) Apply the formula $V = \dfrac{kQ}{r}$

26. (*b*)

27. (*c*) K.E. = VQ and momentum = $\sqrt{2m(KE)} = \sqrt{2mVQ}$

28. (*c*) $PE = q(V_{final} - V_{initial})$
$PE = q\Delta V$ PE decreases if q is +ve increases if q is –ve.

29. (*b*) By conservation of machenical energy

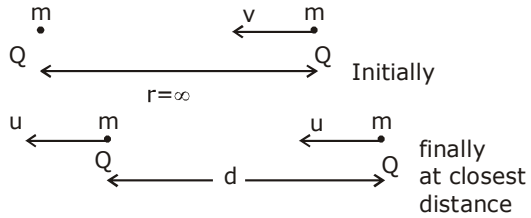
$$\frac{1}{2}mv^2 = \frac{k\,q_1q_2}{r_1} - \frac{k\,q_1q_2}{r_2}\quad \frac{1}{2}(2\times10^{-3})\ v^2$$

$$= 9\times10^9\times10^{-6}\times10^{-3}\left(\frac{1}{1} - \frac{1}{10}\right)$$

or $v^2 = 9\times10^3\times\dfrac{9}{10}$ or $v = 90$ m/sec

30. (*c*) PE may increase may decrease depending on sign of charges.

31. (*b*)



from E.C.

$$\frac{1}{2}mv^2 = 2\left(1/2\,mv^2\right) + \frac{kq^2}{d} \quad ...(1)$$

from M.C. $mv = 2mu \Rightarrow u = v/2$ ...(2)
from (1) and (2)

$$\frac{1}{2}mv^2 = \frac{mv^2}{4} + \frac{kq^2}{d}$$

$$d = \frac{4kq^2}{mv^2}$$

32. (*b*) $U = -QV$

33. (*b*) $E = \dfrac{Kq}{r^2}$; $V = \dfrac{Kq(n-1)}{r}$

$\dfrac{V}{E} = r(n-1)$

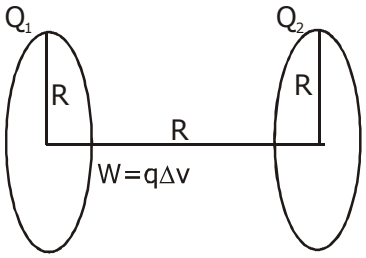
34. (*d*)

35. (*c*) Higher potential (v_1) ⟶ Lower potential (v_2)

$U_1 = -qV_1$ ⟵ qE ⟵ −q ⟶ $U_2 = -qV_2$

$U_1 < U_2$

36. (*b*)



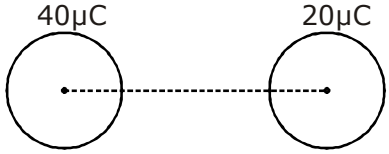
$v_A = \frac{KQ_1}{R} + \frac{KQ_2}{\sqrt{2}R}$ $\qquad v_B = \frac{KQ_2}{R} + \frac{KQ_1}{\sqrt{2}R}$

$$W = q\left[\frac{kQ_2}{R} + \frac{KQ_1}{\sqrt{2}R} - \frac{kQ_1}{R} - \frac{kQ_2}{\sqrt{2}R}\right]$$

$$W = \frac{q}{R4\pi\varepsilon_0}\left[\left(Q_2 + \frac{Q_1}{\sqrt{2}}\right) - \left(Q_1 + \frac{Q_2}{\sqrt{2}}\right)\right]$$

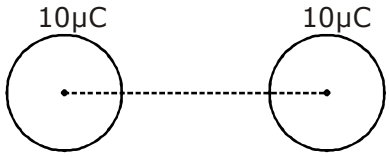
$$W = q(Q_1 - Q_2)(\sqrt{2} - 1)|(\sqrt{2}\,4\pi\varepsilon_0 R)$$

37. (*a*)



$$F_1 = \frac{k(40)(20)}{d^2}$$

After touching the charge on sphere = 10μC



$$F_2 = \frac{k(10)(10)}{d^2}$$

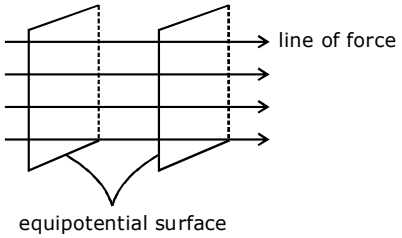
$F_1 : F_2 = 8 : 1$

38. (*a*)

39. (*a*) $\Delta V = E.R$

$\Delta V = 1000 \times 1 \times 10^{-2} = 10$ Volt

40. (*d*)



Angle between both = 90°

41. (*c*) Since B and C are at same potential (lying on a line ⊥ to electric field i.e. equipotential surface)

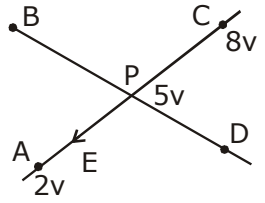
$\therefore \Delta V_{AB} = \Delta V_{AC} = Eb.$

42. (*a*)

43. (*d*) e.f is perpendicular to equipotential surface

m for e.f $= -\frac{1}{2}$

44. (*b*)



$\Delta V = v_A - v_P = 3V$

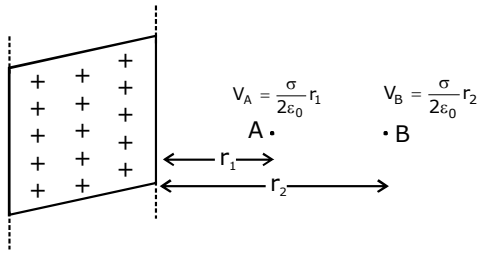
$$\Rightarrow E = \frac{V}{d} = \frac{3}{\sqrt{(0.1)^2 + (0.1)^2}} = 15\sqrt{2}$$

45. (*a*) $E = -\frac{dV}{dx} = -10x - 10$

$\therefore E_{(x=1m)} - 10(1) - 10 = -20$ V/m

46. (*b*) $E \alpha r \Rightarrow \int_0^v dV \alpha \int_0^r r\,dr \Rightarrow V \alpha (-\frac{r^2}{2}) \Rightarrow V \alpha r^2$

47. (*b*)



Given $V_B - V_A = 5$ V

$$\frac{\sigma}{2\varepsilon_0}(r_2 - r_1) = 5\,V$$

$r_2 - r_1 = 0.88$ mm

48. (*c*)

49. (*c*) $P = qd$

$1 \times 10^{-6} \times 2 \times 10^{-2} = 2 \times 10^{-8}$

Maximum torque

$\tau = PE = 2 \times 10^{-2}$ Nm

50. (*a*)

51. (*c*) Since P & Q are axial & equatorial points, so electric fields are parallel to axis at both points.

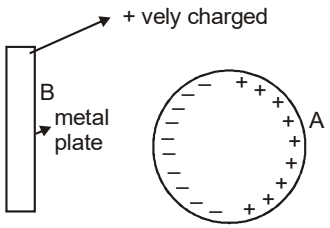
52. (*d*) $\tau_{max} = PE = 4 \times 10^{-8} \times 2 \times 10^{-4} \times 4 \times 10^{8}$

$= 32 \times 10^{-4}$ N-m.

Work done $W = (P.E.)_f - (P.E.)_i$

$= PE - (-PE) = 2PE = 64 \times 10^{-4}$ N-m

53. (*b*)

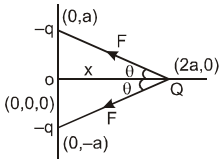


The given diagram shows induction on sphere (metallic) due to metal plate.

Since distance between plate and –ve charge is less than that between plate and +ve charge. electric force acts on object towards plate.

Advanced Level Multiconcept Questions

1. (*b,d*)



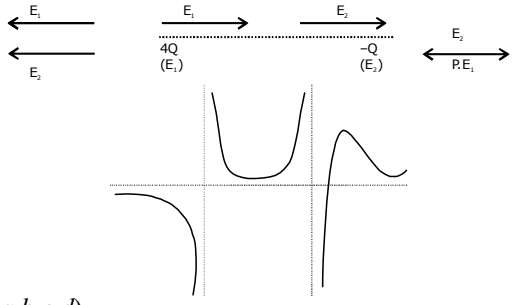
(*i*) From diagram, force on Q at general position x, is given by

$$F_{net} = -2F\cos\theta = -\frac{kQqx}{(a^2+x^2)^{3/2}} \text{ (Towards origin)}$$

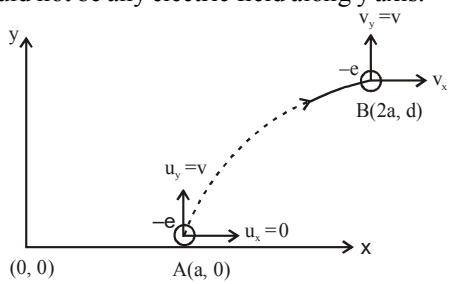
(*ii*) When charge moves from (2a, 0) to origin O, force keeps on acting on Q and becomes zero at O.
$\therefore$ Velocity of Q is max. at O.

(*iii*) $\because$ Motion is SHM for very small displacements. & 2a is not very small os motion is periodic but not SHM.

2. (*a,d*)



3. (*a,b,c,d*)
As velocity along y-axis remains unchanged, so there should not be any electric field along y axis.



As velocity along x axis is increasing, so force on the electron must be along +x direction, so electric field must be towards –x direction.
So force on the electron is :
$F = qE = eE$

acceleration, $a = \frac{eE}{m}$ towards +x direction

From $A \to B$

$S_y = u_y t$

or $d = vt \Rightarrow \therefore t_{A\to B} = \frac{d}{V}$

From : $A \to B$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

or $$a = 0 + \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{d}{V}\right)^2$$

$$\Rightarrow E = \frac{2amV^2}{ed^2} \text{ toward-x direction} \quad ...(1)$$

(*a*) Velocity along x axis at B :
From $A \to B$

$V_x = u_x + a_x t$

or $$V_x = 0 + \left(\frac{eE}{m}\right)\left(\frac{d}{V}\right) \Rightarrow V_x = \frac{eEd}{mV}$$

where, $E = \frac{2amv^2}{ed^2} \Rightarrow \therefore V_x = \frac{2aV}{d}$

(*d*) Net velocity vector at B

$$\vec{V}_B = V_x\hat{i} + V_y\hat{j}$$

$$\vec{V}_B = \frac{2aV}{d}\hat{i} + V\hat{j}$$

(*b*) Rate of work done at B = Power = $\vec{F}\cdot\vec{V}_B$

$$= \left(eE\,\hat{i}\right).\left(\frac{2aV}{d}\hat{i} + V\hat{j}\right)$$

$$= eE\left(\frac{2aV}{d}\right); \text{ where, } E = \frac{2amV^2}{ed^2}$$

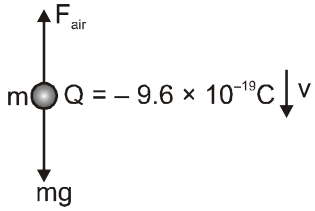
$$\Rightarrow \therefore P = \frac{4ma^2V^3}{d^3}$$

(*c*) Rate of work done at A :

$$P_A = \vec{F}\cdot\vec{V}_A$$

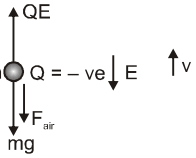
$$= \left(e\vec{E}\,\hat{i}\right)\cdot\left(V\,\hat{j}\right) = 0$$

4. (*b,c*)



$\therefore$ $mg = f_{air}$

(*ii*)



$\therefore$ $QE = mg + f_{air} = 2mg$

$\therefore$ charge is –ve, so electric field 'E' is directed downwards.

& $QE = 2mg$

$\therefore$ $E = \dfrac{2mg}{Q} = \dfrac{2\times1.6\times10^{-18}\times10}{9.6\times10^{-19}} = \dfrac{1}{3}\times10^{2}\ NC^{-1}$

5. (*a,c*)

(*i*) At any point P inside the sphere, electric field

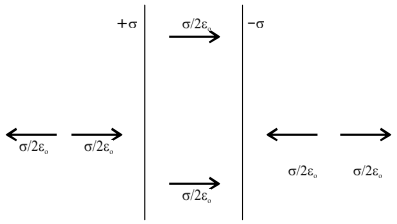
$\Rightarrow E_P = \dfrac{kQr}{R^3}$.

$\therefore$ E_P increases as r increases .

(*ii*) At any point M outside the sphere, $E_M = \dfrac{kQ}{r^2}$

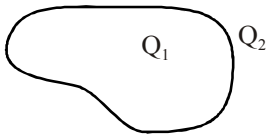
$\therefore E_M$ decreases as r increases.

6. (*a,c*)



7. (*a,b,c*)

$\oint \vec{\varepsilon} \cdot d\vec{A} = \dfrac{q_{in}}{\varepsilon_0}$

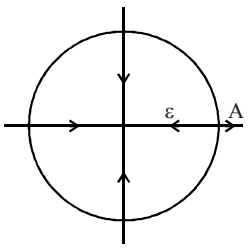


Flux electric field due to charge lie inside or out side the surface. But ϕ is only due to charge lie inside the surface.

8. (*a,b,c*)

$E = 100\,r$

$\oint \vec{\varepsilon} \cdot d\vec{A} = \dfrac{q_{in}}{\varepsilon_0}$



$\varepsilon dA\cos 180° = \dfrac{q_{in}}{\varepsilon_0}$

$\Rightarrow q_{in} = -ve$

$|q_{in}| = EdA\,\varepsilon_0 = 3\times10^{-13}\ C$

9. (*a,c*)

10. (*a,b,c,d*)

$\dfrac{kQ}{(r+5cm)} = 100V$ & $\dfrac{kQ}{(r+10cm)} = 75\ V$

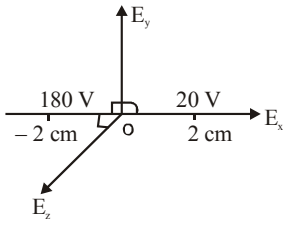
$\therefore$ $Q = \dfrac{5}{3}\times10^{-9}\ C$, $r = 10\ cm$

$\therefore$ $V_{surface} = \dfrac{kQ}{2} = 150V$ $E_{surface} = \dfrac{kQ}{r^2} = 1500\ V/m$

$V_{centre} = \dfrac{3}{2}\,V_{surface} = \dfrac{3}{2}\times150 = 225\ V$

11. (*b,c*)

from given data $E_x = \dfrac{160}{4}\ V/cm = 40\ V/cm$



but $E = \sqrt{E_x^2 + E_y^2 + E_z^2}$ $\Rightarrow$ E may be equal or greater than 40 V/cm ie.

As shown, there can be electric fields $\perp$ to x axis, which will not affect the electric potential difference but can increase net field.

12. (*b,c,d*)

(*a*) $V = \dfrac{KQ}{r} = 0$

(*b*) Depends on distribution of charge .

(*c*) Depends on distribution of charge .

(*d*) F_{net} is zero but τ_{net} may be non zero

13. (*a,d*)

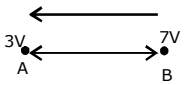
higher density $\Rightarrow$ Higher E

$E_A > E_B$

Electric field lines from higher potential to lower potential.

$V_B > V_A$

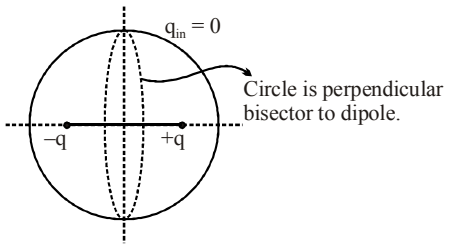
14. (*a,c*)



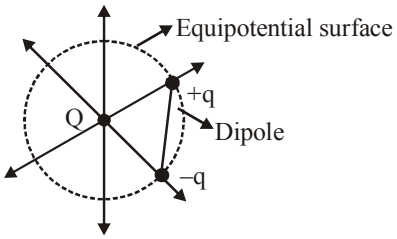
$F = eE \rightarrow$

k.E. = e (7–3) = 4ev

15. (*a,d*)



16. (*a,c*)



In all orientations, dipole experiences force, but does not experience torque if dipole has its dipole moment along or opposite to ELOF.

Dipole can never be in stable equilibrium & work done in moving dipole along an EPS of point charge Q will be zero.

17. (*a,b,d*)

$\vec{\tau} = \vec{P} \times \vec{E}$

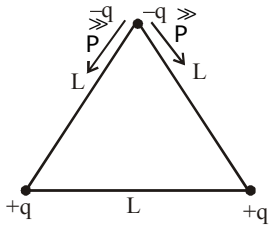
$= \left(2\hat{i} + 3\hat{j}\right) \times \left(3\hat{i} + 2\hat{k}\right) \times 10^{-6} \times 10^{5}$

$= (0.6\,\hat{i} - 0.4\,\hat{j} - 0.9\,\hat{k})$

$\text{P.E.} = -\vec{P}.\vec{E}$

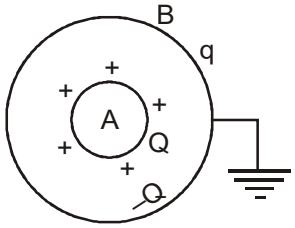
$\text{Max P.E.} = |\vec{P}||\vec{E}|$

18. (*a,d*)



$P_{net}^2 = P^2 + P^2 + 2P^2 \cos 60 = \sqrt{3}\,qL$

19. (*a,c,d*)



(*i*) Due to earthing

Let total charge on B is q.

$V_B = 0 \;\therefore\; \frac{kq}{b} + \frac{kQ}{b} = 0$ or $q = -Q$.

(*ii*) $\therefore$ All charge $q = -Q$

appears on inner surface of B due to induction

$\Rightarrow$ Charge on outer surface of B = 0

$\Rightarrow$ Field between A and B due to B = 0

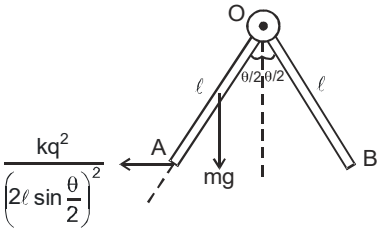
Field between A and B due to A $\neq$ 0

Net field between A and B $\neq$ 0.

20. (*c*) For 30 C charge, angle $\in (5°, 9°) \Rightarrow 7°$

21. (*c*) In (iii) most of the positive charge with run away to the metal knob. So due to less charge on the leaves, the leaves will come closer than before.

22. (*a*)



Applying torque balance about hinge point O.

$$\frac{kq^2}{\left(2\ell \sin\frac{\theta}{2}\right)^2}\left(\ell \cos\frac{\theta}{2}\right) = mg\left(\frac{\ell}{2}\right)\sin\frac{\theta}{2}$$

for small θ, $\sin\frac{\theta}{2} \to \frac{\theta}{2}$, $\cos\frac{\theta}{2} \to 1$

$$\therefore\; \theta = \sqrt{\frac{4kq^2}{mg\ell^2}}$$

23. (*c*) $\because \phi = \frac{Q}{\epsilon_0}$

$\therefore = 2 \times 10^5 \times 8.85 \times 10^{-12}\,C$

$= 1.77\,\mu C$

24. (*b*) $\frac{(1.77 \times 10^{-6} + Q_A)}{\epsilon_0} = -4 \times 10^5$

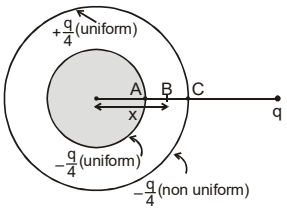
$\Rightarrow Q_A = -5.31 \times 10^{-6}\,C$

25. (*d*) For all values of r, flux ϕ is non-zero i.e. no Gaussian sphere of radius r is possible in which net enclosed charge is zero.

26. (*b*) The inner sphere is grounded, hence its potential is zero. The net charge on isolated outer sphere is zero. Let the charge on inner sphere be q'.

$\therefore$ Potential at centre of inner sphere is zero.

27. (*c*) The region in between conducting sphere and shell is shielded from charges on and outside the outer surface of shell. Hence, charge distribution on surface of sphere and inner surface of shell is uniform.



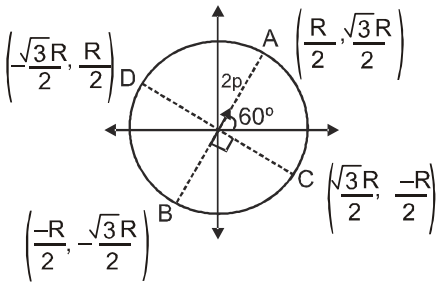
The distribution of induced charge on outer surface of shell depends only on point charge q, hence is nonuniform. The charge distribution on all surfaces, is as shown.

28. (*a*) The electric field at B is $= \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{4x^2}$ towards left.

$$\therefore\; V_C = V_C - V_A = -\int_{2a}^{a} \frac{1}{4\pi\varepsilon_o}\frac{q}{4x^2}dx = \frac{1}{32\pi\varepsilon_o}\cdot\frac{q}{a}$$

29. (*a*) p (*b*) r,s (*c*) p,q (*d*) r,s

The resultant dipole moment has magnitude $\sqrt{(\sqrt{3}\,P)^2 + P^2} = 2P$ at an angle $\theta = \tan^{-1}\frac{\sqrt{3}\,P}{P} = 60°$ with positive x direction.



Diameter AB is along net dipole moment and diameter CD is normal to net dipole moment.

$\therefore$ Potential at A $\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$ is maximum

Potential is zero at C $\left(\frac{\sqrt{3}R}{2}, -\frac{R}{2}\right)$ and D $\left(-\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$

Magnitude of electric field is $\frac{1}{4\pi\varepsilon_0}\frac{4p}{R^3}$ at A $\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$

and B $\left(-\frac{R}{2}, -\frac{\sqrt{3}R}{2}\right)$

Magnitude of electric field is $\frac{1}{4\pi\varepsilon_0}\frac{2p}{R^3}$ at C $\left(\frac{\sqrt{3}R}{2}, -\frac{R}{2}\right)$ and D $\left(-\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$

30. [0191]
$F = F_1 - F_2 = 225 - 34 = 191\ N$

31. [0208]
Use = $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

32. [2.5]

33. [0.5]

$$\tan\theta = \frac{E_y}{E_x} = \frac{\frac{2k\lambda}{y}}{\frac{2k\lambda}{x}} = \frac{x}{y} = \frac{1}{2} = 0.5$$

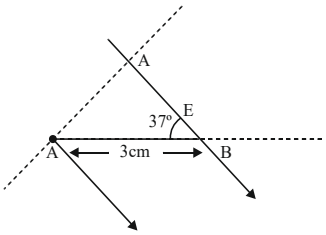
34. [0002]
$W = q\Delta V = 2 \times 10 \times 10^{-6} = 2 \times 10^{-5}$

35. [0024]
$\Delta V = E(AB)$

36. [2.55]
$W = qV_C$

37. [0006] $\Delta V = -\int_{r_A}^{r_B} \vec{E}.\,\vec{dr}$

38. [−9.6]



$\Delta V = Er_{\perp}$
$V_A - V_B = Er_{\perp}$
$= E(AB)$
$= 400 \times 3\cos 37^\circ \times 10^{-2}$
$= 400 \times 3 \times \frac{4}{5} \times 10^{-2}$
$V_A - V_B = 9.60$
$V_B - V_A = -9.6V$

JEE Mains & Advanced Past Years Questions

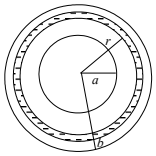
JEE-MAIN PREVIOUS YEAR'S

1. (*a*) From Gauss's law

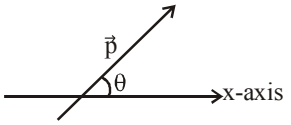
$$E \times 4\pi r^2 = \frac{Q + \int_a^r \left(4\pi x^2 dx\right)\frac{A}{x}}{\in_0}$$

$$\Rightarrow E = \frac{Q}{4\pi r^2} + \frac{A}{2} - \frac{Aa^2}{2r^2}$$

For E to be const., $\frac{Q}{4\pi r^2} = \frac{Aa^2}{2r^2} \Rightarrow A = \frac{Q}{2\pi a^2}$



2. (*a*) So from $\vec{\tau} = \vec{p} \times \vec{E}$



$\tau\hat{k} - \tau\hat{k} = \left(p_x\hat{i} + p_y\hat{j}\right) \times \left(E\hat{i} + \sqrt{3}E\hat{j}\right)$

$= p_x \times \sqrt{3}E\hat{k} + p_y E\left(-\hat{k}\right)$

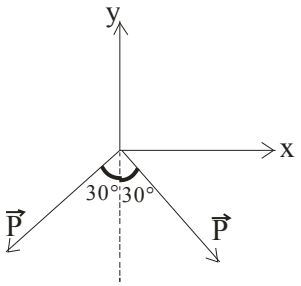
$0 = E\hat{k}\left(\sqrt{3}p_x - p_y\right)$

$\frac{p_y}{p_x} = \sqrt{3}$

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

3. (*d*)



$|\vec{P}| = qL$

$\vec{P}_{net} = 2P\cos 30°\left(-\hat{j}\right) = P\sqrt{3}\left(-\hat{j}\right)$

$= \sqrt{3}\ qL\left(-\hat{j}\right)$

4. (*b*) Electric field

$$E = \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

For maxima $\frac{dE}{dx} = 0$

After solving we get, $\left(x = \pm\frac{R}{\sqrt{2}}\right)$

5. (*a*) $\frac{4kQq}{d^2} + \frac{kQ^2}{d^2} = 0$

$\Rightarrow$ $q = -Q/4$

6. (*b*) $Q = \int \rho 4\pi r^2 dr = \int_0^R \left(\frac{A}{r^2}e^{-\frac{2r}{a}}\right)\left(4\pi r^2\right)dr$

$= 4\pi A\frac{a}{2}\left(1 - e^{\frac{-2R}{a}}\right)$

$\Rightarrow$ $R = \frac{-a}{2}\log\left(1 - \frac{Q}{2\pi Aa}\right)$

7. (*a*) $\vec{E} = \frac{kq_2}{r_2^3}\vec{r}_2 = k\times 10^{-6}$

$\left[\frac{\sqrt{10}}{10\sqrt{10}}(-\hat{i}+3\hat{j}) + \frac{(-25)}{125})(-4\hat{i}+3\hat{j})\right]$

$= (9\times 10^3)\left[\frac{1}{10}(-\hat{i}+3\hat{j}) - \frac{1}{5}(-4\hat{i}+3\hat{j})\right]$

$= (9\times 10^3)\left[\left(-\frac{1}{10}+\frac{4}{5}\right)\hat{i} + \left(\frac{3}{10}-\frac{3}{5}\right)\hat{i}\right]$

$= 9000\left(\frac{7}{10}\hat{i} - \frac{3}{10}\hat{j}\right)$

$= (63\,\hat{i} - 27\,\hat{j})(100)$

8. (*d*) $\because$ $F \propto \frac{1}{r^3}$

Required force = 27 C

9. (*d*) Electric field at p = $2E_1\cos\theta_1 - 2E_2\cos\theta_2$

$= \frac{2K_q}{\left(d^2+D^2\right)}\times\frac{D}{\left(d^2+D^2\right)^{1/2}} - \frac{2K_q}{\left[(2d)^2+D^2\right]}\times\frac{D}{\left[(2d)^2+D^2\right]^{1/2}}$

$= 2KqD\left[\left(d^2+D^2\right)^{-3/2} - \left(4d^2+D^2\right)^{-3/2}\right]$

$= \frac{2KqD}{D^3}\left[\left(1+\frac{d^2}{D^2}\right)^{-3/2} - \left(1+\frac{4d^2}{D^2}\right)^{-3/2}\right]$

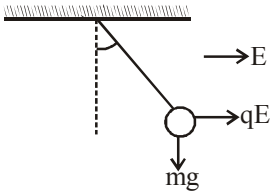
Applying binomial approximation $\because d << D$

$= \frac{2KqD}{D^3}\left[1 - \frac{3}{2}\frac{d^2}{D^2} - \left(1 - \frac{3\times 4d^2}{2D^2}\right)\right]$

$= \frac{2KqD}{D^3}\left[\frac{12}{2}\frac{d^2}{D^2} - \frac{3}{2}\frac{d^2}{D^2}\right]$

$= \frac{9kqd^2}{D^4}$

10. (*c*)



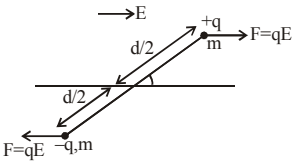
$\tan\theta = \frac{qE}{mg} = \frac{5\times 10^{-6}\times 2000}{2\times 10^{-3}\times 10}$

$\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(0.5)$

11. (*a*) $g_{ff} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$

$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = \sqrt{\frac{\ell}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$

12. (*c*)



moment of inertia $(I) = m\left(\frac{d}{2}\right)^2 \times 2 = \frac{md^2}{2}$

Now by $\tau = I\alpha$

$(qE)(d\sin\theta) = \frac{md^2}{2}.\alpha$

$\alpha = \left(\frac{2qE}{md}\right)\sin\theta$

for small θ

$\Rightarrow$ $\alpha = \left(\frac{2qE}{md}\right)\theta \Rightarrow$ Angular frequency $\omega = \sqrt{\frac{2qE}{md}}$

13. (*c*) $E\,4\pi a^2 = \dfrac{\int_0^a kr\,4\pi^2 dr}{\varepsilon_0}$

$$E = \frac{k\,4\pi a^4}{4\times 4\pi\varepsilon_0}$$

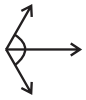
$$2Q = \int_0^R kr\,4\pi r^2 dr$$

$$k = \frac{2Q}{\pi R^4}$$

$$QE = \frac{1}{4\pi\varepsilon_0}\frac{QQ}{(2a)^2}$$

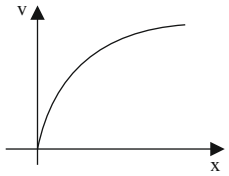
$$R = a8^{1/4}$$

14. (*a*)



$$E_{net} = \frac{4kq}{d^2}\times 2\cos 30^o = \frac{q\sqrt{3}}{\pi\varepsilon_0 d^2}$$

15. (*b*) $V^2 = \dfrac{2qE}{m}x$



16. (*a*) Since is best $\vec{p}.\vec{r} = 0$

$\vec{E}$ must be antiparallel to $\vec{p}$

So, $\vec{E} = -\lambda(\vec{p})$

where λ is a arbitary positive constant

Now $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{A} \parallel \vec{E}$

$$\frac{a}{\lambda} = \frac{b}{3\lambda} = \frac{c}{-2\lambda} = k$$

17. (48)

Flux via ABCD

$$\phi_1 = \int \vec{E}.d\vec{A} = 0$$

Flux via BCEF

$$\phi_2 = \int \vec{E}.d\vec{A}$$

$$\phi_2 = \vec{E}.\vec{A} = (4x\hat{i} - (y^2+1)\hat{j}).4i$$

$$= 16x,\ x = 3$$

$$\phi_2 = 48\frac{N-m^2}{C};\ \phi_1 - \phi_2 = -48\frac{N-m^2}{C}$$

18. (*c*) Magnitude of electric field is constant & the surface is equipotential

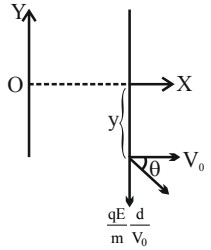
19. (*d*) $\dfrac{E_1}{E_2} = \dfrac{r_1}{r_2}$

$$\frac{V_1}{V_2} = \frac{E_1 r_1}{E_2 r_2} = \frac{r_1}{r_2}\times\frac{r_1}{r_2} = \left(\frac{r_1}{r_2}\right)^2$$

20. (*b*) $\vec{F} = qE\hat{i} + mg\hat{j}$

Since initial velocity is zero. It will move in straight line.

21. (*c*)



$$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{d}{V_0}\right)^2$$

$$y = -\frac{qEd}{mV_0^2}x + C$$

$$\text{At } x = d,\ y = -\frac{qEd^2}{2mV_0^2} \Rightarrow C = \frac{qEd^2}{2mV_0^2}$$

$$y = -\frac{qEd}{mV_0^2}x + \frac{qEd^2}{2mV_0^2}$$

$$y = \frac{qEd^2}{mV_0^2}\left(\frac{d}{2} - x\right)$$

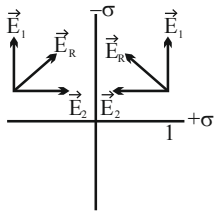
22. (*a*) $E = E_0(1 - ax^2)$

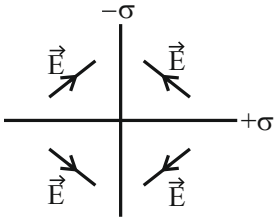
$$\frac{vdv}{dx} = \frac{qE_0}{m}(1 - ax^2)$$

$$\Rightarrow \frac{v^2}{2} = \frac{qE_0}{m}\left[x - \frac{ax^3}{3}\right] = 0$$

$$\Rightarrow x = \sqrt{\frac{3}{a}}$$

23. (*d*)





24. (*c*) For spherical shell

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad (r > R)$$

$$= 0 \qquad (r < R)$$

Force on charge in electric field

$F = qE$

25. (*a*) $\vec{p}$ $\overleftarrow{p^{\bullet}}$

$U = \vec{p}\cdot\vec{E}$

$\vec{E} = \dfrac{2\vec{p}}{4\pi\varepsilon_0 \times a^3}$

$\therefore \quad \Delta U_{loss} = \dfrac{2p^2}{4\pi\varepsilon_0 a^3} = 2\times\dfrac{1}{2}mv^2$

$\Rightarrow \quad v = \dfrac{p}{a}\sqrt{\dfrac{1}{2\pi\varepsilon_0 ma}}$

26. (194.42)

$\because I = \dfrac{1}{2}\varepsilon_0 E_0^2 c$

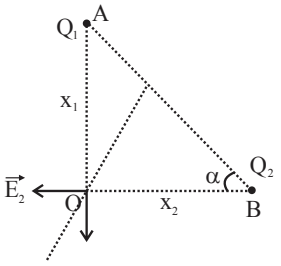
$\Rightarrow \quad E_0 = \sqrt{\dfrac{2I}{\varepsilon_0 c}}$

$\therefore \quad E_{rms} = \dfrac{E_0}{\sqrt{2}} = \sqrt{\dfrac{I}{\varepsilon_0 c}}$

$= \sqrt{\dfrac{315}{\pi}\times\dfrac{1}{8.86\times10^{-12}\times3\times10^8}}$

$= 194.42$

27. (*c*)



Net field along AB at O must be zero

$E_2\cos\alpha = E_1\sin\alpha$

$\dfrac{kQ}{x_2^2}\cdot\dfrac{x_2}{AB} = \dfrac{kQ_1}{x_1^2}\cdot\dfrac{x_1}{AB}$

$\dfrac{Q_1}{Q_2} = \dfrac{x_1}{x_2}$

28. (*a*) Let charges on shells be q_1 and q_2

$q_1 + q_2 = Q$ ...(i)

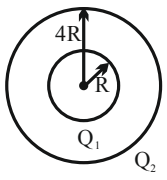
$\dfrac{q_1}{4\pi r^2} = \dfrac{q_2}{4\pi R^2}$ ...(ii)

We get $q_1 = \dfrac{r^2}{r^2+R^2}Q$, $q_2 = \dfrac{R^2}{r^2+R^2}Q$

$V = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_1}{r}+\dfrac{q_2}{R}\right)$

$= \dfrac{1}{4\pi\varepsilon_0}\dfrac{(R+r)}{(R^2+r^2)}Q$

29. (*b*)



$\because \quad \dfrac{Q_1}{R^2} = \dfrac{Q_2}{16R^2}$

$\Rightarrow \quad Q_2 = 16Q_1$

$\left(\dfrac{KQ_1}{R}+\dfrac{KQ_2}{4R}\right)-\left(\dfrac{KQ_1}{4R}+\dfrac{KQ_2}{4R}\right) = V_1 - V_2$

$\Rightarrow \quad V_1 - V_2 = \dfrac{3}{4}\times\dfrac{Q_1}{4\pi\varepsilon_0 R} = \dfrac{3Q_1}{16\pi\varepsilon_0 R}$

30. (*a*) $k = qV$

$\dfrac{1}{2}mv^2 = qV$

$\Rightarrow \quad v = \sqrt{\dfrac{2qV}{m}}$

$\dfrac{V_H}{V_{He}} = \sqrt{\dfrac{q_H m_{He}}{m_H q_{He}}} = 2:1$

31. (*b*) $\Delta u = \dfrac{4q^2}{4\pi\varepsilon_0}\left[\dfrac{1}{3d/2}-\dfrac{1}{d/2}\right]$

$\Delta u = -\dfrac{4q^2}{3\pi\varepsilon_0 d}$

32. (*a*) $\sum q = 0$

$\Rightarrow \quad V = 0$

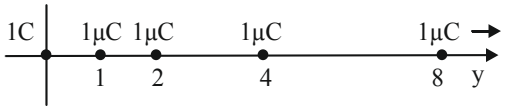
Also, field of one charge will get cancelled due to another symmetrical charge in front of it.

33. (*d*) $\dfrac{1}{2}mv^2 = mgy + \Delta U_{elec.}$

$\Delta U_{elec.} = kQq\left[\dfrac{1}{R}-\dfrac{1}{(R+y)}\right]$

$\Rightarrow \quad v^2 = 2y\left[\dfrac{qQ}{4\pi\varepsilon_0 R(R+y)m}+g\right]$

34. (12)



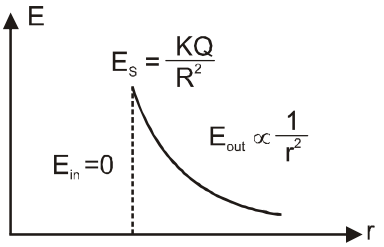
$= 9\times10^9\times10^{-6}\left[1+\left(\dfrac{1}{2}\right)^2+\left(\dfrac{1}{2^2}\right)^2+\left(\dfrac{1}{2^3}\right)^2+\left(\dfrac{1}{2^\infty}\right)^2\right]$

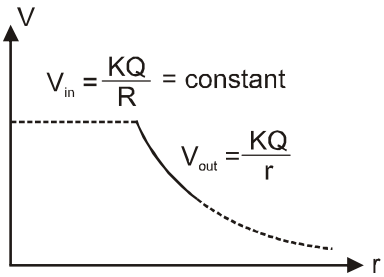
$= 9\times10^9\times10^{-6}\left[\dfrac{1}{1-\dfrac{1}{4}}\right]$

$\therefore \left[S_\infty = \dfrac{a}{1-r}\right]$ for G.P. $= 9\times10^3\times\dfrac{4}{3} = 12\times10^3\,N$

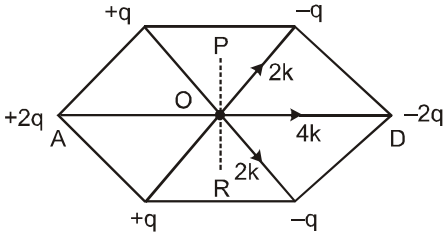
JEE-ADVANCED
PREVIOUS YEAR'S

1. (*d*)



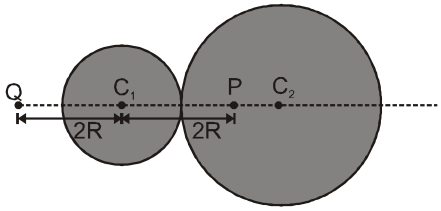


2. (*a,b,c*)



$E_0 = 6$ K (along OD)
$V_0 = 0$
Potential on line PR is zero
PR

3. (*b,d*)



At point P
If resultant electric field is zero
then

$$\frac{KQ_1}{4R^2} = \frac{KQ_2}{8R^3}R$$

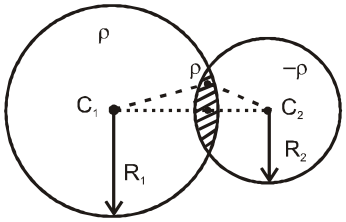
$$\frac{\rho_1}{\rho_2} = 4$$

At point Q
If resultant electric field is zero
then

$$\frac{KQ_1}{4R^2} + \frac{KQ_2}{25R^2} = 0$$

$$\frac{\rho_1}{\rho_2} = -\frac{32}{25}$$ (ρ_1 must be negative)

4. (*c,d*)



For electrostatic field,

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1P} + \frac{(-\rho)}{3\varepsilon_0}\overrightarrow{C_2P}$$

$$= \frac{\rho}{3\varepsilon_0}(\overrightarrow{C_1P} + \overrightarrow{PC_2})$$

$$\vec{E}_P = \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1C_2}$$

For electrostatic potential, Since electric field is non zero so it is not equipotential.

5. (*c*) $E_1 = \frac{KQ}{R^2}$

$$E_2 = \frac{k(2Q)}{R^2} \Rightarrow E_2 = \frac{2kQ}{R^2}$$

$$E_3 = \frac{k(4Q)R}{(2R)^3} \Rightarrow E_3 = \frac{kQ}{2R^2}$$

$E_3 < E_1 < E_2$

6. (*c*) $\frac{Q}{4\pi\in_0 r_0^2} = \frac{\lambda}{2\pi\in_0 r_0} = \frac{\sigma}{2\in_0}$

$Q = 2\pi\sigma r_0^2$ A incorrect

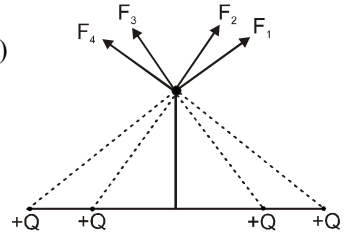
$r_0 = \frac{\lambda}{\pi\sigma}$ B incorrect

$$E_1\left(\frac{r_0}{2}\right) = \frac{4E_1(r_0)}{1}$$

$$E_2\left(\frac{r_0}{2}\right) = 2E_2(r_0) \Rightarrow \text{C correct}$$

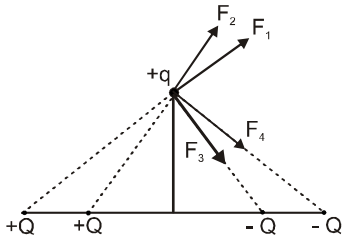
$$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0) \text{ D incorrect}$$

7. (*a*) (P)



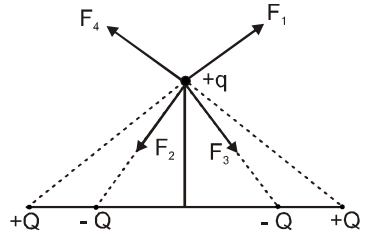
Component of forces along x-axis will vanish. Net force along +ve y-axis

(Q)



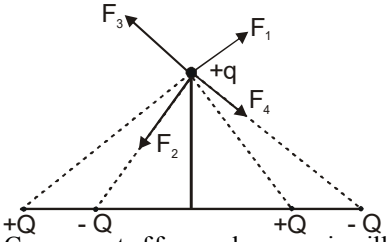
Component of forces along y-axis will vanish. Net force along +ve x-axis

(R)



Component of forces along x-axis will vanish. Net force along -ve y-axis.

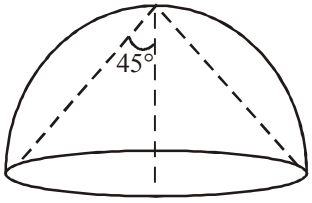
(S)



Component of forces along y-axis will vanish. Net force along -ve x-axis.

Ans. (*a*) P—3, Q—1, R—4,S—2

8. (*c,d*)



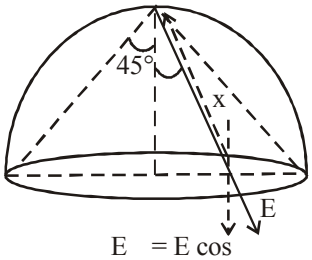
(*a*) ϕ total due to charge Q is $= \dfrac{Q}{\varepsilon_0}$

so ϕ through the curved and flat surface will be less than $\dfrac{Q}{\varepsilon_0}$

(*b*) The component of the electric field perpendicular to the flat surface will decrease so we move away from the centre as the distance increase (magnitude of electric field decreases) as well as the angle between the normal and electric field will increase.

2nd Method

$$x = \frac{R}{\cos\theta}$$



$E = E\cos$

$$E = \frac{KQ}{x^2} = \frac{KQ\cos^2\theta}{R^2}$$

$$E\perp = \frac{KQ\cos^3\theta}{R^2}$$

As we move away from centre $\theta \uparrow$ $\cos\theta$ so$\downarrow$ $E\perp\downarrow$

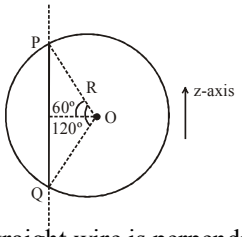
(*c*) Since the circumference is equidistant from 'Q' it will be equuipotential $V = \dfrac{KQ}{\sqrt{2}R}$

(*d*) $\Omega = 2\pi\left(1-\cos\theta\right); \theta = 45°$

$$\phi = -\frac{\Omega}{4\pi}\times\frac{Q}{\varepsilon_0} = -\frac{2\pi\left(1-\cos\theta\right)}{4\pi}\frac{Q}{\varepsilon_0}$$

$$= -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

9. (*a,b*)



Field due to straight wire is perpendicular to the wire & radially outward. Hence $E_z = 0$

Length, PQ = 2R sin 60 = $\sqrt{3}R$ According to Gauss's law

total flux = $\oint \vec{E}.\vec{ds} = \dfrac{q_{in}}{\epsilon_0} = \dfrac{\lambda\sqrt{3}R}{\epsilon_0}$

10. (*b*) (*i*) $E = \dfrac{KQ}{d^2} \Rightarrow E \propto \dfrac{1}{d^2}$

(*ii*) Dipole

$$E = \frac{2kp}{d^3}\sqrt{1+3\cos^2\theta}$$

$E \propto \dfrac{1}{d^3}$ for dipole

(*iii*) For line charge

$$E = \frac{2k\lambda}{d}$$

$$E \propto \frac{1}{d}$$

(*iv*) $E = \dfrac{2K\lambda}{d-\ell} - \dfrac{2K\lambda}{d+\ell} = 2K\lambda\left[\dfrac{d+\ell-d+\ell}{d^2-\ell^2}\right]$

$$E = \frac{2K\lambda(2\ell)}{d^2\left[1-\dfrac{\ell^2}{d^2}\right]}$$

$$E \propto \frac{1}{d^2}$$

(*v*) Electric field due to sheet

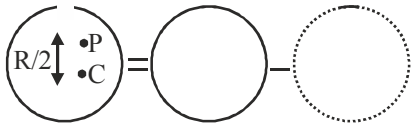
$$\epsilon = \frac{\sigma}{2\epsilon_0}$$

ϵ = v is independent of r

11. (*a*) Let charge on the sphere initially be Q.

$\therefore \quad \frac{kQ}{R} = V_0$

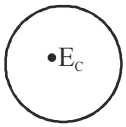
and charge removed = αQ

(*a*) 

and $V_P = \frac{kQ}{R} - \frac{2kQ\alpha}{R} = \frac{kQ}{R}(1-2\alpha)$

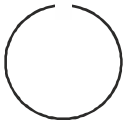
$V_C = \frac{kQ(1-\alpha)}{R}$

$\therefore \quad \frac{V_C}{V_P} = \frac{1-\alpha}{1-2\alpha}$



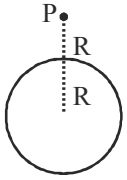
(*b*) $(E_C)_{initial} = \text{zero}$

$(E_C)_{initial} = \frac{k\alpha Q}{R^2}$



$\Rightarrow$ Electric field increases

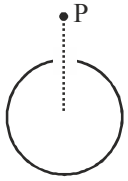
(*c*) $(E_P)_{final} = \frac{kQ}{4R^2} - \frac{k\alpha Q}{R^2}$

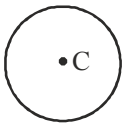


$\Delta E_P = \frac{kQ}{4R^2} - \frac{kQ}{4R^2} + \frac{k\alpha Q}{R^2} = \frac{k\alpha Q}{R^2} = \frac{V_0\alpha}{R}$

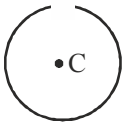
(*d*) $(V_C)_{initial} = \frac{kQ}{R}$

$(V_C)_{final} = \frac{kQ(1-\alpha)}{R}$





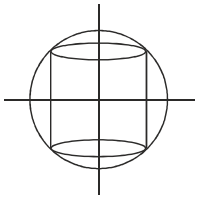
$\Delta V_C = \frac{kQ}{R}(\alpha) = \alpha V_0$



12. (*a,b,d*)

For option (1), cylinder encloses the shell, thus option is correct

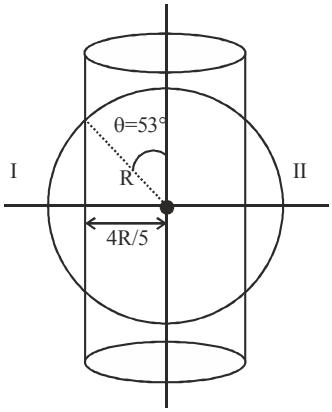
For option (2),



cylinder perfectly enclosed by shell,

thus $\phi = 0$, so option is correct.

for option (3)



$\phi = \frac{2 \times Q}{2\epsilon_0}(1-\cos 53^\circ) = \frac{2Q}{5\epsilon_0}$

For option (4) :

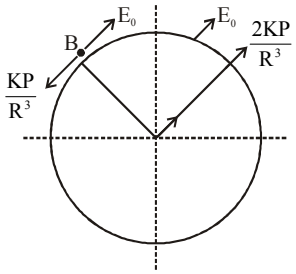
Flux enclosed by cylinder = $\phi = \frac{2Q}{2\epsilon_0}(1-\cos 37^\circ) = \frac{Q}{5\epsilon_0}$

13. (*a,d*)

(1) $\vec{P} = \frac{P_0}{\sqrt{2}}(\hat{i}+\hat{j})$

E.F. at B along tangent should be zero since circle is equi-potential.

So, $E_0 = \frac{K|\vec{P}|}{R^3}$ & $E_B = 0$



So, $R^3 = \frac{KP_0}{E_0} = \left(\frac{P_0}{4\pi\epsilon_0 E_0}\right)$

So $R = \left(\frac{P_0}{4\pi\epsilon_0 E_0}\right)^{1/3}$

So, (*a*) is correct

(*b*) Because E_0 is uniform & due to dipole E.F. is different at different points, so magnitude of total E.F. will also be different at different points

So, (*b*) is incorrect

(*c*) $E_A = \frac{2KP}{R^3} + \frac{KP}{R^3} = 3\frac{KP}{R^3}\frac{P}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

So, (*c*) is wrong

(*d*) $E_B = 0$

So, (*d*) is correct

14. (*b,c*)

$a_y = -400\sqrt{3}\times10^{10}$ [$qE_y = ma_y$]

$$R = 5 = \frac{40\times10^{12}\sin 2\theta}{400\sqrt{3}\times10^{10}}\left[R(\text{range}) = \frac{u^2\sin 2\theta}{a_y}\right]$$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$2\theta = 60^\circ, 120 \Rightarrow \theta = 30^\circ, 60^\circ$

Time of flight $T_1 = \dfrac{2\times2\sqrt{10}\times10^6\times\frac{1}{2}}{400\sqrt{3}\times10^{10}} = \sqrt{\frac{5}{6}}\mu s$ (for $\theta=30^\circ$)

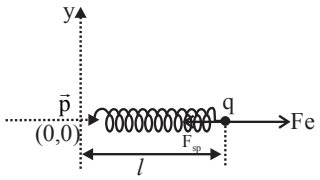
Time of flight $T_2 = \dfrac{2\times2\sqrt{10}\times10^6\times\frac{\sqrt{3}}{2}}{400\sqrt{3}\times10^{10}} = \sqrt{\frac{5}{3}}\mu s$ (for $\theta=60^\circ$)

15. (3.14)

$\Delta\ell \to x$

At ℓ : $Fe = F_{SP}$

$$k\ell = \frac{2kpq}{\ell^3}$$



$$F_{net} = F_{sp} - Fe = k(\ell + x) - \frac{q(2kp)}{(\ell + x)^3}$$

$$= k(x + \ell) - \frac{q(2kp)}{\ell^3(1 + x/\ell)^3}$$

$$kx + k\ell - q\left(\frac{2kp}{\ell^3}\right)\left(1 - \frac{3x}{\ell}\right)$$

$$= kx + k\ell - q\left(\frac{2kp}{\ell^3}\right) + \frac{2kpq}{\ell^3}\cdot\frac{3x}{\ell}$$

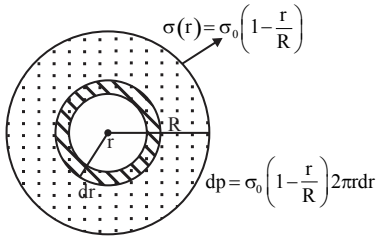
$$F_N = kx + k\ell\left(\frac{3x}{\ell}\right) = 4kx$$

$k_{eq} = 4k \qquad T = 2\pi\sqrt{\dfrac{m}{4k}} = \pi\sqrt{\dfrac{m}{k}}$

$$f = \frac{1}{\pi}\sqrt{\frac{k}{m}}$$

So $\delta = \pi = 3.14$

16. (6.40)



$$\phi = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^R \sigma_0\left(1 - \frac{r}{R}\right)2\pi r\,dr}{\varepsilon_0}$$

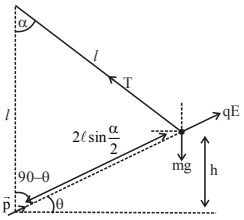
$$\phi = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^{R/4} \sigma_0\left(1 - \frac{r}{R}\right)2\pi r\,dr}{\varepsilon_0}$$

$$\therefore \frac{\phi_0}{\phi} = \frac{\sigma_0 2\pi\int_0^R\left(r - \frac{r^2}{R}\right)dr}{\sigma_0 2\pi\int_0^{R/4}\left(r - \frac{r^2}{R}\right)dr}$$

$$= \frac{\frac{R^2}{2} - \frac{R^2}{3}}{\frac{R^2}{32} - \frac{R^2}{3\times64}} = \frac{32}{5}$$

$= 6.40$

17. (*b*)



$U_i = 0$

$$U_f = \frac{kqP}{\left(2\ell\sin\frac{\alpha}{2}\right)^2} + mgh \qquad \ldots(i)$$

Now, from ΔOAB

$\alpha + 90 - \theta + 90 - \theta = 180 \Rightarrow \alpha = 2\theta$

From ΔABC : $h = 2\ell\sin\left(\frac{\alpha}{2}\right)\sin\theta$

$$h = 2\ell\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right)$$

Now charge is in equilibrium at point B.

So, using sine rule

$$\Rightarrow \frac{mg}{\sin\left[90 + \frac{\alpha}{2}\right]} = \frac{qE}{\sin[180 - 2\theta]}$$

$$\Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{\sin 2\theta}$$

$$\Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{\sin\alpha} \Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$

$$\Rightarrow qE = mg2\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \frac{q2kp}{\left[2\ell\sin\frac{\alpha}{2}\right]^3} = mg2\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \frac{kpq}{\left[2\ell \sin\frac{\alpha}{2}\right]^2} = mg\sin\left(\frac{\alpha}{2}\right)\times\left(2\ell\sin\frac{\alpha}{2}\right)$$

$$\Rightarrow \frac{kpq}{\left[2\ell \sin\frac{\alpha}{2}\right]^2} = mgh \Rightarrow \text{substituting this in equation}$$

...(i)

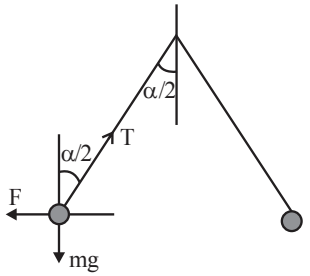
$$U_f = mgh + \frac{kpq}{\left[2\ell \sin\frac{\alpha}{2}\right]^2}$$

$\Rightarrow U_f=2mgh$

$W=\Delta U=Nmgh=N=2$

18. (*a,c*)

The net electric force on any sphere is lesser but by Coulomb law the force due to one sphere to another remain the same.



In equilibrium

$T\cos\frac{\alpha}{2} - mg$ and $T\sin\frac{\alpha}{2} = F$

After immersed is dielectric liquid.

As given no change in angle α.

So $T\cos\frac{\alpha}{2} = mg - V\rho g$

when ρ=800 Kg/m^3

and $T\sin\frac{\alpha}{2} = \frac{F}{e_r}$

$$\therefore \frac{mg}{F} = \frac{mg - V\rho g}{\frac{F}{e_r}}$$

$$\frac{1}{e_r} = 1 - \frac{\rho}{d}$$

d=density of sphere

$$\frac{1}{21} = 1 - \frac{800}{d}$$

d=840

19. (6)

$$E = \frac{V}{d} = \frac{200}{0.01} = 2\times10^4 \text{ V/m}$$

When terminal velocity is achieved

qE=mg

$$\Rightarrow n\times1.6\times10^{-19}\times2\times10^4 = \frac{4\pi}{3}\left(8\times10^{-7}\right)^3\times900\times10$$

$\Rightarrow n \approx 6$

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

2 Capacitance

Topicwise Questions

1. (*c*) Volume of 8 small drops = Volume of big drop

$$8\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3 \Rightarrow R=2r$$

As capacity is r, hence capacity becomes 2 times

2. (*d*) $C=\dfrac{K\varepsilon_0 A}{d}$

3. (*b*) By using $V_{big}=n^{2/3}v_{small}$

$$\Rightarrow \frac{V_{big}}{v_{small}}=(8)^{2/3}=\frac{4}{1}$$

4. (*d*) If the drops are conducting, then

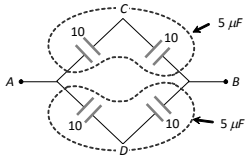
$$\frac{4}{3}\pi R^3=N\left(\frac{4}{3}\pi r^3\right)$$

$\Rightarrow R=N^{1/3}r$. Final charge $Q=Nq$

So final potential $V=\dfrac{Q}{R}=\dfrac{Nq}{N^{1/3}r}=V\times N^{2/3}$

5. (*a*) **6.** (*a*)

7. (*d*) In the given system, no current will flow through the branch CD so it can be removed



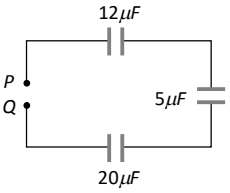
Effective capacitance of the system = 5 + 5 = 10 μF

8. (*b*) The given circuit can be drawn as

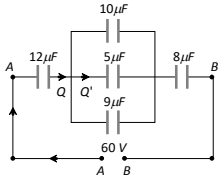
where $C=(3+2)\ \mu F=5\ \mu F$

$$\frac{1}{C_{PQ}}=\frac{1}{5}+\frac{1}{20}+\frac{1}{12}=\frac{20}{60}=\frac{1}{3}$$

$\Rightarrow C_{PQ}=3\ \mu F$



9. (*d*) The given circuit can be redrawn as follows



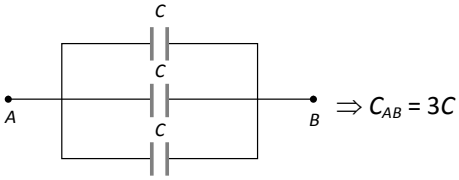
Equivalent capacitance of the circuit $C_{AB}=4\ \mu F$

Charge given by the battery $Q=C_{eq}V=4\times 60=240\ \mu C$

Charge in 5μF capacitor

$$Q'=\frac{5}{(10+5+9)}\times 240=50\ \mu C$$

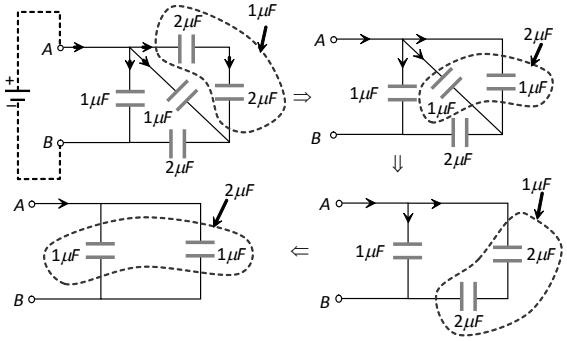
10. (*b*) The given circuit can be redrawn as follows



$\Rightarrow C_{AB}=3C$

11. (*b*) The given arrangement is equivalent to the parallel combination of three identical capacitors. Hence equivalent capacitance $=3C=\dfrac{3\varepsilon_0 A}{d}$

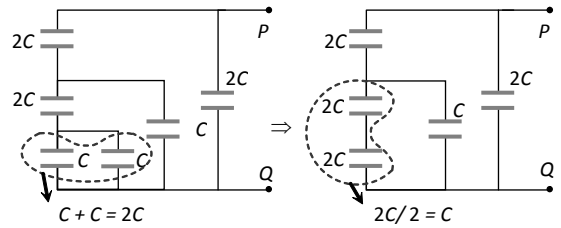
12. (*b*) The given circuit can be simplified as follows

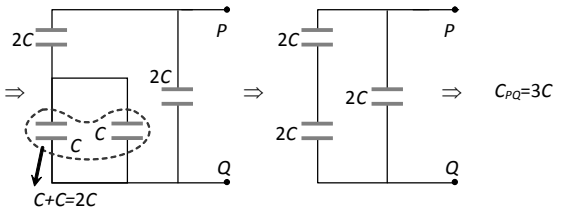


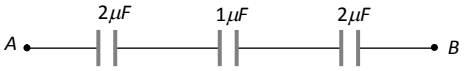
Hence equivalent capacitance between A and B is 2μF.

13. (*d*) Minimum when connected in series and maximum when connected in parallel

14. (*a*)





15. (*d*) 

$$\frac{1}{C}=\frac{1}{2}+\frac{1}{1}+\frac{1}{2}=\frac{1+2+1}{2}=\frac{4}{2}=2$$

$$\Rightarrow \quad C_{AB}=0.5\ \mu F$$

16. (*b*) Charge flows to second capacitor until the potential is same i.e. V/2. So new charge = CV/ 2

17. (*b*) $\frac{1}{C}=\frac{1}{3}+\frac{1}{6} \Rightarrow C=2\ pF$

Total charge = $2\times10^{-12}\times5000=10^{-8}$ C

The new potential when the capacitors are connected in parallel is

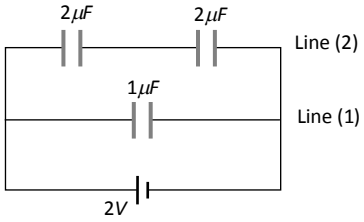
$$V=\frac{2\times10^{-8}}{(3+6)\times10^{-12}}=2222\ V$$

18. (*c*) $\frac{1}{C_{eq}}=\frac{1}{3}+\frac{1}{10}+\frac{1}{15} \Rightarrow C_{eq}=2\ \mu F$

Charge on each capacitor

$Q=C_{eq}\times V \Rightarrow 2\times100=200\ \mu C$

19. (*d*) Potential difference across both the lines is same i.e. 2 V. Hence charge flowing in line 2



$Q=\left(\frac{2}{2}\right)\times2=2\ \mu C$. So charge on each capacitor in line (2) is 2 μC

20. (*c*) Given circuit can be reduced as follows

In series combination charge on each capacitor remain same. So using Q = CV

$\Rightarrow C_1V_1=C_2V_2 \Rightarrow 3\ (1200-V_p)=6\ (V_p-V_B)$

$\Rightarrow 1200-V_p=2V_p\ (\because V_B=0)$

$\Rightarrow 3V_p=1200 \Rightarrow V_p=400$ volt

21. (*d*) $U=\frac{1}{2}CV^2=\frac{1}{2}\times2\times10^{-6}\times(50)^2=25\times10^{-4}$ J

$=25\times10^3$ erg

22. (*c*) $U=\frac{1}{2}CV^2=\frac{1}{2}\times6\times10^{-6}\times(100)^2=0.03$ J

23. (*b*) $\text{Power}=\frac{\frac{1}{2}CV^2}{t}=\frac{1\times40\times10^{-6}\times(3000)^2}{2\times2\times10^{-3}}=90$ kW

24. (*b*)

25. (*b*) In series combination of capacitors, voltage distributes on them, in the reverse ratio of their capacitance i.e.

$$\frac{V_A}{V_B}=\frac{3}{2} \qquad \text{.....(i)}$$

Also $V_A+V_B=10$ .....(ii)

On solving (i) and (ii) $V_A=6V$, $V_B=4V$

26. (*d*) $C=\frac{\varepsilon_0A}{d}$ .....(i)

$$C'=\frac{\varepsilon_0KA}{2d} \qquad \text{.....(ii)}$$

From equation (i) and (ii)

$$\frac{C'}{C}=\frac{K}{2}$$

$$\Rightarrow \ 2=\frac{K}{2} \Rightarrow K=4$$

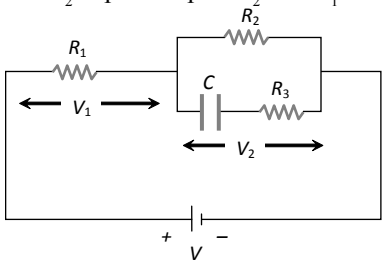
27. (*c*) $C_R=C_1+C_2=\frac{k_1\varepsilon_0A_1}{d}+\frac{k_2\varepsilon_0A_2}{d}$

$$=\frac{2\times\varepsilon_0\frac{A}{2}}{d}+\frac{4\times\varepsilon_0\frac{A}{2}}{d}=2\times\frac{10}{2}+4\times\frac{10}{2}=30\ \mu F$$

28. (*b*) In steady state potential difference across capacitor V_2= potential difference across resistance R_2

$$R_2=\left(\frac{R_2}{R_1+R_2}\right)V$$

Hence V_2 depends upon R_2 and R_1



Learning Plus

1. (*a*) $Q_t=Q_1+Q_2=150\mu C$

$$\frac{Q_1'}{Q_2'}=\frac{C_1}{C_2}=\frac{1}{2} \Rightarrow Q_1'=50\mu C$$

$Q_2'=100\mu C$

25μC charge will flow from smaller to bigger sphere .

2. (*d*) Charge is flow until potential are equal and in charge flow energy is decrease

$$\frac{Q_1}{C_1}=\frac{Q_2}{C_2} \Rightarrow Q_1R_2=Q_2R_1.$$

3. (*d*) Charge / Current flows from higher to lower potential or Q/C ratio.

4. (*d*) $C = \dfrac{k \in_0 A}{d}$ where k = dielectric constant of medium between the plates

A = Area, d = distance between the plates

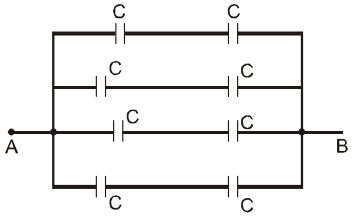
5. (*c*) $Q_1 = 900\mu C$ $\qquad$ $Q_2 = 2500\mu C$

When the two capacitors are connected together let the common potential is V.

$900 + 2500 = (3+5)V$

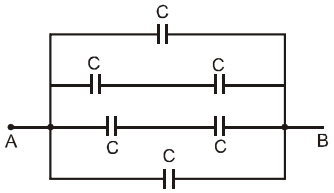
$V = \dfrac{3400}{8} = 425V$

6. (*b*)



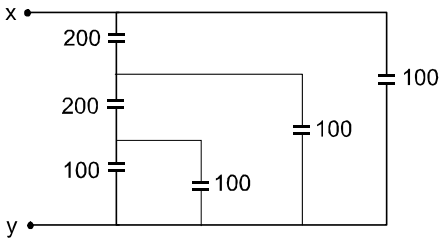
$C_{eq} = \dfrac{4C}{2} = 2C.$

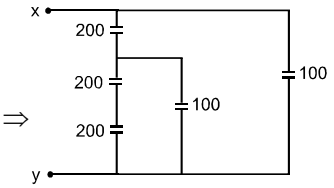
7. (*c*)



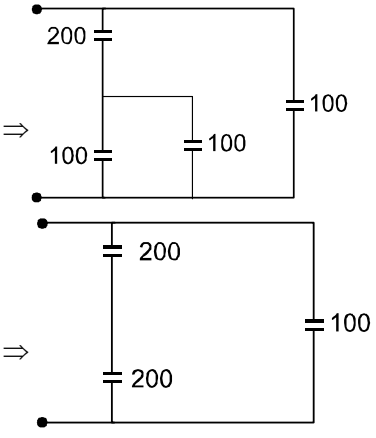
$C_{eq} = C + \dfrac{2C}{2} + C = 3C.$

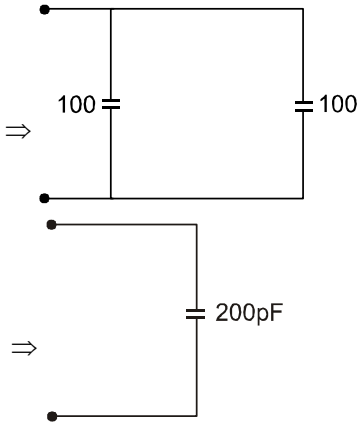
8. (*b*)





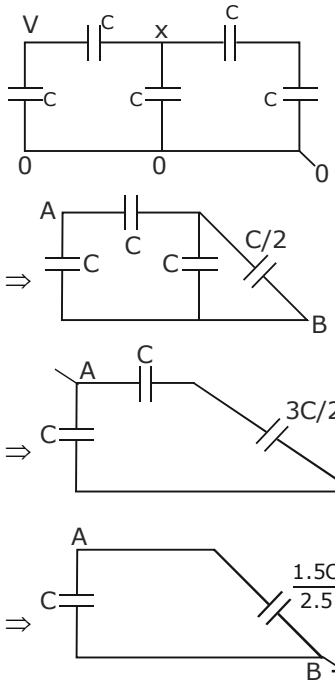
solving by parallel series combinations,





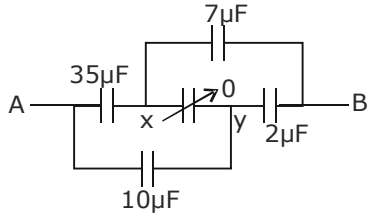
$C_{eq} = 200\ pF$

9. (*b*) Solving the circuit using following steps

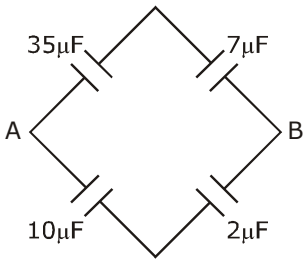


Resultant capacitance of the circuit = 1.6C

10. (*b*)



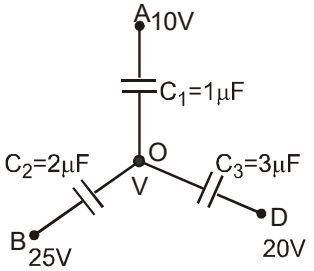
As the resulting circuit is a Wheat stone bridge hence current in 13μF capacitor is zero. Hence the circuit now reduces to



The resultant capacitance is $\frac{35}{6}+\frac{10}{6}=\frac{45}{6}$

$=\frac{15}{2}\ \mu F$

11. (*a*)



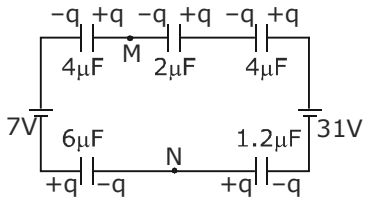
From junction law

$(V-10)1+(V-20)3+(V+25)2=0$

$6V=120$

$V=20$ Volt

12. (*b*) Let q be the charge on all the capacitor



Apply KVL

$31-\frac{q}{4}-\frac{q}{2}-\frac{q}{4}-7-\frac{q}{6}-\frac{q}{12}=0$

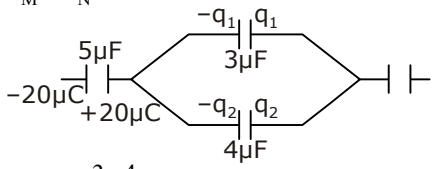
$24=\left[\frac{3+6+3+2+10}{12}\right]q$

$q=12\ \mu C$

Now $V_N+\frac{q}{6}+7+\frac{q}{4}=V_M$

$V_M-V_N=12\ V$

13. (*a*)



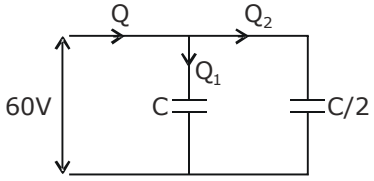
$q_1:q_2=3:4$

$q_1=\frac{3}{7}\times 20\mu C$

14. (*a*) $V_1:V_2=\frac{1}{C_1}:\frac{1}{C_2}=C_1:C_2$

$\frac{V_1}{V_2}=\frac{C_1}{C_2}=\frac{1}{4}$

15. (*d*)



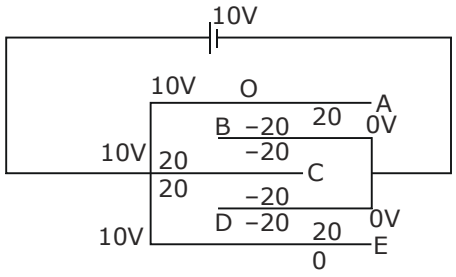
$Q=\frac{3}{2}C\times 60=90C$

$Q_1:Q_2=C_1:C_2=2:1$

$Q_2=\frac{1}{3}\times 90=30\ C$

Potential difference across $C=\frac{30C}{C}=30\ V$

16. (*b*)

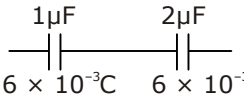


Total charge on plate C = 40 μC

17. (*d*) Maximum charge on 1st capacitor = 6×10^{-3}C.

Maximum charge on 2nd capacitor = 8×10^{-3}C.

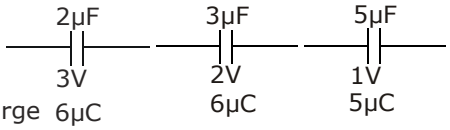
In series the maximum charge they can have is 6×10^{-3}C



Hence maximum voltage =

$V=\frac{6\times10^{-3}}{1\times10^{-6}}+\frac{6\times10^{-3}}{2\times10^{-6}}=9KV$

18. (*b*)



Hence maximum charge that the series can with stand is 5 μC. So break down voltage $=5\times\frac{31}{30}=\frac{31}{6}$ volt

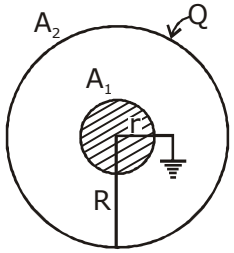
19. (*c*) We know that force between plates is

$\frac{\sigma^2A}{2\epsilon_0}=\frac{Q^2}{2A\epsilon_0}=\frac{C^2V^2}{2A\epsilon_0}$

$=\frac{\epsilon_0^2A^2V^2}{2A\epsilon_0 d^2}=\frac{\epsilon_0 AV^2}{2d^2}$

$C_i=\frac{\epsilon_0 Av^2}{2d^2}\ C_f=\frac{\epsilon_0 Av^2\times4}{2d^2}$

20. (*c*) Let us assume charge on A_1 is q and potential of A_1 is zero as it is earthed.



Potential of A_1 is due to charges Q & q. So we can write the equation as

$V=\frac{KQ}{r}q+\frac{KQ}{R}=0$

$\frac{q}{r}=\frac{-Q}{R}\Rightarrow q=\frac{-Qr}{R}$

21. (*a*) Charge carries electrical energy so capacitor stores electrical energy.

22. (*b*) $W = U_f - U_i = \frac{1}{2}CV_f^2 - \frac{1}{2}CV_i^2 = \frac{1}{2}C(40^2 - 20^2)$

$W = 600\,C$

$W_1 = \frac{1}{2}C(50^2 - 40^2) = \frac{900}{2}C$

$W_1 = \frac{900}{2}\cdot\frac{W}{600} = \frac{3}{4}W$

23. (*a*) Initially

$U_i = \frac{1}{2}CV^2 = \frac{1}{2}\times 0.5\times 10^{-6}\times 10^4 = 0.25\times 10^{-2}\,J$

When the 0.5 μF capacitor is connected to an uncharged capacitor let the common potential is V.

$0.5\times 100 = 0.7\,V$

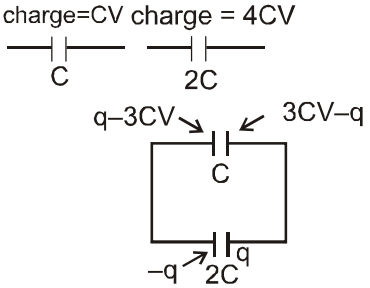
$V = \frac{0.5\times 100}{0.7} = \frac{500}{7}\,Volt$

$U_f = \frac{1}{2}\times 0.7\times 10^{-6}\times\frac{500}{7}\times\frac{500}{7}$

$= 1.78\times 10^{-3}\,J$

$Loss = U_f - U_i = 0.72\times 10^{-3}\,J$

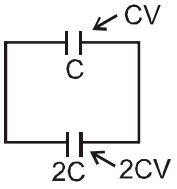
24. (*c*)



Total charge $= 4CV - CV = 3CV$

Now, let it is distributed as shown, potential across the capacitors is same

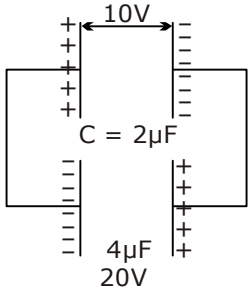
So, $\frac{q}{2C} = \frac{3CV - q}{C} \Rightarrow q = 2CV$



Total potential energy $= \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$

$= \frac{C^2V^2}{2C} + \frac{4C^2V^2}{2\times 2C} = \frac{3CV^2}{2}$

25. (*b*)



Before connection

$Q_1 = 2\times 10 = 20,\ Q_2 = 4\times 20 = 80$

$U_i = \frac{1}{2}2(10)^2 + \frac{1}{2}4(20)^2 = 900\,J$

Since connected as shown

After $Q_{net} = -20 + 80$

Connection $= 60$

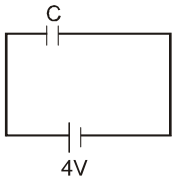
$V = \frac{60}{2+4} = 10\,Volt$

$U_f = \frac{1}{2}6(10)^2 = 300\,J$

Heat generated $= -U_f + U_i = 600\,J$

26. (*b*) $C' = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d} = 2C.$

27. (*c*)



Here, Potential difference on the capacitor will depend on emf of battery i.e., 4V

28. (*a*) Charge or battery $= Q = CV = 4\,C$

Now charge remains same, as battery is disconnected new capacitance $= C' = KC = 8C$

$C'V' = Q$

$V' = \frac{Q}{C'} = \frac{4C}{8C} = \frac{1}{2}V$

29. (*a*) $U_0 = \frac{1}{2}CV^2$ (given)

Now energy $= U' = \frac{1}{2}C'V^2$

$C' = CK$

$U' = \frac{1}{2}CV^2K = U_oK$

30. (*c*) For metal $k = \infty$

Hence from formula.

$C_{eq} = \frac{\epsilon_o A}{d - t + t/k}$

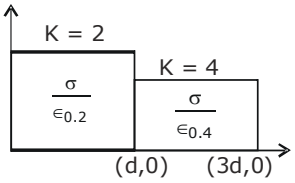
$C = \frac{\epsilon_0 A}{(d-t)}$

31. (*a*) Electric field between two plates of capacitor is given by $\frac{\sigma}{K\epsilon_0}$

When K = 1 then $E = \frac{\sigma}{\epsilon_0}$

then K = K then $E = \frac{\sigma}{K\epsilon_0}$

On increasing dielectric constant electric field decreases.



32. (*b*) $C = \dfrac{\epsilon_0 A}{d - t + \dfrac{t}{k}}$

Now $\dfrac{\epsilon_0 A}{d - t + \dfrac{t}{k}} = \dfrac{3}{2}\dfrac{\epsilon_0 A}{d}$

$\left(d - \dfrac{t}{2}\right) = \dfrac{2d}{3} \Rightarrow \dfrac{t}{d} = \dfrac{2}{3}$

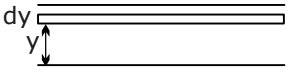
33. (*a*) $V_{max} = E_{max}\, d_{max} = 4000$

$d = \dfrac{4000}{18 \times 10^6}$

Now, $C = \dfrac{\epsilon_0 K A_{min}}{d_{max}} = 7 \times 10^{-2}\,\mu f$

$A = \dfrac{7 \times 10^{-2} \times 10^{-6} \times 4000}{8.85 \times 10^{-12} \times 2.8 \times 18 \times 10^6} = 0.63\ m^2$

34. (*a*)



$dc = \dfrac{\varepsilon_0 A\lambda \sec(\pi y / 2d)}{dy}$

All the elements are in series

Hence $\dfrac{1}{C^n_{eq}} = \int_0^d \dfrac{dy}{\varepsilon_0 A\lambda} \cos\left(\dfrac{\pi y}{2d}\right)$

$= \dfrac{2d}{\varepsilon_0 A\lambda\pi}\left[\sin\left(\dfrac{\pi y}{2d}\right)\right]_0^d$

$C_{eq} = \dfrac{\varepsilon_0 A\lambda\pi}{2d}$

35. (*c*) As the potential difference is constant hence we can say that

$Q_1 = 60\,\mu C = V \times C$ ....(1)

Now there is already 60 μC on the capacitor.

More 120 μC charge flows from battery. Hence net charge on capacitor is

$Q_2 = 180\,\mu C = V \times KC$ ....(2)

$(2)/(1) \Rightarrow 3 = K$

36. (*b*) Charge on 15 μF capacitor A = 1500 μC.

Charge on capacitor B = 100 μC.

When they are connected with dielectric removed from A the capacitor.

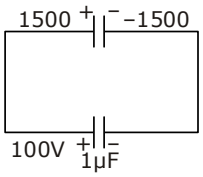
Capacitance of A now becomes 1 μF.

$C_i = \dfrac{\varepsilon_0 A.15}{d} = 15C = 15\mu F,$

$C_f = \dfrac{\varepsilon_0 A}{d}\ C = 1\mu F$

Q remains constant

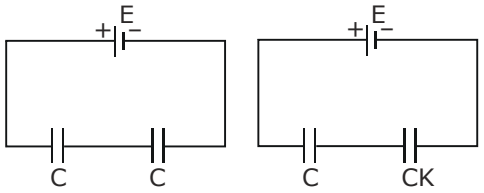
$Q_{net} = C_{eq} \times V_{common}$



$1500 + 100 = 2V$

$V = 800$ Volt

37. (*d*) **Initially** **Finally**



$C^n_{eq} = \dfrac{C}{2}$ $C^n_{eq} = \dfrac{CK}{K+1}$

$q_i = \dfrac{CE}{2}$ $q_f = \dfrac{CEK}{K+1}$

$q_f - q_i = q_{flown} = \dfrac{CEK}{K+1} - \dfrac{CE}{2}$

$= \dfrac{CE(K-1)}{2(K+1)}$

$\dfrac{CE}{2} < \dfrac{CEK}{K+1}$

So charge flows from C to B.

38. (*d*) The two capacitance C_1 & C_2 behave as a series arrangment as both the capacitors have equal charge on them

$C_1 = \epsilon_0 \dfrac{AK_1}{d/2}$

$C_2 = \epsilon_0 \dfrac{AK_2}{d/2}$

$C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

$= \dfrac{\dfrac{\varepsilon_o AK_1}{d/2} \times \dfrac{\varepsilon_o AK_2}{d/2}}{\left(\dfrac{\varepsilon_o AK_1}{d/2}\right) + \left(\dfrac{\varepsilon_o AK_2}{d/2}\right)} = \dfrac{2\varepsilon_o A}{d}\left(\dfrac{K_1 K_2}{K_1 + K_2}\right)$

39. (*b*) Initially

$C = 2.5 = \dfrac{\varepsilon_o A}{d}$

The two capacitanes act as a paralllel connection

$C' = \dfrac{\varepsilon_o A/2}{d} + \dfrac{K\varepsilon_o A/2}{d}$

$5\mu F = \dfrac{\varepsilon_o A}{2d} + \dfrac{K\varepsilon_o A}{2d}$

$5 = \dfrac{2.5}{2} + K\dfrac{2.5}{2}$

$\dfrac{10}{2.5} = K + 1 \Rightarrow K = 3$

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/ NUMERICAL

1. (a, b, d)

Magnitude of charge on the charged capacitor decreases and total charge is conserved.

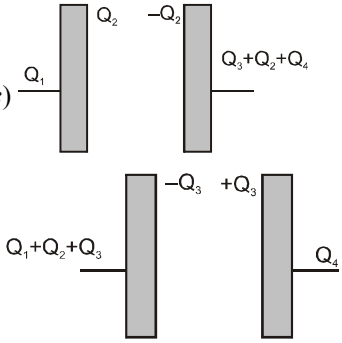
At $V_1 = V_2 \Rightarrow$ no further flow of charge occurs i.e. condition of steady state.

In charge flow energy is consumed in heat.

2. (b, c)

Electric field in the capacitor is same at every where which is equal to V/d. so that force at C and B point is same. Electric field out side the capacitor is zero so that force at A point is zero.

3. (b, c)



Charge on outer surfaces are equal so

$Q_1 = Q_3 + Q_2 + Q_4$ .....(i)

and $Q_1 + Q_2 + Q_3 = Q_4$ .....(ii)

$$V = \left|\frac{Q_2}{C}\right| \text{ or } V = \left|\frac{Q_1 - Q_3 - Q_4}{C}\right|$$

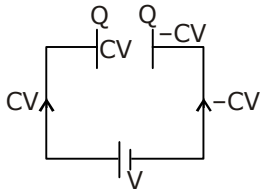
$$V = \left|\frac{Q_3}{C}\right| \text{ or } V = \left|\frac{Q_1 - Q_2 - Q_4}{C}\right|$$

Adding (i) and (ii)

$Q_1 = Q_4$ and $Q_2 = -Q_3$

4. (a, c, d)

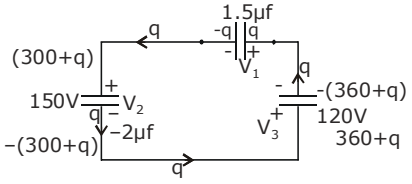
When two plates of capacitor are connected to a battery. The charges get distributed so that the charges on facing surface are equal & opposite. Also the battery does not create or destroy charges it distributes it.



$Q_1 = Q + CV$

$Q_2 = Q - CV$

5. (a, b, c)



$V_1 + V_2 + V_3 = 0$

$$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} = 0$$

$$\frac{300 + q}{2} + \frac{q}{1.5} + \frac{360 + q}{3} = 0$$

$$\frac{900 + 3q + 4q + 720 + 2q}{6} = 0$$

$9q = -1620$

$q = -180$

$Q_1 = 120\mu C$

$Q_2 = 180\mu C$

Flow of charge from right to left through A

6. (a, d)

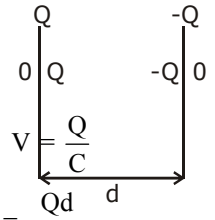
As the capactitance are in series hence charge on both of them will be same. $E = \frac{Q^2}{2C}$

$$V_1 : V_2 = \frac{1}{1} : \frac{1}{2},\ V_1 = \frac{2}{3} \times 15 = 10V$$

$V_2 = 5V$

7. (a, b, c, d) Initially

$$\Rightarrow C = \frac{\epsilon_0 A}{d}$$

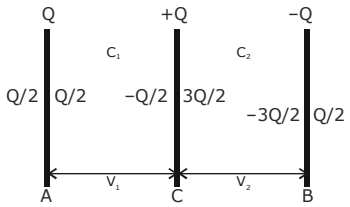


$$= \frac{Qd}{\epsilon_0 A}$$

Finally $C_1 = C_2 = \frac{2\epsilon_0 A}{d}$

$$V_1 = \frac{Q\,d}{2.2\,\epsilon_0 A}$$

$$V_2 = \frac{3Q\,d}{2.2\,\epsilon_0 A}$$



$$V = V_1 + V_2 = \frac{Q\,d}{\epsilon_0 A}$$

$V_f = V_i$

8. (b, c) Isolated $\rightarrow$ Q = constant C $\downarrow$

$$\text{Energy} = \frac{Q^2}{2C} \uparrow, E = \frac{\sigma}{\epsilon_0} = \text{constant}$$

$$\text{Energy density} = \frac{1}{2}\epsilon_0 E^2 = \text{constant}$$

9. (a, c) $C = 2\mu F$

$$C_{eq} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + ...$$

$$C_{eq} = C\left(\frac{1}{1-1/2}\right) = 2\left(\frac{1}{1/2}\right) = 4\mu F \qquad \textbf{Ans}$$

Charge on first row capacitor is $q_1 = 2 \times 10\mu C = 20\mu C$
Charge on second row capacitor is $q_2 = 1 \times 10\mu C = 10\mu C$

Charge on third row capacitor is $q_3 = \frac{1}{2} \times 10\mu C = 5\mu C$

Therefore charge on the capacitor in the first row is more than on any other capacitor.

Energy stored in all capacitor is $= \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (10)^2 = 0.2$ mJ **Ans**

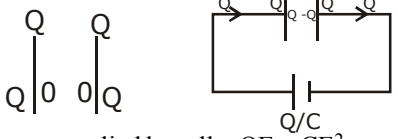
$C = 2\mu F$

$$C_{eq} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + ...$$

$$C_{eq} = C\left(\frac{1}{1-1/2}\right) = 2\left(\frac{1}{1/2}\right) = 4\mu F \qquad \textbf{Ans}$$

10. (a, b, c, d) Initially

After connecting battery



Energy supplied by cell $= QE = CE^2$

11. (b, c, d) $Q_1 = CV_1$

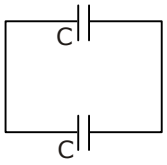
$Q_2 = CV_2$

Net charge = const.

[B correct]

$2CV = C(V_1 + V_2)$

$$V = \frac{V_1 + V_2}{2}$$



[C correct]

As charge flows energy will certainly be lost.

[D correct]

Net charge on the connected plates is equal sum of initial charges because charge is conserved.

12. (a, c) $4 \times 500 - 2 \times 500 = 6 \times V$

13. (a, b, c) $E = \frac{V}{d} \Rightarrow$ remains constant

$C' = KC \Rightarrow$ Increase

$Q' = KQ \Rightarrow$ Increase

$$U = \frac{1}{2} KCV^2 = KU \Rightarrow \text{Increase}$$

14. (a, c, d) Battery connected V = constant

$$U' = \frac{1}{2} KCV^2 = KU \Rightarrow \text{Increase by K–times}$$

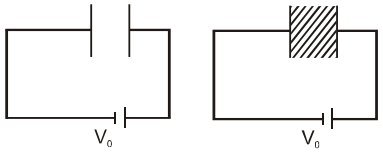
$$E = \frac{V}{d} = \text{constant}$$

$$F = \frac{Q^2}{2 \epsilon_0 A} \Rightarrow F = \frac{C^2V^2}{2 \epsilon_0 A} \Rightarrow F' = \frac{K^2C^2V^2}{2 \epsilon_0 A} = K^2F$$

$\Rightarrow$ Increase by K^2–times

$Q = C'V \Rightarrow Q' = KCV = KQ \Rightarrow$ Increase by K–times.

15. (a, d)



Potential difference $= V_0$ Potential difference $= V_0$

Capacitance = C

Capacitance = KC

[K is the dielectric constant of Slab K > 1]

$Q_0 = CV_0$

New charge $= KC\,V_0$

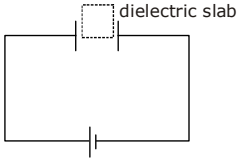
$$\text{Potential Energy} = \frac{1}{2} CV_0^2$$

$$\text{New potential energy} = \frac{1}{2} KC\,V_0^2$$

Correct options are $(a), (d)$.

16. (b, c) $30C_0 = (C_0 + KVC_0).V$

17. (b, c)

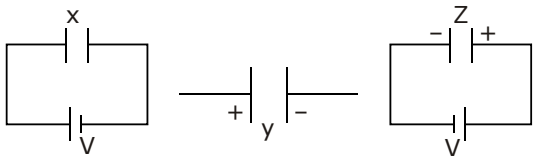


V = const.

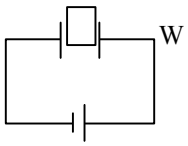
$$C = \frac{\varepsilon_0 kA}{d} \qquad C \uparrow,\ Q = CV \uparrow$$

$$e = \frac{V}{d} = \text{const.}$$

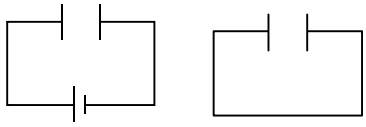
18. (b, c, d)



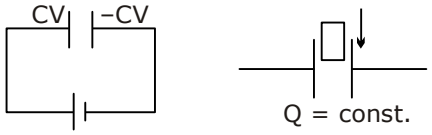
$$\varepsilon = \frac{1}{2} CV^2 \qquad \varepsilon = \frac{1}{2} CV^2 \quad \varepsilon = \frac{1}{2} CV^2$$



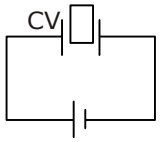
(b) In XWY charge increases



In XYW

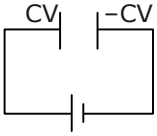


(c)



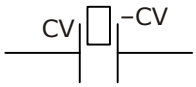
$$\varepsilon = \frac{Q^2}{2C}$$

$$\varepsilon = \frac{k^2C^2V^2}{2KC} = \frac{1}{2}KCV^2$$



$$\varepsilon = \frac{Q^2}{2C} = \frac{C^2V^2}{2KC} = \frac{1}{2}\frac{CV^2}{K}$$

Now insert dielectric



$$\text{W.D.} = U_f - U_i = -\frac{\varepsilon_0 AV^2}{2d}\left(1-\frac{1}{k}\right)$$

19. (*a*, *c*) $t_1 > t_2$

$R_1C_1 > R_2C_2$ for same q_{max}

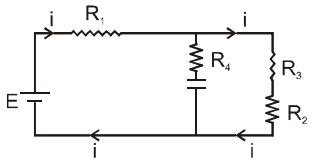
$q_{01} = q_{02} \Rightarrow E_1C_1 = E_2C_2$

If $E_1 = E_2 \Rightarrow C_1 = C_2 \Rightarrow R_1 = R_2$.

20. (*b*, *c*, *d*)

A long time after closing the switch, system comes in steady state and no current flow through capacitor..

Circuit :-



$$i = \frac{E}{R_1 + R_2 + R_3}$$

energy stored in battery $= \frac{1}{2}CV^2$

$$= \frac{1}{2}C\left(\frac{E(R_3+R_2)}{R_1+R_2+R_3}\right)^2$$

21. (*a*, *c*)

$q_{max} = q_{01} = q_{02}$ = Both capacitors are charged up to the same magnitude of charge

$$t_2 > t_1$$
$$R_2C_2 > R_1C_1$$
$$q_{01} = C_1V_1 = q_{02} = C_2V_2$$
$$C_1 \neq C_2$$

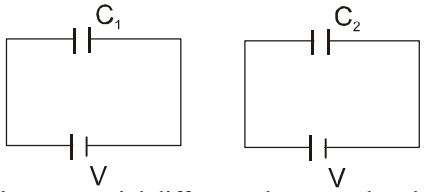
So $V_1 \neq V_2$.

22. (*b*, *d*) During decay of charge in RC circuit

$$I = I_0e^{-t/RC}$$

where $I_0 = \frac{q_0}{RC}$

when $t = 0, I = I_0 = \frac{q_0}{RC}$



Since potential difference between the plates is same initially therefore I same in both the cases at t = 0 and is equal to

$$I = \frac{q_0}{RC} = \frac{V}{R}$$

Also $q = q_0e^{-t/RC}$. When $q = \frac{q_0}{2}$ then $\frac{q_0}{2} = q_0e^{-t/RC}$

$\Rightarrow e^{+t/RC} = 2$.

$$\frac{t}{RC} = \ell n2$$

$\Rightarrow t = RC \log_e 2$

$\Rightarrow t \propto C$. Therefore time taken for the first capacitor (1μF) for discharging 50% of Initial charge will be less.

(*b*), (*d*) are the correct options.

23. (*a*) **24.** (*c*) **25.** (*a*)

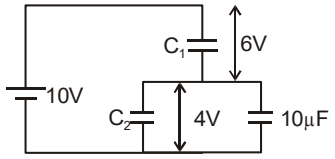
(23 to 25)

When $C_3 = \infty$, there will be no charge on C_2



As $V_1 = 10$ V therefore V = 10 V

From graph when $C_3 = 10\ \mu F$, $V_1 = 6$ V

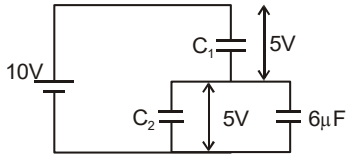


Charge on C_1 = Charge on C_2 + Charge on C_3

$6C_1 = 4C_2 + 40\ \mu C$ .... (1)

Also when $C_3 = 6\ \mu F$, $V_1 = 5V$

Again using charge equation



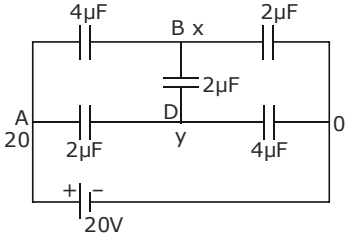
$5C_1 = 5C_2 + 30\ \mu C$ ....(2)

Solving (1) and (2)

$C_1 = 8\ \mu F$

$C_2 = 2\ \mu F$.

26. (*b*, *c*) Let us assume potential at B to be x & D to be y.



$(x-20)4 + (x-y)2 + 2x = 0$

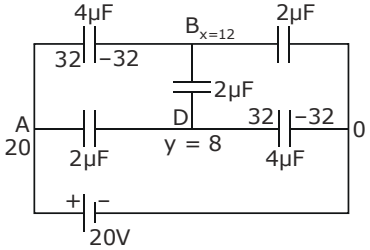
$4x - y = 40$ .....(1)

$2(y-x) + (y-20)2 + y(4) = 0$

$\Rightarrow 4y - x = 20$ .....(2)

Solving (1) and (2)

$x = 12$; $y = 8$



$V_B - V_D = 12 - 8 = 4 > 0$

27. (a, b, c, d)

(a) As from figure $V_A = 20V$

(b) $4(V_A - V_B) + 2(V_D - V_B)$
$= 4(20-12) + 2(8-12)$
$= 32 - 8 = 24 = 2V_B$

(c) $2(V_A - V_D) + 2(V_B - V_D)$
$= 2(20-8) + 2(12-8)$
$= 24 + 8 = 32 = 4V_D$

(d) $V_B + V_D = 12 + 8 = 20 = V_A$

28. (b, c) $V_B = 12 \quad V_D = 8$

29. (c) $q_1 = 4(20-12) = 32\mu C$
$q_2 = 2(20-8) = 24\mu C$
$q_3 = 2(12-8) = 8\mu C$

30. (a) p (b) r (c) q (d) p

The initial charge on capacitor = $CV_i = 2 \times 1\mu C = 2\mu C$

The final charge on capacitor = $CV_f = 4 \times 1\ \mu C = 4\mu C$

$\therefore$ Net charge crossing the cell of emf 4V is
$q_f - q_i = 4 - 2 = 2\ \mu C$

The magnitude of work done by cell of emf 4V is
$W = (q_f - q_i)\,4 = 8\ \mu J$

The gain in potential energy of capacitor is
$\Delta U = \frac{1}{2}C(V_f^2 - V_i^2) = \frac{1}{2}\ 1 \times [4^2 - 2^2]\ \mu J = 6\ \mu J$

Net heat produced in circuit is $\Delta H = W - \Delta U = 8 - 6 = 2\ \mu J$

31. (a) p, q, s (b) p, r, s (c) p, q (d) p, r

(a) For potential difference across each cell to be same

$$E_1 - ir = E_2 + ir \text{ or } i = \frac{E_1 - E_2}{2r} \quad \left(< \frac{E_1 - E_2}{2r+R}\right)$$

Hence potential difference across both cells cannot be same.

Cell of lower emf charges up.

For potential difference across cell of lower emf to be zero

$E_2 + ir = 0$ which is not possible.

Current in the circuit cannot be zero $\quad \because E_1 \neq E_2$.

(b) For potential difference across each cell to be same

$E_1 - ir = E_2 - ir$ which is not possible

No cell charges up.

For potential difference across cell of lower emf to be zero

$E_2 - ir = 0$ and $E_1 - i(r+R) = 0$

or $\frac{E_1}{r+R} = \frac{E_2}{r}$ which is possible. $\quad \because E_1 > E_2$.

Current in the circuit cannot be zero.

(c) Situation is same as in (a) except current decreases from $\frac{E_1 - E_2}{2r+R}$ to zero.

Hence the only option that shall changes is 'current shall finally be zero.'

(d) Situation is same as in (b) except current decreases from $\frac{E_1 + E_2}{2r+R}$ to zero.

Hence the only option that shall changes is 'current shall finally be zero.'

32. [5.2]

33. $C_1 = \frac{A\varepsilon_0 k_1}{d_1}$ & $C_2 = \frac{A\varepsilon_0 k_2}{d_2} \Rightarrow \frac{C_1}{C_2} = \frac{8}{5}$

Series $v = \frac{8}{13}V$

$$E = \frac{V}{l} = \frac{\frac{8}{13}V}{0.5 \times 10^{-3}} = 6.4 \times 10^6$$

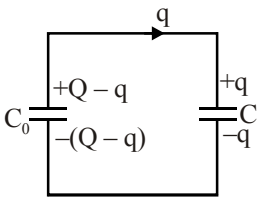
$V = 5200$ Volt

34. [0030] $\frac{q}{C} = \frac{Q-q}{C_0} \Rightarrow q\left(\frac{C+C_0}{CC_0}\right) = \frac{Q}{C_0}$

$$\therefore \quad \frac{q}{C} = \frac{Q}{C+C_0} = \frac{C_0V_0}{C+C_0}$$

$$\Rightarrow \frac{10}{C+10} \times 12 = 3$$

$\therefore$ $C = 30$



35. [0025] $F = \frac{Q^2}{2A\in_0} = \frac{c^2v^2}{2A\in_0} = \frac{\in_0 A}{2d^2}v^2$

$$= \frac{8.85 \times 10^{-12} \times 17.7 \times 10^{-4} \times 500^2}{2 \times (8.85 \times 10^{-3})^2}$$

$$= \frac{10^{-16} \times 25 \times 10^4}{10^{-6}} = 25\mu N$$

36. [0119] $Q = CV$

$$V = -\int_{-3}^{4} E\,dx = -20\int_{-3}^{4}\left(x^2 + \frac{4}{3}\right)dx$$

$$V = -2Q\left[\frac{x^3}{3} + \frac{4x}{3}\ \Big|_{-3}^{4}\right]$$

$$V = -2Q\left[\frac{1}{3}[64+27] + \frac{4}{3}[7]\right]$$

$$\frac{Q}{C} = 3Q\left[\frac{119}{3}\right]$$

$$\frac{1}{C} = 119\ F^{-1}$$

JEE Mains & Advanced Past Years Questions

JEE MAIN PREVIOUS YEAR'S

1. (*c*) $V_{4\mu F} = \frac{12}{4+12} \times 8 = 6V$

$V_{9\mu F} = 2V$

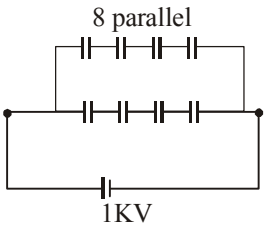
$E = \frac{K(Q_1 + Q_2)}{R^2}$

$= \frac{9\times10^9(4\times10^{-6}\times6 + 9\times10^{-6}\times2)}{(30)^2} = 420\,N/C$

2. (*b*) To hold 1 KV potential difference minimum four capacitors are required in series

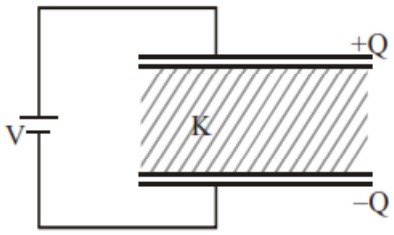
$\Rightarrow C_1 = \frac{1}{4}$ for one series.

So for Ceq to be 2μF, 8 parallel combinations are required.



$\Rightarrow$ Minimum no. of capacitors = 8 × 4 = 32

3. (*d*)



$Q = (kC)V$

$= \left(\frac{5}{3}\times90pF\right)(20\,V)$

$= 3000\,pC$

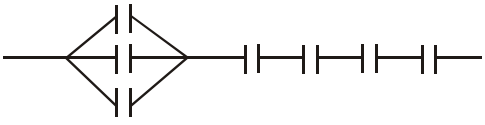
$= 3nC$

induced charges on dielectric

$Q_{ind} = Q\left(1-\frac{1}{K}\right) = 3nC\left(1-\frac{3}{5}\right) = 1.2\,nC$

4. (*b*) $C_{eq} = \frac{6}{13}\mu F$

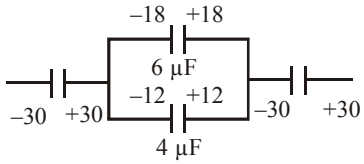
Therefore three capacitors most be in parallel to get 6 in



$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$

$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F$

5. (*d*)



$4V + 6V = 30$

$\Rightarrow V = 3$

6. (Bonus) $C_{eq} = \frac{C_1C_2}{C_1+C_2} + \frac{C_3C_4}{C_3+C_4}$, $C_1 = K_1C$, $C = \frac{\epsilon_0 A/2}{d/2}$

Similarly

$C_2 = K_2C$

$C_3 = K_3C$

$C_4 = K_4C$

$K_{eq}\left(\frac{\epsilon_0 A}{d}\right) = \left(\frac{K_1K_2}{K_1+K_2} + \frac{K_3K_4}{K_3+K_4}\right)\frac{\epsilon_0 A/2}{d/2}$

$\Rightarrow K_{eq} = \frac{K_1K_2}{K_1+K_2} + \frac{K_3K_4}{K_3+K_4}$

No option is correct

7. (*b*) $\theta \propto i$, Let $R_G = R$

$\Rightarrow i_1 = \frac{V}{220+R} = k\times\theta_0$

$\Rightarrow i_2 = \frac{V}{220 + \frac{5\times R}{5+R}} \times \frac{5}{R+5} = k\times\frac{\theta_0}{5}$

$\Rightarrow \frac{1}{220\times(5+R)+5R} \times \frac{5}{1} = \frac{1}{(220+R)\times5}$

$\Rightarrow \frac{1}{45R+220} = \frac{1}{5\times(220+R)}$

$\Rightarrow R = 22\,\Omega$

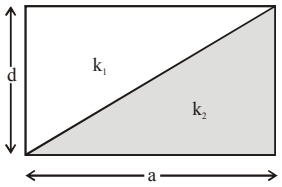
8. (*b*) $W = \frac{Q^2}{2C} - \frac{Q^2}{2CK}$

$= \frac{Q^2}{2C}\left[1-\frac{1}{K}\right]$

$= \frac{1}{2}\times12\times100pJ\left(1-\frac{1}{6.5}\right)$

$= \frac{12\times100\times11}{2\times13}pJ = 507.69pJ$

9. (*a*) $\Rightarrow C_{eq} = \dfrac{\epsilon_0\, k_1 k_2 a^2 \ln\dfrac{k_1}{k_2}}{(k_1 - k_2)d}$



$\Rightarrow C_{eq} = \dfrac{\epsilon_0\, ka^2 \ln\dfrac{1}{k}}{d(1-k)}$

as $k_1 = 1, k_2 = k$

$\Rightarrow C_{eq} = \dfrac{\epsilon_0\, ka^2}{d(k-1)} \ln k$

10. (*a*) $C_{eq} = 0.5\ \mu F = \dfrac{\left(1+\dfrac{4}{3}\right)C}{\dfrac{7}{3}+C} = \dfrac{7C}{\dfrac{7}{3}+C} = \dfrac{1}{2}$

$\Rightarrow \dfrac{14C}{3} = C + \dfrac{7}{3}$

$C = \dfrac{7}{11}$

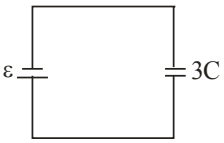
11. (*c*) $E = \dfrac{q}{A\varepsilon_0}$

$\Rightarrow q = EA\varepsilon_0$

$\Rightarrow q = 100 \times 1 \times 8.85 \times 10^{-12}$

$\Rightarrow q = 8.85 \times 10^{-10}\,C$

12. (*a*) Current = slope of q – T graph = 0. [at t = 4 sec]

13. (*b*) 

Before switch is shifted :

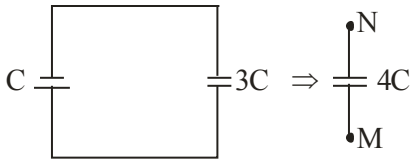
Energy stored,

$U_i = \dfrac{1}{2}CE^2$

and charge stored,

$Q = CE$

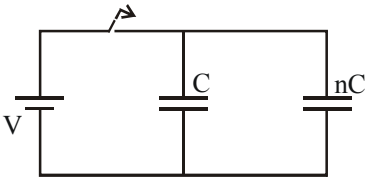
After switch is shifted :



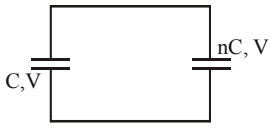
$V_M - V_N = \dfrac{Q}{4C} = \dfrac{CE}{4C} = \dfrac{E}{4}$

Energy stored, $U_f = \dfrac{1}{2}(4C)\left(\dfrac{E}{4}\right)^2 = \dfrac{1}{8}CE^2$

Energy dissipated $= U_i - U_f = \dfrac{3}{8}CE^2$

14. (*c*) 

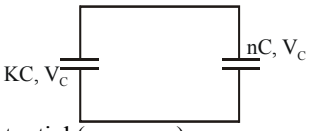
After fully charging, battery is disconnected



Total charge of the system = CV + nCV

= (n + 1)CV

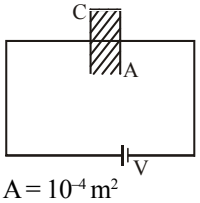
After the insertion of dielectric of constant K



New potential (common)

$V_C = \dfrac{\text{total charge}}{\text{total capacitance}}$

$= \dfrac{(n+1)CV}{KC + nC} = \dfrac{(n+1)V}{K+n}$.

15. (*d*) 

$A = 10^{-4}\,m^2$

$E_{max} = 10^6\,V/m$

$C = 15\ \mu F$

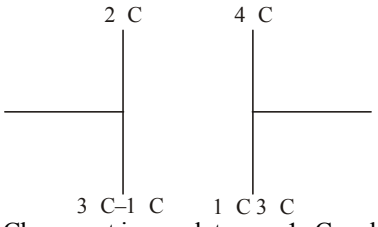
$C = \dfrac{k\varepsilon_0 A}{d}$

$\dfrac{Cd}{\varepsilon_0 A} = k$

$k = \dfrac{15 \times 10^{-12} \times 500 \times 10^{-6}}{8.86 \times 10^{-12} \times 10^{-4}}$

$= \dfrac{15 \times 5}{8.86} = 8.465$

$k \approx 8.5$

16. (*d*) 

Charges at inner plates are 1μC and –1μC

∴ Potential difference across capacitor

$= \dfrac{q}{c} = \dfrac{1\mu C}{1\mu F} = \dfrac{1 \times 10^{-6}\,C}{1 \times 10^{-6}\,\text{Farad}} = 1V$

17. (*a*) Work done = ΔU

$= U_f - U_i$

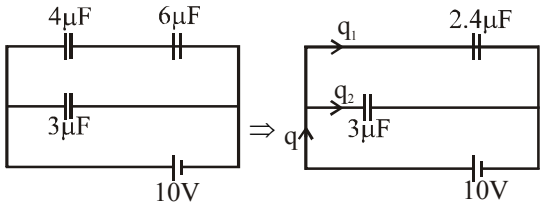
$= \dfrac{q^2}{2C_f} - \dfrac{q^2}{2C_i}$

$= \dfrac{\left(5 \times 10^{-6}\right)^2}{2}\left(\dfrac{1}{2 \times 10^{-6}} - \dfrac{1}{5 \times 10^{-6}}\right)$

$= \dfrac{15}{4} \times 10^{-6}$

$= 3.75 \times 10^{-6}\,J$

18. (*b*)



So total charge flow = q = 5.4 μF × 10V
= 54 μC

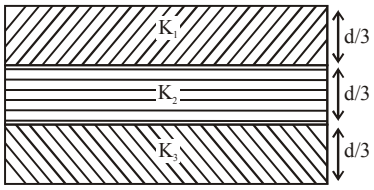
The charge will be distributd in the ratio of capacitance

$\Rightarrow \frac{q_1}{q_2} = \frac{2.4}{3} = \frac{4}{5}$

$\therefore$ 9X = 54 μC $\Rightarrow$ X = 6 μ C

$\therefore$ charge of 4 μF capacitor will be 4X = 4 × 6 μ C
= 24 μC

19. (*a*) I.



$$C_1 = \frac{3\varepsilon_0 AK_1}{d}$$

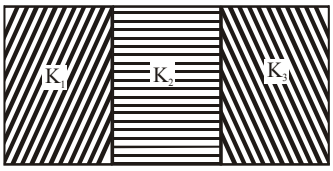
$$C_2 = \frac{3\varepsilon_0 AK_2}{d}$$

$$C_3 = \frac{3\varepsilon_0 AK_3}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow C_{eq} = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3k_1)} \quad \text{....(i)}$$

II.



$$C_1 = \frac{\varepsilon_0 K_1 A}{3d}$$

$$C_2 = \frac{\varepsilon_0 K_2 A}{3d}$$

$$C_3 = \frac{\varepsilon_0 K_3 A}{3d}$$

$$C'_{eq} = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \quad \text{.....(ii)}$$

Now,

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}C_{eq}.V^2}{\frac{1}{2}C'_{eq}V^2} = \frac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}$$

20. (*d*) As q = CV

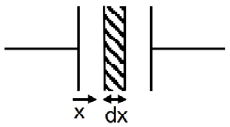
Hence slope of graph will give capacitance. Slope will be more in parallel combination. Hence capacitance in parallel should be 50μF & in series combination must be 8μF.

Only in option 40 μF & 10 μF

$C_{parallel} = 40 + 10 = 50\ \mu F$

$C_{series} = \frac{40 \times 10}{40 + 10} = 8\ \mu F$

21. (*b*) Capacitance of element $= \frac{k \in_0 A}{dx}$



Capacitance of elements $C' = \frac{K(1+\alpha x)\varepsilon_0 A}{dx}$

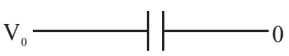
$$\sum \frac{1}{C'} = \int_0^d \frac{dx}{K\varepsilon_0 A(1+\alpha x)}$$

Given – αd << 1

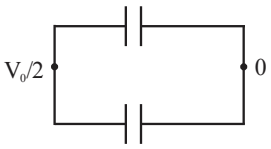
$$\frac{1}{C} = \frac{1}{K\varepsilon_0 A\alpha}\left(\alpha d - \frac{\alpha^2 d^2}{2}\right)$$

$$\frac{1}{C} = \frac{1}{K\varepsilon_0 A}\left(1 - \frac{\alpha d}{2}\right)$$

$$C = \frac{K\varepsilon_0 A}{d}\left(1 + \frac{\alpha d}{2}\right)$$



22. [6]



$V_0 = 20\ V$

Heat lost = $U_i - U_f$

$$= \frac{1}{2}CV_0^2 - 2\left[\frac{1}{2}C\left(\frac{V_0}{2}\right)^2\right]$$

$$= \frac{CV_0^2}{4}$$

$$= \frac{(60 \times 10^{-12})(20)^2}{4}\ J$$

$= 6 + 10^{-9}\ J = 6\ nJ$

23. (*d*) C_1

C_2

1V

Given $C_1 + C_2 = 10\mu F$ .....(i)

$$4\left(\frac{1}{2}C_1V^2\right) = \frac{1}{2}C_2V^2$$

$\Rightarrow 4C_1 = C_2$ .....(ii)

from equation (i) & (ii)

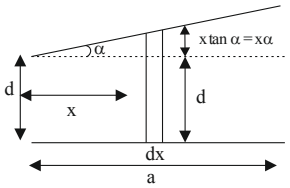
$C_1 = 2\mu F$

$C_2 = 8\mu F$

If they are in series

$$C_{eq.} = \frac{C_1C_2}{C_1 + C_2} = 1.6\ \mu F$$

24. (*a*)



$$dc = \frac{\varepsilon_0 a dx}{d + \alpha x}$$

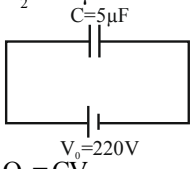
$$\Rightarrow \quad c = \frac{\varepsilon_0 a}{\alpha}\left[\text{In}(d + \alpha x)\right]_0^a$$

$$= \frac{\varepsilon_0 a}{\alpha}\text{In}\left(1 + \frac{\alpha a}{d}\right) \approx \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

25. (*c*) $V_{\text{final}} = \dfrac{C_1 V}{C_1 + C_2}$

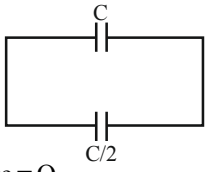
$C_2 = 15\,\mu F$

26. (*d*)



$Q_0 = CV_0$

Now battery is disconnected and a capacitor of $\dfrac{C}{2}$ us connected to C then



$\text{Total Charge} = Q_0$

$$C_{eq} = \frac{3C}{2}$$

$$\therefore \quad \Delta E = \frac{Q_0^2}{2C} - \frac{2Q_0^2}{3C \times 2} \Rightarrow \Delta E = \frac{Q_0^2}{2C}\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad \Delta E = \frac{1}{6} \times 220 \times 220 \times 5 \times 10^{-6}$$

$$= 4 \times 10^{-2}\,\text{J}$$

$\therefore \quad X = 4$

27. (*b*) $Q_3 = C_3 V_3$

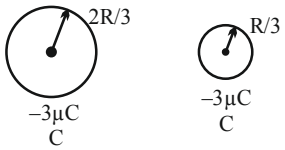
$V_3 = 20V$

$Q_3 = 160\mu C$

$Q = Q_2 + Q_3$

$Q_2 = 590\,\mu C$

28. (*c*)



$C_{eq} = 3C, Q_{\text{total}} = 12 - 3$

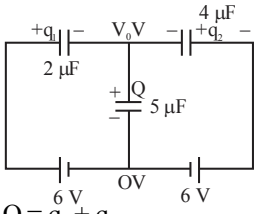
$= 9\mu C$

$$\therefore \quad q_1 = \frac{2C}{3C} \times 9 = 6\mu C$$

$q_2 = 3\mu C$

29. (*c*) $\Delta U = \dfrac{1}{2}\dfrac{C_1 C_2}{C_1 + C_2} V_0^2 = \dfrac{1}{6} C V_0^2$

30. (*a*)



$Q = q_1 + q_2$

$5V_0 = (6 - V_0) + 4(6 - V_0)$

$$V_0 = \frac{36}{11}\text{V}$$

$$Q = 5V_0 = \frac{180}{11}\mu C$$

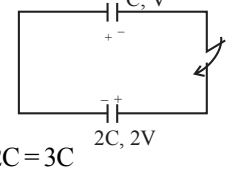
31. (*d*) $U = \dfrac{1}{2}CV^2$

$$\frac{U_1}{U_2} = 2 \Rightarrow \frac{C_1}{C} = 2$$

$$\frac{kx + (\ell - x)}{\ell} = 2$$

$$x = \frac{\ell}{3}$$

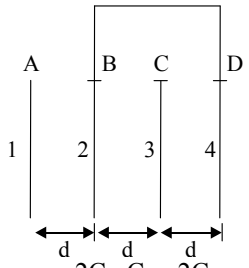
32. (*a*) $V_{\text{common}} = \dfrac{4CV - CV}{3C} = V$



$C_{eq} = C + 2C = 3C$

$\therefore \quad U_f = \dfrac{1}{2} \times (3C) \times V^2 = \dfrac{3}{2}CV^2$

33. (*b*)



$$C_{e_e} = \frac{2C \times C}{2C + C} = \frac{2C}{3}$$

$$= \frac{2}{3}\frac{\epsilon A}{d} = \frac{2}{3} \times \frac{G_0}{d} \times \left(2 \times \frac{3}{2}\right) = \frac{2\varepsilon_0}{d}$$

34. $U_i = \dfrac{1}{2}cu^2 = \dfrac{1}{2} \times 14 \times (12)^2\,\text{pJ} = 1008\text{pJ}$

$$\Rightarrow U_f = \frac{Q^2}{2kC} = \frac{(14 \times 12)^2}{2 \times \pi \times 14} = 144\text{pJ}$$

oscillating energy $= U_i - U_f = 1008 - 144 = 864\text{pJ}$

35. (*c*) $C_1 = \dfrac{K\varepsilon_0 A}{d/2}; C_2 = \dfrac{\varepsilon_0 A}{d/2}$

$$\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K\varepsilon_0 A} + \frac{d}{2\varepsilon_0 A}$$

$$\Rightarrow \frac{1}{C} = \frac{d}{2\varepsilon_0 A}\left(\frac{K+1}{K}\right)$$

$$\Rightarrow C = \frac{2\varepsilon_0, AK}{d(K+1)} = \frac{2 \times 2 \times 3.2}{1 \times 4.2}\varepsilon_0 = 3.04\varepsilon_0$$

JEE ADVANCED PREVIOUS YEAR'S

1. (*c*) $q_3 = \frac{C_3}{C_2 + C_3}.Q$

$q_3 = \frac{3}{3+2} \times 80 = \frac{3}{5} \times 80$

$= 48\,\mu C$

2. (*b*, *d*)

When switch S_1 is released charge on C_1 is $2CV_0$ (on upper plate)

When switch S_2 is released charge on C_1 is CV_0 (on upper plate) and charge on C_2 is CV_0 (on upper plate)

When switch S_3 is released charge on C_1 is CV_0 (on upper plate) and charge on C_2 is $-CV_0$ (on upper plate)

3. (*a*, *d*) $C = \frac{K\varepsilon_0 A}{3d} + \frac{2\varepsilon_0 A}{3d}$

$C_1 = \frac{K\varepsilon_0 A}{3d}$

$\frac{C}{C_1} = \frac{2+K}{K}$ Ans. (D)

$E_1 = E_2 = \frac{V}{d}$

$\Rightarrow \frac{E_1}{E_2} = 1$ Ans. (A)

$Q_1 = C_1 V = \frac{K\varepsilon_0 A}{3d} V$

$Q_2 = C_2 V = \frac{2\varepsilon_0 A}{3d} V$

$\Rightarrow \frac{Q_1}{Q_2} = \frac{K}{2}$

4. (*b*) The line charge & cylinder will behave as capacitor filled with conductor i.e. resistance. It will be like a discharging *RC* circuit.

Hence, (*b*)

5. (*a*, *b*, *c*, *d*)

6. (*d*) $E_C = \frac{1}{2}CV_0^2$; $E_D = V_0 CV_0 - \frac{1}{2}CV_0^2$

$= \frac{1}{2}CV_0^2$

$\therefore E_C = E_D$

7. (*b*) $E_{D_1} = \frac{V_0}{3}\frac{CV_0}{3} - \frac{1}{2}C.\left(\frac{V_0}{3}\right)^2 = \frac{CV_0^2}{9} - \frac{CV_0^2}{18} = \frac{CV_0^2}{18}$

$E_{D_2} = \frac{2V_0}{3}\left[\frac{2CV_0}{3} - \frac{CV_0}{3}\right] - \left[\frac{1}{2}C\left(\frac{2V_0}{3}\right)^2 - \frac{1}{2}C.\left(\frac{V_0}{3}\right)^2\right]$

$= \frac{2V_0}{3}\left[\frac{CV_0}{3}\right] - \frac{1}{2}C\left[\frac{4V_0^2}{9} - \frac{V_0^2}{9}\right]$

$= \left(\frac{2}{9} - \frac{1}{2\times 9}\times 3\right)CV_0^2 = \left(\frac{2}{9} - \frac{1}{6}\right)CV_0^2 = \left(\frac{12-9}{9\times 6}\right)CV_0^2$

$E_{D_2} = \frac{1}{18}CV_0^2$

$E_{D_3} = V_0\left[CV_0 - \frac{2CV_0^2}{3}\right] - \left[\frac{1}{2}CV_0^2 - \frac{1}{2}C\left(\frac{2V_0}{3}\right)^2\right]$

$= \frac{1}{3}CV_0^2 - \frac{1}{2}CV_0^2\left[1 - \frac{4}{9}\right]$

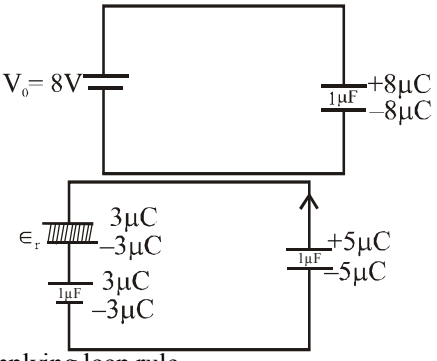
$= \left(\frac{1}{3} - \frac{5}{18}\right)CV_0^2 = \left(\frac{6-5}{18}\right)CV_0^2 = \left(\frac{1}{18}\right)CV_0^2$

Total $= \left(\frac{1}{18} + \frac{1}{18} + \frac{1}{18}\right)CV_0^2$

$= \frac{3}{18}CV_0^2$

$E_D = \frac{3}{9}\left[\frac{1}{2}CV_0^2\right] = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

8. [1.50] $V_0 = 8V$



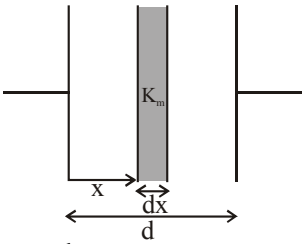
Applying loop rule

$\frac{5}{1} - \frac{3}{\epsilon_r} - \frac{3}{1} = 0$

$\frac{3}{\epsilon_r} = 2$

$\epsilon_r = 1.50$

9. (1.00)



$\delta = dx = \frac{d}{N}$ & $\frac{m}{N} = \frac{x}{d}$

$K_m = K\left(1 + \frac{m}{N}\right)$

$\Rightarrow K_m = K\left(1 + \frac{x}{d}\right)$

$C' = \frac{K_m A \epsilon_0}{dx}$

$\frac{1}{C_{eq}} = \int_0^d \frac{dx}{K_m A \epsilon_0} = \frac{1}{KA\epsilon_0}\int_0^d \frac{dx}{\left(1 + \frac{x}{d}\right)}$

$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{KA\epsilon_0}\left[\ell n\left(1 + \frac{x}{d}\right)\right]_0^d$

$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{KA\epsilon_0}\left[\ell n 2 - \ell n(1)\right]$

$\Rightarrow C_{eq} = \frac{KA\epsilon_0}{d\ell n 2} \Rightarrow \alpha = 1$

3 Current Electricity

Topicwise Questions

1. (*b*) $I = neAv_d$

2. (*b*) Order of drift velocity = 10^{-4} m/sec = 10^{-2} cm/sec

3. (*b*) Density of Cu = 9×10^3 kg/m^3 (mass of 1 m^3 of Cu)

$\because$ 6.0×10^{23} atoms has a mass = 63×10^{-3} *kg*

$\therefore$ Number of electrons per m^3 are

$$= \frac{6.0 \times 10^{23}}{63 \times 10^{-3}} \times 9 \times 10^3 = 8.5 \times 10^{28}$$

Now drift velocity $= v_d = \frac{i}{neA}$

$$= \frac{1.1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times \pi \times (0.5 \times 10^{-3})^2}$$

$= 0.1 \times 10^{-3}$ m/sec

4. (*a*) $v_d = \frac{e}{m} \times \frac{V}{l} \tau$ or $v_d = \frac{e}{m} \cdot \frac{El}{l} \tau$ (Since $V = El$)

$\therefore$ $v_d \propto E$

5. (*b*) $R \propto \frac{1}{A} \Rightarrow R \propto \frac{1}{A} \propto \frac{1}{d^2}$

[d = diameter of wire]

6. (*b*) In the absence of external electric field mean velocity of free electron (V_{rms}) is given by $V_{rms} = \sqrt{\frac{3KT}{m}}$

$\Rightarrow V_{rms} \propto \sqrt{T}$.

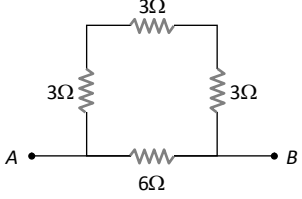
7. (*a*) $\frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4 \Rightarrow \frac{R}{R_2} = \left(\frac{nr}{r}\right)^4 \Rightarrow R_2 = \frac{R}{n^4}$.

8. (*d*) $R = 91 \times 10^2 \approx 9.1\,k\Omega$.

9. (*b*) Resistance of parallel group $= \frac{R}{2}$

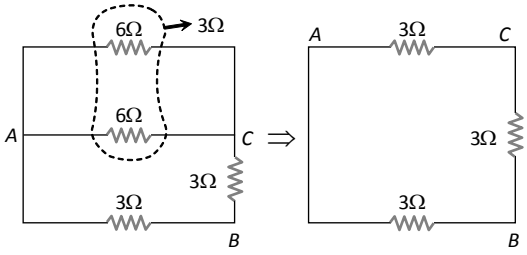
$\therefore$ Total equivalent resistance $= 4 \times \frac{R}{2} = 2R$.

10. (*d*) The circuit reduces to



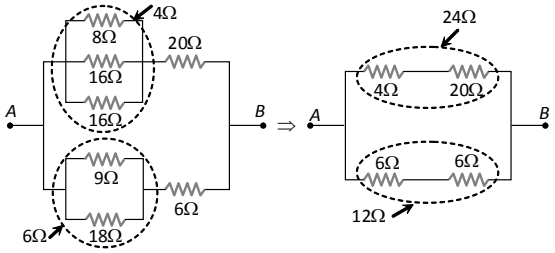
$$R_{AB} = \frac{9 \times 6}{9 + 6} = \frac{9 \times 6}{15} = \frac{18}{5} = 3.6\,\Omega$$

11. (*b*) Given circuit is equivalent to



So the equivalent resistance between points *A* and *B* is equal to $R = \frac{6 \times 3}{6 + 3} = 2\,\Omega$.

12. (*b*)



$$R_{AB} = \frac{24 \times 12}{(24 + 12)} = 8\,\Omega$$

13. (*d*) Three resistances are in parallel.

$\therefore$ $\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$

The equivalent resistance $R' = \frac{R}{3}\,\Omega$.

14. (*c*)

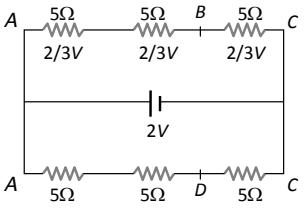
15. (*a*) $R_1 = \frac{\rho_1 l_1}{A}$ and $R_2 = \frac{\rho_2 l_2}{A}$.

In series $R_{eq} = R_1 + R_2$

$$\frac{\rho_{eq.}(l_1 + l_2)}{A} = \frac{\rho_1 l_1}{A} + \frac{\rho_2 l_2}{A} \Rightarrow \rho_{eq} =$$

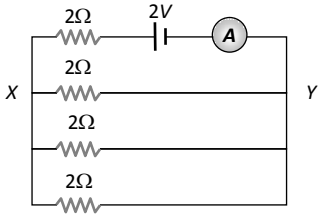
$$\frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2} .$$

16. (*c*) The given circuit can be redrawn as follows



For identical resistances, potential difference distributes equally among all. Hence potential difference across each resistance is $\frac{2}{3}$V and potential difference between *A* and *B* is $\frac{4}{3}$V.

17. (*b*) Resistance across XY $= \frac{2}{3}\Omega$



Total resistance

$= 2 + \frac{2}{3} + \frac{8}{3}\Omega$

Current through ammeter $= \frac{2}{8/3} = \frac{6}{8} = \frac{3}{4}$A

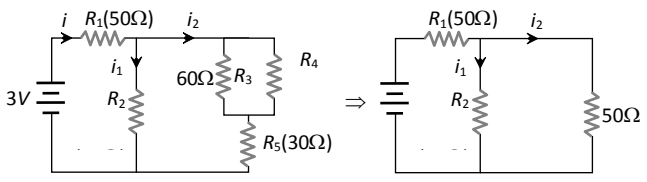
18. (*b*) Current in the given circuit $i = \frac{50}{(5+7+10+3)} = 2A$

Potential difference between A and B $V_A - V_B = 2 \times 12$

$\Rightarrow V_A - 0 = 24V \Rightarrow V_A = 24V.$

19. (*c*) Given circuit is a balanced Wheatstone bridge i.e. potential difference between *B* and *D* is zero. Hence, no current flows between *B* and *D*.

20. (*a*) Equivalent resistance of the given network $R_{eq} = 75\Omega$.



21. (*b*) For no current through galvanometer, we have

$$\left(\frac{E_1}{500+X}\right)X = E \Rightarrow \left(\frac{12}{500+X}\right)X = 2 \Rightarrow X = 100\,\Omega.$$

22. (*a*) Applying Kirchhoff law

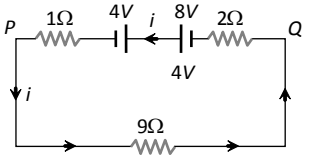
$(2+2) = (0.1+0.3+0.2)i \Rightarrow i = \frac{20}{3}$A

Hence potential difference across *A*

$= 2 - 0.1 \times \frac{20}{3} = \frac{4}{3}$V (less than 2V).

and similarly across B will be zero.

23. (*a*) Applying Kirchoff's voltage law in the given loop.



$-2i + 8 - 4 - 1 \times 9i = 0 \quad -2i + 8 - 4 - 1 \times i - 9i = 0$

$\Rightarrow i = \frac{1}{3}$A

Potential difference across PQ $= \frac{1}{3} \times 9 = 3V$.

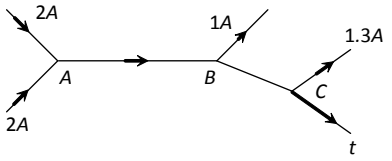
24. (*a*) Given problem is the case of mixed grouping of cells

So total current produced $i = \frac{nE}{R + \frac{nr}{m}}$

Here $m = 100, n = 5000, R = 500\,\Omega$
$E = 0.15$ V and $r = 0.25\Omega$

$\Rightarrow i = \frac{5000 \times 0.15}{500 + \frac{5000 \times 0.25}{100}} = \frac{750}{512.5} \approx 1.5$ A.

25. (*a*) According to Kirchhoff's first law
At junction *A*, $i_{AB} = 2 + 2 = 4A$
At junction *B*,



At junction *C*, $i = i_{BC} - 1.3 = 3 - 1.3 = 1.7$ amp.

26. (*d*) Zero (No potential difference across voltmeter).

27. (*c*) Reading of voltmeter

$= E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = \frac{18 \times 1 + 12 \times 2}{1+2} = 14V.$

28. (*a*) In meter bridge experiment, it is assumed that the resistance of the L shaped plate is negligible, but actually it is not so. The error created due to this is called, end error. To remove this the resistance box and the unknown resistance must be interchanged and then the mean reading must be taken.

29. (*c*) To convert a galvanometer into an ammeter a low value resistance is to be connected in parallel to it called shunt.

30. (*a*) Potential gradient $= \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$

$= \frac{2}{(15+5+0)} \times \frac{5}{1} = 0.5 \frac{V}{m} = 0.005 \frac{V}{cm}$.

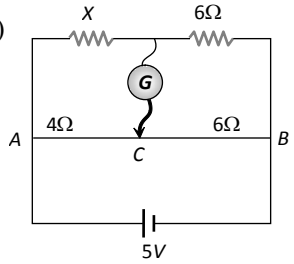
31. (*d*) By using $R = \frac{V}{i_g} - G$

$\Rightarrow R = \frac{100}{5 \times 10^{-3}} - 5 = 19{,}995\Omega$

32. (*c*) By Wheatstone bridge, $\frac{R}{80} = \frac{AC}{BC} = \frac{20}{80} \Rightarrow R = 20\Omega$.

33. (*a*) $E \propto l$ (balancing length)

34. (*c*)



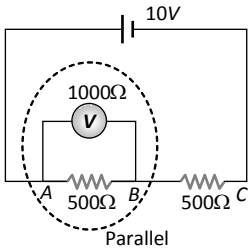
Resistance of the part AC

$R_{AC} = 0.1 \times 40 = 4\Omega$ and $R_{CB} = 0.1 \times 60 = 6\Omega$

In balanced condition $\frac{X}{6} = \frac{4}{6} \Rightarrow X = 4\Omega$

Equivalent resistance $R_{eq} = 5\Omega$ so current drawn from battery $i = \frac{5}{5} = 1A$.

35. (*d*) Resistance between A and B $= \frac{1000 \times 500}{(1500)} = \frac{1000}{3}$



So, equivalent resistance of the circuit

$R_{eq} = 500 + \frac{1000}{3} = \frac{2500}{3}$

$\therefore$ Current drawn from the cell

$i = \frac{10}{(2500/3)} = \frac{3}{250} A$

Reading of voltmeter i.e.
potential difference across 500Ω resistor is 4V.

Learning Plus

1. (*d*) In the presence of an applied electric field ($\vec{E}$) in a metallic conductor. The electrons also move randomly but slowly drift in a direction opposite to $\vec{E}$.

2. (*c*) Given that $v_{d_1} = v$, $v_{d_2} = ?$

We know that

$I = neAv_d$

$\Rightarrow \quad V_d \propto \frac{1}{A} \propto \frac{1}{\frac{\pi d^2}{4}} \propto \frac{1}{d^2}$

$\frac{V_{d_1}}{V_{d_2}} = \frac{(d/2)^2}{d^2} = \frac{1}{4}$

$V_{d_2} = 4V$

3. (*c*) $j = \frac{i}{A}$ current density inversely proportional to area of cross section

4. (*d*) $\frac{R}{\ell} \equiv \frac{R'}{3\ell}$

During stretching volume is constant

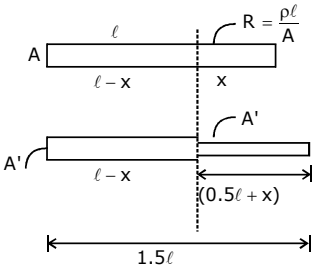
$Al = A'(3l)$

$\Rightarrow A' = A/3$

$\frac{R'}{R} = \frac{\rho\, 3\ell}{A' \frac{\rho\ell}{A}}$, $R' = \frac{3A}{A'} \times R$

Put A' and R from above $R' = R_{new} = 9R = 180\Omega$

5. (*b*) During stretching volume remains constant



R'= 4R

$Ax = A'(0.5\ell + x)$

$$A' = \frac{Ax}{0.5\ell + x} \quad ....(1)$$

$$\Rightarrow \frac{4\rho\ell}{A} = \frac{\rho(\ell - x)}{A} + \frac{\rho(0.5\ell + x)}{A'} \quad ....(2)$$

Put value of A' in equation (2) from equation (1)

$$\Rightarrow \frac{4\rho\ell}{A} = \frac{\rho(\ell - x)}{A} + \frac{\rho(0.5\ell + x)^2}{Ax}$$

$\Rightarrow 4\ell x = \ell x - x^2 + (0.5\ell)^2 + \ell x + x^2$

After solving $x = (1/8)\ell$

6. (*d*) We know that $R = \frac{\rho l}{A}$

$$\left.\begin{aligned} x &= \frac{\rho 4a}{2a^2} = \frac{2\rho}{a} \\ y &= \frac{\rho a}{8a^2} = \frac{\rho}{8a} \\ z &= \frac{\rho(2a)}{4a^2} = \frac{\rho}{2a} \end{aligned}\right\} \text{From } R = \frac{\rho\ell}{A}$$

$x > z > y$

7. (*c*) Given that $v_d' = 2v_d$
$I = neAv_d, A = \pi r^2$

$$I' = neA'v_d', A' = \frac{\pi r^2}{4}$$

$$I' = ne\frac{\pi r^2}{4}v_d'$$

$$I' = ne\,\frac{\pi r^2}{4}.2V_d$$

$I' = I/2$

8. (*b*) $R_1 = R_{01}(1 + \alpha_1 \Delta\theta) = 600(1 + 0.001 \times 30) = 618\,\Omega$
$R_2 = R_{02}(1 + \alpha_2 \Delta\theta) = 300(1 + 0.004 \times 30) = 336\,\Omega$
$R_{eq} = R_1 + R_2 = 618 + 336 = 954\,\Omega$

9. (*c*) $R_{eq} = R_{0eq}(1 + \alpha_{eq}\Delta\theta)$ 954 = 900 (1 + α 30) α =

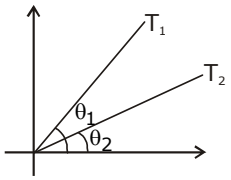
$$\frac{54}{900 \times 30} = \frac{1}{500} \text{ degree}^{-1}$$

10. (*b*) we no that $I = neAv_d$

$$V_d = \frac{I}{neA} \propto \frac{I}{r^2}$$

$$\frac{V_{d_1}}{V_{d_2}} = \left(\frac{I_1}{I_2}\right)\left(\frac{r_2}{r_1}\right)^2 = \left(\frac{4}{1}\right)\left(\frac{2}{1}\right)^2 = 16$$

11. (*b*) $R = \frac{V}{I} \Rightarrow \frac{I}{V} = \frac{1}{R}$

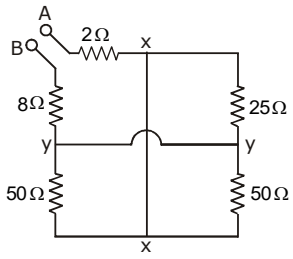


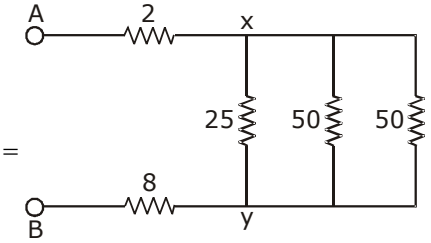
$\because \theta_1 > \theta_2$
$\Rightarrow R_1 < R_2 \Rightarrow T_1 < T_2$

$\because T\uparrow R\uparrow$

12. (*c*) $i = neAV_d$
i is same so
$A\uparrow V_d\downarrow$

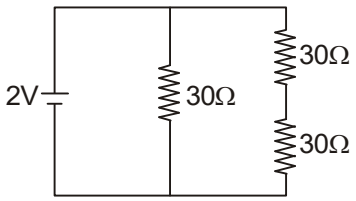
13. (*b*)

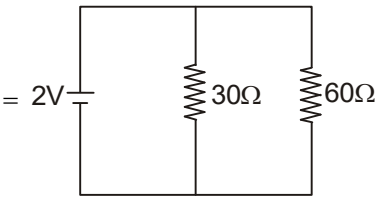


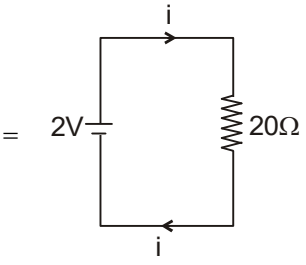


$$R_{eq.} = 2 + \frac{25}{2} + 8 = \frac{45}{2}\Omega$$

14. (*c*) This simplified circuit is shown in the figure.







Therefore, current $i = \frac{2}{20} = \frac{1}{10}$ A

15. (*d*) All resistances are parallel so potential is same

$V = 0.3 \times 20 = 6V$

$i_1 : i_1 : i_3 = \frac{1}{R_1} : \frac{1}{20} : \frac{1}{15}$

$= 60 : 3R_1 : 4R_1$

$\Rightarrow 0.3 = \frac{3R_1}{60 + 7R_1} \times (0.8)$

$\Rightarrow R_1 = 60\,\Omega$

16. (1) $R_{eq} = 2 + \frac{4}{2} + \frac{15}{3} + R_A = 9 + R_A$

$I = \frac{V}{R_{eq}} \Rightarrow 1 = \frac{10}{9 + R_A} \Rightarrow R_A = 1\Omega$

if 4Ω replace by 2Ω resistance then

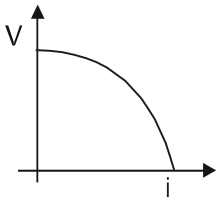
$R_{eq} = 2 + \frac{2}{2} + \frac{15}{3} + 1 = 9\Omega$

$I = \frac{10}{9}$ amp

17. (*d*) Given $r \propto i \Rightarrow r = ki$

$V = E - ir = E - i(ki)$

$V = -i^2k + E$



18. (*b*) (a) $V = E - ir, V < E$

(b) $V = E + ir, V > E$

(c) $V = E$

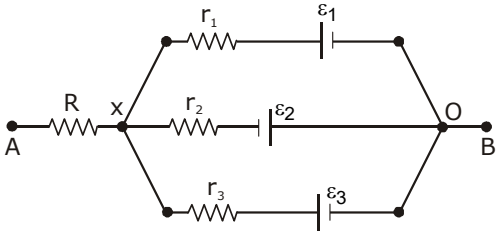
(d) $V = E$

19. (*c*) $E + ir = 12.5$ Volt

$E + (0.5 \times 1) = 12.5$

$E = 12$ volt

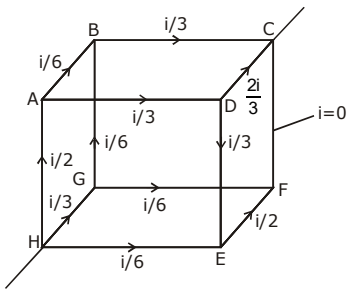
20. (*b*)



$\frac{x - \varepsilon_1}{r_1} + \frac{x - \varepsilon_2}{r_2} + \frac{x - \varepsilon_3}{r_3} = 0$

$x = 2$ volt

21. (*b*)



22. (*a*) $R_{2.5\,W} = \frac{(110)^2}{2.5}\ \Omega$, $R_{100W} = \frac{(110)^2}{100} \Rightarrow R_{2.5} > R_{100}$.

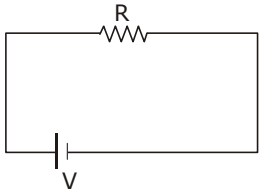
In series current passes through both bulb are same

$P_{2.5} = i^2 R_{2.5}$, $P_{100} = i^2 R_{100}$

$P_{2.5} > P_{100}$ due to $R_{2.5} > R_{100}$ & $\because P_{2.5} > 2.5W$ & $P_{100} < 100$ W (can be verified)

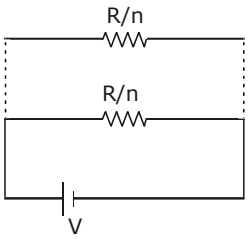
Therefore 2.5 W bulb will fuse

23. (*b*)



Initially $H = \frac{v^2}{R}$

Now after cutting

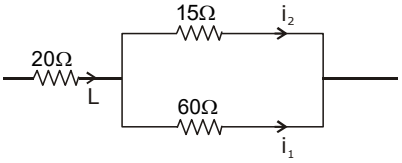


Power in one branch $= \frac{V^2}{R/n} = \frac{nV^2}{R}$

Total power $= \frac{nV^2}{R} + \frac{nV^2}{R} + \ldots = \frac{n^2V^2}{R}$

24. (*b*) As R_{eq} decreases I_{net} increases hence current through X increases but as I_{net} will now be distributed in Y & Z, current in Y decreases.

25. (*b*)



Given $\frac{15 \times i}{75} = 0.75$

Now $i_2 = \frac{60 \times i}{75} = \left[\frac{60 \times 0.75 \times 75}{15}\right] = 3A$

26. (1) $R_{eq} = 10 + \frac{480 \times 20}{480 + 20} = 10 + \frac{96}{5} = \frac{146}{5}$

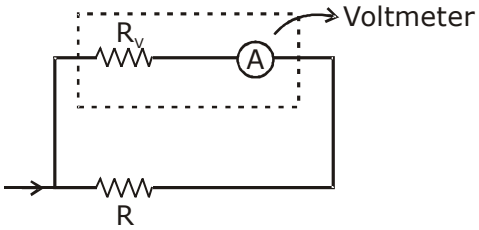
current passes through the battery.

$I = \frac{20 \times 5}{146} = \frac{100}{146} = \frac{50}{73}$ amp.

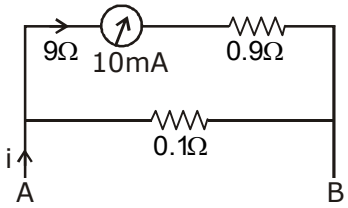
27. (*d*) $\frac{6}{R} = \frac{\ell}{x - \ell}$

$\frac{6}{R} = \frac{30}{20} \Rightarrow R = 4\Omega$

28. (*c*) High resistance in series



29. (*c*) Given for ammeter $i = 10^{-3}A$, $R = 9\Omega$
for given condition circuit shown like



$10 \times 10^{-3} = \frac{0.1}{10} \times i \Rightarrow i = 1$ Ampere

30. (*b*) Voltmeter must be connected in parallel and Ammeter in series with the resistance in circuit.

31. (*a*) $R_1 \times 60 = R_2 \times 40$ ....(1)

$R_1 \times 50 = \frac{R_2 \times 10}{R_2 + 10} \times 50$....(2)

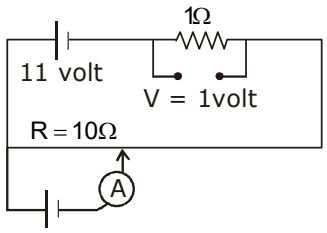
Devide (2) by (1) $\frac{50R_1}{60R_1} = \frac{10R_2 \times 50}{\frac{R_2 + 10}{R_2 \times 40}}$

$R_2 = 5\Omega, R_1 = \frac{10\,\Omega}{3}$

32. (*a*) Potential gradient $x = \frac{6}{1}$

$6\ell = 4 \Rightarrow \ell = \frac{2}{3}$ m

33. (*b*)

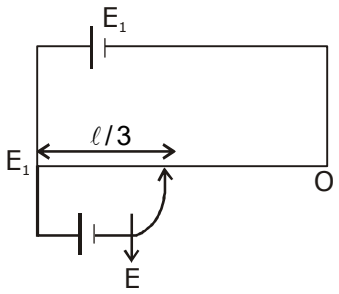


$I = \frac{V}{r + R}$

$I = \frac{11}{10 + 1} = 1$ Amp,

Potential gradient $= x = \frac{11 - 1}{10} = \frac{1 \text{ volt}}{\text{m}}$

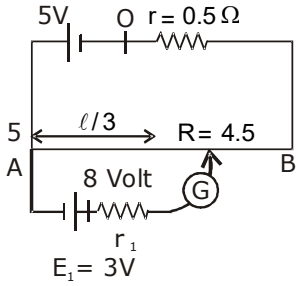
34. (*b*)



$E_1 = 3E$

$1.5\ell \rightarrow 3E$, $E \rightarrow \frac{\ell}{2}$

35. (*d*)



As Battery is connected in reverse order E_1 will not be balanced on entire length of wire AB.

Advanced Level Multiconcept Questions

1. (*a,d*) In series current remain same $I = neAv_d$, $J = I/A$, for constant current

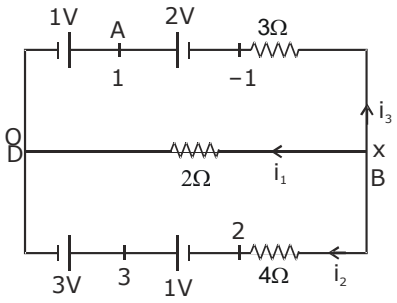
$v_d \propto \frac{1}{A}$ and $J \propto \frac{1}{A}$.

2. (*a,d*) $IR = V = E\ell \Rightarrow I\frac{\rho\ell}{A} = E\ell \Rightarrow \rho = \frac{EA}{I} = \frac{E}{J} =$

$\frac{5 \times 10^{-2}}{10} = 5 \times 10^{-3}\ \Omega$–m

$\sigma = \frac{1}{\rho} = \frac{1}{5 \times 10^{-3}} = 200$ mho/m.

3. (*a,c,d*) In parallel resistance ↓ ∴ i ↑



Let potention of point B is x then from kirchhoff's first law

$i_1+i_2+i_3=0$

$$\frac{x}{2}+\frac{x-2}{4}+\frac{x+1}{3}=0$$

$$\frac{6x+3x-6+4x+4}{12}=0$$

$\Rightarrow 13x=2$

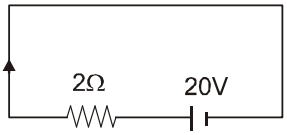
$$x=\frac{2}{13}\text{ volt}$$

4. (*a,c*) (i) $R_{bulb}=\frac{V}{I}=\frac{10}{10\times10^{-3}}=1.\,k\Omega$

(ii) $R_{bulb}=\frac{220}{50\times10^{-3}}=4.4\,k\Omega.$

since increase in temperature increases resistance when it is connected to 220 V mains.

5. (*a,c*)



current flow in circuit is I = 10 amp

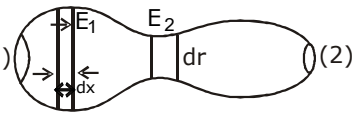
power supplied by the battery is

$= I^2R=(10)^2\times2=200$ W

Potential drop across 4Ω & 6Ω are equal and it is equal to zero.

current in AB wire is 10 amp.

6. (*a,b,c,d*)



$i=neAV_d,\ R=\frac{\rho l}{A}$

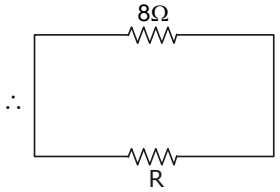
$$E_1=\frac{V}{dx}\Rightarrow=\frac{i.R}{dx}=\frac{i.\rho.dx}{A.dx}$$

$$\frac{i.\rho}{A}=\text{constant}\Rightarrow E_1\propto\frac{1}{A_1}$$

$$\frac{E_1}{E_2}=\frac{A_2}{A_1}$$

$$P=i^2R\Rightarrow i^2\frac{\rho dx}{A}$$

7. (*a,c*) Current should be maximum in 2Ω



$\Rightarrow$ R = 0 (power should be maximum when r = 0)

Power = 72 watt.

8. (*a,c,d*) For non ideal ammeter and voltmeter, ammeter have low resistance and voltmeter have high resistance. Therefore the main current in the circuit will be very low and almost all current will flow through the ammeter. It emf of cell is very high then current in ammeter is very high result of this current the devices may get damaged. If devices are ideal that means resistance of voltmeter is infinity. so that current in the circuit is zero. Therefore ammeter will read zero reading and voltmeter will read the emf of cell.

9. (*b,c*) for 50 V, $R_V=\frac{50}{50\times10^{-6}}=1000\,K\Omega$ in series

for 10 V, $R_V=\frac{10}{50\times10^{-6}}=200\,K\Omega$ in series

for 5 mA, $R_s=\frac{100\times50\times10^{-6}}{5\times10^{-3}}=1\Omega$ in parallel

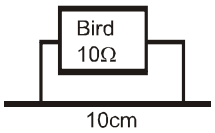
for 10 mA, $R_s=\frac{100\times50\times10^{-6}}{10\times10^{-3}}=\frac{1}{2}\Omega$ in parallel

10. (*a,b*) As emf of E_1 is distributed over the wire AB. Hence A is correct E_2 is balanced by fraction of length of wire $E_1>E_2$.

We only balance potential difference hence B is correct.

11. (*b*)

12. (*a*)

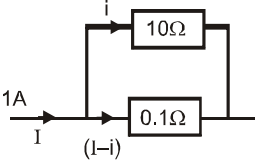


$$r=\left(\frac{1\Omega}{m}\right)(10\text{ cm})\ r=0.1\,\Omega$$

$V_{bird}=(R_{II\ combination})$ current

$$V_{bird}=\left(\frac{10\times0.1}{10+0.1}\right)(1)\cong0.1\text{ V}$$

13. (*c*)

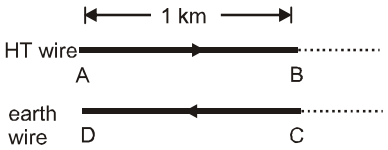


Here $i\times10=(I-i)(0.1)\Rightarrow100\,i=I-i\Rightarrow101\,i=I$

If I = 1A

$$i=\frac{1}{101}\text{ A}\cong0.01\text{ A}$$

14. (*c*)



$V_{AD} = 11\ KV, \quad V_{BC} = ?$

$V_{AD} = V_{AB} + V_{BC} + V_{CD}$

$V_{AB} = IR = (1\ A)\ (1\ km \times 1\Omega/m) = 1000\ V$

$V_{CD} = V_{AB} = 1000\ V$

$V_{BC} = V_{AD} - V_{AB} - V_{CD} = 11000 - 1000 - 1000.$

$V_{BC} = 9000$ Volt

15. (*a*) If critical current through bird is 0.1A then main current

$I = 101\ i$ (As Q.No.5)

$I = 101 \times 0.1 = 10.1\ A$

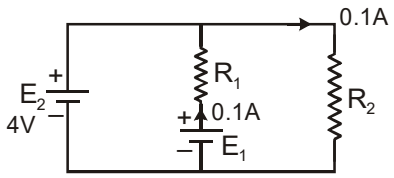
$P_{max} = (11\ KV)\ (10.1\ A) = 111\ KW$

16. (*b*) **17.** (b) **18.** (d)

(19 - 21)

As E_2 is increasing it's current also increases, So, increasing graph is of i_2.

$i_1 = 0.1A, E_2 = 4V, i_2 = 0$

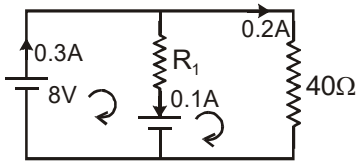


As ;

$0.1\ R_1 + 0.1\ R_2 - E_1 = 0$

$0.1\ R_2 - 4\ V = 0$

$R_2 = 40\ \Omega$



Now; $i_2 = 0.3A, i_1 = -0.1\ A, E_2 = 8V$

Now ; $0.1\ R_1 + E_1 - 8 = 0$

When $E_2 = 6V$, current in E_1 is $i_1 = 0$ (from graph)

$E_1 = 6V$

$\Rightarrow R_1 = \dfrac{4}{0.2} = 20\ \Omega$

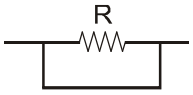
19. (A) q, (B) p, (C) p, (D) q

Drift speed $V_d = \dfrac{J}{ne} = \dfrac{i}{neA}$

$i = \dfrac{V}{R}$ where $R = \dfrac{\rho L}{A}$

$E = \dfrac{V}{L}$ and $P = I^2R$

20. (A) p; (B) q, s; (C) s; (D) p, r, s



short circuited resistor.

In a resistor current always flows from higher potential to lower potential.

In short circuited resistor or ideal cell, energy dissipated is always zero because in short circuited resistor no current flow and in ideal cell has no internal resistance.

Potential difference may be zero across a resistor, non-ideal cell or short circuited resistor.

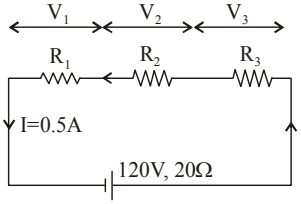
21. [0080]

$V_1 + V_2 + V_3 = 120 - 20\ (0.5)$

$\Rightarrow V_1 + V_2 + V_3 = 110\ V \ldots . (1)$

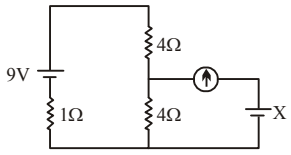
Also $V_1 + V_2 = 60 \Rightarrow V_3 = 110 - 60 = 50V$

Now $V_2 + V_3 = 90 \Rightarrow V_2 = 40V$



$\therefore \quad R_2 = \dfrac{40}{0.5} = 80\Omega$

22. [0004]

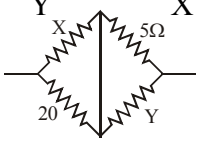


x = P.d. across $4\Omega = 4V$

23. [25]

As bridge is balanced, $\dfrac{x}{20} = \dfrac{5}{Y}$ or $Y = \dfrac{100}{X}$

and Pea of should be $\dfrac{40}{3}\Omega$



Making equation and solving we get $X = 25\Omega$

24. [0020] All the elements of circuit are in parallel arrangement

$$\frac{1}{R_{eq}} = \frac{1}{40} \times \frac{1}{40} \times \frac{1}{40} \times \frac{1}{40} \times \frac{1}{20} \times \frac{1}{20} = \frac{4}{40} \times \frac{2}{40}$$

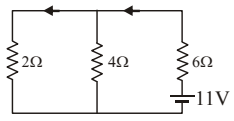
$R_{eq} = 5\Omega$

Power = $V^2/R = 20$ W

25. [0001]

More the resistance less will be power dissipated.

$\therefore$ One should connecte the battery is 6Ω resistor.

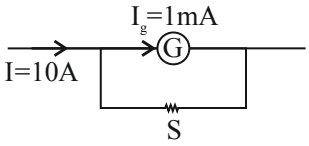


JEE-MAIN PREVIOUS YEAR'S

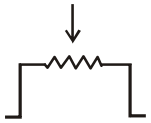
1. (*c*) For conductors, resistance increases with increase in temperature and for semi-conductors, resistance decreases with increase in temperature.

2. (*a*) $I_gR_g=(I-I_g)S$

$$\left(\frac{I_g}{(I-I_g)}\right)\times R_g=0.01\Omega$$

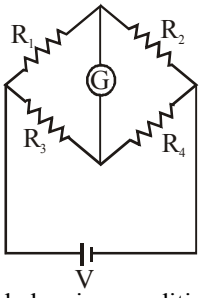


3. (*d*) (1)



On interchanging Cell & Galvanometer.

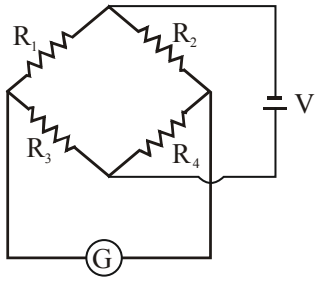
(2)



On balancing condition

$$\frac{R_1}{R_3}=\frac{R_2}{R_4} \quad \text{....(i)}$$

$\Rightarrow$



On balancing condition

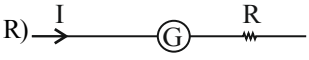
$$\frac{R_1}{R_2}=\frac{R_3}{R_4} \quad \text{.....(ii)}$$

As we see both equation (1) & (2) are same. So 4th statement is false.

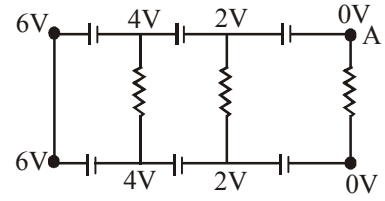
4. (*c*) $V=I(R_g+R)$

$\Rightarrow 10=(5\times10^{-3})(15+R)$

$\Rightarrow R=1985\,\Omega$



5. (2)

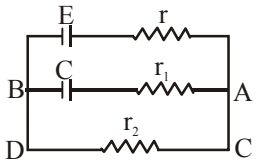


Taking voltage of point A as = 0

Then voltage at other points can be written as shown in figure. Hence voltage across all resistance is zero.

Hence current = 0

6. (*a*) If steady state, current through AB = 0

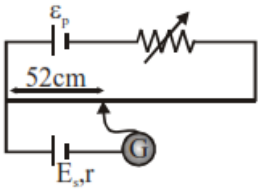


$\Rightarrow V_{AB}=V_{CD}$

$$\Rightarrow V_{CD}=\left(\frac{\varepsilon}{r+r_2}\times r_2\right)$$

$$\Rightarrow Q_C=CV_{AB}=CE\left(\frac{r_2}{r+r_2}\right)$$

7. (*b*) Without shunting condition :

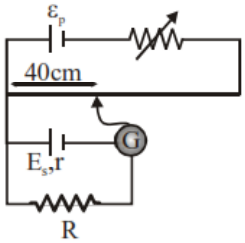


$E_s=52\times x$ .....(1)

when balanced

where, x = potential gradient of wire.

With shunting condition :



On balancing

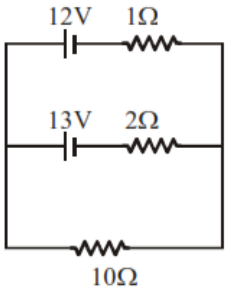
$$E_s-\frac{E_s}{(r+R)}r=40\times x \quad \text{.....(2)}$$

On solving :

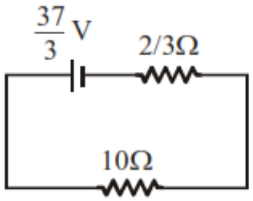
$$\frac{(1)}{(2)}\Rightarrow\frac{1}{1-\frac{r}{r+R}}=\frac{52}{40}$$

$\therefore\ r=1.5\,\Omega$

8. (*a*) $E_{eq} = \dfrac{\frac{12}{1}+\frac{13}{2}}{\frac{1}{1}+\frac{1}{2}} = \dfrac{37}{3}$ volt



$r_{eq} = \dfrac{2\times1}{2+1} = \dfrac{2}{3}\Omega$

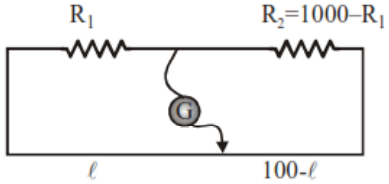


Now its equivalent circuit is :

$i = \dfrac{37/3}{10+\frac{2}{3}} = \dfrac{37}{32}$

$\therefore\ V_{10\Omega} = i\times10 = \dfrac{37}{32}\times10 = \dfrac{370}{32} = 11.56$ volt

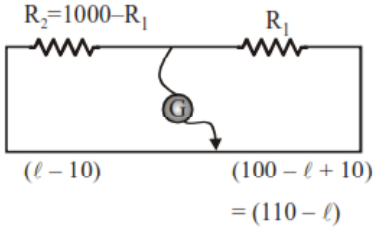
9. (*b*) $R_1 + R_2 = 1000 \Rightarrow R_2 = 1000 - R_1$



On balancing condition

$R_1(100-l) = (1000-R_1)l$ ....(1)

On Interchanging resistance



On balancing condition

$R_1(l-10) = (1000-R_1)(110-l)$ ....(2)

(1) ÷ (2)

$\dfrac{100-l}{l-10} = \dfrac{l}{110-l}$

$\Rightarrow (100-l)(110-l) = l(l-10)$

$\Rightarrow 11000 - 100l - 110l + l^2 = l^2 - 10l$

$\Rightarrow 11000 = 200l$

$l = 55$

Put in eq. (1)

$R_1(100-55) = (1000-R_1)55$

$R_1(45) = (1000-R_1)55$

$\Rightarrow 100\,R_1 = 55\times1000$

$\Rightarrow R_1 = 550\,\Omega$

10. (*c*) $R_g = 20\,\Omega$

No. of divisions $= N_g = N = 30$

Figure of Merit, FOM $= \dfrac{1}{\phi} = 0.005$A / Div.

Current sentivity $=$ CS $= \left(\dfrac{1}{0.005}\right) = \dfrac{\phi}{1}$

Maximum allowable current, $Ig_{max} = 0.005\times30 = 15\times10^{-2} = 0.15$

$\Rightarrow$ $V = 15 = 0.15[20+R]$

$100 = 20 + R$

$R = 80$.

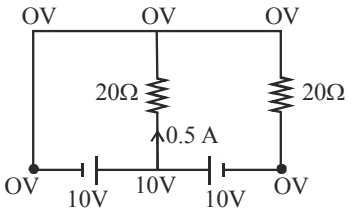
11. (*a*) By colour coding,

$R = 50\times10^2\Omega$

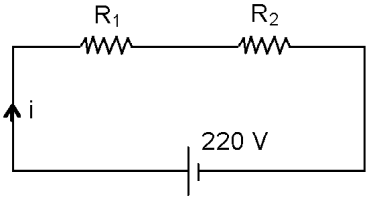
$I^2R = P$

$I = \sqrt{\dfrac{P}{R}} = 20$ mA

12. (*c*)



13. (*a*)



Resistance, $R = \dfrac{V^2}{P}$

$\Rightarrow R_1 = \dfrac{(220)^2}{25} = 1936\ \Omega$

$R_2 = \dfrac{(220)^2}{100} = 484\,\Omega$

$i = \dfrac{220}{R_1+R_2} = \dfrac{1}{11}$A

Power dissipated through $R_1 = P_1 = i^2R_1 = 16$ W

Power dissipated through $R_2 = P_2 = i^2R_2 = 4$ W

14. (*c*) $R = \dfrac{\rho\ell}{A}$

$R = \dfrac{\rho\ell^2}{A\ell} = \dfrac{\rho\ell^2}{V}$

$\dfrac{dR}{R} = \dfrac{2d\ell}{\ell}$

Hence, $\dfrac{dR}{R}\times100 = 1\%$

15. (*b*) $I_3 + I_5 = I_4 \Rightarrow I_3 = I_4 - I_5 = 0.4$ A

$I_1 + I_2 = I_4 \Rightarrow I_2 = I_4 - I_1 = 1.1$ A

$I_3 + I_6 = I_2 + I_1 \Rightarrow I_6 = I_2 + I_1 - I_3 = 0.4$ A

16. (*c*) $\frac{R_1}{R_2}=\frac{2}{3}$ ....(i)

$\frac{R_1+10}{R_2}=1$

$\Rightarrow R_1+10=R_2$ ....(ii)

$\frac{2R_2}{3}+10=R_2 : 10=\frac{R_2}{3}$

$\frac{\frac{30\times R}{30+R}}{30}=\frac{2}{3}$

$R=60\,\Omega$

17. (*d*) $\frac{P}{Q}=\frac{400}{X}$ ...(1)

and $\frac{Q}{P}=\frac{405}{X}$ ...(2)

Solving $X^2=400\times 405$

$\Rightarrow X=402.5\,\Omega$

18. (*b*) Potential difference across AB will be equal to battery equivalent across CD.

$$V_{AB}=V_{CD}=\frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}=\frac{\frac{1}{1}+\frac{2}{1}+\frac{3}{1}}{\frac{1}{1}+\frac{1}{1}+\frac{1}{1}}=\frac{6}{3}=2V$$

19. (*b*) Assuming constant voltage supply

$P=60=\frac{V^2}{R_1+R_2}=\frac{V^2}{2R}$ ....(1) $[R_1=R_2=R]$

And $P'=\frac{V^2}{R_1}+\frac{V^2}{R_2}=\frac{2V^2}{R}=4P=4\times 60=240\,W.$

20. (*b*) Fact based.

21. (*a*) $i=neAV_d$

$1.5=9\times 10^{28}\times 1.6\times 10^{-19}\times 5\times 10^{-6}\times V_d=72\times 10^3\times V_d$

$\Rightarrow V_d=0.02\,mm/s$

22. (*b*) $0.5=\frac{6}{(2+\lambda L)}\lambda x$ ...(1)

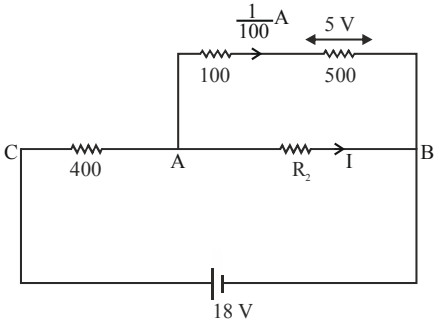
$E_2=\frac{6}{(6+\lambda L)}\lambda x$ ....(2)

So dividing equation (1) and (2)

$\frac{E_2}{0.5}=\frac{2+4}{6+4}=\frac{3}{5}$

$\Rightarrow E_2=0.3$ volt.

23. (*a*)



$V_{AB}=\left(\frac{1}{100}\right)600=6$ volt

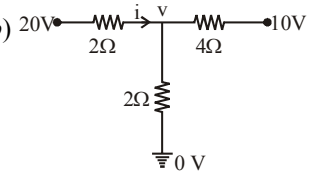
$\Rightarrow V_{AC}=18\,V-6\,V=12$ volt

$I_{AC}=\frac{12}{400}=\frac{3}{100}\Rightarrow I=I_{AC}-\frac{1}{100}$

$\Rightarrow I=\frac{3}{100}-\frac{1}{100}=\frac{2}{100}$

$\Rightarrow R_2=\frac{V_{AB}}{I}=\frac{6\times 100}{2}=300\Omega$

24. (*b*)



From junction law :

$\frac{20-v}{2}+\frac{10-v}{4}=\frac{v}{2}$

$\Rightarrow v=10V$

$\Rightarrow i=\frac{10}{2}=5\,amp$

25. (*a*) $R=\frac{P}{I^2}=\frac{4.4}{4\times 10^{-6}}\Omega$

$P'=\frac{V^2}{R}=\frac{11\times 11\times 4\times 10^{-6}}{4.4}W=11\times 10^{-5}W$

26. (*a*) $R_1=32\times 10=320\,\Omega$

$R_3=\frac{R_4}{R_2}\times R_1=\frac{40\times 320}{80}=160\Omega$

$\therefore$ Colour code of R_3 be Brown, Blue, Brown

27. (*b*) $V_0=i_{g_0}(R_g+R)$

$i_{g_0}=4\times 10^{-4}\times 25=10^{-2}\,A$

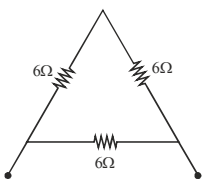
$V_0=2.5\,V$

$R_g+R=\frac{V_0}{i_{g_0}}=\frac{2.5}{10^{-2}}=250\Rightarrow R=200\,\Omega$

28. (*b*) $\frac{30R}{R+30}=30\times 0.95$

$\Rightarrow R=570\,\Omega$

29. (*a*)

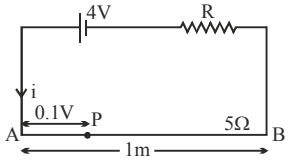


$R_{net}=4\Omega$

30. (*c*) $V_{AJ}=IR_{AJ}$

$$\Rightarrow \frac{E}{2}=\left(\frac{E}{13r}\right)\left(\frac{12r}{L}\right)X \Rightarrow X=\frac{13L}{24}$$

31. (*c*)



$$i=\frac{4}{5+R}$$

$$V_{AB}=i(5)=\frac{20}{5+R}$$

$$V_{AP}=\frac{V_{AB}}{1}(0.1)=\frac{20}{5+R}\left(\frac{0.1}{1}\right)=\frac{2}{5+R}$$

$$\text{Now,}\frac{2}{5+R}=5\times10^{-3}$$

$$\Rightarrow R=395\Omega$$

32. (*c*) $\frac{dR}{d\ell}=\frac{k}{\sqrt{\ell}}$ (k = constant)

$$\int_0^R dR=k\int_0^1\frac{d\ell}{\sqrt{\ell}}$$

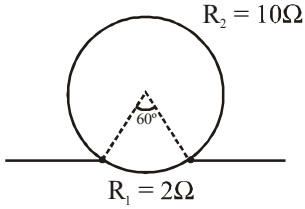
R = 2k = resistance of wire AB.

$$\text{Again, }\int_0^{R/2}dR=k\int_0^L\frac{d\ell}{\sqrt{\ell}}\ L=\text{Length AP}$$

$$\frac{R}{2}=k2L^{1/2}\Rightarrow\frac{R}{4k}=\sqrt{L}=\frac{1}{2}$$

$$\Rightarrow L=\frac{1}{4}m=0.25\ m$$

33. (*b*)



$$\Rightarrow R=\frac{\rho\ell}{A}=\frac{\rho\ell^2}{V}$$

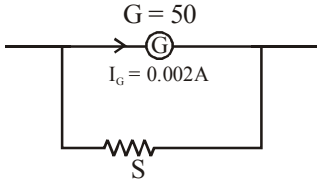
$$R\propto\ell^2$$

R = 12Ω (new resistance of wire)

$R_1=2\Omega$ $R_2=10\Omega$

$$R_{eq}=\frac{10\times2}{10+2}=\frac{5}{3}\Omega$$

34. (*a*)



$$S(0.5-0.002)=50\times0.002$$

$$S=\frac{50\times0.002}{(0.5-0.002)}=\frac{0.1}{0.498}=0.2$$

35. (*d*) $\rho=\frac{m}{ne^2\tau}=\frac{9.1\times10^{-31}}{8.5\times10^{28}\times(1.6\times10^{-19})^2\times25\times10^{-15}}$

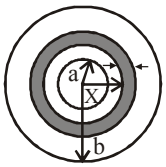
$=0.0167\times10^{-6}$

$=1.67\times10^{-8}\ \Omega m$

36. (*b*) $E_{eq}=\frac{\frac{E_1}{2R_1}+\frac{E_2}{R_2}+\frac{E_3}{2R_1}}{\frac{1}{2R_1}+\frac{1}{R_2}+\frac{1}{2R_1}}=\frac{\frac{2}{2}+\frac{4}{2}+\frac{4}{2}}{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}=\frac{5}{\frac{3}{2}}=\frac{10}{3}=3.3$

37. (*c*) When red is replaced with green, 1st digit changes to 5 so new resistance will be 500Ω.

38. (*a*) $dR=\rho.\frac{dx}{4\pi x^2}$

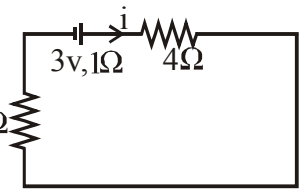


$$\int dR=\rho.\int_a^b\frac{dx}{4\pi x^2}$$

$$R=\frac{\rho}{4\pi}\left[-\frac{1}{x}\right]_a^b$$

$$R=\frac{\rho}{4\pi}\left[\frac{1}{a}-\frac{1}{b}\right]$$

39. (*b*)

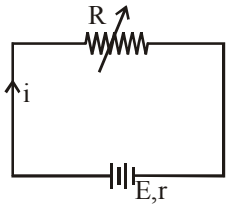


Resistance of wire AB = 400 × 0.01 = 4Ω

$$i=\frac{3}{6}=0.5A$$

Now voltmeter reading = i (Resistance of 50 cm length)
= (0.5A) (0.01 × 50) = 0.25 volt

40. (*d*)



Current $i=\frac{E}{r+R}$

Power generated in R

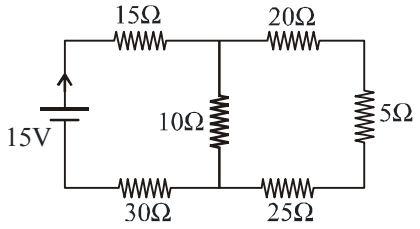
$$P=i^2R$$

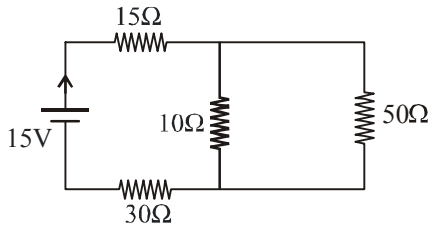
$$P=\frac{E^2R}{(r+R)^2}$$

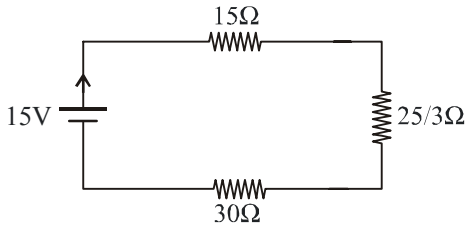
For maximum power, $\frac{dP}{dR}=0$

$$E^2\left[\frac{(r+R)^2\times1-R\times2(r+R)}{(r+R)^4}\right]=0\Rightarrow r=R$$

41. (*c*)



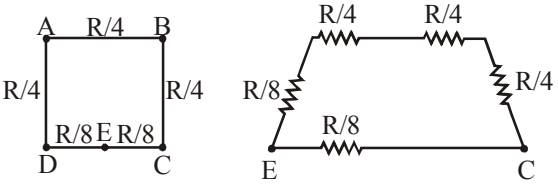




$$R_{eq} = 15 + \frac{25}{3} + 30 = \frac{45+25+90}{3} = \frac{160}{3}$$

$$I = \frac{E}{R_{eq}} = \frac{15\times 3}{160} = \frac{9}{32}\text{amp.}$$

42. (*c*)

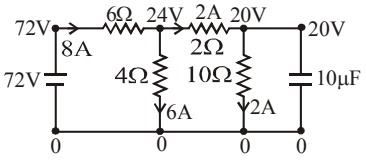


$$\frac{1}{R_{eq}} = \frac{8}{7R} + \frac{8}{R}$$

$$\frac{1}{R_{eq}} = \frac{8+56}{7R}$$

$$R_{eq} = \frac{7R}{64}$$

43. (*c*) Applying point potential method



$Q = CV$

$Q = 10\,\mu F \times 20 = 200\,\mu C$

44. (*b*) $G = 50\,\Omega$

$S = 5000\,\Omega$

$i_g = 4\times 10^{-3}$

$V = i_g(G+S)$

$V = 4\times 10^{-3}(50+5000)$

$= 4\times 10^{-3}(5050) = 20.2$ volt

45. (*b*) $Q = P\times t$

$Q = mc\Delta T + mL$

$$P = \frac{V_{rms}^2}{R}$$

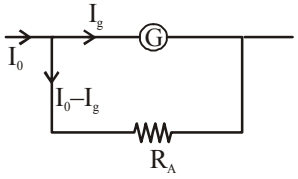
$$4200\times 80 + 2260\times 10^3 = \frac{(200)^2}{20}\times t$$

$t = 1298\text{ sec} = 21.6\text{ min}$

$t = 22\text{ min}$

46. (*b*) When galvanometer is used as an ammeter, shunt is used in parallel with galvanometer.

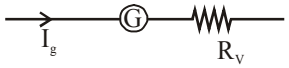
$\therefore$



$$\therefore I_g G = (I_0 - I_g)R_A$$

$$\therefore R_A = \left(\frac{I_g}{I_0 - I_g}\right)G$$

When galvanometer is used as a voltmeter, resistance is used in series with galvanometer.



$I_g(G+R_V) = V = GI_0$ (given $V = GI_0$)

$$\therefore R_V = \frac{(I_0 - I_g)G}{I_g}$$

$$\therefore R_A R_V = G^2 \;\&\; \frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$$

47. (*d*) $20\times 50\times 10^{-6} = 10^{-3}$ Amp.

$$V_1 = \frac{2}{10^{-3}} = 100 + R_1$$

$1900 = R_1$

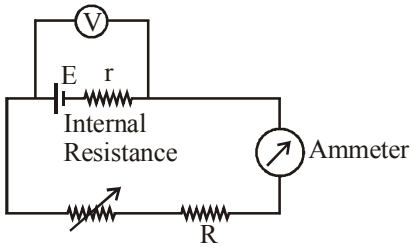
$$V_2 = \frac{10}{10^{-3}} = (2000 + R_2)$$

$R_2 = 8000$

$$V_3 = \frac{20}{10^{-3}} = 10\times 10^3 + R_3$$

$10\times 10^3 = R_3$

48. (*c*)



$V = E - Ir$

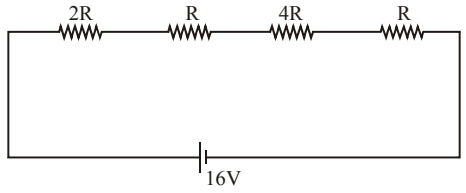
When $V = V_0 = 0 \Rightarrow 0 = E - Ir$

$\therefore$ $E = Ir$

When $I = 0$, $V = E = 1.5$ V

$\therefore$ $r = 1.5\,\Omega$

49. (*a*)



$$P = \frac{16^2}{8R} = 4 \qquad \therefore R = 8\Omega$$

50. (*d*) $\mu = \dfrac{V_d}{E}$ $\qquad E = \rho J$

$$= \frac{1.1\times10^{-3}}{1.7\times10^{-8}\times\dfrac{5}{\pi\times25\times10^{-6}}}$$

$$= \frac{1.1\times10^{-3}\times\pi\times25\times10^{-6}}{1.7\times10^{-8}\times5} \approx 1.01\ m^2/Vs$$

51. (*a*) As $x = \dfrac{R(100-l)}{l}$

$$\text{for (1) } x = \frac{1000\times(100-60)}{60} \approx 667$$

$$\text{for (2) } x = \frac{100\times(100-13)}{13} \approx 669$$

$$\text{for (3) } x = \frac{10\times(100-1.5)}{1.5} \approx 656$$

$$\text{for (4) } x = \frac{1\times(100-1)}{1} \approx 99$$

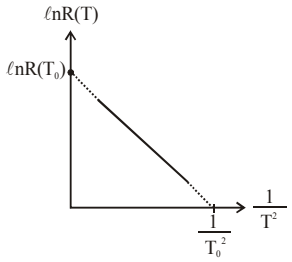
So option (4) is completely different, hence correct

52. (*c*) $\dfrac{\dfrac{1}{T^2}}{\dfrac{1}{T_0^2}} + \dfrac{\ell n\,R(T)}{\ell n\,R(T_0)} = 1$

$\Rightarrow \ell nR(T)$

$$= [\ell nR(T_0)]\left(1-\frac{T_0^2}{T^2}\right)$$

$$\Rightarrow R(T) = R_0 e^{\left(\frac{-T_0^2}{T^2}\right)}$$



53. (Bonus)

$10^{-4}(Rg + 10^6\times2) = 5$

$200 + 10^{-4}Rg = 5$

G = –ve So answer is Bonus

54. (*d*) $R_{eq} = \dfrac{15\times10}{25} + 2 + 2r$

$= 8 + 2r$

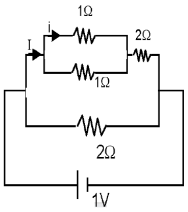
$$i = \frac{3}{8+2r}$$

$$2 = i\,R_{eq} = \frac{3}{8+2r}\times6$$

$16 + 4r = 18$

$\Rightarrow r = 0.5\,\Omega$

55. (*a*)



$$I = \frac{1}{2.5} = 0.4A$$

$$i = \frac{1}{2} = 0.2A$$

56. (*a*) Total power is

$(15\times45) + (15\times100) + (15\times10) + (2\times1000) = 4325\,W$

So current is $= \dfrac{4325}{220} = 19.66A$

Ans. is 20 Amp.

57. [12]

Let the emf of cell is ε internal resistance is 'r' and potential gradient is x.

only cell connected:

$\varepsilon = 560\,x$ ....(1)

After connecting the resistor

$$\frac{\varepsilon\times10}{10+r} = 500x \qquad ...(2)$$

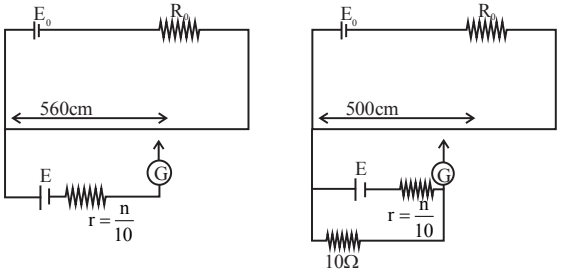
from (1) and (2)

$$\frac{560\times10}{10+r} = 500x$$

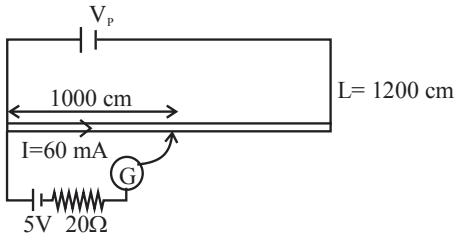
$56 = 50 + 5r$

$$r = \frac{6}{5} = 1.2\Omega$$

$n = 12$



58. (*c*)

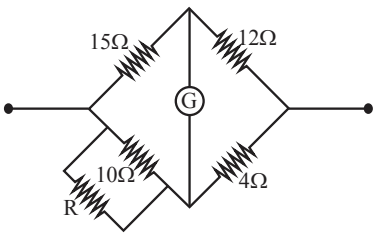


Potential gradient $= \dfrac{5}{1000} = \dfrac{V_P}{1200}$

$V_P = 6\,V$

and $R_P = \dfrac{V_P}{I} = \dfrac{6}{60\times10^{-3}} = 100\Omega$

59. [10]



$$\frac{10R}{10+R}\times 12 = 15\times 4$$

on solving

$R = 10\Omega$

60. (30) $V_1 = \varepsilon_1 - i.r_1$

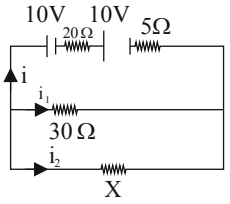
$0 = 10 - i\times 20$

$i = 0.5\,A$

$V_2 = \varepsilon_2 - ir_2$

$= 10 - 0.5\times 5$

$V_2 = 7.5\,V$



$$0.5 = \frac{7.5}{30} + \frac{7.5}{x}$$

$$0.5 = 0.25 + \frac{7.5}{x}$$

$$\frac{7.5}{x} = 0,25;\ x = \frac{7.5}{0.25} = 30$$

61. (*c*) $V_g = i_g R_g = 0.1\,V$

$V = 10\,V$

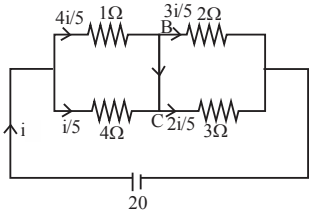
$$R = R_g\left(\frac{V}{V_g} - 1\right)$$

$= 100\times 99 = 9.9\,K\Omega$

62. (*b*) $R_{eff} = \frac{4}{5} + \frac{6}{5} = 2\Omega$

$$i = \frac{20}{2} = 10A$$

$$I = \frac{4i}{5} - \frac{3i}{5} = +\frac{i}{5} = 2A$$



63. [40] $\frac{X}{R} = \frac{75}{25} = 3$

$$R = \frac{\rho\ell}{A} = \frac{4\rho\ell}{\pi d^2}$$

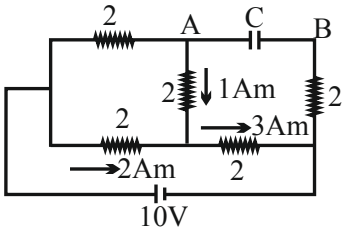
$$R' = \frac{4\rho\left(\frac{\ell}{2}\right)}{\pi\left(\frac{d}{2}\right)^2} = 2R$$

then $\frac{X}{R'} = \left(\frac{100-\ell}{\ell}\right)$

$$\frac{100-\ell}{\ell} = \frac{X}{2R} = \frac{3}{2}$$

$\ell = 40.00\,cm$

64. [8]



After solving the circuit we got the final current distribution as shown in the above diagram. So potential difference between A and B is

$0 + (2\times 3) + (2\times 1) = 8$ volt

65. [3] Potential gradient $= \frac{\text{Potential drop}}{\text{length}}$

$$= \frac{1.02}{100-49}\,V/cm$$

$= 0.02\,V/cm$

66. [4] $\rho_M = 98\times 10^{-8}$

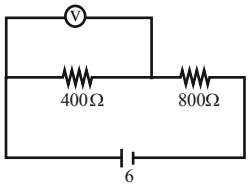
$\rho_A = 2.80\times 10^{-8}$

$\rho_C = 1.72\times 10^{-8}$

$\rho_T = 5.65\times 10^{-8}$

$\rho_M > \rho_T > \rho_A > \rho_C$

67. [4] Voltmeter reading $= \frac{I\times 400R}{400+R}$



$$I = \frac{6}{\frac{400R}{400+R} + 800}$$

Putting the value of R, we get

Reading = 1.95V

68. [1] Theory based

69. [3] Power in external $R = \left(\frac{\varepsilon}{r+R}\right)^2 R$

This power is maximum, when $R = r$

$$r = \int_a^b \rho\frac{dx}{2\pi xl}$$

$$r = \frac{\rho}{2\pi l}\ln\frac{b}{a}$$

70. [2] $V_B = \frac{40\times 60}{100} = 24$ volt

$$V_D = \frac{40\times 110}{200} = 22 \text{ volt}$$

$\therefore\ \Delta V = V_B - V_D = 2$ volt

71. [1] $\varepsilon - i_1R - i_1R = 0$

$8 - 4i_1 - 4i_1 = 0$

$i_1 = 1A$

72. [1] $i^2R = 0.5$

$iR = 2.5$

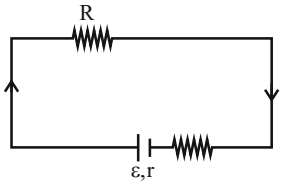
$\Rightarrow i = 0.2\,A$ and

$R = 12.5\,\Omega$

$V = \varepsilon - ir$

$2.5 = 3.0 - 0.2\,r$

$\Rightarrow r = 2.5\,\Omega$

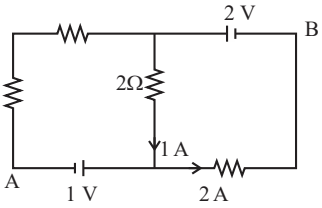


Power dissipated in internal resistance

$= i^2 r = 0.1\,W$

73. [3] Figure of merit $= \dfrac{\text{Current}}{\text{Deflection}}$

74. [1]



$V_B + 2 - 2 \times 1 - 1 = V_A$

$\Rightarrow V_B - V_A = 3 - 2 = 1V$

75. [3] $ig(G + R_1) = 1$

$ig(G + R_1 + R') = 2 \Rightarrow R' = G + R_1$

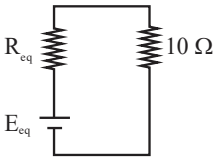
76. [2] $I = \dfrac{P}{V} = \dfrac{(1000)}{220}$

$P_{LOSS} = I^2R_{trans} = \left(\dfrac{1000}{220}\right)^2 \times 2$

$\Rightarrow$ efficiency $= \left(\dfrac{1000}{1000 + P_{LOSS}}\right) \times 100$

$\simeq 96\%$

77. [4]



$\dfrac{1}{R_{eq}} = \dfrac{1}{4} + \dfrac{1}{7} \Rightarrow R_{eq} = \dfrac{28}{11}\Omega$

Circuit can be reduced to

$\dfrac{11E_{eq}}{28} = \dfrac{20}{7} + \dfrac{10}{4}$

$E_{eq} = \dfrac{150}{11}V \quad ; I = \dfrac{\frac{150}{11}}{\frac{28}{11} + 10} = \dfrac{150}{138}A$

$\dfrac{1500}{138} = 10 + 4\,I;\ I = \dfrac{30}{138}A$

78. [1] Ammeter is always connected in series and voltmeter is connected in parallel.

79. [1] $\tau = RC = 10\,\mu s$

For $0 < t < 5\,\mu s$, it will get charged. For $5 < t < 10\,\mu s$ potential is constant and again gets charged after that.

80. [640]

$$\phi = \vec{E} \cdot \vec{A} = \frac{z_0}{5}(2\hat{i} + 3\hat{j}) \cdot (0.4\hat{i}) = \frac{4000}{5}(2 \times 0.4) = 640\,Nm^2C^{-1}$$

81. [3] $I = \dfrac{1}{2} C \in_0 E^2$

$E^2 \propto I$

$I = \dfrac{\text{Power}}{\text{Area}}$

$E^2 \propto \dfrac{P}{A}$

$E \propto \sqrt{P}$

$\dfrac{E'}{E} = \sqrt{\dfrac{60}{100}}$

$E' = \sqrt{\dfrac{3}{5}}E$

So the value of $x = 3$

82. [4] $s = np$

$R_1 + R_2 = n\left[\dfrac{R_1R_2}{R_1 + R_2}\right]$

$\Rightarrow R_1^2 + R_2^2 + 2R_1R_2 = nR_1R_2$

$\Rightarrow R_1^2 + (2 - n)R_1R_2 + R_2^2 = 0$

For real roots, $b^2 - 4ac \geq 0$

$[(2 - n)R_2]^2 - 4 \times 1 \times R_2^2 \geq 0$

$\Rightarrow (2 - 4)^2 R_2^2 \geq 4F_2^2$

$\Rightarrow 2 - n \geq \pm 2$

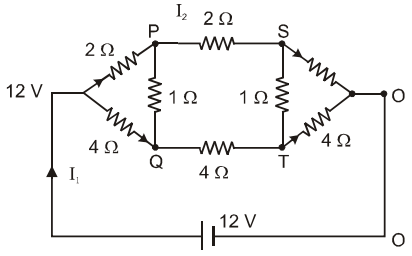
$\Rightarrow 2 - n \geq -2$

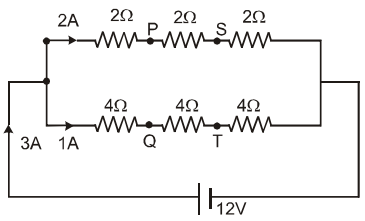
$\Rightarrow n \geq 4$

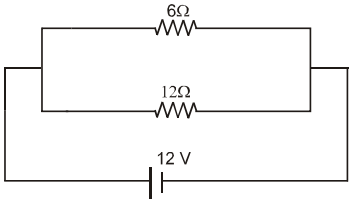
So, minimum value for $n = 4$

JEE-ADVANCED PREVIOUS YEAR'S

1. (*abcd*) Due to input and output symmetry P and Q and S and T have same potential.







$$R_{eq} = \frac{6\times 12}{18} = 4\Omega$$

$$I_1 = \frac{12}{4} = 3A$$

$$I_2 = \left(\frac{12}{6+12}\right)\times 3$$

$I_2 = 2A$
$V_A - V_S = 2\times 4 = 8V$
$V_A - V_T = 1\times 8 = 8V$
$V_P = V_Q \Rightarrow$ Current through PQ $= 0$ (A)
$V_P = V_Q \Rightarrow V_Q > V_S$ (C)
$I_1 = 3A$ (B)
$I_2 = 2A$ (D)

2. (*bd*) In given Kettle

$$R = \rho\frac{L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\rho L}{\pi d^2};\ P = \frac{V^2}{R}$$

In second Kettle $R_1 = \rho\frac{L}{\pi d^2}$ $R_2 = \frac{\rho L}{\pi d^2}$

So $R_1 = R_2 = \frac{R}{4}$

If wires are in parallel equivalent resistance

$$R_P = \frac{R}{8}$$

then power $P_P = 8P$
so it will take 0.5 minute
If wires are in series equivalent resistance

$$R_S = \frac{R}{2}$$

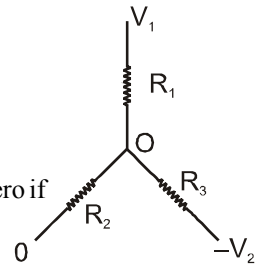
then power $P_S = 2P$
so it will take 2 minutes

3. (*abd*) Potential of Junction O

$$V_0 = \frac{\frac{V_1}{R_1} + 0 - \frac{V_2}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

Current through R_2 will be zero if

$$V_0 = 0 \Rightarrow \frac{V_1}{V_2} = \frac{R_1}{R_3}$$



4. [5] $\frac{6}{1000}(G+4990) = 30$

$$\Rightarrow G + 4990 = \frac{30,000}{6} = 5000 \qquad \Rightarrow G = 10$$

$$\frac{6}{1000}\times 10 = \left(1.5 - \frac{6}{1000}\right)S$$

$$\Rightarrow S = \frac{60}{1494} = \frac{2n}{249} \Rightarrow n = \frac{249\times 30}{1494} = \frac{2490}{498} = 5$$

5. (*c*) For balanced meter bridge

$$\frac{X}{R} = \frac{\ell}{(100-\ell)}$$

$$\frac{X}{40} = \frac{90}{60} \Rightarrow X = 60\Omega \Rightarrow X = R\frac{\ell}{(100-\ell)}$$

$$\frac{\Delta X}{X} = \frac{\Delta\ell}{\ell} + \frac{\Delta\ell}{100-\ell} = \frac{0.1}{40} + \frac{0.1}{60}$$

$\Delta X = 0.25$
so $X = (60 \pm 0.25)\ \Omega$

6. (*ac*)For maximum voltage range across a galvanometer, all the elements must be connected in series. For maximum current range through a galvanometer, all the elements should be connected in parallel.

7. (*a*) Balls are repelled by lower positive plate and hits upper plate where the balls will get negatively charged and will now get attracted to the lower plate which is positively charged. Therefore motion of the balls will be periodic.

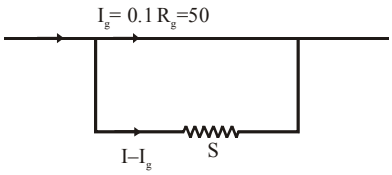
8. (*c*) $\frac{K_q}{r} = V_0 \Rightarrow q = \frac{V_0 r}{K}$

$$\frac{1}{2}\left(\frac{qE}{m}\right)t^2 = h \Rightarrow \frac{1}{2}\frac{V_0 r}{K}\frac{2V_0}{hm}t^2 = h$$

$$t = \frac{h}{V_0}\sqrt{\frac{mK}{r}}$$

Average current, $I_{avg.} = \frac{2q}{t} = \frac{2V_0^2}{h}\frac{r\sqrt{r}}{mK\sqrt{K}}$

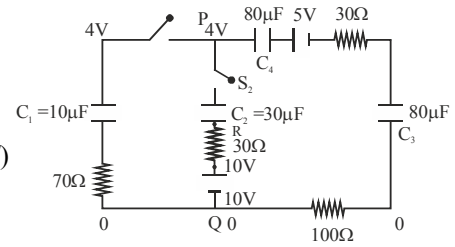
9. [5.55] $n = 50$ tunrs $A = 2\times 10^{-4}\ m^2$
$B = 0.02$ T $\quad K = 10^{-4}$
$Q_m = 0.2$ rad $\quad R_g = 50\ \Omega$
$I_A = 0 - 1.0A\tau = MB = C\theta,\ M = nIA$
$BINA = Cq$
$0.02\times 1\times 50\times 2\times 10^{-4} = 10^{-4}\times 0.2\ 10$
$I_g = 0.1A$
For galvanometer, resistance is to be connected to ammeter in shunt.



$I_g \times R_g = (I - I_g)S$
$0.1\times 50 = (1-0.1)\,S$

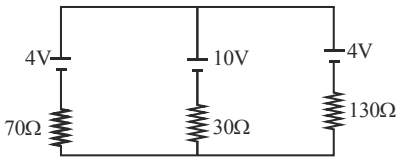
$$S = \frac{50}{9} = 5.55$$

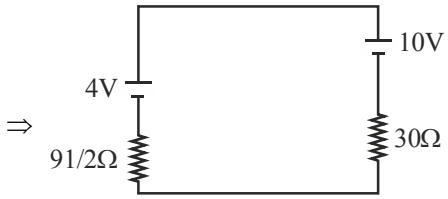
10. (*c,d*)



(1) at t = 0, capacitor C_1 acts as a battery of 4V, C_4 & C_3 of $\frac{1}{2}$ V each, C_2 is shorted

Circuit is





$$\Rightarrow i = \frac{6}{30 + \frac{91}{2}} = 0.079A$$

(2) and (4)

At steady state,

When capacitor is fully charged it behave as open circuit and current through it zero.

Hence, Charge on each capacitor is same.

$Q = C_{eq} V \qquad = (8\,\mu F) \times 5$

$Q = 40\,\mu C$

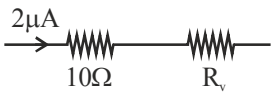
Now, $\qquad V_p - \frac{40}{10} = V_Q$

$V_p - V_Q = 4V$

(3) At t = 0, S_1 is closed, capacitor act as short circuit.

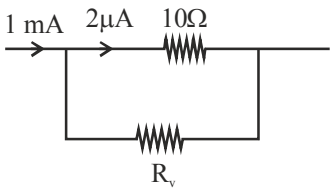
$$i = \frac{V}{R_{eq}} = \frac{5}{200} = 25mA$$

11. (*a,c*)

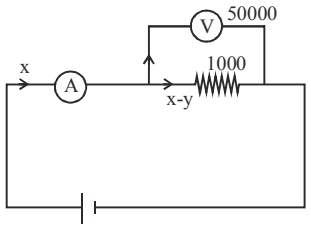


$0.1 = 2 \times 10^{-6}(10 + R_v)$

$\therefore R_V = 49990\,\Omega$



$2 \times 10^{-6} \times 10 = 10^{-3} R_A \therefore = 0.02\,\Omega$



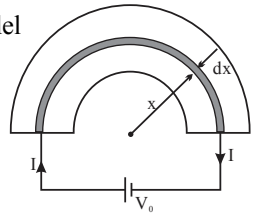
$y\,50000 = (x - y)1000$

$\therefore \quad 51y = x$

$$\text{Reading} = \frac{y50000}{x} = 980$$

12. (*a,c,d*)

All the elements are in parallel



$$\therefore \int \frac{1}{dr} = \int_{R1}^{R_2} \frac{t\,dx}{\rho \pi x}$$

$$\frac{1}{r} = \frac{t}{\pi\rho} \ln\left(\frac{R_2}{R_1}\right)$$

$$\text{Resistance} = \frac{\pi\rho}{t\,\ell n\left(\frac{R_2}{R_1}\right)}$$

$$i = \frac{V_0 t \ln\left(\frac{R_2}{R_1}\right)}{\pi\rho} \qquad (A)$$

$\left(-e\vec{E}\right)$ will be inward direction in order to provide centripetal acceleration. Therefore electric field will be radially outward

$V_{outer} < V_{inner} \qquad (C)$

$$\frac{mV_d^2}{r} = q\vec{E}$$

$$E = \frac{mV_d^2}{qr} \; (I = neAV_d \Rightarrow V_d \propto i)$$

$$\Delta V = \int \vec{E}.\vec{dr}$$

$$\Delta V \propto V_d^2$$

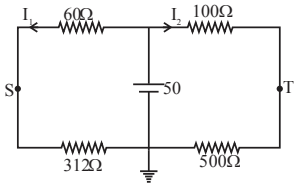
$$\Delta V \propto I^2$$

13. (0.26 to 0.27)

Sol. $R_3' = 300(1 + \alpha \Delta T)$

$= 312\,\Omega$

Now



$$I_1 = \frac{50}{372} \text{ and } I_2 = \frac{50}{600}$$

$V_S - V_T = 312 I_1 - 500 I_2$

$= 41.94 - 41.67$

-0.27 V

PRAYAS 1.0: t.me/realprayasians
PRAYAS 2.0: t.me/realprayasians2

4 Moving Charge and Magnetism

Topicwise Questions

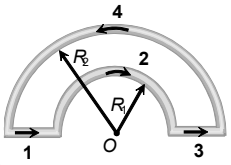
1. (*d*) $dB = \frac{\mu_0}{4\pi}\cdot\frac{id\,l\sin\theta}{r^2} \Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{i(d\vec{l}\times\vec{r})}{r^3}$

2. (*d*) $B = \frac{\mu_0}{4\pi}\frac{(2\pi-\theta)i}{R} = \frac{\mu_0}{4\pi}\frac{\left(2\pi-\frac{\pi}{2}\right)\times i}{R} = \frac{3\mu_0 i}{8R}$

3. (*b*) $i = \frac{q}{T} = \frac{2\times1.6\times10^{-19}}{2} = 1.6\times10^{-19}\,A$

$\therefore\quad B = \frac{\mu_0 i}{2r} = \frac{\mu_0\times1.6\times10^{-19}}{2\times0.8} = \mu_0\times10^{-19}$

4. (*a*) In the following figure, magnetic fields at *O* due to sections 1, 2, 3 and 4 are considered as B_1, B_2, B_3 and B_4 respectively.



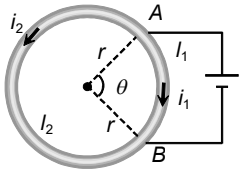
$B_1 = B_3 = 0$

$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{R_1}\otimes$

$B_4 = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{R_2}\odot \qquad As\ |B_2| > |B_4|$

So $B_{net} = B_2 - B_4 \Rightarrow B_{net} = \frac{\mu_0 i}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\otimes$

5. (*d*) Directions of currents in two parts are different, so directions of magnetic fields due to these currents are opposite. Also applying Ohm's law across AB



$i_1R_1 = i_2R_2 \Rightarrow i_1l_2 = i_2l_2$

$\left(\because R = \rho\frac{l}{A}\right)$

Also $B_1 = \frac{\mu_0}{4\pi}\times\frac{i_1l_1}{r^2}$ and $B_2 = \frac{\mu_0}{4\pi}\times\frac{i_2l_2}{r^2}$

$(\because l = r\theta)$

$\therefore\quad \frac{B_2}{B_1} = \frac{i_1l_1}{i_2l_2} = 1$

Hence, two field induction's are equal but of opposite direction. So, resultant magnetic induction at the centre is zero and is independent of θ.

6. (*b*)

7. (*b*) Because for inside the pipe $i = 0$

$\therefore\ B = \frac{\mu_0 i}{2\pi r} = 0$

8. (*c*) The magnetic field in the solenoid along its axis

(i) At an internal point = $\mu_0 ni$

$= 4\pi\times10^{-7}\times5000\times4 = 25.1\times10^{-3}\ Wb/m^2$

(Here n = 50 turns/ cm = 5000 turns / m)

(ii) At one end

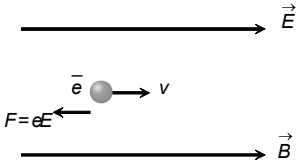
$B_{end} = \frac{1}{2}B_{in}\ \frac{\mu_0 ni}{2} = \frac{25.1\times10^{-3}}{2} = 12.6\times10^{-3}\ Wb/m^2$

9. (*b*) Magnetic field at the centre of solenoid (B)= $\mu_0 ni$

Where *n* = Number of *turns /meter*

$\therefore\quad B = 4\pi\times10^{-7}\times4250\times5 = 2.7\times10^{-2}\ Wb/m^2$

10. (*d*) Since electron is moving is parallel to the magnetic field, hence magnetic force on it $F_m = 0$..



The only force acting on the electron is electric force which reduces it's speed.

11. (*b*) $B = \frac{mv}{qr} = \frac{9\times10^{-31}\times10^6}{1.6\times10^{-19}\times0.1} = 5.6\times10^{-5}\,T$

12. (*b*) This is according to the cross product $\vec{F} = q(\vec{v}\times\vec{B})$ otherwise can be evaluated by the left-hand rule of Fleming.

13. (*d*) $r = \frac{\sqrt{2mK}}{qB} \Rightarrow K \propto \frac{q^2}{m}$

$\Rightarrow \frac{K_p}{K_d} = \left(\frac{q_p}{q_d}\right) \times \frac{m_d}{m_p} = \left(\frac{1}{1}\right)^2 \times \frac{2}{1} = \frac{2}{1}$

$\Rightarrow K_p = 2 \times 50 = 100$ keV.

14. (*a*) Lorentz force is given by

$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{v} \times \vec{B}) = q[\vec{E} + (\vec{v} \times \vec{B})]$

15. (*c*) $r = \frac{1}{B}\sqrt{\frac{2mV}{q}} \Rightarrow r \propto \sqrt{\frac{m}{p}} \Rightarrow \frac{r_x}{r_y} = \sqrt{\frac{m_x}{q_x} \times \frac{q_y}{m_y}}$

$\Rightarrow \frac{R_1}{R_2} = \sqrt{\frac{m_x}{m_y} \times \frac{2}{1}} \Rightarrow \frac{m_x}{m_y} = \frac{R_1^2}{2R_2^2}$

16. (*d*) The component of velocity perpendicular to *H* will make the motion circular while that parallel to *H* will make it move along a straight line. The two together will make the motion helical.

17. (*d*) The deflection produced by the electric field may be nullified by that produced by magnetic field.

18. (*b*) $r = \frac{mv}{qB} \Rightarrow r \propto mv$ (*q* and *B* are constant)

$\because r_A > r_B \Rightarrow m_A v_A > m_B v_B$

19. (*c*) $W = F.D \cos 90^\circ = 0$

20. (*d*) Magnetic field produced by wire at the location of charge is perpendicular to the paper inwards. Hence by applying Fleming's left hand rule, force is directed along OY.

21. (1) $F = \frac{\mu_0}{4\pi}\frac{2 \times i_1 i_2}{a} = \frac{10^{-7} \times 2 \times 5 \times 5}{0.1} = 5 \times 10^{-5}$ N/m

22. (*a*) The magnetic moment of current carrying loop

$M = niA = ni(\pi r^2)$

Hence the work done in rotating it through 180°

$w = MB(1 - \cos\theta) = 2MB = 2(ni\pi r^2)B$

$= 2 \times (50 \times 2 \times 3.14 \times 16 \times 10^{-4}) \times 0.1 = 0.1$ J

23. (*c*) $M = i\pi r^2$

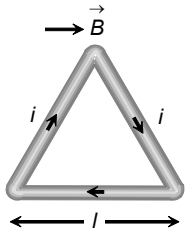
24. (*c*)

25. (*b*) $i = \frac{C\theta}{NAB} \Rightarrow i \propto \theta$

26. (*b*) $w = MB(\cos\theta_1 - \cos\theta_2)$

$= (NiA)B(\cos 0^\circ - \cos 180^\circ) = 2NAIB$

27. (*d*)Since $\theta = 90°$



Hence $\tau = NIAB = 1 \times I \times \left(\frac{\sqrt{3}}{4}l^2\right)B$

$= \frac{\sqrt{3}}{4}l^2BI$

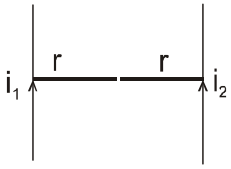
Learning Plus

1. (*c*) $i_1 > i_2$

$\frac{\mu_0}{2r}(i_1 - i_2) = 20$

$\frac{\mu_0}{2r}(i_1 + i_2) = 30$

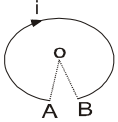
$\frac{i_1 + i_2}{i_1 - i_2} = \frac{3}{2} \Rightarrow \frac{i_1}{i_2} = \frac{5}{1}$



2. (*a*) $B = \frac{\mu_0 i}{4\pi R'}(2\pi - \theta)$

where; $(2\pi - \theta)R' = 2\pi R$

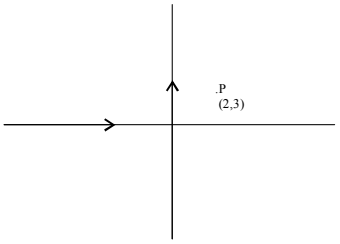
$R' = \frac{2\pi R}{2\pi - \theta}$



$B = \frac{\mu_0 i}{2R}\left(\frac{2\pi - \theta}{2\pi}\right)^2$

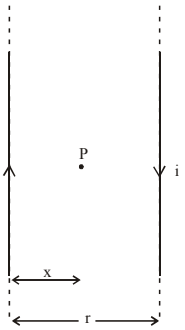
3. (*b*) Zero, because magnetic field due to each wire will be cancelled by another wire.

4. (*c*)



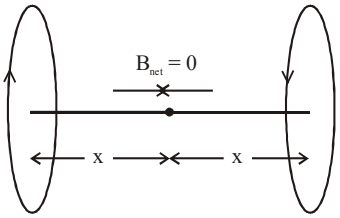
$B_{net} = \frac{\mu_0 I}{2\pi(2)} - \frac{\mu_0 I}{2\pi(3)}$, $B_{net} = \frac{\mu_0 I}{12\pi}\otimes$

5. (*b*)

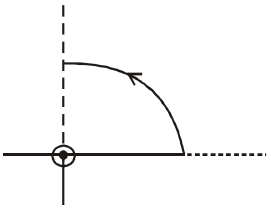


At point P $\frac{\mu_0 i}{2\pi}\left[\frac{1}{x}+\frac{1}{r-x}\right]$

6. (*b*)



7. (*d*)

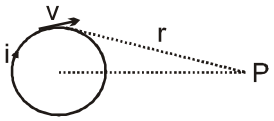


$B = \frac{\mu_0 i}{8R}$

from the above in the given Ques.

$B = \frac{\mu_0 i}{8}\left[\frac{1}{R}+\frac{3}{R'}\right]$

8. (*a*) $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$



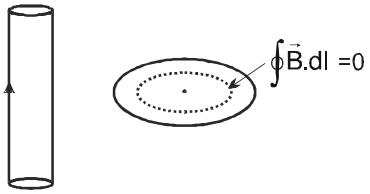
Magnitude fixed but direction keeps on changing

9. (*b*) $\oint\vec{B}.\vec{dl} = \mu_0\frac{i}{\pi R^2}\times\pi r^2 = \frac{\mu_0 i r^2}{R^2}$



$\oint\vec{B}.\vec{dl} = \mu_0 i$

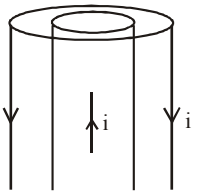
10. (*b*)



$\oint\vec{B}.\vec{dl} = 0$

$B = 0$

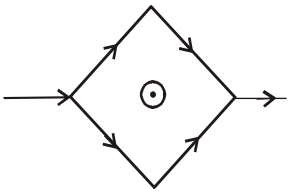
11. (*a*)



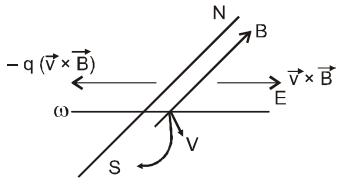
Inside the conductor magnetic field due to both have same direction so we add them.

Out side the conductor magnetic field due to both have opposite direction. so we subtract them.

12. (*d*)



13. (*b*)

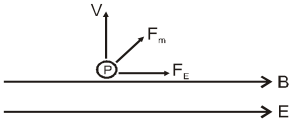


F towards west

So particle will be deflected towards west

14. (*d*) $F_E = qE$, $F_m = qvB$

$R = \frac{mv}{qB}$



Pitch $p = V_{\|} T$

$T = \frac{2\pi R}{V}$

$V_{\|} = 0 + \frac{qE\ t}{m}$

15. (*b*) $R \propto \frac{m}{q}$

$R_p : R_e : R_\alpha = \frac{m_P}{q} : \frac{m_e}{q} : \frac{4m_P}{2q}$.

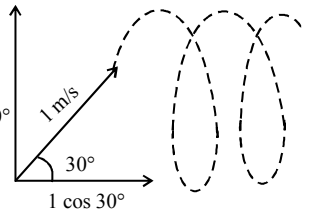
$R = \frac{mv}{qB}$

α-particle has maximum R, so the path followed is B.

16. (*b*) $R = \frac{mv}{qB}$

$R \propto v$

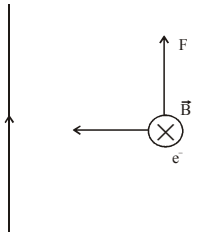
17. (*b*) 1 sin 30°



Pitch = $V_{||}$. T

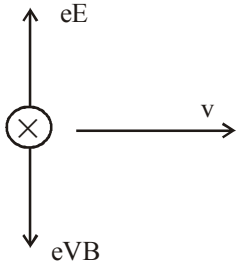
$= V\cos\theta \cdot \frac{2\pi m}{qB} = \frac{\sqrt{3}}{2} \times 2\pi = \sqrt{3}\pi$

18. (*a*)

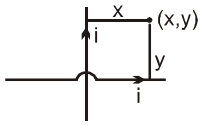


Applying right hand thumb rule.

19. (*b*) Force on electron due to electric field is in positive y-direction so force due to magnetic field should be in negative y-direction. Hence direction of magnetic field should be in –ve z-direction.



20. (*a*) $\frac{\mu_0 i}{2\pi x} = \frac{\mu_0 i}{2\pi y}$

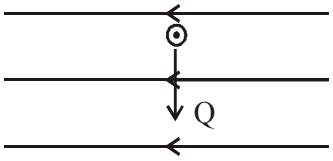


$y = x$

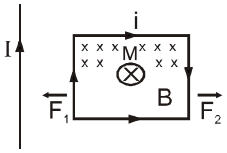
only in first quadrant the fields will be oppositely directed.

21. (*b*) By formula $F = i\left(\vec{\ell} \times \vec{B}\right)$

direction of ℓ in direction of i.



22. (*c*) $\vec{M} \times \vec{B} = 0$



$\tau = 0$

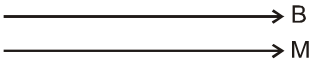
Loop will Not rotate

$F_1 > F_2$

So loop move towards the wire

23. (*b*) $U_i = -MB$

$U_f = MB$

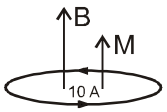


$W = \Delta U = 2MB$

$= 2 \times 2.5 \times 0.2$

$= 1$ J

24. (*a*)



$\vec{\tau} = \vec{M} \times \vec{B} = 0$

25. (*b*) $i = qf = \frac{qv}{2\pi r}$

$T = \frac{2\pi r}{v}$

$M.M. = i\pi r^2 = \frac{qvr}{2}$

Advanced Level Multiconcept Questions

1. (*b,d*)

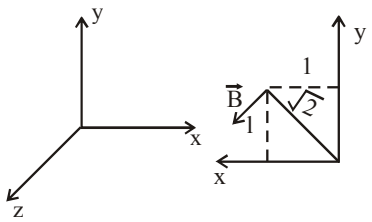
In point A & C

$r = 1$ m

In point B & D

$r = \sqrt{2}$ m

2. (*a,d*)



$B = \frac{\mu_0 i}{2\sqrt{2}\pi}$

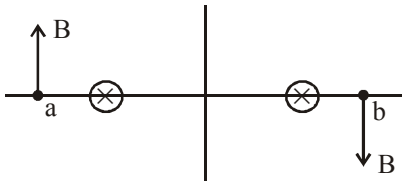
As direction of magnetic field is $\perp$ to line joining wire and point hence angle between xy plane & magnetic field is 45º.

3. (*a,b,c,d*)

(*a*) Direction of magnetic field produced due to the two wires on x axis have opposite direction

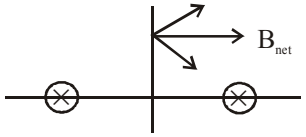
$\Rightarrow B_{net} = 0.$

(*b*)



a & b have only z component.

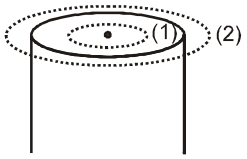
(*c*)



B_{net} has only y component as z component gets cancelled

(*d*) $B_x = 0$ in net B

4. (*b,c,d*)



Loop (1)

$B(2\pi r) = 0$

$B = 0$

Loop (2)

$B(2\pi r) = \mu_0 i$

$B \propto \frac{1}{r}$

5. (*a,d*)

$T = \frac{2\pi m}{qB}; \frac{T_1}{T_2} = 1$

$r_1 = \frac{mV\sin 30°}{qB}; r_2 = \frac{mV\sin 60°}{qB}$

$b = \frac{1}{\sqrt{3}}$

$Pitch_1 = v\cos 30° T_1$

$Pitch_2 = v\cos 60° T_2$

$abc = 1$

$c = \sqrt{3}$

$a = bc$

6. (*c,d*)

W. D. by mag. field is zero

$F_{mg} = q(\vec{v} \times \vec{B})$

7. (*c,d*)

$R = \frac{mv}{eB} = \frac{P}{eB}$

Energy gained = 0

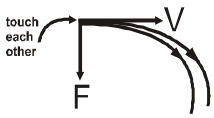
As $W_B = 0$

$F_C = \frac{mv^2}{r} = evB = \frac{ePB}{m}$

8. (*b,d*)

$R = \frac{mv}{qB}$

More q means less R



$\left(\frac{R_1}{R_2}\right) = \left(\frac{q_2}{q_1}\right)$

9. (*a,b,d*)

$\omega_E + \omega_B = \Delta k$

$\Rightarrow qE(2a) = \frac{1}{2} m(2v)^2 - \frac{1}{2} mv^2$

$= \frac{3}{2} mv^2$

$E = \frac{3}{4} \frac{mv^2}{qa}$

At P Rate of work done by $E = qEv = \frac{3}{4} \frac{mv^3}{a}$

At Q Rate of work done by $E = qE(2v)\cos 90° = 0$

At Q Rate of work done by B = 0

10. (*a,b,c*)

$\vec{V}$ constant in direction and may be in magnetude

$\vec{a} = 0$

$q\vec{E} + q(\vec{V} \times \vec{B}) = 0$

Ist posibility

$\vec{E} = 0$ & $\vec{B} = 0$ $\longrightarrow$ V

IInd posibility

$\vec{E} = 0$ & $\vec{V} \parallel \vec{B}$ i.e. $\vec{B} \neq 0$

IIIrd posibility $\longrightarrow$ V

$\longrightarrow$ E

$\vec{E} \parallel \vec{V}$ & B = 0

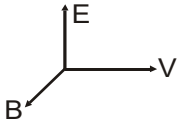
IVth posibility

$\longrightarrow$ B

$\longrightarrow$ V $\quad \vec{E} \parallel \vec{V} \parallel \vec{B}$

$\longrightarrow$ E $\quad \vec{V} \times \vec{B} = 0$

Vth posibility



$q\vec{E} = -q(\vec{V} \times \vec{B})$

11. (*b*) When charge is accelerated by electric field it gains energy for first time $KE_1 = \frac{qV}{2}$

for second time $KE_2 = \frac{3}{2} qV$

for third time $KE_3 = \frac{5}{2} qV$

hence the ratio of radii are

$$r_1 : r_2 : r_3 : \ldots\ldots\ldots\ldots :: \frac{\sqrt{2m\frac{qv}{2}}}{qB} : \frac{\sqrt{2m\frac{3}{2}qv}}{qB} : \ldots\ldots\ldots$$

$$r_1 : r_2 : r_3 \ldots\ldots\ldots :: \sqrt{1} : \sqrt{3} : \sqrt{5} \ldots\ldots\ldots$$

12. (*a*) In one full cycle it gets accelerated two times so change in KE = 2 qV.

13. (*a*) $f = \frac{qB}{2\pi m} \Rightarrow 10^6 = \frac{10^6 B}{2\pi} \Rightarrow 2\pi T.$

14. (*a*) Distance travelled by particle in one time period :

$$\pi(r_1 + r_2) : \pi(r_3 + r_4) : \pi(r_5 + r_6) \ldots\ldots\ldots\ldots$$

$$:: \frac{\sqrt{2m\frac{qV}{2}}}{qB} + \frac{\sqrt{2m\frac{3qV}{2}}}{qB} : \frac{\sqrt{2m\frac{5qV}{2}}}{qB} + \frac{\sqrt{2m\frac{7qV}{2}}}{qB} : \frac{\sqrt{2m\frac{9qV}{2}}}{qB} + \frac{\sqrt{2m\frac{11qV}{2}}}{qB} \ldots\ldots\ldots$$

$$S_1 : S_2 : S_3 \ldots\ldots :: (\sqrt{1} + \sqrt{3}) : (\sqrt{5} + \sqrt{7}) : (\sqrt{9} + \sqrt{11})$$

15. (*c*) Frequency of A.C. depends on charge and mass only so it can be tuned by magnetic field only.

16. (*c*) Inside the cylinder

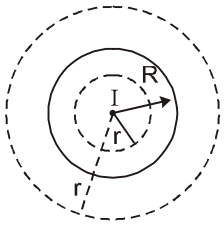
$$B.2\pi r = \mu_0 . \frac{I}{\pi R^2} \pi r^2$$

$$B = \frac{\mu_0 I}{2\pi R^2} . r \quad \ldots\ldots\ldots\ldots (1)$$

outside the cylinder

$$B.2\pi r = \mu_0 I$$

$$\therefore B = \frac{\mu_0 I}{2\pi r} \quad \ldots\ldots\ldots\ldots (2)$$



Inside cylinder B α r and outside B $\alpha \frac{1}{r}$

So at the surface nature of magnetic field changes.

Hence clear from graph, wire 'c' has greatest radius.

17. (*a*) Magnitude of magnetic field is maximum at the surface of wire 'a'.

18. (*a*) Inside the wire

$$B(r) = \frac{\mu_0}{2\pi} . \frac{I}{R^2} . r = \frac{\mu_0 J r}{2}$$

$$\frac{dB}{dr} = \frac{\mu_0 J}{2}$$

i.e. slope $\propto J \propto$ current density

It can be seen that slope of curve for wire a is greater than wire C.

19. (*a*)-p,q,r (*b*)-p,q,r,s (*c*)-r (*d*)-p,q,r,s

The magnetic field is along negative y-direction in p,q,r. Hence z-component of magnetic field is zero in all cases.

The magnetic field at P is $\frac{\mu_0}{4\pi} \frac{i}{d}$ for case (r)

The magnetic field at P is less than $\frac{\mu_0}{2\pi} \frac{i}{d}$ for all cases.

20. (*a*)-p,q; (*b*)-p,r; (*c*)-p; (*d*)-p,q,s

The Force on a magnetic dipole placed in uniform magnetic field is zero. Hence option p is common to all the four situations. Torque on magnetic dipole is $\vec{\tau} = \vec{\mu} \times \vec{B}$ and potential energy of dipole in external magnetic $U = -\vec{\mu} \times \vec{B}$

(*a*) Since $\theta = 0$, therefore $\tau = 0$

(*b*) Since $\theta = \pi/2$, therefore $\tau = \mu B$

(*c*) Since θ is acute, torque is non zero and less than μB in magnitude

(*d*) Since $\theta = \pi$, therefore $\tau = 0$ and $U = \mu B$

21. [0.16] $\vec{F} = q\vec{v} \times \vec{B}$,

Where q has a positive or negative sign, on substituting, we get

$$-0.16\hat{i} - 0.32\hat{j} + -0.64\hat{k}$$

22. [19.3] $T = eV = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2eV}{m}}$$

$$\Rightarrow v_H = \sqrt{\frac{2eV}{m}} \cos\alpha \text{ and}$$

$$v_v = \sqrt{\frac{2eV}{m}} \sin\alpha$$

$$\text{Now, } \frac{mv_v^2}{r} = Bev_v \text{ or } r = \frac{mv_v}{Be}$$

$$\text{and } T = \frac{2\pi r}{v_v} = \frac{2\pi m}{Be}$$

Pitch

$$P = v_H T = \frac{2\pi m}{Be} \sqrt{\frac{2eV}{m}} \cos\alpha = 0.02m$$

23. [19.5] $\frac{-d\phi}{dt} + \frac{Ldt}{dt} = 0$

$$\Rightarrow \frac{d\phi}{dt} = \frac{Ldt}{dt}$$

$$\therefore \Delta\phi = L\Delta I$$

24. [1.18] $\tau = ni\ \vec{A} \times \vec{B}$

$$\Rightarrow \tau = ni\ AB \sin\theta$$

25. [7.14] Magnetic field induction at 0 due to current through ACB is

$$B_1 = \frac{\mu_0}{4\pi} \frac{I\theta}{r}$$

It is acting perpendicular to the paper downwards.

Magnetic field induction at 0 due to current through ABD is

$$B_1 = \frac{\mu_0}{4\pi} \frac{I}{r} (2\pi - \theta)$$

26. [30°] From the figure,

$$\text{Sin}\,\alpha = \frac{d}{R} = \frac{dqB}{mv},$$

As radius of the arc $R = \frac{mv}{qB}$, where v is the velocity of the particle, when it enteres into the field. From intial condition of the problem,

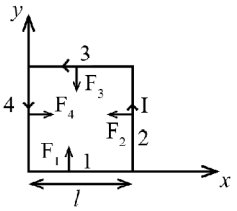
$$qV = \frac{1}{2}mv^2 \text{ or, } v = \sqrt{\frac{2qV}{m}}$$

$$\text{Hence, } \sin\alpha = \frac{dqB}{m\sqrt{\frac{2qV}{m}}} = dB\sqrt{\frac{q}{2mV}}$$

and $\alpha = \sin^{-1}\left(dB\sqrt{\frac{q}{2mV}}\right) = 30°$, on putting the values.

27. Given, [0001] $\vec{B} = B_0\left(1+\frac{x}{l}\right)\hat{K}$

To find Magnetic force at loop=?



sol^N $F_1 = F_3$

$F_4 = IBl.$ {B at $x = 0 \Rightarrow B_0$)

$= IBol$

$F1 = IBl.$ {B at $x = 1 = 2B_0$

$= I2B_0l.$

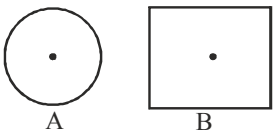
$F_{NCI} = 2B_0lI - IB_0lI = B_0lI$

28. [0001]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*d*) $\frac{B_A}{B_B} = \frac{\frac{\mu_0 i}{2R}}{4\times\frac{\mu_0 i}{4\pi\frac{a}{2}}(\sin 45° + \sin 45°)}$



(where $R = \frac{l}{2\pi}$ and $a = \frac{l}{4}$)

$$\Rightarrow \frac{B_A}{B_B} = \frac{1}{2R}\frac{\pi a}{2}\frac{1}{\sqrt{2}} = \frac{\pi}{4\sqrt{2}}\left(\frac{a}{R}\right)$$

$$= \frac{\pi}{4\sqrt{2}}\left(\frac{l}{4}\right)\left(\frac{2\pi}{l}\right) = \frac{\pi^2}{8\sqrt{2}}$$

2. (*a*) Radius of circular path in magnetic field is given by $R = \frac{\sqrt{2Km}}{qB}$

where K = kinetic energy of particle
m = mass of particle
q = charge on particle
B = magnetic field intensity
R = radius of path

For electron

$$r_e = \frac{\sqrt{2Km_e}}{eB} \quad ...(i)$$

For proton

$$r_p = \frac{\sqrt{2Km_p}}{eB} \quad ...(ii)$$

For α particle

$$r_\alpha = \frac{\sqrt{2Km_\alpha}}{q_\alpha B} = \frac{\sqrt{2K4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB} \quad ...(iii)$$

as $m_e < m_p$ so $r_e < r_p = r_\alpha$

3. (*b*) Dipole moment of circular loop is m

$m_1 = I.A = I.\pi R^2$ {R = radius of the loop}

$$B_1 = \frac{\mu_0 I}{2R}$$

As moment becomes double $\Rightarrow$ radius becomes $\sqrt{2}\,R$ (keeping current constant)

$m_2 = I.\pi\,(\sqrt{2}\,R)^2 = 2.I\pi R^2 = 2m_1$

$$B_2 = \frac{\mu_0 I}{2(\sqrt{2}R)} = \frac{B_1}{\sqrt{2}}$$

$$\Rightarrow \frac{B_1}{B_2} = \sqrt{2}$$

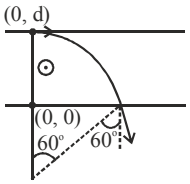
4. (Bonus)
Assuming particle enters from (0, d)

$F = qvB\left(-\sin 60^\circ \hat{i} - \cos 60^\circ \hat{j}\right)$

$F = -\frac{qvB}{2}\left(\sqrt{3}\hat{i} + \hat{j}\right)$

$a = -\frac{qvB}{2m}\left(\sqrt{3}\hat{i} + \hat{j}\right)$

None of the option is correct.



5. (*d*) $r = \frac{mv}{Bq} = \frac{\sqrt{2mqV}}{Bq} = \frac{\sqrt{2\times 9.1\times 10^{-31}\times V}}{B\sqrt{q}}$

$= \frac{\sqrt{2\times 9.1\times 10^{-31}\times 500}}{100\times 10^{-3}\sqrt{1.6\times 10^{-19}}}$

$= \frac{1}{100\times 10^{-3}}\frac{\sqrt{2\times 9.1\times 500\times 10^{-12}}}{\sqrt{1.6}}$

$= \frac{75.4\times 10^{-6}}{100\times 10^{-3}} = 7.5\times 10^{-4}\,m$

6. (*d*) $eE = evB$

$\Rightarrow E = \left(\frac{eBr}{m}\right)B$

$\Rightarrow m = \frac{eB^2r}{E}$

$\Rightarrow m = \frac{(1.6\times 10^{-19})(0.5)^2(0.5\times 10^{-2})}{100} = 2\times 10^{-24}\,kg$

7. (*d*) $B_L = \frac{\mu_0 i}{2R}$

$B_C = \frac{\mu N i}{2(R/N)}$

$\therefore \frac{B_L}{B_C} = \frac{1}{N^2}$

8. (*d*) For shifting of loop along x-direction $PE(x) = -\vec{\mu}\cdot\vec{B}$

$\therefore PE(x) = -\pi a^2 i\frac{\mu_0 I_0}{2\pi x}$

$\therefore U(x) = -\frac{\mu_0 i I_0 a^2}{2x}$ (PE decreases as it comes closer to wire)

So, attractive force $F(x) = \frac{-dU}{dx} = \frac{\mu_0 i I_0 a^2}{2}\left(\frac{-1}{x^2}\right)$

$\therefore F(x) = \frac{\mu_0 i I_0 a^2}{2d^2}$ (Attractive)

$\therefore F \propto \frac{a^2}{d^2}$

9. (*a*) $\because\ T = 2\pi\sqrt{\frac{I}{\mu B}}$

$\frac{T_h}{T_c} = \sqrt{\frac{I_h}{I_c}\times\frac{\mu_c}{\mu_h}}$

$= \sqrt{2\times\frac{1}{2}} = 1$

$T_h = T_c$

10. (*c*) $dM = di\,A$

$= \left(\frac{dq\omega}{2\pi}\right)\pi x^2 = (\rho dx)\frac{\omega}{2\pi}\pi x^2$

$M = \int_0^{\ell} dM = \int_0^{\ell}\frac{\rho_0 x}{\ell}\left(x^2 dx\right)\left(\frac{\omega}{2}\right) = \frac{\rho_0}{2\ell}(2\pi n)\left(\frac{\ell^4}{4}\right) = \frac{\rho_0(\pi n)\ell^3}{4}$

11. (*c*) $B = 2\left[\frac{\mu_0 i}{4\pi d}(\cos\theta_1 - \cos\theta_2)\right]$

$B = 10^{-4}$

$\mu_0 = 4\pi\times 10^{-7},\ \theta_1 = 90^\circ,\ \theta_2 = 180^\circ$

$d = 4\times 10^{-2}$

$\Rightarrow$ = 20 A (into the page)

12. (*a*) $m_p = m$ $\quad q_p = q$ $\quad K_p = qV = K$

$m_\alpha = 4m$ $\quad q_\alpha = 2q$ $\quad K_\alpha = 2qV = 2K$

Radius of circular path,

$r = \frac{mv}{qB} = \frac{\sqrt{2Km}}{qB} = \sqrt{\frac{2mV}{qB^2}}\ \Rightarrow\ \frac{r_P}{r_\alpha} = \frac{1}{\sqrt{2}}$

13. (*a*) $\vec{\tau} = \vec{M}\times\vec{B}$

$\Rightarrow |\vec{\tau}| = MB\sin 90^\circ = C\theta$

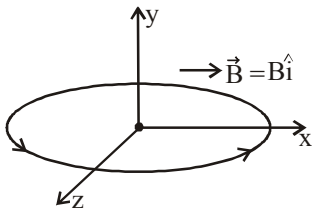
$\Rightarrow MB = C\theta$

$\Rightarrow C\theta = i\,N\,A\,B$

$\Rightarrow 10^{-6}\times\frac{\pi}{180} = 10^{-3}\times 10^{-4}\times 175\times B$

$\Rightarrow B = 10^{-3}$ Tesla.

14. (*a*)

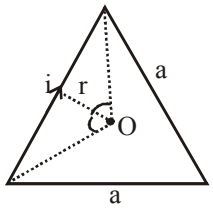


Magnetic moment of coil = $NIA\,\hat{j} = NI\left(\pi r^2\right)\hat{j}$

Torque on loop (coil) = $\vec{M}\times\vec{B} = NI(\pi r^2)B\sin 90^\circ\left(-\hat{k}\right)$

$= NI\pi r^2 B\left(-\hat{k}\right)$

15. (*a*) $B = 3\left[\frac{\mu_0 i}{4\pi r}(\sin 60^\circ + \sin 60^\circ)\right]$



Here, $r = \frac{a}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$

$B = 3\left[\frac{4\pi\times10^{-7}\times10\times2\sqrt{3}}{4\pi\times1}\left[\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right]\right]$

$B = 18\times10^{-6} = 18\mu T$

16. (*b*) $m = NIA = 1\times I\times a^2 = Ia^2$

here a = side of square

Now, $4a = 2\pi r$

$r = \frac{2a}{\pi}$

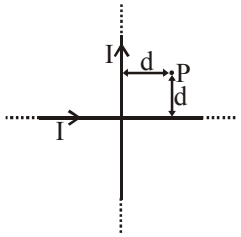
For circular loop

$m' = 1\times I\times\pi r^2$

$= 1\times I\times\pi\times\left(\frac{2a}{\pi}\right)^2 = \left(Ia^2\right)\frac{4}{\pi}$

$m' = \frac{4m}{\pi}$

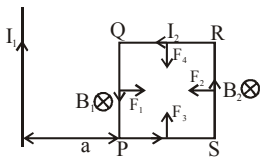
17. (*a*)



Magnetic field at point P

$\vec{B}_{net} = \frac{\mu_0 i}{2\pi d}\left(-\hat{k}\right)+\frac{\mu_0 i}{2\pi d}\left(\hat{k}\right) = 0$

18. (*b*)



F_3 & F_4 cancel each other

Force on PQ will be $F_1 = I_2 B_1 a$

$= I_2\ \frac{\mu_0 I_1}{2\pi a}a\ = \frac{\mu_0 I_1 I_2}{2\pi}$

Force on RS will be $F_2 = I_2 B_2 a$

$= I_2\ \frac{\mu_0 I_1}{2\pi 2a}a$

$= \frac{\mu_0 I_1 I_2}{4\pi}$

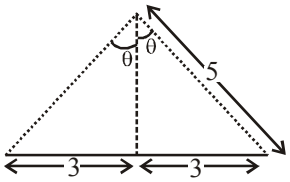
Net force $= F_1 - F_2 = \frac{\mu_0 I_1 I_2}{4\pi}$ repulsion

19. (*b*) $|\vec{\tau}| = |\vec{M}\times\vec{B}|$

$\tau = NI\times A\times B\times\sin45^\circ$

$\tau = 0.27\,Nm$

20. (*d*)



$\sin\theta = \frac{3}{5}$, $d = 4$ cm

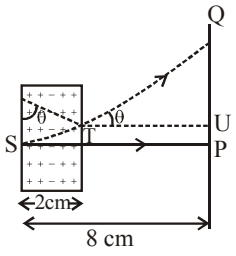
$B = \frac{\mu_0 I}{4\pi d}2\sin\theta = \frac{10^{-7}\times5\times2\times3}{4\times10^{-2}\times5} = \frac{3}{2}\times10^{-5}$

$= 15\times10^{-6} = 15\,\mu T$

21. (*a*) $R = \frac{mv}{qB} = \frac{\sqrt{2m(K.E.)}}{qB}$

$R = \frac{\sqrt{2\times9.1\times10^{-31}\times\left(100\times1.6\times10^{-19}\right)}}{1.6\times10^{-19}\times1.5\times10^{-3}}$

$R = 2.248$ cm



$\sin\theta = \frac{2}{2.248} = 0.89$

$\tan\theta = \frac{QU}{TU}$

$\frac{2}{1.026} = \frac{QU}{6}$

$QU = 11.69$

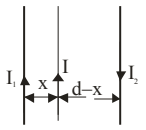
$PU = R(1-\cos\theta) = 1.22$

$d = QU + PU$

22. (*c*) $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

$\Rightarrow r_{He} = r_p > r_e$

23. (*b*)



Net force on wire carrying current I per unit length is

$$\frac{\mu_0 I_1 I}{2\pi x}+\frac{\mu_0 I_2 I}{2\pi(d-x)}=0$$

$$\frac{I_1}{x}=\frac{I_2}{(x-d)} \Rightarrow x=\frac{I_1 d}{I_1-I_2}$$

24. (*c*) Force due to electric field is in direction $-\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

because at t = 0, $E=-\frac{(\hat{i}+\hat{j})}{\sqrt{2}}E_0$

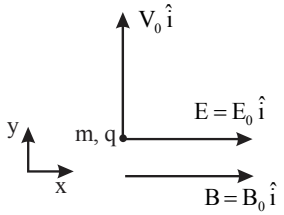
Force due to magnetic field is in direction $q(\vec{v}\times\vec{B})$

and $\vec{v}\parallel\hat{k}$

$\therefore$ it is parallel to $\vec{E}$

$\therefore$ net force is antiparallel to $\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$.

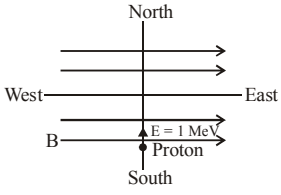
25. (*d*)



$$(2V_0)^2=v_0^2+v_x^2 \quad ; v_x=\sqrt{3}v_0$$

$\therefore$ $\sqrt{3}v_0=0+\frac{eE}{m}t \quad ; t=\frac{mv_0\sqrt{3}}{qE}$

26. (*c*)



$\because$ K.E. $=1.6\times10^{-13}=\frac{1}{2}\times1.6\times10^{-27}v^2$

$v=\sqrt{2}\times10^7$ m/s

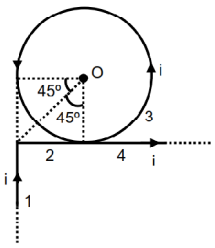
$\therefore$ Bav = ma

$$B=\frac{1.6\times10^{-27}\times10^{12}}{1.6\times10^{-19}\times\sqrt{2}\times10^7}$$

$=0.71\times10^{-3}$ T

So, 0.71 mT

27. (*a*)



$$\vec{B}_0=(\vec{B}_0)_1+(\vec{B}_0)_2+(\vec{B}_0)_3+(\vec{B}_0)_4$$

$$\frac{\mu_0 i}{4\pi R}[\sin90^\circ-\sin45^\circ]\otimes+\frac{\mu_0 i}{2R}\odot+\frac{\mu_0 i}{4\pi R}(\sin45^\circ+\sin90^\circ)\odot$$

$$-\frac{-\mu_0 i}{4\pi R}\left[1-\frac{1}{\sqrt{2}}\right]+\frac{\mu_0 i}{2R}+\frac{\mu_0 i}{4\pi R}\left[\frac{1}{\sqrt{2}}+1\right]\odot$$

$$-\frac{\mu_0 i}{4\pi R}\left[-1+\frac{1}{\sqrt{2}}+2\pi+\frac{1}{\sqrt{2}}+1\right]\odot$$

$$=\frac{\mu_0 i}{4\pi R}\left[\sqrt{2}+2\pi\right]\odot$$

$$=\frac{\mu_0 i}{2\pi R}\left[\frac{1}{\sqrt{2}}+\pi\right]\odot$$

28. (*a*)

(*a*) by work energy theorem

$$W_{mag}+W_{ele}=\frac{1}{2}m(2v)^2-\frac{1}{2}m(v)^2$$

$$0+qE2a=\frac{3}{2}mv^2$$

$$E=\frac{3}{4}\frac{mv^2}{qa}$$

(*b*) Rate of work done at A = power of electric force

$$=qE_0V$$

$$=\frac{3}{4}\frac{mv^2}{a}$$

(*c*) at Q, $\frac{dW}{dt}=0$ for both forces

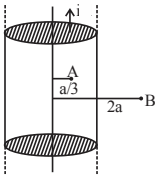
(*d*) $\Delta\vec{L}=(-m2v\,2a\,\hat{k})-(-mva\,\hat{k})$

$|\Delta\vec{L}|=3mva$

29. (*d*) $B_A=\frac{\mu_0 ir}{2\pi a^2}=\frac{\mu_0 i\frac{a}{3}}{2\pi a^2}=\frac{\mu_0 i}{\pi a^2}\frac{a}{6}=\frac{\mu_0 i}{6\pi a}$

$$B_B=\frac{\mu_0 i}{2\pi(2a)}$$

$$\frac{B_A}{B_B}=\frac{4}{6}=\frac{2}{3}$$



30. (*a*) $R_{max} = \frac{R}{2} = \frac{mv_{max}}{e\mu_0 in}$

$$V_{max} = \frac{Re\mu_0 in}{2m}$$

31. (*d*) $\tau = MB\ \sin\theta = I\alpha$

$$\pi R^2 I\ B\theta = \frac{mR^2}{2}\alpha$$

$$\omega = \sqrt{\frac{2\pi IB}{m}} = \frac{2\pi}{T}; \quad T = \sqrt{\frac{2\pi m}{IB}}$$

32. (*b*) $\vec{M} = \vec{M}_1 + \vec{M}_2$

$$\vec{M}_1 = abl\hat{j}$$

$$\vec{M}_2 = abl\hat{k}$$

$$\vec{M} = \sqrt{2}\,abl\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$$

33. (*d*) $(v_0\cos\theta) \times \Delta T = I$

and, $\Delta T = 10 \times \frac{2\pi m}{qB}$

$$\therefore\ 4\times10^5 \times \frac{1}{2} \times \frac{10\times2\pi\times1.67\times10^{-27}}{1.6\times10^{-19}\times0.3} = 1$$

$$\Rightarrow\ I = 0.44\,m$$

34. (*d*) Pitch = $v\cos\theta \times \frac{2\pi m}{qB}$

$$= 4\times10^5 \times \frac{1}{2} \times \frac{2\pi\times1.67\times10^{-27}}{1.69\times10^{-19}\times0.3} = 4\,cm$$

35. (20) $\vec{\mu} = NiA = 500\times3\times10^{-4}\times\frac{5}{10}$

$$\tau = \vec{\mu}\times\vec{\beta} = \mu B\sin\theta\ \ (\because \sin\theta = 1)$$

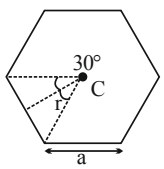
$$\therefore\ B = \frac{15\times10}{10\times500\times3\times5\times10^{-4}}$$

$$B = 20T$$

36. (*a*) $B_C = N.\left(6.\frac{\mu_0 i}{4\pi r}(\sin30° + \sin30°)\right.$

$$r = \frac{\sqrt{3}}{2}a = \frac{300}{4r}\frac{\mu_0 i}{\pi}$$

$$500\sqrt{3}\,\frac{\mu_0 i}{\pi}$$



37. (*d*) $F = q(\vec{V}\times\vec{B})$

$$= (1\times10^{-6})\,[2\hat{i}+3\hat{j}+4\hat{k}] \times [5\hat{i}+3\hat{j}-6\hat{k}] \times 10^{-3}$$

$$= (-30\hat{i}+32\hat{j}-9\hat{k})10^{-9}\,N$$

38. (Bonus)

$$|U_f - U_i| = \frac{1}{2}I\omega^2$$

$$20\times4\times\frac{\sqrt{3}}{2} - 0 = \frac{1}{2}\times(0.8)\omega^2$$

$$\Rightarrow \omega = 13.16\,rad/s$$

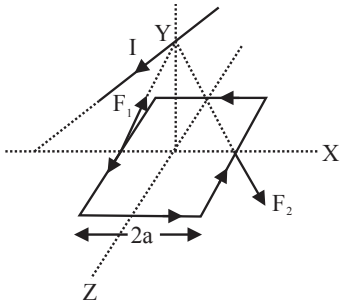
39. (*b*) $B_A = \frac{(\mu_0)(2)}{(2)(2)}\frac{3(\pi/2)}{2\pi}$ ...(1)

$$B_B = \frac{(\mu_0)(3)}{(2)(4)}\frac{5\pi/3}{2\pi}$$ ...(2)

From (1) and (2)

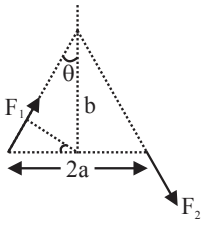
$$\frac{B_A}{B_B} = \frac{6}{5}$$

40. (*a*)



$F_1 = \frac{\mu_0 I^2}{2\pi r}2a$ toward wire

$F_2 = \frac{\mu_0 I^2}{2\pi r}2a$ away for wire



Torque about z-axis

$= F_1 \cdot a\cos\theta + F_2 \cdot a\cos\theta$

$$= 2\left(\frac{\mu_0 I^2 2a}{2\pi r}\right)a\cdot\frac{b}{r}$$

$$= \frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$$

$$= \frac{2\mu_0 I^2 a^2}{\pi b}\ \text{for}\ (b >> a)$$

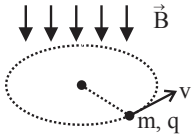
41. (*a*) $F_m^{max} = (q)VB_0$

$$\frac{E_0}{B_0} = c \Rightarrow B_0 = \left(\frac{E_0}{c}\right)$$

$$\Rightarrow F_m^{max} = (1.6 \times 10^{-19})(0.1\,c)\left(\frac{E_0}{c}\right)$$

$$= 4.8 \times 10^{-19} N$$

42. (*a*) $\vec{M} = \frac{1}{2}qvR(-\hat{B})$



$$= \frac{qv}{2}\left(\frac{mv}{qb}\right)\left(\frac{-\vec{B}}{\vec{B}}\right)$$

$$= \frac{mv^2}{2B^2}\vec{B}$$

43. (*c*) $\frac{dv_x}{dt} = \frac{k}{m}v_y$

$$\frac{dv_y}{dt} = \frac{k}{m}v_x$$

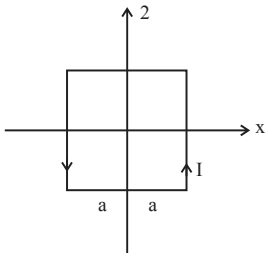
$$\Rightarrow \frac{dv_x}{dv_y} = \frac{v_y}{v_x} \Rightarrow v_x dx = v_y dy$$

$$\Rightarrow v_x^2 = v_y^2 + C$$

$$\vec{v} \times \vec{a}\left(v_x^2 - v_y^2\right)\frac{k}{m}$$

$$\vec{v} \times \vec{a} = \frac{kC}{m}$$

44. (*b*)



$$\tau(t_\perp)r$$

$$f_\perp = (I)\frac{\mu \cdot I}{2\pi\sqrt{a^2+b^2}}\frac{(b)2a}{\sqrt{a^2+b^2}}$$

$$\tau_{total} = \frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$$

45. (*d*) $r = \frac{mv}{qB_0}$

To not collide, $r < d$

$$\Rightarrow r = \frac{mv}{qB_0} < d$$

$$\therefore v_{max} = \frac{qB_0 d}{m}$$

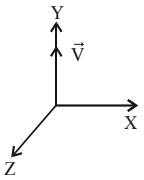
Note: It should be maximum instead of minimum.

46. (*a*) $F = (\vec{E} + \vec{V} \times \vec{B})$

$$\vec{E} + \vec{V} \times \vec{B} = 0$$

47. (*c*)

48. (*a*)



$\because \hat{E} \times \hat{B} = \hat{C}$

$\therefore \hat{E} \times \hat{B}$ should point in the direction of propagation of wave ($y-$ direction here)

$\therefore$ possible combinations are $\left(E_x, B_z\right)$ or $\left(E_z, B_x\right)$

49. (*a*) Force on each wire be along radially outward and equal so, it will take the shape of circle and parallel to the field.

50. (*c*) The proportion of the electric to attractive fields in an electromagnetic wave in free space is equivalent all the time to the speed of light. $E = B \times c$. where: B = Magnetic Field. c = speed of light.

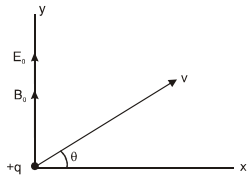
$f = 300\text{MHz}$

$$\vec{B} = 2 \times 10^{-9} T \Rightarrow \vec{E} = \vec{B} \times \vec{V} = (2 \times 10^8 \hat{k}) \times (3 \times 10^8 \hat{i})$$

$$= 6(\hat{k} \times \hat{i}) = 6\hat{j} V/m \Rightarrow \vec{E} = 6\hat{j} V/m$$

JEE-ADVANCED PREVIOUS YEAR'S

1. (*c,d*)



If $\theta = 0°$ then due to magnetic force path is circular but due to force qE_0 ($\uparrow$) q will have accelerated motion along y–axis. So combined path of q will be a helical path with variable pitch so (A) and (B) are wrong.

If $\theta = 10°$ then due to $v\cos\theta$, path is circular and due to qE_0 and $v\sin\theta$, q has accelerated motion along y-axis so combined path is a helical path with variable pitch (C) is correct.

If $\theta = 90°$ then $F_B = 0$ and due to qE_0 motion is accelerated along y-axis. (D)

2. (5) $B_1 = \frac{\mu_o J_a}{2} - \frac{\mu_o J_a}{12}$

$= \left(\frac{\mu_o Ja}{2}\right)\left(1-\frac{1}{6}\right) = \frac{5}{6}\left(\frac{\mu_o Ja}{2}\right) = \frac{5\mu_0 aJ}{12} = \frac{N}{12}\mu_0 aJ$

$N = 5$

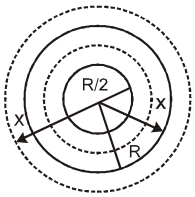
3. (*b*) $\text{Area} = a^2 + 4\times\frac{\pi\left(\frac{a}{2}\right)^2}{2} = a^2 + \frac{\pi a^2}{2}$

$A = \left(1+\frac{\pi}{2}\right) a^2\,\hat{k}$

4. (*d*) **Case-I** $x < \frac{R}{2}$

$|B| = 0$

Case-II $\frac{R}{2} \le x < R$



$\int \vec{B}.d\vec{\ell} = \mu_0 I$

$|B|\,2\pi x = \mu_0\left[\pi x^2 - \pi\left(\frac{R}{2}\right)^2\right] J$

$|B| = \frac{\mu_0 J}{2x}\left(x^2 - \frac{R^2}{4}\right)$

Case-III $x \ge R$

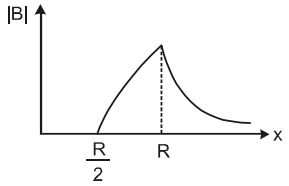
$\int \vec{B}.\,d\vec{\ell} = \mu_0 I$

$|B|\,2\pi x = \mu_0\left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right] J$

$|B| = \frac{\mu_0 J}{2x}\,\frac{3}{2}\,R^2$

$|B| = \frac{3\mu_0 JR^2}{8x}$

so



5. (*a,c*)
Component of final velocity of particle is in positive y direction.
Centre of circle is present on positive y axis. so magnetic field is present in negative z-direction
Angle of deviation is 30° because

$\tan\theta = \frac{v_y}{v_x} = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

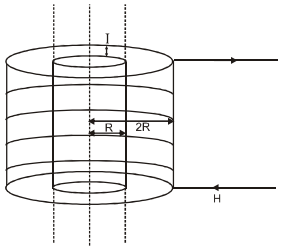
$\omega t = \theta$

$\theta = \frac{QB}{M}t$

$B = \frac{M\theta}{Qt}$

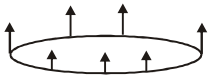
$B = \left(\frac{50M\pi}{3Q}\right)$

6. (*a,d*)

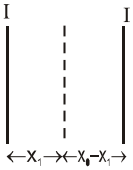


(*a*) For $0 < r < R \Rightarrow B \neq 0$

(*d*) For $r > 2R \Rightarrow B \neq 0$



7. (*c*)



$B_2 = \frac{\mu_{,0} I}{2\pi x_1} + \frac{\mu_0 I}{2\pi(x - x_1)}$ (opposite)

$B_1 = \frac{\mu_{,0} I}{2\pi x_1} - \frac{\mu_0 I}{2\pi(x - x_1)}$ (same)

Case - 1 When current is in the same direction

$B = B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$

$R_1 = \frac{mv}{qB_1}$

Case-2 When current is in oposite direction

$B = B_2 = \frac{9\mu_0 I}{4\pi x_0}$

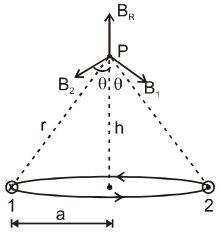
$R_2 = \frac{mv}{qB_2}$

$\frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$

8. (*c*) $\vec{B}_R = \vec{B}$ due to ring

$\vec{B}_1 = \vec{B}$ due to wire - 1

$\vec{B}_2 = \vec{B}$ due to wire - 2



In magnitudes $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi r}$

Resultant of B_1 and $B_2 = 2B_1\cos\theta = \dfrac{\mu_0 Ia}{\pi r^2}$

$B_R = \dfrac{2\mu_0 I\pi a^2}{4\pi r^3}$

For zero magnetic field at P

$\dfrac{\mu_0 Ia}{\pi r^2} = \dfrac{2\mu_0 I\pi a^2}{4\pi r^3}$

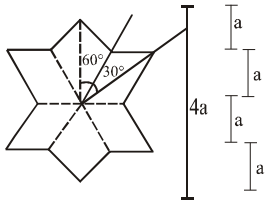
$\Rightarrow$ $h \approx 1.2a$

9. (*b*) Magnetic field at mid point of two wires = $\dfrac{\mu_0 I}{\pi d}\otimes$

Magnetic moment of loop = $I\pi a^2$

Torque on loop = M B sin 150° = $\dfrac{\mu_0 I^2 a^2}{2d}$

10. (*a*)



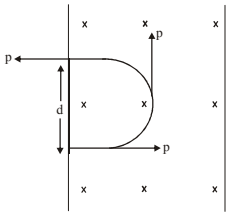
Total Mgnetic Field
at centre = 12 times magnetic field due to one wire

$$B = \frac{12\mu_0 I}{4\pi a}\left[\sin 60° - \sin 30°\right] = \frac{\mu_0 I}{4\pi a}\times 12\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$

$$\Rightarrow\ B = \frac{\mu_0 I}{4\pi a}\times 6\left(\sqrt{3}-1\right)$$

11. (*c,d*)

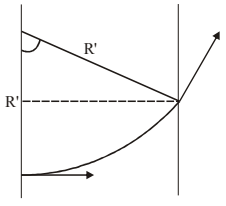
(*a*)



$|\Delta\vec{p}| = \sqrt{2}p$

(*b*) $R' = \dfrac{mv}{QB}$

$d = 2R' = \dfrac{2mv}{QB}$ $\quad d \propto m$



(*c*) $R'(1-\cos\theta) = R$

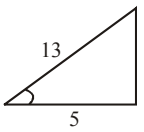
$R'\sin\theta = \dfrac{3R}{2}$

$\dfrac{\sin\theta}{1-\cos\theta} = \dfrac{3}{2}$

$\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \dfrac{3}{2}$

$\cot\dfrac{\theta}{2} = \dfrac{3}{2} \Rightarrow \tan\dfrac{\theta}{2} = \dfrac{2}{3}$

$$\Rightarrow \tan\theta = \frac{2\left(\frac{2}{3}\right)}{1-\frac{4}{9}} = \frac{\frac{4}{3}}{\frac{5}{9}} = \frac{4}{3}\times\frac{9}{5} = \frac{12}{5}$$



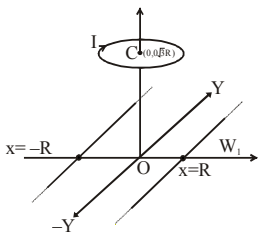
$\sin\theta = \dfrac{12}{13}$

$$R'\left(\frac{12}{13}\right) = \frac{3R}{2};\ R' = \frac{13R}{8} = \frac{P}{QB};\ B = \frac{8P}{13QR}$$

(*d*) $\dfrac{P}{QB} < \dfrac{3R}{2}$

$B > \dfrac{2P}{3QR}$

12. (*a,b,d*)

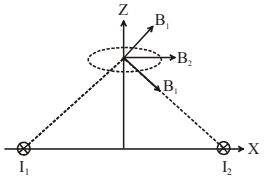


(*a*) at origin, $\vec{B}=0$ due to two wires if $I_1 = I_2$, hence $(\vec{B}_{net})$ at origin is equal to $\vec{B}$ due to ring, which is non-zero.

(*b*) If $I_1 > 0$ and $I_2 < 0$, $\vec{B}$ at origin due to wires will be along $+\hat{k}$ direction and $\vec{B}$ due to ring is along $-\hat{k}$ direction and hence $\vec{B}$ can be zero at origin.

(*c*) If $I_1 < 0$ and $I_2 > 0$, $\vec{B}$ at origin due to wires will is along $-\hat{k}$ and also along $-\hat{k}$ due to ring, hence $\vec{B}$ cannot be zero.

(*d*)

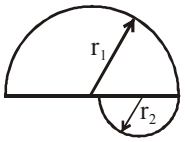


At centre of ring, $\vec{B}$ due two wires is along x-axis, hence z-component is only because of ring which

$\vec{B} = \frac{\mu_0 i}{2R}(-\hat{k})$.

13. [2.00]

(*a*) Average speed along x-axis



$$(v_x) = \frac{\int |\vec{v}_x| dt}{\int dt} = \frac{d_1 + d_2}{t_1 + t_2}$$

(*b*) we have

$$r_1 = \frac{mv}{qB_1}, r_2 = \frac{mv}{qB_2}$$

Since, $B_1 = \frac{B_2}{4}$

$\therefore$ $r_1 = 4r_2$

Time in $B_1 \Rightarrow \frac{\pi m}{qB_1} = t_1$

Time in $B_2 \Rightarrow \frac{\pi m}{qB_2} = t_2$

Total distance along x-axis

$d_1 + d_2 = 2r_1 + 2r_2 = 2(r_1 + r_2) = 2(5r_2)$

Total time $T = t_1 + t_2 = 5t_2$

Average speed = $\frac{10r_2}{5t_2} = 2\frac{mv}{qB_2} \times \frac{qB_2}{\pi m} = 2$

14. (*a,b,d*)

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19 MAGNETISM AND MATTER

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MAGNETISM

Magnetic Field / Magnetic Induction or Flux Density

The total number of lines of force per unit area due to magnetizing field and due to the field induced in the substance is called flux density (B), the unit in which B is measured is Wb/m^2.

The force experienced by a unit north pole of strength 1 Wb placed at a point in a magnetic field is a measure of the magnetic field intensity due to a pole of strength m at a distance r and is given by the following expression

$$\vec{B} = \frac{\mu}{4\pi} \frac{m}{r^2} \hat{r} \text{ Wb/m}^2$$

where μ is the absolute permeability of the medium and is expressed as $\mu = \mu_0 \times \mu_r$

where μ_r is relative permeability of the material and μ_0 is the permeability of the free space or air and is taken as $4\pi \times 10^{-7}$ Wb/A.m.

Magnetic field strength or Magnetizing field

The magnetic field strength or magnetizing field is given by

$\vec{H} = \frac{\vec{B}}{\mu}$ A/m and is independent of the medium.

For a coil having n number of turns per unit length and i_a as the (true) current in the winding then $H = ni_a$ (ampere-turn/meter)

This value of H is independent of the core material.

Intensity of Magnetization (I or J)

The measure of the magnetization of a magnetized specimen is called intensity of magnetization. It is defined as the magnetic moment per unit volume.

$$\text{Thus } I = \frac{\text{magnetic moment}}{\text{volume}}$$

As normally the specimen is small its magnetization can be supposed to be uniform. If the specimen is of uniform cross-section a, magnetic length $2l$, and pole strength is m, then $M = m \times 2l$ and $v = a \times 2l$

$$\therefore I = \frac{m \times 2l}{a \times 2l} = \frac{m}{a} \text{ Wb/m}^2$$

Thus intensity of magnetization is given as pole strength per unit area developed. Its unit is ampere-turn/meter.

Magnetic Susceptibility

The magnetic susceptibility (χ) of a specimen measures the ease with which the specimen can be magnetized and can be defined as the ratio of the intensity of magnetization induced in it and the magnetizing field i.e. $\chi = \frac{I}{H}$

Magnetic Permeability

When a magnetic material is placed in a magnetic field, due to induction it acquires magnetism. The lines of force of the magnetizing field concentrate inside the material and it results in the magnetizing of the material. The measure of the degree to which the lines of force can penetrate or permeate the medium is called absolute permeability of the medium and denoted by μ_a. The permeability is defined as the ratio of the magnetic induction B in the medium to the magnetizing field H i.e. $\mu_a = \mu_0\mu_r = B/H$

When a magnetic material of cross-sectional area A and relative permeability μ_r is placed in a uniform field H, two types of lines of induction pass through it, one due to the magnetizing field H and the other due to the material itself being magnetized by induction. Thus the total flux density B will be given by

$$B = \mu_0 H + \mu_0 I$$

As $\mu_a = \mu_0\mu_r = B/H$

$$\therefore \frac{B}{H} = \frac{\mu_0 H + \mu_0 I}{H} = \mu_0 + \frac{\mu_0 I}{H}$$

$$\text{or } \mu_0\mu_r = \mu_0 + \frac{\mu_0 I}{H}$$

$$\text{or } \mu_r = 1 + \chi \text{ or } \frac{\mu_a}{\mu_0} = 1 + \chi$$

PARA, DIA, FERRO-MAGNETIC SUBSTANCES

Magnetic substances are substances which upon being introduced into an external magnetic field, change so that they themselves become source of an additional magnetic field. Based on their magnetic behaviour substances can be classified into the following three categories.

Paramagnetic Substances

The substances which when placed in a magnetic field acquires a feeble magnetization in the same sense as the applied field are called paramagnetic substances. The examples are platinum, aluminum, manganese, chromium, copper sulphate, iron or nickel salt solutions and crown glass.

Their properties can be summarized as

(i) Such substances in non-uniform magnetic field, experience an attractive force towards the stronger part of the field.

(ii) The permeability μ for a paramagnetic substance is slightly greater than one

(iii) The magnetic susceptibility is small positive value

(iv) For a given temperature χ does not change with variation in H.

(v) The susceptibility varies inversely as the absolute temperature and at higher temperature its value becomes negative.

Diamagnetic substances

The substances, which when placed in a magnetic field acquire feeble magnetization in a direction opposite to that of the applied field are called diamagnetic substances. The examples are bismuth, antimony, water, alcohol and hydrogen. These substances exhibit the following properties.

(i) These substances are repelled by strong magnetic field.

(ii) The permeability μ for diamagnetic substance is less than one but positive.

(iii) Susceptibility for diamagnetic has a small negative value. This value does not vary with field or temperature.

(iv) A diamagnetic substance reduces the flux density B.

Ferromagnetic Substance

Such substances acquire high degree of magnetization in the same sense as the applied magnetic field. The example are: iron, steel, nickel and cobalt. Ferromagnetic substances exhibit the following properties:

(i) They have permeability of the order of hundreds and thousands.

(ii) Susceptibility is also very large and positive.

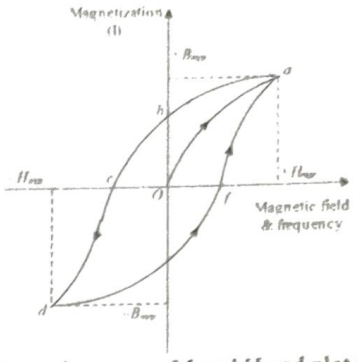
(iii) For small values of H susceptibility, remains constant and for moderate value of H increases rapidly with H and for large value attains a constant value.

(iv) They are attracted even by weak magnet.

(v) As temperature increases the value of χ decreases. Above certain temperature ferromagnetic become ordinary paramagnetics and this temperature is called curie temperature ($\chi \propto 1/T$ is called curie law). For iron, steel and nickel the curie point is 1000°C, 770°C and 360°C respectively.

Hysteresis Loop

If we take a ferromagnetic material in completely demagnetized state and make it to undergo through a cycle of magnetization in which H is increased from zero to a maximum value H_{max}, then decreases to zero, then reversed and again taken to $-H_{max}$, and finally brought back to zero. The variation of B with respect to H can be represented by a closed Hysteresis loop as shown in figure.



To get this graph measure M and H and plot these values. Increase H from zero to H_{max} and draw the cruve oa (the maximum value is known as saturation value). this is the normal magnetization curve. Now decrease H from H_{max} to zero. Intensity of magnetization will not fall as rapidly as it increases and will fall back to b rather than zero giving ab as the back trace of oa. Therefore even when the magnetizing force is made zero or removed, the iron is still magnetic and the flux density ob is called residual magnetism or **retentivity**. Now reverse the magnetizing force H. The value of Intensity of magnetization becomes zero at point c at which the substance is no longer a magnet. Now H is increased to $-H_{max}$ and graph cd is obtained. Change $-H_{max}$ to zero and then to H_{max} again curve dfa is obtained. This lagging of the intensity of magnetization with respect to the magnetizing force H is called **hysteresis** and the close loop graph is known as hysteresis loop. The value of H required to destroy the residual magnetism is called the **coercivity** which is represented by oc.

Energy Loss Due to Hysteresis

To produce a magnetic field a certain amount of energy has to be supplied. This energy is stored in free space where field is established and is returned to circuit when field collapses.

However in case of ferromagnetic substances not all the energy supplied can be returned; part of it is lost in form of heat etc. If the magnetization is carried through a complete cycle, the energy lost is proportional to the area of the hysteresis loop.

When a magnetic material is taken round cycle, there is an energy loss per unit volume of the material given by the area M-H curve.

Properties of Soft and Hard Materials

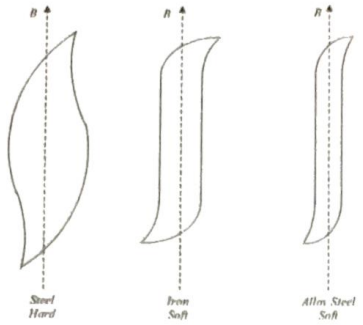
The shape of the B-H curve depends upon the ferromagnetic substance as shown in figure i.e. it is the characteristic of the substance. From the following figure it can be obtained.

(i) The susceptibility is more for soft materials than for hard material.

(ii) Permeability is more for soft materials than for hard materials.

(iii) Soft materials have greater retentivity as compared to hard materials.

(iv) Hysteresis loss for soft materials is less than that of hard materials.



Earth's Magnetic Field (Terrestial Magnetism)

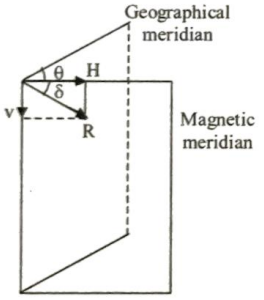
The fact that a freely suspended magnetic compass needle or a bar magnet orients itself roughly along the geographical North-South axis of the earth indicates that there is a magnetic field around the earth. This field is due to circulating electric currents (motion of charged ions of molted substances inside the earth) deep within its interior. We can picture this field as due to a fictitious magnetic dipole deep inside the earth. The earth will not behave like a magnet if it stops rotating.

COMPONENTS OF EARTH'S MAGNETIC FIELD

Following are three main components of earth's magnetic field.

Angle of Declination (θ)

A vertical plane passing through N-S line of a freely suspended magnet is called magnetic meridian and the vertical plane passing through the geographical North-South direction is called geographical meridian. The angle of declination is defined at a place as the angle between magnetic meridian and geographical meridian as shown in figure.



Angle of Dip or Inclination (δ)

The total intensity of earth's magnetic field varies in magnitude as well as in direction from place to place. The angle which the resultant earth's magnetic field makes with the horizontal line in the magnetic meridian is called magnetic dip or inclination. For measuring this angle we use dip circle and at poles $\delta = 90^\circ$ and at equator $\delta = 0$. (see fig.).

Horizontal Component

The resultant magnetic field due to earth B_e can be resolved into two components (i) horizontal component (H), (ii) vertical component (V). From figure the horizontal component of earths magnetic field is the component of total intensity of earths magnetic field in the horizontal direction in magnetic meridian. i.e.

$H = B_e \cos\delta$ and $V = B_e \sin\delta$

BAR MAGNET

Field At an Axial Line of Bar Magnet (End on position)

If $2l$ is the length of the bar magnet and m is the pole strength of each pole. The field due to this bar magnet, at an axial point d distance away from the centre of the magnet, having magnetic dipole moment m is given by

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{(2m)dl}{(d^2 - l^2)^2} \text{ if } l^2 << d^2, \quad B = \left(\frac{\mu_0}{4\pi}\right)\frac{M}{d^3}$$

At a Point on Equatorial Line (Broad side on Position)

If the distance of the point from the centre of the magnet along the equatorial line is d, the field is given by

$$B = \frac{\mu_0}{4\pi}\frac{2ml}{(d^2 + l^2)^{3/2}} \text{ if } l^2 << d^2, \text{ B}$$

$$= \frac{\mu_0}{4\pi}\left(\frac{M}{d^3}\right), \text{ M = Magnetic moment}$$

The ratio of field at axial point and equatorial point at the same distance from the centre is 2.

Force Between Two Small Magnets

When magnets are placed in End-on-position with respect to each other and their centers are x distance apart the force exerted is equal to $F = \frac{\mu_o}{4\pi}\frac{3MM'}{x^4}$. As force acts along the same line so couple is zero.

When magnets are at right angles to each other i.e. broad side-on-position and at a distance of y, the total force exerted is $F = \frac{\mu_o}{4\pi}\frac{3MM'}{y^4}$.

Tangent Law

When a magnet is suspended in a region having two mutually perpendicular field of intensities B and H, the magnet comes to rest position making an angle θ with the direction of H such that

$B = H \tan\theta$

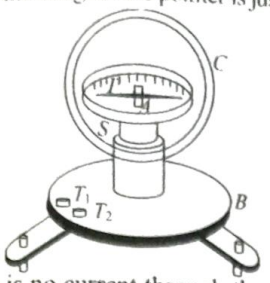
This is known as tangent law.

TANGENT GALVANOMETER

Tangent galvanometer is an instrument to measure an electric current. The essential parts are a vertical circular coil C of conducting wire and a small compass needle A pivoted at the center of the coil. The coil C together with its frame is fixed to a horizontal base B provided with leveling screws. Terminals T_1 and T_2 connected to the coil are provided on this base for connecting the galvanometer to an external circuit. An aluminum pointer P is rigidly attached with the compass needle and perpendicular to it. The compass needle together with the pointer can rotate freely about the vertical axis. The ends of the pointer move over a graduated, horizontal circular scale. The graduation are marked from 0° to 90° in each quadrant.

The scale, the pointer and the compass needle are enclosed in a closed cylindrical box which is placed with its center coinciding with the center of the coil. The box can also be rotated about the vertical axis. The upper surface of the box is made of

glass so that the things inside it are visible. To avoid the errors due to parallax, a plane mirror is fixed at the lower surface of the box. While noting the reading of the pointer, the eye should be properly positioned so that the image of the pointer is just below the pointer.



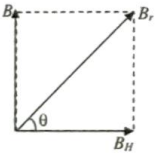
When there is no current through the galvanometer, the compass needle is in magnetic north-south direction. To measure a current with the tangent galvanometer, the base is rotated in such a way that the plane of the coil is parallel to the compass needle. The plane then coincides with the magnetic meridian. The box containing the needle is rotated so that the aluminum pointer reads 0-0 on the scale.

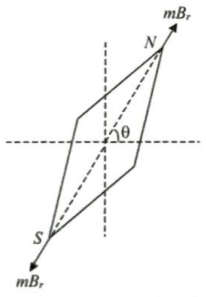
The current to be measured is passed through the coil. The current through the coil produces a magnetic field at the center and the compass needle deflects under its action. The pointer deflects through the same angle and the deflection of both the ends are read from the horizontal scale. the average of these two is calculated.

Suppose the current through the coil is i, the radius of the coil is r and the number of turns in it is n. The magnetic field produced at the center is

$$B = \frac{\mu_o in}{2r} \qquad (1)$$

This field is perpendicular to the plane of the coil. This direction is horizontal and perpendicular to the magnetic meridian and hence to the horizontal component B_H of the earth's magnetic field. The resultant horizontal magnetic field is $B_r = \sqrt{B^2 + B_H^2}$ in a direction making an angle θ with B_H, where $\tan\theta = B/B_H$





If *m* be the pole strength of the needle, the force on the north pole of the needle is mB_r along B_r and on the south pole is mB_r, opposite to B_r. The needle will stay in equilibrium when its length is parallel to B_r, because then no torque is produced by the two forces. Thus, the deflection of the needle from its original position is θ as given by (ii). Using (i) and (ii),

$$B_H \tan\theta = \frac{\mu_o in}{2r} \quad \text{or} \quad i = \frac{2rB_H}{\mu_o n}\tan\theta$$

or $i = K\tan\theta$

where $K = \frac{2rB_H}{\mu_o n}$ is a constant for the given galvanometer at a given place. This constant is called the reduction factor of the galvanometer. The reduction factor may be obtained by passing a known current i through the galvanometer, measuring θ and then using above equation.

The equation (i) is strictly valid only at the center of the coil. The poles of the needle are slightly away from the center. Thus, the length of the needle should be small as compared to the radius of the coil.

TRAIN YOUR BRAIN

Example 1: A tangent galvanometer has 66 turns and the diameter of its coil is 22 cm. It gives a deflection of 45° for 0.10 A current .What is the value of the horizontal component of the earth's magnetic field?

Sol. For a tangent galvanometer

$$i = K\tan\theta = \frac{2rB_H}{\mu_o n}\tan\theta$$

or $$B_H = \frac{\mu_o ni}{2r\tan\theta}$$

$$= \frac{\left(4\pi\times10^{-7}\,\frac{\text{T}-\text{m}}{\text{A}}\right)\times 66\times(0.1\text{ A})}{(22\times10^{-2}\text{ m})(\tan 45^\circ)}$$

$$= 3.8\times10^{-5}\text{ T}$$

Vibration Magnetometer

It is an instrument used for comparing the intensities of earth's magnetic field at two places and the magnetic moments of two magnets. It works on the principle that a freely suspended magnet rests in direction NS parallel to the direction of field. If this magnet is slightly displaced from its equilibrium position and left, the magnet executes SHM about the direction of field as its mean position and the time periods of vibration of the magnet is given by

$$T = 2\pi\sqrt{I/MH}$$

where I is the moment of inertia of the bar magnet, M is the magnetic moment of the magnet and H is intensity of uniform magnetic field (Usually it is horizontal component of earth's magnetic field).

The ratio of horizontal components of earth's field at the two places (H_1 and H_2 respectively) if the same magnet has time period T_1 and T_2 is given by

$$\frac{H_2}{H_1} = \frac{T_1^2}{T_2^2}$$

TRAIN YOUR BRAIN

Example 2: The period of oscillation of a magnet in a vibration magnetometer is 2s. The period of oscillation of a magnet whose magnetic moment is four times that of the first magnet is

(*a*) 0.5 s (*b*) 8 s
(*c*) 5.0 s (*d*) 1 s

Sol. (*d*) $\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}$

$\Rightarrow \quad \frac{2}{T_2} = \sqrt{\frac{4M}{M}} \Rightarrow T_2 = 1$

Example 3: A magnetic needle, suspended horizontally by an unspun silk fibre, oscillates in the horizontal plane, because of restoring force originating mainly from

(*a*) a force of gravity
(*b*) the horizontal component of earth's magnetic field
(*c*) the tension of the silk fibre
(*d*) all the above factors

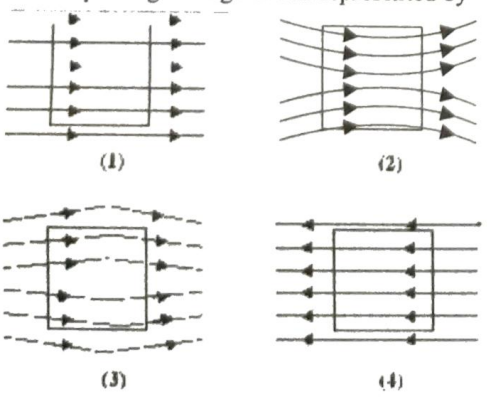
Sol. (*b*) When the needle gets displaced through an angle θ, a restoring couple MH sin θ acts on it, where H is the horizontal component of earth's magnetic field. This couple brings the needle in the direction of magnetic meridian. In this position due to its momentum it moves further in the opposite direction. However due to restoring couple the velocity of the needle keeps on decreasing and ultimately stops for a moment and then returns back and this process is repeated.

Example 4: The arms of deflection magnetometer in broad side on position are placed along

(*a*) N-S direction (*b*) N-E direction
(*c*) S-E direction (*d*) E-W direction

Sol. (*a*) In broad side on position (a position on the perpendicular bisector of the bar magnet) arms of the meter are along N-S direction.

Example 5: A uniform magnetic field, parallel to the plane of the paper, existed in space initially directed from left to right. When a bar of soft iron is placed in the field parallel to it, the lines of force passing through it will represented by



Sol. (2) When a bar of soft iron ($\mu_r > 1$) is placed in a magnetic field the lines of force converge and try to pass through the soft iron so (2) alternate is correct.

Example 6: If a bar magnet of magnetic moment 80 units be cut into two halves of equal lengths along a plane passing through the perpendicular bisector of the magnet, the magnetic moment of the each half will be

(*a*) 20 units (*b*) 460 units
(*c*) 40 units (*d*) 80 units

Sol. (*c*) On cutting a bar magnet the pole strength m remains same but the length is reduced so when cut into halves magnetic moment (M = 2 m*l*) will also become half.

Example 7: At certain place, the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5 oersted. The earth's total magnetic field (in oersted) is

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\frac{1}{2}$
(*c*) 1 (*d*) $\sqrt{3}$

Sol. (*a*) Let the earth's total magnetic field be *I* so the horizontal component I cos δ = H or I = 0.5/cos30° = $\frac{1}{\sqrt{3}}$.

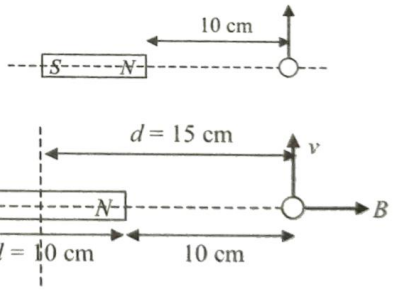
Example 8: Which of the following is more suitable for the core of electromagnets

(*a*) air (*b*) steel
(*c*) soft-iron (*d*) Cu-Ni alloy

Sol. Soft iron is more suitable for core of a electromagnet because of its low hysterisis area so as to minimize the heat losses in each cycle.

Example 9: An electron is moving with a velocity of 2×10^6 m/s in the vicinity of a 10 cm long bar magnet of pole strength 50 Am. What is the force on the electron?

Sol.



B (field at site of electron)

$= \frac{\mu_o}{4\pi} \frac{2Md}{\left(d^2 - l^2\right)^2}$ and $M = 2ml$

or $B = \frac{\mu_o}{4\pi} \frac{4ml}{\left(d^2 - l^2\right)} = 2.5 \times 10^{-3}$ T

$F = qvB \sin\theta$

$\therefore \quad F = (1.6 \times 10^{-19}) \times 2 \times 10^6 \times 2.5 \times 10^{-3} \sin 90°$

$= 8 \times 10^{-16}$ N

Topicwise Questions

1. The intensity of magnetic field due to an isolated pole of strength m_p at a point distant r from it will be (in gs)
   (a) $m_p r^2$ (b) $m_p r^{-2}$
   (c) r^2/m_p (d) m_p/r
2. Nickel shows ferromagnetic property at room temperature. If the temperature is increased beyond Curie temperature, then it will show
   (a) paramagnetism (b) diamagnetism
   (c) anti-ferromagnetism (d) no magnetic property
3. A magnetic needle suspended by a silk-thread is vibrating in the earth's magnetic field. If the temperature of the needle is increased to very high temperature then
   (a) the time period decreases
   (b) the time period remains unchanged
   (c) the time period increases
   (d) the needle stops vibrating
4. Force between two identical bar magnets whose centres are r m apart is 4.8 N when their axes are in the same line. If separation is increased to 2r, the force between them is reduced to
   (a) 2.4 N (b) 0.6 N
   (c) 1.2 N (d) 0.3 N
5. A magnetic needle oscillates in a horizontal plane with a period T at a place where the angle of dip is 60°. When the same needle is made to oscillate in a vertical plane coinciding with the magnetic meridian, its period will be
   (a) $T/\sqrt{2}$ (b) T
   (c) $\sqrt{2}\,T$ (d) 2T
6. If mass is increased 4 times, then the time period of a magnetometer when suspended in earth's magnetic field becomes
   (a) double (b) quadruple
   (c) half (d) decreases 4 times
7. If a piece of metal was thought to be a magnet, which one of the following observations would offer conclusive evidence?
   (a) it repels a known magnet
   (b) it attracts a steel screw driver
   (c) it attracts a known magnet
   (d) none of these
8. A short bar magnet produces magnetic fields of equal induction at two points, one on the axial line and the other on the equatorial line. Then the ratio of their distances is
   (a) $1 : 2^{1/3}$ (b) 1 : 2
   (c) $2^{1/3} : 1$ (d) 1 : 8
9. Ferromagnetic materials used in a transformer must have
   (a) low permeability and high hysteresis loss
   (b) high permeability and low hysteresis loss
   (c) high permeability and high hysteresis loss
   (d) low permeability and low hysteresis loss
10. If a diamagnetic substance is brought near the north or the south pole of a bar magnet, it is
    (a) repelled by the north pole and attracted by the south pole
    (b) attracted by the north pole and repelled by the south pole
    (c) attracted by both the poles
    (d) repelled by both the poles
11. A bar magnet having a magnetic moment of 2×10^4 JT^{-1} is free to rotate in a horizontal plane. A horizontal magnetic field $B = 6 \times 10^{-4}$ T exists in the space. The work done in taking the magnet slowly from a direction parallel to the field to a direction 60° from the field is
    (a) 12 J (b) 6 J
    (c) 2 J (d) 0.6 J
12. The period of oscillation of a vibration magnetometer depends on the following factors: where I is the moment of inertia of the magnet about the axis of suspension, M is the magnetic moment of the magnet and H is the external magnetic field.
    (a) M and I only (b) I and H only
    (c) I, M and H only (d) I and M only
13. The magnet can be completely demagnetized by
    (a) heating it slightly
    (b) dropping it into ice cold water
    (c) a reverse field of appropriate strength
    (d) breaking the magnet into small pieces
14. A bar magnet of magnetic moment 10^4 J/T is free to rotate in a horizontal plane. The work done in rotating the magnet slowly from a direction parallel to a horizontal magnetic field of 4×10^{-5} T to a direction 60° from the field will be
    (a) 0.2 J (b) 2.0 J
    (c) 4.18 J (d) 2×10^2 J
15. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and these parts are then placed on each other with their like poles together. The time period of this combination will be
    (a) 2 s (b) $\frac{2}{3}$ s
    (c) $2\sqrt{3}$ s (d) $\frac{2}{\sqrt{3}}$ s

Learning Plus

1. The area enclosed by a hysteresis loop is a measure of
   (a) retentivity (b) susceptibility
   (c) permeability (d) energy loss per cycle
2. The magnetic susceptibility of an ideal diamagnetic substance is
   (a) −1 (b) 0
   (c) +1 (d) ∞
3. A bar magnet of length 10 cm and having the pole strength equal to 10^{-3} Wb is kept in a magnetic field having magnetic induction B ($=4\pi \times 10^{-3}$ T). It makes an angle of 30° with the direction of magnetic induction. The value of Torque in Newton-metre is
   (a) $2\pi \times 10^{-7}$ (b) $2\pi \times 10^{-5}$
   (c) 0.5 (d) 0.5×10^{-2}
4. In a deflection magnetometer, the time period of a magnet is found to be 8 sec. If the magnetic moment of another magnet is 1/4 of the first one, what is the time period of the latter of it has same M.I. (Moment of Inertia)?
   (a) 8 sec (b) 4 sec
   (c) 2 sec (d) 16 sec
5. Curie temperature above which
   (a) paramagnetic material becomes ferromagnetic material
   (b) ferromagnetic material becomes diamagnetic material
   (c) ferromagnetic material becomes paramagnetic material
   (d) paramagnetic material becomes diamagnetic material
6. The north pole of the earth's magnet is near the geographical
   (a) east (b) west
   (c) north (d) south
7. The angle of dip at a place on the earth gives
   (a) the horizontal component of the earth's magnetic field
   (b) the location of the geographic meridian
   (c) the vertical component of the earth's field
   (d) the direction of the earth's magnetic field
8. A bar magnet is placed in the north south direction with its north pole to the north. In which direction from the centre of the bar magnet will the points of zero magnetic field lie?
   (a) North and South
   (b) East and West
   (c) North East and South West
   (d) North West and South East
9. A magnet of length 0.1 m has a pole strength of 4 Am. If it is making an angle of 45° with the direction of uniform magnetic induction of 4 wb/m², the torque experienced by it is
   (a) 1.13 N-m (b) 1.6 N-m
   (c) 0.8 N-m (d) 1.32 N-m
10. The susceptibility of a diamagnetic substance
    (a) Remains coota with temperature
    (b) first decreases and then increases with increase of temperature
    (c) increase with temperature
    (d) decreases with temperature
11. In the vibration magnetometer, the magnet kept in the stirrup when disturbed vibrates with a period, given by
    (a) $2\pi\sqrt{I/MB}$ (b) $2\pi\sqrt{MB/I}$
    (c) $2\pi\sqrt{MBI}$ (d) $2\pi\sqrt{I/g}$
12. The magnetic lines of force inside a bar magnet
    (a) do not exist
    (b) depend upon the area of cross-section of the bar magnet
    (c) are from south-pole to north-pole of the magnet
    (d) are from north-pole to south-pole of the magnet
13. The materials suitable for making electromagnets should have
    (a) high retentivity and high coercivity
    (b) low retentivity and low coercivity
    (c) high retentivity and low coercivity
    (d) low retentivity and high coercivity

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. Hysteresis loops for two magnetic materials A and B are given below: *[JEE Main-2016]*



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:

(a) A for electromagnets and B for electric generators
(b) A for transformers and B for electric generators
(c) B for electromagnets and transformers
(d) A for electric generators and transformers

2. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is:

[JEE Main-2017]

(a) 6.98 s (b) 8.76 s
(c) 6.65 s (d) 8.89 s

3. A paramagnetic substance in the orm of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is : [JEE Main-2019 (January)]

(a) 3.3×10^{-2} (b) 40.3×10^{-2}
(c) 2.3×10^{-2} (d) 3.3×10^{-4}

4. At some location on earth the hoizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 A m is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is : [JEE Main-2019 (January)]

(a) 3.6×10^{-5} N (b) 1.8×10^{-5} N
(c) 1.3×10^{-5} N (d) 6.5×10^{-5} N

5. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is

[JEE Main-2019 (January)]

(a) 285 A/m (b) 2600 A/m
(c) 520 A/m (d) 1200 A/m

6. A magnet of total magnetic moment $10^{-2}\hat{i}$ A – m^2 is placed in a time varying magnetic field, $B\hat{i}(\cos\omega t)$ where B = 1 Tesla and $\omega = 0.125$ rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is: [JEE Main-2019 (January)]

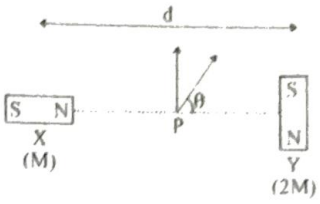
(a) 0.01 J (b) 0.07 J
(c) 0.028 J (d) 0.014 J

7. A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temeprature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is: [JEE Main-2019 (January)]

(a) 3.267×10^{-4} (b) 3.672×10^{-4}
(c) 3.726×10^{-4} (d) 2.672×10^{-4}

8. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing, through their midpoint P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)

[JEE Main-2019 (April)]



(a) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3} \times qv$ (b) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3} \times qv$

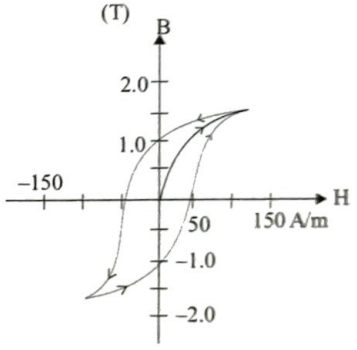
(c) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3} \times qv$ (d) 0

9. A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by: [JEE Main-2019 (April)]

(a) 2.2 (b) 1.8
(c) 0.7 (d) 3.6

10. The figure given experimentally measured B vs H variation in a ferromagnetic material. the retentivity, co-ercivity and saturation, respectively, of the material are:

[JEE Main-2020 (January)]



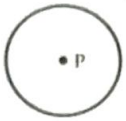
(a) 1.0 T, 50 A/m and 1.5T
(b) 150A/m, 1.0 T and 1.5T
(c) 1.5 T, 50 A/m and 1.0T
(d) 1.5T, 50A/m and 1.0 T

11. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required?

[JEE Main-2020 (September)]

(a) T : Large retentivity, small coercivity
(b) P : Large retentivity, large coercivity
(c) P : Small retentivity, large coercivity
(d) T : Large retentivity, large coercivity

12. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is [JEE Main-2020 (September)]



(a) Much larger than $|\vec{B}|$ and parallel to $\vec{B}$

(b) $\vec{B}$

(c) Much larger than $|\vec{B}|$ and anti-parallel to $\vec{B}$

(d) Zero

13. A paramagnetic sample shows a net magnetisation of 6 A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be $x \times 10^{-2}$ J. Find x: [JEE Main-2020 (September)]

(a) 1 A/m (b) 0.75 A/m

(c) 4 A/m (d) 2.25 A/m

14. A small bar magnet placed with its axis at 30° with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is [JEE Main-2020 (September)]

(a) 9.2×10^{-3} J (b) 6.4×10^{-2} J

(c) 7.2×10^{-2} J (d) 11.7×10^{-3} J

15. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be [JEE Main-2020 (September)]

(a) 5×10^2 Am^2 (b) 0.5×10^2 Am^2

(c) 500×10^2 Am^2 (d) 50×10^2 Am^2

16. Which of the following statements are correct?

(A) Electric monopoles do not exist whereas magnetic monopoles exist.

(B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.

(C) Magnetic field lines are completely confined within a toroid.

(D) Magnetic field lines inside a bar magnet are not parallel.

(E) $\chi = -1$ is the condition for a perfect diamagnetic material, where x is its magnetic susceptibility.

Choose the correct answer from the options given below: [JEE Main-2021 (March)]

(a) (A) and (B) only (b) (B) and (C) only

(c) (C) and (E) only (d) (B) and (D) only

17. A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire. [JEE Main-2021 (March)]

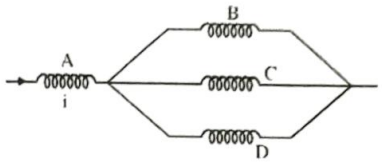
(a) Loop assumes circular shape with its plane normal to the field.

(b) Loop assumes circular shape with its plane parallel to the field.

(c) Wire gets stretched to become straight.

(d) Shape of the loop remains unchanged.

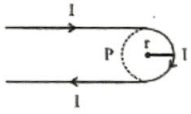
18. Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the center of A is 3 T, the field at the center of C would be : (Assume that the magnetic field is confined with in the volume of respective solenoid). [JEE Main-2021 (March)]



(a) 12T (b) 6T

(c) 9T (d) 1T

19. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle? [JEE Main-2021 (March)]



(a) $\frac{\mu_0 I}{4\pi r}(2-\pi)$ (b) $\frac{\mu_0 I}{2\pi r}(2-\pi)$

(c) $\frac{\mu_0 I}{4\pi r}(2+\pi)$ (d) $\frac{\mu_0 I}{2\pi r}(2+\pi)$

20. A solenoid of 1000 turns per metre has a core with relative permeability 500 . Insulated windings of the solenoid carry an electric current of 5 A. The magnetic flux density produced by the solenoid is:(permeability of free space $= 4\pi \times 10^{-7}$ H / m) [JEE Main-2021 (March)]

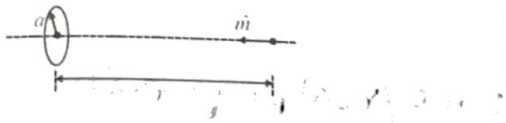
(a) πT (b) $2 \times 10^{-3}\pi$T

(c) $10^{-4}\pi$T (d) $\frac{\pi}{5}$T

Paragraph

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a, with its center at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance $r(\gg a)$ from the center of the loop with its north pole always facing the loop, as shown in the figure below.

The magnitude of magnetic field of a dipole m, at a point on its axis at distance r, is $\frac{\mu_0}{2\pi}\frac{m}{r^3}$, where μ_0 the permeability of free space is. The magnitude of the force between two magnetic dipoles with moments, m_1 and m_2, separated by a distance r on the common axis, with their north poles facing each other, is $\frac{km_1m_2}{r^4}$, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.



1. When the dipole m is placed at a distance r from the center of the loop (as shown in the figure), the current induced in the loop will be proportional to

[JEE Advanced-2021]

(a) $\frac{m}{r^3}$ (b) $\frac{m^2}{r^2}$

(c) $\frac{m}{r^2}$ (d) $\frac{m^2}{r}$

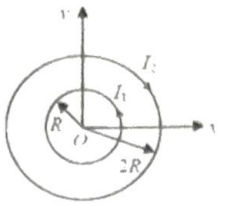
2. The work done in bringing the dipole from infinity to a distance r from the center of the loop by the given process is proportional to [JEE Advanced-2021]

(a) $\frac{m}{r^5}$ (b) $\frac{m^2}{r^5}$

(c) $\frac{m^2}{r^6}$ (d) $\frac{m^2}{r^7}$

3. Two concentric circular loops, one of radius R and the other of radius 2 R, lie in the x y-plane with the origin as their common center, as shown in the figure. The smaller loop carries current I_1 in the anti-clockwise direction and the larger loop carries current I_2 in the clockwise direction, with $I_2 > 2I_1$. $\vec{B}(x,y)$ denotes the magnetic field at a point (x,y) in the x y-plane. Which of the following statement(s) is(are) correct?

[JEE Advanced-2021]



(a) $\vec{B}(x,y)$ is perpendicular to the x y-plane at any point in the plane

(b) $|\vec{B}(x,y)|$ Depends on x and y only through the radial distance $r=\sqrt{x^2+y^2}$

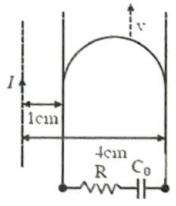
(c) $|\vec{B}(x,y)|$ Is non-zero at all points for $r<R$

(d) $\vec{B}(x,y)$ Points normally outward from the xy-plane for all the points between the two loops

4. A long straight wire carries a current, $I=2$ ampere. A semi-circular conducting rod is placed beside it on two conducting parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1cm and 4cm from the wire. At time $t=0$, the rod starts moving on the rails with a speed $v=3.0\,m/s$ (see the figure). [JEE Advanced-2021]

A resistor $R=1.4\Omega$ and a capacitor $C_0=5.0\mu F$ are connected in series between the rails. At time $t=0$, C_0 is uncharged. Which of the following statement(s) is(are) correct ?

$[\mu_0=4\pi\times10^{-7}$ SI units. Take $\ln 2=0.7]$



(a) Maximum current through R is 1.2×10^{-6} ampere

(b) Maximum current through R is 3.8×10^{-6} ampere

(c) Maximum charge on capacitor C_0 is 8.4×10^{-12} coulomb

(d) Maximum charge on capacitor C_0 is 2.4×10^{-12} coulomb

5. An α -particle (mass 4amu) and a singly charged sulfur ion (mass 32 amu) are initially at rest. They are accelerated through a potential V and then allowed to pass into a region of uniform magnetic field which is normal to the velocities of the particles. Within this region, the α -particle and the sulphur ion move in circular orbits of radii r_α and r_s, respectively. The ratio (r_s/r_α) is [JEE Advanced-2021]

ANSWER KEY

Topicwise Questions

1. (*a*)	**2.** (*a*)	**3.** (*c*)	**4.** (*d*)	**5.** (*c*)	**6.** (*a*)	**7.** (*a*)	**8.** (*c*)	**9.** (*b*)	**10.** (*d*)
11. (*b*)	**12.** (*c*)	**13.** (*c*)	**14.** (*a*)	**15.** (*b*)					

Learning Plus

1. (*d*)	**2.** (*a*)	**3.** (*a*)	**4.** (*d*)	**5.** (*c*)	**6.** (*d*)	**7.** (*d*)	**8.** (*b*)	**9.** (*a*)	**10.** (*a*)
11. (*a*)	**12.** (*c*)	**13.** (*b*)							

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*c*)	**2.** (*c*)	**3.** (*d*)	**4.** (*d*)	**5.** (*c*)	**6.** (Bonus)	**7.** (*a*)	**8.** (*d*)	**9.** (*c*)	**10.** (*a*)
11. (*b*)	**12.** (*d*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)	**16.** (*c*)	**17.** (*a*)	**18.** (*d*)	**19.** (*c*)	**20.** (*a*)

JEE-ADVANCED PREVIOUS YEAR'S

1. (*a*)	**2.** (*c*)	**3.** (*a, b*)	**4.** (*a, c*)	**5.** (*d*)	t.me/realprayasians2

20 ELECTROMAGNETIC INDUCTION

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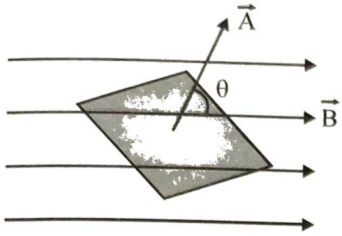
Magnetic Flux & Faraday's Law

$\phi = \vec{B}.\vec{A} = BA\cos\theta$



Units: Weber (SI) or $T \cdot m^2$
Maxwell (CGS)
1 maxwell = 10^{-8} Wb

Faraday's Laws of Electromagnetic Induction

1. An emf is induced in a loop when the number of magnetic field lines *i.e.*, magnetic flux passing through the loop is changing.
2. Magnitude of the emf '*e*' induced in a conducting loop is equal to the rate at which the magnetic flux ϕ_B through that loop changes with time.

$$e \propto \frac{d\varphi_B}{dt}$$

Lenz's Law

The direction of induced emf (i.e., polarity of induced emf) and hence the direction of induced current in a closed circuit is to oppose the cause due to which they are produced. For example, if the flux is increasing, induced emf (and hence induced current) will try to decrease the flux and vice-versa. It is based on energy conservation law.

Methods to Change the Magnetic Flux

Here are the general methods by which we can change the magnetic flux through a coil.

(i) Change the magnitude B of the magnetic field within the coil.

(ii) Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it in or out of the field.

(iii) Change the angle between the direction of the magnetic field B and the plane of coil (For example, by rotating the coil so that field B is first perpendicular to the plane of the coil and then is along that plane).

Note: Changing a magnetic field in a loop induces voltage. If the loop is an electrical conductor forming a closed circuit, then current is induced.

(iv) Induced current $i(t) = \frac{e(t)}{R} = -\frac{1}{R}.\frac{d}{dt}\phi(t)$

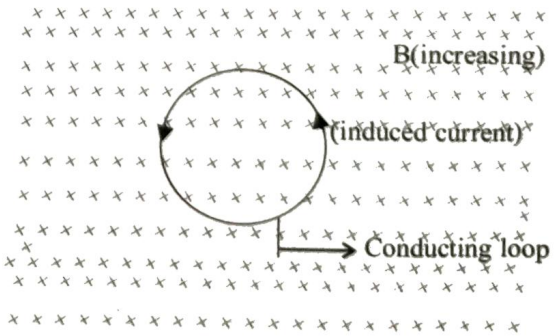
(v) Induced current

$$\Delta q = \int dq = \int i(t)dt = \int \frac{1}{R}\left|\frac{d\phi}{dt}\right|dt = \frac{1}{R}|d\phi|$$

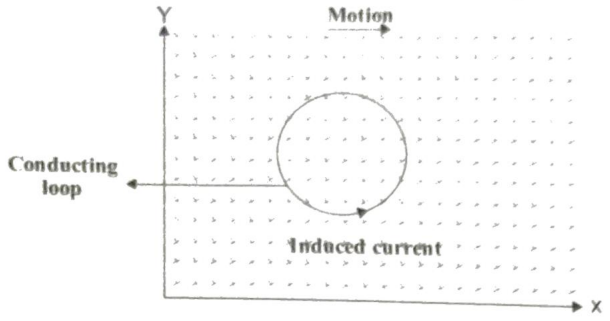
Hence $\Delta q = \frac{1}{R}|\Delta\phi|$, Δq is independent of time interval during which flux is changing.

Applications of Lenz's law shall become more clear by carefully studying the following examples

(1) A conducting loop is kept in a uniform magnetic field B directed into the plane of paper. The magnetic field starts increasing with time. As the magnetic field increases, flux $\left(\phi = \vec{B}.\vec{A}\right)$ starts increasing. Therefore, an induced current will flow into the loop. The current is induced due to increasing field, therefore, it will try to decrease the field. This can happen if induced current produces an outward field in the loop. Therefore, an anticlockwise current appears in the loop.



There exists a non-uniform field $B = B_0X$ (inwards) in x-y plane as shown. The field increases as we move along +x-direction. A conducting loop is placed in the x-y plane. The loop is being moved towards +x-axis. As the loop moves towards +x-axis, the flux linked increases because magnetic field increases. The induced current will try to decrease the field. It can do so by decreasing the magnetic field in the loop. Therefore an anticlockwise current appears in the loop.



TRAIN YOUR BRAIN

Example 1: A current I = 1.5 A is flowing through a long solenoid of diameter 3.2 cm, having 220 turns per cm. At its centre, a 130 turn closely packed coil of diameter 2.1 cm is placed such that the coil is coaxial with the long solenoid. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of emf induced in the coil while the current in the solenoid is changing?

Sol. Initially magnetic flux passing through the coil (one turn),

$$\phi_1 = \vec{B}.\vec{A} = BA\cos 0^\circ$$

$$= \mu_0 nl \times \frac{\pi d^2}{4}$$

$$= 4\pi \times 10^{-7} \times 1.5 \times 22000 \times 3.464 \times 10^{-4}$$

$$= 1.44 \times 10^{-5} \text{ Wb}$$

Finally the flux becomes zero because the current reduces to zero.

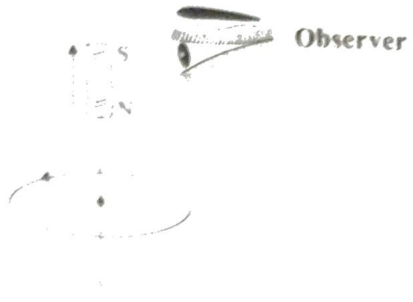
$$\text{Thus, } e = \frac{|\Delta\phi|}{\Delta t} = \frac{1.44 \times 10^{-5}}{25 \times 10^{-3}} = 5.76 \times 10^{-4}$$

The total emf = N × e

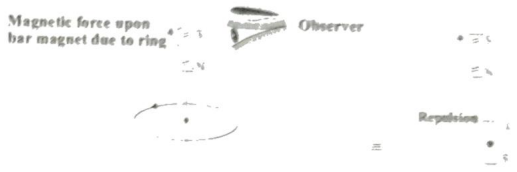
$= 130 \times 5.76 \times 10^{-4}$

$= 75$ mV

Example 2: A bar magnet is dropped through a horizontal aluminum ring along the axis of the ring. What will be direction of induced current in the loop for the observer shown? What will be the direction of magnetic force experienced by the bar magnet?



Sol. As per Lenz's law, induced current in the loop will be such that it will oppose the cause of change in flux (that is motion of the bar magnet towards itself). Thus current flows in the loop so that it is equivalent to a bar magnet with its North facing the approaching North pole of the magnet, thereby repelling it. Thus current for the observer will be anticlockwise.



Thus, acceleration of the bar magnet $a < g$.

Example 3: Consider a coil (of area A, resistance R and number of turns N) held perpendicular to a uniform magnetic field of strength B. The coil is now turned through 180° in time Δt. What is

(i) Average induced emf

(ii) Average induced current

(iii) Total charge that flows through a given cross-section of the coil?

Sol. When plane of coil is perpendicular to the magnetic field, $\theta = 0^\circ$ and after it is rotated through 180°, $\theta = 180^\circ$

$\Rightarrow$ Initial flux = NBA cos0° = NBA and Final flux = NBA cos 180° = – NBA

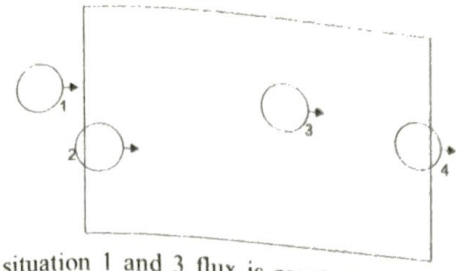
$\Rightarrow$ Change in flux = $|\Delta\phi|$ = NBA –(–NBA)= 2NBA

(i) ε= Average induced emf $= \frac{|\Delta\phi|}{\Delta t} = \frac{2NBA}{\Delta t}$

(ii) Average current $= \frac{\varepsilon}{R} = \frac{2NBA}{R\Delta t}$

(iii) Total charge $(\Delta Q) = \frac{\Delta\phi}{R} \Rightarrow \Delta Q = \frac{2NBA}{R}$

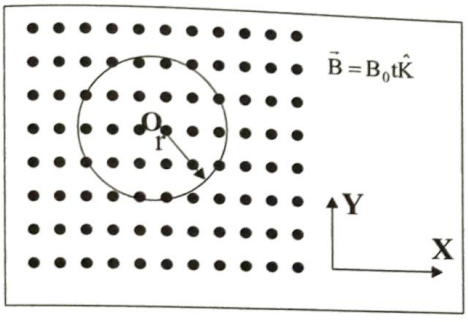
Example 4: A conducting loop is moving from left to right through a region of uniform magnetic field (B). Its four positions are shown below. Show the direction of induced current in all four positions.



Sol. In situation 1 and 3 flux is constant and not changing with time, so there will not be any current induced. In situation 2 the loop is gradually getting into the magnetic field so overlapped area is increasing, so flux is increasing & hence the induced current will have the tendency to create B field in opposite direction i.e. in outward direction. Hence, anticlockwise current will be induced in the loop in situation 2. In situation 4 the loop is gradually getting out of the field, flux is decreasing, so induced current will support the B field. Current induced will be clockwise.

Example 5: There exists a uniform magnetic field $\vec{B} = B_0 t\hat{k}$ in a region. A circular conducting loop of radius r and resistance R is placed with its plane in x-y plane. Determine the current through the loop and sense of the current.

Sol. The flux linked with the loop is



$$\phi = \vec{B}.\vec{A} = (B_0 t)(\pi r^2)\cos 0^\circ$$

$$\phi = B_0 \pi r^2 t$$

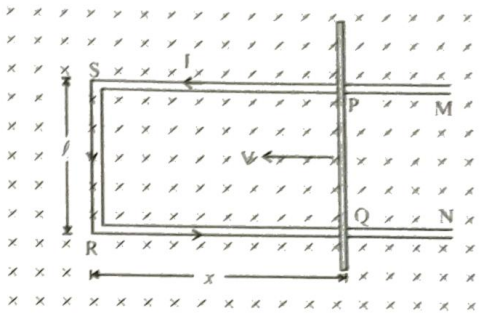
$$\frac{d\phi}{dt} = B_0 \pi r^2$$

$$\varepsilon = \frac{-d\phi}{dt} = -B_0 \pi r^2 \Rightarrow i = \frac{|\varepsilon|}{R} = \frac{B_0 \pi r^2}{R}$$

By Lenz's law, i should be clockwise (as seen by an observer standing on z-axis) so that it can oppose the increase in magnetic field.

MOTIONAL ELECTROMOTIVE FORCE

Let us consider a straight conductor moving in a uniform and time independent magnetic field. Fig. (c) shows a rectangular conductor PQRS in which the conductor PQ is free to move. The rod PQ is moved towards the left with a constant velocity v as shown in the figure. Assume that there is no loss of energy due to friction. PQRS forms a closed circuit enclosing an area that changes as PQ moves. It is placed in a uniform magnetic field **B** which is perpendicular to the plane of this system. If the length RQ = x and RS = ℓ, the magnetic flux Φ_B enclosed by the loop PQRS will be



(c)

Since x is changing with time, the rate of change of flux Φ_B will induce an emf given by:

At any instant flux will be $\phi = B\ell x$

$$\varepsilon = \frac{-d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x)$$

$$= -B\ell\frac{dx}{dt} = B\ell v \qquad \text{.....(A)}$$

It is also possible to explain the motional emf expression in Eq. (A)

by invoking the Lorentz force acting on the free charge carriers of conductor PQ. Consider any arbitrary charge q in the conductor PQ. When the rod moves with speed v, the charge will also be moving with speed v in the magnetic field **B**. The Lorentz force on this charge is qvB in magnitude, and its direction is towards Q. All charges experience the same force, in magnitude and direction, irrespective of their position in the rod PQ.

The work done in moving the charge from P to Q is,

$W = qvB\ell$

Since emf is the work done per unit charge,

$$\varepsilon = \frac{W}{q} = B\ell v$$

This equation gives emf induced across the rod PQ and is identical to Eq. (A)

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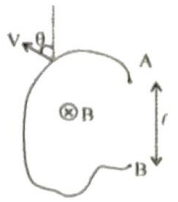
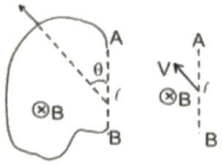
Example 6: Figure shows an irregular shaped wire AB moving with velocity v, as shown.

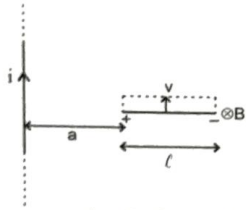


Find the emf induced in the wire.

Sol. The same emf will be induced in the straight imaginary wire joining A and B , which is $Bv\ell \sin\theta$



Example 7: A rod of length ℓ is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the emf induced in the rod , if its nearest end is at a distance 'a' from the wire.

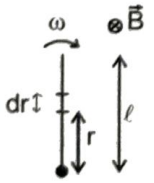


Sol. Consider a segment of rod of length dx , at a distance x from the wire. Emf induced in the segment $dE = \frac{\mu_0 i}{2\pi x} dx.v$

$$\therefore \quad E = \int_a^{a+\ell} \frac{\mu_0 i v dx}{2\pi x} = \frac{\mu_0 i v}{2\pi} \ell n\left(\frac{\ell + a}{a}\right)$$

INDUCED EMF DUE TO ROTATION OF THE ROD

Consider a conducting rod of length ℓ rotating in a uniform magnetic field.



Small segment behave like a rod of length dr having velocity $v = r\omega$

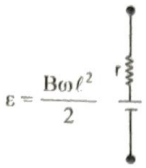
$$d\varepsilon = B\omega r dr$$

emf induced in a small segment

Emf induced in a small segment of length dr, of the rod $= v\,B\,dr = r\omega\,B\,dr$

$$\therefore \quad \text{emf induced in the rod} = \omega B \int_0^{\ell} r dr = \frac{1}{2} B\omega\ell^2$$

equivalent of this rod is as following



$$\text{or} \quad \varepsilon = \frac{d\Phi}{dt}$$

$$\varepsilon = \frac{B\omega\ell^2}{2}$$

$$\varepsilon = \frac{d\Phi}{dt} = \frac{\text{flux through the area swept by the rod in time dt}}{dt}$$

$$= \frac{B\frac{1}{2}\ell^2\omega dt}{dt} = \frac{1}{2} B\omega\ell^2$$

INDUCED ELECTRIC FIELD DUE TO A TIME VARYING MAGNETIC FIELD

Consider a conducting loop placed at rest in a magnetic field $\vec{B}$. Suppose, the field is constant till t = 0 and then changes with time. An induced current starts in the loop at t = 0.

The free electrons were at rest till t = 0 (we are not interested in the random motion of the electrons.) The magnetic field cannot exert force on electrons at rest. Thus, the magnetic force cannot start the induced current. The electrons may be forced to move only by an electric field. So we conclude that an electric field appears at time t = 0.

This electric field is produced by the changing magnetic field and not by charged particles. The electric field produced by the changing magnetic field is non-electrostatic and non-conservative in nature. We cannot define a potential corresponding to this field. We call it induced electric field. The lines of induced electric field are closed curves. There are no starting and terminating points of the field lines.

If $\vec{E}$ be the induced electric field, the force on the charge q placed in the field of $q\vec{E}$. The work done per unit charge as the charge moves through $d\vec{l}$ is E. $d\vec{l}$ .The emf developed in the loop is, therefore,

$$\varepsilon = \oint \vec{E}.d\vec{l}$$

Using Faraday's law of induction,

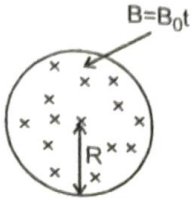
$$\varepsilon = -\frac{d\phi}{dt}$$

$$\text{or,} \quad \oint \vec{E}.d\vec{l} = -\frac{d\phi}{dt} \quad ...(6)$$

The presence of a conducting loop is not necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, the induced electric field is present. If a loop is there, the free electrons start drifting and consequently an induced current results.

CALCULATION FOR INDUCED ELECTRIC FIELD

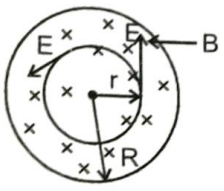
Let electric field at a distance r from axis of changing cylindrical magentic field B, which is parallel to the axis of cylinder?



(i) When $r < R$

let at a distance r electric field is E

$$\varepsilon = \left|\oint \vec{E}.d\vec{l}\right| = \left|-\frac{d\phi}{dt}\right|$$



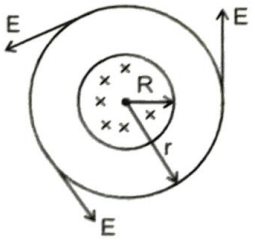
$$E.2\pi r = \left|-\frac{d[B.(\pi r^2)]}{dt}\right|$$

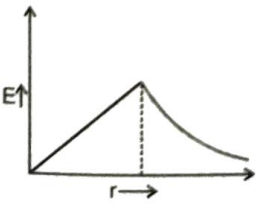
$$E = \frac{r}{2}\frac{dB}{dt} = \frac{B_0 r}{2}$$

(ii) When $r > R$

$$\left|\oint \vec{E}.d\vec{\ell}\right| = \left|\frac{-d\phi}{dt}\right| \Rightarrow E.2\pi r = \frac{d(B\pi R^2)}{dt}$$

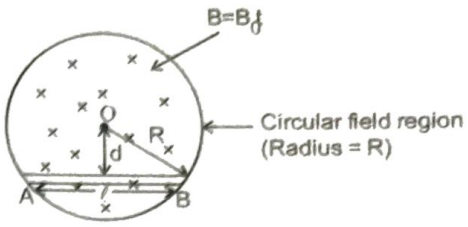
$$\Rightarrow E = \frac{R^2}{2r}.\frac{dB}{dt} = \frac{B_0 R^2}{2r}$$





TRAIN YOUR BRAIN

Example 8: Find the e.m.f induced across rod as shown in the figure.



Sol. $\oint_{OAB} \vec{E}.d\vec{l} = \frac{d\phi}{dt} = \frac{d}{dt}\left(\frac{\ell d}{2}B\right)$

$$\int_{OA} Edl + \int_{AB} Edl + \int_{BO} Edl = \frac{\ell d}{2}\left(\frac{d}{dt}(B)\right)$$

$$\Rightarrow 0 + \int_{AB} Edl + 0 = \frac{B_0 \ell d}{2}$$

$$\Rightarrow V_{AB} = \frac{B_0 \ell d}{2}$$

INDUCTANCE

Inductance is a measure of the ratio of the flux to the current I. Consider two coils, the current through one coil sets up a magnetic field both in the second coil as well as through itself. We have seen that the magnitude of magnetic field $B \propto I$ and thus the flux $\Phi_B \propto I$.

If the geometry of the coils is not varying with time then,

$$\frac{d\Phi_B}{dt} \propto \frac{dI}{dt}$$

The constant of proportionality is called inductance.

Note:

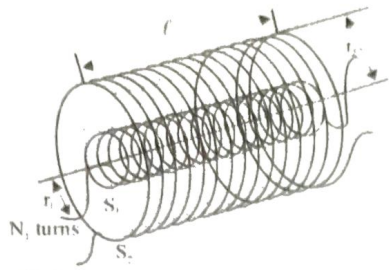
(i) It depends on the geometry of the coil and intrinsic material properties.

(ii) Inductance is a scalar quantity.

(iii) The SI unit of inductance is called henry and is denoted by H.

MUTUAL INDUCTANCE

Consider two long co-axial solenoids each of length ℓ. We denote the radius of the inner solenoid S_1 by r_1 and the number of turns per unit length by n_1. The corresponding quantities for the outer solenoid S_2 are r_2 and n_2 respectively.



We set up a time varying current I_2 through S_2. This sets up a time varying magnetic flux through S_1 which we designate by Φ_1.

$$\Phi_1 = M_{12} I_2 \quad \text{.....(1)}$$

(where M_{12} is the constant of proportionality and is called mutual inductance of the circuit 1 with respect to the circuit 2)

From Faraday's law the induced emf in S_1 is,

$$\varepsilon_1 = -\frac{d\Phi_1}{dt} = -M_{12}\frac{dI_2}{dt}$$

For this simple co-axial solenoids it is possible to calculate M_{12}.

The magnetic field due to the current I_2 is $\mu_0 n_2 I_2$. The flux Φ_1 through S_1 is,

$$\Phi_1 = (\pi r_1^2)(\mu_0 n_2 I_2)(n_1 \ell)$$
$$= \mu_0 n_1 n_2 I_2 \pi r_1^2 \ell \quad \text{.....(2)}$$

where $n_1 \ell$ is the total number of turns of solenoid S_1. Thus from Eq. (1) and Eq. (2),

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 \ell$$

This is a geometrical property of solenoids S_1 and S_2

Note: The current should vary slow enough so that the expressions for a magnetic field due to steady current are applicable.

We now consider the reverse case. A time varying current I_1 is passed through the solenoid S_1 and the associated flux through S_2 is ϕ_2,

$$\phi_2 = M_{21} I \quad \text{.....(3)}$$

M_{21} is called the mutual inductance of the circuit 2 with respect to the circuit 1.

From Faraday's law, the induced emf ε_2 in S_2 is,

$$\varepsilon_2 = -\frac{d\phi_2}{dt} = -M_{21}\frac{dI_1}{dt}$$

Once again for this simple case of coaxial solenoids, it is possible to calculate M_{21}. The flux due to S_1 is confined solely inside S_1 since the solenoids are very long. There is no magnetic field outside S_1, due to the current in S_1. Thus ϕ_2 is

$$\phi_2 = (\pi r_1^2)(\mu_0 n_1 I_1)(n_2 \ell)$$

where $n_2 \ell$ is the total number of turns of S_2. From equation (3)

$$M_{21} = \frac{\phi_2}{I_1}$$
$$= \mu_0 n_1 n_2 \pi r_1^2 \ell = M_{12}$$

Note:

(1) Mutual inductance depends on the geometry of the two circuits.

(2) Mutual inductance depends on the intrinsic magnetic property of the material, for example, the relative permeability.

(3) The equality, $M_{12} = M_{21}$ holds.

SELF-INDUCTANCE

It is also possible that emf is induced in a single isolated coil due to change of flux through the coil by means of varying the current through the same coil. This phenomenon is called self-induction. In this case, flux linkage through a coil of N turns is proportional to the current through the coil and is expressed as

$$N\Phi_B \propto I$$

$$N\Phi_B = LI$$

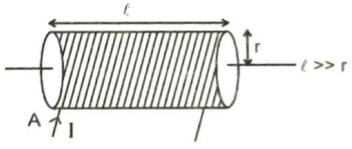
where constant of proportionality L is called self-inductance of the coil. It is also called the coefficient of self-induction of the coil. When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil. Here the induced emf is given by

$$\varepsilon = -\frac{d(N\Phi_B)}{dt}$$

$$\varepsilon = -L\frac{dI}{dt}$$

Thus, the self-induced emf always opposes any change (increase or decrease) of current in the coil.

SELF INDUCTANCE OF SOLENOID



Let the volume of the solenoid be V , the number of turns per unit length be n.

Let a current I be flowing in the solenoid.Magnetic field in the solenoid is given as $B=\mu_0 nI$.The magnetic flux through one turn of solenoid $\phi = \mu_0 n I A$.

The total magnetic flux through the solenoid

$$= N\phi = N\mu_0 nI A = \mu_0 n^2 I A \ell$$

$$\therefore L = \mu_0 n^2 \ell A = \mu_0 n^2 V$$

$$\phi = \mu_0 n I \pi r^2 (n\ell)$$

$$L = \frac{\phi}{I} = \mu_0 n^2 \pi r^2 \ell.$$

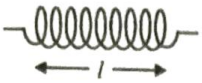
Inductance per unit volume = $\mu_0 n^2$.

Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant.Current can not change suddenly in the inductor.

INDUCTOR

Ideal Inductor: A part of long solenoid having zero resistance, inductance or coefficient of self induction

$$L = \mu_0 n^2 Al = \frac{\mu_0 N^2 A}{l}$$

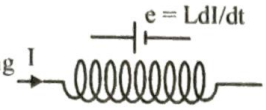


l = length, A = area of cross-section

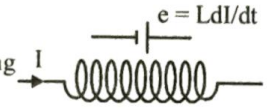
n = number of turns/length, N = total number of turns

Direction of Induced emf

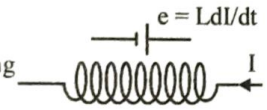
(a) I is increasing



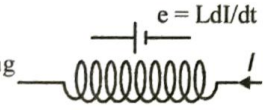
(b) I is decreasing



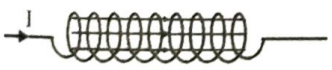
(c) I is increasing



(d) I is decreasing



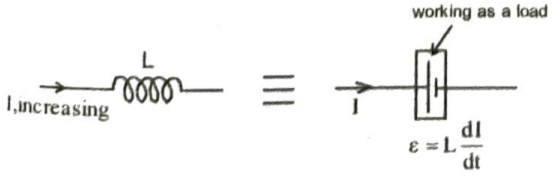
Energy in Inductor



$$\text{Energy } U_B = \frac{1}{2}LI^2, \text{ Energy Density} = \frac{1}{2}\frac{B^2}{\mu_0}$$

ENERGY STORED IN AN INDUCTOR

If current in an inductor at an instant is I and is increasing at the rate dI/dt,the induced emf will oppose the current. Its behaviour is shown in the figure.



Power consumed by the inductor = $IL\frac{dI}{dt}$

Energy consumed in dt time = $IL\frac{dI}{dt}dt$

$\therefore$ total energy consumed as the current increases from 0 to I

$$U = \int_0^I IL\,dI = \frac{1}{2}LI^2$$

$$\Rightarrow U = \frac{1}{2}LI^2$$

Note: This energy is stored in the magnetic field inside the inductor.

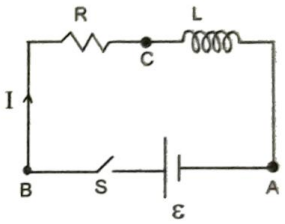
Magnetic energy density

$$= \frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy $U = \int \frac{B^2}{2\mu_0\mu_r}dV$

GROWTH OF CURRENT IN SERIES R–L CIRCUIT

Figure shows a circuit consisting of a cell, an inductor L and a resistor R ,connected in series. Let the switch S be closed at t=0.Suppose at an instant current in the circuit be I which is increasing at the rate dI/dt.

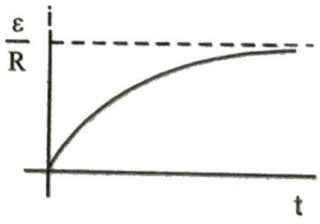


Writing KVL along the circuit ,

we have $\varepsilon - L\frac{dI}{dt} - IR = 0$

On solving we get, $I = \frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})$

The quantity L/R is called time constant of the circuit and is denoted by τ.The variation of current with time is as shown.



Note:

(i) Final current in the circuit = $\frac{\varepsilon}{R}$, which is independent of L.

(ii) After one time constant , current in the circuit =63% of the final current (verify yourself)

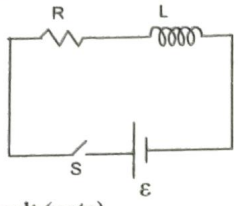
(iii) More time constant in the circuit implies slower rate of change of current.

(iv) If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.

$L_1 I_1 = L_2 I_2$

TRAIN YOUR BRAIN

Example 9: At t = 0 switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made η times lesser ($\frac{L}{\eta}$) then its initial value, find out instant current just after the operation.



Sol. Using above result (note)

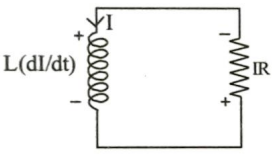
$$L_1 I_1 = L_2 I_2 \Rightarrow i_2 = \frac{\eta\varepsilon}{R}$$

DECAY OF CURRENT IN THE CIRCUIT CONTAINING RESISTOR AND INDUCTOR

Let the initial current in the circuit be I_0. At any time t, let the current be I and let its rate of change at this instant be $\frac{dI}{dt}$.

$$L.\frac{dI}{dt} + IR = 0$$

$$\frac{dI}{dt} = -\frac{IR}{L}$$



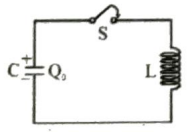
$$\int_{I_0}^{I} \frac{dI}{I} = -\int_{0}^{t} \frac{R}{L}.dt$$

$$\ln\left(\frac{I}{I_0}\right) = -\frac{Rt}{L} \text{ or } I = I_0 e^{\frac{-Rt}{L}}$$

Current after one time constant : $I = I_0 e^{-1} = 0.37\%$ of initial current.

LC OSCILLATIONS

Consider an LC circuit in which a capacitor is connected to an inductor, as shown in Figure.



Suppose the capacitor initially has charge Q_0. When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called **electromagnetic oscillation.**

The total energy in the LC circuit at some instant after closing the switch is

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

The fact that U remains constant hnplies that

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

where $I = -dQ/dt$ and $dI/dt = -d^2Q/dt^2$.

Notice the sign convention we have adopted here. The negative sign implies that the current I is equal to the rate of decrease of change in the capacitor plate immediately after the switch has been closed. The same equation can be obtained by applying the modified Kirchhoff s loop rule clockwise.

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

followed by our definition of current.

The general solution to equation is

$Q(t) = Q_0 \cos(\omega_0 t + \phi)$

where Q_0 is the amplitude of the charge and ϕ is the phase. Here angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The corresponding current in the inductor is

$$I(t) = -\frac{dQ}{dt} = \omega_0 Q_0 \sin(\omega_0 t + \phi) = I_0 \sin(\omega_0 t + \phi)$$

where $I_0 = \omega_0 Q_0$. From the initial conditions $Q(t = 0) = Q_0$ and $I(t = 0) = 0$, the phase ϕ can be determined to $\phi = 0$. Thus, the solutions for the charge and the current in our LC circuit are

$Q(t) = Q_0 \cos \omega_0 t$ and $I(t) = I_0 \sin \omega_0 t$

The time dependence of Q(t) and I(t) are depicted in figure.

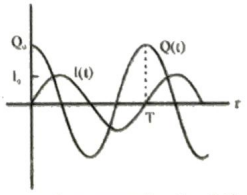


Figure: Charge and current in the *LC* circuit as a function of time

Using Eqs., we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E = \frac{Q^2(t)}{2C} = \left(\frac{Q_0^2}{2C}\right)\cos^2 \omega_0 t \quad \text{and}$$

$$U_B = \frac{1}{2}LI^2(t) = \frac{LI_0^2}{2}\sin^2 \omega t$$

$$= \frac{L(-\omega_0 Q_0)^2}{2}\sin^2 \omega_0 t = \left(\frac{Q^2}{2C}\right)\sin^2 \omega_0 t = \frac{Q_0^2}{2C}$$

The electric and magnetic energy oscillation is illustrated in figure.

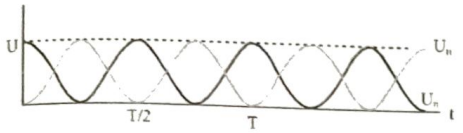


Figure: Electric and magnetic energy oscillations

The mechanical analog of the *LC* oscillations is the mass-spring system, shown in Figure.

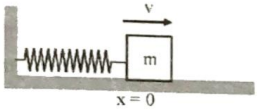


Figure: Mass-spring oscillations

If the mass is moving with a speed v and the spring having a spring constant k is displaced from its equilibrium by x, then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic energy of the mass and the potential energy of the spring, respectively. In the absence of friction, U is conserved and we obtain

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

Using v = dx/dt and dv/dt = d^2x/dt^2, the above equation may be rewritten as

$$m\frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is $x(t) = x_0 \cos(\omega_0 t + \phi)$

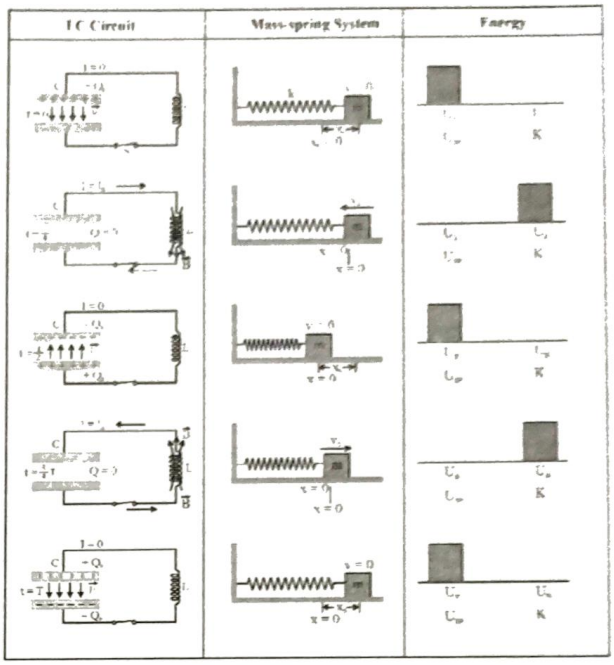
where $\omega_0 = \sqrt{\frac{k}{m}}$

is the angular frequency and x_0 is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system may be written as

$$U = \frac{1}{2}mx_0^2\omega_0^2 \sin^2(\omega_0 t + \phi) + \frac{1}{2}kx_0^2 \cos^2(\omega_0 t + \phi)$$

$$= \frac{1}{2}kx_0^2[\sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi)] = \frac{1}{2}kx_0^2$$

In figure we illustrate the energy oscillations in the LC circuit and the mass spring system (harmonic oscillator).



AC GENERATOR

The Phenomenon of electromagnetic induction has been technologically exploited in many ways. An exceptionally important application is the generation of alternating currents (AC).

BASIC DESIGN

The basic elements of an ac generator are shown in Figure.

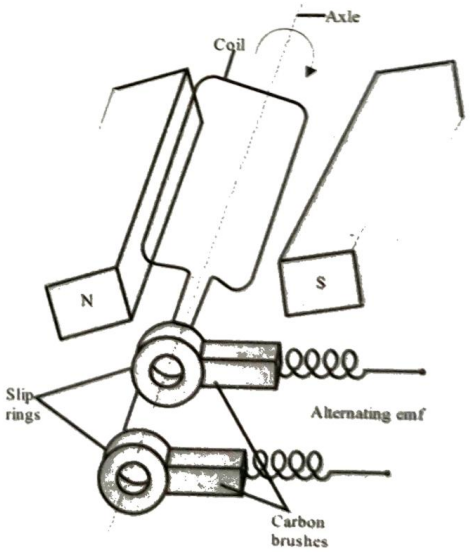


Figure : AC Generator

- It consists of a coil mounted on a rotor shaft.
- The axis of rotation of the coil is perpendicular to the direction of the magnetic field.
- The coil (called armature) is mechanically rotated in the uniform magnetic field by some external means
- The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil.
- The ends of the coil are connected to an external circuit by means of slip rings and brushes.

PRINCIPLE

One method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area.

WORKING

The working of this generator can be seen in the following ways:

- As the coil rotates in a magnetic field $\vec{B}$, the effective area of the loop (the face perpendicular to the field) is $A \cos\theta$, where θ is the angle between $\vec{A}$ and $\vec{B}$. This method of producing a flux change is the principle of operation of a simple ac generator. An ac generator converts mechanical energy into electrical energy.
- When the coil is rotated with a constant angular speed ω, the angle θ between the magnetic field vector $\vec{B}$ and the area vector $\vec{A}$ of the coil at any instant t is $\theta = \omega t$ (assuming $\theta = 0°$ at $t = 0$). As a result, the effective area of the coil exposed to the magnetic field lines changes with time. Hence the flux at any time t is,

$\Phi B = BA \cos\theta = BA \cos\omega t$

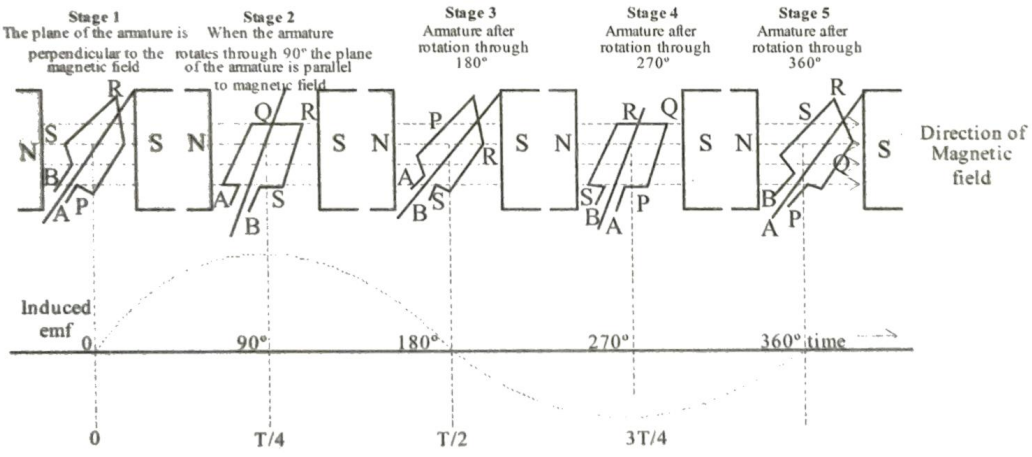
From Faraday's law, the induced emf for the rotating coil of N turns is then,

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t)$$

Thus, the instantaneous value of the emf is $\varepsilon = NBA\omega \sin\omega t$, where $NBA\omega$ is the maximum value of the emf, which occurs when $\sin\omega t = \pm 1$. If we denote $NBA\omega$ as ε_0, then $\varepsilon = \varepsilon_0 \sin\omega t$

- Since the value of the sine function varies between +1 and −1, the sign, or polarity of the emf changes with time.
- We can see from the figure that the emf has its extreme value when $\theta = 90°$ or $\theta = 270°$, as the change of flux is greatest at these points.
- The direction of the current changes periodically and therefore the current is called alternating current (ac). Since $\omega = 2\pi\nu$, the equation $\varepsilon = \varepsilon_0 \sin\omega t$ can be written as $\varepsilon = \varepsilon_0 \sin 2\pi\nu t$, where ν is the frequency of revolution of the generator's coil.

This gives the instantaneous value of the emf and varies between $+\varepsilon_0$ and $-\varepsilon_0$ periodically.



An alternating emf is generated by a loop of wire rotating in a magnetic field

Topicwise Questions

Flux and Faraday's law of EMI Lenz's law

1. In electromagnetic induction, the induced e.m.f. in a coil is independent of
   (*a*) Change in the flux
   (*b*) Time
   (*c*) Resistance of the circuit
   (*d*) None of these

2. A coil having an area A_0 is placed in a magnetic field which changes from B_0 to $4B_0$ in a time interval t. The e.m.f. induced in the coil will be
   (*a*) $\frac{3A_0B_0}{t}$ (*b*) $\frac{4A_0B_0}{t}$
   (*c*) $\frac{3B_0}{A_0t}$ (*d*) $\frac{4B_0}{A_0t}$

3. In a coil of area 10 cm^2 and 10 turns with a magnetic field directed perpendicular to the plane and is changing at the rate of 10^8 gauss/second. The resistance of the coil is 20 ohm. The current in the coil will be
   (*a*) 5 amp (b) 0.5 amp
   (*c*) 0.05 amp (*d*) 5×10^8 amp

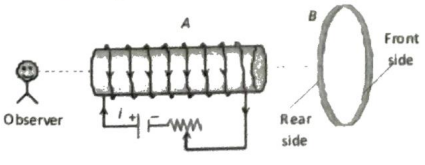
4. A coil having 500 square loops each of side 10 cm is placed normal to a magnetic flux which increases at the rate of 1.0 tesla/second. The induced e.m.f. in volts is
   (*a*) 0.1 (*b*) 0.5
   (*c*) 1 (*d*) 5

5. Lenz's law gives
   (*a*) The magnitude of the induced e.m.f.
   (*b*) The direction of the induced current
   (*c*) Both the magnitude and direction of the induced current
   (*d*) The magnitude of the induced current

6. A coil of 40 Ω resistance has 100 turns and radius 6 mm is connected to ammeter of resistance of 160 ohms. Coil is placed perpendicular to the magnetic field. When coil is taken out of the field, 32 μ C charge flows through it. The intensity of magnetic field will be
   (*a*) 6.55 T (*b*) 5.66 T
   (*c*) 0.655 T (*d*) 0.566 T

7. Lenz's law applies to
   (*a*) Electrostatics
   (*b*) Lenses
   (*c*) Electro-magnetic induction
   (*d*) Cinema slides

8. An aluminium ring B faces an electromagnet A. The current I through A can be altered

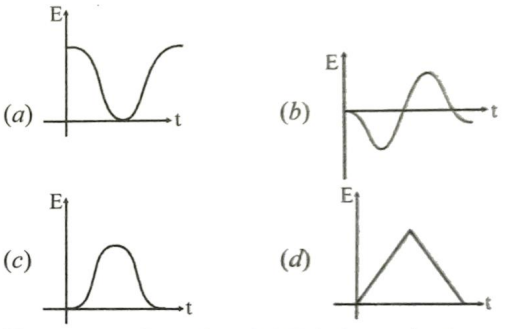


   (*a*) Whether I increases or decreases, B will not experience any force
   (*b*) If I decrease, A will repel B
   (*c*) If I increases, A will attract B
   (*d*) If I increases, A will repel B

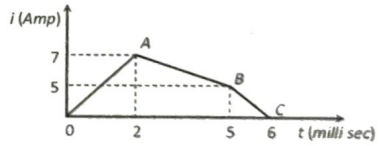
9. A coil having n turns and resistance RΩ is connected with a galvanometer of resistance 4 RΩ. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is
   (*a*) $-\frac{W_2-W_1}{5Rnt}$ (*b*) $-\frac{n(W_2-W_1)}{5Rt}$
   (*c*) $-\frac{(W_2-W_1)}{Rnt}$ (*d*) $-\frac{n(W_2-W_1)}{Rt}$

10. The variation of induced emf (E) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as





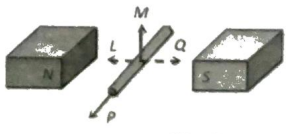
11. The current through a 4.6 H inductor is shown in the following graph. The induced emf during the time interval t = 5 milli-sec to 6 milli-sec will be



   (*a*) 10^3 V (*b*) -23×10^3 V
   (*c*) 23×10^3 V (*d*) Zero

Moving conductor and Motional emf

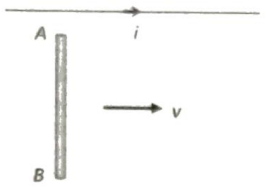
12. An electric potential difference will be induced between the ends of the conductor shown in the diagram, when the conductor moves in the direction



   (*a*) P (*b*) Q
   (*c*) L (*d*) M

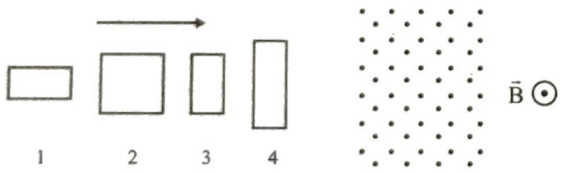
13. A player with 3 *m* long iron rod runs towards east with a speed of 30 *km/hr*. Horizontal component of earth's magnetic field is 4×10^{-5} Wb/m^2. If he is running with rod in horizontal and vertical positions, then the potential difference induced between the two ends of the rod in two cases will be
(*a*) Zero in vertical position and 1×10^{-3} V in horizontal position
(*b*) 1×10^{-3} V in vertical position and zero is horizontal position
(*c*) Zero in both cases
(*d*) 1×10^{-3} V in both cases

14. The current carrying wire and the rod AB are in the same plane. The rod moves parallel to the wire with a velocity *v*. Which one of the following statements is true about induced emf in the rod



(*a*) End A will be at lower potential with respect to B
(*b*) A and B will be at the same potential
(*c*) There will be no induced e.m.f. in the rod
(*d*) Potential at A will be higher than that at B

15. The four wire loops shown have vertical edge lengths of either L, 2L or 3L. They will move with the same speed into a region of uniform magnetic field $\vec{B}$ directed out of the page. Rank them according to the maximum magnitude of the induced emf, greatest to least.



(*a*) 1 and 2 tie, then 3 and 4 tie
(*b*) 3 and 4 tie, then 1 and 2 tie
(*c*) 4, 2, 3, 1
(*d*) 4 then, 2 and 3 tie and then 1

Self Inductance and Mutual Inductance

16. The back e.m.f. induced in a coil, when current changes from 1 ampere to zero in one milli second, is 4 volts, the self inductance of the coil is
(*a*) 1 H (*b*) 4 H
(*c*) 10^{-3} H (*d*) 4×10^{-3} H

17. Two coils of self inductance L_1 and L_2 are placed closer to each other so that total flux in one coil is completely linked with other. If M is mutual inductance between them, then
(*a*) $M = L_1L_2$ (*b*) $M = L_1 / L_2$
(*c*) $M = \sqrt{L_1L_2}$ (*d*) $M = (L_1L_2)^2$

18. When the current in a coil changes from 8 ampere to 2 ampere in 3×10^{-2} second, the e.m.f. induced in the coil is 2 volt. The self inductance of the coil (in millihenry) is
(*a*) 1 (*b*) 5
(*c*) 20 (*d*) 10

19. Two circuits have coefficient of mutual induction of 0.09 henry. Average e.m.f. induced in the secondary by a change of current from 0 to 20 ampere in 0.006 second in the primary will be
(*a*) 120 V (*b*) 80 V
(*c*) 200 V (*d*) 300 V

20. The self inductance of a solenoid of length L, area of cross-section A and having N turns is
(*a*) $\frac{\mu_0 N^2 A}{L}$ (*b*) $\frac{\mu_0 NA}{L}$
(*c*) $\mu_0 N^2 LA$ (*d*) $\mu_0 NAL$

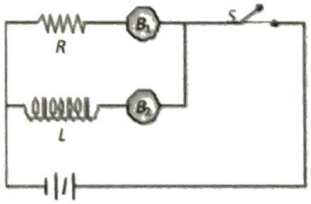
21. The self inductance of a coil is L. Keeping the length and area same, the number of turns in the coil is increased to four times. The self inductance of the coil will now be
(*a*) $\frac{1}{4}L$ (*b*) L
(*c*) 4 L (*d*) 16 L

L-R Circuit

22. Average energy stored in a pure inductor L when a current i flows through it, is
(*a*) Li^2 (*b*) $2Li^2$
(*c*) $\frac{Li^2}{4}$ (*d*) $\frac{Li^2}{2}$

23. Self induction of a solenoid is
(*a*) Directly proportional to current flowing through the coil
(*b*) Directly proportional to its length
(*c*) Directly proportional to area of cross-section
(*d*) Inversely proportional to area of cross-section

24. The adjoining figure shows two bulbs B_1 and B_2 resistor *R* and an inductor *L*. When the switch S is turned off



(*a*) Both B_1 and B_2 die out promptly
(*b*) Both B_1 and B_2 die out with some delay
(*c*) B_1 dies out promptly but B_2 with some delay
(*d*) B_2 dies out promptly but B_1 with some delay

25. A coil of inductance 40 henry is connected in series with a resistance of 8 ohm and the combination is joined to the terminals of a 2 volt battery. The time constant of the circuit is
(*a*) 40 seconds (*b*) 20 seconds
(*c*) 8 seconds (*d*) 5 seconds

L-C Oscillation

26. An oscillator circuit consists of an inductance of 0.5mH and a capacitor of 20 μF. The resonant frequency of the circuit is nearly

(*a*) 15.92 Hz (*b*) 159.2 Hz
(*c*) 1592 Hz (*d*) 15910 Hz

Application (Motor, Dynamic Transformer)

27. A step-down transformer is connected to main supply 200V to operate a 6V, 30W bulb. The current in primary is

(*a*) 3 A (*b*) 1.5 A
(*c*) 0.3 A (*d*) 0.15 A

28. Quantity that remains unchanged in a transformer is

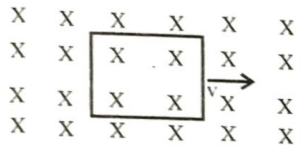
(*a*) Voltage
(*b*) Current
(*c*) Frequency
(*d*) None of these

29. In an inductor of inductance L = 100 mH, a current of I = 10 A is flowing. The energy stored in the inductor is

(*a*) 5 J (*b*) 10 J
(*c*) 100 J (*d*) 1000 J

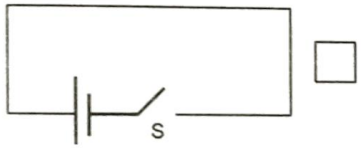
Learning Plus

1. A square frame of wire of side *l* and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop in fig. The current induced in the loop is -



(*a*) 2B*l*υ/R anticlockwise (*b*) B*l*υ/R anticlockwise
(*c*) B*l*υ/R clockwise (*d*) zero

2. Consider the conducting square loop shown in fig. If the switch is closed and after some time it is opened again, the closed loop will show



(*a*) a clockwise current-pulse
(*b*) an anticlockwise current-pulse
(*c*) an anticlockwise current-pulse and then a clockwise current-pulse
(*d*) a clockwise current-pulse and then an anticlockwise current-pulse

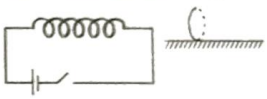
3. A flux of 1m Wb passes through a strip having an area A = 0.02 m^2. The plane of the strip is at an angle of 60° to the direction of a uniform field B. The value of B is-

(*a*) 0.1 T (*b*) 0.058 T
(*c*) 4.0 mT (*d*) none of these

4. The instantaneous flux associated with a closed circuit of 10Ω resistance is indicated by the following equation $\phi = 6t^2 - 5t + 1$, then the value in amperes of the induced current at t = 0.25 sec will be:

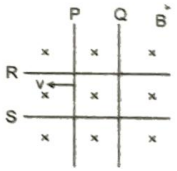
(*a*) 1.2 (*b*) 0.8
(*c*) 6 (*d*) 0.2

5. A horizontal solenoid is connected to a battery and a switch (figure). A conducting ring is placed on a frictionless surface, the axis of the ring being along the axis of the solenoid. As the switch is closed, the ring will



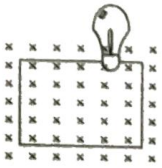
(*a*) move towards the solenoid
(*b*) remain stationary
(*c*) move away from the solenoid
(*d*) move towards the solenoid or away from it depending on which terminal (positive or negative) of the battery is connected to the left end of the solenoid.

6. Two identical conductors P and Q are placed on two frictionless fixed conducting rails R and S in a uniform magnetic field directed into the plane. If P is moved in the direction shown in figure with a constant speed, then rod Q



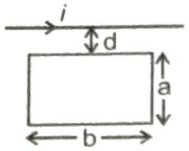
(*a*) will be attracted towards P
(*b*) will be repelled away from P
(*c*) will remain stationary
(*d*) may be repelled or attracted towards P

7. A square wire loop of 10.0 cm side lies at right angles to a uniform magnetic field of 20T. A 10 V light bulb is in a series with the loop as shown in the fig. The magnetic field is decreasing steadily to zero over a time interval Δt. The bulb will shine with full brightness if Δt is equal to



(*a*) 20 ms (*b*) 0.02 ms
(*c*) 2 ms (*d*) 0.2 ms

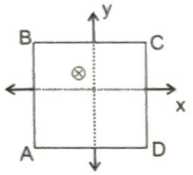
8. A long straight wire is parallel to one edge as in fig. If the current in the long wire is varies in time as $I = I_0 e^{-t\tau}$, what will be the induced emf in the loop?



(a) $\frac{\mu_0 bI}{\pi\tau} ln\left(\frac{d+a}{d}\right)$ (b) $\frac{\mu_0 bI}{2\pi\tau} ln\left(\frac{d+a}{d}\right)$

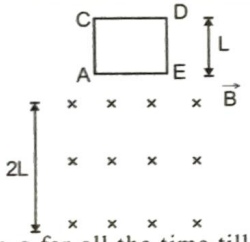
(c) $\frac{2\mu_0 bI}{\pi\tau} ln\left(\frac{d+a}{d}\right)$ (d) $\frac{\mu_0 bI}{\pi\tau} ln\left(\frac{d}{d+a}\right)$

9. A square coil ABCD is placed in x-y plane with its centre at origin. A long straight wire, passing through origin, carries a current in negative z-direction. Current in this wire increases with time.The induced current in the coil is



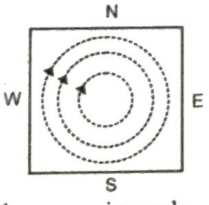
(a) clockwise (b) anticlockwise
(c) zero (d) alternating

10. A square coil ACDE with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length 2L. The acceleration of the coil is



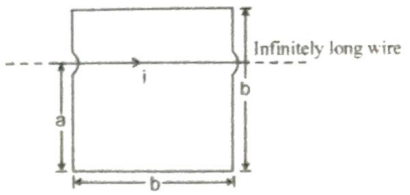
(a) less than g for all the time till the loop crosses the magnetic field completely
(b) less than g when it enters the field and greater than g when it comes out of the field
(c) g all the time
(d) less than g when it enters and comes out of the field but equal to g when it is within the field

11. A metal sheet is placed in a variable magnetic field which is increasing from zero to maximum. Induced current flows in the directions as shown in figure. The direction of magnetic field will be-



(a) normal to the paper, inwards
(b) normal to the paper, outwards.
(c) from east to west
(d) from north to south

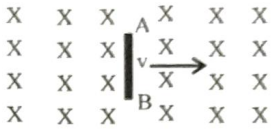
12. For the situation shown in the figure, flux through the square loop is :



(a) $\left(\frac{\mu_0 ia}{2\pi}\right) ln\left(\frac{a}{2a-b}\right)$ (b) $\left(\frac{\mu_0 ib}{2\pi}\right) ln\left(\frac{a}{2b-a}\right)$

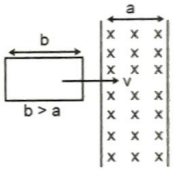
(c) $\left(\frac{\mu_0 ib}{2\pi}\right) ln\left(\frac{a}{b-a}\right)$ (d) $\left(\frac{\mu_0 ia}{2\pi}\right) ln\left(\frac{2a}{b-a}\right)$

13. A conducting rod AB moves with a uniform velocity v in a constant magnetic field as shown in figire.



(a) The rod becomes hot because of Joule heating
(b) The end A becomes positively charged
(c) The end B become positively charged
(d) The rod becomes electrically charged

14. In the given arrangement, the loop is moved with constant velocity v in a uniform magnetic field B in a restricted region of width a. The time for which the emf is induced in the circuit is:



(a) $\frac{2b}{v}$ (b) $\frac{2a}{v}$

(c) $\frac{(a+b)}{v}$ (d) $\frac{2(a-b)}{v}$

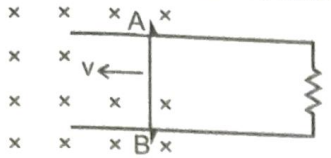
15. A uniform magnetic field exists in region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. A rod of length 5 m is placed along y-axis is moved along x – axis with constant speed 1 m/sec. Then induced e.m.f. in the rod will be :
(a) zero (b) 25 v
(c) 20 v (d) 15 v

16. The magnetic field in a region is given by $\vec{B} = B_0\left(1 + \frac{x}{a}\right)\hat{k}$. A square loop of edge-length d is placed with its edge along x & y axis. The loop is moved with constant velocity $\vec{V} = V_0\hat{i}$. The emf induced in the loop is

(a) $\frac{V_0 B_0 d^2}{a}$ (b) $\frac{V_0 B_0 d^2}{2a}$

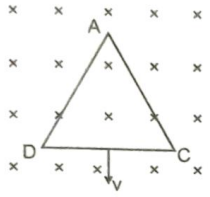
(c) $\frac{V_0 B_0 a^2}{d}$ (d) None of these

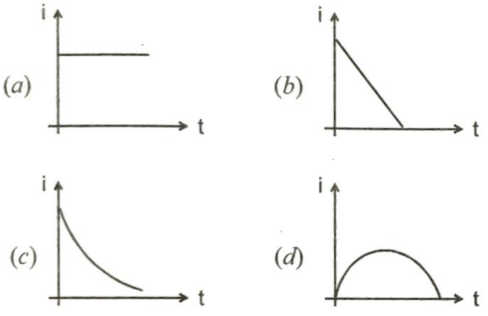
17. Consider the situation shown in fig. The resistanceless wire AB is slid on the fixed rails with a constant velocity. If the wire AB is replaced by a resistanceless semi-circular wire, the magnitude of the induced current will



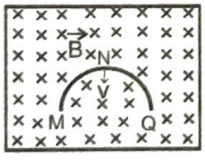
(*a*) increase
(*b*) remain the same
(*c*) decrease
(*d*) increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.

18. An equilateral triangular loop ADC having some resistance is pulled with a constant velocity v out of a uniform magnetic field directed into the paper. At time t = 0, side DC of the loop is at edge of the magnetic field. The induced current (i) versus time (t) graph will be as





19. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is v then the potential difference developed across the ring is:



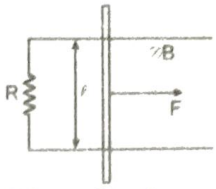
(*a*) zero

(*b*) $\frac{Bv\pi R^2}{2}$ and M is at higher potential

(*c*) π RBV and Q is at higher potential

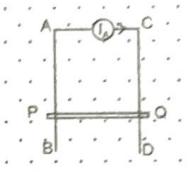
(*d*) 2 RBV and Q is at higher potential.

20. A constant force F is being applied on a rod of length 'ℓ' kept at rest on two parallel conducting rails connected at ends by resistance R in uniform magnetic field B as shown.



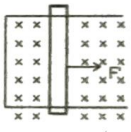
(*a*) the power delivered by force will be constant with time
(*b*) the power delivered by force will be increasing first and then it will decrease
(*c*) the rate of power delivered by the external force will be increasing continuously
(*d*) the rate of power delivered by external force will be decreasing continuously before becoming zero.

21. AB and CD are fixed conducting smooth rails placed in a vertical plane and joined by a constant current source at its upper end. PQ is a conducting rod which is free to slide on the rails. A horizontal uniform magnetic field exists in space as shown. If the rod PQ is released from rest then,



(*a*) The rod PQ may move downward with constant acceleration
(*b*) The rod PQ may move upward with constant acceleration
(*c*) The rod will move downward with decreasing acceleration and finally acquire a constant velocity
(*d*) either (*a*) or (*b*)

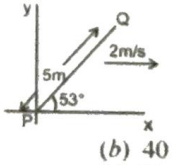
22. A rod closing the circuit shown in figure moves along a U shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicular to the plane. Calculate F if the rate of heat generation in the circuit is Q



(*a*) F = Qv (*b*) $F = \frac{Q}{v}$

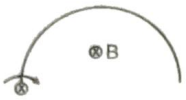
(*c*) $F = \frac{v}{Q}$ (*d*) $F = \sqrt{Qv}$

23. A conducting rod PQ of length 5 m oriented as shown in figure is moving with velocity $(2m/s)\hat{i}$ without rotation in a uniform magnetic field $(3\hat{j}+4\hat{k})$ Tesla. Emf induced in the rod is



(*a*) 32 Volts (*b*) 40 Volt
(*c*) 50 Volt (*d*) None of these

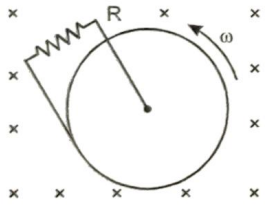
24. A semicircular wire of radius R is rotated with constant angular velocity ω about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced e.m.f. between the ends is :



(*a*) $B\omega R^2/2$ (*b*) $2B\omega R^2$
(*c*) is variable (*d*) none of these

25. The spokes of a wheel are made of metal and their lengths are of one metre. On rotating the wheel about its own axis in a uniform magnetic field of 5×10^{-5} tesla normal to the plane of wheel, a potential difference of 3.14 mV is generated between the rim and the axis. The rotational velocity of the wheel is
(*a*) 63 rev./s (*b*) 50 rev./s
(*c*) 31.4 rev./s (*d*) 20 rev./s

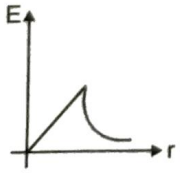
26. Figure shows a conducting disc rotating about its axis in a perpendicular magnetic field B. A resistor of resistance R is connected between the centre and the rim. The radius of the disc is 5.0 cm, angular speed $\omega = 40$ rad/s, B = 0.10 T and R = 1 Ω. The current through the resistor is

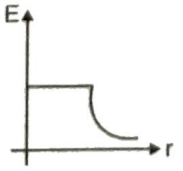


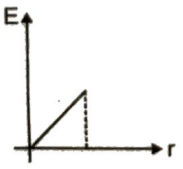
(*a*) 5 mA (*b*) 50 A
(*c*) 5 A (*d*) 10 mA

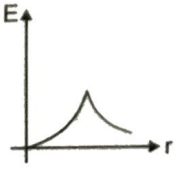
27. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder. If B changes at a constant rate, the graph showing the variation of induced electric field with distance r from the axis of cylinder is



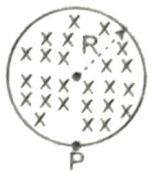
(*a*) 

(*b*) 

(*c*) 

(*d*) 

28. A uniform magnetic field of induction B is confined to a cylindrical region of radius R. The magnetic field is increasing at a constant rate of $\frac{dB}{dt}$ (Tesla/second). An electron of charge q, placed at the point P on the periphery of the field experiences an acceleration :

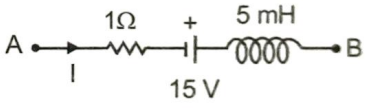


(*a*) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward left (*b*) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward right

(*c*) $\frac{eR}{m}\frac{dB}{dt}$ toward left (*d*) zero

29. A long solenoid of N turns has a self inductance L and area of cross section A. When a current i flows through the solenoid, the magnetic field inside it has magnitude B. The current i is equal to:
(*a*) BAN/L (*b*) BANL
(*c*) BN/AL (*d*) B/ANL

30. The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5A and it is decreasing at a rate of 10^3 As^{-1} then $V_B - V_A$ equals



(*a*) 20 V (*b*) 15 V
(*c*) 10 V (*d*) 5 V

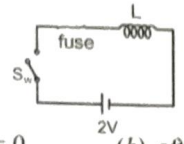
31. On making a coil of copper wire of length ℓ and coil radius r, the value of self inductance is obtained as L. If the coil of same wire, but of coil radius r/2, is made, the value of self inductance will be-
(*a*) 2L (*b*) L
(*c*) 4L (*d*) L/2

32. The coefficient of mutual induction between two coils is 4H. If the current in the primary reduces from 5A to zero in 10^{-3} second then the induced e.m.f. in the secondary coil will be-
(*a*) 10^4 V (*b*) 25×10^3 V
(*c*) 2×10^4 V (*d*) 15×10^3 V

33. A thin copper wire of length 100 metres is wound as a solenoid of length 1 and radius r. Its self inductance is found to be L. Now if the same length of wire is wound as a solenoid of length 1 but of radius r/2, then its self inductance will be-
(*a*) 4L (*b*) 2L
(*c*) L (*d*) L/2

34. A rectangular loop of sides 'a' and 'b' is placed in xy plane. A very long wire is also placed in xy plane such that side of length 'a' of the loop is parallel to the wire. The distance between the wire and the nearest edge of the loop is 'd'. The mutual inductance of this system is proportional to:
(a) a (b) b
(c) 1/d (d) current in wire

35. In the circuit shown, the cell is ideal. The coil has an inductance of 4H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5A. The switch is closed at t = 0. The fuse will blow:

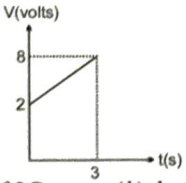


(a) just after t = 0 (b) after 2s
(c) after 5 s (d) after 10s

36. In a series L–R growth circuit, if maximum current and maximum induced emf in an inductor of inductance 3mH are 2A and 6V respectively, then the time constant of the circuit is
(a) 1 ms (b) 1/3 ms
(c) 1/6 ms (d) 1/2 ms

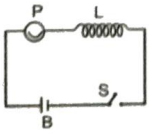
37. The value of coefficient of mutual induction in two coils can be increased by-
(a) placing the coils mutually perpendicular.
(b) keeping the coils near to each other.
(c) keeping the coils considerably apart.
(d) winding the core on the common iron magnetic material and insulating them.

38. A circuit element is placed in a closed box. At time t = 0, constant current generator supplying a current of 1 amp, is connected across the box. Potential difference across the box varies according to graph shown in figure. The element in the box is :



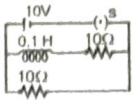
(a) resistance of 2Ω (b) battery of emf 6 V
(c) inductance of 2H (d) capacitance of 0.5F

39. In figure, a lamp P is in series with an iron-core inductor L. When the switch S is closed, the brightness of the lamp rises relatively slowly to its full brightness than it would do without the inductor. This is due to



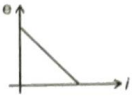
(a) the low resistance of P
(b) the induced-emf in L
(c) the low resistance of L
(d) the high voltage of the battery B

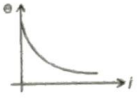
40. In the adjoining circuit, initially the switch S is open. The switch 'S' is closed at t = 0. The difference between the maximum and minimum current that can flow in the circuit is



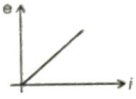
(a) 2 Amp (b) 3 Amp
(c) 1 Amp (d) nothing can be concluded

41. In an L-R circuit connected to a battery of constant e.m.f E switch S is closed at time t = 0. If e denotes the magnitude of induced e.m.f across inductor and i the current in the circuit at any time t. Then which of the following graphs shows the variation of e with *i* ?

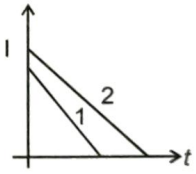
(a) 

(b) 

(c) 

(d) 

42. Two identical inductance carry currents that vary with time according to linear laws (as shown in figure). In which of two inductance is the self induction emf greater?



(a) 1
(b) 2
(c) same
(d) data are insufficient to decide

43. An inductor coil stores U energy when i current is passed through it and dissipates energy at the rate of P. The time constant of the circuit, when this coil is connected across a battery of zero internal resistance is
(a) $\frac{4U}{P}$ (b) $\frac{U}{P}$
(c) $\frac{2U}{P}$ (d) $\frac{2P}{U}$

44 A transformer transforms 220 volts to 11 volts. If the current strength in the primary coil is 5 amp. and that in the secondary, 90 amp, what is the efficiency of the transformer-
(a) 90 % (b) 100 %
(c) 211 % (d) 150 %

45. A transformer is used to light 140 watt 24 volt lamp from 240 volt AC mains, the current in the main cable is 0.7 amp. The efficiency of the transformer is-
(a) 63.8% (b) 84%
(c) 83.3% (d) 48%

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. A conducting ring is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the ring if
(*a*) it is rotated about its axis
(*b*) it is translated
(*c*) it is rotated about a diameter
(*d*) it is deformed

2. Figure shown plane figure made of a conductor located in a magnetic field along the inward normal to the plane of the figure. The magnetic field starts diminishing. Then the induced current

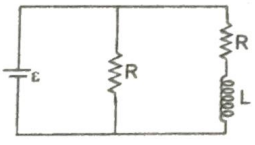


(*a*) at point P is clockwise
(*b*) at point Q is anticlockwise
(*c*) at point Q is clockwise
(*d*) at point R is zero

3. A bar magnet is moved along the axis of copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true?
(*a*) The south pole faces the ring and the magnet moves towards it.
(*b*) The north pole faces the ring and the magnet moves towards it.
(*c*) The south pole faces the ring and the magnet moves away from it.
(*d*) The north pole faces the ring and the magnet moves away from it.

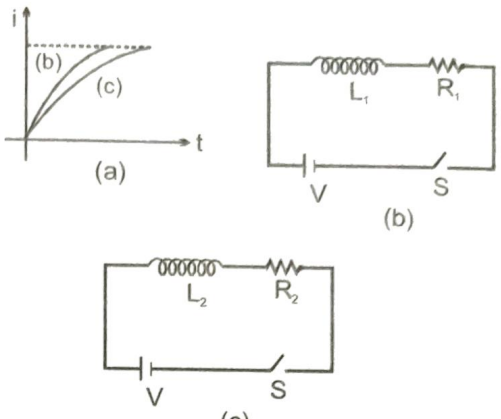
4. A conducting loop rotates with constant angular velocity about its fixed diameter in a uniform magnetic field in a direction perpendicular to that fixed diameter.
(*a*) The emf will be maximum at the moment when flux is zero.
(*b*) The emf will be '0' at the moment when flux is maximum.
(*c*) The emf will be maximum at the moment when plane of the loop is parallel to the magnetic field
(*d*) The phase difference between the flux and the emf is $\pi/2$

5. In the circuit diagram shown



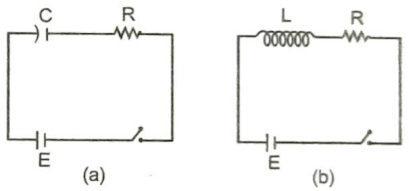
(*a*) time constant is L/R
(*b*) time constant is 2L/R
(*c*) steady state current in inductor is $2\varepsilon/R$
(*d*) steady state current in inductor is ε/R

6. Current growth in two L-R circuits (b) and (c) as shown in figure (a). Let L_1, L_2, R_1 and R_2 be the corresponding values in two circuits. Then



(*a*) $R_1 > R_2$ (*b*) $R_1 = R_2$
(*c*) $L_1 > L_2$ (*d*) $L_1 < L_2$

7. The switches in figures (a) and (b) are closed at t = 0



(*a*) The charge on C just after t = 0 is EC.
(*b*) The charge on C long after t = 0 is EC.
(*c*) The current in L just after t = 0 is E/R.
(*d*) The current in L long after t = 0 is E/R.

8. Two different coils have self inductance 8mH and 2mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are I_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are I_2, v_2 and W_2 respectively. Then :

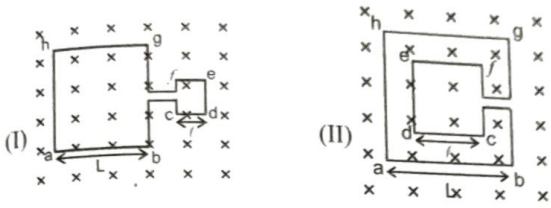
(*a*) $\frac{I_1}{I_2} = \frac{1}{4}$ (*b*) $\frac{I_1}{I_2} = 4$

(*c*) $\frac{W_2}{W_1} = 4$ (*d*) $\frac{V_2}{V_1} = \frac{1}{4}$

9. An LR circuit with a battery is connected at t =0. Which of the following quantities are zero just after the connection?
(*a*) current in the circuit
(*b*) magnetic field energy in the inductor
(*c*) power delivered by the battery
(*d*) emf induced in the inductor

Comprehension–1
Question No. 10 to 13 (4 questions)

The adjoining figure shows two different arrangements in which two square wire frames of same resistance are placed in a uniform constantly decreasing magnetic field B.



10. The value of magnetic flux in each case is given by
(*a*) Case I : $\phi = \pi(L^2 + \ell^2)\,B$; Case II : $\phi = \pi(L^2 - \ell^2)\,B$
(*b*) Case I : $\phi = \pi(L^2 + \ell^2)\,B$; Case II : $\phi = \pi(L^2 + \ell^2)\,B$
(*c*) Case I : $\phi = (L^2 + \ell^2)\,B$; Case II : $\phi = (L^2 - \ell^2)\,B$
(*d*) Case I : $\phi = (L + \ell)^2\,B$; Case II : $\phi = (L - \ell)^2\,B$

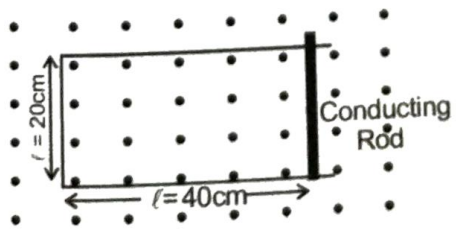
11. The direction of induced current in the case I is
(*a*) from a to b and from c to d
(*b*) from a to b and from *f* to e
(*c*) from b to a and from d to c
(*d*) from b to a and from e to f

12. The direction of induced current in the case II is
(*a*) from a to b and from c to d
(*b*) from b to a and from *f* to e
(*c*) from b to a and from c to d
(*d*) from a to b and from d to c

13. If I_1 and I_2 are the magnitudes of induced current in the cases I and II, respectively, then
(*a*) $I_1 = I_2$ (*b*) $I_1 > I_2$
(*c*) $I_1 < I_2$ (*d*) nothing can be said

Comprehension – 2
Question No. 14 to 16 (3 questions)

Figure shows a conducting rod of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance 1Ω/m. Position of the conducting rod at t = 0 is shown. A time t dependent magnetic field B = 2t Tesla is switched on at t = 0.



14. The current in the loop at t = 0 due to induced emf is
(*a*) 0.16 A, clockwise (*b*) 0.08 A, clockwise
(*c*) 0.08 A, anticlockwise (*d*) zero

15. At t = 0, when the magnetic field is switched on, the conducting rod is moved to the left at constant speed 5 cm/s by some external means. The rod moves remaining perpendicular to the rails. At t = 2s, induced emf has magnitude.
(*a*) 0.12 V (*b*) 0.08 V
(*c*) 0.04 V (*d*) 0.02 V

16. Following situation of the previous question, the magnitude of the force required to move the conducting rod at constant speed 5 cm/s at the same instant t = 2s, is equal to
(*a*) 0.16 N (*b*) 0.12 N
(*c*) 0.08 N (*d*) 0.06 N

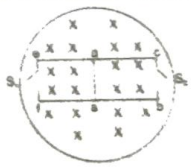
17. A square loop of conducting wire is placed near a long straight current carrying wire as shown. Match the statements in column-I with the corresponding results in column-II.



Column-I
(A) If the magnitude of current I is increased
(B) If the magnitude of current I is decreased
(C) If the loop is moved away from the wire
(D) If the loop is moved towards the wire

Column-II
(p) Induced current in the loop will be clockwise
(q) Induced current in the loop will be anticlockwise
(r) wire will attract the loop
(s) wire will repel the loop

18. The magnetic field in the cylindrical region shown in figure increases at a constant rate of 10.0 mT s. Each side of the square loop abcd and defa has a length of 2.00 cm and resistance of 2.00 Ω. Correctly match the current in the wire 'ad' in four different situations as listed in Column-I with the values given in Column-II.

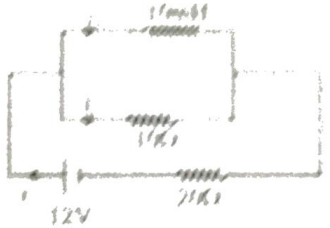


Column-I
(A) the switch S_1 is closed but S_2 is open
(B) S_1 is open but S_2 is closed
(C) both S_1 and S_2 are open
(D) both S_1 and S_2 are closed.

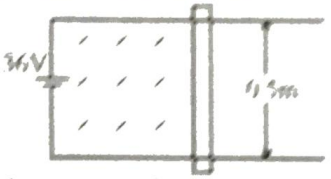
Column-II
(p) 5×10^{-7} A, d to a
(q) 5×10^{-7} A, a to d
(s) No current flows
(r) 2.5×10^{-8} A, d to a

NUMERICAL VALUE BASED

19. In the circuit shown, what is the current (in mA) in the ideal inductor when the current in the battery is 7 mA? (in the direction shown)



20. A uniform magnetic field of 0.6 T is normal to the plane of the figure. The resistance of rod is 2.4 Ω. Mass is 24 gm, and it can slide freely on smooth parallel rails which are perfectly conducting. The whole system is in horizontal plane. If the rod starts from rest, what is its terminal velocity (in m/s)?



21. A copper wire bent in the shape of a semicircle of radius r translates in its plane with a constant velocity v. A uniform magnetic field B exists in the direction perpendicular to the plane of the wire. Find the emf (in volts) induced between the ends of the wire if the velocity is perpendicular to the diameter joining free ends. (**Take** : B = 0.1 T, v = 5 m/s, r = 2m)

22. A rectangular coil of 100 turns measures 40 cm by 25 cm. This coil is placed next to an electromagnet which is switched on, increasing the magnetic field through the coil from 0 to 0.21 T in 50 ms uniformly. The resistance of the coil is 2Ω, what is the induced current (in A) in the coil?

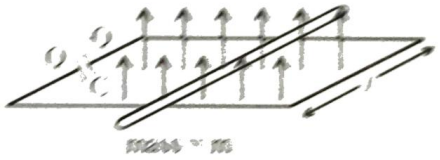
23. A 50 cm long copper wire is moved with a velocity of 2 m/s in a direction perpendicular to a uniform magnetic field of 0.4 T. If the wire is connected to the terminals of a 2 Ω resistor, what is the power (in W) required to maintain the velocity of the wire?

(Assume resistance of copper wire is negligible)

24. A conducting frame is placed in a vertical plane as shown in the figure. The arc, which are part of circle of radius R, is made of a uniform conducting rod and can slide without friction on the frame. The total resistance of the circuit is 6.2 Ω. The rod is connected to a light rope by a massless pulley. A uniform magnetic field of 2 T exists vertically upward. The mass of the rod is 100 g. Find the terminal velocity of rod with which the rod will move vertically upward.

25. An arrangement of wires of one loop and two conducting rails which are resistanceless are joined to a battery. The rails are connected by a rod of mass m and resistance R. Vertical magnetic field is uniform. The rod is initially at rest. It starts moving due to the influence of magnetic field. What should be the mass of the rod (in kg) so that its kinetic energy is steady state is maximum?

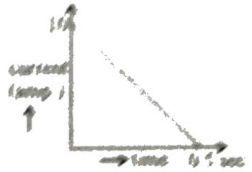
(**Take** : l = 1m, B = 2T, ℰ = 1 volt, R = 1Ω)



JEE Mains & Advanced Past Years Questions

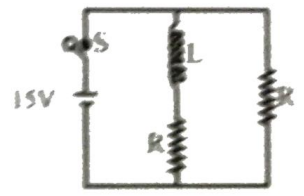
JEE-MAIN PREVIOUS YEAR'S

1. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is : **[JEE Main-2017]**



(a) 250 Wb (b) 275 Wb
(c) 200 Wb (d) 225 Wb

2. In the figure shown, a circuit contains two identical resistors with resistance R = 5 Ω and an inductance with L = 2 mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery, long after the switch is closed? **[JEE Main-2019 (January)]**



(a) 5.5 A (b) 7.5 A
(c) 3 A (d) 6 A

3. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then self inductance of the coil :

[JEE Main-2019 (January)]

(a) decreases by a factor of 9
(b) increases by a factor of 27
(c) increases by a factor of 3
(d) decreases by a factor of $9\sqrt{3}$

4. The self produced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is:

[JEE Main-2019 (January)]

(a) 740 J
(b) 437.5 J
(c) 540 J
(d) 637.5 J

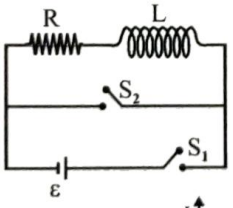
5. There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The ratio of mutual inductance to the self - inductance of the inner-coil is :

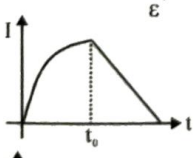
[JEE Main-2019 (January)]

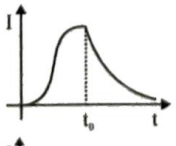
(a) $\frac{n_1}{n_2}$
(b) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$
(c) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$
(d) $\frac{n_2}{n_1}$

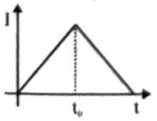
6. In the circuit shown, The switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0) the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't'.

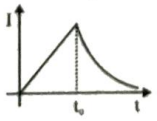
[JEE Main-2019 (January)]



(a) 

(b) 

(c) 

(d) 

7. A conducting circular loop made of a thin wire, has area 3.5×10^{-1} m^2 and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T) \sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to:

[JEE Main-2019 (January)]

(a) 14 mC (b) 7 mC
(c) 21 mC (d) 6 mC

8. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, of 0.3×10^{-4} Wb/m^2. The value of the induced emf in wire is :

[JEE Main - 2019 (January)]

(a) 1.5×10^{-3} V
(b) 1.1×10^{-3} V
(c) 2.5×10^{-3} V
(d) 0.3×10^{-3} V

9. A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is:

[JEE Main-2019 (January)]

(a) 12mV
(b) 6mV
(c) 1mV
(d) 2mV

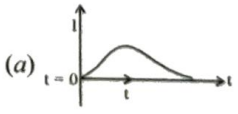
10. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of 10^{-3} Wb passes through 'Q'. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is:

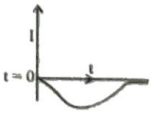
[JEE Main-2019]

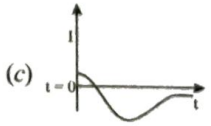
(a) 6.67×10^{-3} Wb
(b) 6.67×10^{-4} Wb
(c) 3.67×10^{-4} Wb
(d) 3.67×10^{-3} Wb

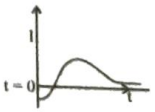
11. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t} (k > 0)$, as a function of time ($t \geq 0$). Counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time by:

[JEE Main-2019 (April)]

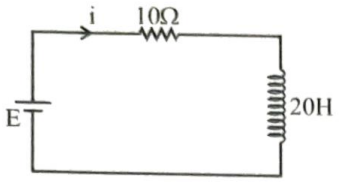
(a) 

(b) 

(c) 

(d) 

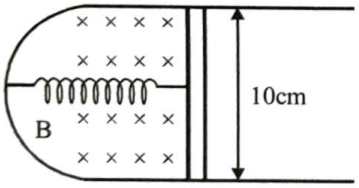
12. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor is : [*JEE Main-2019 (April)*]



(*a*) $\frac{2}{\ell n 2}$ (*b*) $\ell n 2$

(*c*) $2\ell n 2$ (*d*) $\frac{1}{2}\ell n 2$

13. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillation it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to: [*JEE Main-2019 (April)*]



(*a*) 50000 (*b*) 5000

(*c*) 10000 (*d*) 1000

14. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is: (**Take :** $\ell n 5 = 1.6$) [*JEE Main-2019 (April)*]

(*a*) 0.103 s (*b*) 0.016 s

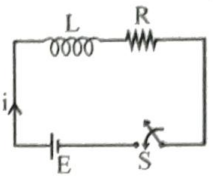
(*c*) 0.002 s (*d*) 0.324 s

15. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to: [*JEE Main-2019 (April)*]

(*a*) $1/L^2$ (*b*) $1/L$

(*c*) L (*d*) L^2

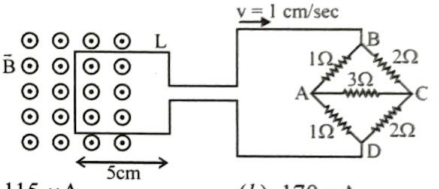
16. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t = \frac{L}{R}$ is [*JEE Main-2019 (April)*]



(*a*) $\frac{EL}{7.3R^2}$ (*b*) $\frac{EL}{2.7R^2}$

(*c*) $\frac{7.3EL}{R^2}$ (*d*) $\frac{2.7EL}{R^2}$

17. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm/sec. At some instant, a part of L is in a uniform magnetic field of 1T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to : [*JEE Main-2019 (April)*]



(*a*) 115 μA (*b*) 170 μA

(*c*) 60 μA (*d*) 150 μA

18. A loop ABCDEFA of straight edges has six corner points A(0, 0, 0), B(5, 0, 0), C(5, 5, 0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k})$ T . The quantity of flux through the loop ABCDEFA (in Wb) is..... [*JEE Main-2020 (January)*]

19. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be: [*JEE Main-2020 (January)*]

(*a*) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

(*b*) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

(*c*) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

(*d*) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

20. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given ϕ. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct? [*JEE Main-2020 (January)*]

(*a*) $\phi i > \phi_0$ (*b*) $\phi i < \phi_0$

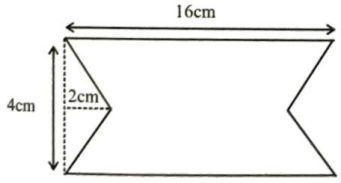
(*c*) $\phi i = -\phi_0$ (*d*) $\phi i = \phi_0$

21. A long solenoid of radius R carries a time(t)- dependent current $I(t) = I_0 t(1 - t)$. A ring of radius 2R is placed coaxially near its middle. During the time interval $0 \le t \le 1$, the induced current (I_R) and the induced EMF(V_R) in the ring change as: [JEE Main-2020 (January)]
(a) Direction of I_R remains unchanged and V_R is zero at t = 0.25
(b) At t = 0.5 direction of I_R reverses and V_R is zero
(c) At t = 0.25 direction of I_R reverses and V_R is maximum
(d) Direction of I_R remains unchanged and V_R is maximum at t = 0.5

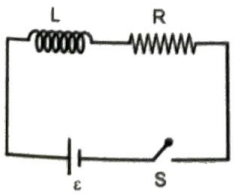
22. An emf of 20 V is applied at time t = 0 to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time $t = \infty$ and at t = 40 s is close to: (Take $e^2 = 7.389$) [JEE Main-2020 (January)]
(a) 1.46
(b) 1.06
(c) 0.84
(d) 1.15

23. A planar loop of wire rotates in a uniform magnetic field. Initially at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at : [JEE Main-2020 (January)]
(a) 5.0 s and 7.5 s
(b) 5.0 s and 10.0 s
(c) 2.5 s and 7.5 s
(d) 2.5 s and 5.0 s

24. At time t = 0 magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5 s, then induced EMF in the loop is : [JEE Main-2020 (January)]



(a) 28μV (b) 36μV
(c) 48μV (d) 56μV

25. 

As shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at t = 0. The total charge that flows from the battery, between t = 0 and $t = t_c$ (t_c is the time constant of the circuit) is : [JEE Main-2020 (January)]

(a) $\frac{\varepsilon L}{R^2}\left(1 - \frac{1}{e}\right)$ (b) $\frac{\varepsilon R}{eL^2}$
(c) $\frac{\varepsilon L}{R^2}$ (d) $\frac{\varepsilon L}{eR^2}$

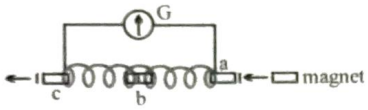
26. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be ..... [JEE Main-2020 (January)]

27. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5}T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2 s. The maximum value of EMF induced (in μV) in the coil will be close to the integer_____. [JEE Main-2020 (September)]

28. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate $dB/dt = 0.032\ Ts^{-1}$. The induced current in the loop is close to (Resistivity of the metal wire is 1.23×10^{-8} Ωm) [JEE Main-2020 (September)]
(a) 0.34 A
(b) 0.61 A
(c) 0.53 A
(d) 0.43 A

29. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at t = 0, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is: [JEE Main-2020 (September)]
(a) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$
(b) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$
(c) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$
(d) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$

30. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil?

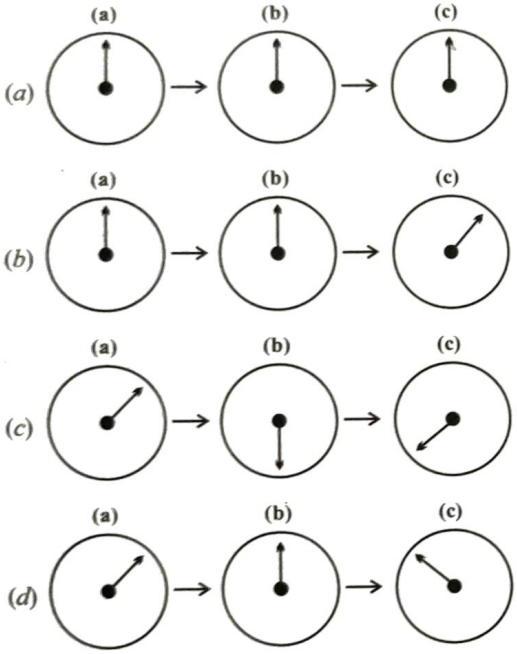


Three positions shown describe:

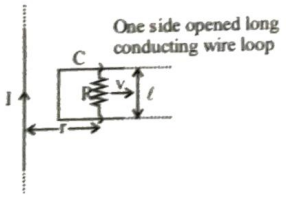
(a) the magnet's entry

(b) magnet is completely inside and

(c) magnet's exit. [*JEE Main-2020 (September)*]



31. An infinitely long, straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown, in the figure. The connector has length and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is [*JEE Main-2020 (September)*]



(a) $\frac{\mu_0}{4\pi}\frac{Iv\ell}{Rr}$ (b) $\frac{\mu_0}{\pi}\frac{Iv\ell}{Rr}$

(c) $\frac{\mu_0}{2\pi}\frac{Iv\ell}{Rr}$ (d) $\frac{2\mu_0}{\pi}\frac{Iv\ell}{Rr}$

32. Two concentric circular coils, C_1 and C_2 are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$ A where t is in s. The emf induced in C_1 (in mV), at the instant t = 1 s is $\frac{4}{x}$. The value of x is _____. [*JEE Main-2020 (September)*]

33. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2 A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is _____.

[*JEE Main-2020 (September)*]

34. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is:

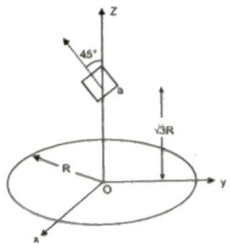
[*JEE Main-2021 (March)*]

(a) infinite (b) $\frac{L}{R}\ln 10$

(c) $\frac{L}{R}\ln 2$ (d) $\frac{L}{R}\ln 5$

JEE-ADVANCED PREVIOUS YEAR'S

1. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a ($a << R$) having two turns is placed with its center at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is: [*IIT-JEE-2012*]



2. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement (s) is (are): [*IIT-JEE-2012*]

(a) the emf induced in the loop is zero if the current is constant.

(b) The emf induced in the loop is finite if the current is constant.

(c) The emf induced in the loop is zero if the current decreases at a steady rate.

(d) The emf induced in the loop is finite if the current decreases at a steady rate.

Paragraph for Question No. 3 and 4

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionally constant γ.

[JEE ADVANCED-2013]

3. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is :

(*a*) $\frac{BR}{4}$ (*b*) $\frac{BR}{2}$

(*c*) BR (*d*) 2BR

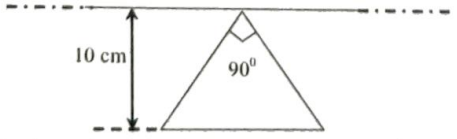
4. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is :

(*a*) $-\gamma BQR^2$ (*b*) $-\gamma\frac{BQR^2}{2}$

(*c*) $\gamma\frac{BQR^2}{2}$ (*d*) γBQR^2

5. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90^0 vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counter clockwise direction and increased at a constant rate of 10 As^{-1}. Which of the following statement(s) is (are) true?

[JEE Advanced-2016]



(*a*) If the loop is rotated at a constant angular speed about the wire, an additional *emf* of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(*b*) The induced current in the wire is in opposite direction to the current along the hypotenuse

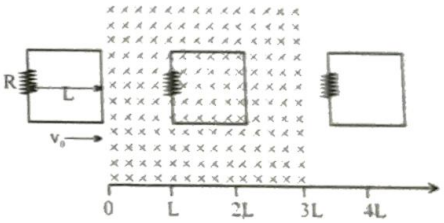
(*c*) The magnitude of induced *emf* in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(*d*) There is a repulsive force between the wire and the loop

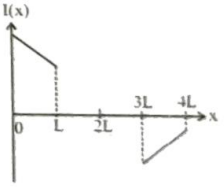
6. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is **[JEE Advanced-2016]**

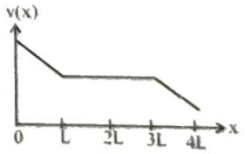
7. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter –clockwise current is taken as positive.

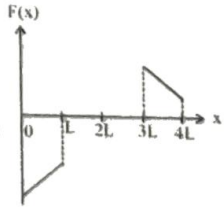
[JEE Advanced-2016]

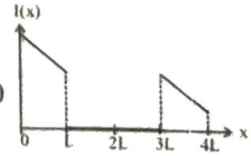


Which of the following schematic plot(s) is (are) correct? (Ignore gravity)

(*a*)


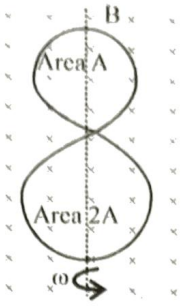
(*b*)


(*c*)


(*d*)


8. A circular insulated copper wire loop is twistd to form two loops of are A and 2A as shown in the figure. At the point of crossong the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotation about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?

[*JEE Advanced-2017*]



(*a*) The net emf induced due to both the loops is proportional to cos ωt
(*b*) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper
(*c*) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone
(*d*) The emf induced in the loop is proportional to the sum of the area of the two loops

Comprehesion (9, 10 and 11) by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in column 1,2 and 3, respectively. The queantities E_0, B_0 are positive in magnitude.

Column-1	**Column-2**	**Column-3**
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	**(i)** $\vec{E} = E_0\hat{z}$	**(P)** $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	**(ii)** $\vec{E} = -E_0\hat{y}$	**(Q)** $\vec{B} = B_0\hat{x}$
(III) Electron with $\vec{v} = 0$	**(iii)** $\vec{E} = -E_0\hat{x}$	**(R)** $\vec{B} = B_0\hat{y}$
(IV) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	**(iv)** $\vec{E} = E_0\hat{x}$	**(S)** $\vec{B} = B_0\hat{z}$

9. In which case will the particle move in a straight line with constant velocity ? [*JEE Advanced-2017*]
(*a*) (IV)(i)(S) (*b*) (III)(ii)(R)
(*c*) (II)(iii)(S) (*d*) (III)(iii)(P)

10. In which case will the particle describe a helical path with axis along the positive z-direction?

[*JEE Advanced-2017*]

(*a*) (IV)(i)(S) (*b*) (II)(ii)(R)
(*c*) (III)(iii)(P) (*d*) (IV)(ii)(R)

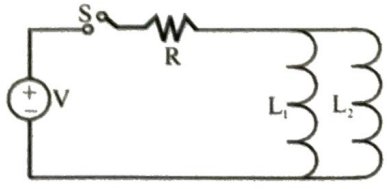
11. In which case would the particle move in a straight along the negative directin of y-axis (i.e. move along $-\hat{y}$)?

[*JEE Advanced-2017*]

(*a*) (III)(ii)(R) (*b*) (IV)(ii)(S)
(*c*) (III)(ii)(P) (*d*) (II)(iii)(Q)

12. A source of constant voltage V is connected to a resistance R and two ideal inductor L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct?

[*JEE Advanced-2017*]



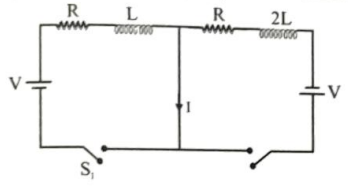
(*a*) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(*b*) At t = 0, the current through the resistance R is $\frac{V}{R}$

(*c*) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(*d*) The ratio of the currents through L_1 and L_2 is fixed at all times (t > 0)

13. In the figure below, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. the current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statement(s) is (are) true? *[JEE Advanced-2018]*



(*a*) $I_{max} = \frac{V}{2R}$ (*b*) $I_{max} = \frac{V}{4R}$

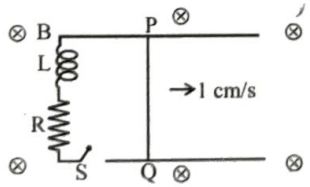
(*c*) $\tau = \frac{L}{R}\ln 2$ (*d*) $\tau = \frac{2L}{R}\ln 2$

14. A 10 cm long perfectly conducting wire PQ is moving, with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$ A, where the value of x is_____.

[**Assume** the velocity of wire PQ remains constant (1 cm/s) after key S is closed.

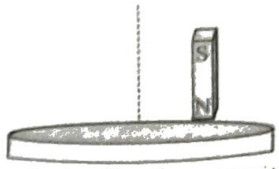
Given : $e^{-1} = 0.37$, where e is base of the natural logarithm]

[JEE Advanced-2019]



15. A light disc made of aluminium (a nonmagnetic material) is kept horizontally and is free to rotate about its axis as shown in the figure. A strong magnet is held vertically at a point above the disc away from its axis. On revolving the magnet about the axis of the disc, the disc will (figure is schematic and not drawn to scale)-

[JEE Advanced-2020]



(*a*) rotate in the direction opposite to the direction of magnet's motion

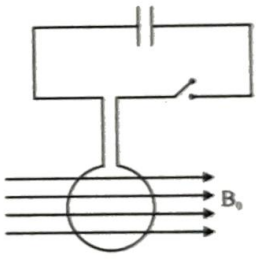
(*b*) rotate in the same direction as the direction of magnet's motion

(*c*) not rotate and its temperature will remain unchanged

(*d*) not rotate but its temperature will slowly rise

16. A circular coil of radius R and N turns has negligible resistance. As shown in the schematic figure, its two ends are connected to two wires and it is hanging by those wires with its plane being vertical. The wires are connected to a capacitor with charge Q through a switch. The coil is in a horizontal uniform magnetic field B_0 parallel to the plane of the coil. When the switch is closed, the capacitor gets discharged through the coil in a very short time. By the time the capacitor is discharged fully, magnitude of the angular momentum gained by the coil will be (assume that the discharge time is so short that the coil has hardly rotated during this time)

[JEE Advanced-2020]

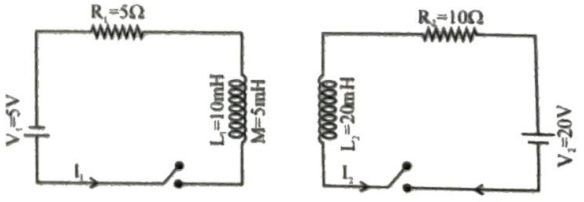


(*a*) $\frac{\pi}{2}NQB_0R^2$ (*b*) πNQB_0R^2

(*c*) $2\pi NQB_0R^2$ (*d*) $4\pi NQB_0R^2$

17. The inductors of two LR circuits are placed next to each other, as shown in the figure. The values of the self-inductance of the inductors, resistances, mutual-inductance and applied voltages are specified in the given circuit. After both the switches are closed simultaneously, the total work done by the batteries against the induced EMF in the inductors by the time the currents reach their steady state values is_____ mJ.

[JEE Advanced-2020]



ANSWER KEY

Topicwise Questions

1. (*c*)	**2.** (*a*)	**3.** (*a*)	**4.** (*d*)	**5.** (*b*)	**6.** (*d*)	**7.** (*c*)	**8.** (*d*)	**9.** (*b*)	**10.** (*b*)
11. (*c*)	**12.** (*d*)	**13.** (*b*)	**14.** (*d*)	**15.** (*d*)	**16.** (*d*)	**17.** (*c*)	**18.** (*d*)	**19.** (*d*)	**20.** (*a*)
21. (*d*)	**22.** (*d*)	**23.** (*c*)	**24.** (*c*)	**25.** (*d*)	**26.** (*c*)	**27.** (*d*)	**28.** (*c*)	**29.** (*a*)	

Learning Plus

1. (*d*)	**2.** (*d*)	**3.** (*a*)	**4.** (*d*)	**5.** (*c*)	**6.** (*a*)	**7.** (*a*)	**8.** (*b*)	**9.** (*c*)	**10.** (*d*)
11. (*b*)	**12.** (*c*)	**13.** (*b*)	**14.** (*b*)	**15.** (*b*)	**16.** (*a*)	**17.** (*b*)	**18.** (*b*)	**19.** (*d*)	**20.** (*d*)
21. (*d*)	**22.** (*b*)	**23.** (*a*)	**24.** (*b*)	**25.** (*d*)	**26.** (*a*)	**27.** (*a*)	**28.** (*a*)	**29.** (*a*)	**30.** (*b*)
31. (*a*)	**32.** (*c*)	**33.** (*c*)	**34.** (*a*)	**35.** (*d*)	**36.** (*a*)	**37.** (*d*)	**38.** (*d*)	**39.** (*b*)	**40.** (*c*)
41. (*a*)	**42.** (*a*)	**43.** (*c*)	**44.** (*a*)	**45.** (*c*)					

Advanced Level Multiconcept Questions

1. (*c*,*d*) **2.** (*a*, *c*, *d*) **3.** (*b*, *c*) **4.** (*a*, *b*, *c*, *d*) **5.** (*a*, *d*) **6.** (*b*, *d*) **7.** (*b*, *d*) **8.** (*a*, *c*, *d*)
9. (*a*, *b*, *c*) **10.** (*c*) **11.** (*c*) **12.** (*b*) **13.** (*b*) **14.** (*a*) **15.** (*b*) **16.** (*c*)
17. (*a*) q, s (*b*) p, r (*c*) p, r (*d*) q, s **18.** (*a*) q (*b*) p(*c*) s (*d*) s **19.** [300 mA] **20.** [1200] **21.** [0002] **22.** [64A] **23.** [0625]
24. [0120] **25.** [4]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*a*)	**2.** (*d*)	**3.** (*c*)	**4.** (*b*)	**5.** (*d*)	**6.** (Bonus)	**7.** (Bonus)	**8.** (*b*)	**9.** (*a*)	**10.** (*b*)
11. (*d*)	**12.** (*c*)	**13.** (*b*)	**14.** (*b*)	**15.** (*b*)	**16.** (*b*)	**17.** (*b*)	**18.** (175)	**19.** (*c*)	**20.** (*c*)
21. (*b*)	**22.** (*b*)	**23.** (*d*)	**24.** (*d*)	**25.** (*d*)	**26.** 10	**27.** (15)	**28.** (*b*)	**29.** (*b*)	**30.** (*d*)
31. (*c*)	**32.** (05.00)	**33.** (33.00)	**34.** (*c*)						

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JEE-ADVANCED PREVIOUS YEAR'S

1. (7)	**2.** (*a*, *c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*a*, *d*)	**6.** (8)	**7.** (*b*, *d*)	**8.** (*b*, *c*)	**9.** (*c*)	**10.** (*a*)
11. (*a*)	**12.** (*a*, *c*, *d*)	**13.** (*b*, *d*)	**14.** (0.63)	**15.** (*b*)	**16.** (*b*)	**17.** (55.00)			

21 ALTERNATING CURRENT

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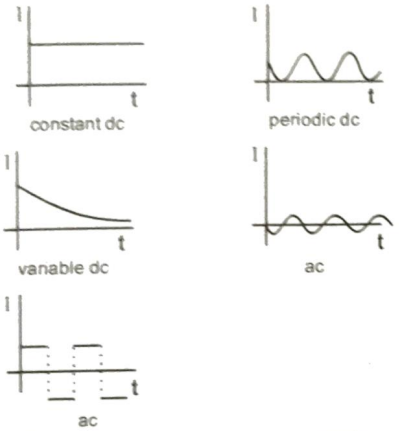
ALTERNATING CURRENT

AC AND DC CURRENT

A current that changes its direction periodically with 't' is called alternating current (AC). If a current maintains its direction constant it is called as direct current (DC).

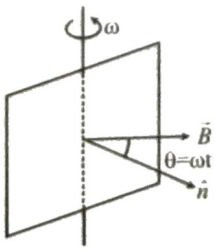


If a function suppose current, varies with time as $I = I_0 \sin(wt+f)$, it is called sinusoidally varying function.Here I_0 is the peak current or maximum current and I is the instantaneous current. The factor (wt+f) is called phase. w is called the angular frequency, its unit is rad/s.Also w =2pf, where f is called the frequency, its unit is s^{-1} or Hz. Also frequency f = 1/T where T is called the time period.

BASIC PRINCIPLE OF AC GENERATION

Alternating voltage is generated by rotating a coil of conducting wire in a strong magnetic field. The magnetic flux linked with the coil changes with time and an alternating emf is thus induced. Instantaneous flux linked with coil is

$$\phi = N(\vec{B}.\vec{A}) = NBA\cos(\omega t + \theta_0)$$



where A = area of the coil (in m^2)

B = magnetic field (in tesla)

N = number of turns in the coil

ω = angular frequency $= \frac{2\pi}{T} = 2\pi f$ (in rad s^{-1})

f = frequency (in hertz)

θ_0 = initial phase angle.

With the change of time $\cos(\omega t + \theta_0)$ changes consequently an emf V is induced. According to Faraday's law

$$\varepsilon = \frac{-d\varphi}{dt} = -\frac{d}{dt}[NBA\cos(\omega t + \theta_0)]$$

$$= NBA\,\omega\sin(\omega t + \theta_0)$$

$$V = V_0 \sin(\omega t + \theta_0)$$

Here V_0 = voltage amplitude of sinusoidal voltage or the peak value of ac voltage

where $V_0 = NBA\omega$

AVERAGE VALUES OF AC VOLTAGE AND AC CURRENT

Time average volue of voltage (from $t = t_1$ to $t = t_2$)

$$\frac{\int_{t_1}^{t_2} V(t)\ dt}{\int_{t_1}^{t_2} dt} = \frac{\int_{t_1}^{t_2} V(t)\ dt}{t_2 - t_1} = \langle V(t) \rangle$$

AC voltage or currrents are commonly sinusoidal (sine or cosine function) and their mean values for complete cycle is zero.

(i) **Average value for positive half cycle (or rectified average value):**

$$V = V_0 \sin\omega t$$

$$\therefore \quad (V)_{av} = \frac{\int_0^{T/2} V_0 \sin\omega t\ dt}{\int_0^{T/2} dt}$$

$$= \frac{2}{T}\int_0^{T/2} V_0 \sin\omega t \, dt = \frac{2}{\pi}V_0 = 0.637V_0 .$$

This is also known as the **rectified average value of a sinusoidal voltage**

(ii) ***Root Mean Square Value*** (V_{rms} or I_{rms}) **:**

$$V_{rms} \text{ (from } t = t_1 \text{ to } t = t_2) = \sqrt{\frac{\int_{t_1}^{t_2} V^2(t)\, dt}{\int_{t_1}^{t_2} dt}}$$

rms value for a complete cycle of sinusoidal voltage

$V = V_0 \sin\omega t$

$$V_{rms} = \sqrt{\frac{\int_0^T V_0^2 \sin^2\omega t \, dt}{T}} = \frac{V_0}{\sqrt{2}}$$

rms value for a complete cycle of sinusoidal current

$I = I_0 \sin\omega t$

and $$I_{rms} = \sqrt{\frac{\int_0^T I_0^2 \sin^2\omega t \, dt}{T}} = \frac{I_0}{\sqrt{2}}$$

or RMS value $= \dfrac{\text{Peak value}}{\sqrt{2}}$

How do we measure sinusoidally varying voltage or current?

Moving coil galvanometer measure steady currents but if we pass sinusoidal current through them, the needle may wiggle at low fequency but its average deflection is zero. Hot wire instruments are commonly used to measure the rms values (also known as **virtual values**).

It must be emphasied here that metres used for ac voltage and current measurement are always calibrated to read rms values, not peak (or maximum) or rectified average values. The usual domestic power supply "220 - volt ac" has rms voltage of 220 V. The voltage amplitude or peak value is

$$V_0 = \sqrt{2}V_{rms} = \sqrt{2}(220V) = 311V .$$

TRAIN YOUR BRAIN

Example 1: If a domestic appliance draws 2.5 A from a 220-V, 60-Hz power supply, find

(a) The average current for full cycle

(b) The average of the square of the current

(c) The current amplitude

(d) The supply voltage amplitude.

Sol. (a) The average of sinusoidal AC values over any whole number of cycles is zero.

(b) RMS value of current $= I_{rms} = 2.5$ A

$\therefore (I^2)_{av} = (I_{rms})^2 = 6.25A^2$

(c) $I_{rms} = \dfrac{I_0}{\sqrt{2}}$

$\therefore$ Current amplitude $= \sqrt{2}I_{rms} = \sqrt{2}(2.5A) = 3.5$ A

(d) $V_{rms} = 220V = \dfrac{V_0}{\sqrt{2}}$

$\therefore$ Supply voltage amplitude

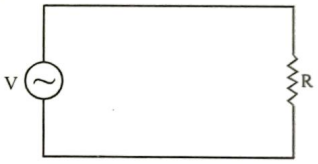
$V_0 = \sqrt{2}(V_{rms}) = \sqrt{2}(220V) = 311$ V.

AC VOLTAGE APPLIED TO A RESISTOR

A resistor connected to a source ε of ac voltage as shown in the ciruit digram . The symbol for an ac source in a circuit diagram is $\ominus$. For simplicity, we consider a source which produces sinusoidally varying potential difference across its terminals. Let this potential difference, also called ac voltage, be given by

$V = V_0 \sin\omega t$ ...(i)

where V_0 is the amplitude of the sinusoidal voltage and ω is its angular frequency.



AC voltage applied to a resistor

The instantaneous potential drop across the resistor R is

$V_0 \sin\omega t = IR$

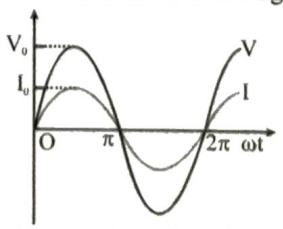
or $I = \dfrac{V_0}{R}\sin\omega t$

$I = I_0 \sin\omega t$ ...(ii)

where I is the instantaneous current and the current amplitude I_0 is given by

$$I_0 = \frac{V_0}{R} \quad \text{...(iii)}$$

Equation (iii) is just Ohm's law which for resistors work equally well for both ac and dc voltages. The voltage across a pure resistor and the current through it, given by equation (i) and (ii) are plotted as a function of time in figure. Note, in particular that both V and I reach zero, minimum and maximum values at the same time. Clearly, the voltage and current are in phase for a circuit containing pure resistance.



In a pure resistor, the voltage and current are in phase. The minima with zero and maxima occur at the same respective times.

We see that, like the applied voltage, the current varies sinusoidally and has corresponding positive and negative values during each cycle. Thus, **the sum of the instantaneous current values over one complete cycle is zero, and the average current is zero**. The fact that the average current is zero, however, does not mean that the average power is zero and that there is no dissipation of electrical energy. As you know, joule heating is given by I^2R and depends on I^2 (which is always positive whether I is positive or negative) and not on I. Thus there is Joule heating and dissipation of electrical energy when an ac current passes through a resistor.

The instantaneous power dissipated in the resistor is

$$P = I^2 R = I_0^2 R \sin^2 \omega t \quad ...(iv)$$

The average value of Power P over a cycle is

$$\bar{P} = \frac{1}{2} I_0^2 R = I_{rms}^2 R \quad ...(v)$$

Where the bar over a letter(here, P) denotes its average value.

To express ac power in the same form as dc power ($P = I^2R$), as special value of current is used. It is called, root mean square (rms) or **effective current** and is denoted by I_{rms}.

Similarly, we define the rms voltage or **effective voltage**

From equation [iii], we have

$$V_0 = I_0 R \quad ...(vi)$$

$$\text{or} \quad \frac{V_0}{\sqrt{2}} = \frac{I_0}{\sqrt{2}} R \quad ...(vii)$$

$$\text{or} \quad V_{rms} = I_{rms} R \quad ...(viii)$$

In terms of rms values, the equation for power and relation between current and voltage in ac circuits are essentially the same as those for the dc case.

In fact, the I_{rms} or rms current is the equivalent dc current that would produce the same average power loss as the alternating current. Equation [v] can also be written as

$$\bar{P} = V_{rms}^2 / R = I_{rms}^2 R \text{ (since } V_{rms} = I_{rms} R) \quad ...(ix)$$

TRAIN YOUR BRAIN

Example 2: A bulb is rated 60 W at 220 V/50 Hz. Find the maximum value of instantaneous current through the filament?

Sol. $V_0 = 220\sqrt{2} = 311 \text{ V}$

$$R = \frac{220^2}{P} = \frac{220 \times 220}{60} = \frac{2420}{3} = 806.67\,\Omega$$

$$I_0 = \frac{V_0}{R} = \frac{311}{806.67} = 0.39\text{A}$$

Example 3: A light bulb is rated at 200 W for a 220 V supply. Find

(a) The resistance of the bulb;

(b) The peak voltage of the source; and

(c) The rms current through the bulb.

Sol. (a) We are given P = 100 W and V_{rms} = 220 V. The resistance of the bulb is

$$R = \frac{V_{rms}^2}{P} = \frac{(220\text{V})^2}{200\text{W}} = 242\Omega$$

(b) The peak voltage of the source is

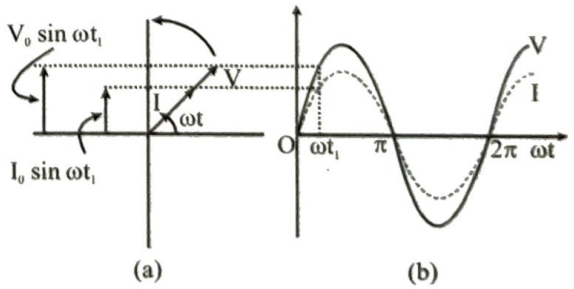
$$V_0 = \sqrt{2} V_{rms} = 311\text{V}$$

(c) Since, $P = I_{rms} V_{rms}$

$$I_{rms} = \frac{P}{V_{rms}} = \frac{200\text{W}}{220\text{V}} = 0.90\text{A}$$

REPRESENTATION OF AC CURRENT AND VOLTAGE BY ROTATING VECTORS – PHASORS

In the previous section, we saw that the current through a resistor is in phase with the ac voltage. But this is not so in the case of an inductor, a capacitor or a combination. In order to show phase relationship between voltage and current in an ac circuit, we use the motion of **PHASORS**. The analysis of an ac circuit is facilitated by the use of a phasor diagram. A phasor is a vector which rotates about the origin with angular speed ω, as shown in figure. The vertical components of phasors V and I represent the sinusoidally varying quantities V and I. The magnitudes of phasors V and I represent the amplitudes or the peak values V_0 and I_0 of these oscillating quantities. Figure (a) shows the voltage and their relationship at time t_1 for the case of an ac source connected to a resistor i.e., corresponding to the circuit show in figure. The projection of voltage and current phasors on vertical axis, i.e., $V_0 \sin \omega t$ and $I_0 \sin \omega t$, respectively represent the instaneous value of voltage and current at that instant. As they rotate with frequency ω, curves in figure(b) are generated which respresent the sinusoidal variation of voltage and current with time.



(a) A phasor diagram for the circuit in figure
(b) Graph of V and I versus ωt

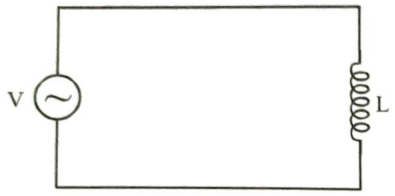
From figure (b) we see that phasors V and I for the case of a resistor are in the same phase. This is so for all times. This means that the phase angle between the voltage and the current is zero.

AC VOLTAGE APPLIED TO AN INDUCTOR

An ac source connected to an inductor as shown in the circuit below. Usually, inductors have appreciable resistance in their windings, but we shall assume that this is ideal inductor (having zero resistance). Thus, the circuit is a purely inductive ac circuit. Let the voltage across the source be $V = V_0 \sin \omega t$.

Using the Kirchhoff's loop rule. $\sum \varepsilon(t) = 0$, and since there is no resistor in the circuit.

$$V - L\frac{dI}{dt} = 0 \qquad ...(x)$$



An AC source connected to an inductor

where the second term is the self-induced emf in the inductor; and L is the self-inductance of the coil. Combining equation (i) and (x), we have

$$\frac{dI}{dt} = \frac{V}{L} = \frac{V_0}{L}\sin \omega t \qquad ...(xi)$$

$$dI = \frac{V_0}{L}\sin \omega t\, dt$$

$$I = -\frac{V_0}{\omega L}\cos(\omega t)$$

Using $-\cos(\omega t) = \sin\left(\omega t - \frac{\pi}{2}\right)$, we have

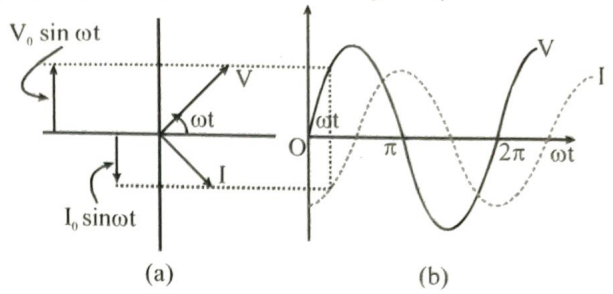
$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(xii)$$

where $I_0 = \frac{V_0}{\omega L}$ is the amplitude of the current. The quantity ωL is analogous to the resistance and is called **inductive reactance**, denoted by X_L:

$$X_L = \omega L = 2\pi fL \qquad ...(xiii)$$

The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a pure inductive circuit in the same way as does the resistance in a pure resistive circuit. The inductive reactance is directly proportional to the inductance frequency of the voltage source.

A comparison of equation (i) and (ii) for the source voltage and the current in an inductor shows that the current lags the voltage by $\frac{\pi}{2}$ or one-quarter $\left(\frac{1}{4}\right)$ cycle. Figure shows the voltage and the current phasors in the present case at instant t_1. The current phasor I is $\frac{\pi}{2}$ behind the voltage phasor V. When rotated with frequency ω counter-clockwise, they generate the voltage and current given by equation [ii] and [xii], respectively and as shown in figure (b)



(a) A phasor diagram for the circuit in figure
(b) Graph of V and I versus ωt.

We see that the current reaches its maximum value later than the voltage by one-fourth of a period $\left[\frac{T}{4} = \frac{\pi/2}{\omega}\right]$. You have seen that an inductor has reactance that limits current similar to resistance in a dc circuit. Does it also consume power like a resistance? Let us try to find out.

The instantaneous power supplied to the inductor is

$$P_L = IV = I_m \sin\left(\omega t - \frac{\pi}{2}\right) V_0 \sin(\omega t) = -I_0 V_0 \cos(\omega t).$$

$$\sin(\omega t) = -\,-\frac{I_0 V_0}{2}\sin(2\omega t)$$

So, the average power over a complete cycle is zero since the average of $\sin(2\omega t)$ over a complete cycle is zero. Thus, the average power supplied to an inductor over one complete cycle is zero.

Physically, this result means the follows. During the first quarter of each current cycle, the flux through the inductor builds up and sets up a magnetic field and energy is stored in the inductor. In the next quarter of cycle, as the current decrease, the flux decreases and the stored energy is returned to the source. Thus, in each half cycle, the energy which is withdrawn from the source is returned to it without any dissipation of power.

TRAIN YOUR BRAIN

Example 4: A pure inductor of 50.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Sol. The inductive reactance.

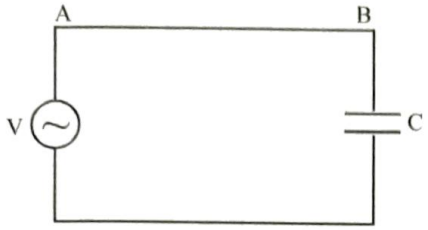
$$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 50 \times 10^{-3}\Omega$$
$$= 15.7\Omega$$

The rms current in the circuit is

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220V}{15.7\Omega} = 14.01A.$$

AC VOLTAGE APPLIED TO A CAPACITOR

An ac source ε connected to a capacitor only, a purely capacitive ac circuit is as shown.



An ac source connected to a capacitor

When the capacitor is connected to an ac source, as in figure, it limits or regulates the current, but does not completely prevent the flow of charge. The capacitor is alternately charged and discharged as the current reverses each half cycle. Let q(t) be the charge on the capacitor at any time t. The instantaneous voltage V(t) across the capacitor is

$$V(t) = \frac{q(t)}{C} \qquad ...(xiv)$$

From the Kirchhoff's loop rule, the voltage across the source and the capacitor are equal,

$$V_0 \sin \omega t = \frac{q}{C}$$

To find the current, we use the relation $I = \frac{dq}{dt}$

$$I = \frac{d}{dt}(V_0 C \sin \omega t) = \omega C V_0 \cos(\omega t)$$

Using the relation, $\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$, we have

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad ...(xv)$$

where the amplitude of the oscillating current is

$$I_0 = \frac{V_0}{(1/\omega C)}$$

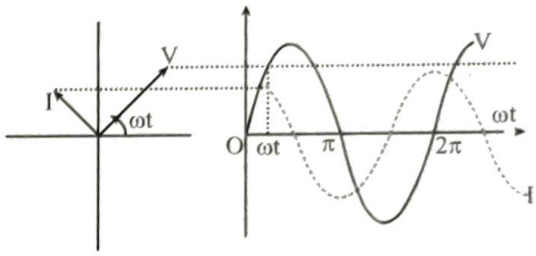
Comparing it to $I_0 = \frac{V_0}{R}$ for a purely resistive circuit, we find that $(1/\omega C)$ plays the role of resistance. It is called **capacitive reactance** and is denoted by $\mathbf{X_C}$,

$$X_C = 1/\omega C = 1/2\pi fC \qquad ...(xvi)$$

So that the amplitude of the current is

$$I_0 = \frac{V_0}{X_C} \qquad ...(xvii)$$

The dimension of capacitive reactance is the same as that of resistance and its SI unit is Ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as does the resistance in a purely resistive circuit. But it is inversely proportional to the frequency and the capacitance.



A comparison of equation (xv) with the equation of source voltage equation (i) shows that the current in a capacitor leads the voltage by $\pi/2$. Figure shows the phasor diagram at an instant t. Here the current phasor I is $\frac{\pi}{2}$ rad ahead of the voltage phasor V as they rotate counter clockwise. Figure shows the variation of voltage and current with time. We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.

The instantaneous power supplied to the capacitor is

$$P_C = IV = I_0 \cos(\omega t).\ V_0 \sin(\omega t)$$
$$= I_0 V_0 \cos(\omega t) \sin(\omega t)$$
$$= \frac{I_0 V_0}{2} \sin(2\omega t) \qquad ...(xviii)$$

So, as in the case of an inductor, the average power Since average of sin 2ωt over a complete cycle is zero. As discussed in the case of an inductor, the energy stored by a capacitor in each quarter period is returned to the source in the next quarter period.

Thus, we see that in the case of an inductor. the current lags the voltage by 90° and in the case of a capacitor, the current leads the voltage by 90°.

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Example 5: 30.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current.

Sol. The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = 106\Omega$$

The rms current is

$$I_{rms} = \frac{V_{rms}}{X_C} = 2.08A$$

The peak current is

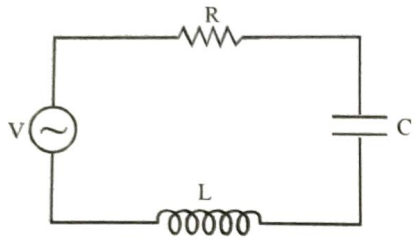
$$I_0 = \sqrt{2} I_{rms} = 2.96A$$

This current oscillates between 2.96A and – 2.96A and is ahead of the voltage by 90°.

If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

AC VOLTAGE APPLIED TO A SERIES LCR CIRCUIT

Figure shows a series LCR circuit connected to an ac source ε. As usual, we take the voltage of the source to be $V = V_0 \sin \omega t$.



A series LCR circuit connected to an ac source

If q is the charge on the capacitor and I the current, at time t, we have, from Kirchhoff's loop rule:

$$L\frac{dI}{dt} + IR + \frac{q}{C} = V \qquad ...(xix)$$

We want to determine the instantaneous current I and its phase relationship to the applied alternating voltage V. We shall solve this problem by two methods. First, we use the technique of phasors and in the second method, we solve equation [xx] analytically to obtain the time –dependence of I.

PHASOR-DIAGRAM SOLUTION

From the circuit shown in figure we see that the resistor, inductor and capacitor are in series. therefore, the ac current in each element is the same, having the same amplitude and phase. Let it be

$$I = I_0 \sin(\omega t + \phi) \qquad ...(xx)$$

where ϕ is the phase difference between the voltage across the source and the current in the circuit. On the basis of that we construct a phasor diagram for the present case.

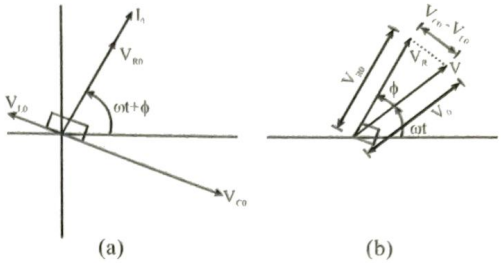
Let I be the phasor representing the current in the circuit as given by equation [xxi]. Further, let V_L, V_R, V_C, and V represent the voltage across the inductor, resistor, capacitor and the source, respectively. From previous section, we know that V_R is in phase with I, V_C is $\frac{\pi}{2}$ rad behind I and V_L is $\frac{\pi}{2}$ rad ahead of I. V_R, V_C and I are shown in figure (a) with appropriate phase-relations.

The length of these phasors or the amplitude of V_R, V_C and V_L are :

$$V_{R0} = I_0 R \ , \ V_{C0} = I_0 X_C, \ V_{L0} = I_0 X_L \qquad ...(xxi)$$

The voltage equation (xx) for the circuit can be written as

$$V = V_L + V_R + V_C \qquad ...(xxii)$$



(a) (b)

(a) Relation between the phasors. (b) Relation between the phasors for the circuit in

This relation is represented in figure (b). Since $\vec{V}_{C0}$ and $\vec{V}_{L0}$ are always along the same line and in opposite direction, they can be combined vectorially into a single phasor $(\vec{V}_C + \vec{V}_L)$ which has a magnitude $|\vec{V}_{C0} - \vec{V}_{L0}|$. Since, $\vec{V}_0$ is represented as the hypotenuse of a right- angle triangle

$$V_0^2 = V_{R0}^2 + (V_{C0} - V_{L0})^2$$

Substituting the values of V_{R0}, V_{C0} and V_{L0} from equation [xxii] into the above equation, we have

$$V_0^2 = (I_0 R)^2 + (I_0 X_C - I_0 X_L)^2$$
$$= I_0^2 [R^2 + (X_C - X_L)^2]$$

$$\text{or} \quad I_0 = \frac{V_0}{\sqrt{R^2 + (X_C - X_L)^2}} \qquad ...[xxiii\ (a)]$$

By analogy to the resistance in a circuit, we introduce the impedance Z in ac circuit :

$$I_0 = \frac{V_0}{Z} \qquad ...[xxiii\ (b)]$$

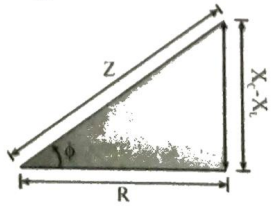
where $Z = \sqrt{R^2 + (X_C - X_L)^2}$ ...(xxiv)

Since phasor I is always parallel to phasor V_R, the phase angle ϕ is the angle between V_R and V and can be determined from figure:

$$\tan\phi = \frac{V_{C0} - V_{L0}}{V_{R0}}$$

from the impedance triangle

$$\tan\phi = \frac{X_C - X_L}{R} \qquad ...(xxv)$$



impedance triangle

Equations (xxv) and (xxiv) are shown in figure. This is called **Impedance triangle** which is a right triangle with Z as its hypotenuse.

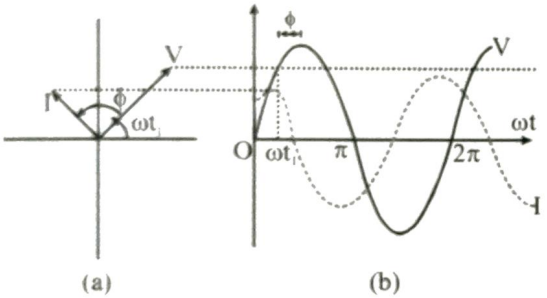
Equation (xxiii (a)) gives the amplitude of the current and figure gives the phase angle. With these, equation (xxi) is completely specified.

If $X_C > X_L$, ϕ is positive and the circuit is capacitive. Consequently, the voltage across the source lags the current.

If $X_C < X_L$, ϕ is negative and the circuit is inductive. Consequently, the voltage across the source leads the current.

Figure shows the phasor diagram and variation of V and I with ωt for the case $X_C > X_L$.

Thus, we have obtained the amplitude and phase of current for an LCR series circuit using the technique of phasors. But this method of analyzing ac circuits suffers from certain disadvantages. First, the phasor diagram states nothing about the initial condition. One can take any arbitr ary value of t and draw different phasors which show the relative angle between different phasors. The solution so obtained is called the **steady-state solution.**



(a) Phasor diagram of V and I. (b) Graphs of V and I versus ωt for a series LCR circuit where $X_C > X_L$.

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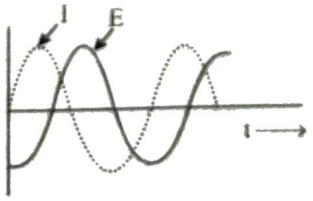
Example 6: When an AC source of e.m.f. $E = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the e.m.f. E and the current I in the circuit is observed to be $\pi/4$, as shown in the figure. If the circuit consists possibly only of R-C in series. What will be the relation between the two elements of the circuit?



Sol. Given $E = E_0 \sin(100\,t)$. Comparing this with $E = E_0 \sin \omega t$, we have $\omega = 100\,\text{rad s}^{-1}$. It follows from the figure that the current leads the e.m.f. which is true only for R-C circuit, and not for R-L circuit. Hence the circuit does not contain an inductor. For R-C circuit, the phase difference between E and I is given by

$$\tan\phi = \frac{1}{\omega RC} \qquad ...(1)$$

Given $\phi = \pi/4$. Also $\omega = 100\,\text{rad s}^{-1}$. Using these values in (1), we get

$$\tan\left(\frac{\pi}{4}\right) = \frac{1}{100\,RC} \text{ or } RC = \frac{1}{100}$$

Example 7: An LCR series circuit with 100 Ω resistance is connected to an ac source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current in the LCR circuit.

Sol. When capacitance is removed, the circuit becomes L-R with,

$$\tan\phi = \frac{X_L}{R} \quad \text{i.e.,} \quad X_L = R\tan\phi = 100\sqrt{3}\,\Omega$$

and when inductance is removed the circuit becomes C-R with,

$$\tan\phi = \frac{X_C}{R} \quad \text{i.e.,} \quad X_C = R\tan\phi = 100\sqrt{3}\,\Omega$$

as here $X_L = X_C$ so the circuit is in series resonance and hence as $X = X_L - X_C = 0$,

i.e., $Z = \sqrt{R^2 + X^2} = R$.

$$\text{So, } I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{200}{100} = 2\text{ A}$$

RESONANCE

An interesting characteristic of the series RLC circuit is the phenomenon of resonance. The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the system's natural frequency. If such a system is driven by an energy source at a frequency that is near the natural frequency, the amplitude of oscillation is found to be large.

For an RLC circuit driven with voltage of amplitude V_0 and frequency ω, we found that the current amplitude is given by

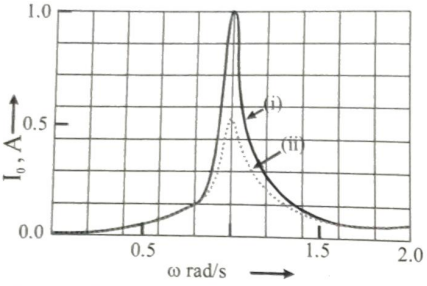
$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_C - X_L)^2}}$$

with $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$. So if ω is varied, then at a particular frequency $\omega_0, X_C = X_L$, the impendance is minimum. This frequency is called the **resonant frequency** :

$$\text{then} \quad \frac{1}{\omega_0 C} = \omega_0 L \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}} \qquad ...(xxvi)$$

At resonant frequency, the current amplitude is maximum; $I_0 = \frac{V_0}{R}$

The variation of I_0 with ω in a RLC series circuit with L = 1.00 mH, C = 1.00 nF for two values of R: (i) R = 100 Ω and (ii) R = 200 Ω are shown in the figure. For the source applied V_0=100 V . ω_0 for this case is $\left(\frac{1}{\sqrt{LC}}\right) = 1.00 \times 10^6$ rad/s.



Variation of I_0 with ω for two cases: (i) R = 100Ω, (ii)R = 200Ω, L = 1.00mH. C = 1.00nF and V_0 = 100V for both cases.

We see that the current amplitude is maximum at the resonant frequency.

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Example 8: Resonance frequency of a circuit is f. If the capacitance is made 4 times the initial value, then find the resonance frequency ?

Sol. $f = \frac{1}{2\pi\sqrt{LC}}$ i.e. $f \propto \frac{1}{\sqrt{C}} \rightarrow \frac{1}{\sqrt{4}} = \frac{1}{2}$ time

POWER IN AC CIRCUITS: THE POWER FACTOR

We have seen that a voltage $V = V_0 \sin\omega t$ applied to a series RLC circuit drives a current in the circuit given by $I = I_0 \sin(\omega t + \phi)$ where

$$I_0 = \frac{V_0}{Z} \text{ and } \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Therefore, the instantaneous power P supplied to the source is

$$P = VI = (V_0 \sin\omega t) \times [I_0 \sin(\omega t + \phi)]$$

$$= \frac{V_0 I_0}{2}[\cos\phi - \cos(2\omega t + \phi)] \quad ...(xxvii)$$

The average power over a cycle is given by the average of the two terms in R.H.S. of equation [xxxvii]. Second term average is zero, therefore,

$$\bar{P} = \frac{V_0 I_0}{2}\cos\varphi = \frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\varphi$$

$$= V_{rms} I_{rms} \cos\phi \quad ...[xxviii (a)]$$

This can also be written as,

$$\bar{P} = I_{rms}^2 Z\cos\phi \quad ...[xxviii (b)]$$

So, the average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between them. The quantity $\cos\phi$ is called the **power factor**. Let us discuss the following cases: from impedence triangle $\cos\phi = \frac{R}{z}$.

Case (I) Resistive circuit:
If the circuit contains only pure R, it is called resistive. In that case $\phi = 0$, $\cos\phi = 1$. There is maximum power dissipation.

Case (II) Purely Inductive or Capacitive Circuit :
If the circuit contains only an inductor or capacitor, we know that the phase difference between voltage and current is $\frac{\pi}{2}$. Therefore, $\cos\phi = 0$, and no power is dissipated even though a current is flowing in the circuit. This current is sometimes referred to as **wattless current**.

Case(III) LCR Series Circuit:
In an LCR series circuit, power dissipated is given by equation [xxviii] where $\phi = \tan^{-1}\frac{X_C - X_L}{R}$. So, ϕ may be non-zero (except $\frac{\pi}{2}$) in a RL or RC or RCL circuit. Even in such cases, power is dissipated only in the resistor.

Case (IV) Power Dissipated at Resonance in LCR Circuit:
At resonance $X_C - X_L = 0$, and $\phi = 0$. Therefore, $\cos\phi = 1$, and $P = I_0^2 z = I_0^2 R$. That is, maximum power is dissipated in a circuit (through R) at resonance.

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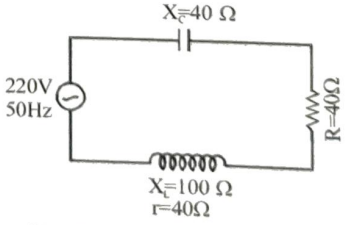
Example 9: Find the power factor of the circuit shown in figure?



Sol. $R = 40 + 40 = 80\,\Omega$,

$X_L - X_C = 100 - 40 = 60\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{80^2 + 60^2} = 100$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{80}{100} = 0.8$.

Example 10: A current of 4 A flows in a coil when connected to a 12 dc source. If the same coil is connected to a 12 V, 50 rad/s ac source a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.

Sol. In case of a coil as $Z = \sqrt{R^2 + \omega^2 L^2}$

i.e., $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$

so when dc is applied

$$I = \frac{V}{R} \quad \text{i.e.,} \quad R = \frac{12}{4} = 3\Omega \qquad \ldots (i)$$

and when ac is applied,

$$I = \frac{V}{Z} \quad \text{i.e.,} \quad Z = \left(\frac{V}{I}\right) = \left(\frac{12}{2.4}\right) = 5\Omega$$

or, $R^2 + X_L^2 = 5^2 \quad (\text{as } Z = \sqrt{R^2 + X_L^2})$

so, $X_L^2 = 5^2 - R^2 = 5^2 - 3^2 = 4^2 \quad$ i.e., $\quad X_L = 4\Omega$

but as $X_L = \omega L, \quad L = \frac{X_L}{\omega} = \frac{4}{50} = 0.08$

Now when the capacitor is connected to the above circuit in series,

As $X_C = \frac{1}{\omega C} = \frac{1}{50 \times 2500 \times 10^{-6}} = \frac{10^3}{125} = 8\ \Omega$

So, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (4-8)^2} = 5\ \Omega$

and hence $I = \frac{V}{Z} = \frac{12}{5} = 2.4\,A$

so, $P_{av} = V_{rms} I_{rms} \cos\phi = (I_{rms} \times Z) \times I_{rms} \times \left(\frac{R}{Z}\right)$

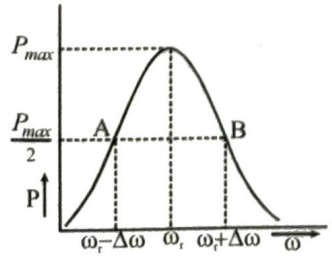
i.e., $P_{av} = I_{rms}^2 R = (2.4)^2 \times 3 = 17.28\ W$.

Band Width and Q-Factor:

Angular frequency variation of power in LCR series circuit.

$$P = P_{max} \sqrt{\frac{R^2}{\left[R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2\right]}}$$

Graph between P & ω as shown in the figure.



$$\omega_1 = +\frac{R}{2L} + \left(\omega_r^2 + \frac{R^2}{4L^2}\right)^{1/2} \quad \text{and} \quad \omega_2 = -\frac{R}{2L} + \left(\omega_r^2 + \frac{R^2}{4L^2}\right)^{1/2}$$

Now, $\omega_1 - \omega_2 = \frac{R}{L}$

or $(\omega_r + \Delta\omega) - (\omega_r - \Delta\omega) = \frac{R}{L}$ or $2\Delta\omega = \frac{R}{L}$.

The frequency interval between half maximum power points is known as **band width**.

The ratio of resonance frequency and band width is known as **quality factor (Q)**.

$$\therefore \quad Q = \frac{\omega_r}{2\Delta\omega} = \frac{\omega_r L}{R}.$$

Q factor is a measure of the sharpness of resonance. Resonance will be sharp if the value of bandwidth $(2\Delta\omega)$ is small. This is of course possible only when the power-frequency curve fall steeply around $\omega = \omega_r$.

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Example 11: A resistance R and inductance L and a capacitor C all are connected in series with an AC supply. The resistance of R is 16 ohm and for a given frequency, the inductive reactance of L is 24 ohm and capacitive reactance of C is 12 ohm. If the current in the circuit is 5 amp., find

(a) The potential difference across R, L and C
(b) The impedance of the circuit
(c) The voltage of AC supply
(d) Phase difference between applied source voltage and current drawn

Sol. (a) Potential difference across the resistance

$V_R = iR = 5 \times 16 = 80$ Volt

Potential difference across the inductor

$V_L = iX_L = 5 \times 24 = 120$ Volt

Potential difference across the capacitor

$V_C = iX_C = i \times (1/\omega C) = 5 \times 12 = 60$ Volt

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{[(16)^2 + (24-12)^2]} = 20$ ohm

(c) The voltage of AC supply is given by

$E = I\,Z = 5 \times 20 = 100$ volt

(d) Phase angle

$$\phi = \tan^{-1}\left[\frac{\omega L - (1/\omega C)}{R}\right] = \tan^{-1}\left[\frac{24-12}{16}\right]$$

$= \tan^{-1}(0.75) = 36°46'$.

Example 12: Obtain the resonant freqency and Q-factor of a series LCR circuit with $L = 3.0$ H, $C = 27\mu F$ and $R = 7.4\Omega$. It is desired to improve the sharpness of the resonance of the circuit by reducing its full width at half maximum by a factor of 2. Suggest a suitable way.

Sol. L = 3 henry, $C = 27\times10^{-6}F$, $R = 7.4\Omega$

Resonant angular frequency is given by

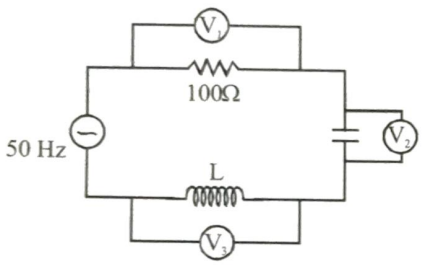
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3\times27\times10^{-6}}} rad\ s^{-1}$$

$$= \frac{1}{9\times10^{-3}} rad\ s^{-1} = \frac{1000}{9} rad\ s^{-1} = 111\ rad\ s^{-1}$$

$$Q = \frac{\omega_0 L}{R} = \frac{111\times3}{7.4} = 45$$

To double Q without changing ω_0, Reduce R to 3.7Ω.

Example 13: A series L-C-R circuit is connected to an AC source of 50 Hz. as shown in figure. If the readings of the three voltmeters V_1, V_2 and V_3 are 65 V, 415 V and 204 V respectively, calculate,



(i) The current in the circuit
(ii) The value of inductor
(iii) The value of the capacitor C and
(iv) The value of C (for the same L) required to produce resonance.

Sol. (i) Here $V_R = i_{rms} R$,
where i_{rms} is the rms value of current in the circuit.

$$\therefore\ i_{rms} = \frac{V_R}{R} = \frac{65}{100} = 0.65\,amp$$

(ii) $V_L = i_{rms} \times X_L$ or $X_L = \frac{V_L}{i_{rms}}$

$$\therefore\ X_L = \frac{204}{0.65} = 313.85\Omega$$

Now, $X_L = \omega L = 2\pi fL$ or $L = \frac{X_L}{2\pi f}$

$$\therefore\ L = \frac{313.85}{2\pi\times50} = 1.0\ \text{henry}$$

(iii) $X_C = \frac{V_C}{i_{rms}} = \frac{415}{0.65} = 638.46\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

$$\therefore\ C = \frac{1}{2\pi\times50\times638.46} = 5\times10^{-6} = 5\mu F$$

(iv) Let 'C' be the capacitance of capacitor that will produce resonance with inductor L = 1.0 henry. Then

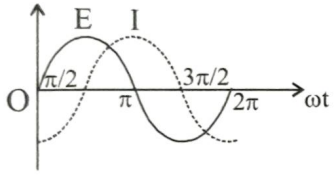
$$f = \frac{1}{2\pi\sqrt{(LC')}} \text{ or } C' = \frac{1}{4\pi^2 f^2 L}$$

$$= \frac{1}{4\pi^2\times(50)^2\times1.0} = 10.1\times10^{-6}F = 10.1\mu F$$

Topicwise Questions

Average peak and RMS value

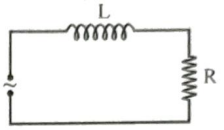
1. The r.m.s. value of alternating current is :
   (a) Double of peak value
   (b) Half of peak value
   (c) $\frac{1}{\sqrt{2}}$ times of peak value
   (d) Equal to peak value
2. The peak value of alternating potential is E_0 then r.m.s. value of the same will be :
   (a) $\frac{E_0}{2}$ (b) $\sqrt{E_0}$
   (c) $\frac{E_0}{\sqrt{2}}$ (d) $E_0\sqrt{2}$
3. The variation of the instantaneous current (I) and the instantaneous emf (E) in a circuit is as shown in fig. Which of the following statements is correct?



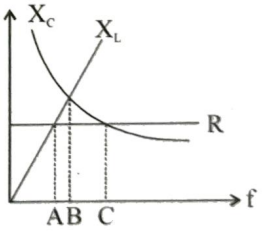
   (a) the voltage lags behind the current by $\pi/2$
   (b) the voltage leads the current by $\pi/2$
   (c) the voltage and the current are in phase
   (d) the voltage leads the current by π

AC source with RLC connected in Series

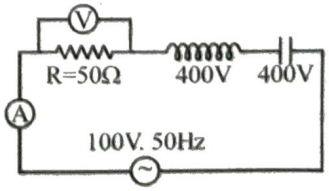
4. In an A.C. circuit, the reactive reactance X_L is :
   (a) $2\pi f L$ (b) $\frac{1}{(2\pi f L)}$
   (c) $\frac{\pi f L}{2}$ (d) $\frac{2}{\pi f L}$
5. If an inductive circuit the equation of A.C., is $i = i_0 \sin \omega t$ then:
   (a) $E = E_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ (b) $E = E_0 \sin\left(\omega t - \frac{\pi}{2}\right)$
   (c) $E = E_0 \sin \omega t$ (d) None of these
6. A coil has reactance of 100Ω, when frequency is 50Hz. If the frequency becomes 150 Hz., then the reactance will be:
   (a) 100 Ω (b) 300 Ω
   (c) 450 Ω (d) 600 Ω
7. In a circuit the frequency is $f = \frac{1000}{2\pi}$ Hz and the inductance is 2 henry, then the reactance will be :
   (a) 200 Ω (b) 200 μΩ
   (c) 2000 Ω (d) 2000 μΩ
8. Which one of the following has not the same unit
   (a) $\sqrt{LC}$ (b) $\frac{1}{\sqrt{LC}}$
   (c) RC (d) $\frac{L}{R}$
9. An inductor and a resistor in series are connected to an A.C. supply of variable frequency. As the frequency of the source is increased, the phase angle between current and the potential difference across source will be:



   (a) First increase and then decrease
   (b) First decrease and then increase
   (c) Go on decreasing
   (d) Go on increasing
10. If the current through an inductor of inductance I, is given by $I = I_0 \sin \omega t$, then the voltage across inductor will be:
    (a) $I_0 \omega L \sin\left(\omega t - \frac{\pi}{2}\right)$ (b) $I_0 \omega L \sin\left(\omega t + \frac{\pi}{2}\right)$
    (c) $I_0 \omega L \sin(\omega t - \pi)$ (d) None of these
11. The figure shows variation of R, X_L and X_C with frequency f in a series L, C, R circuit. Then for what frequency point, the circuit is inductive

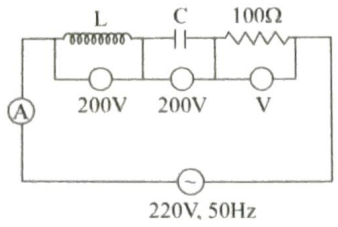


    (a) A (b) B
    (c) C (d) All points
12. In given LCR circuit the voltage across the terminals of a resistance and current will be



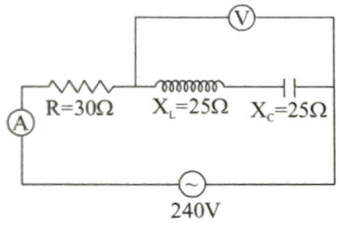
    (a) 400 V, 2A (b) 800 V, 2A
    (c) 100 V, 2A (d) 100V, 4A

13. The readings of ammeter and voltmeter in the following circuit are respectively



(a) 2A, 200V (b) 1.5 A, 100 V
(c) 2.7A, 220 V (d) 2.2 A, 220 V

14. In the circuit shown in figure neglecting source resistance the voltmeter and ammeter reading will respectively be



(a) 0V, 3A (b) 150 V, 3A
(c) 150 V, 6A (d) 0V, 8A

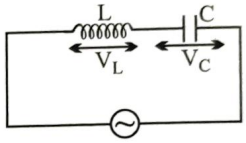
Power consumed in an AC circuit

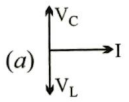
15. If a choke coil of negligible resistance works on 220V source and 5mA current is flowing through it, then the loss of power in choke coil is:
(a) Zero (b) 11 watt
(c) 44×10^3 watt (d) 1.1 watt

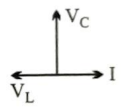
16. In which of the following case power factor will be negligible :
(a) Inductance and resistance both high
(b) Inductance and resistance both low
(c) Low resistance and high inductance
(d) High resistance and low inductance

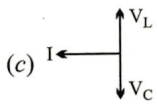
17. In a circuit having a resistance of 100Ω connected in series with a capacitive reactance of 100Ω to an alternating voltage source, the current:
(a) Leads voltage by 90°
(b) Leads voltage by 45°
(c) Lags behind voltage by 90°
(d) Lags behind voltage by 45°

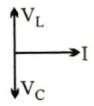
18. The current I, potential difference V_L across the inductor and potential difference V_C across the capacitor in circuit as shown in the figure are best represented vectorially as



(a) 

(b) 

(c) 

(d) 

Learning Plus

1. The peak value of an alternating e.m.f given by $E = E_0 \cos \omega t$, is 10 volt and frequency is 50 Hz. At time $t = (1/600)$ sec, the instantaneous value of e.m.f is :
(a) 10 volt (b) $5\sqrt{3}$ volt
(c) 5 volt (d) 1 volt

2. An AC voltage is given by :

$$E = E_0 \sin \frac{2\pi t}{T}$$

Then the mean value of voltage calculated over time interval of T/2 seconds :
(a) is always zero (b) is never zero
(c) is $(2e_0/\pi)$ always (d) may be zero

3. An alternating current changes from a complete cycle in 1μs, then the frequency in Hz will be–
(a) 10^{-6} (b) 50
(c) 100 (d) 10^6

4. An ac circuit, the current is given by $i = 4 \sin (100 \pi t + 30°)$ ampere. The current becomes maximum first time (after t = 0) at t equal to –
(a) (1/200) sec (b) (1/300) sec
(c) (1/50) sec (d) None of these

5. The value of alternating e.m.f. is $e = 500 \sin 100 \pi t$, then the frequency of this potential in Hz is–
(a) 25 (b) 50
(c) 75 (d) 100

6. The average value or alternating current for half cycle in terms of I_0 is
(a) $\frac{2I_0}{\pi}$ (b) $\frac{I_0}{\pi}$
(c) $\frac{I_0}{\sqrt{2}}$ (d) 0

7. The phase difference between the alternating current and voltage represented by the following equation $I = I_0 \sin \omega t$, $E = E_0 \cos(\omega t + \pi/3)$, will be –

(a) $\frac{\pi}{3}$ (b) $\frac{4\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{6}$

8. A 100 volt AC source of angular frequency 500 rad/s is connected to a LCR circuit with L = 0.8 H, C = 5 μF and R = 10 Ω, all connected in series. The potential difference across the resistance is

(a) $\frac{100}{\sqrt{2}}$ volt (b) 100 volt

(c) 50 volt (d) $50\sqrt{3}$

9. In an L-R series circuit ($L = \frac{175}{11}$ mH and R = 12Ω), a variable emf source ($V = V_0 \sin \omega t$) of $V_{rms} = 130\sqrt{2}$ V and frequency 50 Hz is applied. The current amplitude in the circuit and phase of current with respect to voltage are respectively. (Use $\pi = 22/7$)

(a) 14.14A, 30° (b) $10\sqrt{2}$ A, $\tan^{-1}\frac{5}{12}$

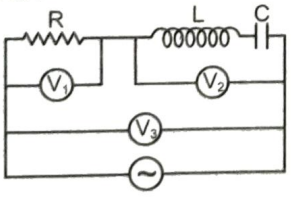
(c) 10 A, $\tan^{-1}\frac{5}{12}$ (d) 20 A, $\tan^{-1}\frac{5}{12}$

10. In an AC circuit the potential differences across an inductance and resistance joined in series are respectively 16 V and 20 V. The total potential difference across the circuit is

(a) 20 V (b) 25.6 V

(c) 31.9 V (d) 53.5 V

11. A resistor R, an inductor L, a capacitor C and voltmeters V_1, V_2 and V_3 are connected to an oscillator in the circuit as shown in the adjoining diagram. When the frequency of the oscillator is increased, upto resonance frequency, the voltmeter reading (at resonance frequency) is zero in the case of :



(a) voltmeter V_1 (b) voltmeter V_2

(c) voltmeter V_3 (d) all the three voltmeters

12. A coil of inductance 0.1 H is connected to an alternating voltage generator of voltage E = 100 sin (100t) volt. The current flowing through the coil will be –

(a) $I = 10\sqrt{2}\sin(100t)$ A (b) $I = 10\sqrt{2}\cos(100t)$ A

(c) $I = -10\sin(100t)$ A (d) $I = -10\cos(100t)$ A

13. Alternating current lead the applied e.m.f. by π/2 when the circuit consists of –

(a) only resistance

(b) only capacitor

(c) only an inductance coil

(d) capacitor and resistance both

14. A resistance of 50Ω, an inductance of 20/π henry and a capacitor of 5/π μF are connected in series with an A.C. source of 230 volt and 50Hz. The impedance of circuit is–

(a) 5Ω (b) 50Ω

(c) 5KΩ (d) 500Ω

15. The potential difference between the ends of a resistance R is V_R between the ends of capacitor is $V_C = 2V_R$ and between the ends of inductance is $V_L = 3V_R$, then the alternating potential of the source in terms of V_R will be–

(a) $\sqrt{2}\ V_R$ (b) V_R

(c) $V_R/\sqrt{2}$ (d) $5V_R$

16. In a series resonant L–C–R circuit, if L is increased by 25% and C is decreased by 20%, then the resonant frequency will

(a) Increase by 10%

(b) Decrease by 10%

(c) Remain unchanged

(d) Increase by 2.5%

17. A capacitor is a perfect insulator for :

(a) direct current

(b) alternating current

(c) direct as well as alternating current

(d) None of these

18. In an A.C. circuit, a resistance of 3Ω, an inductance coil of 4Ω and a condenser of 8Ω are connected in series with an A.C. source of 50 volt (R.M.S.). The average power loss in the circuit will be

(a) 600 watt (b) 500 watt

(c) 400 watt (d) 300 watt

19. In LR circuit the a.c. source has voltage 220 V. If the potential difference across the inductance is 176 volts, the p.d. across the resistance will be:

(a) 44 V (b) 396 V

(c) 132 V (d) $\sqrt{(250\times176)}$ V

20. The potential difference V across and the current I flowing through an instrument in an AC circuit are given by:

$V = 5\cos\omega t$ volt

$I = 2\sin\omega t$ volt

The power dissipated in the instrument is :

(a) zero (b) 5 watt

(c) 10 watt (d) 2.5 watt

21. A sinusoidal AC current flows through a resistor of resistance R. If the peak current is I_p, then average power dissipated is :

(a) $I_p^2 R \cos\theta$ (b) $\frac{1}{2} I_p^2 R$

(c) $\frac{4}{\pi} I_p^2 R$ (d) $\frac{1}{\pi^2} I_p^2 R$

22. If the frequency of the source e.m.f. in an AC circuit is n, the power varies with a frequency :

(a) n (b) 2n

(c) n/2 (d) zero

23. Power factor may be equal to 1 for :

(a) pure inductor (b) pure capacitor

(c) L, R circuit (d) An LCR circuit

24. If the power factor changes from $\frac{1}{2}$ to $\frac{1}{4}$ then what is the increase in impedance in AC?

(a) 20% (b) 50%

(c) 25% (d) 100%

25. A power transformer (step up) with an 1 : 8 turn ratio has 60 Hz, 120 V across the primary; the load in the secondary is $10^4\ \Omega$. The current in the secondary is

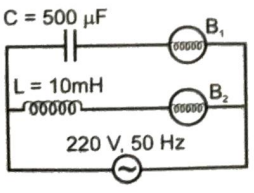
(a) 96 A (b) 0.96 A

(c) 9.6 A (d) 96 mA

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/ NUMERICAL

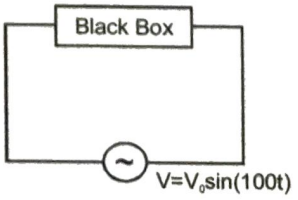
1. In a series LCR circuit with an AC source (E_{rms} = 50 V and $\nu = 50/\pi$ Hz), R = 300 Ω, C = 0.02mF, L = 1.0 H, Which of the following is correct?
   (a) the rms current in the circuit is 0.1 A
   (b) the rms potential difference across the capacitor is 50 V
   (c) the rms potential difference across the capacitor is 14.1 V
   (d) the rms current in the circuit is 0.14 A
2. A pure inductance of 1 henry is connected across a 110 V, 70 Hz source. Then correct option are (Use $\pi = 22/7$):
   (a) reactance of the circuit is 440 Ω
   (b) current of the circuit is 0.25 A
   (c) reactance of the circuit is 880 Ω
   (d) current of the circuit is 0.5 A
3. In an AC series circuit when the instantaneous source voltage is maximum, the instantaneous current is zero. Connected to the source may be a
   (a) pure capacitor
   (b) pure inductor
   (c) pure resistor
   (d) combination of pure an inductor and pure capacitor
4. In the circuit shown in figure, if both the bulbs B_1 and B_2 are identical:



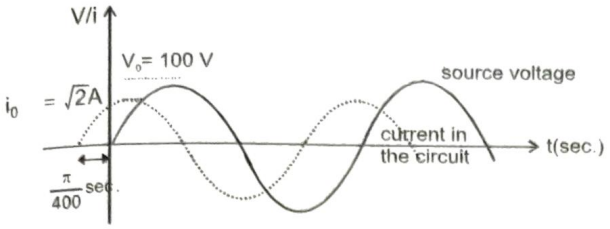
   (a) their brightness will be the same
   (b) B_2 will be brighter than B_1
   (c) as frequency of supply voltage is increased the brightness of bulb B_1 will increase and that of B_2 will decrease.
   (d) only B_2 will glow because the capacitor has infinite impedance
5. An AC source supplies a current of 10 A (rms) to a circuit, rms voltage of source is 100 V. The average power delivered by the source:
   (a) must be 1000 W
   (b) may be less than 1000 W
   (c) may be greater than 1000 W
   (d) may be 1000 W
6. 11 kW of electric power can be transmitted to a distant station at (i) 220 V or (ii) 22000 V. Which of the following is correct?
   (a) first mode of transmission consumes less power
   (b) second mode of transmission consumes less power
   (c) first mode of transmission draws less current
   (d) second mode of transmission draws less current

Comprehension-1 (7 to 9)

A voltage source $V = V_0 \sin(100\ t)$ is connected to a black box in which there can be either one element out of L, C, R or any two of them connected in series.



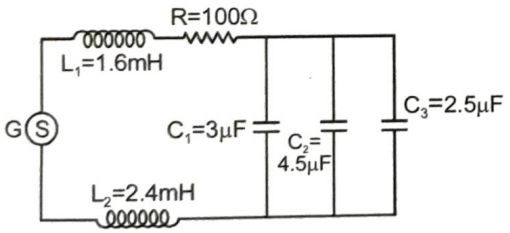
At steady state. the variation of current in the circuit and the source voltage are plotted together with time, using an oscilloscope, as shown



7. The element(s) present in black box is/are :
   (*a*) only C (*b*) L C
   (*c*) L and R (*d*) R and C
8. Values of the parameters of the elements, present in the black box are -
   (*a*) R = 50 Ω , C = 200 μf (*b*) R = 50 Ω , L = 2 mμ
   (*c*) R = 400 Ω , C = 50 μf (*d*) None of these
9. If AC source is removed, the circuit is shorted for some time so that capacitor is fully discharsed and then a battery of constant EMF is connected across the black box. At t = 0, the current in the circuit will -
   (*a*) increase exponentially with time constant = 0.02 sec.
   (*b*) decrease exponentially with time constant = 0.01 sec.
   (*c*) oscillate with angular frequency 20 rad/sec
   (*d*) first increase and then decrease

Comprehension-2 (10 to 14)

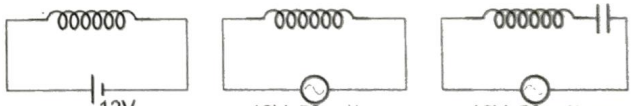
An ac generator G with an adjustable frequency of oscillation is used in the circuit, as shown.



10. Current drawn from the ac source will be maximum if its angular frequency is -
    (*a*) 10^5 rad/s (*b*) 10^4 rad/s
    (*c*) 5000 rad/s (*d*) 500 rad/s
11. To increase resonant frequency of the circuit, some of the changes in the circuit are carried out. Which change(s) would certainly result in the increase in resonant frequency?
    (*a*) R is increased.
    (*b*) L_1 is increased and C_1 is decreased.
    (*c*) L_2 is decreased and C_2 is increased.
    (*d*) C_3 is removed from the circuit.
12. If the ac source G is of 100 V rating at resonant frequency of the circuit, then average power supplied by the source is -
    (*a*) 50 W (*b*) 100 W
    (*c*) 500 W (*d*) 1000 W
13. Average energy stored by the inductor L_2 (Source is at resonance frequency) is equal to
    (*a*) zero (*b*) 1.2 mJ
    (*c*) 2.4 mJ (*d*) 4 mJ
14. Thermal energy produced by the resistance R in time duration 1 μs, using the source at resonant condition, is
    (*a*) 0 J (*b*) 1 μJ
    (*c*) 100 μJ
    (*d*) not possible to calculate from the given information
15. Match the Physical quantities given in Column-I with the parameters they depend on as given in Column-II.

Column-I	Column-II
(*a*) Inductance of a coil	(p) Depends on resistivity
(*b*) Capacitance	(q) Depends on shape
(*c*) Impedance of a coil	(r) Depends on medium inserted
(*d*) Reactance of a capacitor	(s) Depends on external voltage source

16. A steady current 4 A flows in an inductor coil when connected to a 12 V DC source as shown in figure 1. If the same coil is connected to an ac source of 12 V, 50 rad/s, a current of 2.4 A flows in the circuit as shown in figure 2. Now after these observations, a capacitor of capacitance $\frac{1}{50}$F is connected in series with the coil as shown in figure 3 with the same AC source :



(Figure 1) (Figure 2) (Figure 3)

Column-I	Column-II (in S.I units)
(*a*) The inductance of the coil (nearly)	(p) 24
(*b*) The resistance of the coil	(q) 3
(*c*) Average power (nearly)	(r) 0.08
(*d*) Total reactance	

NUMERICAL VALUE BASED

17. An RLC circuit includes a 1.6 H inductor and a 250 μF capacitor rated at 400 V. The circuit is connected across a sine-wave generator whose peak voltage is 32V. What minimum resistance must the circuit have to ensure that the capacitor voltage does not exceed its rated value when the generator is at the resonant frequency? Fill 10R in OMR sheet.

18. If an AC voltage 100 V is applied between points A and B, then current of 1 A and phase difference between current and voltage will be $\Delta\phi = 37°$. If the same voltage is applied between points B and C, then current of 5 A and $\Delta\phi = 53°$. What will be the impedance (in Ω) in the chain, if the same voltage is applied between points A and C? Round off to nearest integer.

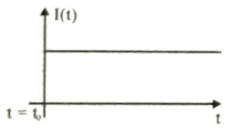


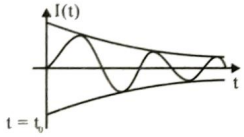
19. A coil is connected to an alternating emf of voltage 24 V and of frequency 50 Hz. The reading on the ammeter connected to the coil in series is 10 mA. If a 1μF capacitor is connected to the coil in series the ammeter shows 10 mA again. What would be the approx reading on a dc ammeter (in A) if the coil was connected to a 180 V dc voltage supply? (**Take** $\pi^2 = 10$)

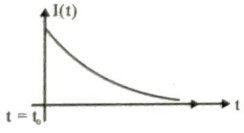
JEE Mains & Advanced Past Years Questions

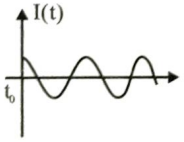
JEE-MAIN PREVIOUS YEAR'S

1. A series LR circuit is connected to a voltage source with $V(t) = V_0 \sin \omega t$. After very large time, current (t) behaves as $\left(t_0 >> \frac{L}{R}\right)$: [*JEE Mains-2016*]

(*a*) 

(*b*) 

(*c*) 

(*d*) 

2. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor need for it to work is close to: [*JEE Mains-2016*]

(*a*) 80 H (*b*) 0.08 H

(*c*) 0.044 H (*d*) 0.065 H

3. A sinusoidal voltage of peak value 283 V and angular frequency 320/s is applied to a series LCR circuit. Given that R = 5Ω, L = 25 mH and C = 1000 μF. The total impedance, and phase difference between the voltage across the source and the current will respectively be: [*JEE Main-2017*]

(*a*) 10 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$ (*b*) 7 Ω and 45°

(*c*) 10 Ω and $\tan^{-1}\left(\frac{8}{3}\right)$ (*d*) 7Ω and $\tan^{-1}\left(\frac{5}{3}\right)$

4. A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $I = I_0 \cos(\omega t)$. The emf induced in the smaller inner loop is nearly: [*JEE Main-2017*]

(*a*) $\frac{\pi\mu_0 I_0}{2}\cdot\frac{a^2}{b}\omega\sin(\omega t)$ (*b*) $\frac{\pi\mu_0 I_0}{2}\cdot\frac{a^2}{b}\omega\cos(\omega t)$

(*c*) $\pi\mu_0 I_0 \frac{a^2}{b}\omega\sin(\omega t)$ (*d*) $\frac{\pi\mu_0 I_0 b^2}{a}\omega\cos(\omega t)$

5. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance.

The quality factor, Q is given by: [*JEE Main-2018*]

(*a*) $\frac{\omega_0 R}{L}$ (*b*) $\frac{R}{(\omega_0 C)}$

(*c*) $\frac{CR}{\omega_0}$ (*d*) $\frac{\omega_0 L}{R}$

6. In an a.c. circuit, the instantaneous e.m.f. and current are given by $e = 100 \sin 30\, t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively. [*JEE Main-2018*]

(*a*) $\frac{1000}{\sqrt{2}}, 10$ (*b*) $\frac{50}{\sqrt{2}}, 0$

(*c*) 50, 0 (*d*) 50, 10

7. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns, given the output power at 230 V. If the current in the primary of the transformer is 5A, and its efficiency is 90%, the output current would be:

[*JEE Main-2019* (January)]

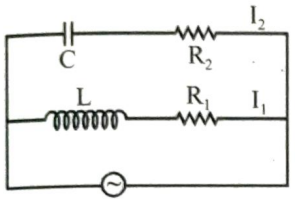
(*a*) 20 A (*b*) 40A

(*c*) 45A (*d*) 25A

8. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is: [*JEE Main-2019* (January)]

(*a*) 5.65×10^2 J (*b*) 2.26×10^3 J

(*c*) 5.17×10^2 J (*d*) 3.39×10^3 J

9. 

In the above circuit, $C = \frac{\sqrt{3}}{2}\mu F$, $R_2 = 20\Omega$, $L = \frac{\sqrt{3}}{10}H$ and $R_1 = 10\Omega$. Current in L-R_1 path is I_1 and C-R_2 path is I_2. The voltage of A.C. source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is [*JEE Main-2019* (January)]

(*a*) 60° (*b*) 30°

(*c*) 90° (*d*) 0°

10. An alternating voltage $v(t) = 220 \sin 100\pi t$ volt is applied to a purely resistance load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is:

[*JEE Main-2019* (April)]

(*a*) 2.2 ms (*b*) 5 ms

(*c*) 3.3 ms (*d*) 7.2 ms

11. A circuit connected to an ac source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this?

[*JEE Main-2019* (April)]

(*a*) RC circuit with R = 1 kΩ and C = 1μF

(*b*) RL circuit with R = 1 kΩ and L = 1mH

(*c*) RL circuit with R = 1 kΩ and L = 10 mH

(*d*) RC circuit with R = 1 kΩ and C = 10 μF

12. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10A, then the input voltage and current in the primary coil are:

[*JEE Main-2019* (April)]

(*a*) 220 V and 10A (*b*) 440 V and 5A

(*c*) 440 V and 20A (*d*) 220 V and 20A

13. In LC circuit the inductance L = 40 mH and capacitance C = 100 μF. If a voltage V(t) = 10sin (314t) is applied to the circuit, to the current in the circuit is given as:

[*JEE Main-2020* (January)]

(*a*) 0.52 cos 314 t (*b*) 10 cos 314 t

(*c*) 5.2 cos 314 t (*d*) 0.52 sin 314 t

14. An inductance coil has a reactance of 100 Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is

[*JEE Main-2020* (September)]

(*a*) 6.7×10^{-7} H (*b*) 5.5×10^{-5} H

(*c*) 1.1×10^{-1} H (*d*) 1.1×10^{-2} H

15. A 750Hz, 20 V (rms) source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2 J/°C) will get heated by 10°C. (Assume no loss of heat to the surroudings) is close to

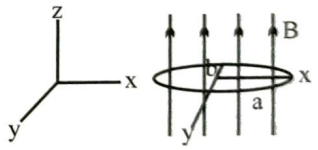
[*JEE Main-2020* (September)]

(*a*) 348 s (*b*) 418 s

(*c*) 245 s (*d*) 365 s

16. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is

[*JEE Main-2020* (September)]



(*a*) $\frac{\pi abB\omega}{R}$ (*b*) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$

(*c*) Zero (*d*) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$

17. In a series LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as $\left(\frac{n}{3\pi}\right)\mu F$ then value of n is _____.

[*JEE Main-2020* (*September*)]

18. An AC circuit has R = 100 Ω, C = 2 μF and L = 80 mH, connected in series. The quality factor of the circuit is

[*JEE Main-2020* (September)]

(*a*) 20 (*b*) 2

(*c*) 0.5 (*d*) 400

19. An In a series LCR circuit, the inductive reactance (x_L) is 10Ω and the capacitive reactance (X_C) is 4Ω. The resistance (R) in the circuit is 6Ω. The power factor of the circuit is:

[*JEE Main-2021* (March)]

(*a*) $\frac{1}{2}$ (*b*) $\frac{\sqrt{3}}{2}$

(*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\frac{1}{2\sqrt{2}}$

20. An A C source rated 220V, 50Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is:

[*JEE Main-2021* (March)]

(*a*) 2.5 ms (*b*) 25 ms

(*c*) 2.5 s (*d*) .25 ms

21. In a series LCR resonance circuit, if we change the resistance only, from a lower to higher value:

[*JEE Main-2021* (March)]

(*a*) The bandwidth of resonance circuit will increase.

(*b*) The resonance frequency will increase.

(*c*) The quality factor will increase.

(*d*) The quality factor and the resonance frequency will remain constant.

22. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved?

[*JEE Main-2021* (March)]

(*a*) Both, inducting reactance and current will be doubled.

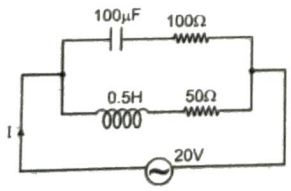
(*b*) Inductive reactance will be doubled and current will be halved.

(*c*) Both, inductive reactance and current will be halved.

(*d*) Inductive reactance will be halved and current will be doubled.

JEE-ADVANCED PREVIOUS YEAR'S

1. In the given circuit, the AC source has ω = 100 rad/s. considering the inductor and capacitor to be ideal, the correct choice (s) is(are) [*IIT JEE-2012*]



(*a*) The current through the circuit, I is 0.3 A

(*b*) The current through the circuit, I is $0.3\sqrt{2}$ A.

(*c*) The voltage across 100Ω resistor = $10\sqrt{2}$ V

(*d*) The voltage across 50Ω resistor = 10V

Paragraph for Questions (2 & 3)

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the currents and voltages mentioned are rms values.

[*JEE Advanced-2013*]

2. If the direct transmission method with a cable of resistance 0.4 Ω km^{-1} is used, the power dissipation (in %) during transmission is:

(*a*) 20 (*b*) 30

(*c*) 40 (*d*) 50

3. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is:

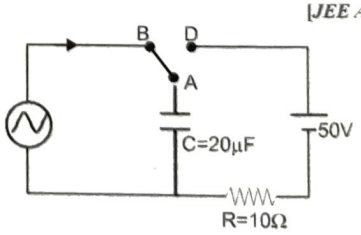
(*a*) 200 : 1 (*b*) 150 : 1

(*c*) 100 : 1 (*d*) 50 : 1

4. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and alternating current $I(t) = I_0\cos(\omega t,)$, with I_0 = 1A and ω = 500 rad s^{-1} starts flowing in it with the initial direction shown in the figure.

At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards on ly A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20μ, R = 10Ω and the battery is ideal with emf of 50V, identify the correct statement (s)

[*JEE Advanced-2014*]



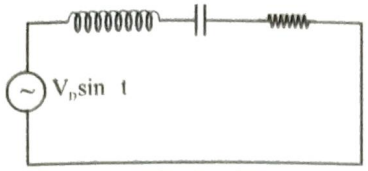
(*a*) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C.

(*b*) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(*c*) Immediately after A is connected to D. the current in R is 10A.

(*d*) $Q = 2 \times 10^{-3}$ C.

5. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin\omega t$ as shown. Which of the following options is/are correct? [*JEE Advanced-2017*]



(*a*) The current will be in phase with the voltage if $\omega = 10^4$ rad.s^{-1}

(*b*) At $\omega >> 10^6$ rad.s^{-1}, the circuit behaves like a capacitor

(*c*) The frequency at which the current will be in phase with the voltage is independent of R

(*d*) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

6. The instantaneous voltages at three terminals marked X, Y and Z are given by : $V_X = V_0 \sin\omega t$,

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and } V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between point X and Y and then between Y and Z. The reading (s) of the voltmeter will be [*JEE Advanced-2017*]

(*a*) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$ (*b*) $V_{XY}^{rms} = V_0\sqrt{\frac{1}{2}}$

(*c*) independent of the choice of the two terminals

(*d*) $V_{XY}^{rms} = V_0$

ANSWER KEY

Topicwise Questions

1. (*c*) **2.** (*c*) **3.** (*b*) **4.** (*a*) **5.** (*a*) **6.** (*b*) **7.** (*c*) **8.** (*b*) **9.** (*d*) **10.** (*b*)
11. (*c*) **12.** (*c*) **13.** (*d*) **14.** (*d*) **15.** (*a*) **16.** (*c*) **17.** (*b*) **18.** (*d*)

Learning Plus

1. (*b*) **2.** (*d*) **3.** (*d*) **4.** (*b*) **5.** (*b*) **6.** (*a*) **7.** (*d*) **8.** (*b*) **9.** (*d*) **10.** (*b*)
11. (*b*) **12.** (*d*) **13.** (*b*) **14.** (*b*) **15.** (*a*) **16.** (*c*) **17.** (*a*) **18.** (*d*) **19.** (*c*) **20.** (*a*)
21. (*b*) **22.** (*b*) **23.** (*d*) **24.** (*d*) **25.** (*d*)

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. (*a, b*) **2.** (*a, b*) **3.** (*a, b, d*) **4.** (*b, c*) **5.** (*b, d*) **6.** (*b, d*) **7.** (*d*) **8.** (*a*) **9.** (*b*) **10.** (*c*)
11. (*d*) **12.** (*b*) **13.** (*b*) **14.** (*d*) **15.** (*a*) q, r (*b*) q, r (*c*) p, q, r, s (*d*) q, r, s **16.** (*a*) r, (*b*) q, (*c*) p, (*d*) q
17. [0064] **18.** [0119] **19.** [0.1]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*d*) **2.** (*d*) **3.** (*b*) **4.** (*a*) **5.** (*d*) **6.** (*a*) **7.** (*c*) **8.** (*c*) **9.** (Bonus) **10.** (*c*)
11. (*d*) **12.** (*b*) **13.** (*a*) **14.** (*d*) **15.** (*a*) **16.** (*b*) **17.** (400) **18.** (*b*) **19.** (*c*) **20.** (*a*)
21. (*a*) **22.** (*d*)

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JEE-ADVANCED PREVIOUS YEAR'S

1. (*a, c*) **2.** (*b*) **3.** (*a*) **4.** (*c, d*) **5.** (*c, d*) **6.** (*a, c*)

22 ELECTROMAGNETIC WAVES

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INTRODUCTION

Light may be described as a wave. The wave equation for light propagating in x-direction in vacuum may be written as

$$E = E_o \sin \omega(t - x/c)$$

where E is the sinusoidally varying electric field at position x at time t.

The constant c is the speed of light in vacuum.

There is also a sinusoidally varying magnetic field associated with electric field when light propagates. The magnetic field is perpendicular to the direction of propagation as well as to the electric field E. It is given by

$$B = B_o \sin \omega(t - x/c)$$

Such a combination of mutually perpendicular electric and magnetic fields is referred to as an electromagnetic wave in vacuum.

MAXWELL'S DISPLACEMENT CURRENT

Maxwell discovered the concept of displacement current. Displacement current is that which results due to the change in electric field. The generalized form of

Ampere's law $\oint \vec{B}.\vec{dl} = \mu_0 I$, which includes both conduction current I_C (current flowing through the conducting wire) and displacement current I_D as given by Maxwell i.e.

$$\text{i.e., } \oint \vec{B}.\vec{dl} = \mu_0(I_C + I_D)$$

where $I_C = \dfrac{dq}{dt}$, where $\dfrac{dq}{dt}$ is rate of flow of charge.

and $I_D = \varepsilon_o \dfrac{d\Phi_E}{dt}$

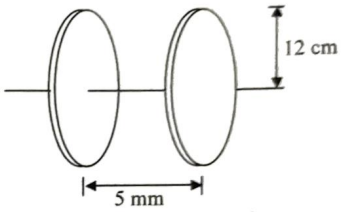
Here Φ_E is electric flux.

Therefore modified Ampere's law may be expressed as

$$\int \vec{B}.\vec{dl} = \mu_o\left(I_C + \varepsilon_o \frac{d\Phi_E}{dt}\right)$$

TRAIN YOUR BRAIN

Example 1: Figure shows a capacitor made of two circular plates each of radius 12 cm and separated by 5.0 mm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and is equal to 0.15 A.



(a) Calculate the capacitance and the rate of change of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's current rule valid at each plate of the capacitor? Explain.

Given that $e_o = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

Sol. Here, d = 5.0 mm = 5.0×10^{-3} m; R = 12 cm = 12×10^{-2} m, I = 0.15 A.

$\varepsilon_o = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

Now, $A = \pi R^2 = \pi(12 \times 10^{-2})^2 = 1.44\ \pi \times 10^{-2}\ m^2$

(*a*) The capacitance of parallel plate capacitor is given by

$$C = \frac{\varepsilon_o A}{d} = \frac{8.85 \times 10^{-12} \times 1.44\pi \times 10^{-2}}{5 \times 10^{-3}} = 80.1 \times 10^{-12}\ F$$

Now, $q = CV$

Therefore, $\dfrac{dq}{dt} = C\dfrac{dV}{dt}$ or $I = C\dfrac{dV}{dt}$

or $\dfrac{dV}{dt} = \dfrac{I}{C} = \dfrac{0.15}{80.1 \times 10^{-12}} = 1.873 \times 10^9\ Vs^{-1}$

(*b*) Displacement current is equal to conduction current i.e. 0.15 A.

(*c*) Yes, Kirchhoff's current law will be valid at each plate of the capacitor, provided electric current means the sum of the conduction and displacement currents.

MAXWELL'S EQUATIONS

The four basic laws of electricity and magnetism i.e. Gauss's law in electrostatics, Gauss's law in magnetism, Faraday's law of electromagnetic induction and Maxwell-Ampere's circuital law are called Maxwell's equations.

1. **Gauss's Law in Electrostatics**

$$\oint \vec{E}.d\vec{s} = q/\varepsilon_o$$

2. **Gauss's Law in Magnetism**

$$\oint \vec{B}.d\vec{s} = 0$$

3. **Faraday's Law of Electromagnetic Induction**

$$\varepsilon = -\frac{d\Phi_B}{dt} \text{ or } \oint \vec{E}.d\vec{l} = -\frac{d\Phi_B}{dt}$$

4. **Maxwell-Ampere's circuital Law**

$$\oint \vec{B}.d\vec{l} = \mu_o\left(I_C + \varepsilon_o \frac{d\Phi_E}{dt}\right)$$

NATURE OF ELECTROMAGNETIC WAVES

Maxwell's concept of displacement current led to the conclusion that an electric field varying with time at a point produces magnetic field at that point. This symmetry in the law of electricity and magnetism leads to the conclusion that a time varying electric field gives rise to a time varying magnetic field and vice versa. Electric and magnetic fields in electromagnetic wave is of transverse nature.

At any instant, the electric and magnetic fields varying sinusoidally with x can be expressed by equation

$$E_y = E_o \sin(x - ct) \quad ...(1)$$

and $B_Z = B_o \sin(x - ct)$ ...(2)

Here E_o and B_o are the amplitudes of electric and magnetic fields along y-axis and z-axis respectively.

Faraday's law of electromagnetic induction states that

$$\oint \vec{E}.d\vec{l} = -\frac{d\Phi_B}{dt} \quad ...(3)$$

From equation (3) it can be proved that for a plane electromagnetic wave propagating along x-axis,

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad ...(4)$$

i.e. a magnetic field varying with time gives rise to an electric field varying in space.

From equation (1), we have

$$\frac{\partial E_y}{\partial x} = E_o \cos(x - ct)$$ and from equation (2), we have

$$\frac{\partial B_y}{\partial t} = -cB_o \cos(x - ct)$$

Substituting for $\frac{\partial E_y}{\partial x}$ and $\frac{\partial B_z}{\partial t}$ in equation (1)

$$E_o = cB_o \quad \text{or } c = \frac{E_o}{B_o}$$

It shows that the velocity of electromagnetic waves is equal to the ratio of amplitudes of electric and magnetic fields.

Velocity of electromagnetic waves is also given by

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

TRAIN YOUR BRAIN

Example 2: Green light of mercury has a wavelength 5.5 $\times 10^{-5}$ cm.

(*a*) What is the frequency in MHz and period in μs in vacuum?

(*b*) What is the wavelength in glass, if refractive index of glass is 1.5?

Given c = 3×10^8 m/s.

Sol. Here, wavelength $\lambda = 5.5 \times 10^{-5}$ cm $= 5.5 \times 10^{-7}$ m

Velocity of light, $c = 3 \times 10^8$ m/s

(*a*) If ν is the frequency, then

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{5.5 \times 10^{-7}} \text{ Hz}$$

$$= \frac{3 \times 10^8}{5.5 \times 10^{-7} \times 10^6} = 5.45 \times 10^8 \text{ MHz}$$

Time period,

$$T = \frac{1}{\nu} = \frac{\lambda}{c} = \frac{5.5 \times 10^{-7}}{3 \times 10^8} = 1.8 \times 10^{-15} \times 10^6 \ \mu s$$

$= 1.8 \times 10^{-9}$ μs

(*b*) Now, refractive index

$$\mu = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in glass}} = \frac{c}{v}$$

Therefore, velocity of light in glass $v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5}$

$= 2 \times 10^8$ m/s

The wavelength of light in glass, $\lambda = \frac{v}{\nu} = vT$

$= 2 \times 10^8 \times 1.8 \times 10^{-15}$

$= 3.6 \times 10^{-7}$ m

Energy Density of Electromagnetic Waves

(*a*) The average energy density of electric field is

$$u_E = \frac{1}{2}\varepsilon_o E^2 = \frac{1}{2}\varepsilon_o \left(E_o/\sqrt{2}\right)^2 = \frac{1}{4}\varepsilon_o E_o^2$$

(*b*) The average energy density of magnetic field is,

$$u_B = \frac{B^2}{2\mu_o} = \frac{\left(B_o/\sqrt{2}\right)^2}{2\mu_o} = \frac{B_o^2}{4\mu_o}$$

(*c*) $u_E = u_B$

(*d*) Total average energy density

$$u_E + u_B = 2u_E = 2u_B = \frac{1}{2}\varepsilon_o E_o^2 = \frac{1}{2}\frac{B_o^2}{\mu_o}$$

The units of u_E and u_B are J m^{-3}.

Intensity of Electromagnetic Wave

The energy of electromagnetic wave crossing per unit time per unit area perpendicular to the direction of propagation of wave is called the intensity of electromagnetic wave.

The intensity of electromagnetic wave is

$$I = \frac{P_{av}}{4\pi r^2} = u_{av} \times c = \frac{1}{2}\varepsilon_o E_o^2 c = \frac{1}{2\mu_o} B_o^2 c = \frac{1}{2\mu_o} \cdot \frac{E_o^2}{c}$$

Momentum of Electromagnetic Wave

The electromagnetic wave during its propagation has linear momentum associated with it. The linear momentum carried by the portion of wave having energy U is given by

$$p = U/c$$

If the electromagnetic wave incident on a material surface is completely absorbed, it will deliver energy U and momentum p = U/c to the surface.

If the incident wave is totally reflected from the surface, the momentum delivered to the surface = U/c – (–U/c) = 2U/c. Due to which the electromagnetic waves incident on a surface exert a force on the surface.

Poynting Vector

When an electromagnetic wave advances, the electromagnetic energy flows in the direction of $\vec{E} \times \vec{B}$. The total energy flowing perpendicularly per second per unit area into the surface in free space is called a Poynting Vector $\vec{S}$,

where $\vec{S} = c^2 \varepsilon_o \left(\vec{E} \times \vec{B}\right) = \left(\vec{E} \times \vec{B}\right)/\mu_o$.

The S.I. unit of S is watt/(metre)^2.

PROPERTIES OF EM WAVES

(i) The direction of variation of electric field E and magnetic field B are perpendicular to each other as well as to the direction of propagation.

(ii) Speed of EM wave is 3×10^8 m/s in space, in any other medium it depends on the electric and magnetic properties of the medium and is independent of the amplitude of field variation.

(iii) The electric and magnetic field variation are in phase.

(iv) EM waves are produced by accelerating charged particle. An oscillating charge in an LC circuit also produces EM waves. These waves require no medium for its propagation.

EM WAVE SPECTRUM

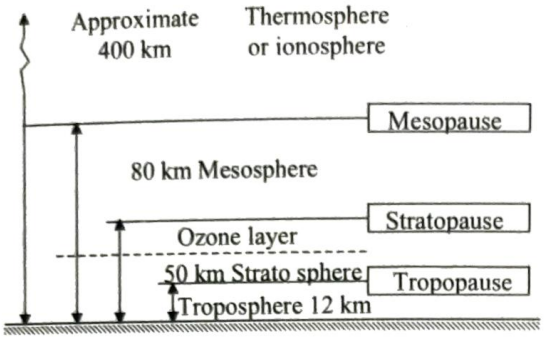
The whole spectrum of electromagnetic waves and the modern terminology used for various sections of the spectrum are shown in table. Various regions do not have sharply defined boundaries.

Nature of waves	Frequency Range	Wavelength Range
Audio Waves	20 Hz to 20 kHz	1.5×10^4 m to 1.5×10^7 m
Radio Waves	20 Hz to 10^9 Hz	0.3 m to 1.5×10^7 m
Micro Waves	10^9 Hz to 3×10^{11} Hz	1 mm to 0.3 mm
Infra-red Waves	3×10^{11} Hz to 4×10^{14} Hz	7500 Å to 1 mm
Visible radiation	4×10^{14} Hz to 8×10^{14} Hz	3750 Å to 7500 Å
Ultra violet Radiation	8×10^{14} Hz to 3×10^{17} Hz	100 Å to 3750 Å
X-rays	3×10^{17} Hz to 3×10^{19} Hz	0.1 Å to 100 Å
Gamma rays	3×10^{19} Hz to 3×10^{22} Hz	0.0001 Å to 0.1 Å

Various layers of our atmosphere

Various layers in which the atmosphere has been divided are shown in figure. The actual boundaries are not sharp and the distances are approximate.

Ozone layer: The upper region of stratosphere is rich in ozone and is called ozone layer. The ozone layer absorbs ultraviolet radiation from the sun and prevents them from reaching the earth's surface, otherwise these can cause damage to life.



Green house effect.

The phenomenon of heating of earth's atmosphere due to trapping of infra-red rays, which are radiated from earth's surface, by carbon dioxide layer in the atmosphere is called **green house effect.**

TRAIN YOUR BRAIN

Example 3: An electric charge oscillating with a frequency of 1 kilo cycles per second can radiate electromagnetic waves of wavelength

(*a*) 100 km (*b*) 200 km
(*c*) 300 km (*d*) 400 km

Sol. $\lambda = \frac{c}{\nu} = \frac{3\times10^8 \text{ ms}^{-1}}{1000 \text{ s}^{-1}} = 3 \times 10^5 \text{ m} = 300 \text{ km}$

$\therefore$ (*c*)

Example 4: Which of the following has zero average value in a plane electromagnetic wave

(*a*) kinetic energy (*b*) magnetic field
(*c*) electric field (*d*) both (*b*) and (*c*)

Sol. In electromagnetic wave, the average value of electric field or magnetic field is zero.

$\therefore$ (*d*)

Example 5: When capacitor is connected to the circuit, the current, when key is pressed, is

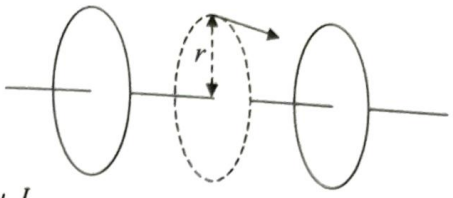
(*a*) zero
(*b*) maximum
(*c*) any transient value
(*d*) depends on capacitor used

Sol. The current = charge / time.

When capacitor is connected to the circuit and key is pressed, initially the rate of flow of charge to the capacitor will be maximum because after storing some charge, the charge in the capacitor will repel the more like charge coming to capacitor. Hence the rate of flow of charge in the circuit will decrease.

$\therefore$ (*b*)

Example 6: *An expression for the magnetic field strength B at the point between the capacitor plates indicates in fig. Express B in terms of the rate of change of the electric field strength i.e. dE/dt between the plates*



(*a*) $\frac{\mu_o I}{2\pi r}$ (*b*) $\frac{\epsilon_o \mu_o r}{2} dE/dt$

(*c*) zero (*d*) $\frac{\mu_o I}{2r}$

Sol. $B = \frac{\mu_o}{4\pi}\frac{2i_D}{r} = \frac{\mu_o}{4\pi}\frac{2}{r} \times \varepsilon_o \frac{d\phi_E}{dt} = \frac{\mu_o}{2\pi r}\varepsilon_o \frac{d}{dt}\left(E\pi r^2\right)$

$= \frac{\mu_o \varepsilon_o \pi r^2 dE}{2\pi r dt} = \frac{\mu_o \varepsilon_o r}{2}\frac{dE}{dt}$

$\therefore$ (*b*)

Example 7: Figure in above question shows a capacitor made of two circular plates each of radius 12 cm and separated by 4 mm. The capacitor is being charged by an external source not shown in the fig. The charging current is constant and equal to 0.15 A. The capacitance of capacitor is

(*a*) 10 pF (*b*) 100 pF
(*c*) 1000 pF (*d*) 10^4 pF

Sol. $C = \frac{\varepsilon_o A}{d} = \frac{1}{4\pi\times9\times10^9} \times \frac{\pi(0.12)^2}{\left(4\times10^{-3}\right)} = 10^{-10}$ F

$= 100$ pF

Example 8: In the above question displacement current is

(*a*) 15 A (*b*) 0.15 A
(*c*) 1.5 A (*d*) 1.5×10^{-9} A

Sol. $I_D = I_C$ in magnitude.

$\therefore$ (*b*)

Example 9: According to the Maxwell's equation the velocity of light in any medium is expressed as

(*a*) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ (*b*) $\frac{1}{\sqrt{\mu\varepsilon}}$

(*c*) $\sqrt{\mu/\varepsilon}$ (*d*) $\sqrt{\mu_0/\varepsilon}$

Sol. Velocity of light in a medium,

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\therefore$ (*b*)

Example 10: The displacement current is

(*a*) $\varepsilon_0 d\phi_E/dt$ (*b*) $\frac{\varepsilon_0}{R} d\Phi_E/dt$

(*c*) $\varepsilon_0 E/R$ (*d*) $\varepsilon_0 q\, C/R$

Sol. $I_D = \varepsilon_o \frac{d\phi_E}{dt}$

$\therefore$ (*a*)

Example 11: The magnetic field between the plates of a capacitor when $r < R$ is given by

(a) $\frac{\mu_0 I_D r}{2\pi R^2}$ (b) $\frac{\mu_0 I_D}{2\pi R}$

(c) $\frac{\mu_0 I_D}{2\pi r}$ (d) zero

Sol. Consider a loop of radius $r (<< R)$ between the two circular plates, placed coaxially with them. The area of loop = πr^2.

By symmetry magnetic field $\vec{B}$ is equal in magnitude at all points on the loop. If I_D is the displacement current crossing the loop and I_D is the total displacement current between plates

$$I_D = \frac{I_D}{\pi R^2} \times \pi r^2$$

Using Ampere Maxwell's law we have,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_D \text{ or}$$

$$2\pi r B = \mu_0 I_D \frac{\pi r^2}{\pi R^2} \text{ or } B = \frac{\mu_0 I_D r}{2\pi R^2}$$

$\therefore$ (a)

Example 12: The magnetic field at a point between the plates of a capacitor at a perpendicular distance R from the axis of the capacitor of plate radius R having displacement current I_D is given by

(a) $\frac{\mu_0 I_D r}{2\pi R^2}$ (b) $\frac{\mu_0 I_D}{2\pi R}$

(c) $\frac{\mu_0 I_D}{2\pi r}$ (d) zero

Sol. According to Ampere Maxwell's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_D$$

or $B 2\pi R = \mu_0 I_D$

or $B = \frac{\mu_0 I_D}{2\pi R}$

$\therefore$ (b)

Example 13: A parallel plate capacitor with plate area A and separation between the plates d is charged by a constant current i. Consider a plane surface of area A/3 parallel to the plates and drawn symmetrically between the plates. Find the displacement current through this area.

Sol. Let Q be the charge on the capacitor at time t. The electric field intensity at a point in between the plates of the capacitor is

$$E = \frac{Q}{\varepsilon_0 A}$$

The electric flux through the area considered is

$$\phi_E = \frac{Q}{\varepsilon_0 A} \cdot \frac{A}{3} = \frac{Q}{3\varepsilon_0}$$

The displacement current is

$$i_D = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{Q}{3\varepsilon_0}\right) = \varepsilon_0 \times \frac{1}{3\varepsilon_0} \frac{dQ}{dt} = \frac{i}{3}$$

Example 14: Find the energy stored in a 120 cm length of a laser beam operating at 6m W.

Sol. Time taken by laser beam to move through a distance 120 cm is

$$t = \frac{120}{c} = \frac{120}{3 \times 10^8} = 4 \times 10^{-6} \text{ s}$$

The energy contained in the 120 cm length of laser beam is

$U = P \times t = 6m \text{ W} \times (4 \times 10^{-6} \text{ s})$

$= (6 \times 10^{-3} \text{ J s}^{-1}) \times (4 \times 10^{-6} \text{ s}) = 24 \times 10^{-12} \text{ J}$

Example 15: Find the amplitude of the electric field in a parallel beam of light of intensity 10 watt/m^2.

Sol. Intensity of plane electromagnetic wave is

$$I = u_{av} c = \frac{1}{2}\varepsilon_0 E_0^2 c$$

or $E_0 = \left(\frac{2I}{\varepsilon_0 c}\right)^{1/2} = \left(\frac{2 \times 10}{8.85 \times 10^{-12} \times 3 \times 10^8}\right)^{1/2}$

$= 86.8 \text{ N/C}$

Example 16: In a plane electromagnetic wave the electric field oscillates sinusoidally at a frequency 3×10^{10} Hz and amplitude 50 V/m.

(a) What is the wavelength of the wave?

(b) What is the amplitude of oscillating magnetic field?

Sol. Here, $\nu = 3 \times 10^{10}$ Hz, $E_0 = 50$ V/m

(a) Wavelength, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3 \times 10^{10}} = 10^{-2} \text{ m} = 1 \text{ cm}$

(b) Amplitude of oscillating magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{50}{3 \times 10^8} = 1.67 \times 10^{-7} \text{ T}$$

Example 17: A plane electromagnetic wave propagating in the x-direction has a wavelength of 6.0 mm. The electric field is in the y-direction and its maximum magnitude is 33 V/m. Write suitable equations for the electric and magnetic field as a function of x and t.

Sol. Here, $\lambda = 6.00 \text{ mm} = 6 \times 10^{-3} \text{ m}$, $E_0 = 33$ V/m.

$$\omega = 2\pi\nu = \frac{2\pi c}{\lambda} = \frac{2\pi \times 3 \times 10^8}{6 \times 10^{-3}} = \pi \times 10^{11} \text{ rad/s.}$$

The maximum magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{33}{3 \times 10^8} = 11 \times 10^{-8} \text{ T}$$

The equation for the electric field, along y-axis in the electromagnetic wave is

$$E_y = E_0 \sin\omega\left(t - \frac{x}{c}\right) = 33 \sin \pi \times 10^{11}(t - x/c)$$

The equation for the magnetic field along z-axis in the electromagnetic wave is

$$B_z = B_0 \sin\omega\left(t - \frac{x}{c}\right) = 11 \times 10^{-8} \sin \pi \times 10^{11} (t - x/c)$$

Topicwise Questions

EM WAVES, DISPLACEMENT CURRENT, INTENSITY, ENERGY DENSITY

1. The displacement current flows in a dielectric of a capacitor, when potential difference across its plates
   (*a*) is increasing with time
   (*b*) is constant with time
   (*c*) becomes zero
   (*d*) becomes infinite
2. If a free electron is placed in the path of plane electromagnetic wave, it will start moving along
   (*a*) center of earth (*b*) equator of earth
   (*c*) magnetic field (*d*) electric field
3. A plane electromagnetic wave is incident on a material surface. If the wave delivers momentum p and energy E to the material surface, then
   (*a*) $p = 0, E = 0$ (*b*) $p \neq 0, E \neq 0$
   (*c*) $p \neq 0, E = 0$ (*d*) $p = 0, E \neq 0$
4. Ampere's circuital law holds good for
   (*a*) conduction current (*b*) displacement current
   (*c*) both (*d*) none of these
5. The conduction current in ideal case through a circuit is zero when charge on capacitor is
   (*a*) zero
   (*b*) maximum
   (*c*) any transient value
   (*d*) depends on capacitor used
6. The frequency of visible light is of the order of
   (*a*) 10^{15} Hz (*b*) 10^{10} Hz
   (*c*) 10^{6} Hz (*d*) 10^{4} Hz
7. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic field vectors of electromagnetic waves then the direction of propagation of electromagnetic wave is along the direction of
   (*a*) $\vec{E}$ (*b*) $\vec{B}$
   (*c*) $\vec{E} \times \vec{B}$ (*d*) none of these
8. The magnetic field between the plates of capacitor when $r > R$ is given by
   (*a*) $\frac{\mu_0 I_D r}{2\pi R^2}$ (*b*) $\frac{\mu_0 I_D}{2\pi R}$
   (*c*) $\frac{\mu_0 I_D}{2\pi r}$ (*d*) zero
9. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1}. The wavelength of the wave is
   (*a*) 24×10^{-10} m (*b*) 1.5×10^{-2} m
   (*c*) 4.16×10^{8} m (*d*) 3×10^{8} m
10. In a plane electromagnetic wave propagating in space has an electric field of amplitude 9×10^{3} V/m, then the amplitude of the magnetic field is
    (*a*) 2.7×10^{12} T (*b*) 9×10^{-3} T
    (*c*) 3×10^{-4} T (*d*) 3×10^{-5} T
11. Instantaneous displacement current of 1.0 A in the space between the parallel plate of 1 μF capacitor can be established by changing potential difference of
    (*a*) 10^{-6} V/s (*b*) 10^{6} V/s
    (*c*) 1 V/s (*d*) 0.1 V/s
12. A point source of electromagnetic radiation has an average power output of 800 W. The maximum value of electric field at a distance 4.0 m from the source is
    (*a*) 64.7 V/m (*b*) 57.8 V/m
    (*c*) 56.72 V/dm (*d*) 54.77 V/m
13. The average energy density of an e.m. wave given by $E = (50\ N/C) \sin(\omega t - kx)$ will be nearly
    (*a*) 10^{-8} J/m^3 (*b*) 10^{-7} J/m^3
    (*c*) 10^{-6} J/m^3 (*d*) 10^{-5} J/m^3
14. A plane electromagnetic wave
    $E_z = 100 \cos(6 \times 10^8 t + 4x)$ V/m
    propagates in a medium of refractive index
    (*a*) 1.5 (*b*) 2.0
    (*c*) 2.4 (*d*) 4.0

Learning Plus

1. Which of the following statement is false for the properties of electromagnetic waves?
   (*a*) Both electric and magnetic field vectors attain the maxima and minima at the same place and same time.
   (*b*) The energy in electromagnetic wave is divided equally between electric and magnetic vectors.
   (*c*) Both electric and magnetic field vectors are parallel to each other and perpendicular to the direction of propagation of wave.
   (*d*) These waves do not require any material medium for propagation.

2. The electric field of an electromagnetic wave in free space is given by $\vec{E} = 10\cos(10^7 t + kx)\hat{j}$ V/m, where t and x are in seconds and metres respectively. It can be inferred that
(i) the wavelength λ is 188.4 m.
(ii) the wave number k is 0.33 rad/m
(iii)the wave amplitude is 10 V/m
(iv)the wave is propagating along +x direction
Which one of the following pairs of statements is correct?
(a) (iii) and (iv) (b) (i) and (ii)
(c) (ii) and (iii) (d) (i) and (iii)

3. The dimensions of $(\mu_0\varepsilon_0)^{-1/2}$ are:
(a) $[L^{1/2}T^{-1/2}]$ (b) $[L^{-1}T]$
(c) $[LT^{-1}]$ (d) $[L^{1/2}T^{1/2}]$

4. The electric and the magnetic fields, associated with an e.m. wave, propagating along the +z-axis, can be represented by
(a) $\left[\vec{E} = E_0\hat{i}, \vec{B} = B_0\hat{j}\right]$ (b) $\left[\vec{E} = E_0\hat{k}, \vec{B} = B_0\hat{i}\right]$
(c) $\left[\vec{E} = E_0\hat{j}, \vec{B} = B_0\hat{i}\right]$ (d) $\left[\vec{E} = E_0\hat{j}, \vec{B} = B_0\hat{k}\right]$

5. The electric field associated with an EM wave in vacuum is given by $\vec{E} = \hat{i}\ 40\cos\left(kz - 6\times10^8 t\right)$, where E, z and t are in volt/m, meter and seconds respectively. The value of wave vector k is
(a) 2 m^{-1} (b) 0.5 m^{-1}
(c) 6 m^{-1} (d) 3 m^{-1}

6. The decreasing order of wavelengths of infrared, microwave, ultraviolet and gamma rays is
(a) microwave, infrared, ultraviolet, gamma rays
(b) gamma rays, ultraviolet, infrared, microwaves
(c) microwaves, gamma rays, infrared, ultraviolet
(d) infrared, microwaves, ultraviolet,gamma rays

7. The ratio of amplitude of magnetic field to the amplitude of electric field for an electromagnetic wave propagating in vacuum is equal to
(a) the speed of light in vacuum
(b) reciprocal of speed of light in vacuum
(c) the ratio of magnetic permeability to the electric susceptibility of vacuum
(d) unity

8. An electromagnetic wave of frequency υ = 3.0 MHz passes from vacuum into a dielectric medium with relative permittivity ε = 4.0 .Then
(a) Wavelength is doubled and frequency becomes half.
(b) Wavelength is halved and frequency remains unchanged.
(c) Wavelength and frequency both remains unchanged.
(d) Wavelength is doubled and frequency remains unchanged.

9. The condition under which a microwaves oven heats up a food item containing water molecules most efficiently is
(a) Microwaves are heat waves, so always produce heating.
(b) Infra-red waves produce heating in a microwave oven.
(c) The frequency of the microwaves must match the resonant frequency of the water molecules.
(d) The frequency of the microwaves has no relation with natural frequency of water molecules

10. Out of the following options which one can be used to produce a propagating electromagnetic wave
(a) A chargeless particle
(b) An accelerating charge
(c) A charge moving at constant velocity
(d) A stationary charge

11. A 100Ω resistance and a capacitor of 100Ω reactance are connected in series across a 220 V source. When the capacitor is 50% charged, the peak value of the displacement current is :
(a) 2.2 A (b) 11A
(c) 4.4 A (d) $11\sqrt{2}$ A

12. In an electromagnetic wave in free space the root mean square value of the electric field is $E_{rms} = 6\ Vm^{-1}$. The peak value of the magnetic field is:
(a) 2.83×10^{-8} T (b) 0.70×10^{-8} T
(c) 4.23×10^{-8} T (d) 1.41×10^{-8} T

13. An EM wave is propagating in a medium with a velocity $\vec{V} = V\hat{i}$. The instantaneous oscillating electric field of this EM wave is along +y axis. Then the direction of oscillating magnetic field of the EM wave will be along
(a) –y direction (b) +z direction
(c) –z direction (d) –x direction

14. Which colour of the light has the longest wavelength?
(a) red (b) blue
(c) green (d) violet

15. The ratio of contributions made by the electric field and magnetic field components to the intensity of an electromagnetic wave is: (c = speed of electromagnetic waves)
(a) 1 : 1 (b) 1 : c
(c) 1 : c^2 (d) c : 1

16. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then
(a) $\vec{X}\parallel\vec{B}$ and $\vec{k}\parallel\vec{B}\times\vec{E}$ (b) $\vec{X}\parallel\vec{E}$ and $\vec{k}\parallel\vec{E}\times\vec{B}$
(c) $\vec{X}\parallel\vec{B}$ and $\vec{k}\parallel\vec{E}\times\vec{B}$ (d) $\vec{X}\parallel\vec{E}$ and $\vec{k}\parallel\vec{B}\times\vec{E}$

17. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is:
(a) 3V/m (b) 6V/m
(c) 9V/m (d) 12 V/m

18. During the propagation of electromagnetic waves in a medium :
(a) Electric energy density is double of the magnetic energy density.
(b) Electric energy density is half of the magnetic energy density.
(c) Electric energy density is equal to the magnetic energy density.
(d) Both electric and magnetic energy densities are zero.

19. Match List-I (Electromagnetic wave type) with List-II (Its association/application)and select the correct option from the choices given below the lists :

List-I		List-II	
(a)	Infrared waves	(i)	To treat muscular strain
(b)	Radio waves	(ii)	For broadcasting
(c)	X-rays	(iii)	To detect fractureof bones
(d)	Ultraviolet	(iv)	Absorbed by the ozone layer of the atmosphere

(a) (a)→ (iv); (b)→ (iii); (c)→ (ii); (d)→ (i)
(b) (a)→ (i); (b)→ (ii); (c)→ (iv); (d)→ (iii)
(c) (a)→ (iii); (b)→ (ii); (c)→ (i); (d)→ (iv)
(d) (a)→ (i); (b)→ (ii); (c)→ (iii); (d)→ (iv)

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVISOUS YEAR'S

1. A 27 mW laser beam has a cross - sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by:[**Given :** permittivity of space $\varepsilon_0 = 9 \times 10^{-12}$ SI units, speed of light $= c = 3 \times 10^8$ m/s]: [*JEE Main-2019 (January)*]
(a) 2 kV/m (b) 0.7 kV/m
(c) 1 kV/m (d) 1.4 kV/m

2. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index' 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by: [*JEE Main - 2019 (January)*]
(a) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (b) $\left(\sqrt{n}, \sqrt{n}\right)$
(c) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

3. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8 m/s)
$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$ then the maximum electric field associated with it is: [*JEE Main - 2019 (January)*]
(a) 6×10^4 N/C (b) 3×10^4 N/C
(c) 4×10^4 N/C (d) 4.5×10^4 N/C

4. The electric field of a plane polarized electromagnetic wave in free space at time t = 0 is given by an expression
$\vec{E}(x,y) = 10\hat{j}\cos[6x + 8z]$
The magnetic field $\vec{B}(x, z, t)$is given by : (c is the velocity of light) [*JEE Main - 2019 (January)*]
(a) $\frac{1}{c}\left(6\hat{k} + 8\hat{i}\right)\cos[(6x - 8z + 10ct)]$
(b) $\frac{1}{c}\left(6\hat{k} - 8\hat{i}\right)\cos[(6x + 8z - 10ct]$
(c) $\frac{1}{c}\left(6\hat{k} + 8\hat{i}\right)\cos[(6x + 8z - 10ct)]$
(d) $\frac{1}{c}\left(6\hat{k} - 8\hat{i}\right)\cos[(6x + 8z + 10ct)]$

5. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\hat{j}$ V/m. The corresponding magnetic field $\vec{B}$, at that point will be [*JEE Main - 2019 (January)*]
(a) $18.9 \times 10^{-8}\ \hat{k}T$ (b) $2.1 \times 10^{-8}\ \hat{k}T$
(c) $6.3 \times 10^{-8}\ \hat{k}T$ (d) $18.9 \times 10^8\ \hat{k}T$

6. The energy associated with electric field is (U_E) and with magnetic field is (U_B)for an electromagnetic wave in free space. Then: [*JEE Main - 2019 (January)*]
(a) $U_E = \frac{U_B}{2}$ (b) $U_E > U_B$
(c) $U_E < U_B$ (d) $U_E = U_B$

7. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is E = 6V m^{-1} along y-direction. Its corresponding magnetic field component, B would be : [JEE Main-2019 (April)]

(a) 6×10^{-8} T along z-direction

(b) 6×10^{-8} T along x-direction

(c) 2×10^{-8} T along z-direction

(d) 2×10^{-8} T along y-direction

8. The magnetic field of an electromagnetic wave is given by: [JEE Main-2019 (April)]

$$\vec{B} = 1.6\times10^{-6}\cos\left(2\times10^{7}z + 6\times10^{15}t\right)\left(2\hat{i}+\hat{j}\right)\frac{Wb}{m^2}$$

The associated electric field will be :-

(a) $\vec{E} = 4.8\times10^{2}\cos\left(2\times10^{7}z + 6\times10^{15}t\right)\left(\hat{i}-2\hat{j}\right)\frac{V}{m}$

(b) $\vec{E} = 4.8\times10^{2}\cos\left(2\times10^{7}z - 6\times10^{15}t\right)\left(2\hat{i}+\hat{j}\right)\frac{V}{m}$

(c) $\vec{E} = 4.8\times10^{2}\cos\left(2\times10^{7}z - 6\times10^{15}t\right)\left(-2\hat{j}+\hat{i}\right)\frac{V}{m}$

(d) $\vec{E} = 4.8\times10^{2}\cos\left(2\times10^{7}z + 6\times10^{15}t\right)\left(-\hat{i}+2\hat{j}\right)\frac{V}{m}$

9. The magnetic field of a plane electromagnetic wave is given by : [JEE Main-2019 (April)]

$\vec{B} = B_0\hat{i}\left[\cos(kz-\omega t)\right] + B_1\hat{j}\cos(kz+\omega t)$ where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at z = 0 is closest to :

(a) 0.9 N (b) 0.1 N

(c) 3×10^{-2}N (d) 0.6 N

10. A plane electromagnetic wave having a frequency ν= 23.9 GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave? [JEE Main-2019 (April)]

(a) $\vec{B} = 2\times10^{-7}\sin\left(0.5\times10^{3}z + 1.5\times10^{11}t\right)\hat{i}$

(b) $\vec{B} = 2\times10^{-7}\sin\left(1.5\times10^{2}x + 0.5\times10^{11}t\right)\hat{j}$

(c) $\vec{B} = 2\times10^{-7}\sin\left(0.5\times10^{3}z - 1.5\times10^{11}t\right)\hat{i}$

(d) $\vec{B} = 60\sin\left(0.5\times10^{3}x + 1.5\times10^{11}t\right)\hat{k}$

11. An electromagnetic wave is represented by the electric field $\vec{E} = E_0\hat{n}\sin\left[\omega t + (6y-8z)\right]$. Taking unit vectors in x, y and z directions to be $\hat{i},\hat{j},\hat{k}$, the direction of propogation $\hat{s}$, is : [JEE Main-2019 (April)]

(a) $\hat{s} = \frac{4\hat{j}-3\hat{k}}{5}$ (b) $\hat{s} = \frac{3\hat{i}-4\hat{j}}{5}$

(c) $\hat{s} = \left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$ (d) $\hat{s} = \frac{-4\hat{k}+3\hat{j}}{5}$

12. The electric field of a plane electromagnetic wave is given by [JEE Main-2019 (April)]

$\vec{E} = E_0\hat{i}\cos(kz)\cos(\omega t)$

The corresponding magnetic field $\vec{B}$ is then given by

(a) $\vec{B} = \frac{E_0}{C}\vec{j}\sin(kz)\cos(\omega t)$

(b) $\vec{B} = \frac{E_0}{C}\vec{j}\sin(kz)\sin(\omega t)$

(c) $\vec{B} = \frac{E_0}{C}\vec{k}\sin(kz)\cos(\omega t)$

(d) $\vec{B} = \frac{E_0}{C}\vec{j}\cos(kz)\sin(\omega t)$

13. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3 \times 10^{-8}\sin(1.6 \times 10^{3}x + 48 \times 10^{10}t)\,\hat{j}$ T, then what will be expression for electric field? [JEE Main-2020 (January)]

(a) $\vec{E} = (3 \times 10^{-8}\sin(1.6 \times 10^{3}x + 48 \times 10^{10}t)\,\hat{j}$ V/m)

(b) $\vec{E} = (3 \times 10^{-8}\sin(1.6 \times 10^{3}x + 48 \times 10^{10}t)\,\hat{i}$ V/m)

(c) $\vec{E} = (60\sin(1.6 \times 10^{3}x + 48 \times 10^{10}t)\,\hat{k}$ V / m)

(d) $\vec{E} = (9\sin(1.6 \times 10^{3}x + 48 \times 10^{10}t)\,\hat{k}$ V / m)

14 A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B} = 5\times10^{-8}\hat{j}$ T. The corresponding electric field $\vec{E}$ is (Speed of light $c = 3 \times 10^{8}ms^{-1}$) [JEE Main-2020 (January)]

(a) $1.66\times10^{-16}\hat{i}$ V / m (b) $15\hat{i}$ V/m

(c) $-15\hat{i}$ V/m (d) $-1.66\times10^{-16}\hat{i}$ V/m

15. The electric fields of two plane electromagnetic plane waves in vacuum are given by :

$\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$ and $\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$

At t = 0, a particle of charge q is at origin with a velocity $\vec{v} = 0.8c\hat{j}$ (c is the speed of the light in vaccum). The instantaneous force experienced by the particle is :

[*JEE Main-2020 (January)*]

(*a*) $E_0q(0.8\hat{i} + \hat{j} + 0.2\hat{k})$ (*b*) $E_0q(-0.8\hat{i} + \hat{j} + \hat{k})$

(*c*) $E_0q(0.4\hat{i} - 3\hat{j} + 0.8\hat{k})$ (*d*) $E_0q(0.8\hat{i} - \hat{j} + 0.4\hat{k})$

16. A plane electromagnetic wave is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$, with its polarization along the direction $\hat{k}$. The correct from of the magnetic field of wave would be (here B_0 is an appropriate constant):

[*JEE Main-2020 (January)*]

(*a*) $B_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(*b*) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t + k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(*c*) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(*d*) $B_0\hat{k}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

17. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i} - 2\hat{j}$ respectively. What is the unit vector along direction of propagation of the wave? [*JEE Main-2020 (September)*]

(*a*) $\frac{1}{\sqrt{5}}(\hat{i} + 2\hat{j})$ (*b*) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

(*c*) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ (*d*) $\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$

18. A plane electromagnetic wave, has frequency of 2.0×10^{10} Hz and its energy density is 1.02×10^{-8} J/m^3 in vacuum. The amplitude of the magnetic field of the wave is close to ($\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{Nm^2}{C^2}$ and speed of light $= 3 \times 10^8$ ms^{-1}) [*JEE Main-2020 (September)*]

(*a*) 110 nT (*b*) 140 nT

(*c*) 190 nT (*d*) 160 nT

19. The electric field of a plane electromagnetic wave propagating along the x direction in vaccum is $\vec{E} = E_0\hat{j}\cos(\omega t - kx)$. The magnetic field $\vec{B}$, at the moment t = 0 is- [*JEE Main-2020 (September)*]

(*a*) $\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos(kx)\hat{j}$ (*b*) $\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos(kx)\hat{k}$

(*c*) $\vec{B} = \frac{E_0}{\sqrt{\mu_0\varepsilon_0}}\cos(kx)\hat{k}$ (*d*) $\vec{B} = \frac{E_0}{\sqrt{\mu_0\varepsilon_0}}\cos(kx)\hat{j}$

20. The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0(\hat{x} + \hat{y})\sin(kz - \omega t)$. Its magnetic field will be given by [*JEE Main-2020 (September)*]

(*a*) $\frac{E_0}{c}(\hat{x} - \hat{y})\cos(kz - \omega t)$ (*b*) $\frac{E_0}{c}(\hat{x} - \hat{y})\sin(kz - \omega t)$

(*c*) $\frac{E_0}{c}(-\hat{x} + \hat{y})\sin(kz - \omega t)$ (*d*) $\frac{E_0}{c}(\hat{x} - \hat{y})\sin(kz - \omega t)$

21. Choose the correct option relating wavelengths of different parts of electromagnetic wave spectrum

[*JEE Main-2020 (September)*]

(*a*) $\lambda_{x-rays} < \lambda_{micro\,waves} < \lambda_{radio\,waves} < \lambda_{visible}$

(*b*) $\lambda_{visible} > \lambda_{x-rays} > \lambda_{radio\,waves} > \lambda_{micro\,waves}$

(*c*) $\lambda_{radio\,waves} > \lambda_{micro\,waves} > \lambda_{visible} > \lambda_{x-rays}$

(*d*) $\lambda_{visible} < \lambda_{micro\,waves} < \lambda_{radio\,waves} < \lambda_{x-rays}$

22. For a plane electromagnetic wave, the magnetic field at a point x and time t is [*JEE Main-2020 (September)*]

$\vec{B}(x, t) = [1.2 \times 10^{-7}\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{k}]$T.

The instantaneous electric field $\vec{E}$ corresponding to $\vec{B}$ is (speed of light $c = 3 \times 10^8$ ms^{-1})

(*a*) $\vec{E}(x,t) = [36\sin(1 \times 10^3 x + 1.5 \times 10^{11} t)\hat{i}]\frac{V}{m}$

(*b*) $\vec{E}(x,t) = [36\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{k}]\frac{V}{m}$

(*c*) $\vec{E}(x,t) = [-36\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{j}]\frac{V}{m}$

(*d*) $\vec{E}(x,t) = [36\sin(1 \times 10^3 x + 0.5 \times 10^{11} t)\hat{j}]\frac{V}{m}$

23. The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3 m is E. The electric field intensity produced by the radiation coming from 60 W at the same distance is $\sqrt{\frac{x}{5}}$E. Where the value of x = ---_____. [*JEE Main-2021 (March)*]

ANSWER KEY

Topicwise Questions

1. (*a*)	**2.** (*d*)	**3.** (*b*)	**4.** (*a*)	**5.** (*a*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*d*)
11. (*b*)	**12.** (*d*)	**13.** (*a*)	**14.** (*b*)						

Learning Plus

1. (*c*)	**2.** (*d*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*a*)	**7.** (*b*)	**8.** (*b*)	**9.** (*c*)	**10.** (*b*)
11. (*a*)	**12.** (*a*)	**13.** (*b*)	**14.** (*a*)	**15.** (*a*)	**16.** (*b*)	**17.** (*b*)	**18.** (*c*)	**19.** (*d*)	

JEE Mains & Advanced Past Years Questions

JEE-MAINS PREVIOUS YEAR'S

1. (*d*)	**2.** (*c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*b*)	**6.** (*d*)	**7.** (*c*)	**8.** (*d*)	**9.** (*d*)	**10.** (*c*)
11. (*c*)	**12.** (*b*)	**13.** (*d*)	**14.** (*b*)	**15.** (*a*)	**16.** (*c*)	**17.** (*c*)	**18.** (*d*)	**19.** (*b*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** [3]							

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23 RAY OPTICS AND OPTICAL INSTRUMENTS

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INTRODUCTION

Electromagnetic radiations belonging to the wavelength range about 400 nm to 750 nm is called light. Human eye (retina) has the sensitivity to detect electromagnetic wave of this range. Speed of light in vacuum is $c = 2.99792458 \times 10^8\ ms^{-1} = 3 \times 10^8\ ms^{-1}$ which is highest speed attainable in nature.

Sir **Isaac Newton**, held the theory that light was made up of tiny particles. In **1678, Dutch Physicist,** Christian Huygens, believed that light was made up of waves vibrating up and down perpendicular to the direction of propagation of the light, and therefore formulated a way of visualising wave propagation. This is known as **"Huygens' Principle".**

At the time, some of the experiments conducted on light theory, both the Wave theory and Particle theory, had some unexplained phenomena. Newton could not explain the phenomenon of light interference, In **1803, Thomas Young** studied the interference of light waves by shining light through a screen with two slits equally separated, the light emerging from the two slits, spread out according to Huygen's principle. Later in **1815, Augustin Fresnel** supported Young's experiments with mathematical calculations. In **1900, Max Planck** proposed the existence of a light quantum, a finite packet of energy which depends on the frequency.

In **1905, Albert Einstein** had proposed a solution to the problem of observations made on the behaviour of light having characteristics of both Wave and Particle theory. From work of Planck on emission of light from hot bodies, Einstein suggested that light is composed of tiny particles called photons, and each photon has energy.

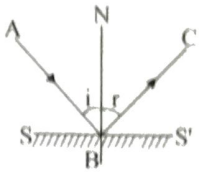
We can say that light has dual nature in some situation it behaves like wave and in some situation it behaves like particle.

REFLECTION

When the light returns to same medium after touching a boundary is called reflection.

Laws of Reflection

[1]The incident ray AB, the reflected ray BC and normal (BN) to the surface [SS'] of reflection at the point of incident lies in the same plane.

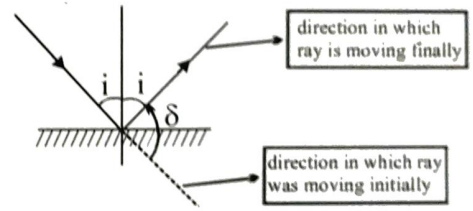


[2] The angle of incidence [the angle between normal and incident ray] and the angle of reflection [the angle between the reflected ray and normal] are equal

$\angle i = \angle r$

Deviation by Plane Mirror

A Plane mirror deviates the light through an angle $\delta = 180° - 2i$ i is angle of incidence



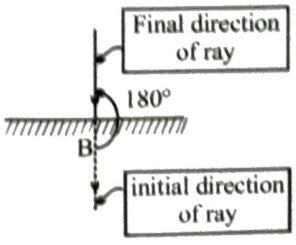
$$\boxed{\delta = 180° - 2i}$$

in anti clockwise direction.

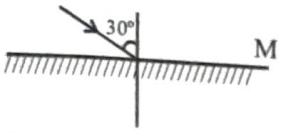
(*a*) $\delta_{min} = 0$ for $i = 90°$



(*b*) $\delta_{max} = 180°$ for $i = 0$



Example 1: Find the angle of deviation (both clockwise and anticlockwise) suffered by a ray incident on a plane mirror, at an angle of incidence 30°.



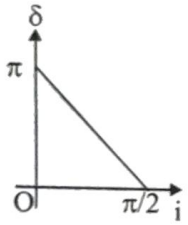
Sol. $d = 180° - 2i$

$= 180° - 60° = 120°$ anticlockwise

$\delta = 240°$ clockwise

Graphical variation between Angle of incidence and deviation :

1. Slope is negative
2. As i increase deviation decreases.

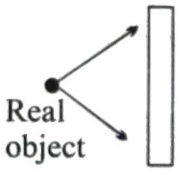


OBJECT & IMAGE

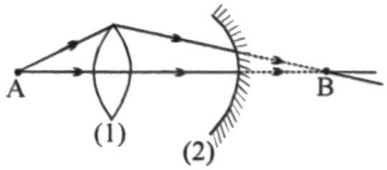
OBJECT : Point of intersection of incident rays is called object.

Object is defined for incident ray :

Real Object: If the incident ray diverges from a point and fall on a optical element it is called real object for that optical element.



Virtual Object: If there is converging set of rays and it is interrupted by an optical element then the point of convergence is the point of virtual object behind the optical element.

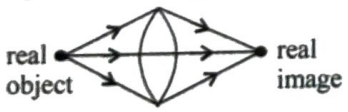


(1) A is real object for lens

(2) B is virtual object for mirror.

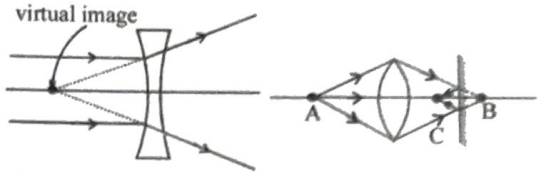
Image : The light ray after reflection or refraction meet or appear to meet at a point that point is called image.

Real Image : If the refracted or reflected rays are converging then point of convergence is called real image.



Note : Thus the real image can be taken on the screen.

Virtual image : If the refracted or reflected rays diverges then the point from where they appear to come is called virtual image.



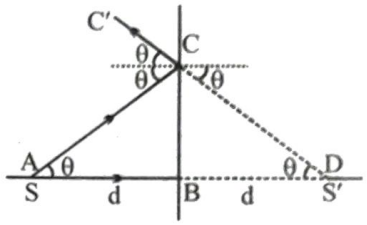
A → object for lens - real

B → image for lens - real

B → object for mirror - virtual

C → image for mirror - real image

Image formation by a plane mirror :



To trace image of S, take minimum two rays

(1) AB which is reflected normally.

(2) Ray AC which is reflected as CC'. The two reflected ray, ray does not meet so extended backward to form image at S'.

ΔABC and ΔBCD are Congruent

so $BS' = d$.

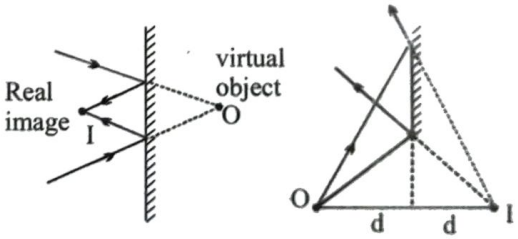
from this we can conclude that

(1) distance of object from mirror = distance of image from the mirror.

(2) the line joining a point object and its image is normal to the reflecting surface.

(3) the size of image is same as that of the object.

(4) for a real object the image is virtual and for a virtual object the image is real.



It is not necessary for a image formation that object must be infront of mirror.

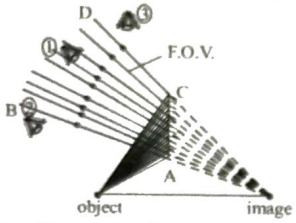
- In this case to form an image we have to extend the mirror and image is formed behind the mirror at same distance as object. Object and image lie on the line normal to extension of mirror.

FIELD OF VIEW

Formation of image and viewing of image are two different things. it is not necessary that every formed image can be viewed.

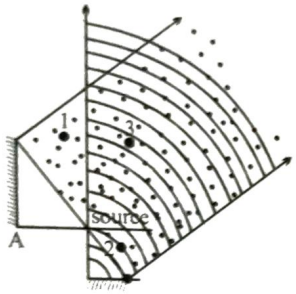
Field of view : the image can only be viewed when the eye lies in the field of the view.

The incident rays after reflection are only between AB and CD. Image can be seen in this area only.



Only Eye 1 will see the Image.

Draw the field of view for source for each mirror :



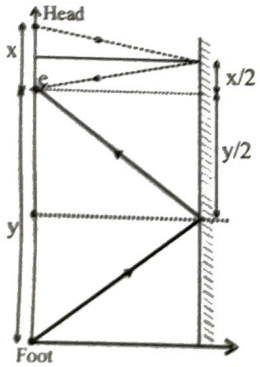
(1) at point 1 only image formed by mirror A can be seen

(2) at point 2 only image formed by mirror B can be seen

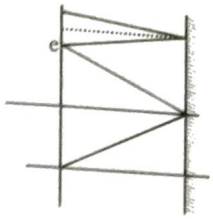
(3) at point 3 image formed by mirror A & B can be seen

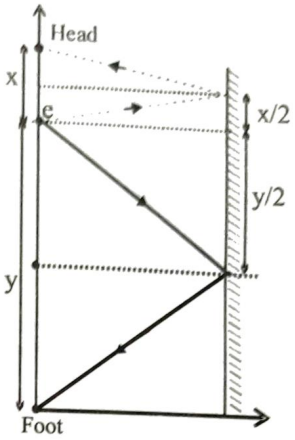
Minimum size of plane mirror Required to view the full image :

Method-1 : So, from the ray diagram we can see that minimum length required will be equal to $\frac{x+y}{2}$ or half the height of man.



Method-2 : *If ray is coming from head and goining to the eye, then ray from eye must go to head and if foot is seen then ray from eye must go to foot. So the whole body must lie in the field of view of eye.*

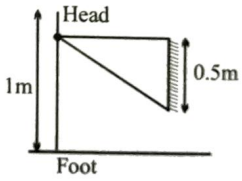


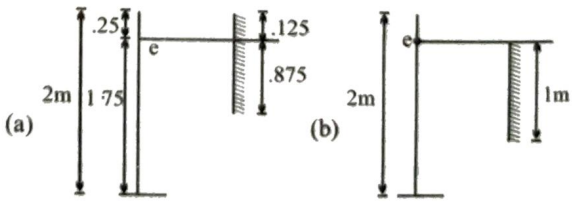


Note :

It is not necessary that if the size of mirror is half the size of man then whole man can be seen. It must be kept in a proper position half the size from eye to head of mirror must lie above the eye and half the size from eye to foot of mirror must lie below the eye.

In this example in spite of size of mirror is half the size of man full size of man cannot be seen by himself.

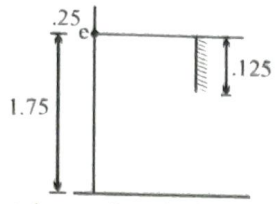




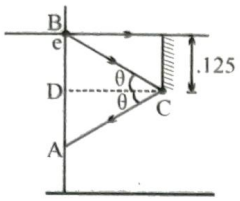
In case (a) whole image of man can be seen but in case (b) inspite of size of mirror being half of size of man whole image cannot be seen.

TRAIN YOUR BRAIN

Example 2: Find the length of man than can be seen by man.



Sol. Don't try to take ray from body to eye it will be tough to solve the problem in that way.



Draw the field of view of eye

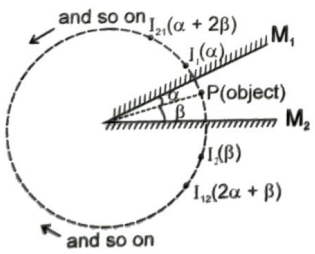
part AB of the body is in the field of view of eye. So part AB can view eye. So eye can view part AB from reversibility of light ΔACD and ΔDCB is congruent so AD = DB = .125.

So length of man seen = .25 m.

Locating all the images formed by two plane mirrors

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha+\beta$ as shown in figure.

Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1 ,denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly image of object P formed by M_2 ,denoted by I_2 ,will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.



Now I_2 will act as an object for M_1 which is at an angle $(\alpha+2\beta)$ from M_1 . Its image will be formed at an angle $(\alpha+2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21}, and so on. Think when this process will stop. Hint: The virtual image formed by a plane mirror must not bein front of the mirror or its extension.

Number of images formed by two inclined mirrors

(i) If $\frac{360^\circ}{\theta}$ = even number;

number of image = $\frac{360^\circ}{\theta} - 1$

(ii) If $\frac{360^\circ}{\theta}$ = odd number;

number of image = $\frac{360^\circ}{\theta} - 1$,

if the object is placed on the angle bisector.

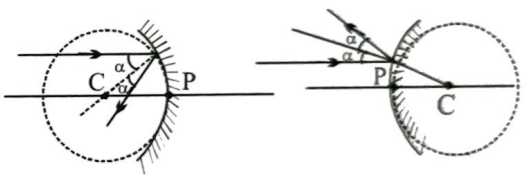
(iii) If $\frac{360^\circ}{\theta}$ = odd number;

number of image = $\frac{360^\circ}{\theta}$,

if the object is not placed on the angle bisector.

(iv) If $\frac{360^\circ}{\theta} \neq$ integer,then count the number of images as explained above.

REFLECTION FROM CURVED SURFACE



1. **Centre of curvature [C]**

   It is the centre of sphere of which mirror is a part.

2. **Radius of curvature [R]**

   It is the radius of the sphere of which mirror is part.

3. **Pole [P]**

   It is the geometrical centre of spherical reflecting surface

4. **Principal axis**

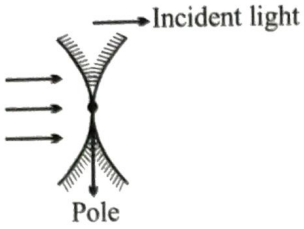
   It is the straight line joining the centre of curvature to pole.

5. **Focus [F]**

   When a narrow beam of light rays, parallel to the principal axis and close to it is incident on surface of mirror the reflected beam is found to converge or appear to diverge from a point on the principle axis the point is called the focus for spherical mirror f = R/2.

6. **APERTURE :** The size of mirror is called aperture

SIGN CONVENSION



(*a*) all the distances are measured from pole.

(*b*) distance measured in the direction of incident rays are taken as positive.

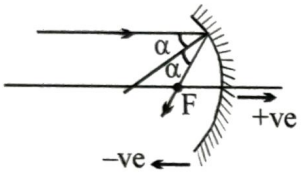
(*c*) distance measured in direction opposite to that of the incident rays are taken as negative

(*d*) distances above the principle axis are taken as positive

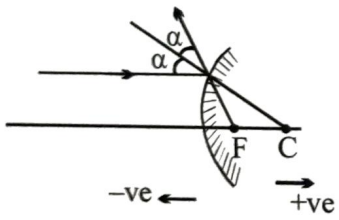
(*e*) distances below the principle axis are taken as negative.

(*f*) angles measured from the normal in the anticlockwise sense are positive while that in the clockwise sense are negative.

Concave mirror focal length is negative

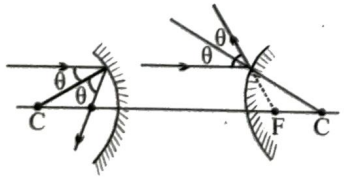


Convex mirror focal length is positive

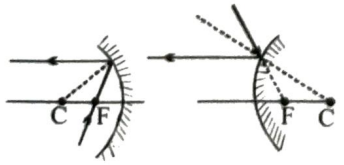


Ray Tracing in a Spherical Mirror

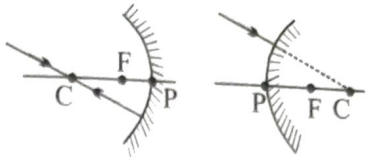
(*a*) Ray parallel to principle axis and closer to it (called paraxial rays) passes or appear to pass through focus after reflection.



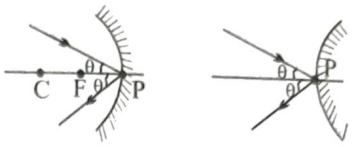
(*b*) A ray of light passing through focus or appears to pass through focus becomes parallel to principle axis after reflection.



(*c*) A ray of light passing through or appear to pass through centre of curvature is reflected back as it is perpendicular to mirror.



(*d*) A ray of light hitting pole is reflected making equal angle with principle axis.



Relation between focal length and radius of curvature

Take a ray AB parallel to principle axis. The ray will follow the rule of reflection i.e. angle of incident = angle of reflection and will pass through point P. Line joining the centre of curvature to the point B will be normal to the mirror at point B. Drawing a line PD perpendicular to line BC. In triangle BCP.

$\angle PBC = i$ (law of reflection)

$\angle BCP = i$ (as AB is parallel to CP)

So triangle BCP will be isosceles and PD will bisect BC

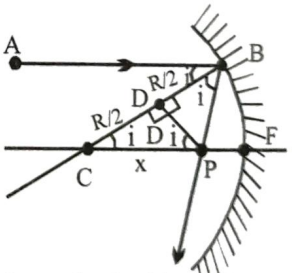
$CD = DB = \frac{R}{2}$, as $BC = R$

Taking CP as x

$x \cos i = R/2$

$x = \frac{R}{2\cos i}$

$PF = R - \frac{R}{2\cos i}$



$PF = R - \frac{R}{2}\sec i$ (This is not the focal length.)

$i \to 0$ for paraxial rays. [Paraxial rays are very near to principle axis]

$f = R - \frac{R}{2} = \frac{R}{2}$



1. So rays parallel to principle axis after reflection do not meet at a single point.
2. The incident ray which parallel and for from principle axis after reflection do not meet at focus.
3. Only the paraxial rays meet at focus.
4. ***x* is Positive, hence object is virtual.**
   ***x* is Negative, hence object is real.**

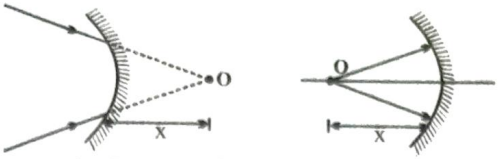
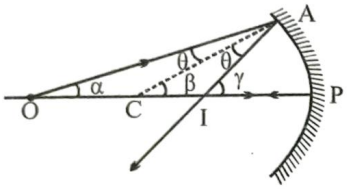


Image negative hence real. Image positive hence virtual



RELATION BETWEEN u, v AND R FOR SPHERICAL MIRRORS

Consider the situation shown in figure. A point object is placed at the point O of the principal axis of a concave mirror. A ray OA is incident on the mirror at A. It is reflected in the direction AI. Another ray OP travels along the principal axis. As PO is normal to the mirror at P, the ray is reflected back along PO. The reflected rays PO and AI intersect at I where the image is formed.



Let C be the centre of curvature. The line CA is the normal at A. Thus, by the laws of reflection, $\angle OAC = \angle CAI$. Let α, β, γ and θ denote the angles AOP, ACP, AIP and OAC respectively. As the exterior angle in a triangle equals the sum of the two opposite interior angles, we have

from triangle OAC $\beta = \alpha + \theta$ ....(i)

and from triangle OAI $\gamma = \alpha + 2\theta$ ....(ii)

Eliminating θ from (i) and (ii), $2\beta = \alpha + \gamma$ ...(iii)

If the point A is close to P, the angles α, β and γ are small and we can write

$$\alpha \approx \frac{AP}{PO}, \ \beta = \frac{AP}{PC} \text{ and } \gamma = \frac{AP}{PI}.$$

As C is the centre of curvature, the equation for β is exact whereas the remaining two are approximate. Putting in (iii),

$$2\frac{AP}{PC} = \frac{AP}{PO} + \frac{AP}{PI} \text{ or, } \frac{1}{PO} + \frac{1}{PI} = \frac{2}{PC}.$$

The pole P is taken as the origin and the principal axis as the X-axis. The rays are incident from left to right. We take the direction from left to right as the positive X-positive. The point O, I and C are situated to the left of the origin P in the figure. The quantities u, v and R are, therefore, negative. As the distances PO, PI and PC are positive, PO = – u, PI = – v and PC = – R. Putting in (iv),

$$\frac{1}{-u} + \frac{1}{-v} = \frac{2}{-R} \quad \text{or} \quad \frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

Although equation is derived for a special situation shown in figure, it is also valid in all other situations with a spherical mirror. This is because we have taken proper care of the signs of u, v and R appearing in figure.

Relation between the Focal length and the radius of Curvature

If the object O in figure is taken at a large distance, the rays coming from O and incident on the mirror become almost parallel. The image is then formed close to the focus. Thus, if $u = \infty$, $v = f$. Putting in,

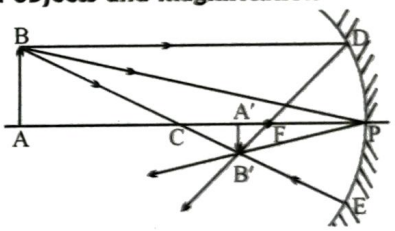
$$\frac{1}{\infty} + \frac{1}{f} = \frac{2}{R} \quad \text{or,} \quad f = \frac{R}{2}$$

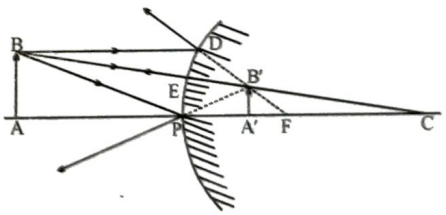
Equation may also be written as

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

The focus is midway between the pole and the centre of curvature.

Extended objects and magnification





Suppose an object AB is placed on the principal axis of a spherical mirror with the length AB perpendicular to the principal axis (figure). Consider two rays BD and BE, the first parallel to the principal axis and the other directed towards the centre of curvature. The ray BD will go through the focus F after reflection. The ray BE will return along EB as it hits the mirror normally. The image of B is formed at the intersection of these two reflected rays. Thus, the image B' of the point B is traced. If we drop a perpendicular B'A' on the principal axis, it can be shown that A' is the image of A and A'B' is the image of AB. Figures (a) and (b) show the construction in two different situations.

Lateral Magnification

The ratio $\frac{\text{height of the image}}{\text{height of the objective}}$ is called lateral or transverse magnification. The height of the object (placed perpendicular to the principal axis) is taken to be positive. If the image is also on the same side of the principal axis, its height is also positive. The image is then erect. Figure shows an example. If the image is inverted, its height is taken as negative.

Consider the ray BP in figure hitting the mirror at the pole P. The reflected ray passes through B' as B' is the image of B. The principal axis is the normal at P to the mirror. By the laws of reflection,

$\angle BPA = \angle A'PB'$

Thus, the right-angled triangles ABP and A'B'P are similar. Thus,

$$\frac{A'B'}{AB} = \frac{PA'}{PA} \quad \text{.....(i)}$$

In figure; $A'B' = -h_2$, $AB = h_1$, $PA' = -u$ and $PA = -v$. Equation (i) gives

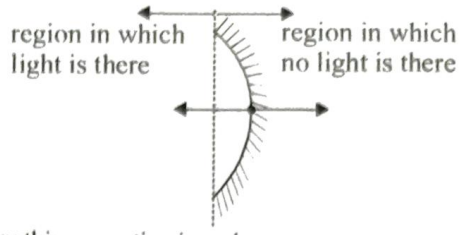
$$\frac{-h_2}{h_1} = \frac{-v}{-u} \quad \text{or,} m = \frac{h_2}{h_1} = -\frac{v}{u}.$$

Since proper signs are used, this same reflected is also valid in all other situations. For example. in figure, $A'B' = +h_2$, $AB = +h_1$. $PA' = +v$ and $PA = -u$.

Equation (i) gives

$$\frac{h_2}{h_1} = \frac{v}{-u} \quad \text{or,} \quad m = -\frac{v}{u}.$$

Position sizes and nature of images formed by spherical mirror



(i) anything negative is real

(ii) any thing positive is virtual

$$m = \frac{h_2}{h_1} = \frac{-v}{u}; h_2 = -\left(\frac{v}{u}\right)h_1$$

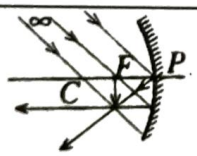
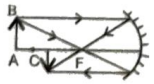
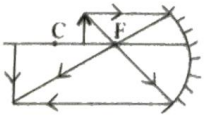
(*a*) If v and u have same sign, h_2 will have opposite sign of h_1, so image will be inverted w.r.t object.

(*b*) If v and u have opposite sign h_2 will have same sign as h_1 so image will be erect with respect to object.

(*a'*) if $|v|>|u|$, $|h_2|>h_1$ so image will be magnified.

(*b'*) if $|v|<|u|$, $|h_2|<|h_1|$ so image will be diminished.

IMAGE FORMED BY THE CONCAVE MIRROR

S.No.	Position of object	Ray diagram	Position of image	Nature of image	Size of image
1.	At infinity		at focus	real and inverted	very small
2.	Between infinity and centre of curvature		between focus and centre of curvature	real and inverted	small
3.	At centre of curvature		at centre of curvature	real and inverted	equal to
4.	Between focus and centre of curvature		between centre of curvature and infinity	real and inverted	enlarged

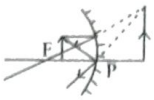
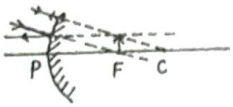
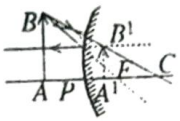
5.	At focus		at infinity	real and inverted	very large
6.	Between pole and focus		between poles and focus	virtual and erect	enlarged

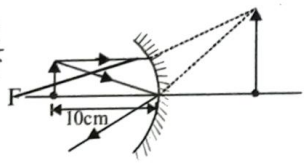
IMAGE FORMED BY THE CONVEX MIRROR

1.	At infinity		at focus	virtual and erect	very small
2.	Between pole and infinity		between focus & pole	virtual and erect	small

TRAIN YOUR BRAIN

Example 3: An object 1 cm high is placed at 10 cm in front of a concave mirror of focal length 15 cm. Find the position, height and nature of image. Also find magnification.

Sol. $u = -10$ cm $\quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$



$$v = \frac{1}{v}-\frac{1}{10}=\frac{-1}{15}$$

$f = -15$ cm $\qquad \frac{1}{v}=\frac{1}{10}-\frac{1}{15}$

$v \Rightarrow 30$ cm $\qquad m=\frac{-1}{v}=\frac{-30}{-10}=3$

[a] image is virtual $\qquad$ [b] magnified

Example 4: The image of a real object in a convex mirror is 4 cm from mirror. If the mirror has a radius of curvature 24 cm. find the position of object and magnification.

Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{4}+\frac{1}{u}=\frac{1}{12}$$

$u = -6$ cm

$$m=\frac{-v}{u}=-\frac{4}{-6}=+\frac{2}{3}$$

Example 5: when an object is placed at a distance of 25 cm from a mirror, the magnification is m_1. The object is moved 15 cm farther away with respect to the earlier position and the magnification becomes m_2. If $\frac{m_1}{m_2}=4$, then calculate focal length of mirror and type of mirror.

Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}-\frac{1}{40}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{40}+\frac{1}{f}$$

$$\frac{1}{v}=\frac{40+f}{40f}$$

$$v=\frac{40f}{40+f}$$

$$m_2=\frac{-v}{-40}=\frac{f}{40+f}$$

$$m_1=\frac{f}{25+f}$$

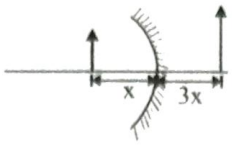
$$\frac{m_1}{m_2}=\frac{f+40}{f+25}=4$$

$f = -20$ cm

Negative sign shows that mirror is concave.

Example 6: A concave mirror has a focal length of 36 cm. At what position should an object be placed for its image to be erect and magnified by a factor of three.

Sol. $f = -36$ cm



$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$-\frac{1}{x}+\frac{1}{3x}=\frac{1}{f}$$

$$\frac{1}{x}\left[-1+\frac{1}{3}\right]=\frac{1}{f}$$

$$\frac{1}{x}\left[\frac{-3+1}{3}\right]=\frac{1}{-36}$$

$$x=\frac{+36\times 2}{3}=+24 \text{ cm}$$

Example 7: An object is 60 cm from concave mirror produces a real image 15 cm from mirror. What is radius of curvature of mirror.

Sol. $u = -60$ cm

$v = -15$ cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow f=-12 \text{ cm}$$

$\therefore$ $R = 24$ cm

Example 8: An object of height 0.5 cm is placed 18 cm from a curved mirror. The erect image is 2 cm high. What is focal length of mirror ?

Sol. erect and magnified only when mirror is concave

$u = -18$ cm

$$m=\frac{2}{0.5}=\frac{-v}{-18}$$

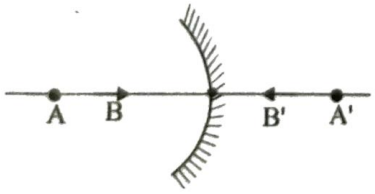
$v = 72$ cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{72}-\frac{1}{18}=\frac{1}{f}$$

$f = -24$ cm

Longitudinal Magnification



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{-1}{v^2}dv+\frac{1}{u^2}du=0$$

$$\frac{-dv}{v^2}=\frac{du}{u^2}$$

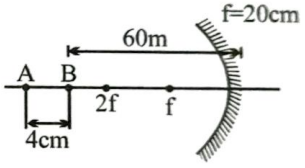
$$\frac{dv}{du}=-\left(\frac{v^2}{u^2}\right)=-m^2$$

So, longitudinal magnification is square of lateral magnification.

Mathematically du implies small change in position of object dv implies corresponding small change in position of image. If a small object lies along principal axis du may indicate the size of its image along principle axis. In this case $\frac{dv}{du}$ is called longitudinal magnification . Negative sign indicates inversion of image irrespective of nature of image of mirror.

TRAIN YOUR BRAIN

Example 9: Find length of image formed if object is shown as AB.



Sol. As object size is equal

$u = -60$, $f = -20$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}-\frac{1}{60}=-\frac{1}{20}=\frac{1-3}{60}=\frac{-20}{60}=\frac{-1}{30}$$

$v = -30$ cm

$$\frac{\ell}{4}=-m^2=-\left(\frac{1}{2}\right)^2$$

$$\ell=\frac{4}{4}=1 \text{ cm}.$$

REFRACTION

When light rays goes from one medium to other there is change in speed of light due to which there is bending of light.

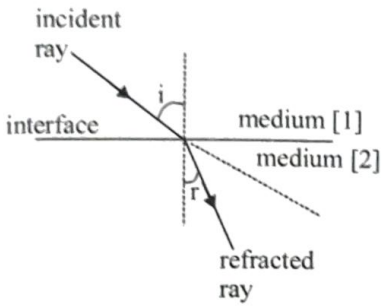
When light goes from rarer to denser it bends towards the normal. when goes from denser to rarer the rays bend away from the normal

(*i*) when light travels from one medium to other frequency remains constant because frequency is characteristic of source of light. Let v_1 and v_2 be speed of light in medium [1] and medium [2] respectively. Further if λ_1 and λ_2 are wavelength of light respectively im medium [1] and [2] respectively

$$f = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

when light enters in a medium where its speed increases, it bends away from normal and medium is said to be optically rarer.

Refractive index [R. I.] :



Refractive index of a medium with respect to another medium is defined as ratio between sine of angle of refraction.

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \mu_{21}$$

Refractive index of medium [2] with respect to medium [1]

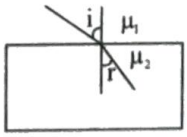
[a] Absolute R.I. : This is refractive index of any medium with respect to air or vacuum

$$\mu = \frac{\text{speed of light in air}}{\text{speed of light in medium}} = \frac{c}{v}$$

[b] Relative R.I.:

Relative refractive index of two media is equal to the ratio of their absolute refractive index

$$\mu_2 = \frac{\mu_2}{\mu_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2}$$



$\mu_1 \sin i = \mu_2 \sin r$

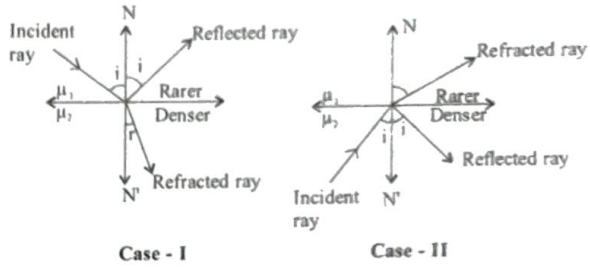
$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$$

When a light travels from one transparent medium to other, it deviates from its original path. The phenomenon of change in path of light at the medium of separation as it goes from one medium to other medium is called as refraction.

Laws of Refraction :

1. The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.
2. **Snell's Law** : The bending of light at the medium of separation follows Snell's law. In the figure if NN' represents normal at the medium of separation and 'i' and 'r' are the angles of incidence and refraction respectively then for pair of media and for a given colour

$$\frac{\sin i}{\sin r} = {}_1\mu_2 = \text{Constant.}$$



$\Rightarrow \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$; ${}_1\mu_2$ is called as refractive index of second medium w.r.t. first medium.

μ_1 and μ_2 are the absolute refractive indices of rarer and denser medium.

Hence, in the first case $\mu_1 \sin i = \mu_2 \sin r$ and in the second case $\mu_2 \sin i = \mu_1 \sin r$ are applicable. When light ray travels from rarer to denser media it bends towards normal and when it travels from denser to rarer media it bends away from normal.

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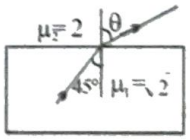
Example 10: A ray is incident at angle 45° in medium of refractive index $\mu_1 = \sqrt{2}$. Find angle of emergence θ.

Sol. Applying Snell's law

$\mu_1 \sin 45° = \mu_2 \sin \theta$



$\frac{\mu}{\sqrt{2}} = \mu_2 \sin \theta$

$1 = 2 \sin \theta$

$\sin \theta = \frac{1}{2}$

$\theta = 30°$.

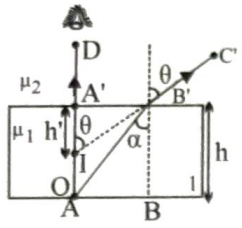
Refraction at plane surface and apparent shift

Object is kept in the medium 1 having refractive index μ_1 object O is kept at a depth h and it is seen from medium 2 having refractive index μ_2 as shown in diagram.

For image formation take a ray AB' at an incident angle α the ray emerges at an refracted angle θ. Another ray is taken as AA' normally it emerges undeviated.

These two refracted rays B'C' and A'D are produced backward to produce the image at I.

Applying Snell's law at B



$\mu_1 \sin \alpha = \mu_2 \sin \theta$

If normal incidence is there i.e. if O the object is viewed normally angle of incidence will be small and $\sin \theta \approx \theta \approx \tan \theta$

$\sin \alpha \approx \alpha \approx \tan \alpha$

$\mu_1 \tan \alpha = \mu_2 \tan \theta$

In $\Delta AB'B$, $\tan \alpha = \dfrac{AB}{h}$

In $\Delta IA'B'$, $\tan \theta = \dfrac{A'B'}{h'}$

$$\mu_1 \frac{AB}{h} = \mu_2 \frac{A'B'}{h'}$$

$AB = A'B'$

$$h' = \frac{\mu_2}{\mu_1} h$$

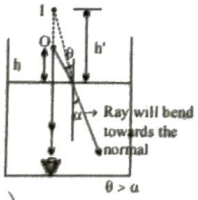
$$\text{shifting } \Delta h = h - h' = h\left[1 - \frac{\mu_2}{\mu_1}\right]$$

μ_2 is refractive index in which ray is going after the refraction and μ_1 is refractive index of medium is which ray was before the refraction.

h and h' are distances from the refractive surface and is independent of distance of eye from refracting surface.

So to apply this formulae

1. Take a ray from object to eye.
2. $$\frac{\text{Image distance from refracting surface}}{\text{Object distance from refracting surface}} = \frac{\mu \text{ of medium in which ray was finally}}{\mu \text{ of medium in which ray is initially}}$$
3. $h' = \dfrac{\mu_2}{\mu_1} h$



$\mu_2 > \mu_1$

$h' > h$

$$\Delta h = h - h' = \left(1 - \frac{\mu_2}{\mu_1}\right) h$$

μ_2 is refractive index of observer and μ_1 is refractive index of object.

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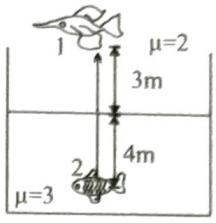
Example 11: Find the distance of 1 as seen by 2 and distance of 2 as seen by 1



Sol.(*a*) distance of 1 as seen by 2

Bird is object so take a ray from bird to eye of fish.

Ray is initially in medium of $\mu_1 = 2$ and finally in medium of refractive index $\mu_2 = 3$

$$\frac{h'}{h} = \frac{\mu_2}{\mu_1}$$

$$\frac{h'}{3} = \frac{3}{2}$$

$$h' = \frac{9}{2} = 4.5$$

This is the distance of bird from refracting surface as seen by fish which is independent of distance of fish from refractive surface.

$d = 4.5 + 4 = 8.5$ cm

(b) Object is fish so take a ray from fish to bird

$h' = 4 \times 2/3 = 8/3$

$$d = 3 + \frac{8}{3} = \frac{17}{3} \text{ cm}$$

APPARENT SHIFT DUE TO PARALLEL FACED SLAB

An object O is kept on left side of glass slab ABCD at a distance x from left side of AB.

Prove that total shift is independent of x.

Distance of image I_1 from first refracting surface

$x' = \dfrac{\mu_s}{\mu_m} x$ distance of I_1 from second refractive $(t + x')$

I_1 is object for second refracting surface and I_2 is image for second refracting surface

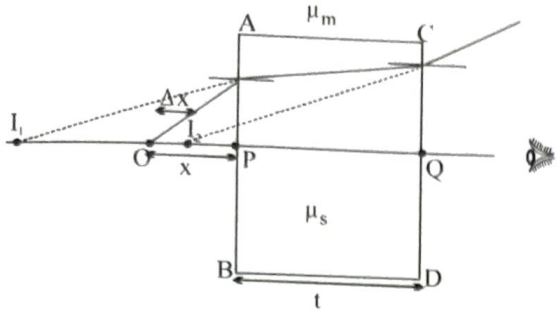
Distance of image from second refracting surface

$$QI_2 = \frac{\mu_M}{\mu_S}\left[t + \frac{\mu_S}{\mu_M} x\right] = \frac{\mu_M}{\mu_S} \times t + x$$

distance of object from second refracting surface

$= t + x$

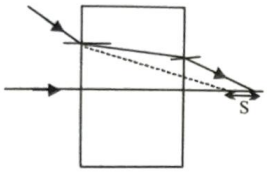
$$\text{shift} = t + x - \frac{\mu_M}{\mu_S} \times t - x = t\left[1 - \frac{\mu_M}{\mu_S}\right]$$



[a] **If it is positive, shifting is in the direction of incident ray and**

[b] **If it is negative, shifting will be opposite to incident rays.**

If object is virtual then also this is valid.



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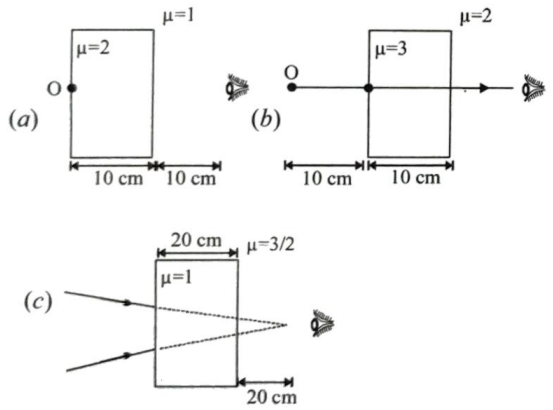
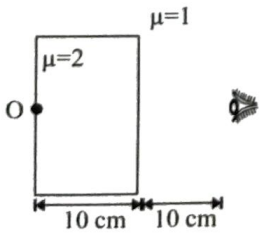
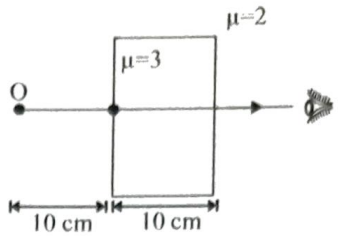
Example 12: Find the shifting of image in following cases



Sol. (a) $\Delta x = 10\left[1-\frac{\mu_m}{\mu_s}\right]=10\left[1-\frac{1}{2}\right]=5$ cm

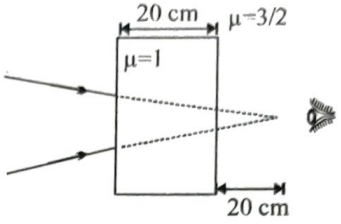


(b) $\Delta x = 10\left[1-\frac{\mu_m}{\mu_s}\right]=10\left[1-\frac{2}{3}\right]=\frac{10}{3}$ cm



(c) Shifting is in the direction of incident ray

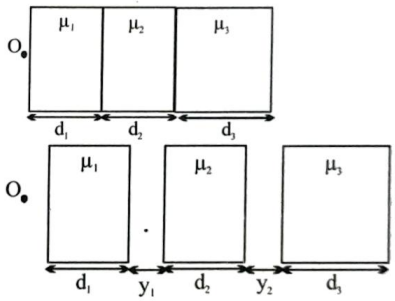
$$\Delta x = 20\left[1-\frac{3}{2\times 1}\right]=20\times\frac{-1}{2}=-10 \text{ cm}$$



Refraction from Multiple Slabs

Shift due to individual slab in a medium and shift due to composite slab is equal.

both the following systems are same, hence shifting will be same in both the cases.



Surrounding medium is same for both so

$$\Delta x = d_1\left(1-\frac{1}{\mu_1}\right)+d_2\left(1-\frac{1}{\mu_2}\right)+d_3\left(1-\frac{1}{\mu_3}\right)$$

We are considering independent shifting and adding them all.

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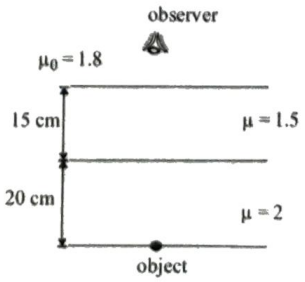
Example 13: Find the apparent depth as seen by observer. [x in this case is zero.]



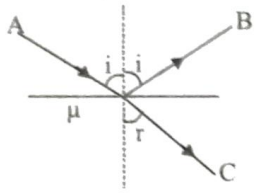
Sol. $\Delta x = 20\left(1-\frac{\mu_O}{2}\right)+15\left(1-\frac{\mu_O}{1.5}\right)$

$=20\left(1-\frac{1.8}{2}\right)+15\left(1-\frac{1.8}{1.5}\right)$

$=2-3=-1$ cm

Depth seen $= 35 + 1 = 36$ cm

Example 14: In the figure shown the refracted ray C and reflected ray B makes an angle 90° with each other, find incident angle i in terms of μ.



Sol. $90° - i + 90° - r = 90°$

$i + r = 90°$

$1\sin i = \mu \sin r$

$\sin i = \mu \sin (90° - i)$

$\tan i = \mu$

$i = \tan^{-1} \mu$

TOTAL INTERNAL REFLECTION (TIR)

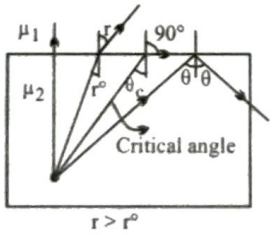
The phenomenon of total internal reflection occurs when light travels from a medium of high refractive index to a medium of lower refractive index. When the angle of incidence increases the percentage of ray getting reflected increases and refraction decreases at a particular angle called the critical angle (θ_c), the refracted ray just grazes the boundary between the two media.

using Snells law.

$\mu_2 \sin\theta_c = \mu_1 \sin 90°$

$\sin\theta_c = \frac{\mu_1}{\mu_2}$

$\theta_c = \sin^{-1}\frac{\mu_1}{\mu_2}$



for angle of incidence greater than (θ_c) the light is reflected back in the media of higher refractive index. This phenomenon is called total internal reflection.

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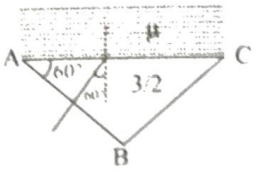
Example 15: What should be refractive index of liquid so that TIR does occur

Sol.



$\frac{3}{2}\sin 60° = \mu \sin 90°$

$\frac{3\sqrt{3}}{4} = \mu$ incident angle is 60°

$\mu < \frac{3\sqrt{3}}{4}$

Self Practice Problems

1. What will happen if angle is increased from 60° ?

Ans. If angle is increased from 60° angle of incidence becomes greater than critical angle so TIR occurs.

2. What will happen if μ is increased keeping incident angle 60°?

Ans. If μ is increased critical angle is increased so angle of incidence will be less than critical angle so ray will go in another media.

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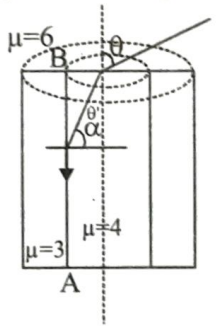
Example 16: Find angle θ so that TIR occurs at interface AB

Sol.



$4\sin\alpha = 3\sin 90°$

$\sin\alpha = \frac{3}{4}$

$\cos\alpha = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\sin\theta' = \cos\alpha = \frac{\sqrt{7}}{4}$

$6\sin\theta = 4\sin\theta'$

$6\sin\theta = \frac{4\sqrt{7}}{4}$

$\sin\theta = \frac{\sqrt{7}}{6}$

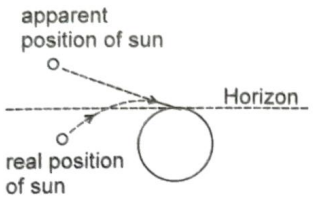
Self Practice Problems

3. What will happen if θ is increased ?

Ans. If θ is increased α decreases. It becomes less than critical angle and ray goes in outer cylinder.

Some interesting facts about light :

(1) The Sun Rises before it actually rises and sets after it actually sets : The atmosphere is less dense as its height increase, and it is also known that the index of refraction decrease with a decrease in density. So, there is a decrease of the index of refraction with height. Due to this the light rays bend as they move in the earth's atmosphere



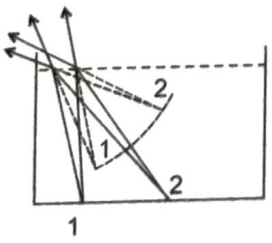
(2) The Sun Is Oval Shaped At The Time Of Its Rise And Set :
The rays diverging from the lower edge of the sun have to cover a greater thickness of air than the rays from the upper edge. Hence the former are refracted more than the latter, and so the vertical diameter of the sun appears to be a little shorter than the horizontal diameter which remains unchanged.

(3) The Stars Twinkle But Not The Planets :
The refractive index of atmosphere fluctuates by a small amount due to various reasons. This causes slight variation in bending of light due to which the apparent position of star also changes, producing the effect of twinkling.

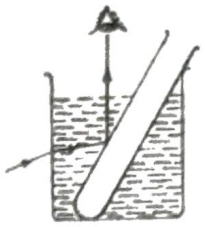
(4) Glass Is Transparent, But Its Powder Is White:
When powdered, light is reflected from the surface of innumerable small pieces of glass and so the powder appears white. Glass transmits most of the incident light and reflects very little hence it appears transparent.

(5) Greased Or Oiled Paper is Transparent, But Paper is White : The rough surface of paper reflects incident light and so it appears white. When oiled or greased, very little reflection takes place and most of the light is allowed to pass and hence it appears transparent.

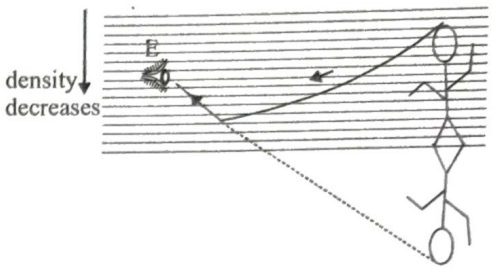
(6) An Extended Water Tank Appears Shallow At The Far End :



(7) A Test Tube Or A Smoked Ball Immersed In Water appears Silvery White When Viewed From The Top :
This is due to Total internal reflection



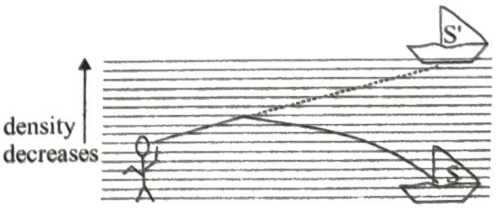
Mirage



It is an optical illusion created due to phenomenon of total internal reflection. This is seen in hot region. In hot area like deserts surface of earth is very hot, so air in lower region of atmosphere is hot as compared to that of in higher region. This result in variation of density with height and it increases as we go up. In this situation atmosphere can be assumed to be made up of large number of thin layers of air.

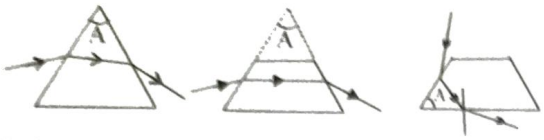
A beam of light starting from an object say tree and travelling downward find itself going from denser to rarer medium. Therefore its angle of incidence goes on increasing gradually till it surpasses the critical value and reflected back due to total internal reflection. A virtual image of object is seen. Due to disturbance of air the mirage is formed in nature. Thus giving an illusion for the presence of water which is actually not there.

Looming



This effect occurs when the density of air decreases more repidly with increasing height than it does under normal conditions. This situation sometimes happens in cold region particularly in the visnity of the cold surface of sea or of a lake. Light rays starting from 'S' are curved downward and on entering the eye the rays apper to apper to cone from S'. Thus giving the impression that the ship is floating in air. This effect is called superior mirage.

REFRACTION THROUGH PRISM



(*a*) **Refracting faces :** The plane surfaces which faces incident and emergent ray are called refracting faces.

(*b*) **Refracting angle :** The dihedral angle between two refracting faces is called refracting angle. This is also known as apex angle or prism angle.

(*c*) **Refracting edge :** The two refractive surface meet each other in a line called refracting edge.

(*d*) **Principle section :** The section of a prism by a plane perpendicular to the refracting edge is called principle section.

Note : The glass slab can be considered as limiting case of prism with prism angle zero.

Derivation for angle of deviation

$\delta = i - r_1 + e - r_2$

$= (i + e) - (r_1 + r_2)$

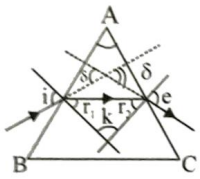
$= (i + e) - A$

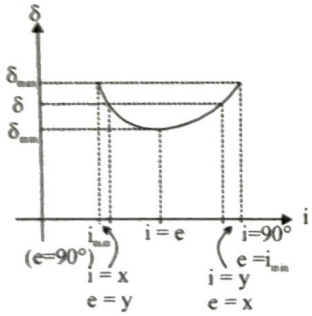
$r_1 + r_2 = A$

$\sin i = \mu \sin r_1$

$\mu \sin r_2 = \sin e$

$e = \sin^{-1}[\mu \sin (A - r_1)]$





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Example 17: A ray of light is incident at an angle of 60° on the face of prism having refracting angle 30°. The ray emerging out of the prism makes an angle 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges.

Sol. $i = 60°, A = 60°, \delta = 60°$

$\delta = i + i' - 30°$

$30° = 60° + i' - 30°$

$i' = 0°$

Example 18: The refracting angle of a glass prism is 30°. A ray is incident onto one of the faces perpendicular to it. Find the angle δ between the incident ray and the ray that laves the prism.

The refractive index of glass is $n = 1.5$, $\sin^{-1}\left(\frac{3}{4}\right) = 48.59°$

Sol. $\delta = (i + i') - A$

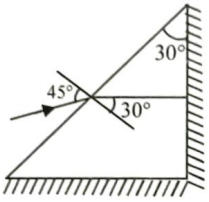
$\frac{3}{2}\sin 30° = 1 \sin i$

$\sin i = \frac{3}{4}\ i' = 48.59°$

$\delta = (48.59 - 30°) = 18.6°$

Example 19: One face of a prism with a refractive angle of 30° is coated with silver. A ray incident on another face at an angle of 45° is refracted and reflected from the silver coated face and ratraces its path .What is the refractive index of the prism?

Sol.



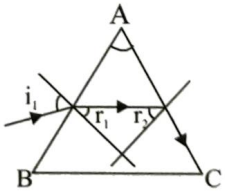
$1 \sin 45° = \mu \sin 30°$

$\frac{1}{\sqrt{2}} = \mu \times \frac{1}{2}$

$\mu = \sqrt{2}$

Condition of Grazing Emergence

$\sin i = \mu \sin r_1$



$\sin i = \mu \sin (A - r_2)$

$\sin i = \mu \sin A \cos r_2 - \mu \cos A \sin r_2$

$= \sqrt{1 - \mu^2} \sin A - \cos A$

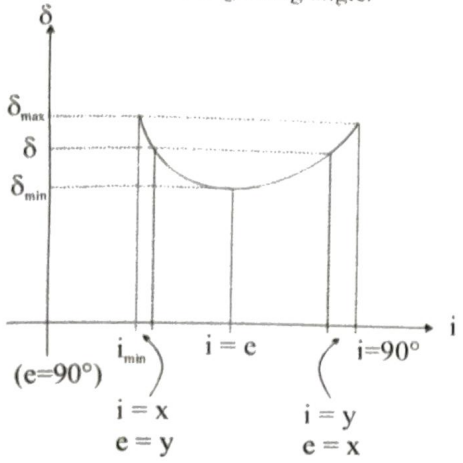
$i = \sin^{-1}\left[(1 - \mu)\sin A - \cos A\right]$

$\mu \sin r_2 = 1$

$\sin r_2 = \frac{1}{\mu}$

$r_2 = \frac{\sqrt{1 - \mu_2}}{\mu}$

The light will emerge only if the angle of incidence is greater than the condition of grazing angle.



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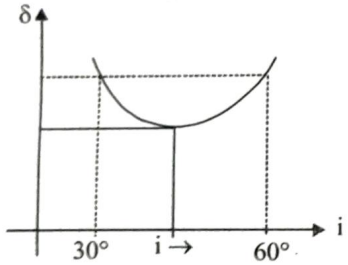
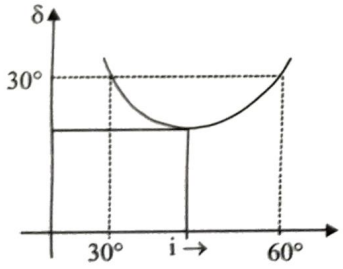
Example 20: From the graph of angle of deviation δ versus angle of incidence i, find the prism angle



Sol.



from the graph

$\delta = i + e - A$

$30° = 30° + 60° - A$

$\therefore$ $A = 60°$

Use the result : If i and e are interchanged then we get same value of δ

Condition for minimum deviation

From the graph we can see that at the angle of minimum deviation.

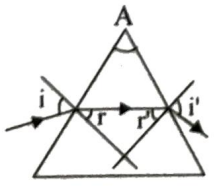
$i = i'$

so $r = r'$

$\delta_{min} = 2i - A$

$r = \frac{A}{2}$

$\sin i = \mu \sin r$



$\frac{\sin\frac{\delta_m + A}{2}}{} = \mu \sin r$

$\mu = \frac{\sin\frac{\delta_m + A}{2}}{\sin r} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\frac{A}{2}}$

In case of thin prism and small angle of incidence

$\mu = \frac{\sin\left(A + \frac{\delta_{min}}{2}\right)}{\sin\frac{A}{2}} \Rightarrow \mu = \frac{A + \delta_{min}}{A}$

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Example 21: A prism with apex angle A = 60° produces a minimum deviation of 30° . Find the refractive index of the material

Sol. $\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$

Example 22: The angle of minimum deviation for a glass prism with $\mu = \sqrt{3}$ equals the refracting angle of prism. What is angle of prism?

Sol. $\delta_m = A$

$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\mu = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$

$\sqrt{3} = 2\cos\frac{A}{2}$

$\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$

$\frac{A}{2} = 30° \Rightarrow A = 60°$

Example 23: The angle of minimum deviation for a glass prism with $n = \sqrt{3}$ equals the refracting angle of the prism. What is the angle of the prism ?

Sol. $\delta = 2i - 2r$

$A = 2i - 2r$

$2r = A$

$A = 2i - A$

$i = A$ $\qquad$ $r = \frac{A}{2}$

Applying Snells law at first surface 1 sini = μ sinr

$$\sin A = \sqrt{3}\sin\frac{A}{2}$$

$$2\sin\frac{A}{2}\cos\frac{A}{2} = \sqrt{3}\sin\frac{A}{2}$$

$$\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{A}{2} = 30°$$

$$A = 60°$$

Condition for no Emergence

Ray will not come out of the second surface if minimum value of r_2 is greater than the critical angle for two medium r_2 is minimum when i_1 is maximum.

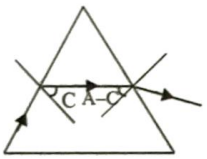
There is two angle of incidence for which there is same angle of deviation. maximum angle of incidence can be 90°.

For small angle and small angle of incidence.

$A - C > C$

$A > 2C$

$C < A/2$



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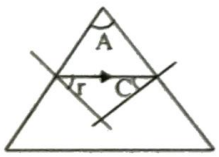
Example 24: A ray incident on the face of a prism is refracted and escaped through an adjacent face. What is the maximum permissible angle of the prism, if it is made of glass with a refractive index of μ =1.5 ?

Sol. $r < C$

$$\frac{A}{2} < C$$



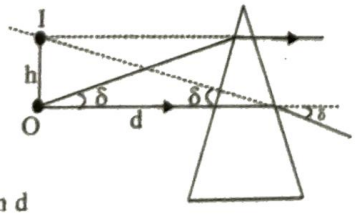
$A = 2r$

$A < 2C$

$$r = \frac{A}{2}$$

$$A < 2\sin^{-1}\frac{2}{3}; \qquad A < 83.62$$

Image formed by a prism

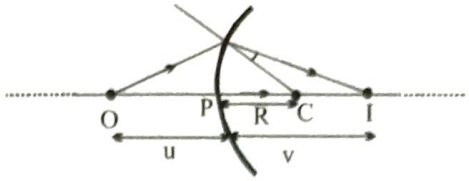


$$\tan\delta = \frac{h}{d}$$

if δ is small h = d tan d

$C < A/2$

REFRACTION ON CURVED SURFACE



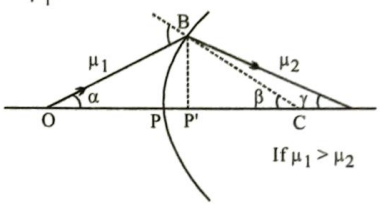
Center of curvature : Centre of the curved part is called centre of curvature of medium.

Principle Axis : The line joining the object and centre of curvature of curved part is called principle axis.

Pole : The point where the curved part cuts the principle axes called pole.

RELATION BETWEEN OBJECT AND IMAGE DISTANCE

Consider the point object O placed in a medium with refractive index equal to μ_1.



As $\mu_1 \sin i = \mu_2 \sin r$, and for small aperture i, $r \to \theta$

$\Rightarrow \mu_1 i = \mu_2 r \Rightarrow i = \alpha + \beta, \beta = \gamma + p$

$\Rightarrow \mu_1(\alpha + \beta) = \mu_2(\beta - \gamma) \Rightarrow \mu_1\alpha + \mu_2\gamma = (\mu_1 - \mu_2)\beta$

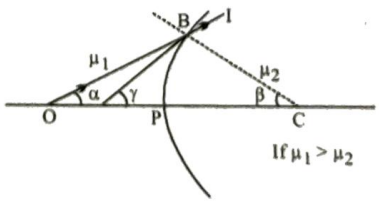
As aperture is small, $\alpha = \tan\alpha, \beta = \tan\beta, \gamma = \tan\gamma$

$\Rightarrow \mu_1 \tan\alpha + \mu_2 \tan\gamma = (\mu_2 - \mu_1)\tan\beta$

$$\Rightarrow \frac{\mu_1}{P'O} + \frac{\mu_2}{P'I} = \frac{\mu_2 - \mu_1}{P'C}$$

Applying sign convention i.e.,

u = P'O, v = P'I, R = P'C



$\therefore$ For a spherical surface, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

The symbols should be carefully remembered as : μ_2 = refractive index of the medium into which light rays are entering; μ_1 = refractive index of the medium from which light rays are coming. Care should also be taken while applying the sign convention to R.

Lateral Magnification for refracting Spherical Surface

Lateral magnification,

$$m = \frac{\text{Image height}}{\text{Object height}} = \frac{-(A'B')}{AB}$$

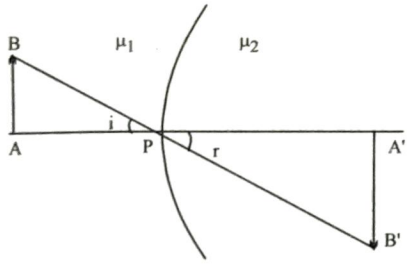
Now, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

For small angles of incidence and thus refraction, sin i = tan i and sin r = tan r

$\Rightarrow \frac{\tan i}{\tan r} = \frac{\mu_2}{\mu_1}$; in triangles ABP and A'B'P, we have

$$\frac{AB/PA}{A'B'/P'A'} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{A'B'}{AB} = \frac{-\mu_1 PA'}{\mu_2 PA} = \frac{-\mu_1 v}{\mu_2(-u)} = \frac{\mu_1 v}{\mu_2 u}$$



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Example 25: A spherical surface of radius R separates two medium of refractive indices μ_1 and μ_2, as shown in figure. where should an object be placed in the medium 1 so that a real image is formed in medium 2 at the same distance?

Sol. Let x be the distance of the object from the pole in medium 1.

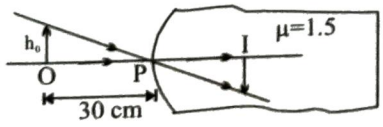
here we have , u = – x, v = + x and R = + R

Using the equation we get,

$$\frac{\mu_2}{+x} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{-R} \text{ or } x = \left(\frac{\mu_2 + \mu_1}{\mu_2 - \mu_1}\right) R$$

Note that the real image is formed only when $\mu_2 > \mu_1$

Example 26: A small object of height 0.5 cm is placed in front of a convex surface of glass (μ = 1.5) of radius of curvature 10 cm. Find the height of the image formed in glass.



Sol. According to Cartesian sign convention

u = – 30 cm, R = + 10 cm, μ_1 = 1, μ_2 = 1.5

Applying equation, we get

$$\frac{1.5-1}{v} - \frac{1}{-30} = \frac{1.5-1}{+10}$$

or v = 90 cm (real image)

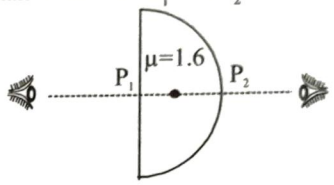
Let h_i be the height of the image, then

$$\frac{h_i}{h_0} = \frac{\mu_1 v}{\mu_2 u} = \frac{(1)(90)}{(1.5)(-30)} = -2$$

$h_i = -2h_0 = -2(0.5) = -1$ cm

The negative sign shows that the image is inverted.

Example 27: A plastic hemisphere has a radius of curvature of 8 cm and an index of refraction of 1.6 on the axis halfway between the plane surface and the spherical one is A small object O. The distance of the two images viewed from two sides.Find distance between P_1 and P_2



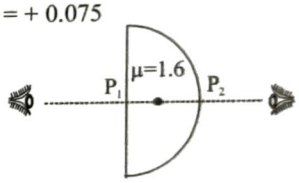
Sol. $I_1 = \frac{4}{1.6} = 2.5$ cm from P_1

when viewed from other sides

$$\frac{1}{v} + \frac{1.6}{4} = \frac{1-1.6}{-8} = -.075 = +0.075$$

$$\frac{1}{v} = -.475$$

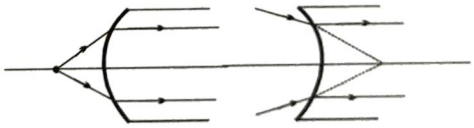
v = – 3.07 cm from P_2



Principal Focii

First Principle Focus :

It is the point at which an object must be kept so final image is formed at infinity.

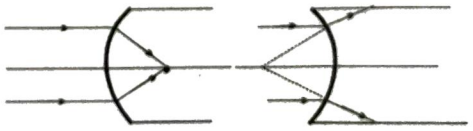


$$\frac{\mu_2}{\infty} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$u = -\frac{\mu_1 R}{\mu_2 - \mu_1}$$

Second Principle Focus :

Second principle focus of the surface is the point at which a beam coming parallel to principle axis or appears to meet after refraction.



$$\frac{\mu_2}{v} - \frac{1}{\infty} = \frac{\mu_2 - \mu_1}{R} \quad \frac{\mu_2}{v} - \frac{1}{\infty} = \frac{\mu_2 - \mu_1}{-R}$$

$$v = \frac{\mu_2 R}{\mu_2 - \mu_1} \qquad v = \frac{-\mu_2 R}{\mu_2 - \mu_1}$$

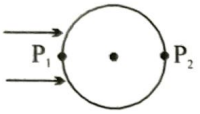
The distance between second focus and the pole is called second principle focus length.

For curved mirror first principle focus and second principle focus is same.

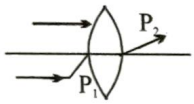
LENS

A lens is defined as portion of transparent refracting medium bounded by two surface in which one of the refracting surface should be curved.

THICK LENS : If the two poles of the refracting surface are separated by a significant distance it is called **thick lens**.

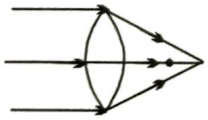


THIN LENS : If the two poles of the two refracting surface are separated by negligible distance it is called **thin lens**.

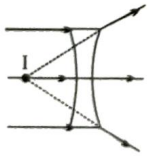


Thin lenses are of following **two types**

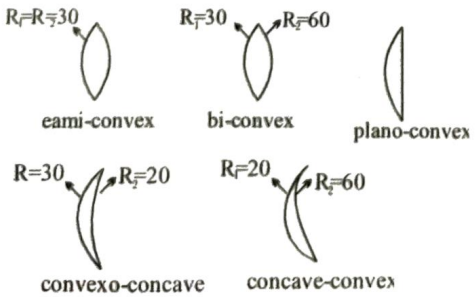
(i) Converging lens : A lens is said to be converging if the width of beam decreases after refraction through it.

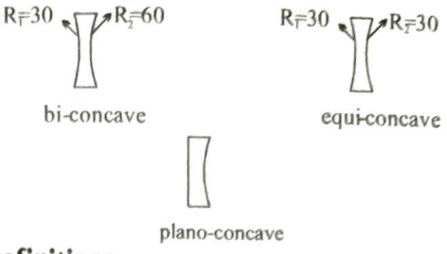


(ii) Diverging lens : A lens is said to be diverging lens if the width of the beam increases after refraction.



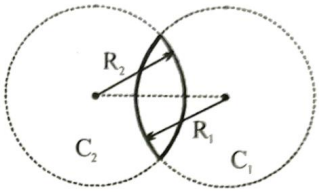
Naming a Lens : while naming lens large radius of curvature is named first





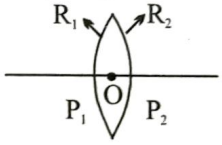
Basic Definitions

(i) Centre of curvature C_1 and C_2 : There are two centres of curvature for a lens, one each belonging to both the surfaces. Centre of curvatures of surface of a lens is defined as the centre of that sphere of which that surface forms a part.



(ii) Radius of curvature R_1 and R_2 : There are two radii of curvature of lens, one each belonging to both the surfaces. Radius of curvature of surface of lens is defined as the radius of sphere of which the surface is a part.

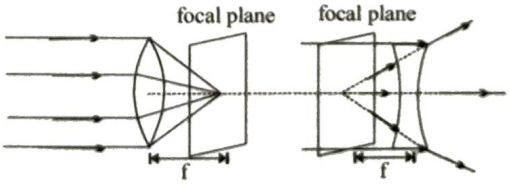
(iii) Optical centre (O) : It is the point laying on the principal axis of the lens within it or outside, such that a ray of light passing though it goes undeviated.



$$\frac{P_1O}{P_2O} = \frac{R_1}{R_2}$$

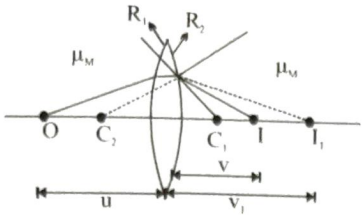
usually in thin lenses O is taken in middle of lens.

1. **Principal Axis :** A line joining the two centres of curvature and passing through the optical centre is called principal axis.
2. **Principal Focus :** Principal focus of a lens is a point at which a beam of light coming parallel to principal axis actually meets or appear to meet after refraction through the lens.



3. **Secondary focus :** It is the point of principle axis at which object is kept so that image is formed at infinity.
4. **Focal length (f) :** Focal length of a lens is defined as the distance between principal focus And its optical centre.
5. **Focal plane :** It is a plane passing through the principal focus and perpendicular to the principal axis.

LENS MAKER'S FORMULA FOR THIN LENS



For first surface : O is the object and I_1 is the image and for second surface I_1 is object and I is image.

$$\frac{\mu_L}{v_1}-\frac{\mu_M}{u}=\frac{\mu_L-\mu_M}{R_1}$$

For Second Surface

$$\frac{\mu_M}{v}-\frac{\mu_L}{v_1}=\frac{\mu_M-\mu_L}{R_2}$$

$$\mu_M\left[\frac{1}{v}-\frac{1}{u}\right]=(\mu_M-\mu_L)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_L}{\mu_M}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

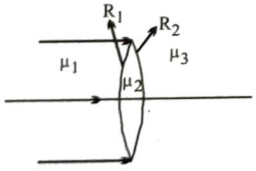
For $u\to\infty$ $v=f$

$$\frac{1}{f}=\left(\frac{\mu_L}{\mu_M}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

so lens formula becomes

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

This formula is valid only when two sides has same medium



If two sides have different medium, the above formula will not be valid.

Magnification

Let h_2 is height of final image h' is height of intermediate h is height of initial object.

$$m=\frac{h_2}{h_1}=\frac{h_2}{h'}\times\frac{h'}{h_1}$$

$$=m_1\times m_2$$

$$=\frac{\mu_2 v_1}{\mu_1 u}\times\frac{\mu_1 v}{\mu_2 v_1}=\frac{v}{u}$$

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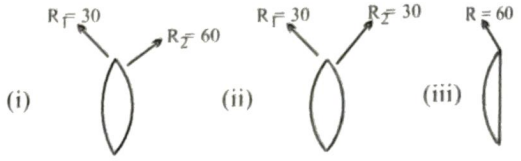
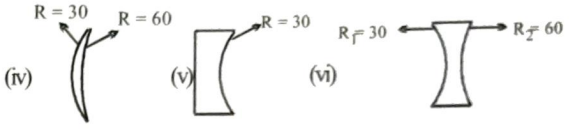
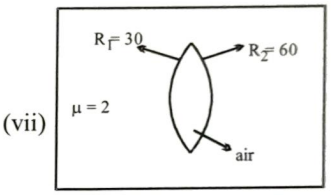
Example 28: Find focal length for diagram (i) to (vii).

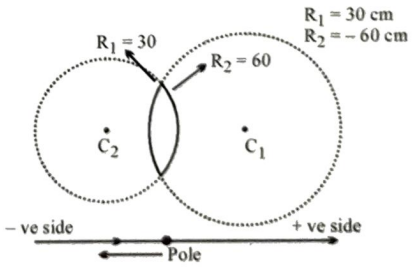






Take $\mu_L=\frac{3}{2}$ consider the lens to be in air.

Sol. (i) $\frac{1}{f}=\left[\frac{\mu_L}{\mu_m}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

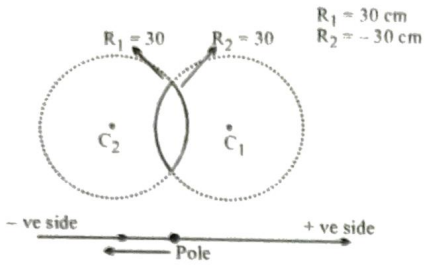


$$=\left[\frac{3}{2}-1\right]\left[\frac{1}{30}+\frac{1}{60}\right]$$

$$=\frac{1}{2}\left[\frac{1}{30}+\frac{1}{60}\right]$$

F = 40 cm

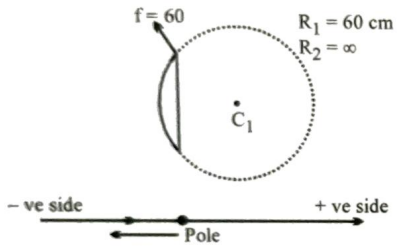
(ii) $\frac{1}{f}=\left[\frac{3}{2}-1\right]\left[\frac{1}{30}+\frac{1}{30}\right]$



$\frac{1}{f}=\frac{1}{2}\times\frac{2}{30}=\frac{1}{30}$

$f = 30$

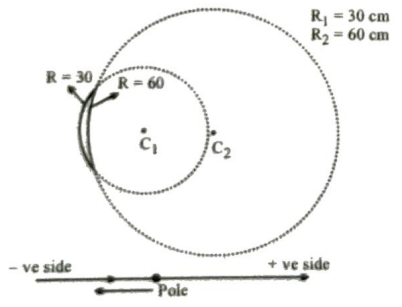
(iii) $\frac{1}{f}=\left[\frac{3}{2}-1\right]\left[\frac{1}{60}-\frac{1}{\infty}\right]$



$=\frac{1}{2}\times\frac{1}{60}=\frac{1}{120}$

$f = 120$ cm

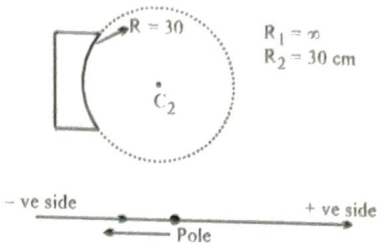
(iv) $\frac{1}{f}=\left[\frac{3}{2}-1\right]\left[\frac{1}{30}-\frac{1}{60}\right]$



$=\frac{1}{2}\left[\frac{1}{60}\right]$

$f = 120$ cm

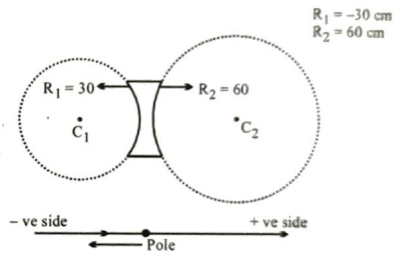
(v) $\frac{1}{f}=\left[\frac{3}{2}-1\right]\left[\frac{1}{\infty}-\frac{1}{30}\right]$



$=\frac{1}{2}\times\frac{-1}{30}=\frac{-1}{60}$

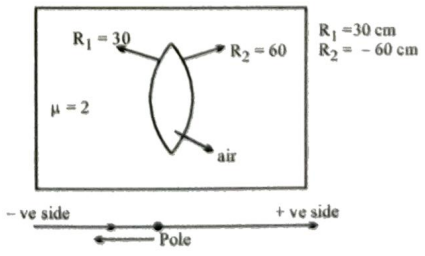
$f = -60$ cm

(vi) $\frac{1}{f}=\left[\frac{3}{2}-1\right]\left[-\frac{1}{30}-\frac{1}{60}\right]$



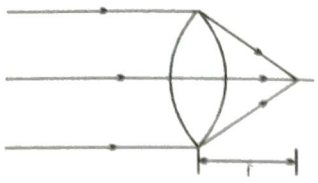
$f = -40$ cm

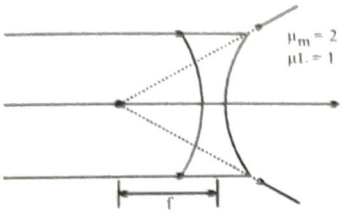
(vii) $\frac{1}{f}=\left[\frac{1}{2}-1\right]\left[\frac{1}{30}+\frac{1}{60}\right]$



$=-\frac{1}{2}\times\left[\frac{2+1}{60}\right]=\frac{-1}{2}\times\frac{1}{30}=\frac{-1}{60}$

$f = -60$ cm

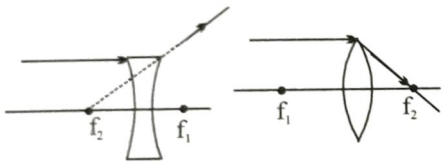




for converging lens secondary focal length is positive and for diverging lens secondary focal length is negative.

Ray diagram for lens

[1] Ray parallel to principle axis



A ray, initially parallel to the principal Axis will pass through focus or appear to pasts through focus.

[2] Ray passing through focus.

A ray which initially passes through focus will emerge from the lens parallel to the principal axis.



[3] Ray passing through optical center

A ray passing through, the optical centre of the lens goes undeviated as it passes through optical centre as it passes through slab region.



$\frac{1}{u}$ AND $\frac{1}{v}$ GRAPH FOR CONVERGING AND DIVERGING LENS

In the case of refraction

If v is positive image is real

If v is negative image is virtual

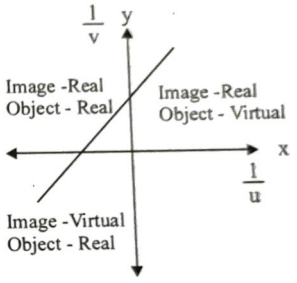
If u is positive object is virtual

If u is negative object is real

$\frac{1}{u}, \frac{1}{v}$ GRAPH

For converging lens

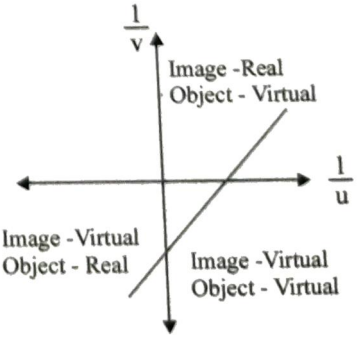
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$ (focal length is positive)



So, the converging lens does not formed virtual image of virtual object.

For Diverging lens

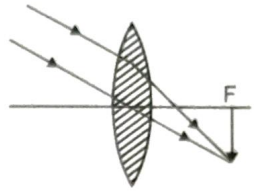
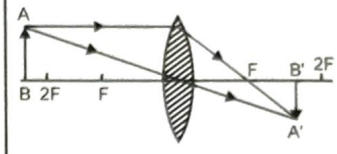
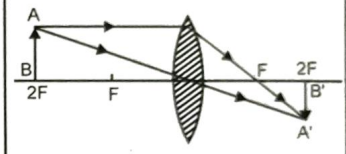
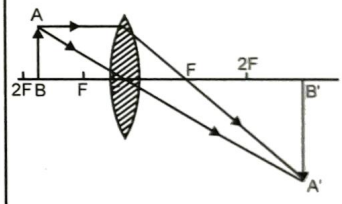
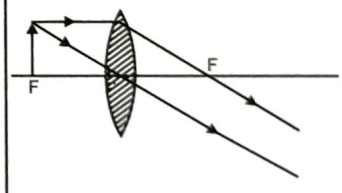
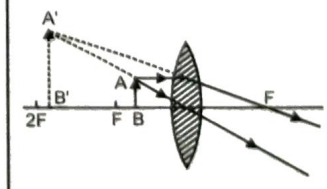
$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$$



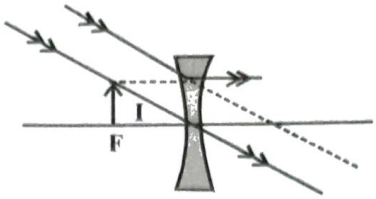
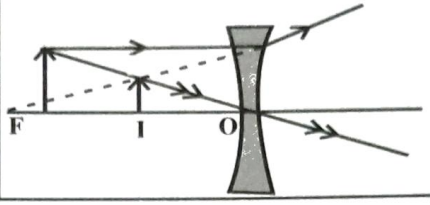
* So, diverging lens does not form real image of a real object.

POSITION, SIZE AND NATURE OF IMAGE FROM LENS

Convex Lens

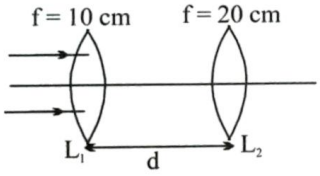
Position of object	Position of image	Real/ virtual	Inverted/ erect	Magnification and size of image	Sign of magni-fication	RAY DIAGRAM
at infinity ($u = \infty$)	at focus ($v = f$)	real	inverted	$m < 1$ greatly diminished	negative	
beyond $2f$ ($u > 2f$)	between f and $2f$ ($f < v < 2f$)	real	inverted	$m < 1$ diminished	negative	
at $2f$ ($u = 2f$)	at $2f$ ($v = 2f$)	real	inverted	$m = 1$ same size	negative	
between f and $2f$ ($f < u < 2f$)	beyond $2f$ ($v > 2f$)	real	inverted	$m > 1$ magnified	negative	
at f ($u = f$)	at infinity ($v = \infty$)	real	inverted	$m = \infty$ magnified	negative	
between optical centre and focus ($u < f$)	at a distance greater than the object distance and on the same side as object	virtual	erect	$m > 1$ magnified	positive	

For Divergent or Concave Lens

S.No.	Position of Object	Position of image	Ray – Diagram	Nature of image	Size
1.	At infinity	At F		Virtual, erect	Highly diminished (m << +1)
2.	In front of lens	Between F & optical centre		Virtual, erect	Diminished (m < +1)

TRAIN YOUR BRAIN

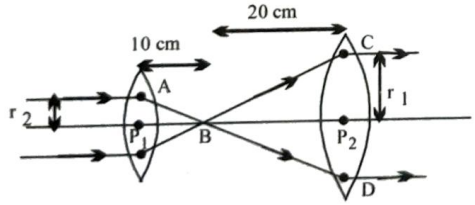
Example 29: Figure shows two converging lenses. Incident rays are parallel to principal axis. What should be the value of d so that final rays are also parallel. If initially beam diameter is 20 cm. Find the final beam diameter and what will be the effect on intencity of light.



Sol. Final rays should be parallel. For this the II focus of L_1 must coincide with I focus of L_2

$d = 10 + 20 = 30$ cm

here the diameter of ray beam becomes wider.



ΔAP_1B and ΔBP_2C will be similar hence $\frac{AP_1}{P_1B} = \frac{CP_2}{P_2B}$

Hence, $CP_2 = 40$ cm

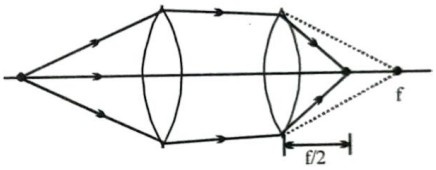
As diameter of beam increases intensity decreases.

Example 30: Two thin converging lenses are placed on a common axis so that the center of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left lens. Where will its image be? What is the lateral magnification? The focal length of each lens is *f*.

Sol. For first lens object is at – 2f so image will be at 2f.

For seconds lens u = f

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\quad m = -\frac{v}{u}$

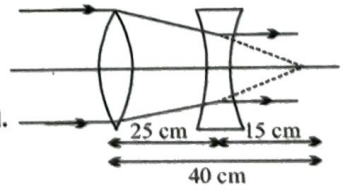


$\frac{1}{v} - \frac{1}{f} = \frac{1}{f}$

$m = -\frac{1}{2}$ as $v = \frac{f}{2}$ & $u = f$

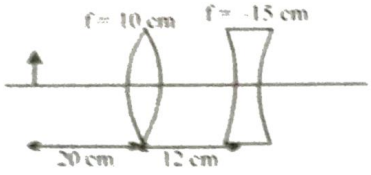
Example 31: A parallel beam of rays is incident on a convergent lens with a focal length of 40 cm. where a divergent lens with a focal length of 15 cm be placed for the beam of rays to remain parallel after passing through the two lens.

Sol.



Example 32: An object of height 2 cm is 20 cm from a converging lens (f = 10 cm), which is located at a distance 12 cm in front of a diverging lens (f = –15 cm). Find the location of the final image and its transverse magnification. Draw a ray Diagram.

Sol.



For first lens image distance will be 2f = 20 cm so image will be formed at 2f.

Image for first lens will be object for second lens.

For second lens.

u = 8 cm, f = – 15 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{18}=-\frac{1}{15}$$

$$\frac{1}{v}=\frac{1}{18}-\frac{1}{15}=17.14 \text{ cm}$$

$$m=\pm\frac{17.14}{8}=2.14 \text{ cm}$$

Example 33: A magnifying lens has focal length of 10 cm

(*a*) Where should the object be placed if the image is to be at 30 cm from the lens? What will be the magnification?

(*b*) Image is erect, virtual and four times the size of object.

Sol. (*a*) v = 30 cm, f = 10 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{30}-\frac{1}{u}=\frac{1}{10}$$

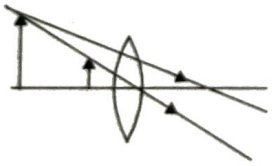
$$\frac{1}{u}=\frac{1}{30}-\frac{1}{10}$$

$$u=-15 \text{ cm}$$

$$m=\frac{u}{v}=\frac{30}{-15}=-2 \text{ cm}$$

One more case is there when image is virtual and erect then u = – x; v = – 3x.

(b) u = –x, v = – 4x



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$-\frac{1}{4x}+\frac{1}{x}=\frac{1}{10}$$

$$\frac{3}{4x}=\frac{1}{10}$$

$$x=\frac{30}{4}=7.5 \text{ m}$$

TRAIN YOUR BRAIN

Example 34: A convex lens of 10 cm focal length is combined with a concave lens of 6 cm focal length.

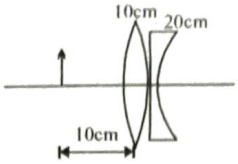
Find the focal length of the combination.

Sol. Here $f_1 = 10$ cm, $f_2 = -6$ cm, F = ?

Use the formula, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{10}-\frac{1}{6}=\frac{-1}{15}$

$\therefore$ F = – 15 cm

Example 35: Find the lateral magnification produced by the combination of lenses shown in the figure.



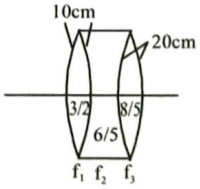
Sol. $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20} \Rightarrow f = +20$

$$\therefore \frac{1}{v}-\frac{1}{-10}=\frac{1}{20}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{20}-\frac{1}{10}=\frac{-1}{20}=-20 \text{ cm}$$

$$\therefore m=\frac{-20}{-10}=2$$

Example 36: Find the focal length of equivalent system.



Sol. $\frac{1}{f_1}=\left(\frac{3}{2}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right)=\frac{1}{2}\times\frac{2}{10}=\frac{1}{10}$

$$\frac{1}{f_2}=\left(\frac{6}{5}-1\right)\left(\frac{-1}{10}-\frac{1}{20}\right)=\frac{1}{5}\left(\frac{-30}{10\times 20}\right)=\frac{-3}{100}$$

$$\frac{1}{f_3}=\left(\frac{8}{5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right)=\frac{3}{50}$$

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}=\frac{1}{10}+\frac{-3}{100}+\frac{3}{50} \quad f=\frac{100}{13}$$

DISPLACEMENT METHOD TO FIND FOCAL LENGTH OF CONVERGING LENS

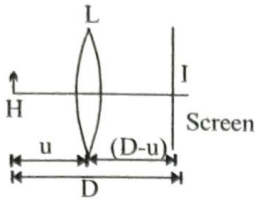
If a thin converging lens of focal length 'f' is placed between an object and a screen fixed at a distance D apart and if D > 4f, then there are two positions of the lens at which a sharp image of the object is formed on the screen.

It the object is at a distance 'u' from the lens, the distance of image from the lens v = (D – u).,

So from lens formula.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

we have, $\frac{1}{(D-u)}-\frac{1}{-u}=\frac{1}{f}$



i.e. $u^2 - Du + Df = 0$

So that $u = \frac{1}{2}[D \pm \sqrt{D(D-4f)}]$ ...(i)

Now, there are three possibilities

(*a*) If $D < 4f$: u will be imaginary, so physically no position of lens is possible.

(*b*) If $D = 4f$: In this situation $u = D/2 = 2f$. So only one position is possible and in this situation,

$v = D - u = 4f - 2f = 2f (= u)$

(*c*) If $D > 4f$: In this situation both the roots of equation (i) will be real.

i.e. $u_1 = \frac{1}{2}[D - \sqrt{D(D-4f)}]$

$$v_1 = D - u_1 = \frac{1}{2}\left[D + \sqrt{D^2 - 4Df}\right]$$

$$m_1 = \frac{v_1}{u_1} = \frac{I_1}{O} = \frac{[D + \sqrt{D^2 - 4Df}]}{[D - \sqrt{D^2 - 4Df}]}$$

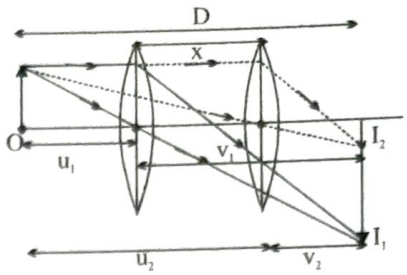
and $u_2 = \frac{1}{2}[D + \sqrt{D(D-4f)}]$

$$v_2 = D - u_2 = \frac{1}{2}\left[D - \sqrt{D^2 - 4Df}\right]$$

$$m_2 = \frac{v_2}{u_2} = \frac{I_2}{O} = \frac{[D - \sqrt{D^2 - 4Df}]}{[D + \sqrt{D^2 - 4Df}]}$$

m_1 will be more than 1 and m_2 will be less than 1.

So, if $d > 4f$, there are two positions of lens at distance u_1 and u_2 from the object for which real image is formed on the screen. This method is called **'Displacement method".**



$$\frac{I_1}{O} \times \frac{I_2}{O} = 1 \Rightarrow O^2 = I_1 I_2 \Rightarrow O = \sqrt{I_1 I_2}$$

[i] If the distance between two positions of the lens is x,

$x = u_2 - u_1 = \sqrt{D(D-4f)}$ from eq. (ii),

i.e. $x^2 = D^2 - 4Df$

so, $f = (D^2 - x^2)/4D$ ....(iii)

$$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \left(\frac{D+x}{D-x}\right)$$

and $m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \left(\frac{D-x}{D+x}\right)$....(iv)

And hence

(*a*) $m_1 \times m_2 = (I_1 I_2 / O^2) = 1$

i.e. $O = \sqrt{I_1 I_2}$

(*b*) $$\frac{m_1}{m_2} = \frac{I_1}{I_2} = \left(\frac{D+x}{D-x}\right) \times \left(\frac{D+x}{D-x}\right) = \left(\frac{D+x}{D-x}\right)^2$$

$$m_1 - m_2 = \left(\frac{D+x}{D-x}\right) - \left(\frac{D-x}{D+x}\right) = \frac{4Dx}{D^2 - x^2}$$

with the help of equation (iii) it transforms to,

$m_1 - m_2 = \frac{x}{f}$ i.e. $f = \frac{x}{m_1 - m_2}$

TRAIN YOUR BRAIN

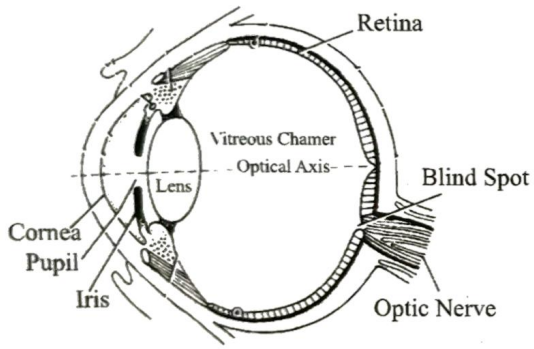
Example 37: A lens with a focal length of 16 cm produces a sharp image of an object in two positions, which are 60 cm apart. Find the distance from the object to the screen

Sol. $f = \frac{D^2 - x^2}{4D}$

$$16 = \frac{D^2 - (60)^2}{4 \times D} = 100 \text{ cm.}$$

STRUCTURE OF EYE

Light enters the eye through a curved front surface, the cornea. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye-lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information. The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles. For example, when the muscle is released, the focal length is about 2.5 cm and (for a normal eye) objects at infinity are in sharp focus on the retinas. When the object is brought closer to the eye, in order to maintain the same image-lens distance ($\cong$ 2.5 cm), the focal length of the eye-lens becomes shorter by the action of the ciliary muscles. This property of the eye in called accommodation. If the object is too close to the eye, the lens cannot curve enough to focus the image on to the retina, and the image is blurred.



Human Eye

The closest distance for which the lens can focus light on the retina is called the least distance of distinct vision, or the near point. The standard value (for normal vision) taken here is 25 cm. (Often the near point is given the symbol D.)

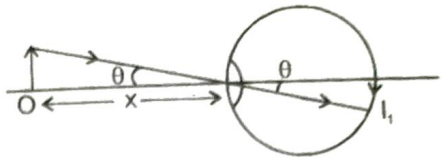
DEFECTS OF VISION

Regarding eye, it is nothing that:

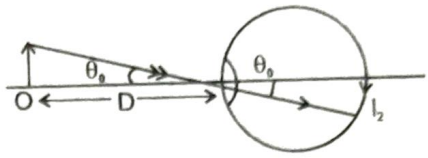
(1) In eye convex eye-lens forms real inverted and diminished image at the retina by changing its convexity (the distance between eye lens and retina is fixed)

(2) The human eye is most sensitive to yellow green light having wavelength 5550 A^0 and least to violet (4000 A^0) and red (7000 A^0)

(3) The size of an object as perceived by eye depends on its visual-angle when object is distant its visual angle θ and hence image I_1 at retina is small (it will appear small) and as it is brought near to the eye its visual angle θ_0 and hence size of image I_2 will increase.



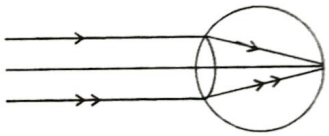
(A)



(B)

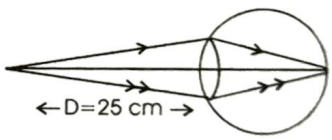
(4) The far and near point for normal eye are usually taken to be infinity and 25 cm respectively ie., normal eye can see very distant object clearly but near objects only if they are at distance greater than 25 cm from the eye. The ability of eye to see objects from infinite distance to 25 cm from it is called Power of accommodation.

(5) If object is at infinity i.e., parallel beam of light enters the eye is least strained and said to be relaxed or unstrained. However, if the object is at least distance of distint vision (L.D.D.V] i.e., D (=25 cm) eye is under maximum strain and visual angle is maximum.



Relaxed or normal eye

(A)



Maximum strained eye

(B)

(6) The limit of resolution of eye is one minute ie., two object will not be visible distintely to the eye if the angle substanded by them on the eye is lesser than one minute.

(7) The persistance of vision is (1/10) sec i.e., If time interval between two consecutive light pulses is lesser than 0.1 sec eye cannot distinguish them separately. This fact is taken into account in motion pictures.

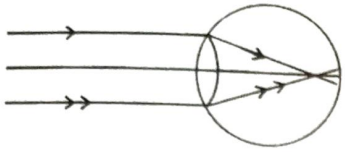
In case of eye following are the common defects of vision.

MYOPIA

[or short-sightedness or near - sightedness]

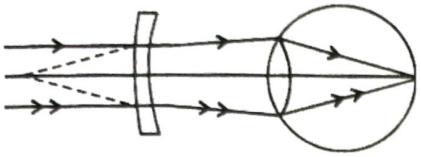
In it distant objects are not clearly visible. i.e. Far Point is at a distance lesser than

Infinity and hence image of distant object is formed before the retina.



Defective-eye

(A)



Corrected-eye

(B)

This defect is (i.e., negative focal length or power) which forms the image of distant object at the far point of patient - eye [which is lesser than ∞] so that in this case from lens formula we have

$$\frac{1}{-\text{F.P}} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$$

And if the object is at ∞ then $P = \frac{1}{f} = \frac{1}{-\text{F.P.}}$ .... (1)

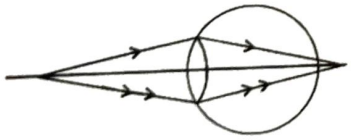
where F.P is farther point of eye

HYPERMETROPIA

[Or Long-sightendness or far-sightendness]

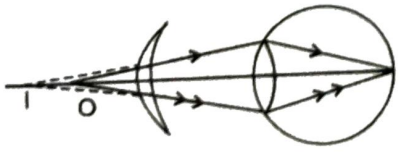
In this near object are not clearly visible i.e., Near Point is at a distance greater than 25 cm and hence image of near object is formed behind the retina.

This defect is remedied by using spectacles having convergent lens (i.e., positive focal length of power) which the image of near objects at the Near Point of the



Defective-eye

(A)



Corrected-eye

(B)

Patient-eye (which is more than 25 cm). So that in this case from lens formula we have

$$\frac{1}{-\text{N.P.}} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$$

If object is placed at D = 25 cm = 0.25 m

$$P = \frac{1}{f} = \left[\frac{1}{0.25} - \frac{1}{\text{N.P.}}\right] \qquad \text{.....(2)}$$

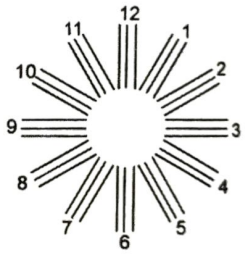
where N.P is near point of eye

PRESBYOPIA

In this both near and far objects are not clearly visible i.e., far point is lesser than infinity and near point greater than 25 cm. It is an old age disease as at old age ciliary muscles lose their elasticity and so can not change the focal length of eye-lens effectively and hence eye loses its power of accommodation.

ASTIGMATISM

In this due to imperfect spherical nature of eye-lens, the focal length of eye lens is two orthogonal directions becomes different and so eye cannot see object in two orthogonal directions clearly simultaneously. This defect is directional and is remedied by using cylindrical lens in particular direction. If in the spectacle of a person suffering from astigmatism, the lens is slightly rotated the arrangement will get spoiled.



Self Practice Problems

1. For a normal eye, the least distance of distinct vision is
   (*a*) 0.25 m (*b*) 0.50 m
   (*c*) 25 m (*d*) Infinite
2. For the myopic eye, the defect is cured by
   (a) Convex lens (*b*) Concave lens
   (c) Cylindrical lens (*d*) Toric lens
3. Lens used to remove long sightedness (hypermetropia) is
   or
   A person suffering from hypermetropia requires which type of spectacle lenses
   (*a*) Concave lens
   (*b*) Plano- concave lens
   (*c*) Convexo- concave lens
   (*d*) Convex lens
4. Image formed on the retina is
   (*a*) Real and inverted (*b*) Virtual and erect
   (*c*) Real and erect (*d*) Virtual and inverted

Ans. 1. (*a*) **2.** (*b*) **3.** (*d*) **4.** (*a*)

DISPERSION OF LIGHT

When a beam of light (containing several wavelengths) falls on one face of a prism, it splits into its constituent colors. This phenomenon of splitting of light into its constituent colours is called dispersion of light and the band of colours obtained on a screen is called spectrum. The cause of dispersion is variation of refractive index with wavelength of light. An approximate empirical relation as proposed by Cauchy is given by

$$\mu(\lambda) = A + \frac{B}{\lambda^2}$$

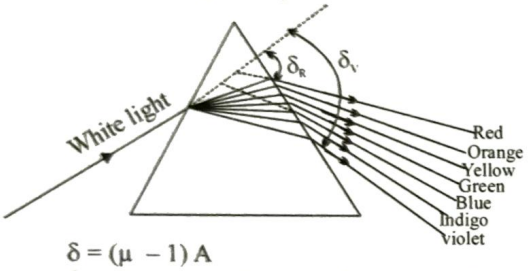
where A and B are known as Couchy's constant. The value of A and B depends on material of prism.

We know that $\lambda_{red} > \lambda_{violet}$

$\therefore \quad \mu_{red} < \mu_{violet}$

Hence $\delta_{red} < \delta_{violet}$

The difference in the deviations suffered by two colours in passing through a prism gives the angular dispersion for these colours. The angle between the vilolet and red colours is known as angular dispersion. We know that for small angle of prism, deviation is given by



$\delta = (\mu - 1)A$

δ_v = Deviation of violet colour = $(\mu_v - 1)A$

δ_R = Deviation of red colour = $(\mu_R - 1)A$

Hence, Angular dispersion (A.D) = $\delta_V - \delta_R$

$= (\mu_R - \mu_R)A$

It is clear from above relation that angular dispersion depends upon (i) the nature of material of the prism and (ii) the angle of the prism. This is also defined as the rate of change of angle of deviation with wavelength i.e. A.D. $= \frac{d\delta}{d\lambda}$

Dispersive power of a prism is defined as the ratio between angular dispersion to mean deviation produced by the prism. ω = Dispersive power

$$= \frac{\delta_v - \delta_R}{\delta_y} = \frac{\mu_v - \mu_R}{\mu_y - 1} = \frac{d\mu}{\mu_y - 1}$$

Where $d\mu$ denotes the difference between the refractive indices of material of prism for violet and red light. It is also defined as dispersion per unit deviation. Yellow colour is taken as mean colour.

Also $\mu_y = \frac{\mu_v - \mu_R}{2}$ or $\frac{d\mu}{\mu_y - 1}$

Dispersion without deviation

Let us consider a crown glass prism combined with a flint glass prism in position as shown in figure. Let A and A' be the angles of crown glass prism and flint glass prism respectively. Let μ_0, μ and μ_r be the refractive indices of the crown glass for violet, yellow and red colours respectively.

Let μ_0', μ' and μ_r', be the corresponding values for the flint glass prism

Let δ and δ' be the deviations suffered by yellow light through crown glass prism and flint glass prism respectively.

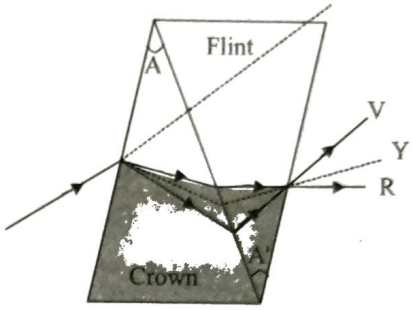
If the combination does not produce any deviation, then

$\delta + \delta' = 0$

or $(\mu - 1)A + (\mu' - 1)A' = 0$

or $(\mu' - 1)A' = -(\mu - 1)A$

or $A' = -\left(\frac{\mu - 1}{\mu' - 1}\right)A$



This is the condition for no deviation. The negative sign indicates that the two prisms are to be placed in position as shown in figure.

Net angular dispersion = $[(d_v - d_r) + (d'_v - d'_r)]$

$= (\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A'$

$= A\left[(\mu_v - \mu_r) + (\mu'_v - \mu'_r)\frac{A'}{A}\right]$

$= A\left[(\mu_v - \mu_r) + \left(\frac{\mu - 1}{\mu' - 1}\right)(\mu'_v - \mu'_r)\right]$

$= (\mu - 1)A\left[\frac{\mu_v - \mu_r}{\mu - 1} + \frac{\mu'_v - \mu'_r}{\mu - 1}\right] = \delta(\omega - \omega')$

Net angular dispersion = $\delta(\omega - \omega')$

Here ω and ω' are the dispersive powers of crown glass and flint glass respectively. As the dispersive powers ω and ω' are not equal, in such a combination there will be resultant dispersion and the final dispersed beam is parallel to the incident beam.

Since $\omega' > \omega$ therefore the net angular dispersion is negative. This explains why the order of colours in the spectrum due to combination is opposite to that in the crown glass prism.

Deviation without Dispersion

For the combination of prism shown in figure, if there is to be no angular dispersion, then

$(\delta_v - \delta_r) + (\delta'_v - \delta'_r) = 0$

or $(\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A' = 0$

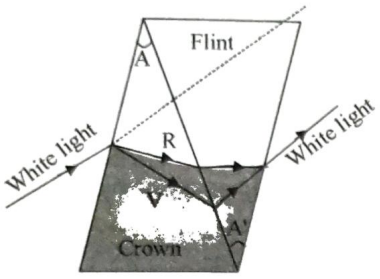
or $(\mu'_v - \mu'_r)A' = -(\mu_v - \mu_r)A$

or $A' = -\left(\frac{\mu_v - \mu_r}{\mu'_v - \mu'_r}\right)A$

This is the condition for achromatism i.e., the condition for no dispersion. This condition can be written in another form as given below.

From equation (1),

$$(\mu-1)A\frac{\mu_v-\mu_r}{\mu-1}+(\mu'-1)A\frac{\mu'_v-\mu'_r}{\mu'-1}=0$$



or $\delta\omega + \delta'\omega' = 0$

or $\frac{\omega}{\omega'} = -\frac{\delta'}{\delta}$

Since $\omega' > \omega \ \therefore \ \delta > \delta'$

or $(\mu - 1)A > (\mu' - 1)A'$

But, $(\mu - 1) < (\mu' - 1) \therefore A > A'$

So, the crown glass prism should have a larger angle than the flint glass prism

Net deviation $= \delta + \delta'$

$$= (\mu-1)A+(\mu'-1)A' = (\mu-1)A\left[1+\frac{(\mu'-1)A'}{(\mu-1)A}\right]$$

$$= (\mu-1)A\left[1-\frac{\mu'-1}{\mu-1}\times\frac{\mu_v-\mu_r}{\mu'_v-\mu'_r}\right] \quad \text{[Using equation (2)]}$$

$$= (\mu-1)A\left[1-\frac{\mu_v-\mu_r}{\mu-1}\times\frac{\mu_v-\mu_r}{\mu-1}\times\frac{\mu'-1}{\mu'_v-\mu'_r}\right] = \omega\left(1-\frac{\omega}{\omega'}\right)$$

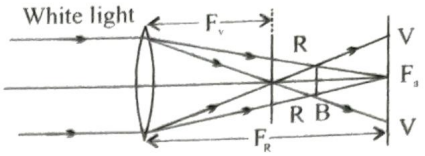
Since $\omega' > \omega$, therefore the net deviation is in the direction of the deviation produced by crown glass prism.

Chromatic aberration

The image of an object in white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colours. For a single lens.

$$\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

And an μ of lens in maximum for vilolet while minimum for red, violet is focused nearest to the lens whereas red farthest from it as shown in figure.



As a result of this in case of convergent lens, at F_v centre of image will be violet and focused while extreme end will be red and blurred, whereas at F_R reverse is the case, i.e., centre will be red and focused while extreme end will be violet and blurred. The difference between F_v and F_R is a measure of longitudinal chromatic aberration, i.e.,

L.C.A $= f_R - f_v = -dv$

with $df = f_V - f_R$ .....(1)

However, as for a single lens,

$$\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad .....(2)$$

i.e., $$-\frac{df}{f^2} = d\mu\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad .....(3)$$

So dividing equation (3) and (2)

$$-\frac{df}{f} = \frac{d\mu}{(\mu-1)} = \omega\left[\text{as } \omega\frac{d\mu}{(\mu-1)}\right] \quad ....(4)$$

And hence, from equation (1) and (4),

L.C.A. $= -df = \omega f$ .....(5)

Now, as for a single lens neither f nor ω can be zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism

In case of two thin lenses in contact.

$$\frac{1}{F} = \frac{1}{f_1}+\frac{1}{f_2} \qquad \text{i.e. } -\frac{dF}{F^2} = -\frac{df_1}{f_1^2}-\frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if $dF = 0$

i.e. $$\frac{df_1}{f_1^2}+\frac{df_2}{f_2^2} = 0$$

equation (5) reduces to

$$\frac{\omega_1 f_1}{f_1^2}+\frac{\omega_2 f_2}{f_2^2} = 0$$

i.e. $$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2} = 0 \quad .....(6)$$

This condition is called condition of achromatism (for two thin lenses in contact) and the lens combination which satisfies this condition achromatic lens. From this condition, i.e., from equation (6) it is clear that in case of achromatic doublet:

(1) The two lenses must be of different materials

since, if $\omega_1 = \omega_2$, $\frac{1}{f_1}+\frac{1}{f_2}=0$ i.e. $\frac{1}{F}=0$ or $F=\infty$

i.e., combination will not behave as a lens, but as a plane glass plate.

(2) As ω_1 and ω_2 are positive quantities, for equation (6) to hold, f_1 and f_2 must be of opposite nature, i.e. if one of the lenses is convex the other must be concave.

(3) If the achromatic combination is convergent,

$$f_C < f_D \text{ and as } -\frac{f_C}{f_D}=\frac{\omega_C}{\omega_D}, \omega_C < \omega_D$$

i.e., a convergent achromatic doublet, convex lens has lesser focal length and dispersive power than divergent one.

TRAIN YOUR BRAIN

Example 38: The refractive indices of flint glass for red and violet light are 1.613 and 1.632 respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5^0.

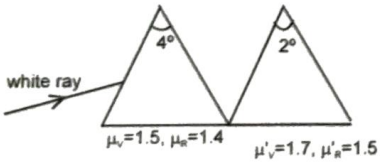
Sol. Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.
The dispersion $= \delta_v - \delta_r = (\mu_v - \mu_r)A$
$= (1.632 - 1.613) \times 5^0 = 0.095^0$.

Example 39: Refractive index of glass for red and violet colours are 1.50 and 1.60 respectively. Find
(*a*) the ref. index for yellow colour, approximately
(*b*) Dispersive power of the medium.

Sol. (*a*) $\mu_r \simeq \frac{\mu_v+\mu_R}{2} = \frac{1.50+1.60}{2} = 1.55$

(*b*) $\omega = \frac{\mu_v-\mu_R}{\mu_r-1} = \frac{1.60-1.50}{1.55-1} = 0.18.$

Example 40: If two prisms are combined, as shown in figure, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Sol. Both prisms will turn the light rays towards their bases and hence in same direction. Therefore turnings caused by both prisms are additive.
Total angular dispersion
$= \theta + \theta' = (\mu_v - \mu_R)A + (\mu'_v - \mu'_R)A'$
$= (1.5 - 1.4)\,4^0 + (1.7 - 1.5)2^0 = 0.8^0$
Total deviation
$= \delta + \delta'$
$= \left(\frac{\mu_v+\mu_R}{2}-1\right)A + \left(\frac{\mu'_v+\mu'_R}{2}-1\right)A'$
$= \left(\frac{1.5+1.4}{2}-1\right)0.4^0 + \left(\frac{1.7+1.5}{2}-1\right)0.2^0$
$= (1.45 - 1)\,0.4^0 + (1.6 - 1)\,0.2^0$
$= 0.45 \times 0.4^0 + 0.6 \times 0.2^0$
$= 1.80 + 1.2 = 3.0^0$ Ans.

Example 41: Two thin prisms are combined to form an achromatic combination. For I prism $A = 4^0$, $\mu_R = 1.35$, $\mu_Y = 1.40$, $\mu_v = 1.42$. for II prism $\mu'_R = 1.7$, $\mu'_Y = 1.8$ and $\mu'_R = 1.9$ find the prism angle of II prism and the net mean deviation.

Sol. Condition for achromatic combination.
$\theta = \theta'$
$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$
$\therefore\ A' = \frac{(1.42-1.35)4^0}{1.9-1.7} = 1.4^0$
$\delta_{Net} = \delta \sim \delta' = (\mu_Y - 1)A \sim (\mu'_Y - 1)A'$
$= (1.40 - 1)\,4^0 \sim (1.8 - 1)\,1.4^0 = 0.48^0.$

Example 42: A crown glass prism of angle 5^0 is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.5, 1.6 and 1.7 respectively for crown glass and 1.8,2.0 and 2.2 for flint glass.

Sol. The deviation produced by the crown prism is
$\delta = (\mu - 1)A$
and by the flint prism is :
$\delta' = (\mu' - 1)A'$.
The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is :
$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'$. .....(1)

(*a*) If the net deviation for the mean ray is zero,
$(\mu - 1)A = (\mu' - 1)A'$.

or, $A' = \frac{(\mu-1)}{(\mu'-1)}A = \frac{1.6-1}{2.0-1}\times 5^0 = 3^0$

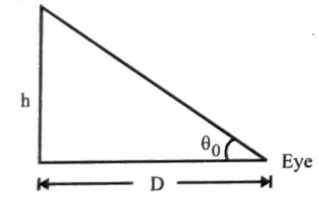
(*b*) The angular dispersion produced by the crown prism is:
$\delta_v - \delta_r = (\mu_v - \mu_r)A$
and that by the flint prism is,
$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$
The net angular dispersion is,
$(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A$
$= (1.7 - 1.5) \times 5^0 - (2.2 - 1.8) \times 3^0$
$= -0.2^0$.
The angular dispersion has magnitude 0.2^0.

OPTICAL INSTRUMENT

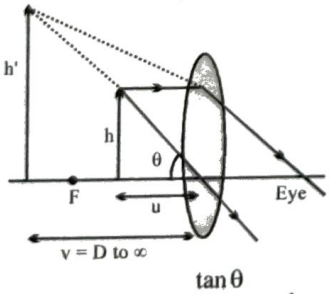
Microscopes

The apparent size of an object is determined by the size of its retinal image which, in turn, if the eye is unaided, depends upon the angle subtended by the object at the eye. When one wants to examine a small object in detail, one brings it close to the eye in order that the angle subtended and the retinal image may be as laarge as possible. Since the eye cannot focus sharply on objects closer than the nearer point, a given object subtends the maximum possible angle at an unaided eye when placed at this point. By placing a converging lens in front of the eyee, the accommodation may, in effect, be increased. The object may then be brought closer to the eye than the near point and will subtend a correspondingly larger angle. The optical instrument which is used to increase the visual angle of eye is termed as microscope.

(i) Simple Microscope or Magnifier or Magnifying Glass:
A converging lens used for the above mentioned purpose is termed as magnifier or simple microscope. The magnifier forms a virtual image of the object and eye looks at this virtual image, i.e. we can say the object is placed in between the focus and optical center of lens and eye close to it on the other side. Since a normal eye can focus sharply on an object anywhere between the near point and infinity, the image can be seen clearly if it is formed anywhere within this range.
Magnifying power or angular magnification of an object instrument is defined as the ratio of visual angle with instrument to the maximum visual angle for clear vision when the eye is unaided [i.e. when the object is at the near point].



Unaided eye



γ = Magnifying power = $\frac{\tan\theta}{\tan\theta_0}$, where θ and θ_0 are clearly shown in figure.

$\tan\theta = \frac{h'}{v}$, where v = D to ∞ and D is the least distance of distinct vision.

$\tan\theta_0 = \frac{h}{D}$

$\gamma = \frac{h'}{v} \times \frac{D}{h}$, where $\frac{h'}{v} = \frac{h}{u}$, u being the object distance from the lens.

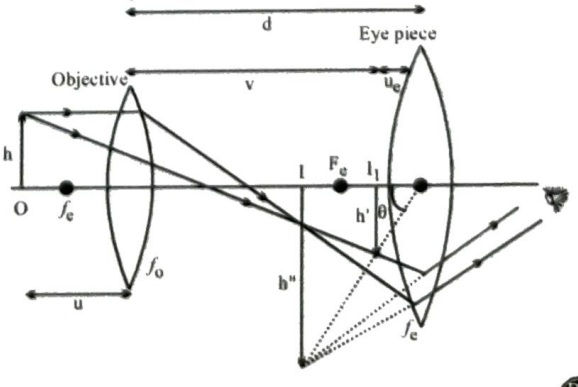
For $v = D$, $\gamma = \left[1 + \frac{D}{f}\right]$. In this situation, the magnifying power is maximum and eye is under maximum strain.

For $v = \infty$, $\gamma = \frac{D}{f}$. In this situation, magnifying power is minimum and eye is the least strained.

Thus, angular magnification of a simple magnifier of focal length 10 cm is 2.5 × (as D = 25 cm), i.e., height of retinal image of an object viewed through the magnifier is 2.5 times as great as, when viewed with unaided eye at minimum distance of distinct vision,i.e. the near point. [This is the situation when eye is the least strained, the most general case.]

- It seems that can be increaseed as much as desired by decreasing *f*, but actually it is not possible due to aberrations of simple lens.
- Simple magnifier is an essential part of most of the optical instruments (compound microscope or telescopes) in the form of eyepiece or oculars.

(ii) Compound Microscope :
When an angular magnification higher than that attainable with a simple magnifier is desired, it is necessary to use a compound microscope, simply termed as a microscope.
It consists of two convergent lenses of short focal lengths and apertures arranged coaxially. One is termed as objective while the other is eyepiece or ocular, object is placed near to objective while eye piece is arranged in such a way that it is facing the eye. The objective has smaller aperture and focal length as compared to the eye piece. The separation between the objective and eye piece can be varied. Construction and ray diagram outlining the basic principle of a mircoscope in shown in figure.



An object O is placed just beyound the focus of the objective, whose image I_1 is formed by the objective within the focus and optical center of eye piece.

This image I_1 is real, inverted and enlarged image w.r.t. object O. The eye piece then forms a virtual image of the object between D and ∞. This final image is inverted and enlarged w.r.t. the object O.

Angular magnification or magnifying power of the instrument is $\gamma = \frac{\tan\theta}{\tan\theta_0}$

$\tan\theta_0 = \frac{h'}{u_e}$ and $\tan\theta_0 = \frac{h}{D}$

So, $\gamma = \frac{h'}{h} \times \frac{D}{u_e} = m\gamma_e = \frac{-v}{u} \times \gamma_e$

where m is the lateral magnification produced by the objective and γ_e is the angular magnification produced by the eye piece,

- When final image is formed at the least distance of distinct vision (D), the eye is most strained and $\gamma = \frac{-v}{u}\left[1+\frac{D}{f_e}\right]$ with $L = v + \frac{f_e D}{f_e + D}\gamma$ is maximum in this mode.
- In general, as f_0 is small and object is close to objective first focus i.e., u ; f_0 and L ; v as $u_e << v$, so for normal unchanged.
- In this case, γ is negative. Hence, the final image is inverted.
- In normal adjustment, if eye piece and objective lenses are interchanged, the angular magnification remains unchanged.
- Due to small apertures of lenses, spherical aberration is decreased to a great extent.
- F_0 is taken smaller than f_e to increase the field of view and to increase brightness of the image.
- With respect to microscope, the minimum distance between two lines at which they are just distinct is called limit of resolution (Δx) and the reciprocal of limit of resolution is called resolving power.

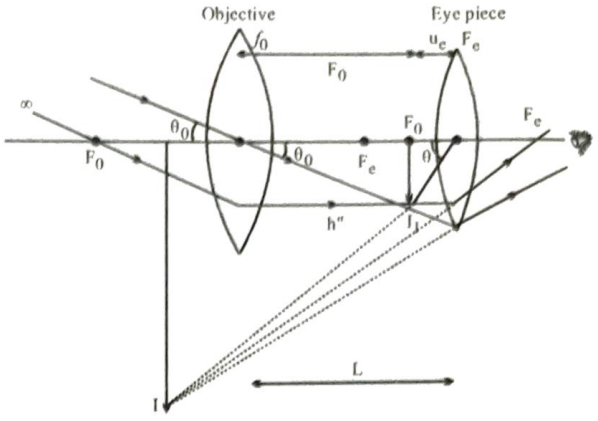
Resolving power $= \frac{1}{\Delta x} \propto \frac{1}{\lambda}$

Telescopes

(i) Refracting Telescope :

The optical system of a refracting telescope is essentially the same as that of a compound microscope. In both instruments, the image formed by the objective is viewed through the ocular (eye piece), the difference is that telescope is used to examine large objects at farther distances while microscope is used to examine small objects close to hands.

(ii) Astronomical Telescope :



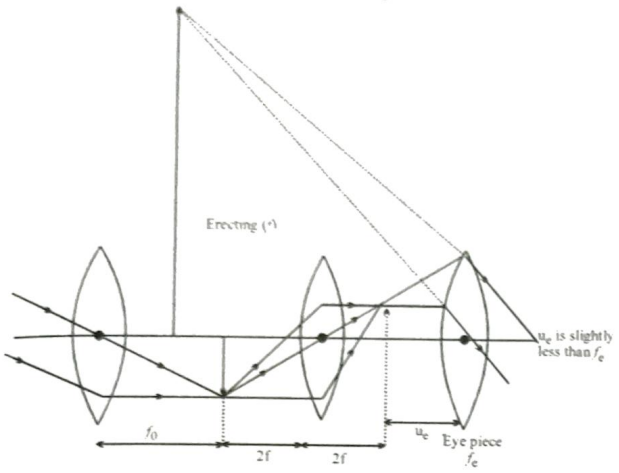
The working diagram of astronomial telescope is as shown in figure.

- While seeing the object through telescope, the object is between ∞ and 2F of the objective which forms real, inverted and diminished image within the focus and optical center of eye piece which then forms its virtual erect (w.r.t. intermediate image), and magnified image at a distance between D to ∞ from the eye. This means final image is inverted (w.r.t object), at a distance between D to ∞ from the eye.
- In a telescope, the aperture and focal length of objective are greater than that of eye piece.
- Magnifying power, $\gamma = \frac{\tan\theta}{\tan\theta_0} = \frac{y/u_e}{y/f_0} = \frac{f_0}{u_e}$.
- In general case, final image is formed at ∞ and hence y $= \frac{-f_0}{u_e}$. In this case, eye is the least strained and magnification power is minimum with length of the tube. $L = f_0 + f_e$.
- In normal adjustment (i.e., final image is at ∞), to have large magnifying power, f_0 must be as large as practically possible while has to be kept small.
- Resolving power of a telescope depends on the aperture of the objective and wavelength of light

Resolving power $\propto \frac{\text{Aperture of objective}}{\text{Wavelength}}$

(iii) Terrestrial Telescope

While an inverted image is not a disadvantage if the instrument is to be used for astronomial observation, it is desirable that a terrestrial telescope shall form an erect image. This can be accomplished by using an erecting lens in the astronomical telescope.



Magnifying power for terrestrial telescope is $\gamma_e = \frac{f_0}{f_e}$

[For relaxed eye]

Length of tube, $L = f_0 + 4f + f_e$.

(iv) Reflecting Telescope

It is an astronomial telescope, in which objective lens is replaced by a concave mirror.

The reflecting telescope is cheap, light and portable as compared to astronomial telescope.

TRAIN YOUR BRAIN

Example 43: A man with normal near point (25 cm) reads a book with small print using a magnifying a thin convex lens of focal length 5 cm. (a) What is the closest farest distance at which he can read the book when viewing through the magnifying glass? (b) What is the maximum and minimum MP possible using the above simple microscope?

Sol. *(a)* As for normal eye far and near point are ∞ and 25cm respectively, so for magnifier $v_{max} = -\infty$ and $v_{min} = -25\text{cm}$. However, for a lens as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \text{i.e.,} \quad u = \frac{f}{\left(\frac{f}{v}\right) - 1}$$

So u will be minimum when $v = \text{min} = -25\text{cm}$

i.e., $$(u)_{min} = \frac{5}{\left(\frac{-5}{25}\right) - 1} = -\frac{25}{6} = -4.17\text{cm}$$

And u will be maximum when $v = \text{max} = \infty$

So, the closest and farest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) An in case of simple magnifier $MP = \left(\frac{D}{u}\right)$. So MP will be minimum when u = max = 5 cm

i.e., $$(MP)_{min} = \frac{-25}{-5} = 5 \quad \left[= \frac{D}{f}\right]$$

And MP will be maximum when $u = \text{min} = \left(\frac{25}{6}\right)\text{cm}$

i.e., $$(MP)_{max} = \frac{-25}{-\left(\frac{25}{6}\right)} = 6\left[= 1 + \frac{D}{f}\right]$$

Example 44: The focal length of the objective and eyepiece of a microscope are 2 cm and 5 cm respectively and the distance between them is 20 cm. Find the distance of object from the objective, when the final image seen by the eye is 25 cm from the eyepiece. Also find the magnifying power.

Sol. Given $f_0 = 2$ cm, $f_e = 5$ cm

$|v_0| + |u_e| = 20$ cm

$\therefore \quad v_e = -25$ cm

From lens formula $\frac{1}{f_e} = \frac{1}{v_0} - \frac{1}{u_e}$

$$\frac{1}{u} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5}$$

$\therefore \quad u_e = -\frac{25}{6}$ cm

Distance of real image from objective

$$v_0 = 20 - |u_e| = 20 - \frac{25}{6}$$

$$= \frac{120 - 25}{6} = \frac{95}{6} \text{ cm}$$

Now $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$

given $\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{(95/6)} - \frac{1}{2}$

i.e., $\frac{1}{u_0} = \frac{6}{95} - \frac{1}{2} = \frac{12 - 95}{190} = -\frac{83}{190}$

$\therefore \quad u_0 = -\frac{190}{83} = -2.3$ cm

Magnifying power $M = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

$$= -\frac{95/6}{(190/83)}\left(1 + \frac{25}{3}\right) = -41.5$$

Example 45: A telescope consists of two convex lens of focal length 16 cm and 2 cm. What is angular magnification of telescope for relased eye? What is the separation between the lenses?

If object subtends an angle of 0.5° on the eye, what will be angle subtended by its image ?

Sol. Angular magnification

$$M = \frac{\alpha}{\beta} = \frac{F}{f} = \frac{16}{2} = 8 \text{ cm}$$

Separation between lenses

$= F + f = 16 + 2 = 18$ cm

Here $\alpha = 0.5°$

$\therefore$ Angular subtended by image

$\beta = M\alpha = 8 \times 0.5° = 4°$

Example 46: The magnifying power of the telescope if found to be 9 and the separation between the lenses is 20 cm for relased eye. What are the focal lengths of component lenses ?

Sol. Magnification $M = \frac{F}{f}$

Separaton between lenses

$d = F + f$

Given $\frac{F}{f} = 9$ i.e., $F = 9f$.....(1)

and $F + f = 20$ .....(2)

Putting value of F from (1) in (2), we get

$9f + f = 20 \Rightarrow 10f = 20$

$\Rightarrow \frac{20}{10} = 2$cm

$\therefore F = 9f = 9 \times 2 = 18$ cm

$\therefore F = 18$ cm, $f = 2$ cm

RAINBOW

(*a*) The seven coloured curved strip formed as a result of dispersion of light through water droplets which are suspended in the atmosphere after rains, is known as rainbow.

(*b*) The rainbow is of two types -

(i) Primary rainbow

(ii) Secondary rainbow

(*c*) Difference between primary and secondary rainbow

	Primary rainbow	Secondary rainbow
(i)	In this, the incident ray of light undergoes total internal reflection once.	It this the incident ray undergoes total internal reflection twice
(ii)	The order of the colours is from red to violet	The order of colours is from violet to red.
(iii)	It is formed by one total internal reflection and dispersion	It is formed by two internal reflections and dispersion.

Limit of resolution and resolving power of optical instruments –

(i) Resolving power –

(a) The ability of an optical instrument to produce separate diffraction patterns of two nearby objects is known as resolving power.

(b) The ability of an optical instrument to show two closely lying objects or spectral lines as separate, is known as its resolving power.

(ii) Limit of resolution – The reciprocal of resolving power is defined as the limit of resolution.

(iii) Rayleigh's limit of resolution – The distance between two object points, when the central maximum of diffraction pattern of one coincides with the first minimum of diffraction pattern of another, is defined as the Rayleigh's limit of resolution.

Resolving power (R.P.) of telescope

$$R.P = \frac{a}{1.22\lambda} = \frac{1}{\Delta\theta} = \frac{D}{d}$$

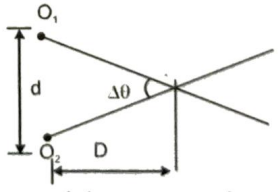
a = diameter of the aperture of objective

λ = wavelength of light used

$\Delta\theta$ = Limit of resolution

d = distance of two objects

D = distance of objects from objective lens



The resolving power of an electron microscope is 4×10^3 times that of an ordinary microscope.

Resolving power (R.P.) of microscope

$$R.P. = \frac{2\mu\sin\theta}{1.22\lambda}$$

μ = refractive index of medium.

Topicwise Questions

Plane Mirror

1. A ray of light incident on a plane miror at an angle of incidence of 30°. The deviation produced by the mirror is-
(a) 30° (b) 60°
(c) 90° (d) 120°
2. The image of a real object formed by a plane mirror is-
(a) Erect, real and of equal size
(b) Erect, virtual and of equal size
(c) Inverted, real and of equal size
(d) Inverted, virtual and of equal size
3. An object is at a distance of 0.5 m in front of a plane mirror. Distance between the object and image is
(a) 0.5 m (b) 1 m
(c) 0.25 m (d) 1.5 m
4. A man of height 160 cm. stands in front of a plane mirror. His eyes are at a height of 150 cm. from the floor. Then the minimum length of the plane mirror for him to see his full length image is
(a) 85 cm (b) 170 cm
(c) 80 cm (d) 340 cm

Curved Mirror

5. Which of the following could not produce a virtual image
(a) Plane mirror
(b) Convex mirror
(c) Concave mirror
(d) All the above can produce a virtual image
6. The field of view is maximum for
(a) Plane mirror (b) Concave mirror
(c) Convex mirror (d) Cylindrical mirror
7. A spherical mirror forms an erect image three times the linear size of the object. If the distance between the object and the image is 80 cm, the focal length of the mirror is
(a) 15 cm (b) –15 cm
(c) –30 cm (d) 40 cm
8. A convex mirror has a focal length f. A real object is placed at a distance f in front of it from the pole, then it produces an image at-
(a) Infinity (b) f
(c) f/2 (d) 2f
9. The image formed by convex mirror of focal length 30 cm is a quarter of the size of the object. Then the distance of the object from the mirror, is-
(a) 30 cm (b) 90 cm
(c) 120 cm (d) 60 cm
10. The largest distance of the image of a real object from a convex mirror of focal length 10 cm can be-
(a) 20 cm
(b) Infinite
(c) 10 cm
(d) Depends on the position of the object
11. In case of concave mirror, the minimum distance between a real object and its real image is-
(a) f (b) 2f
(c) 4f (d) Zero
12. An object is placed at a distance of 40 cm in front of a concave mirror of focal length 20cm. The image produced is :
(a) virtual and inverted
(b) real and erect
(c) real inverted and diminished
(d) real, inverted and of same size as the object.
13. Sun subtends an angle of 0.5° at the pole of a concave mirror of radius of curvature 15 m. The diameter of the image of the sun formed by the mirror is
(a) 8.55 cm (b) 7.55 cm
(c) 6.55 cm (d) 5.55 cm
14. Which of the following can form erect, virtual, diminished image?
(a) plane mirror (b) concave mirror
(c) convex mirror (d) none of these
15. The focal length of a concave mirror is 20 cm. Determine where an object must be placed to form an image magnified two times when the image is real-
(a) 30cm from the mirror (b) 10cm from the mirror
(c) 20cm from the mirror (d) 15cm from the mirror
16. A point source is placed 15 cm away from a convex mirror. A virtual image is formed at a distance of 6 cm. The radius of curvature of the mirror is :
(a) 4.3 cm (b) 8.6 cm
(c) 10 cm (d) 20 cm
17. A person sees his virtual image by holding a mirror very close to the face. When he moves the mirror away from his face, the image becomes inverted. What type of mirror he is using
(a) Plane mirror (b) Convex mirror
(c) Concave mirror (d) None of these

Refraction From Plane Surface

18. v_1 is velocity of light in first medium, v_2 is velocity of light in second medium, then refractive index of second medium with respect to first medium is
(a) v_1/v_2 (b) v_2/v_1
(c) $\sqrt{v_1/v_2}$ (d) $\sqrt{v_2/v_1}$
19. Velocity of light in water, glass and vacuum have the values v_w, v_g and v_c respectively. Which of the following relations is true?
(a) $v_w = v_g = v_c$ (b) $v_w > v_g$ but $v_w < v_c$
(c) $v_w = v_g$ but $v_w < v_c$ (d) $v_c = v_w$ and $v_w < v_g$

20. Time taken to cross a 4 mm window glass of refractive index 1.5 will be-

(*a*) 2×10^{8} sec (*b*) 2×10^{8} sec

(*c*) 2×10^{11} sec (*d*) 2×10^{11} sec

21. The wavelength of monochromatic blue light in air is 420 nm. What will be its wavelength in water? (μ of water is 4/3)

(*a*) 280 nm (*b*) 560 nm

(*c*) 315 nm (*d*) 400 nm

22. Monochromatic light is refracted from air into the glass of refractive index μ. The ratio of the wavelength of incident and refracted waves is

(*a*) 1 : μ (*b*) 1 : μ^2

(*c*) μ : 1 (*d*) 1 : 1

23. The refractive index of water and glass with respect to air are 1.3 and 1.5 respectively. What will be the refractive index of glass with respect to water-

(*a*) $\frac{15}{13}$ (*b*) $\frac{13}{15}$

(*c*) 2 (*d*) 3

24. Refractive index for a material for infrared light is

(*a*) Equal to that of ultraviolet light

(*b*) Less than that for ultraviolet light

(*c*) Equal to that for red colour of light

(*d*) Greater than that for ultraviolet light

25. If ${}_i\mu_j$ represents refractive index when a light ray goes from medium i to medium j, then the product ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3$ is equal to

(*a*) ${}_3\mu_1$ (*b*) ${}_3\mu_2$

(*c*) $({}_1\mu_4)^{-1}$ (*d*) ${}_4\mu_2$

26. A monochromatic beam of light passes from a denser medium into a rarer medium. As a result

(*a*) Its velocity increases

(*b*) Its velocity decreases

(*c*) Its frequency decreases

(*d*) Its wavelength decreases

27. When a small lamp is held 1.5m above the surface of water in a tank, its image formed by reflection at the surface appears to coincide with the image of the bottom of the tank. (μ of water = 4/3). The depth of the tank is

(*a*) 2 m (*b*) 1.5 m

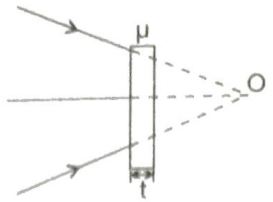
(*c*) 1 m (*d*) 4 m

28. An air bubble in a glass slab (μ = 1.5) is 6 cm deep as viewed from one face and 4 cm deep as viewed from the other face. The thickness of the glass slab is

(*a*) 6.67 cm (*b*) 10 cm

(*c*) 15 cm (*d*) Data is incomplete

29. A beam of light is converging towards a point. A plane parallel plate of glass of thickness t refractive index μ is introduced in the path of the beam. The convergent point is shifted by (assume near normal incidence):



(*a*) $t\left(1-\frac{1}{\mu}\right)$ away (*b*) $t\left(1+\frac{1}{\mu}\right)$ away

(*c*) $t\left(1-\frac{1}{\mu}\right)$ nearer (*d*) $t\left(1+\frac{1}{\mu}\right)$ nearer

30. A rectangular tank of depth 8 meter is full of water (μ = 4/3), the bottom is seen at the depth

(*a*) 6 m (*b*) 8/3 m

(*c*) 8 cm (*d*) 10 cm

31. An object is placed at 24 cm distance above the surface of a lake. If water has refractive index of 4/3, then at what distance from lake surface, a fish will sight the object-

(*a*) 32 cm above the surface of water

(*b*) 18 cm over the surface of water

(*c*) 6 cm over the surface of water

(*d*) 6 cm below the surface of water

Total Internal Reflection

32. Total internal reflection occurs in waves, when wave enters-

(*a*) Glass from air (*b*) Air from vaccum

(*c*) Water from air (*d*) Air from water

33. The wavelength of light in two liquids 'x' and 'y' is 3500 Å and 7000 Å. Then the critical angle of x relative to y will be

(*a*) 60° (*b*) 45°

(*c*) 30° (*d*) 15°

34. A light wave travels from glass to water. The refractive index for glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively. The value of the critical angle will be:

(*a*) $\sin^{-1}\left(\frac{1}{2}\right)$ (*b*) $\sin^{-1}\left(\frac{9}{8}\right)$

(*c*) $\sin^{-1}\left(\frac{8}{9}\right)$ (*d*) $\sin^{-1}\left(\frac{5}{7}\right)$

35. To an observer on the earth the stars appears to twinkle. This can be ascribed to

(*a*) The fact that stars do not emit light continuosly

(*b*) Frequent absorption of star light by their own atmosphere

(*c*) Frequent absorption of star light by the earth's atmosphere

(*d*) The refractive index fluctuations in the earth's atmosphere

36. The wavelength of light diminishes μ times (μ = 1.33 for water) in a medium. A driver from inside water looks at an object whose natural colour is green. He sees the object as
(a) Green (b) Blue
(c) Yellow (d) Red

37. We see the sun a little before it rises on the horizon and a little after it sets below the horizonal. This is a consequence of the phenomenon of
(a) Total internal reflection
(b) Refraction
(c) Dispersion
(d) Scattering of light

38. Mirages are observed in deserts due to phenomenon of
(a) Interference of light
(b) Total internal reflection
(c) Scattering of light
(d) Double reflection of light

39. When a light ray travels into a single medium then refractive index is
(a) directly proportional to wavelength of light
(b) inversely proportional to wavelength of light
(c) inversely proportional to the square of wavelength of light
(d) directly proportional to the square of wavelength of light

Prism

40. The ratio of the refractive index of red light to blue light in air is
(a) Less than unity
(b) Equal to unity
(c) Greater than unity
(d) Less as well as greater than unity depending upon the experimental arrangement

41. The refractive index of a piece of transparent quartz is the greatest for
(a) Red light (b) Violet light
(c) Green light (d) Yellow light

42. A ray of light is incident normally on one of the faces of a prism apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is:
(a) 15° (b) 22.5°
(c) 0° (d) 12.5°

43. A glass prism of refracting angle 60° gives a minimum deviation of 30°. What is the refractive index of the glass -
(a) 1.5 (b) $\sqrt{1.5}$
(c) $\sqrt{2}$ (d) can not be determined

44. The refractive index of the material of prism of 60° angle is $\sqrt{2}$. At what angle the ray of light be incident on it so that minimum deviation takes place?
(a) 45° (b) 60°
(c) 30° (d) 75°

45. A ray of monochromatic light is incident on one refracting face of a prism of angle 75°. It passes through the prism and is incident on the other face at the critical angle. If the refractive index of the material of the prism is $\sqrt{2}$, the angle of incidence on the first face of the prism is
(a) 30° (b) 45°
(c) 60° (d) 0°

46. A ray of light is incident at angle i on a surface of a prism of small angle A and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, the angle of incidence i is nearly equal to :
(a) A/μ (b) A/(2 μ)
(c) μ A (d) μ A/2

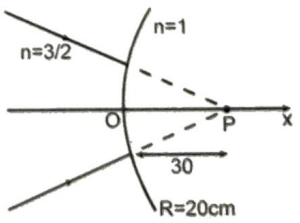
47. When light rays are incident on a prism at an angle of 45°, the minimum deviation is obtained. If refractive index of the material of prism is $\sqrt{2}$, then the angle of prism will be
(a) 30° (b) 40°
(c) 50° (d) 60°

48. The refractive indedx of a prism for a monochromatic wave is $\sqrt{2}$ and its refracting angle is 60°. For minimum deviation, the angle of incidence will be
(a) 30° (b) 45°
(c) 60° (d) 75°

Refraction at Curved Surface

49. There is a small black dot at the centre C of a solid glass sphere of refractive index μ. When seen from outside, the dot will appear to be located:
(a) away from C for all values of μ
(b) at C for all values of μ
(c) at C for μ = 1.5, but away from C for $\mu \neq 1.5$
(d) at C only for $\sqrt{2} \leq \mu \leq 1.5$.

50. The image for the converging beam after refraction through the curved surface is formed at:



(a) x = 40 cm (b) $x = \frac{40}{3}$ cm
(c) $x = -\frac{40}{3}$ cm (d) $x = \frac{180}{7}$ cm

51. An object is placed at a distance of 20 cm, in rarer medium, from the pole of a convex spherical refracting surface of radius of curvature 10 cm. If the refrective index of the rarer medium is 1 and of the refracting medium is 2, then the position of the image is at-

(*a*) (40/3) cm from the pole & inside the denser medium

(*b*) 40 cm from the pole & inside the denser medium.

(*c*) (40/3) cm from the pole & outside the denser medium

(*d*) 40 cm from the pole & outside the denser medium.

Lens

52. An object is placed at a distance of 5 cm from a convex lens of focal length 10 cm, then the image is

(*a*) real, diminished and at a distance of 10 cm from the lens

(*b*) real, enlarged and at a distance of 10 cm from the lens

(*c*) virtual, enlarged and at a distance of 10 cm from the lens

(*d*) virtual, diminished and at a distance of 10/3 cm from the lens.

53. A convex lens of focal length f will form a magnified real image of an object if the object is placed

(*a*) anywhere beyond 2f

(*b*) anywhere beyound f

(*c*) between f and 2f

(*d*) between lens and f

54. A lens behaves as a converging lens in air and a diverging lens in water. The refractive index of the material is (refractive index of water = 1.33)

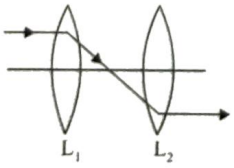
(*a*) equal to unity

(*b*) equal to 1.33

(*c*) between unity and 1.33

(*d*) greater than 1.33

55. In the figure given below, there are two convex lens L_1 and L_2 having focal length of f_1 and f_2 respectively. The distance between L_1 and L_2 will be



(*a*) f_1 (*b*) f_2

(*c*) $f_1 + f_2$ (*d*) $f_1 - f_2$

56. A biconvex lens with equal radii of curvature has refractive index 1.6 and focal length 10 cm. Its radius of curvature will be:

(*a*) 20 cm (*b*) 16 cm

(*c*) 10 cm (*d*) 12 cm

57. Inside water, an air bubble behave-

(*a*) Always like a convex lens

(*b*) Always like a concave lens

(*c*) Always like a slab of equal thickness

(*d*) Sometimes concave and sometimes like a convex lens

58. A glass lens is placed in a medium in which it is found to behave like a glas plate. Refractive index of the medium will be :

(*a*) Greater than the refractive index of glass

(*b*) Smaller than the refractive index of glass

(*c*) Equal to refractive index of glass

(*d*) No case will be possible from above

59. The radius of curvature of convex surface of a thin plano-convex lens is 15 cm and refractive index of its material is 1.6. The power of the lens will be

(*a*) +1D (*b*) –2D

(*c*) +3D (*d*) +4D

60. A convex lens of glass (μ = 1.5) is immersed in water. Compared to its power in air, its power in water-

(*a*) increases (*b*) decreases

(*c*) remain same (*d*) nothing can be predicted

61. A convex lens of power 4D and a concave lens of power 3D are placed in contact, the equivalent power of combination:

(*a*) 1D (*b*) $\frac{3}{4}$D

(*c*) 7D (*d*) $\frac{4}{3}$ D

62. Two thin lenses of power +5D and –2D are placed in contact with each other. Focal length of the combination will behave like a-

(*a*) Convex lens of focal length 3m

(*b*) Concave lens of focal length 0.33m

(*c*) Convex lens of focal length 0.33m

(*d*) None of the above

63. A divergent lens for real object will produce

(*a*) Always a virtual image

(*b*) Always real image

(*c*) Sometimes real and sometimes virtual

(*d*) None of the above

Dispersion

64. A person can't see the objects clearly placed at a distance more than 40 cm. He is advised to use a lens of power -

(*a*) + 2.5 D (*b*) – 2.5 D

(*c*) + 0.4 D (*d*) – 0.4 D

65. Dispersive power of a prism depends on-
(a) Material (b) Prism angle
(c) Shape of prism (d) Angle on incidence

66. Two thin lenses, one convex of focal length 30 cm and the other concave of focal length 10cm are put into contact. If this combination is equivalent to an achromatic lens then the ratio of dispersive powers (ω_1/ω_2) of above two lenses is -
(a) 1/3 (b) – 3
(c) 3 (d) – 1/3

67. When light is passed through a prism, the colour which deviates least is:
(a) Red (b) violet
(c) Blue (d) Green

68. When white light passes through a glass prism, one gets spectrum on the other side of the prism. In the emergent beam, the ray which is deviating least is or Deviation by a prism is lowest for
(a) Violet ray (b) Green ray
(c) Red ray (d) Yellow ray

69. Astigmatism (for a human eye) can be removed by using
(a) Concave lens
(b) Convex lens
(c) Cylindrical lens
(d) Prismatic lens

70. If refractive index of red, violet and yellow lights are 1.42, 1.62 and 1.50 respectively for a medium, its dispersive power will be -
(a) 0.4 (b) 0.3
(c) 0.2 (d) 0.1

Optical Instrument

71. The resolving power of a telescope is more when its objective lens has
(a) greater focal length
(b) smaller focal length
(c) greater diameter
(d) smaller diameter

72. The convex lens is used in-
(a) Microscope
(b) Telescope
(c) Projector
(d) All of the above

73. The magnifying power of a simple microscope can be increased if an eyepiece of :
(a) shorter focal length is used
(b) longer focal length is used
(c) shorter diameter is used
(d) longer diameter is used

74. A simple microscope has a focal length of 5 cm. The magnification at the least distance of distinct vision is-
(a) 1 (b) 5
(c) 4 (d) 6

75. In a compound microscope, the intermediate image is -
(a) virtual, erect and magnified
(b) real, erect and magnified
(c) real, inverted and magnified
(d) virtual, erect and reduced

76. An astronomical telescope has an eyepiece of focal-length 5 cm. If the angular magnification in normal adjustment is 10, when final image is at least distance of distinct vision (25cm) from eye piece, then angular magnification will be :
(a) 10 (b) 12
(c) 50 (d) 60

77. If in compound microscope m_1 and m_2 be the linear magnification of the objective lens and eye lens respectively, then magnifying power of the compound microscope will be
(a) $m_1 - m_2$ (b) $\sqrt{m_1 + m_2}$
(c) $(m_1 + m_2)/2$ (d) $m_1 \times m_2$

78. For which of the following colour, the magnifying power of a microscope will be maximum :
(a) White colour
(b) Red colour
(c) Violet colour
(d) Yellow colour

79. If the focal length of objective and eye lens are 1.2 cm and 3 cm respectively and the object is put 1.25 cm away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is :
(a) 150 (b) 200
(c) 250 (d) 400

80. An astronomical telescope has a magnifying power 10. The focal length of the eye piece is 20 cm. the focal length of the objective is-
(a) 2 cm (b) 200 cm
(c) (1/2) cm (d) (1/200) cm

Learning Plus

1. A straight line joining the object point and image point is always perpendicular to the mirror
   (*a*) if mirror is plane only
   (*b*) if mirror is concave only
   (*c*) if mirror is convex only
   (*d*) none of these
2. Mark the correct options.
   (*a*) If the incident rays are converging, we have a real object.
   (*b*) If the final rays are converging, we have a real image.
   (*c*) The image of a virtual object is called a virtual image.
   (*d*) If the image is virtual, the corresponding object is called a virtual object.
3. In case of concave mirror, the minimum distance between a real object and its real image is:
   (*a*) f (*b*) $2f$
   (*c*) $4f$ (*d*) zero
4. Two plane mirrors are inclined to each other at an angle 60°. If a ray of light incident on the first mirror is parallel to the second mirror, it is reflected from the second mirror
   (*a*) Perpendicular to the first mirror
   (*b*) Parallel to the first mirror
   (*c*) Parallel to the second mirror
   (*d*) Perpendicular to the second mirror
5. An object is initially at a distance of 100 cm from a plane mirror. If the mirror approaches the object at a speed of 5 cm/s. Then after 6 s the distance between the object and its image will be :
   (*a*) 60 cm (*b*) 140 cm
   (*c*) 170 cm (*d*) 150 cm
6. Images of an object placed between two plane mirrors whose reflecting surfaces make an angle of 90° with one another lie on a :
   (*a*) straight line
   (*b*) zig-zag curve
   (*c*) circle
   (*d*) ellipse
7. A point object at 15 cm from a concave mirror of radius of curvature 20 cm is made to oscillate along the principal axis with amplitude 2 mm. The amplitude of its image will be
   (*a*) 2 mm (*b*) 4 mm
   (*c*) 8 mm (*d*) 16 mm
8. The images of clouds and trees in water always less bright than in reality
   (*a*) because water is forming the image dirty
   (*b*) because there is an optical illusion due to which the image appears to be less bright
   (*c*) because only a portion of the incident light is reflected and quite a large portion goes mid water
   (*d*) because air above the surface of water contains a lot of moisture
9. In image formation from spherical mirrors, only paraxial rays are considered because they:
   (*a*) are easy to handle geometrically
   (*b*) contain most of the intensity of the incident light
   (*c*) form nearly a point image of a point source
   (*d*) show minimum dispersion effect.
10. The distance of an object from the focus of a convex mirror of radius of curvature ' a ' is ' b '. Then the distance of the image from the focus is:
    (*a*) $b^2 / 4a$ (*b*) a / b^2
    (*c*) $a^2 / 4b$ (*d*) $4b / a^2$
11. Which of the following can form erect, virtual, diminished image?
    (*a*) plane mirror (*b*) concave mirror
    (*c*) convex mirror (*d*) none of these
12. I is the image of a point object O formed by spherical mirror, then which of the following statements is **incorrect**:
    (*a*) If O and I are on same side of the principal axis, then they have to be on opposite sides of the mirror.
    (*b*) If O and I are on opposite side of the principal axis, then they have to be on same side of the mirror.
    (*c*) If O and I are on opposite side of the principal axis, then they can be on opposite side of the mirror as well.
    (*d*) If O is on principal axis then I has to lie on principal axis only.
13. A convex mirror has a focal length f. A real object, placed at a distance f in front of it from the pole, produces an image at
    (*a*) $2f$ (*b*) $f/2$
    (*c*) f (*d*) ∞
14. A convex mirror has a focal length = 20 cm. A convergent beam tending to converge to a point 20 cm behind convex mirror on principal axis falls on it. The image if formed at
    (*a*) infinity (*b*) 40 cm
    (*c*) 20 cm (*d*) 10 cm
15. A concave mirror gives an image three times as large as the object placed at a distance of 20 cm from it. For the image to be real, the focal length should be:
    (*a*) 10 cm (*b*) 15 cm
    (*c*) 20 cm (*d*) 30 cm

16. If an object is 30 cm away from a concave mirror of focal length 15 cm, the image will be
(a) erect (b) virtual
(c) diminished (d) of same size

17. The largest distance of the image of a real object from a convex mirror of focal length 20 cm can be :
(a) 20 cm
(b) infinite
(c) 10 cm
(d) depends on the position of the object.

18. What is the distance of a needle from a concave mirror of focal length 10 cm for which a virtual image of twice its height is formed?
(a) 2.5 cm (b) 5 cm
(c) 8 cm (d) 9.1 cm

19. A candle is kept at a distance equal to double the focal length from the pole of a convex mirror. its magnification will be :
(a) –1/3 (b) 1/3
(c) 2/3 (d) –2/3

20. Which one of the following statements are **incorrect** for spherical mirrors.
(a) a concave mirror forms only virtual images for any position of real object
(b) a convex mirror forms only virtual images for any position of a real object.
(c) a convex mirror forms only a virtual diminished image of an object placed between its pole and the focus
(d) a concave mirror forms a virtual magnified image of an object placed between its pole and the focus.

21. A vessel of depth 2d cm is half filled with a liquid of refractive index μ_1 and the upper half with a liquid of refractive index μ_2. The apparent depth of the vessel seen perpendicularly is

(a) $\left(\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right)$ d (b) $\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)$ d

(c) $\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)$ 2d (d) $\left(\frac{1}{\mu_1\mu_2}\right)$ 2d

22. A ray of light from a denser medium strike a rarer medium. The angle of reflection is r and that of refraction is r'. The reflected and refracted rays make an angle of 90° with each other. The critical angle will be
(a) $\sin^{-1}$ (tan r) (b) $\tan^{-1}$ (sin r)
(c) $\sin^{-1}$ (tan r') (d) $\tan^{-1}$ (sin r')

23. A plane glass slab is placed over various coloured letters. The letter which appears to be raised the least is:
(a) violet (b) yellow
(c) red (d) green

24. A plane glass slab is placed over various coloured letters. The letter which apears to be raised the least is :
(a) violet (b) yellow
(c) red (d) green

25. An under water swimmer is at a depth of 12 m below the surface of water. A bird is at a height of 18 m from the surface of water, directly above his eyes. For the swimmer, the bird appears to be at a distance X from the surface of water. (Refractive index of water is 4/3.) The value of X is
(a) 24 m (b) 12 m
(c) 18 m (d) 9 m

26. A bird is flying 3 m above the surface of water. If the bird is diving vertically down with speed = 6 m/s, his apparent velocity as seen by a stationary fish underwater is
(a) 8 m/s (b) 6 m/s
(c) 12 m/s (d) 4 m/s

27. The wavelength of light in vacuum is 6000 A° and in a medium it is 4000 A°. The refractive index of the medium is :
(a) 2.4 (b) 1.5
(c) 1.2 (d) 0.67

28. A ray of light passes from vacuum into a medium of refractive index n. If the angle of incidence is twice the angle of refraction, then the angle of incidence is:
(a) $\cos^{-1}$ (n/2) (b) $\sin^{-1}$ (n/2)
(c) 2 $\cos^{-1}$ (n/2) (d) 2 $\sin^{-1}$ (n/2)

29. The critical angle of light going from medium A to medium B is θ. The speed of light in medium A is v. The speed of light in medium B is:

(a) $\frac{v}{\sin\theta}$ (b) v sin θ
(c) v cot θ (d) v tan θ

30. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and fish is 12 cm below the surface, the radius of the circle in cm is
(a) $12 \times 3 \times \sqrt{5}$ (b) $4 \times \sqrt{5}$
(c) $12 \times 3 \times \sqrt{7}$ (d) $12 \times 3 / \sqrt{7}$

31. Critical angle of light passing from glass to air is minimum for
(a) red (b) green
(c) yellow (d) violet

32. If the critical angle for the medium of prism is C and the angle of prism is A, then there will be no emergent ray when -
(a) $A < 2C$ (b) $A = 2C$
(c) $A > 2C$ (d) $A \geq 2C$

33. A ray of light is incident at angle i on a surface of a prism of small angle A & emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, the angle of incidence i is nearly equal to:

(a) $\frac{A}{\mu}$ (b) $\frac{A}{(2\mu)}$

(c) μA (d) $\mu\frac{A}{2}$

34. A beam of monochromatic light is incident at i = 50° on one face of an equilateral prism, the angle of emergence is 40°, then the angle of minimum deviation is :

(a) 30° (b) < 30°

(c) ≤ 30° (d) ≥ 30°

35. The angle of a prism is 60° and the index of refraction of glass with air is 1.5. If the angle of incidence on the first face is I_1 and the angle of emergence at the second face is I_2, then the prism produces minimum deviation when

(a) $I_1 = 0$ (b) $I_1 > I_2$

(c) $I_1 < I_2$ (d) $I_1 = I_2$

36. A prism having refractive index $\sqrt{2}$ and refracting angle 30°, has one of the refracting surfaces polished. A beam of light incident on the other refracting surface will retrace its path if the angle of incidence is:

(a) 0° (b) 30°

(c) 45° (d) 60°

37. A prism of refractive index $\sqrt{2}$ has refracting angle 60°. In order that a ray suffers minimum deviation it should be incident at an angle

(a) 45° (b) 90°

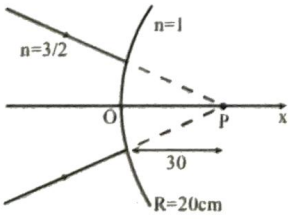
(c) 30° (d) None

38. A ray of light is incident at an angle 60° on the face of a prism having refractive angle 30°.The ray emerging out of the prism makes an angle 30° with the incident ray θ through which it emerges from the surfece.

(a) 0° (b) 30°

(c) 45° (d) 60°

39. The image for the converging beam after refraction through the curved surface (in the given figure) is formed at:



(a) x = 40 cm (b) $x = \frac{40}{3}$ cm

(c) $x = -\frac{40}{3}$ cm (d) $x = \frac{180}{7}$ cm

40. When a lens of power P (in air) made of material of refractive index μ is immersed in liquid of refractive index μ_0. Then the power of lens is:

(a) $P\frac{\mu - \mu_0}{\mu - 1}$ (b) $\frac{\mu - 1}{\mu - \mu_0}P$

(c) $\frac{\mu - \mu_0}{\mu - 1} \cdot \frac{P}{\mu_0}$ (d) None of these

41. Two symmetric double convex lenses A and B have same focal length, but the radii of curvature differ so that $R_A = 0.9\ R_B$. If $n_A = 1.63$, find n_B.

(a) 1.7 (b) 1.6

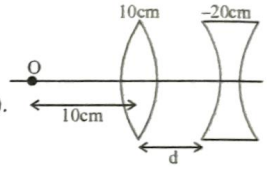
(c) 1.5 (d) 4/3

42. A convexo - concave diverging lens is made of glass of refractive index 1.5 and focal length 24 cm. Radius of curvature for one surface is double that of the other. Then radii of curvature for the two surfaces are (in cm):

(a) 6, 12 (b) 12, 24

(c) 3, 6 (d) 18, 36

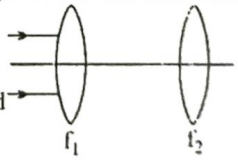
43. What should be the value of distance d so that final image is formed on the object itself. (focal lengths of the lenses are as given in the figure).



(a) 10 cm (b) 20 cm

(c) 5 cm (d) none of these

44. Parallel beam of light is incident on a system of two convex lenses of focal lengths f_1 = 20 cm and f_2 = 10 cm. What should be the distance between the two lenses so that rays after refraction from both the lenses pass undeviated



(a) 60 cm (b) 30 cm

(c) 90 cm (d) 40 cm

45. A thin lens of focal length f and its aperture diameter d, forms a real image of intensity I. Now the central part of the aperture upto diameter $(\frac{d}{2})$ is blocked by an opaque paper. The focal length and image intensity would change to

(a) $\frac{f}{2}, \frac{I}{2}$ (b) $f, \frac{I}{4}$

(c) $\frac{3f}{4}, \frac{I}{2}$ (d) $f, \frac{3I}{4}$

46. A biconvex lens has a focal length of 10 cm. It is cut in half and two pieces are placed as shown. The focal length of the final combination is:



(*a*) 10 (*b*) 20
(*c*) 40 (*d*) Not a lens

47. Two plano-convex lenses each of focal length 10 cm & refractive index $\frac{3}{2}$ are placed as shown in the figure. In the space left, water $\left(\text{R.I.}=\frac{4}{3}\right)$ is filled. The whole arrangement is in air. The optical power of the system is (in dioptre):



(*a*) 6.67 (*b*) - 6.67
(*c*) 33.3 (*d*) 20

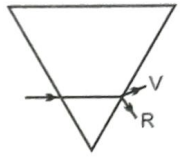
48. A lens of power + 2.0 D is placed in contact with another lens of power – 1.0 D. The combination will behave like
(*a*) a converging lens of focal length 100 cm
(*b*) a diverging lens of focal length 100 cm
(*c*) a converging lens of focal length 50 cm
(*d*) a diverging lens of focal length 50 cm.

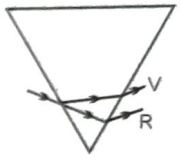
49. A medium has $n_v = 1.56$, $n_r = 1.44$. Then its dispersive power is :

(*a*) $\frac{3}{50}$ (*b*) $\frac{6}{25}$
(*c*) 0.03 (*d*) None of these

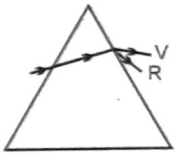
50. The refractive index of flint glass for blue line is 1.6333 and red line is 1.6161, then dispersive power of the glass is :
(*a*) 0.0276 (*b*) 0.276
(*c*) 2.76 (*d*) 0.106

51. Which of the following diagrams shows correctly the dispersion of white light by a prism?

(*a*)


(*b*)


(*c*)

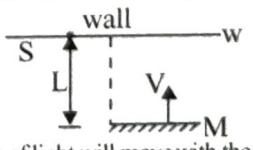

(*d*)


Advanced Level Multiconcept Questions

MCQ/STATEMENT/COMPREHENSION/ MATCHING

1. Which of the following statements are **incorrect** for spherical mirrors.
(*a*) a concave mirror forms only virtual images for any position of real object
(*b*) a convex mirror forms only virtual images for any position of a real object
(*c*) a concave mirror forms only a virtual diminished image of an object placed between its pole and the focus
(*d*) a convex mirror forms a virtual enlarged image of an object if it lies between its pole and the focus.

2. A flat mirror M is arranged parallel to a wall W at a distance L from it as shown in the figure. The light produced by a point source S kept on the wall is reflected by the mirror and produces a light patch on the wall. The mirror moves with velocity v towards the wall.



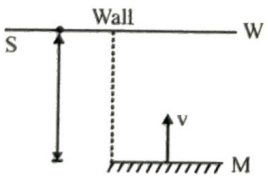
(*a*) The patch of light will move with the speed v on the wall.
(*b*) The patch of light will not move on the wall.
(*c*) As the mirror comes closer the patch of light will become larger and shift away from the wall with speed larger than v.
(*d*) The width of the light patch on the wall remains the same.

3. A man wants to photograph a white donkey as a Zebra after fitting a glass with black streaks onto the lens of his camera.
   (*a*) The image will look like a white donkey on the photograph.
   (*b*) The image will look like a Zebra on the photograph
   (*c*) The image will be more intense compared to the case in which no such glass is used.
   (*d*) The image will be less intense compared to the case in which no such glass is used.

4. Two plane mirrors are inclined to each other with their reflecting faces making acute angle. A light ray is incident on one plane mirror. The total deviation after two successive reflections is:
   (*a*) independent of the initial angle of incidence
   (*b*) independent of the angle between the mirrors
   (*c*) dependent on the initial angle of incidence
   (*d*) dependent on the angle between the mirrors.

5. A man of height 170 cm wants to see his complete image in a plane mirror (while standing). His eyes are at a height of 160 cm from the ground.
   (*a*) Minimum length of the mirror = 80 cm
   (*b*) Minimum length of the mirror = 85 cm
   (*c*) Bottom of the mirror should be at a height 80 cm or less
   (*d*) Bottom of the mirror should be at a height 85 cm

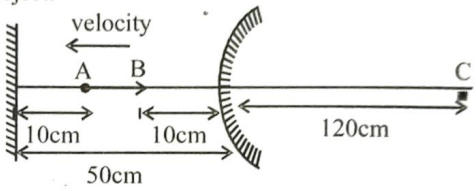
6. A flat mirror M is arranged parallel to a wall W at a distance *l* from it. The light produced by a point source S kept on the wall is reflected by the mirror and produces a light spot on the wall. The mirror moves with velocity v towards the wall.



   (*a*) The spot of light will move with the speed v on the wall
   (*b*) The spot of light will not move on the wall
   (*c*) As the mirror comes closer the spot of light will become larger and shift away from the wall with speed larger then v
   (*d*) The size of the light on the wall remains the same

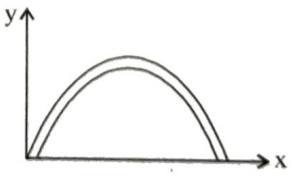
7. An object is kept on the principal axis of a convex mirror of focal length 10 cm at a distance of 10 cm from the pole. The object starts moving at a velocity 20 mm/sec towards the mirror at angle 30° with the principal axis. What will be the speed of its image and direction with the principal axis at that instant.
   (*a*) speed $= 5\frac{\sqrt{7}}{4}$ mm/sec
   (*b*) speed $= \frac{5\sqrt{7}}{2}$ mm/sec
   (*c*) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ with the principal axis
   (*d*) None of these

8. In the figure shown consider the first reflection at the plane mirror and second at the convex mirror. AB is object.



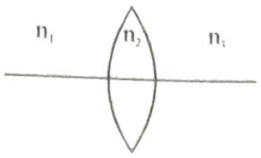
   (*a*) the second image is real, inverted of 1/5th magnification
   (*b*) the second image is virtual and erect with magnification 1/5
   (*c*) the second image moves towards the convex mirror.
   (*d*) the second image moves away from the convex mirror.

9. A reflecting surface is represented by the equation $Y = \frac{2L}{\pi}\sin\left(\frac{\pi x}{L}\right)$, $0 \le x \le L$. A ray travelling horizontally becomes vertical after reflection. The coordinates of the point (s) where this ray is incident is



   (*a*) $\left(\frac{L}{4}, \frac{\sqrt{2}L}{\pi}\right)$
   (*b*) $\left(\frac{L}{3}, \frac{\sqrt{3}L}{\pi}\right)$
   (*c*) $\left(\frac{3L}{4}, \frac{\sqrt{2}L}{\pi}\right)$
   (*d*) $\left(\frac{2L}{3}, \frac{\sqrt{3}L}{\pi}\right)$

10. An equiconvex lens of refractive index n_2 is placed such that the refractive index of the surrounding media is as shown. Then the lens :



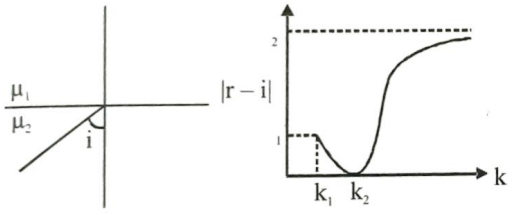
(*a*) must be diverging if n_2 is less than the arithmetic mean of n_1 and n_3

(*b*) must be converging if n_2 is greater than the arithmetic mean of n_1 and n_3

(*c*) may be diverging if n_2 is less than the arithmetic mean of n_1 and n_3

(*d*) will neither be diverging nor converging if n_2 is equal to arithmetic mean of n_1 and n_3

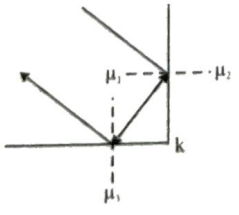
11. The figure shows ray incident at an angle $i = \pi/3$. If the plot drawn shown the variation of $|r - i|$ versus $\mu_1/\mu_2 = k$, (r = angle of refraction)



(*a*) the value of k_1 is $\frac{2}{\sqrt{3}}$ (*b*) the value of $\theta_1 = \pi/6$

(*c*) the value of $\theta_2 = \pi/3$ (*d*) the value of k_2 is 1

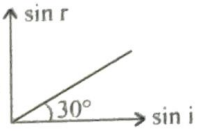
12. In the diagram shown, a ray of light is incident on the interface between 1 and 2 at angle slightly greater than critical angle. The light surfers total internal reflection at this interface. After that the light ray falls at the interface of 1 and 3, and again it suffers total internal reflection. Which of the following relations should hold true?



(*a*) $\mu_1 > \mu_2 < \mu_3$ (*b*) $\mu_1^2 - \mu_2^2 > \mu_3^2$

(*c*) $\mu_1^2 - \mu_3^2 > \mu_2^2$ (*d*) $\mu_1^2 + \mu_2^2 > \mu_3^2$

13. A ray of monochromatic light is incident on the plane surface of separation between two media *x* and *y* with angle of incidence '*i*' in the medium x and angle of refraction 'r' in the medium *y*. The graph shows the relation between sin r and sin i.



(*a*) the speed of light in the medium y is $(3)^{1/2}$ times than in medium x.

(*b*) the speed of light in the medium y is $(1/3)^{1/2}$ times than in medium x.

(*c*) the total internal reflection can take place when the incidence is in x.

(*d*) the total internal reflection can take place when the incidence is in y.

14. An equilateral prism deviates a ray through 40° for two angles of incidence differing by 20°. The possible angle of incidences are :

(*a*) 40° (*b*) 50°

(*c*) 20° (*d*) 60°

15. For the refraction of light through a prism

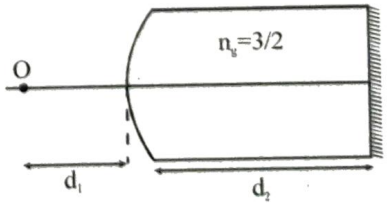
(*a*) For every angle of deviation there are two angles of incidence.

(*b*) The light travelling inside an equilateral prism is necessarily parallel to the base when prism is set for minimum deviation.

(*c*) There are two angles of incidence for maximum deviation.

(*d*) Angle of minimum deviation will increase if refractive index of prism is increased keeping the outside medium unchanged if $\mu_p > \mu_s$.

16. In the figure shown a point object O is placed in air on the principal axis. The radius of curvature of the spherical surface is 60 cm. I_f is the final image formed after all the refractions and reflections.



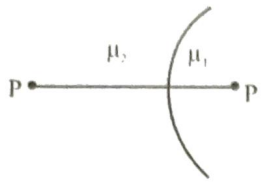
(*a*) If d_1 = 120 cm, then the 'I_f' is formed on 'O' for any value of d_2.

(*b*) If d_1 = 240 cm, then the 'I_f' is formed on 'O' only if d_2 = 360 cm.

(*c*) If d_1 = 240 cm, then the 'I_f' is formed on 'O' for all values of d_2.

(*d*) If d_1 = 240 cm, then the 'I_f' cannot be formed on 'O'.

17. Two refracting media are separated by a spherical interface as shown in the figure. PP′ is the principal axis, μ_1 and μ_2 are the refractive indices of medium of incidence and medium of refraction respectively. Then:



(*a*) if $\mu_2 > \mu_1$, then there cannot be a real image of real object.

(*b*) if $\mu_2 > \mu_1$, then there cannot be a real image of virtual object.

(*c*) if $\mu_1 > \mu_2$, then there cannot be a virtual image of virtual object.

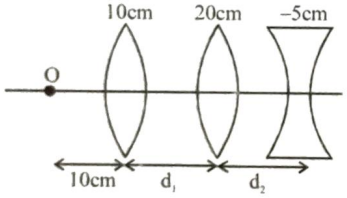
(*d*) if $\mu_1 > \mu_2$, then there cannot be a real image of real object.

18. Which of the following quantities related to a lens depend on the wavelength of the incident light?

(*a*) power (*b*) focal length

(*c*) chromatic aberration (*d*) radii of curvature

19. The values of d_1 & d_2 for final rays to be parallel to the principal axis are : (focal lengths of the lenses are written above the respective lenses in the given figure)



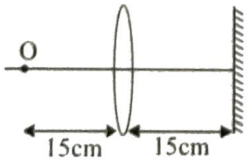
(*a*) $d_1 = 10$ cm, $d_2 = 15$ cm]

(*b*) $d_1 = 20$ cm, $d_2 = 15$ cm

(*c*) $d_1 = 30$ cm, $d_2 = 15$ cm

(*d*) None of these

20. An object O is kept infront of a converging lens of focal length 30 cm behind which there is a plane mirror at 15 cm from the lens as shown in the figure.



(*a*) the final image is formed at 60 cm from the lens towards right of it

(*b*) the final image is at 60 cm from lens towards left of it

(*c*) the final image is real

(*d*) the final image is virtual.

21. A man wishing to get a picture of a Zebra photographed a white donkey after fitting a glass with black streaks onto the objective of his camera.

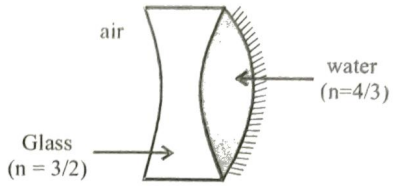
(*a*) the image will look like a white donkey on the photograph.

(*b*) the image will look like a Zebra on the photograph.

(*c*) the image will be more intense compared to the case in which no such glass is used.

(*d*) the image will be less intense compared to the case in which no such glass is used.

22. The radius of curvature of the left & right surface of the concave lens are 10 cm & 15 cm respectively. The radius of curvature of the mirror is 15 cm.



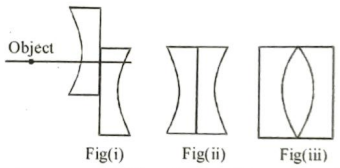
(*a*) equivalent focal length of the combination is −18cm

(*b*) equivalent focal length of the combination is +36cm

(*c*) the system behaves like a concave mirror

(*d*) the system behaves like a convex mirror.

23. If a symmetrical biconcave thin lens is cut into two identical halves. They are placed in different ways as shown:



(*a*) three images will be formed in case (i)

(*b*) two images will be formed in the case (i)

(*c*) the ratio of focal lengths in (ii) & (iii) is 1

(*d*) the ratio of focal lengths (in) (ii) & (iii) is 2

24. A convex lens forms an image of an object on screen. The height of the image is 9 cm. The lens is now displaced until an image is again obtained on the screen. The height of this image is 4 cm. The distance between the object and the screen is 90 cm.

(*a*) The distance between the two positions of the lens is 30 cm.

(*b*) The distance of the object from the lens is its first position is 36cm.

(*c*) The height of the object is 6cm.

(*d*) The focal length of the lens is 21.6 cm.

25. A narrow beam of white light goes through a slab having parallel faces

(*a*) The light never splits in different colours

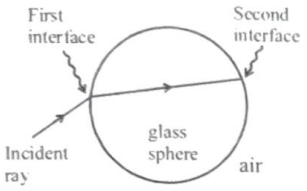
(*b*) The emergent beam is white

(*c*) The light inside the slab is split into different colours

(*d*) The light inside the slab is white

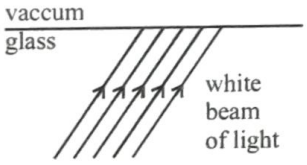
26. By properly combining two prisms made of different materials, it is possible to
(*a*) have dispersion without average deviation
(*b*) have deviation without dispersion
(*c*) have both dispersion and average deviation
(*d*) have neither dispersion nor average deviation

27. **Statement-1 :** A ray is incident from outside on a glass sphere surrounded by air as shown. This ray may suffer total internal reflection at second interface.



Statement-2 : For a ray going from denser to rarer medium, the ray may suffer total internal reflection.
(*a*) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(*b*) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(*c*) Statement-1 is True, Statement-2 is False
(*d*) Statement-1 is False, Statement-2 is True.

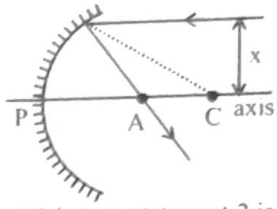
28. **Statement- 1:** A white parallel beam of light is incident on a plane glass- vacuum interface as shown. The beam may not undergo dispersion after suffering deviation at the interface (The beam is not incident normally on the interface.)



Statement-2: Vacuum has same refractive index for all colours of white light.
(*a*) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(*b*) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(*c*) Statement-1 is True, Statement-2 is False
(*d*) Statement-1 is False, Statement-2 is True.

29. **Statement-1 :** If a source of light is placed in front of rough wall its image is not seen.
Statement-2 : The wall does not reflect light.
(*a*) Statement-1 is true, statement-2 is true; statement-2 is correct explanantion for satement-1
(*b*) Statement-1 is true, statement-2 is true; statement-2 is NOT correct explanantion for satement-1
(*c*) Statement - 1 is true, Statement - 2 is false.
(*d*) Statement - 1 is false, statement - 2 is true.

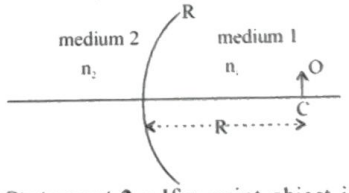
30. **Statement-1 :** As the distance x of a parallel ray from axis increases, focal length decreases
Statement-2 : As x increases, the distance from pole to the point of intersection of reflected ray with principal axis decreases



(*a*) Statement-1 is true, statement-2 is true; statement-2 is correct explanantion for satement-1
(*b*) Statement-1 is true, statement-2 is true; statement-2 is NOT correct explanantion for satement-1
(*c*) Statement-1 is true, Statement-2 is false.
(*d*) Statement-1 is false, statement-2 is true.

31. **Statement-1 :** A point object moves near the principal axis of a fixed spherical mirror along a straight line. Then the image formed by the spherical mirror also moves along a straight line.
Statement-2 : For an incident ray on a fixed spherical mirror there is a fixed reflected ray. If a point object moves along this incident ray, its image will always lie on the given reflected ray. Further an incident ray can be drawn from the moving point object in its direction of velocity towards the mirror.
(*a*) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(*b*) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(*c*) Statement-1 is True, Statement-2 is False
(*d*) Statement-1 is False, Statement-2 is True

32. **Statement--1 :** A spherical surface of radius of curvature R separates two media of refractive index n_1 and n_2 as shown. If an object O (a thin small rod) is placed upright on principal axis at a distance R from pole (i.e., placed at centre of curvature), then the size of image is same as size of object.



Statement-2 : If a point object is placed at centre of curvature of spherical surface separating two media of different refractive index, then the image is also formed at centre of curvature, i.e., image distance is equal to object distance.
(*a*) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(*b*) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(*c*) Statement-1 is True, Statement-2 is False
(*d*) Statement-1 is False, Statement-2 is True

33. **Statement-1 :** When an object dipped in a liquid is viewed normally, the distance between the image and the object is indepedent of the height of the liquid above the object.

Statement-2 : The normal shift is independent of the location of the slab between the object and the observer.

(*a*) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

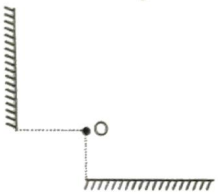
(*b*) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(*c*) Statement-1 is True, Statement-2 is False

(*d*) Statement-1 is False, Statement-2 is True

34. **Statement-1 :** When two plane mirrors are kept perpendicular to each other as shown (O si the point object), 3 image will be formed.

Statement-2 : In case of multiple reflection, image of one surface can act as an object for the next surface.



(*a*) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(*b*) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(*c*) Statement-1 is True, Statement-2 is False

(*d*) Statement-1 is False, Statement-2 is True

35. **Statement-1 :** A piece of paper placed at the position of a real image of a virtual object of intense light will burn after sufficient time.

Statement-2 : A virtual object is that point where the incident rays appear to converge and a real image is that point at which reflected/ refracted rays actually converge.

(*a*) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(*b*) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(*c*) Statement-1 is True, Statement-2 is False

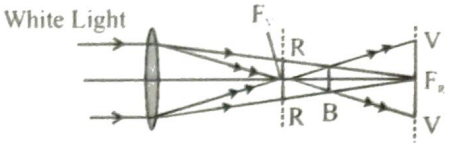
(*d*) Statement-1 is False, Statement-2 is True

Comprehension – 1 (Q. No. 36 to 40)

Chromatic Aberration: The image of a white object in white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colours. As R.I. μ of lens is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it as shown in figure.

As a result of this, in case of convergent lens if a screen is placed at F_V centre of the image will be violet and focused while sides are red and blurred. While at F_R, reverse is the case, i.e., centre will be red and focused while sides violet and blurred. The difference between f_V and f_R is a measure of the longitudinal chromatic aberration (L.C.A), i.e.,

L.C.A. $= f_R - f_V = -df$ with $df = f_V - f_R$.....(1)



However, as for a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad .....(2)$$

$$\Rightarrow \quad -\frac{df}{f^2} = d\mu\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad .....(3)$$

Dividing Eqn. (3) by (2) ;

$$-\frac{df}{f} = \frac{d\mu}{(\mu - 1)} = \omega \quad \left[\omega = \frac{d\mu}{(\mu - 1)}\right]$$

= dispersive power .....(4)

And hence, from Eqns. (1) and (4),

L.C.A. $= -df = \omega f$

Now, as for a single lens neither f nor ω can be zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism:

In case of two thin lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \text{ i.e., } -\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if $dF = 0$

$$\text{i.e., } \quad \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

which with the help of Eqn. (4) reduces to

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0 \text{ i.e., } \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad .....(5)$$

This condition is called condition of achromatism (for two thin lenses in contact) and the lens combination which satisfies this condition is called achromatic lens, from this condition, i.e., from Eqn. (5) it is clear that in case of achromatic doublet:

(*a*) The two lenses must be of different materials.

Since, if $\omega_1 = \omega_2$, $\frac{1}{f_1} + \frac{1}{f_2} = 0$ i.e., $\frac{1}{F} = 0$ or $F = \infty$

i.e., combination will not behave as a lens, but as a plane glass plate.

(*b*) As ω_1 and ω_2 are positive quantities, for equation (5) to hold, f_1 and f_2 must be of opposite nature, i.e. if one of the lenses is converging the other must be diverging.

(*c*) If the achromatic combination is convergent,

$$f_C < f_D \text{ and as } -\frac{f_C}{f_D} = \frac{\omega_C}{\omega_D}, \omega_C < \omega_D$$

i.e., in a convergent achromatic doublet, convex lens has lesser focal length and dispersive power than the divergent one.

36. Chromatic aberration in the formation of images by a lens arises because :

(*a*) of non-paraxial rays.

(*b*) the radii of curvature of the two sides are not same.

(*c*) of the defect in grinding.

(*d*) the focal length varies with wavelength.

37. Chromatic aberration of a lens can be corrected by :

(*a*) providing different suitable curvatures of its two surfaces.

(*b*) proper polishing of its two surfaces.

(*c*) suitably combining it with another lens.

(*d*) reducing its aperture.

38. A combination is made of two lenses of focal lengths f and f' in contact ; the dispersive powers of the materials of the lenses are ω and ω'. The combination is achromatic when :

(*a*) $\omega = \omega_0, \omega' = 2\omega_0, f' = 2f$

(*b*) $\omega = \omega_0, \omega' = 2\omega_0, f' = f/2$

(*c*) $\omega = \omega_0, \omega' = 2\omega_0, f' = -f/2$

(*d*) $\omega = \omega_0, \omega' = 2\omega_0, f' = -2f$

39. The dispersive power of crown and flint glasses are 0.02 and 0.04 respectively. An achromatic converging lens of focal length 40 cm is made by keeping two lenses, one of crown glass and the other of flint glass, in contact with each other. The focal lengths of the two lenses are:

(*a*) 20 cm and 40 cm (*b*) 20 cm and –40 cm

(*c*) –20cm and 40 cm (*d*) 10 cm and –20cm

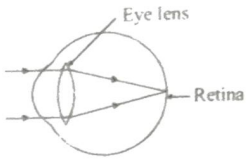
40. Chromatic aberration in a spherical concave mirror is proportional to :

(*a*) f (*b*) f^2

(*c*) $1/f$ (*d*) None of these

Comprehension–2 (Q. No. 41 to 44)

The ciliary muscles of eye control the curvature of the lens in the eye and hence can alter the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained the curvature of lens increases (that means radius of curvature decreases) and focal length decreases. For a clear vision the image must be on retina. The image distance is therefore fixed for clear vision and it equals the distance of retina from eye-lens. It is about 2.5 cm for a grown-up person (Refer the figure below).



A person can theoretically have clear vision of objects situated at any large distance from the eye. The smallest distance at which a person can clearly see is related to minimum possible focal length. The ciliary muscles are most strained in this position. For an average grown-up person minimum distance of object should be around 25 cm.

A person suffering for eye defects uses spectacles (eye glass). The function of lens of spectacles is to form the image of the objects within the range in which person can see clearly. The image of the spectacle-lens becomes object for eye-lens and whose image is formed on retina.

The number of spectacle-lens used for the remedy of eye defect is decided by the power of the lens required and the number of spectacle-lens is equal to the numerical value of the power of lens with sign. For example power of lens required is +3D (converging lens of focal length $\frac{100}{3}$ cm) then number of lens will be + 3.

For all the calculations required you can use the lens formula and lens maker's formula. Assume that the eye lens is equiconvex lens. Neglect the distance between eye lens and the spectacle lens.

41. Minimum focal length of eye lens of a normal person is

(*a*) 25 cm (*b*) 2.5 cm

(*c*) $\frac{25}{9}$ cm (*d*) $\frac{25}{11}$ cm

42. Maximum focal length of eye lens of normal person is

(*a*) 25 cm (*b*) 2.5 cm

(*c*) $\frac{25}{9}$ cm (*d*) $\frac{25}{11}$ cm

43. A nearsighted man can clearly see object only upto a distance of 100 cm and not beyond this. The number of the spectacles lens necessary for the remedy of this defect will be.

(*a*) +1 (*b*) – 1

(*c*) + 3 (*d*) – 3

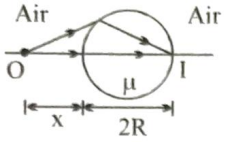
44. A farsighted man cannot see object clearly unless they are at least 100 cm from his eyes. The number of the spectacles lens that will make his range of clear vision equal to an average grown up person

(*a*) + 1 (*b*) – 1

(*c*) + 3 (*d*) – 3

Comprehension–3 (Q. No. 45 to 47)

The figure, shows a transparent sphere of radius R and refractive index μ. An object O is placed at a distance x from the pole of the first surface so that a real image is formed at the pole of the exactly opposite surface.



45. If x = 2R, then the value of μ is
(*a*) 1.5 (*b*) 2
(*c*) 3 (*d*) None of these

46. If x = ∞, then the value of μ is
(*a*) 1.5 (*b*) 2
(*c*) 3 (*d*) none of these

47. If an object is placed at a distance R from the pole of first surface, then the real image is formed at a distance R from the pole of the second surface. The refractive index μ of the sphere is given by
(*a*) 1.5 (*b*) 2
(*c*) $\sqrt{2}$ (*d*) none of these

Comprehension–4 (Q. No. 48 to 50)

A turnip sits before a thin converging lens, outside the focal point of the lens. The lens is filled with a transparent gel so that it is flexible; by squeezing its ends toward its center [as indicated in figure(*a*)], you can change the curvature of its front and rear sides.

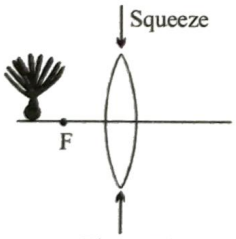


Figure (a)

48. When you squeeze the lens, the image.
(*a*) moves towards the lens
(*b*) moves away from the lens
(*c*) shifts up
(*d*) remains as it is

49. When you squeeze the lens, the lateral height of image.
(*a*) increases (*b*) decreases
(*c*) remains same (*d*) data insufficient

50. Suppose that a sharp image must be formed on a card which is at a certain distance behind the lens [figure(*b*)], while you move the turnip away from the lens, then you should

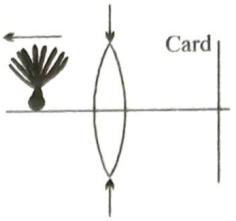


Figure (b)

(*a*) decrease the squeeze of the lens
(*b*) increase the squeeze of the lens
(*c*) keep the card and lens as it is.
(*d*) move the card away from the lens

Comprehension–5 (Q. No. 51 to 53)

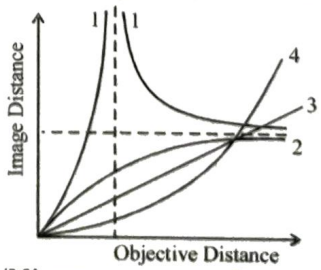
A prism of refractive index $\sqrt{2}$ has refracting angle 60°. Answer the following questions

51. In order that a ray suffers minimum deviation it should be incident at an angle :
(*a*) 45° (*b*) 90°
(*c*) 30° (*d*) None

52. Angle of minimum deviation is :
(*a*) 45° (*b*) 90°
(*c*) 30° (*d*) None

53. Angle of maximum deviation is :
(*a*) 45°
(*b*) $\sin^{-1}(\sqrt{2}\sin 15°)$
(*c*) $30° + \sin^{-1}(\sqrt{2}\sin 15°)$
(*d*) none

54. A small particle is placed at the pole of a concave mirror and then moved along the principal axis to a large distance. During the motion, the distance between the pole of the mirror and the image is measured. The procedure is then repeated with a convex mirror, a concave lens and a convex lens. The graph is plotted between image distance versus object distance. Match the curves shown in the graph with the mirror or lens that is corresponding to it. (Curve 1 has two segments)



Lens/Mirror	Curve
(*a*) Converging lens	(p) 1
(*b*) Converging Mirror	(q) 2
(*c*) Diverging Lens	(r) 3
(*d*) Diverging Mirror	(s) 4

55. Column-II shows the optical phenomenon that can be associated with optical components given in Column-I. Note that Column-I may have more than one matching options in Column-II.

Column-I	Column-II
(i) Convex mirror	(A) Dispersion
(ii) Converging lens	(B) Deviation
(iii) Thin prism	(C) Real image of real object
(iv)Glass slab	(D) Virtual image of real object.

NUMERICAL VALUE BASED

56. For an achromatic combination, three different lenses are combined. Dispersive power of their material are 0.066, 0.055 and 0.040 and their main focal lengths are – 22 cm, –11 cm and 'f' respectively then find the value of 'f' (in cm.)

57. An object is located at a distance x from a converging lens (Focal length f = 18 cm) on its optic axis. A plane mirror is attached to the lens on the other side of the lens. The final image is formed at a distance of 2f from the lens on the same side as the object. What is the distance of the object from the lens? (in cm).

58. Two objects are placed on the principal axis of a thin converging lens. One is 10 cm from the lens and the other is on the other side of the lens at a distance of 40 cm from the lens. The images of both objects are in the same plane. What is the focal length of the lens (in cm)?

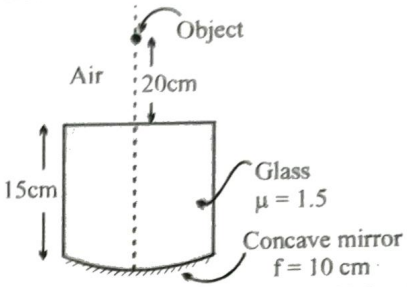
59. The plane surface of a Plano-convex lens is silvered and it then acts like concave mirror of 30 cm focal length. Now the silvering is removed and the lens is used to make the image of an object placed on its principal axis. The image is seen to be inverted with a magnification of 2. At what distance from the lens (in cm) was the image formed?

60. A fish is rising up vertically inside a pond with velocity of 4 cm/s, and notices a bird, which is diving vertically downward and its velocity appears to be 16 cm/s (to the fish). What is the real velocity (in cm/s) of the diving bird, if refractive index of water is 4/3.

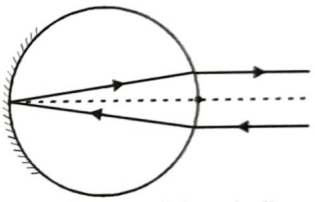
61. Plane glass slab is kept between a model and a plane mirror. The mirror is 50 cm away from the model. The glass slab has a thickness of 15 cm and refractive index of 1.5. At what distance (in cm) from herself will the model see her image?

62. At the bottom of the vessel filled with water, lies a flat mirror. A man leaning over the vessel, sees an image of his face in the mirror at a distance d = 25 cm from himself when the distance from the face to the surface is L = 5 cm. Determine the depth of the vessel in cm. refractive index of water is 4/3.

63. After all possible reflection and refractions the final image is located at a distance 'x' cm from air-glass interface. Then value of 7x is



64. A retro-reflector is an optical device which reflects light coming from infinity back directly. Recently "Scotchlite" sphere have been invented which behave as a retroreflector (figure). The index of refraction for such sphere is



65. A plane mirror is suspended vertically at the center of a large thin-walled spherical flask filled with water. The diameter of the flask is 10 inches. An observer whose eye is 35 inches from the mirror as shown in figure tries to see an image of his own eye. What is distance of the image from the eye (in inches). The effect of the thin glass walls of the flask may be neglected. Take $\mu_{water} = \frac{4}{3}$.



JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears: [*JEE Main-2016*]

(*a*) 10 times taller.

(*b*) 10 times nearer.

(*c*) 20 times taller.

(*d*) 20 times nearer.

2. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, *plot, it was found that a ray incident at angle* 35^0, suffers a deviation of 40^0 and that it emerges at angle 79^0. In that case which of the following is closest to the maximum possible value of the refractive index? [*JEE Main-2016*]

(*a*) 1.5 (*b*) 1. 6

(*c*) 1.7 (*d*) 1.8

3. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is: [*JEE Main-2017*]

(*a*) Real and at a distance of 40 cm from the divergent lens

(*b*) Real and at a distance of 6 cm from the convergent lens

(*c*) Real and at a distance of 40 cm from convergent lens

(*d*) Virtual and at a distance of 40 cm from convergent lens.

4. A conves lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is : [*JEE Main - 2019 (January)*]

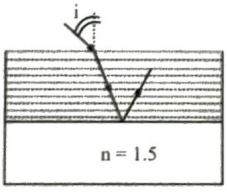
(*a*) 1.1 cm away from the lens

(*b*) 0

(*c*) 0.55 cm towards the lens

(*d*) 0.55 cm away from the lens

5. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is [*JEE Main - 2019 (January)*]



(*a*) $\sqrt{\frac{5}{3}}$ (*b*) $\frac{3}{\sqrt{5}}$

(*c*) $\frac{5}{\sqrt{3}}$ (*d*) $\frac{4}{3}$

6. Two plane mirrors are inclined to each other such that a ray of incident to the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be: [*JEE Main - 2019 (January)*]

(*a*) 45° (*b*) 60°

(*c*) 75° (*d*) 90°

7. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as : [*JEE Main - 2019 (January)*]

(*a*) $\mu_1 + \mu_2 = 3$ (*b*) $2\mu_1 + \mu_2 = 1$

(*c*) $3\mu_1 + \mu_2 = 1$ (*d*) $2\mu_1 + \mu_2 = 1$

8. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus. [*JEE Main - 2019 (January)*]

(*a*) 1 cm (*b*) 2 cm

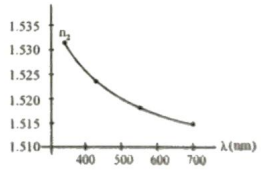
(*c*) 4.0 cm (*d*) 3.1 cm

9. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence [*JEE Main - 2019 (January)*]

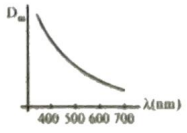
(*a*) 90° (*b*) 30°

(*c*) 60° (*d*) 45°

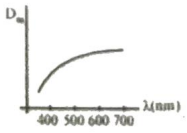
10. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following, graph is the correct one, if D_m is the angle of minimum deviation? [*JEE Main - 2019 (January)*]



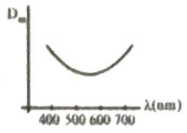
(*a*)



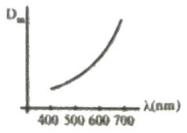
(*b*)



(*c*)

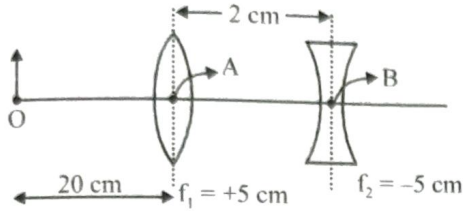


(*d*)



11. What is the position and nature of image formed by lens combination shown in figure? (f_1, f_2 are focal lengths)

[JEE Main - 2019 (January)]



(a) 70 cm from point B at left; virtual

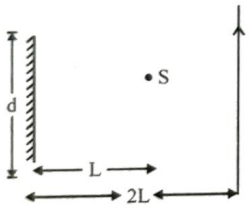
(b) 40 cm from point B at right; real

(c) $\frac{20}{3}$ cm from point B at right; real

(d) 70 cm from point B at right; real

12. A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vetically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance 2 L as shown below. The distance over which the man can see the image of the light source in the mirror.

[JEE Main - 2019 (January)]



(a) d (b) 2d

(c) 3d (d) $\frac{d}{2}$

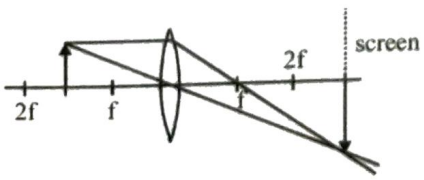
13. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1 radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be:

[JEE Main - 2019 (January)]

(a) $f_1 - f_2$ (b) $\frac{R}{\mu_2 - \mu_1}$

(c) $\frac{2f_1f_2}{f_1 + f_2}$ (d) $f_1 + f_2$

14. Formation of real image using a biconvex lens is shown below :



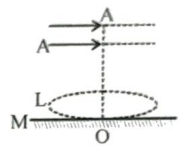
If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen? [JEE Main - 2019 (January)]

(a) Image disappears (b) Magnified image

(c) Erect real image (d) No change

15. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_1 is put between the lens and the mirror, the pin has to be moved to A′, such that OA′ = 27 cm, to get its inverted real image at A′ itself. The value of μ will be:

[JEE Main 2019(April)]



(a) $\sqrt{2}$ (b) $\frac{4}{3}$

(c) $\sqrt{3}$ (d) $\frac{3}{2}$

16. A convex lens of focal length 20 cm produces images of the same magnification 2 when an objects is kept at two distances x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is :- [JEE Main 2019(April)]

(a) 5 : 3 (b) 2 : 1

(c) 4 : 3 (d) 3 : 1

17. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600nm. coming from a distant object, the limit of resolution of the telescope is close to:

[JEE Main 2019(April)]

(a) 1.5×10^{-7} rad (b) 2.0×10^{-7} rad

(c) 3.0×10^{-7} rad (d) 4.5×10^{-7} rad

18. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be:

[JEE Main 2019(April)]

(a) 40 cm from the convergent mirror, same size as the object

(b) 20 cm from the convergent mirror, same size as the object

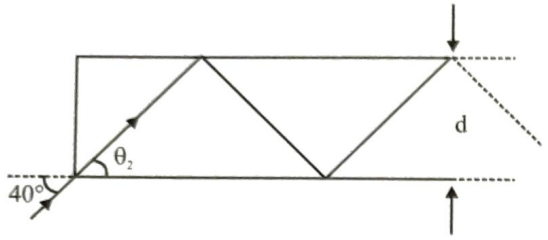
(c) 20 cm from the convergent mirror, twice the size of the object

(d) 40 cm from the convergent lens, twice the size of the object

19. In figure, the optical fiber is $l = 2$ m long and has a diameter of $d = 20\mu m$. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40°$, the number of reflection it makes before emerging from the other end is close to:

[*JEE Main 2019 (April)*]

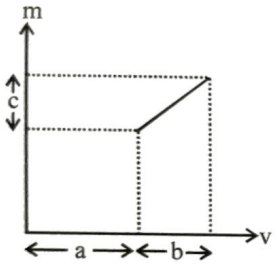
(refractive index of fibre is 1.31 and sin 40° = 0.64)



(*a*) 55000 (*b*) 57000

(*c*) 66000 (*d*) 45000

20. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used? [*JEE Main 2019 (April)*]



(*a*) $\frac{b^2c}{a}$ (*b*) $\frac{b^2}{ac}$

(*c*) $\frac{a}{c}$ (*d*) $\frac{b}{c}$

21. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star:

[*JEE Main 2019(April)*]

(*a*) 305×10^{-9} radian (*b*) 152.5×10^{-9} radian

(*c*) 610×10^{-9} radian (*d*) 457.5×10^{-9} radian

22. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be:

[*JEE Main 2019 (April)*]

(*a*) 20 cm (*b*) 10 cm

(*c*) 25 cm (*d*) 30 cm

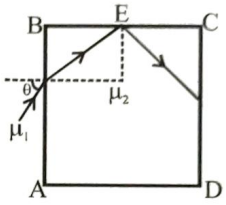
23. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is : [*JEE Main 2019 (April)*]

(*a*) 1.60 m (*b*) 0.24 m

(*c*) 0.16 m (*d*) 0.32 m

24. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index μ_1 ($\mu_1 < \mu_2$). A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC.

[*JEE Main 2019 (April)*]



The θ must satisfy :

(*a*) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (*b*) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

(*c*) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$ (*d*) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

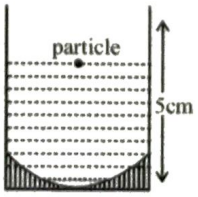
25. The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be : [*JEE Main 2019 (April)*]

(*a*) 0.24 μm (*b*) 0.48 μm

(*c*) 0.12 μm (*d*) 0.38 μm

26. A concave mirror has radius or curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to : (Refractive index of water = 1.33)

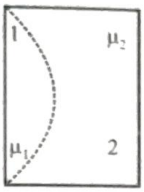
[*JEE Main 2019 (April)*]



(*a*) 8.8 cm (*b*) 11.7 cm

(*c*) 6.7 mm (*d*) 13.4 cm

27. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that 2 is μ_2, then the focal length of the combination is: [JEE Main 2019 (April)]



(a) $\frac{R}{2-(\mu_1-\mu_2)}$ (b) $\frac{2R}{\mu_1-\mu_2}$

(c) $\frac{2R}{2(\mu_1-\mu_2)}$ (d) $\frac{R}{\mu_1-\mu_2}$

28. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to : [JEE Main-2020 (January)]

(a) 2 mm (b) 33 mm

(c) 12 mm (d) 22 mm

29. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_ℓ, then the ratio f_ℓ/f is closed to the integer: [JEE Main-2020 (January)]

(a) 1 (b) 5

(c) 17 (d) 9

30. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is _____. [JEE Main-2020 (January)]

31. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be: [JEE Main-2020 (January)]

(a) 15° (b) 30°

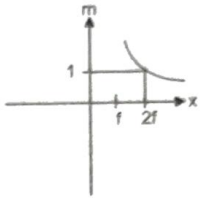
(c) 60° (d) 45°

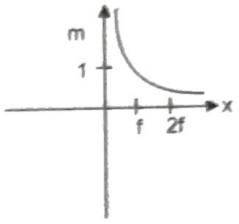
32. The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eye piece? [JEE Main-2020 (January)]

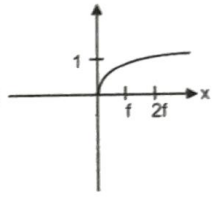
(a) 30 cm (b) 10 cm

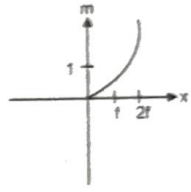
(c) 40 cm (d) 20 cm

33. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by (Graphs are drawn schematically and are not to scale) [JEE Main-2020 (January)]

(a)


(b)


(c)


(d)


34. A vessel of depth 2h is half filed with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be. [JEE Main-2020 (January)]

(a) $\frac{h}{3\sqrt{2}}$ (b) $\frac{3}{4}h\sqrt{2}$

(c) $\frac{h}{\sqrt{2}}$ (d) $\frac{h}{2(\sqrt{2}+1)}$

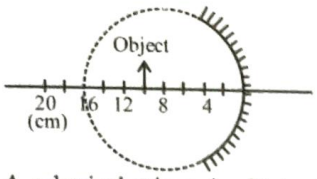
35. There is a small source of light at some depth below the surface of water (refractive index = 4/3) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly); [Use the fact that surface area of a spherical cap of height h and radius of curvature r is $2\pi rh$] [JEE Main-2020 (January)]

(a) 50% (b) 34%

(c) 17% (d) 21%

36. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is _____. [JEE Main-2020 (September)]

37.



A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object? (Figure drawn as schematic and not to scale)

[JEE Main-2020 (September)]

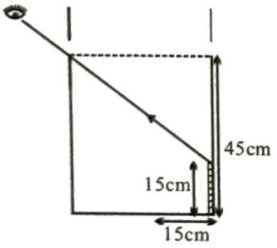
(*a*) Inverted, real and unmagnified
(*b*) Inverted, real and magnified
(*c*) Erect, virtual and magnified
(*d*) Erect, virtual and unmagnified

38. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is _____.

[JEE Main-2020 (September)]

39. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is N/100, where N is an integer, the value of N is _____.

[JEE Main-2020 (September)]



40. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)$D where N is an integer, the value of N is _____.

[JEE Main-2020 (September)]

41. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is _____.

[JEE Main-2020 (September)]

42. A prism of angle A = 1° has a refractive index μ = 1.5. A good estimate for the minimum angle of deviation (in degrees) is close to $\frac{N}{10}$. Value of N is _____.

[JEE Main-2020 (September)]

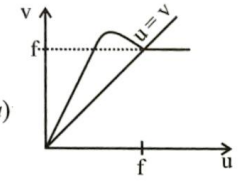
43. A compound microscope consists of an objective lens of focal length 1 cm and an eyepiece of focal length 5 cm with a separation of 10 cm.
The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm.
The value of n is _____.

[JEE Main-2020 (September)]

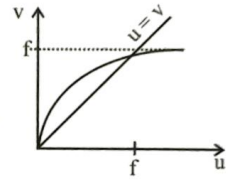
44. For a concave lens of focal length f, the relation between object and image distances u and v, respectively, from its pole can best be represented by (u = v is the reference line)

[JEE Main-2020 (September)]

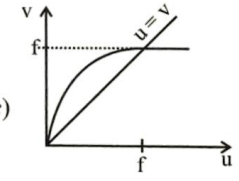
(*a*)



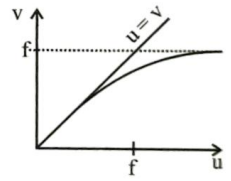
(*b*)



(*c*)



(*d*)



45. A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a planoconvex lens made of the same material with power 1.5 P is

[JEE Main-2020 (September)]

(*a*) $\frac{3R}{2}$ (*b*) 2R

(*c*) $\frac{R}{3}$ (*d*) $\frac{R}{2}$

46. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved, and the least count of the screw gauge, are respectively.

[JEE Main-2020 (September)]

(*a*) Positive, 0.1 mm (*b*) Positive, 10 μ m
(*c*) Negative, 2 μ m (*d*) Positive, 0.1 μ m

47. A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is [*JEE Main-2020 (September)*]

(*a*) 2.6 m from the mirror, real
(*b*) 1 m from the mirror, real
(*c*) 2.6 m from the mirror, virtual
(*d*) 1 m from the mirror, virtual

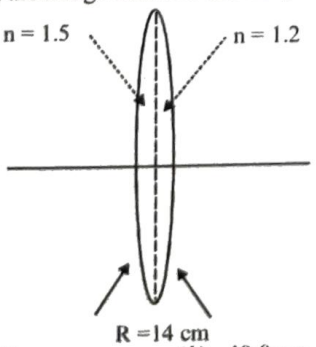
48. A galaxy is moving away from the earth at a speed of $286 kms^{-1}$. The shift in the wavelength of a redline at 630nm is $x \times 10^{-10}$ m. The value of x, to the nearest integer, is
Take the value of speed of Light c, as $3 \times 10^8 ms^{-1}$ [*JEE Main-2021 (March)*]

49. The image of an object placed in air formed by a convex refracting surface is at a distance of 10 behind the surface. The image is real and is at $\frac{2^{rd}}{3}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. The value of x' is [*JEE Main-2021 (March)*]

50. A fringe width of 6mm was produced for two slits separated by 1mm apart. The screen is placed 10m away. The wavelength of light used is x'nm. The value of x ' to the nearest integer is [*JEE Main-2021 (March)*]

JEE-ADVANCED

1. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be [*JEE Advanced-2012*]



(*a*) –280.0 cm (*b*) 40.0 cm
(*c*) 21.5 cm (*d*) 13.3 cm

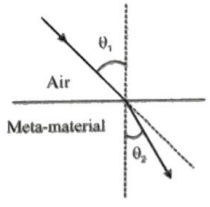
Comprehension (Q. No. 2 and 3)

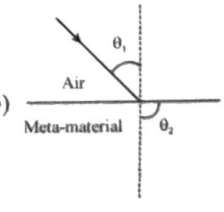
Most materials have the refractive index, n > 1. So, when a light ray from air enters a naturally occurring material, then by Snells' law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$ where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

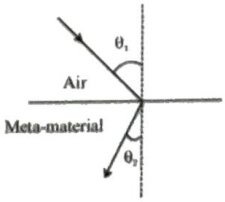
2. Choose the correct statement. [*JEE Advanced-2012*]

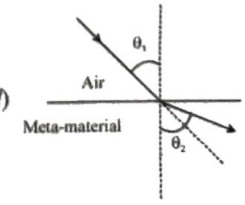
(*a*) The speed of light in the meta-material is v = c|n|
(*b*) The speed of light in the meta-material is $v = \frac{c}{|n|}$
(*c*) The speed of light in the meta-material is v = c.
(*d*) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air.

3. For light incident from air on a meta-material, the appropriate ray diagram is: [*JEE Advanced-2012*]

(*a*)


(*b*)


(*c*)


(*d*)


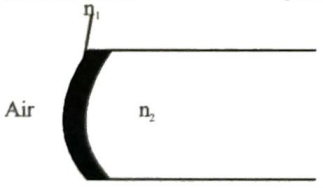
4. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is : [JEE Advanced-2013]

(a) 1 m (b) 2 m
(c) 3 m (d) 6 m

5. A ray of light travelling in the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$. The angle of incidence is : [JEE Advanced-2013]

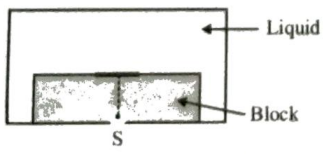
(a) 30° (b) 45°
(c) 60° (d) 75°

6. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$. as shown in the figure. Rays of light of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then [JEE Advanced-2014]



(a) $|f_1| = 3R$ (b) $|f_1| = 2.8R$
(c) $|f_2| = 2R$ (d) $|f_2| = 1.4R$

7. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is [JEE Advanced-2014]



(a) 1.21 (b) 1.30
(c) 1.36 (d) 1.42

8. When d ≈ a but wires are not touching the loop, it is found that the net magnetic filed on the axis of the loop is zero at a height h above the loop. In that case [JEE Advanced-2014]

(a) current in wire 1 and wire 2 is the direction PQ and RS, respectively and h ≈ a
(b) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ a
(c) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ 1.2 a
(d) current in wire 1 and wire 2 is the direction PQ and RS, resepectively and h ≈ 1.2 a

9. Consider d >> a, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop) [JEE Advanced-2014]

(a) $\frac{\mu_0 I^2 a^2}{d}$ (b) $\frac{\mu_0 I^2 a^2}{2d}$

(c) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (d) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

10. Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surface is r and the refractive index of all lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists. [JEE Advanced-2014]

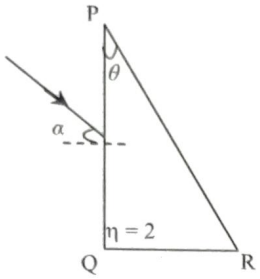
List-I		List-II
P.		1. 2r
Q.		2. r/2
R.		3. –r
S.		4. r

Code :

(a) P-1, Q-2, R-3,S-4 (b) P-2, Q-4, R-3, S-1
(c) P-4-,Q-1, R-2,S-3 (d) P-2, Q-1, R-3, S-4

11. A parallel beam of light is incident from air at an angle á on the side PQ of a right angled triangular prism of refractive index n = $\sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when á has a minimum value of 45°. The angle è of the prism is

[JEE Advanced-2016]

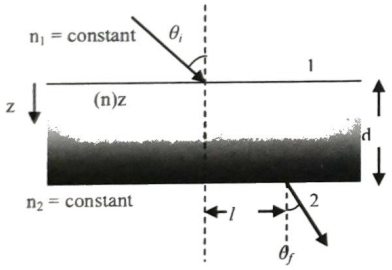


(*a*) 15° (*b*) 22.5°

(*c*) 30° (*d*) 45°

12. A transparent slab of thickness *d* has a refractive index n(z) that increases with z . Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 ($> n_1$), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement *l*.

[JEE Advanced-2016]



Which of the following statement(s) is (are) true?

(*a*) $n_1 \sin\theta_i = (n_1 - n_2) \sin\theta_f$

(*b*) *l* is independent of n_2

(*c*) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(*d*) *l* is dependent on n(z)

13. A plano –convex lens is made of a material of refractive index *n*. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to the reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is (are) true?

[JEE Advanced-2016]

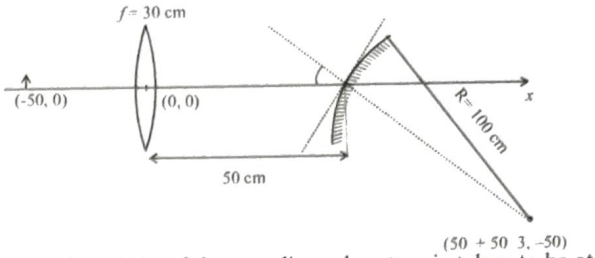
(*a*) The refractive index of the lens is 2.5

(*b*) The radius of curvature of the convex surface is 45cm

(*c*) The faint image is erect and real

(*d*) The focal length of the lens is 20 cm

14. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50cm. The mirror is tilted such that the axis of the mirror is at an angle è = 30° to the axis of the lens, as shown in the figure. **[JEE Advanced-2016]**



If the origin of the coordinated system is taken to be at the centre of the lens, the coordinates (in cm) of the Points v(x,y) at which the image is formed are

(*a*) (25, 25√3) (*b*) (0, 0)

(*c*) (125 / 3, 25/√3) (*d*) (50 – 25√3, 25)

15. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

[JEE Advanced-2017]

(*a*) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism.

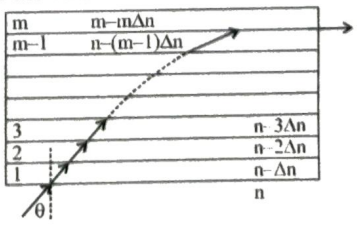
(*b*) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(*c*) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(*d*) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is

$$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right].$$

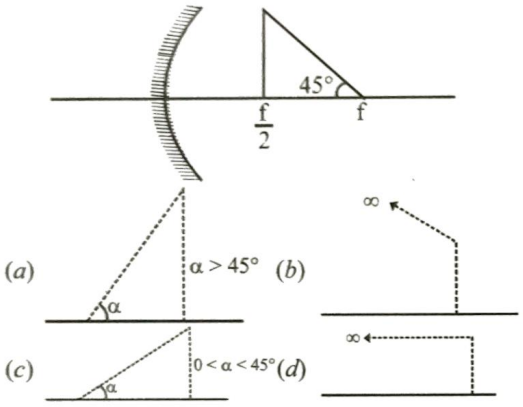
16. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle θ = 30°. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the mth slab and Δn = 0.1 (see the figure). The ray is refracted out parallel to the interface between the $(m - 1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m? **[JEE Advanced-2017]**



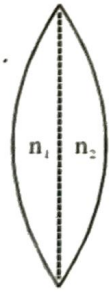
17. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is _____.

[*JEE Advanced-2018*]

18. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire ? (These figures are not to scale.)? [*JEE Advanced-2018*]



19. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statement(s) is/are : [*JEE Advanced - 2019*]



(*a*) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the concex surfaces are replaced by concave surfaces of the same radius of curcature.

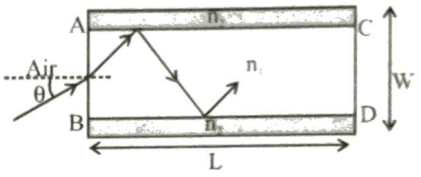
(*b*) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(*c*) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place)

(*d*) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta n}{f} > 0$

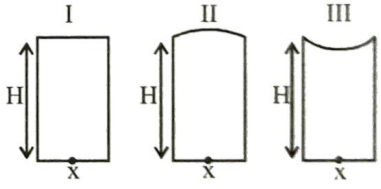
20. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If $L >> W$, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is ____.

[Speed of light $c = 3 \times 10^8$ m/s] [*JEE Advanced - 2019*]



21. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surfaces shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are

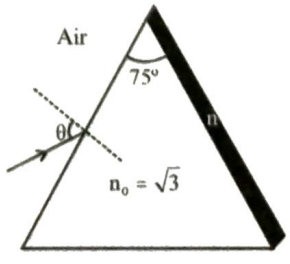
[*JEE Advanced - 2019*]



(*a*) $H_3 > H_1$ (*b*) $0.8\,\text{cm} < (H_2 - H_1) < 0.9\,\text{cm}$

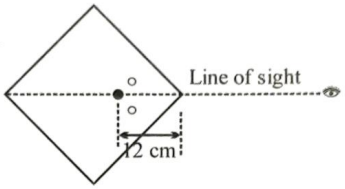
(*c*) $H_2 > H_3$ (*d*) $H_2 > H_1$

22. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$ The other refracting surface of a prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \leq 60°$. The value of n^2 is_____. [*JEE Advanced - 2019*]



23. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is_____. [*JEE Advanced - 2019*]

24. A large square container with thin transparent vertical walls and filled with water (refractive index $\frac{4}{3}$) is kept on a horizontal table. A student holds a thin straight wire vertically inside the water 12 cm from one of its corners, as shown schematically in the figure. Looking at the wire from this corner, another student sees two images of the wire, located symmetrically on each side of the line of sight as shown. The separation (in cm) between these images is _____.
[*JEE Advanced - 2020*]



ANSWER KEY

Topicwise Questions

1. (*d*)	**2.** (*b*)	**3.** (*b*)	**4.** (*c*)	**5.** (*d*)	**6.** (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*c*)
11. (*d*)	**12.** (*d*)	**13.** (*c*)	**14.** (*c*)	**15.** (*a*)	**16.** (*d*)	**17.** (*c*)	**18.** (*a*)	**19.** (*b*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** (*a*)	**24.** (*b*)	**25.** (*c*)	**26.** (*a*)	**27.** (*a*)	**28.** (*c*)	**29.** (*a*)	**30.** (*a*)
31. (*a*)	**32.** (*d*)	**33.** (*c*)	**34.** (*c*)	**35.** (*d*)	**36.** (*a*)	**37.** (*a*)	**38.** (*b*)	**39.** (*c*)	**40.** (*a*)
41. (*b*)	**42.** (*a*)	**43.** (*c*)	**44.** (*a*)	**45.** (*b*)	**46.** (*c*)	**47.** (*d*)	**48.** (*b*)	**49.** (*b*)	**50.** (*a*)
51. (*b*)	**52.** (*c*)	**53.** (*c*)	**54.** (*c*)	**55.** (*c*)	**56.** (*d*)	**57.** (*b*)	**58.** (*c*)	**59.** (*d*)	**60.** (*b*)
61. (*a*)	**62.** (*c*)	**63.** (*a*)	**64.** (*b*)	**65.** (*a*)	**66.** (*c*)	**67.** (*a*)	**68.** (*c*)	**69.** (*c*)	**70.** (*a*)
71. (*c*)	**72.** (*d*)	**73.** (*a*)	**74.** (*d*)	**75.** (*c*)	**76.** (*b*)	**77.** (*d*)	**78.** (*c*)	**79.** (*b*)	**80.** (*b*)

Learning Plus

1. (*d*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*b*)	**6.** (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*c*)
11. (*c*)	**12.** (*c*)	**13.** (*b*)	**14.** (*a*)	**15.** (*b*)	**16.** (*d*)	**17.** (*a*)	**18.** (*b*)	**19.** (*b*)	**20.** (*a*)
21. (*b*)	**22.** (*a*)	**23.** (*c*)	**24.** (*c*)	**25.** (*a*)	**26.** (*a*)	**27.** (*b*)	**28.** (*c*)	**29.** (*a*)	**30.** (*d*)
31. (*d*)	**32.** (*c*)	**33.** (*c*)	**34.** (*b*)	**35.** (*d*)	**36.** (*c*)	**37.** (*a*)	**38.** (*a*)	**39.** (*a*)	**40.** (*c*)
41. (*a*)	**42.** (*a*)	**43.** (*a*)	**44.** (*b*)	**45.** (*d*)	**46.** (*a*)	**47.** (*a*)	**48.** (*a*)	**49.** (*b*)	**50.** (*a*)
51. (*b*)									

Advanced Level Multiconcept Questions

MCQ/STATEMENT/COMPREHENSION/MATCHING

1. (*a,c,d*)	**2.** (*b,d*)	**3.** (*a,d*)	**4.** (*a,d*)	**5.** (*b,c*)	**6.** (*b,d*)	**7.** (*b,c*)	**8.** (*b,c*)	**9.** (*b,d*)	**10.** (*a,b,d*)
11. (*b,c,d*)	**12.** (*b,c,d*)	**13.** (*b, d*)	**14.** (*a,d*)	**15.** (*b, c, d*)	**16.** (*a,b*)	**17.** (*a,c*)	**18.** (*a,b,c*)	**19.** (*a,b,c*)	**20.** (*b,c*)
21. (*a,d*)	**22.** (*a,c*)	**23.** (*a,c*)	**24.** (*b,c,d*)	**25.** (*b,c*)	**26.** (*a,b,c*)	**27.** (*d*)	**28.** (*b*)	**29.** (*c*)	**30.** (*d*)
31. (*a*)	**32.** (*d*)	**33.** (*d*)	**34.** (*d*)	**35.** (*a*)	**36.** (*d*)	**37.** (*c*)	**38.** (*d*)	**39.** (*b*)	**40.** (*d*)
41. (*d*)	**42.** (*b*)	**43.** (*b*)	**44.** (*c*)	**45.** (*c*)	**46.** (*b*)	**47.** (*b*)	**48.** (*a*)	**49.** (*b*)	**50.** (*a*)
51. (*a*)	**52.** (*c*)	**53.** (*c*)	**54.** (*a*)-p; (*b*)-p; (*c*)-q; (*d*)-q				**55.** (i)-*b,d*; (ii)-*a,b,c,d*; (iii)-*a,b,d*; (iv)-*d*		
56. [5]	**57.** (*a,b*)	**58.** [16]	**59.** 180	**60.** [9]	**61.** [90]	**62.** [10]	**63.** [10]	**64.** [2]	**65.** [21]

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*d*)	**2.** (*a*)	**3.** (*c*)	**4.** (*d*)	**5.** (*b*)	**6.** (*b*)	**7.** (*a*)	**8.** (*d*)	**9.** (*c*)	**10.** (*a*)
11. (*d*)	**12.** (*c*)	**13.** (*b*)	**14.** (*a*)	15. (*b*)	**16.** (*d*)	**17.** (*c*)	**18.** (*b*)	**19.** (*b*)	**20.** (*d*)
21. (*a*)	**22.** (*b*)	**23.** (*d*)	**24.** (*b*)	**25.** (*a*)	**26.** (*a*)	**27.** (*d*)	**28 .** (*d*)	**29 .** (*d*)	**30.** [60]
31. (*b*)	**32.** (*b*)	**33.** (*a*)	**34.** (*b*)	**35.** (*c*)	**36.** [90]	**37.** (*a*)	**38.** (*a*)	**39.** [158]	**40.** (176)
41. (06.25)	**42.** (05)	**43.** (50.00)	**44.** (*d*)	**45.** (*c*)	**46.** (*b*)	**47.** (*a*)	**48.** [6]	**49.** [30]	**50.** [600]

JEE-ADVANCED

1. (*b*)	**2.** (*b*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)	**6.** (*a,c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*b*)
11. (*a*)	**12.** (*b,c,d*)	**13.** (*a,d*)	**14.** (*a*)	**15.** (*a,c,d*)	**16.** (8)	**17.** (130)	**18.** (*d*)	**19.** (*a,c,d*)	
20. (50.00)	**21.** (*c,d*)	**22.** (1.50)	**23.** (0.69)	**24.** (*b*)					

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WAVE OPTICS

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INTRODUCTION

(i) In a wave motion, there is some quantity which changes its value with time and space. The quantity that changes with space and time, in terms of which the wave equation should be written. For example light is transverse wave in which the electric field and magnetic field fluctuate in the direction perpendicular to the propagation of light. If the light is propagating is x - direction electric and magnetic field will be in y-z plane and both perpendicular to each other. The light wave can be represented by electric or magnetic field if we represent it in terms of electric field it is

$E = E_0 \sin(\omega t - kx)$. The average intensity at a point is proportional to $I = kE_0^2$

(ii) The light waves need no material medium to travel. They can propagate in vacuum.

(iii) Because light waves are transverse, they can be polarized. If a plane wave of light is travelling along the x-direction, the electric field may being the y-direction or along the z, direction or in any other direction in the y-z, plane. The equation of such a light wave may be written as

$$E = E_0 \sin\omega (t - x/v)$$

Where E, is the magnitude of the electric field at point x at time t.

(iv) The speed of light in vacuum is always the same and is very nearly equal to 3×10^8s.

(v) The frequency of visible light varies from about 3800×10^{11} Hz to about 7800×10^{11} Hz. The corresponding wavelengths (obtained from $\lambda = c/v$) are 380 nm to 780 nm. Light of wavelength close to 780 nm appears red and that close to 380 nm appears violet.

(vi) Light of single wavelength is called monochromatic light.

(vii) White light itself is a mixture of light of all wavelengths from about 380 nm to about 780 nm in appropriate proportion.

So the light can be represented by electric field or magnetic field. In this chapter we will represent it by electric field

WAVEFRONTS

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront.

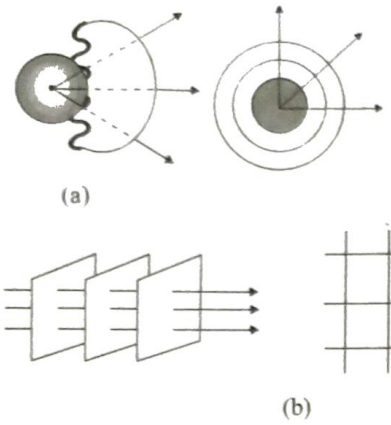
Figure (a) shows light waves from a point source forming a spherical wavefront in three dimensional space. The energy travels outwards along straight lines emerging from the source. i.e.. radii of the spherical wavefront. These lines are the rays. Notice that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (b)

A linear source such as a slit illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wavefronts may be regarded as planar.



(a)

(b)

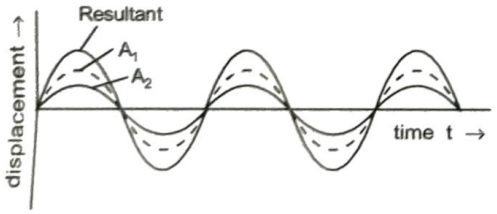
Figure : Wavefronts and the corresponding rays in two cases: (a) diverging spherical wave. (b) plane wave. The figure on the left shows a wave (e.g.. light) in three dimensions. The figure on the right shows a wave in two dimensions (a water surface).

COHERENT SOURCE

The two sources of light, whose frequencies are same and the phase difference between the waves emitted by which remains constant with respect to time are defined as coherent sources.

PRINCIPLE OF SUPERPOSITION

Two or more Progressive waves can travel simultaneously in a medium without affecting the motion of one another, Therefore, the resultant displacement of each particle of the medium at any instant is equal to the vector sum of the displacements produced by the two waves separately. This principle is called principle of superposition.



Resultant displacement

$$\vec{y} = \vec{y}_1 + \vec{y}_2$$

Meaning of principle of superposition

The principle of superposition means that if a number of waves are travelling in a medium, then each one travels independently as if the other waves were not present at all; the shape and other characteristics of any wave are not changed.

INTERFERENCE OF TWO WAVES

When two waves of same frequency travel in a medium simultaneously in the same direction then, due to their superposition, the resultant intensity at any point of the medium is different from the sum of intensities of the two waves. At some points the intensity of the resulatant wave is very large while at some other points it is very small or zero. This phenomenon is called the **interference of waves.**

Mathematical Interpretation of interference of two waves:

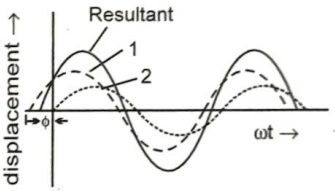
Let us consider two simple harmonic progressive waves of the same frequency travelling in the same direction. Let a_1 and a_2 be the amplitudes of the waves and ϕ the phase difference between them at any point in the medium. If the displacements due to these waves at that point be y_1 and y_2 then.

$y_1 = a_1 \sin \omega t$ —— (i)

and $y_2 = a \sin (\omega t + \phi)$—— (ii)

By the principle of superposition, the resultant displacement at the point is given by

$y = y_1 + y_2$ or $y = R \sin (\omega t + \theta)$



$R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos \phi$.

The intensity is directly proportional to the square of the amplitude. Hence the resultant intensity I is given by

$I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$. and phase angle $\phi = \tan^{-1} \frac{a_2 \sin \varphi}{a_1 + a_2 \cos \varphi}$

Thus the resultant intensity at any point depends upon the phase difference ϕ between the two waves at that point.

Comment: if ϕ phase diff is equivalent to Δx path difference or Δt time difference, then remember

$$\frac{\varphi}{2\pi} = \frac{x}{\lambda} = \frac{\Delta t}{T}$$

Two types of interference:

(1) Constructive (2) Destructive.

Law of conservation of energy in constructive & destructive interference

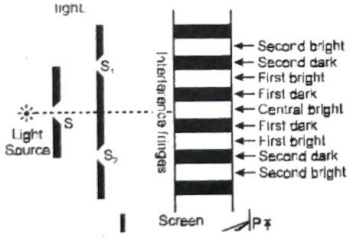
Energy distributes ununiformly by interference. Neither energy is produced nor destroyed. It changes from constructive to destructive & from destructive to constructive. Energy redistributes. It follows the law of conservation of energy.

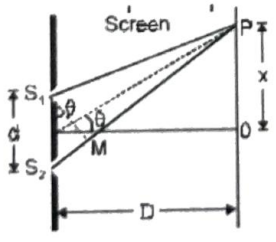
CONDITIONS FOR SUSTAINED INTERFERENCE PATTERN

(1) The source of light must be monochromatic.
(2) Two sources of light must be coherent.
(3) Frequencies of two waves must be same.
(4) The amplitudes of two waves must be nearly equal.
(5) The distance between two light sources must be small.
(6) The two coherent sources must be narrow.
(7) If the two light waves are polarized then their state of polarisation must be same.
(8) The two light waves must travel in the same direction.
(9) The vibrations of two waves must be in the same direction.
(10) The distance between the source & the screen must be large.

EXPERIMENT OF YOUNG'S DOUBLE-SLIT

(1) This experiment was performed to show practical form of interference of light.
(2) S_1 & S_2 slit behaves as a two coherent sources.
(3) On Screen, Bright & Dark portion are alternatively found.
(4) Bright portion is called Bright fringe & dark portion is called **dark fringe.**
(5) Central fringe is always bright.
(6) Energy is conserved in interference of light.
(7) It is explained on the basis of Huygen's principal.
(8) This experiment verifies the wave nature of light.





(a) At a point on screen to find dark or bright fringe, it depends upon path difference between S_1P & S_2P light waves.
(b) Two types of path difference between light waves:
(i) Geometrical path difference.
(ii) Optical path difference.

(c) In above experiment optical path difference = 0 and S_1P & S_2P has geometrical path difference so, Total path difference = Geometrical difference

$$S_2P - S_1P = \frac{xd}{D} \quad (\text{note : } \sin\theta \approx \tan\theta \approx Q)$$

Waves at p is $y_1 = a_0 \sin\omega t$, $y_2 = a_0 \sin(\omega t + \phi)$ where

$\phi = \frac{2\pi}{\lambda}(S_2P - S_1P)$ amplitude of resultant wave at P is $a = 2a_0 \cos(\phi/2)$ and intensity of the resultant wave at point P is given by $I = 4I_0 \cos^2(\phi/2)$

BRIGHT FRINGES

If n^{th} bright fringe forms at point P, then for Bright fringe:

$$S_2P - S_1P = \frac{X_n d}{D} = n\lambda \Rightarrow X_n = \frac{n\lambda D}{d}$$

so distance between central fringe & n^{th} bright fringe:

$$X_n = \frac{n\lambda D}{d}$$

n = 1 first bright fringe
n = 2 Second bright fringe.

DARK FRINGE

If nth dark fringe forms at a point P, then for dark fringe.

$$\frac{X_n d}{D} = \frac{(2n-1)}{2}\lambda$$

where n = 1, 2, 3 .....

$$X_n = \frac{(2n-1)\lambda D}{2d}$$

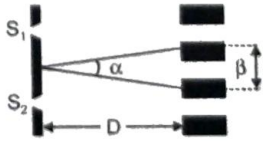
n = 1 First dark fringe.
n = 2 Second dark fringe.

DARK FRINGE

Fringe width: The distance between two consecutive bright fringe or two consecutive dark fringe is called fringe width.

$$\beta = \frac{\lambda D}{d}$$

Angular width of fringe:



$\alpha D = \beta$
$\alpha = \beta/D$

Comments on young's interference experiment

(i) Energy in conserved in interference. This indicates that energy is redistributed from destructive interference region to the constructive interference region.

(ii) If the entire arrangement of young's double slit experiment is immersed in water then fringe width decreases

$$\frac{\beta_{water}}{\beta_{air}} = \frac{\lambda_{water}}{\lambda_{air}} = \frac{1}{{}_a\mu_w} = \frac{1}{(4/3)}$$

(iii) If white light is used in placed of monochromatic light in young's double slit experiment.
- (a) central fringe is white
- (b) Coloured fringe around the central white fringe
- (c) Inner edge of the dark fringe is red, while the outer edge is violet (or blue)
- (d) Inner edge of white fringe is violet (or blue) and the outer edge in red.

(iv) If a filter allowing only λ_{red} (or λ_1) is placed in front of slit s_1 and filter allowing only λ_{blue} (or λ_2) is placed is front of slit s_2, then there does not occur any interference (refer to point no. 3 of conditions)

(v) If a thin glass plate or mica sheet is placed in front of one of the slits central fringe shifts towards the slit in front of which glass plate is placed. If t is the thickness of the glass or mica sheet and μ is the refractive index then the optical path = μt
so extra path difference $(\mu - 1)t$
if the central fringe now appears at the location of previously formed nth bright fringe then $(\mu - 1)\, t = n\lambda$ if the central fringe appears at the position of previously formed nth dark fringe then

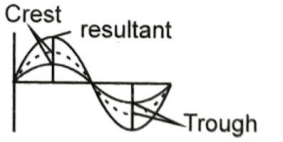
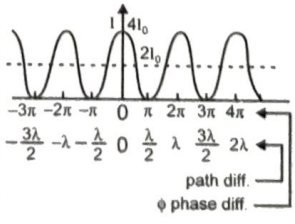
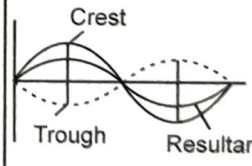
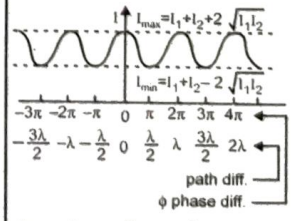
$$(\mu - 1)\, t = (2n - 1)\, \frac{\lambda}{2}$$

(vi) If the width of the slit S increased : the degree of spatial coherence decreases. As a result the interference pattern gradually disappears. Similar effect occurs if d between s_1 and s_2 is increased.

(vii) The fringe visibility : $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

V_{max} : if $I_1 = I_2 = I_0$ or $I_{min} = 0$
If widths of slits s_1 and s_2 are unequal the brightness of the bright fringe and the darkness of the dark fringe decreases.
if $I_1 >> I_2$ then $I_{max} \approx I_{min}$

(viii) When waves from two coherent sources S_1 and S_2 interfere in space the shape of the fringe is hyperbolic with foci at S_1 and S_2.

CONSTRUCTIVE ($I > I_1 + I_2$)	DESTRUCTIVE ($I < I_1 + I_2$)
1. The phase difference between two waves is an even multiple of π i.e. $\delta = 2n\pi$ where n = 0, 1, 2	The phase difference between two waves is an odd multiple of π. $\delta = (2n - 1)\pi$ where n = 1, 2, 3
2. The path difference between two waves is an even multiple of $\lambda/2$. $\Delta = 2n(\lambda/2)$ where n = 0,1, 2	The path difference between two waves is an odd multiple of $\lambda/2$. $\Delta = (2n - 1)\lambda/2$, where n = 1, 2
3. The time interval between two waves is even multiple of T/2 $\theta = 2n\left(\frac{T}{2}\right)$, n = 0, 1, 2	The time interval between two waves is an odd an multiple of T/2 $\theta = (2n-1)T/2$, n =1, 2, 3
4. The resultant amplitude of wave is equal to the sum of amplitudes of individual waves $A = a_1 + a_2$ if$\phi = 0, 2\pi, 4\pi,....2n\pi$	The resultant amplitude of wave is equal to the difference of amplitudes of two waves $A = a_1 - a_2$ if$\phi = \pi, 3\pi, 5\pi....(2n-1)\pi$
5. The resultant intensity is more than the sum of intensities of individual waves $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $= (\sqrt{I_1} + \sqrt{I_2})^2$   Intensity of resultant wave (interfernce) pattern when the waves have equal intensities.	The resultant intensity is less than the sum of intensities due to individual waves $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ $= (\sqrt{I_1} - \sqrt{I_2})^2$   Intesity of resultant wave (interference pattern when the intensity of the two waves are not equal). Note that the average intensity is $I = \dfrac{I_{max} + I_{min}}{2} = I_1 + I_2$

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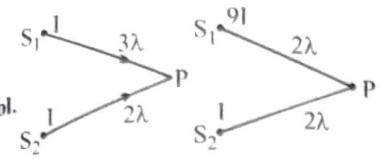
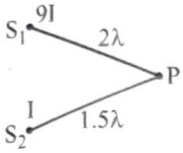
Example 1: Find intensity at P.

Sol.

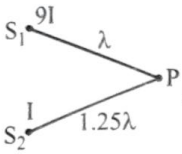


$I_r = \left(\sqrt{I} + \sqrt{I}\right)^2 = 4\ I\ I_r = \left(\sqrt{9I} + \sqrt{I}\right)^2 = 16I$

Similarly



$I_r = \left(\sqrt{9I} - \sqrt{I}\right)^2 = \left(2\sqrt{I}\right)^2 = 4I$

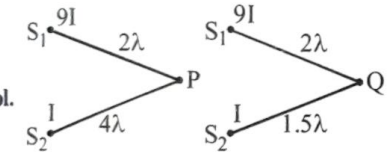


$\Delta x = 0.25\lambda\ ;\ \phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{\pi}{2}$

$I_r = 9I + I + \sqrt{9I}\ \sqrt{I} \cos\frac{\pi}{2} = 10I.$

Example 2: Find the ratio of intensity at P & Q.

Sol.



$\frac{I_P}{I_Q} = \frac{(3I+I)^2}{(3I-I)^2} = 4.$

Example 3: Two coherent sources are 0.3 mm apart. They are 0.9m away from the screen. The second dark fringe is at a distance of 0.3 cm from the centre. Find the distance of fourth bright fringe from the centre. Also find the wavelength of light used.

Sol. Given, $d = 0.3 \times 10^{-3}$ m, $D = 0.9$ m

$\frac{3\lambda D}{2d} = 0.3 \times 10^{-2}$ cm

(the distance of second dark fringe)

$\therefore\ \frac{\lambda D}{d} = (0.3 \times 10^{-2})\left(\frac{2}{3}\right)$

$= 0.2 \times 10^{-2}$ m $= 0.2$ cm

(i) Distance of fourth bright fringe from centre

$= \frac{4\lambda D}{d} = 0.8$ cm

(ii) $\lambda = \left(\frac{d}{D}\right)(0.2 \times 10^{-2})$ m $= \left(\frac{0.3\times10^{-3}}{0.9}\right)$

$(0.2 \times 10^{-2}) = 6.67 \times 10^{-7}$ m.

Example 4: In a Young's double slit set up the wavelength of light used is 546 nm. The distance of screen from slits is 1 m. The slit separation is 0.3 mm.

(a) Compare the intensity at a point P distant 10 mm from the central fringe where the intensity is I_0.

(b) Find the number of bright fringes between P and the central fringe.

Sol. Given, $\lambda = 546$ nm $= 5.46 \times 10^{-7}$ m, $D = 1.0$ m and $d = 0.3$ mm $= 0.3 \times 10^{-3}$ m

(a) At a distance $y = 10$ mm $= 10 \times 10^{-3}$ m from central fringe, the path difference will be,

$\Delta x = \frac{yd}{D} = \frac{(10\times10^{-3})(0.3\times10^{-3})}{1.0}$

$= 3.0 \times 10^{-6}$ m

The corresponding phase difference between the two interfering beams will be $\phi = \frac{2\pi}{\lambda} \times \Delta x$

$= \left(\frac{2\pi}{5.46\times10^{-7}}\right)(3.0 \times 10^{-6})$ radian $= 1978°$

$\therefore\ \frac{\phi}{2} = 989°$

$\therefore\ I = I_0 \cos^2\frac{\phi}{2} = I_0 \cos^2(989) = 3.0 \times 10^{-4}\ I_0.$

(b) Fringe width $\beta = \frac{\lambda D}{d} = \frac{(5.46\times10^{-7})(1.0)}{0.3\times10^{-3}}$

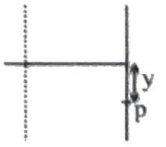
$\beta = 1.82$ mm

$\therefore$ no of bright fringes $= \frac{y}{\beta} = \frac{10}{1.82} = 5.49$

Therefore, number of bright fringes between P and central fringe will be 5 (excluding the central fringe).]

Example 5: In a double slit pattern ($\lambda = 6000$ Å), the first order and tenth order maxima fall at 12.50 mm and 14.75 mm from a particular reference point. If λ is changed to 5500 Å, find the position of zero order and tenth order fringes, other arrangements remaining the same.

Sol. $\frac{D\lambda}{d} = y + 12.5$



$\frac{10D\lambda}{d} = y + 14.75$

$9D = 2.25$ mm

$\beta = 0.25\ ;\ y = -12.25$

$\beta = \frac{D\lambda_1}{d}\ ;\ \beta' = \frac{D\lambda_2}{d}$

$\frac{\beta}{\beta'} = \frac{6000}{5500}$

$$\Rightarrow \beta' = \frac{55}{60}\beta = \frac{11}{12}\beta$$

$$= \frac{11}{12} \times \frac{2025}{9} = 0.229 = 0.23$$

$y'' = 12.25 \times 0.23 \times 10 = 14.55$ mm

$d = 0.3$ mm

$$0.3 = \frac{1.5D\lambda}{d}$$



$$y = \frac{4D\lambda}{d}$$

$$\frac{y}{3} = \frac{4}{1.5}$$

$$\Rightarrow y = \frac{4}{1.5} \times 3 = 0.8 \text{ cm.}$$

$$\lambda = \frac{d}{D}(0.2 \times 10^{-2})$$

$= 6.67 \times 10^{-7}$ m.

The intencity of the slit will be praportional to its area

If both the slit has same area intensity will be same.

Example 6: If in an YDSE width of on hole is double of other then find.

(a) Intensity ratio of two slits.

(b) Ratio of maximum and minimum intensity ratio on screen.

Sol. $$\frac{I_1}{I_2} = \frac{a_1}{a_2} = 2$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{2}I + \sqrt{I}\right)^2}{\left(\sqrt{2}I - \sqrt{I}\right)^2} = \frac{\left(\sqrt{2}+1\right)^2}{\left(\sqrt{2}-1\right)^2}$$

Example 7: Find the intensity at $y = \frac{D\lambda}{3d}$ if intensity of each source = I

Sol. $$f = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} \times \frac{dy}{D}$$

$$\frac{2\pi}{\lambda} \times \frac{d}{D} \times \frac{Dx}{3d} = \frac{2\pi}{3}$$

$I_r = I + I + 2I\cos\phi = I$

Example 8: Find the distance y from the central maxima at which intensity is 75% of maximum intensity

Sol. $Ir = I + I + 2I\cos\phi = I$

$3I = 2I + 2I\cos\phi$

$I = 2I\cos\phi$

$\cos\phi = 1/2$

$\phi = \pi/3$

$$\frac{2\pi}{\lambda} \times \frac{dy}{D} = \frac{2\pi}{3}$$

$$y = \frac{D\lambda}{6d}$$

Y.D.S.E. WITH WHITE LIGHT

The central maxima will be white because all wavelengths will constructively interfere here. However slightly below (or above) the position of central maxima fringes will be coloured for example if P is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{violet}}{2} = 190 \text{ nm,}$$

completely destructive interference will occur for violet light. Hence we will have a line devoid of violet colour that will appear reddish, And if

$$S_2P - S_1P = \frac{\lambda_{red}}{2} \simeq 350 \text{ nm,}$$

completely destructive interference for red light results and the line at this position will be violet. The coloured Fringes disappear at points far away from the central white fringe ; for these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination for exanple if

$$S_2P - S_1P = 3000 \text{ nm,}$$

then constructive interference will occur For wavelengths $\lambda = 3000$ nm. In the visible region these wavelength are 750 nn (red), 600 nm (yellow). 500 nm (greenish-yellow), 430 nrn (violet). Clearly such a light will appear white to the unaided eye.

Thus with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical inrerference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of cenral maxima in such cases.

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Example 9: In Youngs double slit experiment mixture of two light having wavelenght $\lambda_1 = 3000$Å and $\lambda_2 = 9000$Å are used. If d = 1 mm and D = 1 m find the distance from central maxima at which there is complete darckness.

Sol. There is complete darkness where minima of two waves coinsides.

Point at which minima coincides

$$y = \frac{(2n+1)D\lambda_1}{2d} = \frac{(2m+1)D\lambda_2}{2d}$$

$$\frac{2n+1}{2m+1} = \frac{\lambda_2}{\lambda_1} = \frac{3}{1}$$

3rd minima of smaller wavelength and 1st minima of bigger wavelength coincides.

GEOMETRICAL PATH AND OPTICAL PATH

$$\mu_1 = \frac{c}{v_1}, \qquad \mu_2 = \frac{c}{v_2}$$

$$\boxed{\frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}}, \qquad \boxed{\frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1}}.$$

Actual distance travelled by light in a medium is called geometrical path (Δx). Consider a light wave given by the equation.

$$E = E_0\sin(\omega t - kx + \phi)$$

If the light travels by Δx, its phase changes by $k\Delta x = \frac{\omega}{v}\Delta x$, where ω, the frequency of light does not depend on the medium, but v, the speed of light depends on the medium as $v = c/\mu$.

Consequently, change in phase,

$$\Delta\phi = k\Delta x = \frac{\omega}{C}(\mu\Delta x)$$

It is clear that a wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum, i.e. path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

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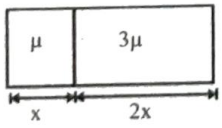
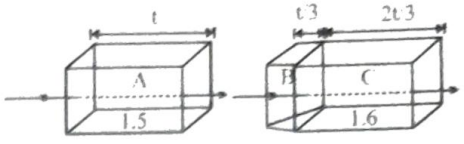
Example 10: Find the optical length of system in vaccum.



Sol. $\mu x + 6\,\mu x = 7\mu x$

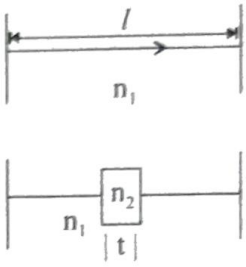
Example 11: Two transparent slabs have the same thickness as shown in figure. One is made of material A of refractive index 1.5. The other is made of two materials B and C with thickness in the ratio 1:2 The refractive index of C is 1.6. If a monochromatic parallel beam passing through the slabs has the same number of wavelengths inside both, the refractive index of B is



Sol. $1.5\,t = \frac{\mu t}{3} + \frac{3.2}{3}t$

$\mu = 1.3$

Example 12: Find the path difference in medium of refractive index n_1.



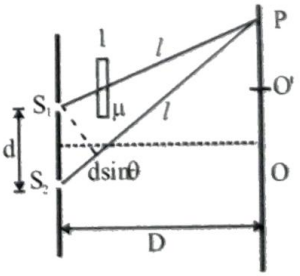
Sol. equivalent length w.r.t. $n_1 = (l - t) + \frac{n_2 t}{n_1}$

$$= l + t\left[\frac{n_2}{n_1} - 1\right], \Delta x = t\times\left[\frac{n_2}{n_1} - 1\right]$$

$$\phi = \frac{2\pi\times t}{\lambda_1}\left[\frac{n_2}{n_1} - 1\right]$$

DISPLACEMENT OF FRINGES

Displacement of fringe on introduction of a glass slab in the path of the light coming out of the slits.



path difference at point P

$\Delta x = [d\sin\theta + l\,] - [l - t + \mu t]$

$= d\sin\theta - t(\mu - 1)$

To find the intensity at a point find $\phi = k\Delta x$ and use the formula for intensity

For max.

$$n\lambda = \frac{dy}{D} - t(\mu - 1)$$

$$y_{max} = \frac{nD\lambda}{d} + \frac{Dt}{d}(\mu - 1)$$

$$y_{min} = \left(\frac{2n-1}{2}\right)\frac{D\lambda}{d} + \frac{Dt}{d}(\mu - 1)$$

Notice that this shift is in the direction of the slit in front of which the glass slab is placed. If the glass slab is placed ahead of the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed ahead of the lower slit the fringe pattern gets shifted downwards.

(a) So each maxima is shifted by $\frac{Dt}{d}(\mu - 1)$.

(b) So each minima is shifted by $\frac{Dt}{d}(\mu - 1)$

(c) Whole fringe pattern is shifted by $\frac{Dt}{d}(\mu - 1)$

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Example 13: In YDSE on introduction of slab of refractive μ and thick t to one of slit.

(a) –3rd order maxima comes to central maxima find wavelength.

(b) On intorduction of slab 3 maxima crosses point P and minima is formed at P find wavelength.

(c) On introduction of slab to lower slit 3 maxima cross the point P and the point at which intensity is 75% of maximum intensity comes to Point P. Find wavelength.

Sol. (a) $\frac{3D\lambda}{d} = \frac{t(\mu-1)D}{d}$

$$\lambda = \frac{t(\mu-1)}{3}$$

(b) $\frac{3.5D\lambda}{d} = \frac{t(\mu-1)D}{d}; \lambda = \frac{t(\mu-1)}{3.5}$

(c) $\frac{3D\lambda}{d} + \frac{D\lambda}{6d} = \frac{Dt(\mu-1)}{d}$

$$3\lambda + \frac{\lambda}{6} = t(\mu-1)$$

$$\frac{19\lambda}{6} = t(\mu-1)$$

$$\lambda = \frac{6t(\mu-1)}{19}$$

WAVE NATURE OF LIGHT : DIFFRACTION & POLARISATION

DIFFRACTION

Meaning of Diffraction

It is the spreading of waves round the corners of an obstacle

Defination of Diffraction

The phenomenon of bending of light waves around the sharp edges of opaque obstacles or aperture and their encroachment in the geometrical shadow of obstacle or aperture is defined as diffraction of light.

Necessary Conditions of Diffraction of Waves

The size of the obstacle (a) must be of the order of the wavelength of the waves (λ).

$$\frac{a}{\lambda} > 1 \text{ or } \frac{a}{\lambda} \approx 1$$

Greater the wave length of wave higher will be its degree of diffraction. This is the reason that diffraction of sound & radio waves is easily observed but for diffraction of light, additional arrangement has to be made.

$\lambda_{sound} > \lambda_{light}$

Wave length of sound is nearly equal to size of obstacle.

If size of obstactle is a & wavelength is λ then,

S.No.	a V/S λ	Diffraction
(1)	$a << \lambda$	Not possible
(2)	$a >> \lambda$	Not possible
(3)	$a \approx \lambda$	Possible

Interpretation of Diffraction

As a result of diffraction, maxima & minima of light intensities are found which has unequal intensities. Diffraction is the result of superposing of waves from infinite number of coherent sources on the same wavefront after the wavefront has been distorted by the obstacle.

Example of Diffraction

(a) When an intense source of light is viewed with the partially opened eye, colours are observed in the light.

(b) Sound produced in one room can be heard in the nearby room.

(c) Appearance of a shining circle around the section of sun just before sun rise.

(d) Colored spectrum is observed if a light source at far distance is seen through a thin cloth

Two types of diffraction

(a) Fresnel Diffraction : Fresenel diffraction which involves non-plane (spherical) wavefronts, so that the source s and the point p (where diffraction effect is to be observed) are to be at a finite distance from the diffracting obstacle

(b) **Fraunhofer Diffraction:** Fraunhofer diffraction deals with wavefronts that are plane on arrival and an effective viewing distance of infinity. If follows that fraunhofer diffraction is an important special case of fresnel diffraction. In youngs double slit experiment, we assume the screen to be relatively distant, that we have fraunhofer conditions.

DIFFERENCE BETWEEN INTERFERENCE & DIFFRACTION OF LIGHT

S.No.	Interference	Diffraction
1.	Two coherent sources are necessary	One coherent source is necessary
2.	All fringes has same width	Fringes has unequal width
3.	Width of bright fringes is equal to other fringes	Width of bright fringes is just doubled to other fringe
4.	All bright fringes has equal intensity	As order of bright fringes increases, intensity goes down
5.	**For bright fringe :** (a) Path difference $\Delta x = n\lambda$ (b) Phase difference $\Delta\phi = 2n\pi$	**For bright fringe :** $\Delta x = (2n-1)\lambda/2$ $\Delta\phi = (2n-1)\pi$
6.	**For dark fringes :** (a) Path difference $\Delta x = (2n-1)\lambda/2$ (b) Phase difference $\Delta\phi = (2n-1)\pi$	**For dark fringes:** $\Delta x = n\lambda$ $\Delta\phi = 2n\pi$

RESOLVING POWER (R.P)

A large number of images are formed as a consequence of light diffraction from a source. If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved R.P. of an optical instrument is its ability to distinguish two neighbouring points.

Linear R.P. = $d/\lambda D$

Angular R.P. = d/λ

D = Observed distance

d = Distance between two points

Telescope:

Limit of resolution = $\theta = \sin^{-1}\frac{1.22\lambda}{a}$;

For small angles $\theta = \frac{1.22\lambda}{a}$

Resolving power = $\frac{1}{\text{limit of resolution}}$

Microscope:

Limit of resolution (the smallest distance between two objects) = $x_{min} = \frac{1.22\lambda}{2\mu\sin\theta}$

Prism:

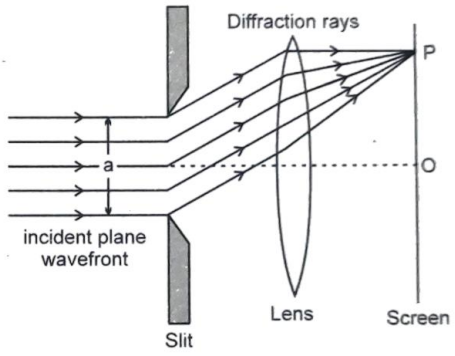
R.P. = $t(d\mu/d\lambda) = \lambda/d\lambda$

Eye:

The limit of resolution of human eye is 1' of arc (One minute of arc)

FRAUNHOFER DIFFRACTION FOR SINGLE SLIT

In this diffraction pattern central maxima is bright on the both side of it, maxima & minima occurs symmetrically



(1) For Diffraction Maxima

$a\sin\theta = (2n-1)\lambda/2$

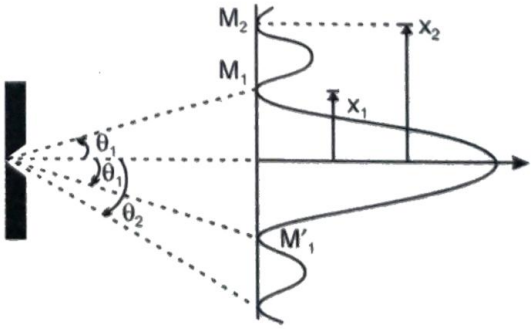
(2) For Diffraction Minima

$a\sin\theta = n\lambda$

(3) The maxima or minima is observed due to the superposition of waves emerging from infinite secondary sources between A & B points of slit.

(4) Fringe width

The distance between two secondary minima formed on two sides of central maximum is known as the width of central maximum



$$W = \frac{2f\lambda}{a}$$

f = focal distance of convex lens
a = width of slit

$$\text{Angular width} = W_\theta = \frac{2\lambda}{a}$$

RECTLINEAR MOTION OF LIGHT

(1) Rectlinear motion of light can be explained by diffraction of light.
(2) If size of obstacle is the order of wavelength of light, then diffraction of light takes place & its rectlinear motion of light is not possible.
(3) If size of obstacle is much greater than wave length of light, then rectlinear motion of light is observed.

POINTS TO REMEMBER

(1) Greymaldy discovered the diffraction.
(2) Intensity of diffraction pattern decreases, if size of obstacle is increased.
(3) Superposition of waves causes both diffraction & Interference. Superposition of secondary waves originated from two coherent sources generates interference. Superposition of secondary wavelets generated from same wavefront, is called diffraction.
(4) Interference fringes has equal width but diffraction fringes has unequal width.
(5) In diffraction pattern, intensity of bright fringes is different but for interference , it is equal.
(6) Intensities of dark fringes of interference is zero but for diffraction, it is not equal to zero.
(7) Effect of diffraction can be observed in only geometrical shadow of end region.
(8) In diffraction bright central fringe has double width, in comparison to others.
(9) In fresenel diffraction nor wavefront is plane, neither rays are parallel.

POLARISATION

Unpolarised light

(a) An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$. However, because of all directions are equally probable the resulting electromagnetic wave is a superposition of waves produced by individual atomic sources. This wave is called unpolarised light.

(b) All the vibrations of an unpolarised light at a given instant can be resolved in two mutually perpendicular directions and hence an unpolarised light is equivalent to superposition of two mutually perpendicular identical plane polarised light.

Plane polarised light

(a) If somehow we confine the vibrations of electric vector in one direction perpendicular to the direction of wave motion, the light is said to be plane polarised and the plane containing the direction of vibration and wave motion is called plane of polarisation.
(b) If an unpolarised light is converted into plane polarised light, its intensity reduces to half.
(c) Polarisation is a convincing proof of wave nature of light.

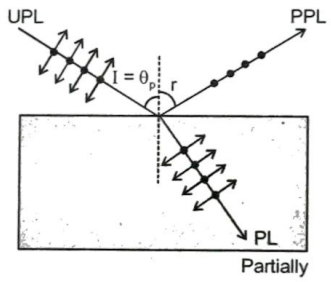
Partially polarised light

If in case of unpolarised light, electric vector in some plane is either more or less; then in its perpendicular plane, the light is said to be partially polarised.

Methods of polarisation

(a) By Reflection: Brewster discovered that when light is incident at a particular angle on a transparent substance, the reflected light is completely plane polarised with vibrations in a plane perpendicular to the plane of incidence. This specific angle of incidence is called polarising angle θ_p and is related to the refractive index μ of the material through the relation:

$$\tan \theta_p = \mu \qquad \ldots.(1)$$

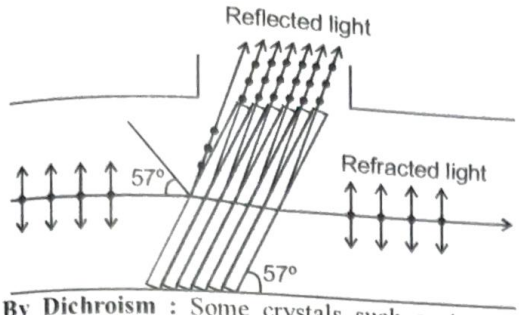


This is known as Brewster's law.

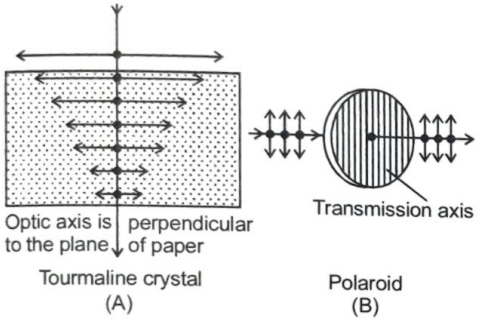
In case of polarisation by reflection:

(1) For $I = \theta_p$, refracted light is plane polarised.
(2) For $I = \theta_p$, reflected and refracted rays are perpendicular to each other.
(3) For $I <$ or $> \theta_p$, both reflected and refracted light become partially polarised.

(b) By Refraction: In this method, a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident at polarising angle. Since, in one reflection about 15 % of the light with vibration perpendicular to plane of paper is reflected, therefore after passing through a number of plates as shown in figure emerging light will become plane polarised with vibrations in the plane of paper.

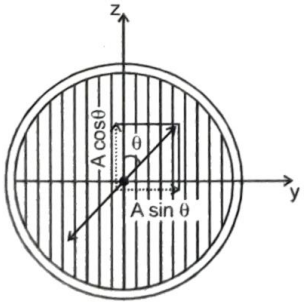


(c) By Dichroism : Some crystals such as tourmaline and sheets of iodosulphate of quinone have the property of strongly absorbing the light with vibrations perpendicular to a specific direction (called transmission axis) transmitting the light with vibrations parallel to it. This selective absorption of light is called **dichroism**. So if unpolarised light passes through proper thickness of these, the transmitted light will be plane polarised with vibrations parallel to transmission axis. Polariods work on this principle.



Intensity of Light Emerging from a Polariod:

If plane polarised light of intensity $I_0 (= KA^2)$ is incident on a polariod and its vibrations of amplitude A make an angle θ with transmission axis, as polariod will pass only those vibrations which are parallel to its transmission axis, i.e., A cos θ, so the intensity of emergent light will be



$I = K(A \cos\theta)^2 = KA^2 \cos^2\theta$

or $I = I_0 \cos^2\theta$ [as $I_0 = KA^2$]....(2)

This law is called **Malus law**.

From this it is clear that:

(1) If the incident light is unpolarised, then as vibrations are equally probable in all directions (in a plane perpendicular to the direction of wave-motion), θ can have any value from 0 to 2π and $(\cos^2\theta)_{av} = \frac{1}{2}$.

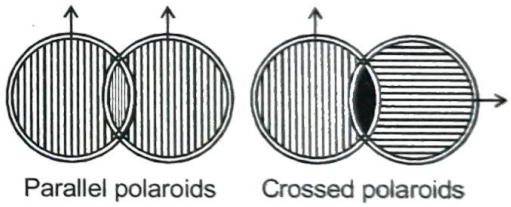
$$I = \frac{1}{2} I_0$$

i.e., If an unpolarised light is converted into plane polarised light its intensity becomes half.

(2) If light of intensity I_1 emerging from one polaroid called **polariser** is incident on a second polaroid (usually called **analyser**) the intensity of the light emerging from the second polaroid in accordance with Malus law will be given by

$I_2 = I_1 \cos^2\theta'$

where θ' is the angle between the transmission axis of the two polariods.



So if the two polaroids have their transmission axes parallel to each other, i.e., $\theta' = 0°$,

$I_2 = I_1 \cos^2 0° = I_1$

and if the two polaroids are crossed, i.e., have their transmission axes perpendicular to each other, $\theta' = 90°$.

$I_2 = I_1 \cos^2 90° = 0$

So, if an analyser is rotated from 0° to 90° with respect to polariser, the intensity of emergent light changes from maximum value I_1 to minimum value zero.

Topicwise Questions

1. By corpuscular theory of light, the phenomenon which can be explained is
   (a) Refraction (b) Interference
   (c) Diffraction (d) Polarisation
2. The idea of the quantum nature of light has emerged in an attempt to explain
   (a) Interference
   (b) Diffraction
   (c) Radiation spectrum of a black body
   (d) Polarisation
3. Two coherent monochromatic light beams of intensities I and 4I are superposed. The maximum and minimum possible intensities in the resulting beam are
   (a) 5I and I (b) 5I and 3I
   (c) 9I and I (d) 9I and 3I
4. The similarity between the sound waves and light waves is
   (a) Both are electromagnetic waves
   (b) Both are longitudinal waves
   (c) Both have the same speed in a medium
   (d) They can produce interference
5. For constructive interference to take place between two monochromatic light waves of wavelength λ, the path difference should be
   (a) $(2n-1)\frac{\lambda}{4}$ (b) $(2n-1)\frac{\lambda}{2}$
   (c) $n\lambda$ (d) $(2n+1)\frac{\lambda}{2}$
6. The ratio of intensities of two waves are given by 4 : 1. The ratio of the amplitudes of the two waves is
   (a) 2 : 1 (b) 1 : 2
   (c) 4 : 1 (d) 1 : 4
7. If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, the amplitude of the resulting wave will be about
   (a) 7 (b) 6
   (c) 5 (d) 3.5
8. The dual nature of light is exhibited by
   (a) Photoelectric effect
   (b) Refraction and interference
   (c) Diffraction and reflection
   (d) Diffraction and photoelectric effect

YDSE with monochromatic light

9. In the interference pattern, energy is
   (a) Created at the position of maxima
   (b) Destroyed at the position of minima
   (c) Conserved but is redistributed
   (d) None of the above
10. The maximum intensity of fringes in Young's experiment is I. If one of the slit is closed, then the intensity at that place becomes I_0. Which of the following relation is true?
    (a) $I = I_0$
    (b) $I = 2I_0$
    (c) $I = 4I_0$
    (d) There is no relation between I and I_0
11. In a Young's double slit experiment, the central point on the screen is
    (a) Bright
    (b) Dark
    (c) First bright and then dark
    (d) First dark and then bright
12. The slits in a Young's double slit experiment have equal widths and the source is placed symmetrically relative to the slits. The intensity at the central fringes is I_0. If one of the slits is closed, the intensity at this point will be
    (a) I_0 (b) $I_0/4$
    (c) $I_0/2$ (d) $4I_0$
13. In Young's double slit experiment how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe if $\lambda = 2000$ Å abd $d = 7000$ Å.
    (a) 12 (b) 7
    (c) 18 (d) 4

YDSE with glass slab, optical path

14. In Young's double slit experiment, a mica slit of thickness t and refractive index μ is introduced in the ray from the first source S_1. By how much distance the fringes pattern will be displaced
    (a) $\frac{d}{D}(\mu-1)t$ (b) $\frac{D}{d}(\mu-1)t$
    (c) $\frac{d}{(\mu-1)D}$ (d) $\frac{D}{d}(\mu-1)$
15. If a transparent medium of refractive index $\mu = 1.5$ and thickness $t = 2.5 \times 10^{-5}$ m is inserted in front of one of the slits of Young's Double Slit experiment, how much will be the shift in the interference pattern? The distance between the slits is 0.5 mm and that between slits and screen is 100 cm
    (a) 5 cm (b) 2.5 cm
    (c) 0.25 cm (d) 0.1 cm

16. When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the two interfering waves of light, then the path difference changes by

(a) $(\mu + 1)t$ (b) $(\mu - 1)t$

(c) $\frac{(\mu + 1)}{t}$ (d) $\frac{(\mu - 1)}{t}$

17. When one of the slits of Young's experiment is covered with a transparent sheet of thickness 4.8 mm, the central fringe shifts to a position originally occupied by the 30th bright fringe. What should be the thickness of the sheet if the central fringe has to shift to the position occupied by 20th bright fringe

(a) 3.8 mm (b) 1.6 mm

(c) 7.6 mm (d) 3.2 mm

Diffraction of Light and Polarization of Light

18. The penetration of light into the region of geometrical shadow is called

(a) Polarisation

(b) Interference

(c) Diffraction

(d) Refraction

19. If we observe the single slit Fraunhofer diffraction with wavelength λ and slit width d, the width of the central maxima is 2θ. On decreasing the slit width for the same λ

(a) θ increases

(b) θ remains unchanged

(c) θ decreases

(d) θ increases or decreases depending on the intensity of light

Polarization of light

20. Figure represents a glass plate placed vertically on a horizontal table with a beam of unpolarised light falling on its surface at the polarising angle of 57° with the normal. The electric vector in the reflected light on screen S will vibrate with respect to the plane of incidence in a



(a) Vertical plane

(b) Horizontal plane

(c) Plane making an angle of 45° with the vertical

(d) Plane making an angle of 57° with the horizontal

21. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is

(a) Zero (b) I_0

(c) $\frac{1}{2}I_0$ (d) $\frac{1}{4}I_0$

Learning Plus

1. Ratio of intensities of two light waves is given by 4 : 1. The ratio of the amplitudes of the waves is:

(a) 2 : 1 (b) 1 : 2

(c) 4 : 1 (d) 1 : 4

2. The contrast in the fringes in any interference pattern depends on :

(a) Fringe width

(b) Wavelength

(c) Intensity ratio of the sources

(d) Distance between the sources

3. The path difference between two waves $y_1 = A_1 \sin \omega t$ and $y_2 = A_2 \cos(\omega t + \phi)$ will be

(a) $(\lambda/2\pi)\phi$ (b) $(\lambda/2\pi)(\phi + \pi/2)$

(c) $(2\pi/\lambda)(\phi - \pi/2)$ (d) $(2\pi/\lambda)\phi$

4. Interference event is observed in

(a) Only transverse waves

(b) Only longitudinal waves

(c) Both types of waves

(d) Not observed in both type of waves

5. The nature of light which is verified by the interference event is –

(a) Particle nature (b) Wave nature

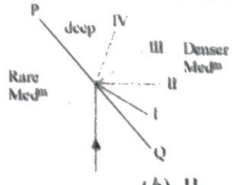
(c) Dual nature (d) Quantum nature

6. Two coherent sources have intensity ratio of 100 : 1, and are used for obtaining the phenomenon of interference. Then the ratio of maximum and minimum intensity will be–

(a) 100 : 1 (b) 121 : 81

(c) 1 : 1 (d) 5 : 1

7. Figure, shows wave fronts in still water, moving in the direction of the arrow towards the interface PQ between a shallow region and a deep (denser) region. Which of the lines shown may represent one of the wave fronts in the deep region?



(a) I (b) II

(c) III (d) IV

8. Plane microwaves from a transmitter are directed normally towards a plane reflector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima, the detector travels a distance 0.13 m. If the velocity of light is 3×10^8 m/s, find the frequency of the transmitter.

(*a*) 1.5×10^{10} Hz (*b*) 10^{10} Hz
(*c*) 3×10^{10} Hz (*d*) 6×10^{10} Hz

9. In a YDSE: D = 1 m, d = 1 mm and λ = 500 n m. The distance of 1000th maxima from the central maxima is:

(*a*) 0.5 m (*b*) 0.577 m
(*c*) 0.495 m (*d*) does not exist

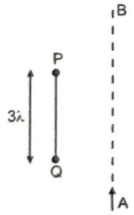
10. The path difference between two wavefronts emitted by coherent sources of wavelength 5460 Å is 2.1 micron. The phase difference between the wavefronts at that point is –

(*a*) 7.692 (*b*) 7.692 π
(*c*) $\frac{7.692}{\pi}$ (*d*) $\frac{7.692}{3\pi}$

11. In a Young's double slit experiment, d = 1 mm, λ = 6000 Å & D = 1 m. The slits produce same intensity on the screen. The minimum distance between two points on the screen having 75 % intensity of the maximum intensity is:

(*a*) 0.45 mm (*b*) 0.40 mm
(*c*) 0.30 mm (*d*) 0.20mm

12. Two coherent light sources each of wavelength λ are separated by a distance 3λ. The total number of minima formed on line AB which runs from $-\infty$ to $+\infty$ is:



(*a*) 2 (*b*) 4
(*c*) 6 (*d*) 8

13. In Young's double slit experiment, the distance of the n-th dark fringe from the centre is –

(*a*) $n\left(\frac{\lambda D}{2d}\right)$ (*b*) $n\left(\frac{2d}{\lambda D}\right)$
(*c*) $(2n-1)\frac{\lambda D}{2d}$ (*d*) $(2n-1)\frac{4d}{\lambda D}$

14. The fringe width in a Young's double slit experiment can be increased, if we decrease

(*a*) Width of the slits
(*b*) Separation of the slits
(*c*) Wavelength of the light used
(*d*) Distance between slits and screen

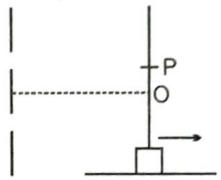
15. In YDSE how many maxima can be obtained on the screen if wavelength of light used is 200nm and d = 700 nm.

(*a*) 12 (*b*) 7
(*c*) 18 (*d*) none of these

16. In a young double slit experiment D equals the distance of screen and d is the separation between the slit. The distance of the nearest point to the central maximum where the intensity is same as that due to a single slit, is equal to

(*a*) $\frac{D\lambda}{d}$ (*b*) $\frac{D\lambda}{2d}$
(*c*) $\frac{D\lambda}{3d}$ (*d*) $\frac{2D\lambda}{d}$

17. In a Young's Double slit experiment, first maxima is observed at a fixed point P on the screen. Now the screen is continuously moved away from the plane of slits. The ratio of intensity at point P to the intensity at point O (centre of the screen)

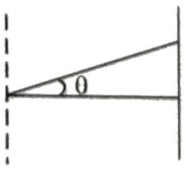


(*a*) remains constant
(*b*) keeps on decreasing
(*c*) first decreases and then increases
(*d*) First decreases and then becomes constant

18. In a double slit experiment, the separation between the slits is d = 0.25 cm and the distance of the screen D = 100 cm from the slits. If the wavelength of light used is λ = 6000 Å and I_0 is the intensity of the central bright fringe, the intensity at a distance $x = 4 \times 10^{-5}$ m from the central maximum is

(*a*) I_0 (*b*) $I_0/2$
(*c*) $3I_0/4$ (*d*) $I_0/3$

19. Two slits are separated by 0.3 mm. A beam of 500 nm light strikes the slits producing an interference pattern. The number of maxima observed in the angular range $-30° < \theta < 30°$.



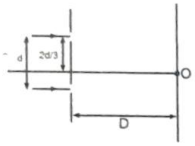
(*a*) 300 (*b*) 150
(*c*) 599 (*d*) 149

20. Two monochromatic and coherent point sources of light are placed at a certain distance from each other in the horizontal plane. The locus of all those points in the horizontal plane which have constructive interference will be
(*a*) a hyperbola (*b*) family of hyperbolas
(*c*) family of straight lines (*d*) family of parabolas

21. White light is incident normally on a glass plate (in air) of thickness 500 nm and refractive index of 1.5. The wavelength (in nm) in the visible region (400 nm - 700 nm) that is strongly reflected by the plate is:
(*a*) 450 (*b*) 600
(*c*) 400 (*d*) 500

22. In Young's double slit experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of n for which n^{th} bright band due to red light coincides with $(n + 1)^{th}$ bright band due to blue light, is
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

23. In the figure shown if a parallel beam of white light is incident on the plane of the slits then the distance of the nearest white spot on the screen from O is: [assume $d << D$, $\lambda << d$]



(*a*) 0 (*b*) d/2
(*c*) d/3 (*d*) d/6

24. In a YDSE, the central bright fringe can be identified.
(*a*) as it has greater intensity than the other bright fringes
(*b*) as it is wider than the other bright fringes
(*c*) as it is narrower than the other bright fringes
(*d*) by using white light instead of single wavelength light.

25. In a YDSE both slits produce equal intensities on the screen. A 100 % transparent thin film is placed in front of one of the slits. Now the intensity of the geometrical centre of system on the screen becomes 75 % of the previous intensity. The wavelength of the light is 6000Å and $\mu_{glass} = 1.5$. The thickness of the film cannot be:
(*a*) 0.2 μm (*b*) 1.0 μm
(*c*) 1.4 μm (*d*) 1.6 μm

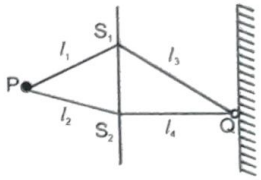
26 In Young's double slit arrangement, water is filled in the space between screen and slits. Then :
(*a*) fringe pattern shifts upwards but fringe width remains unchanged.
(*b*) fringe width decreases and central bright fringe shifts upwards.
(*c*) fringe width increases and central bright fringe does not shift.
(*d*) fringe width decreases and central bright fringe does not shift.

27. Light of wavelength λ in air enters a medium of refractive index μ. Two points in this medium, lying along the path of this light, are at a distance x apart. The phase difference between these points is :

(*a*) $\frac{2\pi\mu x}{\lambda}$ (*b*) $\frac{2\pi x}{\mu\lambda}$

(*c*) $\frac{2\pi(\mu-1)x}{\lambda}$ (*d*) $\frac{2\pi x}{(\mu-1)\lambda}$

28. Two identical narrow slits S_1 and S_2 are illuminated by light of wavelength λ from a point source P. If, as shown in the diagram above the light is then allowed to fall on a screen, and if n is a positive integer, the condition for destructive interference at Q is that



(*a*) $(l_1 - l_2) = (2n + 1)\,\lambda/2$
(*b*) $(l_3 - l_4) = (2n + 1)\,\lambda/2$
(*c*) $(l_1 + l_2) - (l_2 + l_4) = n\lambda$
(*d*) $(l_1 + l_3) - (l_2 + l_4) = (2n + 1)\,\lambda/2$

29. In a biprism experiment the distance of source from biprism is 1 m and the distance of screen from biprism is 4 meters. The angle of refraction of biprism is 2×10^{-3} radians. μ of biprism is 1.5 and the wavelength of light used is 6000 Å. How many fringes will be seen on the screen?
(*a*) 4 (*b*) 5
(*c*) 3 (*d*) 6

30. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate with the curved surface downwards. Monochromatic light is incident normally from the top. The observed interference fringes from this combination do not follow one of the following statements.
(*a*) the fringes are straight and parallel to the length of the piece.
(*b*) the line of contact of the cylindrical glass piece and the glass plate appears dark.
(*c*) the fringe spacing increases as we go outwards.
(*d*) the fringes are formed due to the interference of light rays reflected from the curved surface of the cylindrical piece and the top surface of the glass plate.

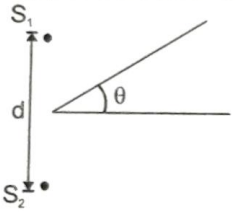
31. A thin film of thickness t and index of refraction 1.33 coats a glass with index of refraction 1.50. What is the least thickness t that will strongly reflect light with wavelength 600 nm incident normally?
(*a*) 225 nm (*b*) 300 nm
(*c*) 400 nm (*d*) 450 nm

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. If the source of light used in a Young's Double Slit Experiment is changed from red to blue, then
   (*a*) the fringes will become brighter
   (*b*) consecutive fringes will come closer
   (*c*) the number of maxima formed on the screen increases
   (*d*) the central bright fringe will become a dark fringe.

2. In an interference arrangement similar to Young's double-slit experiment, the slits S_1 & S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance d = 150.0 m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity then $I(\theta)$ for $0 \le \theta \le 90°$ is given by



   (*a*) $I(\theta) = \frac{I_0}{2}$ for $\theta = 30°$

   (*b*) $I(\theta) = \frac{I_0}{4}$ for $\theta = 90°$

   (*c*) $I(\theta) = I_0$ for $\theta = 0°$

   (*d*) $I(\theta)$ is constant for all values of θ

3. White light is used to illuminate the two silts in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance (d > > b) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are :

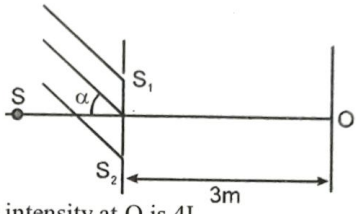
   (*a*) $\lambda = \frac{b^2}{d}$ (*b*) $\lambda = \frac{2b^2}{d}$

   (*c*) $\lambda = \frac{b^2}{3d}$ (*d*) $\lambda = \frac{2b^2}{3d}$

4. In a Young's double-slit experiment, let A and B be the two slits. A thin film of thickness t and refractive index μ is placed in front of A. Let β = fringe width. The central maximum will shift :
   (*a*) towards A (*b*) towards B
   (*c*) by $t(\mu - 1)\beta/\lambda$ (*d*) by $\mu t \beta/\lambda$

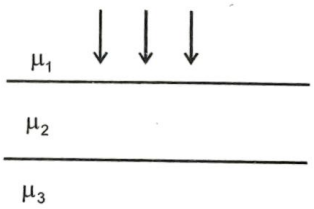
5. In a standard YDSE appratus a thin film (μ = 1.5, t = 2.1 μm) is placed in front of upper slit. How far above or below the centre point of the screen are two nearest maxima located? Take D = 1 m, d = 1 mm, λ = 4500 Å. (Symbols have usual meaning)
   (*a*) 1.5 mm (*b*) 0.6 mm
   (*c*) 0.15 mm (*d*) 0.3 mm

6. A parallel beam of light (λ =5000 Å) is incident at an angle $\alpha = 30°$ with the normal to the slit plane in a young's double slit experiment. Assume that the intensity due to each slit at any point on the screen is I_0. Point O is equidistant from S_1 & S_2. The distance between slits is 1 mm.



   (*a*) the intensity at O is $4I_0$
   (*b*) the intensity at O is zero
   (*c*) the intensity at a point on the screen 4m from O is $4I_0$
   (*d*) the intensity at a point on the screen 4m from O is zero

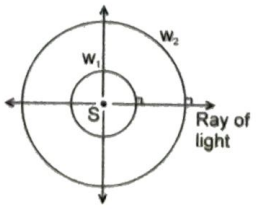
7. Consider a case of thin film interference as shown. Thickness of film is equal to wavelength of light is μ_2.



   (*a*) Reflected light will be maxima if $\mu_1 < \mu_2 < \mu_3$
   (*b*) Reflected light will be maxima if $\mu_1 < \mu_2 > \mu_3$
   (*c*) Transmitted light will be maxima if $\mu_1 > \mu_2 > \mu_3$
   (*d*) Transmitted light will be maxima if $\mu_1 > \mu_2 < \mu_3$

Comprehension-1 (8 to 12)

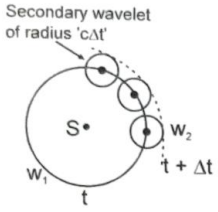
Huygen was the first scientist who proposed the idea of wave theory of light. He said that the light propagates in form of wavefronts. A wavefront is an imaginary surface at every point of which waves are in the same phase. For example the wavefronts for a point source of light is collection of concentric spheres which have centre at the origin. w_1 is a wavefront. w_2 is another wavefront.



The radius of the wavefront at time 't' is 'ct' in this case where 'c' is the speed of light. The direction of propagation of light is perpendicular to the surface of the wavefront. The wavefronts are plane wavefronts in case of a parallel beam of light.



Huygen also said that every point of the wavefront acts as the source of secondary wavelets. The tangent drawn to all secondary wavelets at a time is the new wavefront at that time. The wavelets are to be considered only in the forward direction (i.e. the direction of propagation of light) and not in the reverse direction. If a wavefront w_1 at time t is given, then to draw the wavefront at time $t + \Delta t$ take some points on the wavefront w_1 and draw spheres of radius '$c\Delta t$'. They are called secondary wavelets.



Draw a surface w_2 which is tangential to all these secondary wavelets. w_2 is the wavefront at time '$t + \Delta t$'. Huygen proved the laws of reflection and laws of refraction using concept of wavefronts.

8. A point source of light is placed at origin, in air. The equation of wave front of the wave at time t, emitted by source at t = 0, is (take refractive index of air as 1)

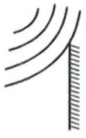
(*a*) $x + y + z = ct$

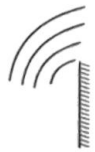
(*b*) $x^2 + y^2 + z^2 = t^2$

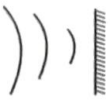
(*c*) $xy + yz + zx = c^2t^2$

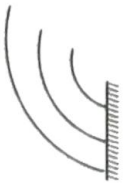
(*d*) $x^2 + y^2 + z^2 = c^2t^2$

9. Spherical wave fronts shown in figure, strike a plane mirror. Reflected wave fronts will be as shown in



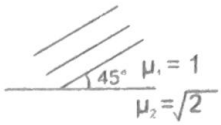
(*a*) 

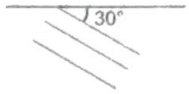
(*b*) 

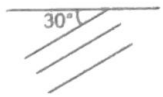
(*c*) 

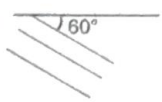
(*d*) 

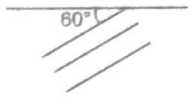
10. Wavefronts incident on an interface between the media are shown in the figure. The refracted wavefronts will be as shown in



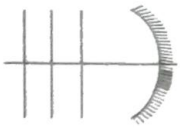
(*a*) 

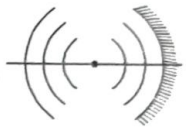
(*b*) 

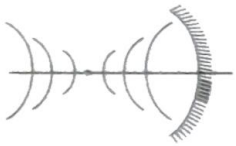
(*c*) 

(*d*) 

11. Plane wavefronts are incident on a spherical mirror as shown. The reflected wavefronts will be



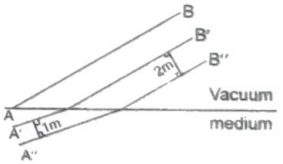
(*a*) 

(*b*) 

(*c*) 

(*d*) 

12. Certain plane wavefronts are shown in figure. The refractive index of medium is



(*a*) 2

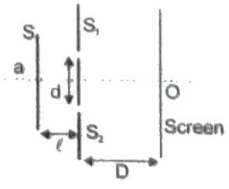
(*b*) 4

(*c*) 1.5

(*d*) Cannot be determined

Comprehension-2 (13 to 15)

The figure shows a schematic diagram showing the arrangement of Young's Double Slit Experiment



13. Choose the correct statement(s) related to the wavelength of light used

(*a*) Larger the wavelength of light larger the fringe width

(*b*) The position of central maxima depends on the wavelength of light used

(*c*) If white light is used in YDSE, then the violet colour forms its first maxima closest to the central maxima

(*d*) The central maxima of all the wavelengths coincide

14. If the distance D is varied, then choose the correct statement(s)

(*a*) The angular fringe width does not change

(*b*) The fringe width changes in direct proportion

(*c*) The change in fringe width is same for all wavelengths

(*d*) The position of central maxima remains unchanged

15. If the distance d is varied, then identify the correct statement

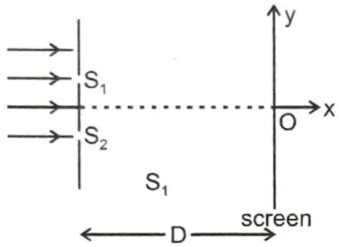
(*a*) the angular width does not change

(*b*) the fringe width changes in inverse proportion

(*c*) the positions of all maxima change

(*d*) the positions of all minima change

16. A monochromatic parallel beam of light of wavelength λ is incident normally on the plane containing slits S_1 and S_2. The slits are of unequal width such that intensity only due to one slit on screen is four times that only due to the other slit. The screen is placed perpendicular to x-axis as shown. The distance between slits is d and that between screen and slit is D. Match the statements in Column-I with results in Column-II.

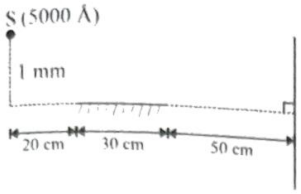


Column-I	Column-II
(*a*) The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{1}{9}$th of maximum intensity.	(p) $\frac{D\lambda}{3d}$
(*b*) The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{3}{9}$th of maximum intensity.	(q) $\frac{D\lambda}{d}$
(*c*) The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{5}{9}$th of maximum intensity.	(r) $\frac{2D\lambda}{d}$
(*d*) The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{7}{9}$th of maximum intensity.	(s) $\frac{3D\lambda}{d}$

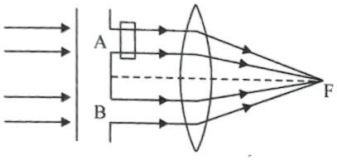
NUMERICAL VALUE BASED

17. A certain crude oil has an index of refraction of 1.25. A ship dumps $1 m^3$ of this oil into the ocean and oil spreads into a thin uniform film on water surface $\left(\mu = \frac{4}{3}\right)$. When light of wavelength 500 nm is incident on it, the reflected light shows maxima. What is the area (in km^2) covered by the spilled oil? Assume that the thickness of film is minimum required to satisfy this criterion.

18. Find the total number of fringes formed on screen in the Lloyd's mirror arrangement shown.



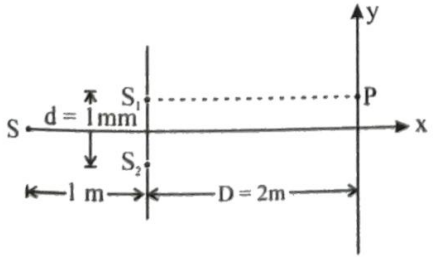
19. In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $I_1 = 10/\pi$ W/m^2 is incident normally on two circular apertures A and B of radii $r_1 = 0.001$ m and $r_2 = 0.002$ m respectively. A perfectly transparent film of thickness $t = 2000$ Å and refractive index $n = 1.5$ for the wavelength $\lambda = 6000$ Å is placed in front of aperture A (see figure). Calculate the power (in μW) received at the focal spot F of the lens. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.



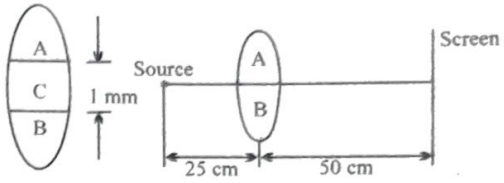
20. In a YDSE setup, one of the slit width is four times that of the other. The separation between the slits is $d = 1$ mm and the distance between slits and screen is $D = 5$m. A monochromatic light of wavelength $\lambda =$ 6000Å incidents on slits normally. The intensity of light only due to narrow slit on screen is I_0. If the intensity of light on screen at a point infront of the narrow slit is nI_0, find n?

21. In a modified YDSE the source S of wavelength 5000Å oscillates about axis of setup according to the equation y $= 0.5 \sin\left(\frac{\pi}{6}\right)$ t, where y is in millimeter and t in second.

At what time t will the intensity at P, a point exactly in front of slit S_1, be maximum for the first time?



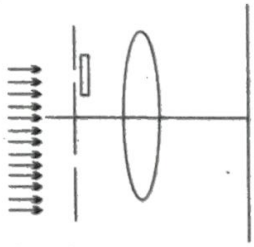
22. A convex lens of focal length 50 cm is cut along the diameter into two identical halves A and B and in the process a layer C of the lens thickness 1 mm is lost. Then the two halves A and B are put together to form a composite lens. Now infront of this composite lens a source of light emitting wavelength $\lambda = 6000$ Å is placed at a distance of 25 cm as shown in the figure. Behind the lens there is a screen at a distance 50 cm from it. Find the fringe width of the interference pattern obtained on the screen.



23. A radio receiver is set up on a mast in the middle of a calm lake to track the radio signal from a satellite orbiting the earth. As the satellite rises above the horizon, the intensity of the signal varies periodically. The intensity is at a maximum when the satellite is $\theta_1 = 0.01$ radian above the horizon and then again at $\theta_2 = 0.03$ radian above the horizon. What is the wavelength λ (in meters) of the satellite signal? The receiver is $h = 4.0$ m above the lake surface.

24. Visible light of variable wavelength is incident normally on a thin sheet of plastic in air. The reflected light has a minima only for $\lambda = 512$ nm and $\lambda = 640$ nm in the visible spectrum. what is the minimum thickness (in μm) of the film ($\mu = 1.28$)?

25. The intensity received at the focus of the lens is I when no glass slab has been placed in front of the slit. Both the slits are of the same dimension and the plane wavefront incident perpendicularly on them, has wavelength l. On placing the glass slab, the intensity reduces to 3I/4 at the focus. Find out the minimum thickness of the glass slab (in Å) if its refractive index is 3/2. **Given** $l = 6933$Å, $\mu = 1.5$.



26. Light of wavelength 627 nm illuminates two slits. What is the minimum path difference (in nm) between the waves from the slits for the resultant intensity to fall to 25% of the central maximum?

JEE-MAIN
PREVIOUS YEAR'S

1. The box of a pin hole camera of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength ë the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when: [*JEE Main-2016*]

(*a*) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(*b*) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(*c*) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(*d*) $a = \frac{\lambda^2}{L} = b_{min} = \sqrt{4\lambda L}$

2. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is: [*JEE Main-2017*]

(*a*) 9.75 mm
(*b*) 15.6 mm
(*c*) 1.56 mm
(*d*) 7.8 mm

3. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.) [*JEE Main-2018*]

(*a*) 50 μm (*b*) 75 μm
(*c*) 100 μm (*d*) 25 μm

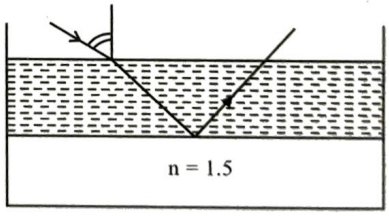
4. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is : [*JEE Main-2018*]

(*a*) 30° (*b*) 45°
(*c*) 60° (*d*) 0°

5. A plane polarized light is incident on a polarizer with its pass axis making angle θ with x-axis, as shown in the figure. At four different values of θ, θ = 8°, 38°, 188° and 218°, the observed intensities are same. What is the angle between the direction of polarization and x-axis. [*JEE Main-2018 (Online)*]

(*a*) 203° (*b*) 45°
(*c*) 98° (*d*) 128°

6. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i.(see figure) is for a bam of light entering the liquid, the light reflected from the liquid glass interface is nver completely polarized. For this to happen, the minimum value of μ is : [*JEE Main-2019 (January)*]



(*a*) $\frac{3}{\sqrt{5}}$ (*b*) $\frac{5}{\sqrt{3}}$

(*c*) $\sqrt{\frac{5}{3}}$ (*d*) $\frac{4}{3}$

7. In a double - slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44μm and a width of 4.05μm. The number of bright fringes between the first and the second diffraction minima is: [*JEE Main-2019 (January)*]

(*a*) 10 (*b*) 05
(*c*) 04 (*d*) 09

8. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $1/8^{th}$ of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to: [JEE Main-2019 (January)]

(a) 0.74 (b) 0.85
(c) 0.94 (d) 0.80

9. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field for the wave propagating in the glass medium will be: [JEE Main-2019 (January)]

(a) 30 V/m (b) 10 V/m
(c) 24 V/m (d) 6 V/m

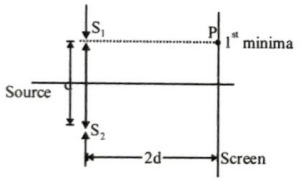
10. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is: [JEE Main-2019 (January)]

(a) 640 (b) 320
(c) 321 (d) 641

11. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio: [JEE Main-2019 (January)]

(a) 16 : 9 (b) 25 : 9
(c) 4 : 1 (d) 5 : 3

12. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)? [JEE Main-2019 (January)]



(a) $\frac{\lambda}{2(\sqrt{5}-2)}$ (b) $\frac{\lambda}{(\sqrt{5}-2)}$

(c) $\frac{\lambda}{2(5-\sqrt{2})}$ (d) $\frac{\lambda}{(5-\sqrt{2})}$

13. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are: [JEE Main-2019 (January)]

(a) 625 nm to 500 nm
(b) 380 nm to 525 nm
(c) 380 nm to 500 nm
(d) 400 nm to 500 nm

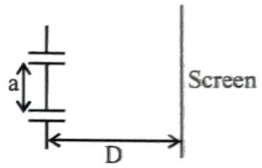
14. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be: [JEE Main-2019 (April)]

(a) 4 (b) 2
(c) 9 (d) 18

15. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be: [JEE Main-2019 (April)]

(a) $(\sqrt{3}+1)^4 : 16$ (b) 9 : 1
(c) 4 : 1 (d) 25 : 9

16. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gest shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be : [JEE Main-2019 (April)]



(a) $\frac{2\lambda}{a(\mu-1)}$ (b) $\frac{\lambda}{a(\mu-1)}$

(c) $\frac{2n\lambda}{a(\mu-1)}$ (d) $\frac{n\lambda}{a(\mu-1)}$

17. A system of three polarizers P_1, P_2, P_3, is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly) : [JEE Main-2019 (April)]

(a) 16.00 (b) 1.80
(c) 5.33 (d) 10.67

18. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used): [JEE Main-2019 (April)]

(a) $\frac{\lambda}{2(\mu-1)}$ (b) $\frac{\lambda}{(2\mu-1)}$

(c) $\frac{2\lambda}{(\mu-1)}$ (d) $\frac{\lambda}{(\mu-1)}$

19. Visible light of wavelength 6000×10^{-8} cm falls normally on single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1, then θ_1 is close to : [*JEE Main-2020 (January)*]

(*a*) 25° (*b*) 30°

(*c*) 20° (*d*) 45°

20. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer-analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero is : [*JEE Main-2020 (January)*]

(*a*) 45° (*b*) 90°

(*c*) 71.6° (*d*) 18.4°

21. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's moduls of wire is 16×10^{11} Nm^{-2}, the extension of wire over its natural length is: [*JEE Main-2020 (January)*]

(*a*) 0.02 mm (*b*) 0.01 mm

(*c*) 0.03 mm (*d*) 0.04 mm

22. In a Young' double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a soruce of light of wavelength 589 nm is used and the interference pattern is observed on screen kept 1.5 m away. The separation between the successive bright fringes on the screen is: [*JEE Main-2020 (January)*]

(*a*) 4.9 mm (*b*) 3.9 mm

(*c*) 5.9 mm (*d*) 6.9mm

23. In a double-slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$ th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is: [*JEE Main-2020 (January)*]

(*a*) 0.568 (*b*) 0.760

(*c*) 0.853 (*d*) 0.672

24. An electron (mass m) with initial velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wave length at time t is given by: [*JEE Main-2020 (January)*]

(*a*) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$ (*b*) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E^2t^2}{2m^2v_0^2}}}$

(*c*) $\frac{\lambda_0}{\sqrt{2+\frac{e^2E^2t^2}{m^2v_0^2}}}$ (*d*) $\frac{\lambda_0\sqrt{2}}{\sqrt{1+\frac{e^2E^2t^2}{m^2v_0^2}}}$

25. The aperture diameter of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 Å, the minimum separation between abjects on the surface of moon, so that they are just resolved, is close to: [*JEE Main-2020 (January)*]

(*a*) 600 m (*b*) 20 m

(*c*) 200 m (*d*) 60 m

26. Three harmonic waves having equal frequency v and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to: [*JEE Main-2020 (January)*]

(*a*) $5.8\ I_0$ (*b*) $3\ I_0$

(*c*) I_0 (*d*) $0.2\ I_0$

27. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of a wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be [*JEE Main-2020 (September)*]

(*a*) 28 (*b*) 24

(*c*) 30 (*d*) 18

28. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close is [*JEE Main-2020 (September)*]

(*a*) 1.27 μm (*b*) 2.05 μm

(*c*) 2.87 μm (*d*) 2 μm

29. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is [*JEE Main-2020 (September)*]

(*a*) $\frac{2\pi}{\lambda}(n_2L_1 - n_1L_2)$ (*b*) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$

(*c*) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$ (*d*) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$

30. In a Young's double slit experiment, light of 500 nm is used to produce and interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to [JEE Main-2020 (September)]

(a) 0.17° (b) 1.7°

(c) 0.57° (d) 0.07°

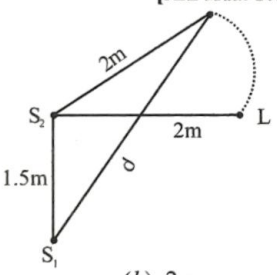
31. Orange light of wavelength 6000×10^{-10} m illuminates a single slit of width 0.6×10^{-4} m. The maximum possible number of diffraction minima produced on both sides of the central maximum is _____. [JEE Main-2020 (September)]

32. A beam of plane polarised light of large crosssectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser (cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to [JEE Main-2020 (September)]

(a) 1.0×10^{-5} J (b) 1.0×10^{-4} J

(c) 5.0×10^{-4} J (d) 1.5×10^{-4} J

33. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 1$ m, in phase. S_1 and S_2 are placed 1.5 m apart (see fig). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2 m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is [JEE Main-2020 (September)]

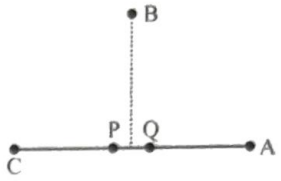


(a) 5 m (b) 2 m

(c) 12 m (d) 3 m

34. A Young's double-slit experiment is performed using monochromatic light of wavelength λ. The intensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is _____. [JEE Main-2020 (September)]

35. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio [JEE Main-2020 (September)]



(a) 0 : 1 : 2 (b) 4: 1 : 0

(c) 2 : 1 : 0 (d) 0 : 1 : 4

36. A galaxy is moving away from the earth at a speed of 286 kms^{-1}. The shift in the length of a redline at 630 nm is $x \times 10^{-10}$ m. The value of x, to the nearest integer, is

Take the value of speed of Light c, as 3×10^8 ms^{-1} [JEE Main-2021 (March)]

37. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wave length of light used is x' nm. The value of x' to the nearest integer is [JEE Main-2021 (March)]

JEE-ADVANCED PREVIOUS YEAR'S

1. Young's double slit experiment is carried out by using green, red and blue light, one color at time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then [IIT JEE-2012]

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$

(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

2. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is: [IIT JEE-2012]

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$

(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

3. A light source, which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then [JEE Advanced -2014]

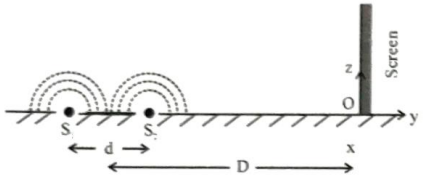
(a) $\beta_2 > \beta_1$

(b) $m_1 > m_2$

(c) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(d) The angular separation of fringes for λ_1 is greater than λ_2

4. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for $z > 0$) at a distance D = 3 from the mid –point of S_1S_2, as shown schematically in the figure. The distance between the sources d = 0.06003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is (are) true of the intensity pattern on the screen? [*JEE Advanced -2016*]



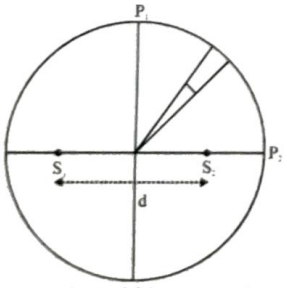
(*a*) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

(*b*) Straight bright and dark bands parallel to the x-axis

(*c*) Semi circular bright and dark bands centered at point O

(*d*) The region very close to the point O will be dark

5. Two coherent monochromatic point sources S_1 and S_2 of wavelength λ = 600 nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is Δq. Which of the following is /are correct? [*JEE Advanced-2017*]



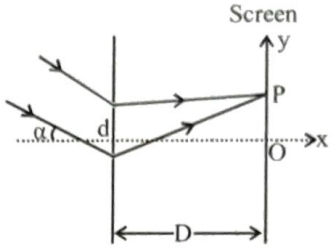
(*a*) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

(*b*) A dark spot will be formed at the point P_2

(*c*) At P_2 the order of the fringe will be maximum

(*d*) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

6 In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1m. A parallel beam of light of wavelength 600nm is incident on the slits at angle a as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct? [*JEE Advanced-2019*]



(*a*) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

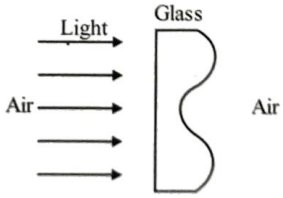
(*b*) Fringe spacing depends on α

(*c*) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P

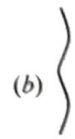
(*d*) For $\alpha = 0$, there will be constructive interference at point P.

7. A parallel beam of light strikes a piece of transparent glass having cross section as shown in the figure below. Correct shape of the emergent wave front will be (figures are schematic and not drawn to scale):

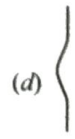
[*JEE Advanced-2020*]



(*a*) 

(*b*) 

(*c*) 

(*d*) 

ANSWER KEY

Topicwise Questions

1. (*a*) **2.** (*c*) **3.** (*c*) **4.** (*d*) **5.** (*c*) **6.** (*d*) **7.** (*b*) **8.** (*d*) **9.** (*c*) **10.** (*c*)
11. (*a*) **12.** (*b*) **13.** (*b*) **14.** (*b*) **15.** (*b*) **16.** (*b*) **17.** (*d*) **18.** (*c*) **19.** (*a*) **20.** (*a*)
21. (*c*)

Learning Plus

1. (*a*) **2.** (*c*) **3.** (*b*) **4.** (*c*) **5.** (*b*) **6.** (*b*) **7.** (*a*) **8.** (*a*) **9.** (*b*) **10.** (*b*)
11. (*d*) **12.** (*c*) **13.** (*c*) **14.** (*b*) **15.** (*b*) **16.** (*c*) **17.** (*c*) **18.** (*c*) **19.** (*c*) **20.** (*b*)
21. (*b*) **22.** (*b*) **23.** (*d*) **24.** (*d*) **25.** (*d*) **26.** (*d*) **27.** (*a*) **28.** (*d*) **29.** (*b*) **30.** (*c*)
31. (*a*)

Advanced Level Multiconcept Questions

1. (*b*, *c*) **2.** (*a*, *c*) **3.** (*a*, *c*) **4.** (*a*, *c*) **5.** (*c*, *d*) **6.** (*a*, *c*) **7.** (*a*, *d*) **8.** (*d*) **9.** (*c*) **10.** (*b*)
11. (*a*) **12.** (*a*) **13.** (*a*, *c*, *d*) **14.** (*a*, *b*, *d*) **15.** (*b*, *d*) **16.** (*a*) q, r, s (*b*) p, q, r, s (*c*) q, r, s (*d*) p,q,r,s
17. [0005] **18.** [0012] **19.** [0007] **20.** [0007] **21.** [0001] **22.** [0.6 mm] **23.** [0.16 m] **24.** [0001] **25.** [2311] **26.** [209]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*c*) **2.** (*d*) **3.** (*d*) **4.** (*b*) **5.** (*a*) **6.** (*a*) **7.** (*b*) **8.** (*b*) **9.** (*c*) **10.** (*d*)
11. (*b*) **12.** (*a*) **13.** (*a*) **14.** (*a*) **15.** (*b*) **16.** (*d*) **17.** (*d*) **18.** (*d*) **19.** (*a*) **20.** (*d*)
21. (*c*) **22.** (*c*) **23.** (*c*) **24.** (*b*) **25.** (*d*) **26.** (*a*) **27.** (*a*) **28.** (*a*) **29.** (*c*) **30.** (*c*)
31. (198) **32.** (*b*) **33.** (*d*) **34.** (09) **35.** (*c*) **36.** (6) **37.** (600)

JEE-ADVANCED PREVIOUS YEAR'S

t.me/realprayasians2

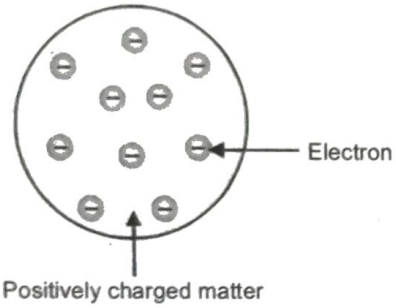
1. (*d*) **2.** (*b*) **3.** (*a*, *b*, *c*) **4.** (*c*, *d*) **5.** (*a*, *c*) **6.** (3) **7.** (*a*)

25 ATOMS, DUAL NATURE OF RADIATION AND MATTER

t.me/realprayasians

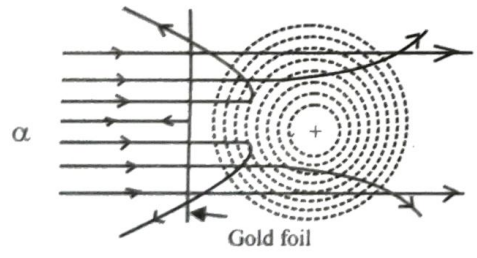
THOMSON'S ATOMIC MODEL

J.J. Thomson suggested that atoms are just positively charged lumps of matter with electrons embedded in them like raisins in a fruit cake. Thomson's model called the **'Plum pudding'** model is shown in figure.



Thomson played an important role in discovering the electron by passing current through gas discharge tube discovering cathode rays. His idea was taken seriously.
But the real atom turned out to be quite different.

Rutherford's α-Scattering Experiment

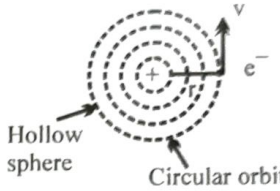


Result of the Experiment

- Majority of α-particles passes undeviated through gold foil that means most of the space inside the atom is empty.
- Very few α-particles deflect at an angle larger than 90° & even some at 180° which is possible only in case when there exists a solid positive mass confined in a very small space.

RUTHERFORD'S MODEL

- The whole positive charge & almost whole mass of an atom (leaving aside the mass of revolving e^- in various circular orbits) remains concentrated in a nucleus of radius of the order of 10^{-15}m.

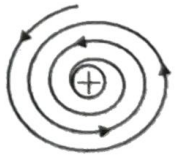


- Total positive charge of protons = total negative charge of electrons. Hence, the atom is electrically neutral.
- The necessary centripetal force for revolving e^- round the nucleus in circular orbit is provided by coulomb's electrostatic force of attraction

$$\frac{mv^2}{r} = \frac{k(ze)(e)}{r^2}$$

REASON OF FAILURE OF MODEL

- It could not explain the line spectrum of H-atom.
  Justification: According to Maxwell's electromagnetic theory every accelerated moving charged particle radiates energy in the form of electromagnetic waves & therefore during revolution of e^- in circular orbit its frequency will continuously vary (i.e. decrease) which will result in the continuous emission of lines & therefore spectrum of atom must be continuous but in reality, one obtains line spectrum for atoms.



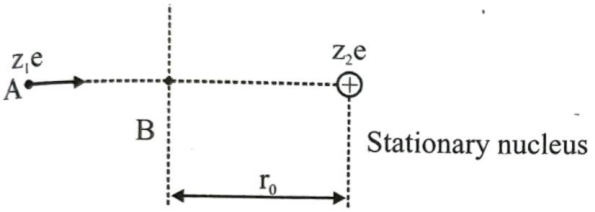
- It could not explain the stability of atoms.
  Justification - Since revolving electron will continuously radiates energy & therefore radii of circular path will continuously decrease & in a time of about 10^{-8} sec revolving electron must fall down in a nucleus by adopting a spiral path

Application of Rutherford's Model

Determination of distance of closest approach: When a positively charged particle approaches towards stationary nucleus then due to repulsion between the two particle the kinetic energy of positively charged particle gradually decreases and a stage comes when its kinetic energy becomes zero & from where it again starts retracing its original path.

Definition: The distance of closest approach is the minimum distance of a stationary nucleus with a positively charged particle making head on collision from a point where its kinetic energy becomes zero.

Suppose a positively charged particle A of charge q_1 $(=z_1e)$ approaches from infinity towards a stationary nucleus of charge z_2e then,



Let at point B, kinetic energy of particle A becomes zero then by the law of conservation of energy at point A & B,

$$TE_A = TE_B$$

$$KE_A + PE_A = KE_B + PE_B$$

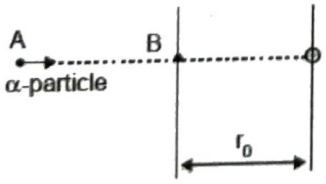
$$E + 0 = 0 + \frac{k(z_1e)(z_2e)}{r_0} \quad \text{(in joule)}$$

$$\therefore r_0 = \frac{k(z_1e)(z_2e)}{E} \quad m$$

TRAIN YOUR BRAIN

Example 1: An α-particle with kinetic energy 10 MeV is heading towards a stationary point-nucleus of atomic number 50. Calculate the distance of closest approach.

Sol. $\because TE_A = TE_B$



$$\therefore 10 \times 10^6 e = \frac{K \times (2e)(50e)}{r_0}$$

$$r_0 = 1.44 \times 10^{-14} \text{ m}$$

$$r_0 = 1.44 \times 10^{-4} \text{ Å}$$

Bohr's Atomic Model

In 1913 Niels Bohr developed a physical theory of atomic hydrogen from which the Rydberg formula could be derived. Bohr's model for atomic hydrogen is based on certain assumptions, which are as follows :

(*i*) In the hydrogen atom electron revolves around the nucleus in circular orbits.

(*ii*) Electron revolves only in those orbits around nucleus where the angular momentum of electron is an integral multiple of $\frac{h}{2\pi}$. These orbits are called stationary orbits. This assumption is called Bohr's quantization rule.

(*iii*) The energy of electron can take only definite values in a given stationary orbit. The electron can jump from one stationary orbit to other. If it jumps from an orbit of higher energy to an orbit of lower energy it emits a photon of radiation. Similarly an electron can take energy from a source and jumps from a lower energy orbit to a higher energy orbit.

In both the cases energy of radiation involved is given by the Einstein-Planck equation.

$$\Delta E = \frac{hc}{\lambda}$$

Energy of a Hydrogen Atom

Let the mass of electron be m and it is revolving in an orbit or radius r, then using the quantization rule we get

$$mvr = n\,\frac{h}{2\pi}, \qquad \text{...(i)}$$

Where n is a positive integer

Also, from the equation of motion of electron we have

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \qquad \text{...(ii)}$$

Though Z = 1 for hydrogen, however generally we leave Z in the equation, as the theory is equally applicable to other atoms with all but one of their electrons are removed.

Solving (i) and (ii)

$$v = \frac{Ze^2}{2\varepsilon_0 hn} \qquad \text{...(iii)}$$

$$\text{and } r = \frac{\varepsilon_0 h^2 n^2}{\pi m Ze^2} \qquad \text{...(iv)}$$

So allowed radii are proportional to n^2. Putting Z = 1 and n = 1 the quantity $\frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53$ Å is called Bohr's radius and is radius of smallest circle allowed to the electron. Using equation (iii) we write kinetic energy of the electron in n^{th} orbit is

$$K = \frac{1}{2}mv^2 = \frac{mZ^2e^4}{8\varepsilon_0^2h^2n^2} \quad ...(v)$$

The potential energy of the atom is

$$U = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{mZ^2e^4}{4\varepsilon_0^2h^2n^2}$$

So total energy of the atom is
E = K + U

$$E = -\frac{mZ^2e^4}{8\varepsilon_0^2h^2n^2} \quad ...(vi)$$

In deriving the energy of an atom we have considered kinetic energy of electron and potential energy of the electron nucleus pair.

The kinetic energy of electron, potential energy and total energy of an atom are related with each other in same manner as in gravitation. That is E = U/2, E = – K, K = – U/2

From (vi), the total energy of the atom in the state n = 1 is $E_1 = -\frac{mZ^2e^4}{8\varepsilon_0^2h^2}$

For hydrogen atom Z = 1 and we get
E_1 = –13.6 eV, this is the energy of electron when it moves in the smallest allowed orbit. It is also evident from equation (vi) that energy of the atom in the nth energy state is proportional to $\frac{1}{n^2}$. So we can write.

$$E_n = -\frac{13.6}{n^2}eV$$

So we get energy in the state n = 2 is –3.4 eV. In the state n = 3 it is –1.5 eV etc. The state of an atom with the lowest energy is called its ground state and states with higher energies are called excited states.

Important results for a hydrogen like atom are

$$\text{Velocity} = \frac{Ze^2}{2\varepsilon_0 hn} = \frac{c}{137}\left(\frac{Z}{n}\right) m/s = 2.18 \times 10^6\left(\frac{Z}{n}\right) \quad ...(1)$$

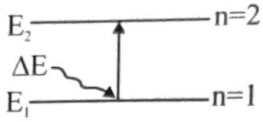
$$\text{Radius} = \frac{\varepsilon_0 h^2 n^2}{\pi m Ze^2} = 0.53\left(\frac{n^2}{Z}\right)Å \quad ...(2)$$

$$\text{Energy} = -\frac{mZ^2e^4}{8\pi\varepsilon_0 h^2 n^2} = -13.6\left(\frac{Z^2}{n^2}\right) eV$$

(1) Binding Energy - Binding energy of an electron is the amount of energy released while forming an atom.

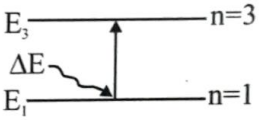
(2) Ionization Energy & Ionization Potential- The minimum energy required to remove an electron from hydrogen or hydrogen like atom from ground state is called its ionization energy & corresponding potential through which an electron is accelerated for this is called ionization potential
I.E. = $E_\infty - E_1 = -E_1$
= Binding energy of e^-

(3) Excitation Energy & Excitation Potential: The minimun energy required to excite an atom is called excitation energy of the particular excited state & corresponding potential is called excitation potential.



$\Delta E + E_1 = E_2$

$\Delta E = E_2 - E_1$
↑
Excitation Energy



$\Delta E + E_1 = E_3$

$\Delta E = E_3 - E_1$

If excitation energy & ionisation energy are represented in eV then corresponding value in volt is termed as excitation potential & ionisation potential respectively.

For example: Excitation energy & Ionisation energy for H-atom are 10.2 eV & 13.6 eV respectively & therefore 10.2V & 13.6V are excitation & ionisation potential respectively.

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Example 2: A α particle after passing through a potential difference of V volt collides with a nucleus. If the atomic number of the nucleus is Z then the distance of closest approach of α-particle to the nucleus will be-

(*a*) $14.4\frac{Z}{V}$ Å (*b*) $14.4\frac{Z}{V}$ m

(*c*) $14.4\frac{Z}{V}$ cm (*d*) All of these

Sol. $E_k = U$, $2eV = \frac{k(Ze)(2e)}{d}$

$$d = \frac{kZe}{V} = 9 \times 10^9 \times 1.6 \times 10^{-19}\left(\frac{Z}{V}\right),$$

$$d = 14.4 \times 10^{-10}\left(\frac{Z}{V}\right) m = 14.4\left(\frac{Z}{V}\right) Å$$

Example 3: The radius of hydrogen atom in the ground state is 5.3×10^{-11} m. If this atom collides with an electron then its values becomes 21.2×10^{-11} m. The value of principal quantum number will be-

(*a*) 2 (*b*) 16

(*c*) 3 (*d*) 4

Sol. $\because \frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2$, $\therefore n_2^2 = \frac{n_1^2 r_2}{r_1}$

$$\therefore n_2 = n_1\sqrt{\frac{r_2}{r_1}} \Rightarrow n_2 = 1\sqrt{\frac{21.2\times10^{-11}}{5.3\times10^{-11}}}$$

$\Rightarrow n_2 = 2$

Example 4: An electron revolves round a nucleus of charge Ze. In order to excite the electron from the state n = 3 to n = 4, the energy required is 66.0 eV. Z will be -

(*a*) 25 (*b*) 10

(*c*) 4 (*d*) 5

Sol. Energy required for transition in H-like atom

$$= 13.6 \times z^2\left(\frac{1}{3^2}-\frac{1}{4^2}\right) \text{ eV}$$

$$66.0 \text{ eV} = Z^2 \times 13.6 \times \frac{7}{144}$$

$$Z^2 = \frac{66}{0.66} = 100, \quad Z = 10$$

Example 5: A hydrogen atom rises from its n = 1 state to the n = 4 state by absorbing energy. If the potential energy of the atom in the n = 1 state be –13.6 eV, then potential energy in the n = 4 state will be -

(*a*) 3.4 eV (*b*) –1.54 eV

(*c*) 0.85 eV (*d*) –0.85 eV

Sol. $E_n = -\frac{13.6}{n^2}$

E_4 = Energy of 4th state $= -\frac{13.6}{4^2}$

$$\Rightarrow E_4 = -\frac{13.6}{16} = -0.85 \text{ eV}$$

Example 6: The wavelength of first spectral line of sodium is 5896 Å . The first excitation potential of sodium atom will be ($h = 6.63 \times 10^{-34}$ Js)

(*a*) 4.2 V (*b*) 3.5 V

(*c*) 2.1 V (*d*) None of these

Sol. The energy of first excitation of sodium is

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{5896\times10^{-10}} J$$

$= 3.37 \times 10^{-19}$ J = 2.1 eV

Example 7: The radius of first orbit of hydrogen atom is 0.53 Å. The radius of its fourth orbit will be-

(*a*) 0.193 Å (*b*) 4.24 Å

(*c*) 2.12 Å (*d*) 8.48 Å

Sol. $r_n = 0.53\ n^2$, $n = 4$

$\Rightarrow r_4 = 0.53 \times 16$ $\Rightarrow r_4 = 8.48$ Å

Type of Line Spectrum

(*a*) **Emission line spectrum:** When an atomic gas or vapour at a pressure less than the atmospheric pressure is excited by passing electric discharge, the emitted radiation has a spectrum which contains certain specific bright lines only. These emission lines constitute emission spectrum. These are obtained when electron jumps from excited states to lower states. The wavelength of emission lines of different elements are different. For one element the emission spectrum is unique. It is used for the determination of composition of an unknown substance.

(*b*) **Absorption line spectrum:** When white light is passed through a gas, the gas is found to absorb light of certain wavelength. the bright background on the photographic plate is then crossed by dark lines that corresponds to those wavelengths which are absorbed by the gas atoms.

The absorption spectrum consists of dark lines on bright background. These are obtained due to absorption of certain wavelengths, resulting into transition of atom from lower energy states to higher energy states. (The emission spectrum consists of bright lines on dark background.)

The spectrum of sunlight has dark lines called **Fraunhoffer lines**. These are produced when the light coming out of the interior (core) of the sun, passes through the envelope of cooler gas. The cooler gas absorbs light of certain wavelengths corresponding to the elements present in the cooler gas. This results into dark lines (absence of wavelength) on brighter backgound. Study of Fraunhoffer lines is used to determine the elements (composition) of the star.

Determination of no. of Spectral Lines (Theoretical) in Emission & in Absorption Transitions:

No. of emission spectral lines - If the electron is excited to state with principal quantum number n then from the nth state, the electron may go to $(n-1)^{th}$ state , ..., 2nd state or 1st state. So there are (n – 1) possible transitions starting from the nth state. The electron reaching $(n-1)^{th}$ state may make (n – 2) different transitions. Similarly for other lower states. The total no. of possible transitions is

$$(n-1) + (n-2) + (n-3) + \ldots 2 + 1 = \frac{n(n-1)}{2}$$

No. of absorption spectral lines - Since at ordinary temperatures, almost all the atoms remain in their lowest energy level (n = 1) & so absorption transition can start

only from n = 1 level (not from n = 2 , 3 , 4, ... levels). Hence, only Lyman seires is found in the absorption spectrum of hydrogen atom (which as in the emission spectrum, all the series are found)

No. of absorption spectral lines = (n – 1)

Remember : The absorption spectrum of sun has Balmer series also besides the Lyman series. Many H-atoms remain in n = 2 also due to very high temperature.

EXPLANATION OF H-SPECTRUM & SPECTRAL LINE FORMULA

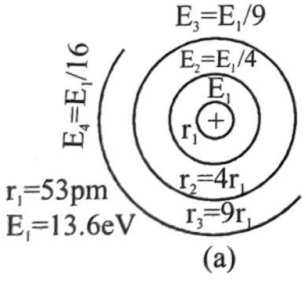
Hydrogen spectra: Now on the basis of Bohr's model of hydrogen atom it is possible to explain the spectra of hydrogen. If an electron jumps from m^{th} orbit to the nth orbit, the energy of the atom changes from E_m to E_n. The extra energy $E_m - E_n$ is emitted as a photon of electromagnetic radiation. The corresponding wavelength λ is given by

$$\frac{1}{\lambda} = \frac{E_m - E_n}{hc} = \frac{mZ^2e^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

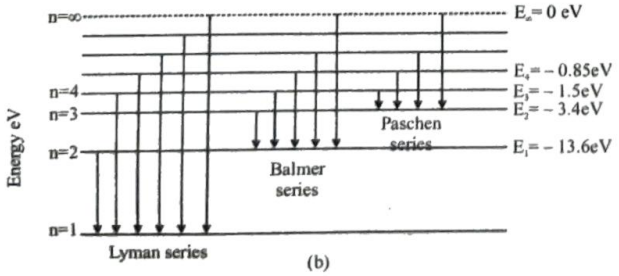
$$= RZ^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \quad ...(4)$$

where $R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$ is called the Rydberg constant. Putting the values of different constant, the Rydberg constant R comes out to be $1.0973 \times 10^7\ m^{-1}$.

So on the basis of energy levels involved in the transition we can divide the entire hydrogen spectrum in various series.



(a)



(b)

Lyman series: When an electron jumps from any of the higher states to the ground state (n = 1), the series of spectral lines emitted fall in uv region and is called as Lyman series. The wavelength λ of any line of the series can be given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right),\ n = 2, 3, 4.....$$

Balmer series: When an electron makes a transition from any of the higher states to the state with n = 2 (first excited state), the series of spectral lines emitted fall in visible region and is called Balmer series. The wavelength of any of Balmer lines is given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{2^2} - \frac{1}{n^2}\right),\ n = 3, 4, 5, ...$$

Paschen series: When an electron jumps from any of the higher states to the state with n = 3 (2nd excited state), the series of spectral lines emitted fall in near infra-red region and is called Paschen series. The wavelength λ for any of the line of Paschen series is given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{3^3} - \frac{1}{n^2}\right),\ n = 4, 5, 6, ...$$

Bracket series: When an electron jumps from any of the higher states to the state with n = 4, (3rd excited state) the series of spectral lines emitted fall in far infrared region and constitude Bracket series. The wavelength λ of any spectral lines of Bracket series is given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{4^2} - \frac{1}{n^2}\right),\ n = 4, 5, 6,7, ...$$

Pfund series: Pfund series is constituted by spectral lines emitted when electron jumps from any of the higher energy states to the state with n = 5 (4th excited state).

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{5^2} - \frac{1}{n^2}\right),\ n = 6, 7, 8, ...$$

Figure shows schematically the allowed orbits together with the energies of the hydrogen atom. It also shows the allowed energies separately.

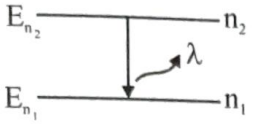
General Points for Spectral Lines in every Spectral Series:

(1) Wavelength of first line is maximum & last line is minimum.

(2) As the order of spectral series increases, wavelength also usually increases
$\lambda_{PF} > \lambda_{BR} > \lambda_P > \lambda_B > \lambda_L$

(3) Frequency of energy emission in lyman transitions are highest among all other series.

Concept of Recoiling of an Atom & Determination of Momentum & Energy for Recoil Atoms: When ever any electron makes transition from any higher energy state to any lower energy state then photon is emitted & due to which in the back side atom is recoiled. The atom is recoiled by sharing some energy from the energy evolved during electronic transition.



If m = mass of recoil atom
v = velocity of recoil atom

Then $\frac{1}{2}mv^2 + \frac{hc}{\lambda} = E_{n_2} - E_{n_1} = \Delta E$

Recoil momentum of atom = $\frac{h}{\lambda}$ = momentum of photon

Recoil energy of atom = $\frac{p^2}{2m}$

Shortcoming's of Bohr's Atomic Model

(*i*) It is valid only for one electron atoms.
e.g : H, He^+, Li^{+2}, Na^{+10} etc.

(*ii*) Orbits were taken as circular but according to Sommerfield these are elliptical.

(*iii*) Intensity of spectral lines could not be explained.

(*iv*) Nucleus was taken as stationary but it also rotates on its own axis.

(*v*) It could not be explained the minute structure in spectrum line.

(*vi*) This does not explain the zeeman effect (splitting up of spectral lines in magnetic field) & stark effect (splitting up in electric field)

(*vii*) This does not explain the doublets in the spectrum of some of the atoms like sodium (5890Å & 5896Å)

IMPORTANT FORMULAE

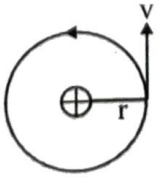
(1) Time period (T):

distance = time × speed

$2\pi r = T \times v$

$T = \frac{2\pi r}{v}$

$T_n = \frac{n^3h^3}{4\pi k^2z^2me^4}$ $\quad T_n \propto \frac{n^3}{z^2}$



(2) Frequency of revolution

$f_n = \frac{v_n}{2\pi r_n} = \frac{4\pi k^2z^2me^4}{n^3h^3}$; $f_n \propto \frac{z^2}{n^3}$

(3) Momentum of electron

$P_n = \frac{2\pi mkze^2}{nh}$; $P_n \propto \frac{Z}{n}$

(4) Angular velocity of electron

$\omega_n = \frac{8\pi^3k^2z^2me^4}{n^3h^3}$, $\omega_n \propto \frac{z^2}{n^3}$

(5) Current (I)

$I = \frac{e}{T} = ev = \frac{ev}{2\pi r}$; $I \propto \frac{z^2}{n^3}$

(6)

4	———	–0.85 eV
3	———	–1.51 eV
2	———	–3.4 eV
n=1	———	–13.6 eV

Energy level values for H-atom

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Example 8: The wavelength of the first line of lymen series for hydrogen is identical to that of the second line of Balmer series for some hydrogen like ion x. Energies of two levels of x will be : (Ground state binding energy of hydrogen atom = 13.6 eV)

(*a*) –54.4 eV, –6.07 eV (*b*) –13.6 eV, –3.4 eV
(*c*) –3.4 eV, –13.6 eV (*d*) –54.4 eV, –13.6 eV

Sol. We know that

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For first line of Lyman series in hydrogen atom

$$\frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$$

For second line of Balmer series of hydrogen like ion x

$$\frac{1}{\lambda_2} = Z^2R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3Z^2R}{16}$$

Given that, $\lambda_1 = \lambda_2$

$$\therefore \frac{3R}{4} = \frac{3Z^2R}{16} \Rightarrow Z = 2$$

Energy of nth state of ion X is given by

$$E_x = -\frac{13.4}{n^2} \times Z^2,\ (E_x)_1 = -\frac{13.4\times 4}{1}$$

$$= -54.4 \text{ eV}$$

$$(E_x)_2 = -\frac{13.4\times 4}{4} = -13.6 \text{ eV}$$

Example 9: The wavelength of the first member of the Balmer series in hydrogen spectrum is 6563 Å. Calculate the wavelength of first member of Lyman series in the same spectrum.

(*a*) 1000 Å (*b*) 1215.37 Å

(*c*) 1512 .37 Å (*d*) None of these

Sol. For the first member of the Balmer series

$$\bar{\nu} = \frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= \frac{5}{36} R \qquad ...(i)$$

For the first member of Lyman series

$$\bar{\nu} = \frac{1}{\lambda_2} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$= \frac{3R}{4} \qquad ...(ii)$$

Dividing eq. (1) by eq.(2) , we get

$$\frac{\lambda_2}{\lambda_1} = \frac{5}{27} \text{ or } \lambda_2 = \frac{5}{27} \lambda_1$$

$$\lambda_2 = \frac{5\times 6563}{27} = 1215.37 \text{ Å}$$

Example 10: An electron makes transition from n = 4 state to n = 1 state in hydrogen atom. The momentum of recoil hydrogen atom in kg-m/s will be-

(*a*) 12.75×10^{-19} (*b*) 13.6×10^{-19}

(*c*) 6.8×10^{-27} (*d*) zero

Sol. According to law of conservation of momentum, momentum of recoil atom

$$= \text{momentum of photon} = \frac{E_4 - E_1}{C}$$

$$\Rightarrow P = \frac{1275\times 1.6\times 10^{-19}}{3\times 10^8}$$

$$= 6.8 \times 10^{-27} \text{ kg-m/s}$$

DUAL NATURE OF LIGHT

Light have both wave as well as as particle character. Some of the character can be explained by wave and some can be explained by particle nature of light.

Wave Nature : Young's double slit experiment.

Particle Nature : Photoelectric experiment cannot be explained by wave nature.

According to particle nature of light, light is made up of small particles called photons.

(*a*) A photon always travels at a speed c = 299, 792, 458 m/s $\approx 3.0 \times 10^8$ m/s in vacuum. This is true for any frame of reference used to observe the photon.

(*b*) The mass of a photon is not defined in the sense of Newtonian mechanics. We shall ignore this concept. We simply state that the rest mass of a photon is zero.

(*c*) Each photon has a definite energy and a definite linear momentum.

(*d*) Let 'E' and 'p' be the energy and linear momentum of a photon of light, and 'v' and λ be the frequency and wavelength of the same light when it behaves as a wave. Then,

$$E = h\mu = hc/\lambda$$

and $$p = h/\lambda = E/c$$

where 'h' is a universal constant known as the Planck constant and has a value 6.626×10^{-34} J–s

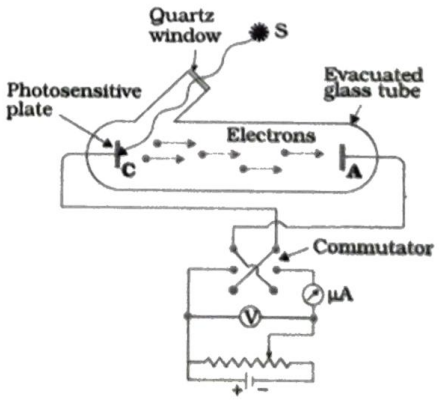
$= 4.136 \times 10^{-15}$ eV -s

Thus, all photons of light of a particular wavelength λ have the same energy $E = hc/\lambda$ and same momentum $p = h/\lambda$

(*e*) A photon may collide with a material particle. The total energy and the total momentum remain conserved in such a collision. The photon may get absorbed and / or a new photon may be emitted. Thus, the number of photons may not be conserved.

(*f*) If the intensity of light of a given wavelength is increased, there is an increase in the number of photons crossing a given area in a given time. The energy of each photon remains the same.

PHOTO ELECTRIC EFFECT



When light of sufficiently small wavelength is incident on a metal surface, electrons are ejected from the metal. This phenomenon is called photoelectric effect and the electrons ejected are called photoelectrons. An experimental setup was arranged to study photoelectric effect, and the results obtained from the experiment are

(*i*) When light of sufficiently small wavelength falls on a metal surface, the metal emits photoelectrons. This emission of photoelectrons is instantaneous.

(*ii*) The photoelectric current i.e., the number of photoelectrons emitted per second depends on the intensity of the incident light.

(*iii*) The maximum kinetic energy with which electrons come out of the metal depends only on frequency of incident light and is independent of intensity of light.

(*iv*) There is a threshold wavelength for a given metal such that if the wavelength of incident light is greater than the threshold, there will be no emission of photoelectrons.

IMPORTANT DEFINITIONS

Threshold Frequency (ν_0): Minimum frequency of incident photon below which no ejection of photoelectrons from a metal surface can take place. It is known as **threshold frequency** for that metal. Its value is constant for a particular metal but may be different for different metals.

Threshold wavelength (λ_0): Maximum wavelength of incident photon above which there will be no photoelectric emission from a metal surface is known as threshold wavelength

$$\lambda_0 = \frac{c}{\nu_0}$$

Work Function or Threshold Energy (ϕ):

(*i*) The minimum energy of incident photon below which no ejection of photo electron from a metal surface will take place is known as work function or threshold energy for that metal .

$$\phi = h\nu_0 = \frac{hc}{\lambda_0}$$

(*ii*) Work function is the characteristic of given metal

Laws of Photo Electric Effect: On the basis of experiments Lenard gave following laws regarding photo emission.

(*a*) Rate of photo electron's emission does not depend upon frequency or wavelength of light (or photon) and in other words it does not depend upon energy of incident light.

(*b*) Rate of photo electron's emission depends upon intensity of light which is incident on metal surface

(*c*) Photo electric current depends upon intensity of light but does not depend upon frequency/wavelength or energy.

(*d*) Kinetic energy of emitted electron depends upon frequency or wavelength of incident light. With increasing frequency of incident light, kinetic energy of photo electrons increases but with increasing wavelength it decreases.

So $\nu\uparrow\ \lambda\downarrow$ K.E. of Emitted electrons $\uparrow$

$\nu\downarrow\ \lambda\uparrow$ K.E. of Emitted electrons $\downarrow$

(*e*) Kinetic energy of emitted photo electrons does not depend upon intensity of light.

(*f*) Value of threshold frequency or threshold wavelength depends upon photo sensitive nature of metal.

(*g*) There is no time lag between emission of electron and incidence of photon i.e. the electrons are emitted out as soon as the light falls on metal surface.

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Example 11: The work function of silver is 5.26×10^{-19} J. Calculate its threshold wavelength-

(*a*) 3674 Å (*b*) 3467 Å

(*c*) 3647 Å (*d*) 3764 Å

Sol. Threshold wavelength $= \lambda_0 = \frac{hc}{\varphi}$

$$\therefore \lambda = \frac{6.6\times10^{-34}\times3\times10^{8}}{5.26\times10^{-19}} = 3.764 \times 10^{-7} \text{ m}$$

$\lambda = 3764$ Å

Example 12: The work function of Na is 2.3 eV. What is the maximum wavelength of light that will cause photo electrons to be emitted from sodium?

(*a*) 539 mm (*b*) 0.539 mm

(*c*) 539 nm (*d*) 0.539 nm

Sol. The threshold wavelength $\lambda_0 = \frac{hc}{\varphi}$

($\because \phi = h\nu_0 = hc/\lambda_0$)

and $hc = 1.24 \times 10^{-6}$ (eV) m

$$\lambda_0 = \frac{1.24\times10^{-6}}{2.3} \text{ m ;}$$

$\lambda_0 = 0.539 \times 10^{-6}$ m = 539 nm

EINSTEINS EQUATION OF PHOTO ELECTRIC EFFECT

Soon after the publication of results of photoelectric effect, efforts were made to explain the result. Wave theory which considered light as wave failed on all counts to explain photoelectric effect. The main cause for this failure was inability of wave theory to consider the energy of light quantised and not distributed continuously. In 1900 Planck proposed that radiation from a hot body consists of small packets of energy called **'quantas'**. The energy of a quanta is given by $h\mu$ (μ being frequency of radiation). Einstein, getting a hint from this, proposed that light wave also consist of packets of energy or quantas whose energy is also given by '$h\mu$'. He called these quantas as photons.

Einstein postulated that a photon of incident light interacts with a metal electron and transfers its energy to electron in two ways. A part of the energy of the incident photon is used up in liberating the metal electron against the attractive forces of surrounding ions inside the metal ; the remaining energy is spent in giving kinetic energy to ejected photoelectrons. If μ be the frequency of incident light, ϕ be the minimum energy required to liberate an electron from the surface and E_k be the maximum kinetic energy of the emitted free electrons, then

$$h\nu = \phi + E_k \qquad ...(1)$$

ϕ is called the work function and it obviously depends on the nature of metal. Equation (1) is called Einstein photoelectric equation.

Einstein's Equation Explain Following Concepts:

(*a*) With increasing frequency of light, kinetic energy of electrons increases. Similarly with decreasing wavelength, Kinetic energy of electrons increases.

If ν_0 is threshold frequency then maximum kinetic energy

$$E_{max} = h\nu - h\nu_0$$

$$\Rightarrow \frac{1}{2} m v_{max}^2 = h(\nu - \nu_0)$$

So maximum velocity of photo electrons :-

$$\Rightarrow v_{max} = \sqrt{\frac{2h(\nu - \nu_0)}{m}}$$

m- mass of electron.
ν- frequency of incident light
ν_0- threshold frequency
λ_0- threshold wavelength
λ - incident wavelength

$$\Rightarrow E_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$\Rightarrow \frac{1}{2} m v_{max}^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

(*b*) If $\nu = \nu_0$ or $\lambda = \lambda_0$ then $v = 0$

(*c*) $\nu < \nu_0$ or $\lambda > \lambda_0$

$\Rightarrow$ There will be no emission of photo electrons.

(*d*) When Intensity of light is increased it means number of photons have been increased. It does not affect energy of photons. Hence rate of emission increases but there will be no change in kinetic energy of electrons. With increasing number of emitted electrons, value of photo electric current increases.

TRAIN YOUR BRAIN

Example 13: Light of wavelength 4000 Å is incident on a metal whose work function is 2eV. Calculate the maximum possible kinetic energy of the photo electrons.

(*a*) 3.09 eV (*b*) 1. 9 eV
(*c*) 1.09 eV (*d*) None of these

Sol. Energy of the incident photon = hc / λ

$$= \frac{19.8\times10^{-19}}{4\times1.6\times10^{-19}} = 3.09 \text{ eV}$$

Kinetic energy of the emitted electron

$E_k = h\nu - \phi = 3.09 - 2.00 = 1.09$ eV

Example 14: The threshold wavelength of a metal is 5800 Å. If wavelength of incident light is 4500Å, then the maximum kinetic energy of photoelectrons would be-

(*a*) 0.62 eV (*b*) 26 eV
(*c*) 62 eV (*d*) 0.26 eV

Sol. $E_{k_{max}} = \frac{hc[\lambda_0 - \lambda]}{\lambda_0\lambda}$

$$= 6.62\times10^{-34}\times3\times10^{8} \frac{[5800\times10^{-10} - 4500\times10^{-10}]}{5800\times4500\times10^{-20}}$$

$$= 9.9 \times 10^{-20} \text{ J}$$

$$E_{k_{max}} = \frac{9.9\times10^{-20}}{1.6\times10^{-19}} = 0.62 \text{ eV}$$

$$\Rightarrow v_{max} = \sqrt{\frac{2hc(\lambda_0 - \lambda)}{m\lambda\lambda_0}}$$

PHOTO ELECTRIC CURRENT

Intensity of light (I):

(*a*) It is the quantity of light energy falling normally on a unit surface area in unit time.

$$\text{or } I = \frac{E}{A.t}$$

where I = Intensity of light in $\frac{W}{m^2}$

E = total energy incident = $nh\nu = n\frac{hc}{\lambda}$

n = no. of photons
A = area of surface
t = time of exposure

(*b*) Intensity of light is proportional to saturation current

(*c*) For point source of light $I \propto \frac{1}{r^2}$

(*d*) For line source of light $I \propto \frac{1}{r}$

where r is the distance of the point from the light source.

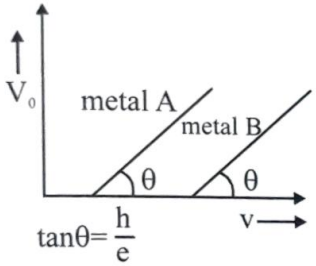
STOPPING POTENTIAL

This is the smallest magnitude of anode potential which just stops the electron with maximum kinetic energy from reaching the anode.

As $K.E_{max} = h\nu - \phi$

so, if stopping potential for a given photoelectric emission is V_0 then $eV_0 = KE_{max} = h\nu - \phi$

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$$

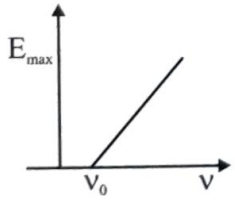


If we plot a curve between V_0 and ν we can get the value of Planck's constant by measuring slope.

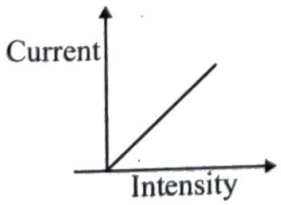
GRAPHS

(*a*) Kinetic energy V/s frequency :

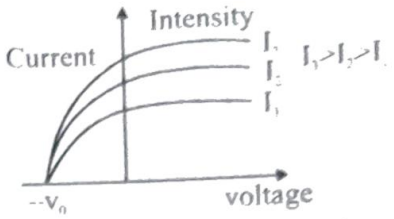
At $\nu = \nu_0$ $E_{max} = 0$



(*b*) Saturated Current V/s Intensity :

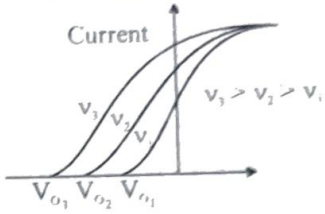


(*c*) Potential V/s current : (ν : constant)



⇒ Stopping potential does not depend upon intensity of light.

(*d*) Photo electric current V/s potential (Intensity constant):



Retarding potential

TRAIN YOUR BRAIN

Example 15: If one photon has 25 eV energy and work function of material is 7 eV then value of stopping potential will be-

(*a*) 32 V (*b*) 18 V
(*c*) 3.3 V (*d*) zero

Sol. $V_0 = \frac{E - \varphi_0}{e} = \frac{25 - 7}{e}$ eV

$V_0 = 18$ V

Example 16: Two different photons of energies, 1 eV and 2.5 eV, fall on two identical metal plates having work function 0.5 eV, Then the ratio of maximum KE of the electrons emitted from the two surface is-

(*a*) 1 : 2 (*b*) 1 : 4
(*c*) 2 : 1 (*d*) 4 : 1

Sol. $K_{1_{max}} = h\nu_1 - \phi = 1 - 0.5 = 0.5$ eV

$K_{2_{max}} = 2.5 - 0.5 = 2.0$ eV

Thus $K_{1_{max}} : K_{2_{max}} = 0.5 : 2 = 1 : 4$

Example 17: A monochromatic source of light operating at 200 W emits 4×10^{20} photons per second. Find the wavelength of light.

(*a*) 400 mm (*b*) 200 n
(*c*) 4×10^{-10} Å (*d*) None

Sol. The energy of each photon $= \frac{200}{4 \times 10^{20}}$

$= 5 \times 10^{-19}$ J

Wavelength $\lambda = \frac{hc}{E}$

$$= \frac{(6.63\times10^{-34})\times(3\times10^{8})}{5\times10^{-19}}$$

$\Rightarrow \lambda = 4.0 \times 10^{-7} = 400$ nm

Example 18: Which metal will be suitable for a photo electric cell using light of wavelength 4000Å. The work functions of sodium and copper are respectively 2.0 eV and 4.0 eV.

(*a*) sodium (*b*) copper

(*c*) Both (*a*) and (*b*) (*d*) None of both

Sol. $\because \lambda_0 = \frac{hc}{\phi}$

$$\therefore (\lambda_0)_{sodium} = \frac{6.6\times10^{-34}\times3\times10^{8}}{2\times1.6\times10^{-19}} = 6188 \text{ Å}$$

$$\because \lambda_0 \propto \frac{1}{\phi} \Rightarrow \frac{(\lambda_0)_{sodium}}{(\lambda_0)_{copper}} = \frac{(\phi)_{copper}}{(\phi)_{sodium}}$$

$$\Rightarrow (\lambda_0)_{copper} = \frac{2}{4} \times 6188 = 3094 \text{ Å}$$

To eject photo-electrons from sodium the longest wavelength is 6188 Å and that for copper is 3094 Å. Hence for light of wavelength 4000 Å, sodium is suitable.

Example 19: Slope of $V_0 - \nu$ curve is-

(*a*) e (*b*) $\frac{h}{e}$

(*c*) ϕ_0 (*d*) h

Sol. $V_0 = \frac{h\nu}{e} - \frac{h\nu_0}{e}$

Put it in the form of $y = mx - c$,

here $V_0 = y$, $\nu = x$, $\frac{h\nu_0}{e} = c$

$$\therefore y = \left(\frac{h}{e}\right)x - c$$

$$\therefore m = \frac{h}{e}$$

Example 20: A radio station is transmitting waves of wavelength 300 m, If diffracting power of transmitter is 10 kW, then numbers of photons diffracted per second is-

(*a*) 1.5×10^{35} (*b*) 1.5×10^{31}

(*c*) 1.5×10^{29} (*d*) 1.5×10^{33}

Sol. $P = 10 \times 10^3$ watt

$n = ?$

$\lambda = 300$ m

$$P = \frac{nhc}{\lambda t}$$

$$10^4 = \frac{6.62\times10^{-34}\times3\times10^{8}\times n}{300\times1}$$

$$n = \frac{300\times10^{4}}{6.62\times10^{-34}\times10^{8}} = 1.5 \times 10^{31}$$

Example 21: Light of wavelength 332 Å incidents on metal surface (work function = 1.07 eV). To stop emission of photo electron, retarding potential required to be-

(*a*) 3.74 V (*b*) 2.67 V

(*c*) 1.07 V (*d*) 4.81 V

Sol. $\phi_0 = 1.07$ eV $= 1.07 \times 1.6 \times 10^{-19}$ J

$\lambda = 332 \times 10^{-10}$ m

$$eV_0 = \frac{hc}{\lambda} - \phi_0 = V_0 = \frac{hc}{e\lambda} - \frac{\varphi_0}{e}$$

$$V_0 = \frac{6.62\times10^{-34}\times3\times10^{8}}{332\times10^{-10}\,e} - \frac{1.07\times1.6\times10^{-19}}{e}$$

$V_0 = 3.74 - 1.07 = 2.67$ volt.

Example 22: Light of wavelength 5000 Å falls on a sensitive surface. If the surface has received 10^{-7} Joule of energy, then what is the number of photons falling on the surface ?

(*a*) 25×10^{11} (*b*) 25×10^{12}

(*c*) 0.25×10^{11} (*d*) 2.5×10^{11}

Sol. Let the energy of one photon = hc/λ,

$\therefore$ Energy of n photons $E = nhc/\lambda$

$$\because 10^{-7} = \frac{n\times6.6\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}}$$

$$n = \frac{5000\times10^{-10}\times10^{-7}}{19.8\times10^{-26}} = 0.25 \times 10^{12}$$

$n = 2.5 \times 10^{11}$

Example 23: The wavelength of a photon is 4000 Å. Calculate its energy.

(*a*) 49.5×10^{-19} J (*b*) 495×10^{-19} J

(*c*) 0.495×10^{-19} J (*d*) 4.95×10^{-19} J

Sol. $E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}}$

$= 4.95 \times 10^{-19}$ J

Example 24: When ultraviolet light of energy 6.2 eV incidents on a aluminimum surface, it emits photo electrons. If work function for aluminium surface is 4.2 eV, then kinetic energy of emitted electrons is-

(*a*) 3.2×10^{-19} J (*b*) 3.2×10^{-17} J

(*c*) 3.2×10^{-16} J (*d*) 3.2×10^{-11} J

Sol. $E_k = E - \phi_0 = 6.2 - 4.2 = 2.0$ eV,

$E_k = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ J

Example 25: Using light of wavelength 6000 Å stopping potential is obtained 2.4 volt for photo electric cell. If light of wavelength 4000 Å is used then stopping potential would be-

(*a*) 2.9 V (*b*) 1.9 V

(*c*) 3.43 V (*d*) 9.4 V

Sol. $V_0 = \frac{hc}{e\lambda} - \frac{\varphi_0}{e}$,

$$2.4 = \frac{hc}{6000\times10^{-10}e} - \frac{\varphi_0}{e} \quad ....(i)$$

$$V_0 = \frac{hc}{4000\times10^{-10}e} - \frac{\varphi_0}{e} \quad ....(ii)$$

Eq. (1) - Eq.(2)

$2.4 - V_0$

$$= \frac{hc}{e}\left[\frac{1}{6000\times10^{-10}} - \frac{1}{4000\times10^{-10}}\right]$$

$$2.4 - V_0 = \frac{6.62\times10^{-34}\times3\times10^{8}}{10^{-7}1.6\times10^{-19}}\left[\frac{4-6}{24}\right]$$

$$V_0 = 2.4 + \frac{6.62\times10^{-34}\times3\times10^{8}}{12\times10^{-7}\times1.6\times10^{-19}}$$

$V_0 = 2.4 + 1.03 = 3.43$ V

Example 26: When light source is placed at 1 m distant from photo electric cell, then value of stopping potential is obtained 4 volt. If it is placed at 4 m distant, then value of stopping potential becomes-

(*a*) 2 volt (*b*) 1 volt

(*c*) 4 volt (*d*) 16 volt

Sol. Stopping potential does not depend upon distance from light source.

Example 27: When monochromatic light of wavelength λ illuminates a metal surface then stopping potential for photo electric current is $3V_0$. If wavelength changes to 2λ then stopping potential becomes V_0. Threshold wavelength for photo electric emission is-

(*a*) 4 λ (*b*) 8 λ

(*c*) 4/3 λ (*d*) 6 λ

Sol. $\frac{hc}{\lambda} - \phi_0 = 3V_0$ ...(i)

$$\frac{hc}{2\lambda} - \phi_0 = V_0 \quad ...(ii)$$

eq. (1) - eq.(2) : $\frac{hc}{\lambda}\left[1-\frac{1}{2}\right] = 2V_0$

$$\frac{hc}{2\lambda} = 2V_0$$

$$\Rightarrow \lambda = \frac{hc}{4V_0} \quad \Rightarrow 4\lambda = \frac{hc}{V_0}$$

DE-BROGLIE WAVELENGTH OF MATTER WAVE

A photon of frequency ν and wavelength λ has energy.

$$E = h\nu = \frac{hc}{\lambda}$$

By Einstein's energy mass relation, $E = mc^2$ the equivalent mass m of the photon is given by,

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c} \quad ...(i)$$

$$\text{or } \lambda = \frac{h}{mc}$$

$$\text{or } \lambda = \frac{h}{p} \quad ...(ii)$$

Here p is the momentum of photon. By analogy De-Broglie suggested that a particle of mass m moving with speed v behaves in some ways like waves of wavelength λ (called De-Broglie wavelength and the wave is called matter wave) given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad ... (iii)$$

where p is the momentum of the particle. Momentum is related to the kinetic energy by the equation,

$$p = \sqrt{2Km}$$

and a charge q when accelerated by a potential difference V gains a kinetic energy $K = qV$. Combining all these relations Eq. (iii), can be written as,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}}$$

(de-Broglie wavelength) ...(iv)

De-Broglie Wavelength for an Electron

If an electron (charge = e) is accelerated by a potential of V volts, it acquires a kinetic energy,

$K = eV$

Substituting the values of h, m and q in Eq. (iv), we get a simple formula for calculating de-Broglie wavelength of an electron.

$$\lambda(\text{in Å}) = \sqrt{\frac{150}{V(\text{in volts})}} \quad ...(v)$$

De-Broglie Wavelength of a Gas Molecule:

Let us consider a gas molecule at absolute temperature T. Kinetic energy of a gas molecule is given by

$$\text{K.E.} = \frac{3}{2}kT \text{ ; k = Boltzman constant}$$

$$\therefore \lambda_{\text{gas molecule}} = \frac{h}{\sqrt{3mkT}}$$

TRAIN YOUR BRAIN

Example 28: An electron is accelerated by a potential difference of 50 volt. Find the de-Broglie wavelength associated with it.

Sol. For an electron, de-Broglie wavelength is given by,

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{50}}$$

$$= \sqrt{3} = 1.73 \text{ Å}$$ **Ans.**

Example 29: If there is an increase in linear dimensions of the objects, the associated de-Broglie wavelength -

(*a*) increases
(*b*) decreases
(*c*) remains unchanged
(*d*) depends on the density of object

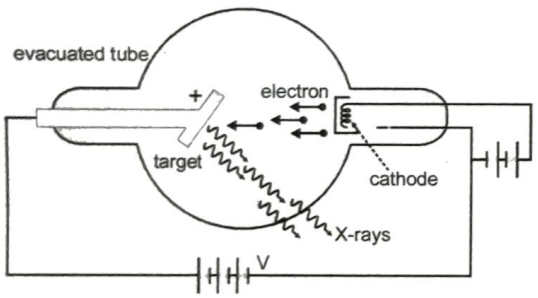
Ans. (*c*)

Sol. As long as momentum does not change de-Broglie wavelength remains unchanged.

X-RAYS

ROENTGEN EXPERIMENT

Roentgen discovered X-ray. While performing experiment on electric discharge tube Roentgen observed that when pressure inside the tube is 10^{-3} mm of Hg and applied potential is kept 25 KV then some unknown radiation are emitted by anode.



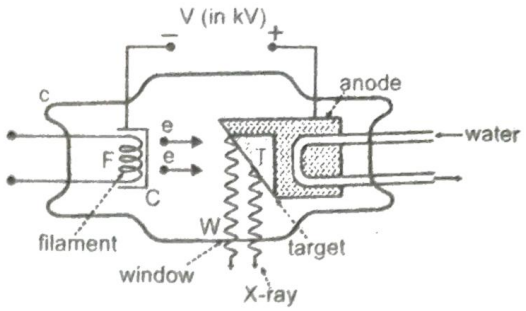
These are known as X-ray.

To Produce X-ray Three Things are Required

(*i*) Source of electron
(*ii*) Means of accelerating these electron to high speed
(*iii*) Target on which these high speed electron strike

COOLIDGE METHOD

Coolidge developed thermoionic vacuum X-ray tube in which electron are produced by thermoionic emission method. Due to high potential difference electrons (emitted due to thermoionic method) move towards the target and strike the atom of target due to which X-ray are produced. Experimentally it is observed that only 1% or 2% kinetic energy of electron beam is used to produce X-ray rest of energy is wasted in form of heat.



Characteristic of Target

(*i*) The atomic number of target should be high.
(*ii*) The thermal conductivity of target should be high.
(*iii*) The melting point of target should be high.

Control of Intensity

The intensity of X-ray depend on number of electrons striking the target and number of electron depend on temperature of filament which can be controlled by filament current.

Thus intensity of X-ray depend on current flowing thorugh filament.

Control of Penetrating Power

The Penetrating power of X-ray depend on the energy of incident electron. The energy of electron can be controlled by applied potential difference. Thus penetrating power of X-ray depend on applied potential difference.

Thus the intensity of X-ray depend on current flowing through filament while penetrating power depend on applied potential difference

Properties of X-Rays

(1) X-rays are electromagnetic waves with wavelength of the order of 0.1 nm ($\approx$ 1 Å)
(2) They are invisible.
(3) They travel in straight line.
(4) They travel with speed of light.
(5) X-rays are not deflected by magnetic and electric field.
(6) X-rays carry no charge.
(7) Wavelength of gama rays < wavelength of X-rays < Wavelength of ultra violet rays
(8) They shows all the important properties of light rays, like ; reflection, refraction , interference, diffraction and polarization etc.

(9) They produce illumination on falling upon fluorescent material.

(10) They have ionisation power. Therefore when they passes through a gas , the gas is ionised.

(11) They have penetration power. They penetrate through different depth into different substance, e.g. wood, cardboard, thin metal sheet , flesh etc. Penetration power depends upon wavelength of x-rays and nature of material.

If $\lambda \uparrow$ Penetration power $\downarrow$ ($\because$ energy $\downarrow$)

If $\lambda \downarrow$ Penetration power $\uparrow$ ($\because$ energy $\uparrow$)

(12) X-rays do not pass through heavy metal (Lead, Calcium, Barium Sulphate) and bones. If such objects is placed in their path , they cast their shadow.

(13) They affect photographic plates.

(14) They show photoelectric effect and compton effect.

(15) Long exposure to X-rays is injurious for human body.

(16) When x-rays falls on living creatures, it produces strain.

Classification of X-Rays

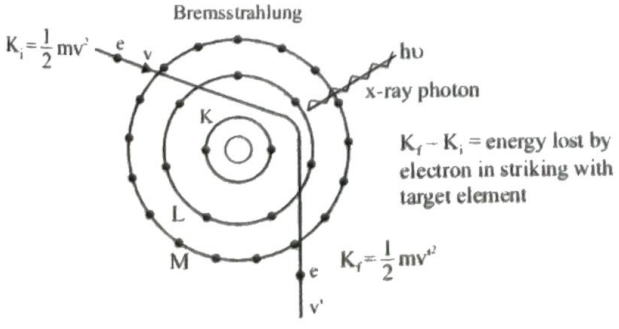
X-ray emitted from target can be divided in two categories.

(*a*) Continuous X-ray

(*b*) Characteristics X-ray

As the electron enters into the target material, it loses its kinetic energy & may come to rest inside the metal. The electron before finally being stopped, makes several collisions with the atoms in the target. At each collision one of the following two types of X-rays may get form.

(*a*) **Continuous X-Ray:** When high energy electrons (accelerated by coolidge tube potential) strike the target element they are deflected by coulomb attraction of nucleus & due to numerous glancing collisions with the atoms of the target, they lose energy which appears in the form of electromagnetic waves (bremsstrahlung or braking radiation) & the remaining part increases the kinetic energy of the colliding particles of the target. The energy received by the colliding particles goes into heating the target. The electron makes another collision with its remaining energy

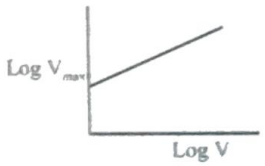


1. When high energetic electrons enter into target material, they are decelerated. In this process emission of energy take place. Spectrum of this energy is continuous. This is also called **Bremsstrahlung**.
2. Continuous x-rays are also called decelerated radiations.
3. Continuous spectrum (υ or λ) depends upon potential difference between filament and target.
4. It does not depend upon nature of target material.
5. If V is the potential difference & υ is the frequency of emitted x-ray photon then

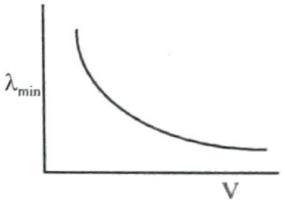
(A) When incident electron loses its complete energy in 1st collision with target atom then photon of maximum frequency (or minimum wavelength) will be emitted & in this case

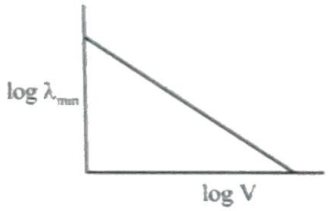
$$eV = \frac{1}{2} mv^2 = h\upsilon_{max} = \frac{hc}{\lambda_{min}} \quad , \quad \upsilon_{max} = \frac{eV}{h}$$





$\because \upsilon = \frac{c}{\lambda}$ $\quad \therefore \lambda_{min} = \frac{c}{v_{max}} = \frac{hc}{eV}$





$$\lambda_{min} \propto \frac{1}{v}$$

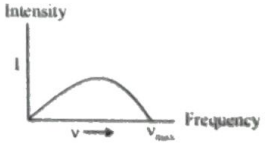
λ_{min} = Also called cutoff wavelength.

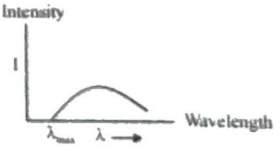
(B) When incident electron does not lose any of its energy in collision with target atom.

$$eV = \frac{1}{2} mv^2 = h\upsilon = \frac{hc}{\lambda} = 0$$

$\lambda = \infty$

and therefore Wavelength of continuous x-ray photon ranges from certain minimum to infinity & therefore x-ray spectrum is not monochromatic.





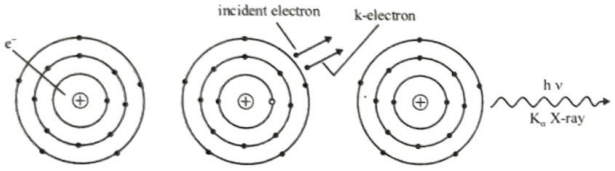
(*b*) **Characteristics x-rays:** We shall now discuss what happens if the electron knocks out an inner electron from the atom with which it collides.

(*i*) The electrons in an atom occupy different quantum states characterized by the quantum numbers n, ℓ, m_ℓ, m_s. The energy primarily depends on the principle quantum number n.

(*ii*) The two electrons corresponding to n = 1 are said to be in K shell, those corresponding to n = 2 are in L shell etc.

(*iii*) Suppose, the incident electron knocks out an electron from the K shell. This will create a vacancy in the K shell in the sense that now there is only one electron with n = 1, whereas two could be accommodated by Pauli exclusion principle. An electron from a higher energy state may make a transition to this vacant state. When such a transition takes place, the difference of energy ΔE is converted into an X-ray photon of wavelength $\lambda = hc / \Delta E$.

(*iv*) X-rays emitted due to electronic transition from a higher energy state to a vacancy careated in the K shell are called K series X-rays.



(*v*) As the electrons in K shell are most tightly bound, maximum energy is to be given to the atom to knock out an electron from the K shell. That is why, in figure, the energy level of the atom with a vacancy in the K shell is shown highest.

The energies E_K, E_L ..... etc. are characteristic properties of the material. For different materials, the values of these energies will be different. The values of $E_K - E_L$, $E_L - E_M$ etc. also have definite values for a given material. The wavelengths of the X-rays emitted corresponding to these transitions are

$$\lambda_{k_\alpha} = \frac{hc}{E_K - E_L}$$

The shells have energies $-E_K$, $-E_L$, $-E_M$ etc

$$\lambda_{k_\beta} = \frac{hc}{E_K - E_M}$$

$$\lambda_{L_\beta} = \frac{hc}{E_L - E_M}$$

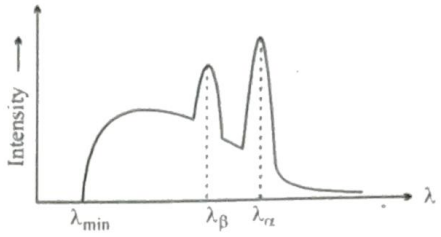
Hence, these energies E_K, E_L, E_M, etc are the magnitudes of energies.

These wavelength, therefore, have definite values for a particular material. The X-rays emitted in this way are the characteritic X-rays shown in figure. They are so named because their wavelengths may be used to identify the element from which they originate.

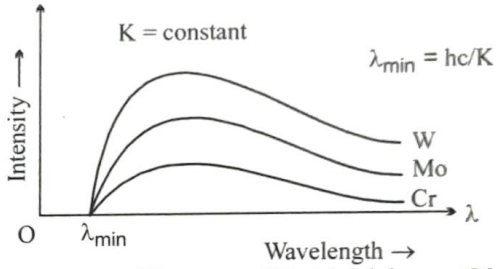
(*vi*) Characterstic x-rays spectrum is a line spectrum.

(*vii*) In this spectrum frequency or wavelength of lines of series does not change with accelerating voltage.

Graph of Intensity of X-rays versus wavelengths:



The X-ray emission when stream of fast moving electrons strike a target of heavy element. λ_α, λ_β corresponds to characteristic X-rays.



Different targets of Tungsten (W), Molybdenum (Mo) and Chromium (Cr) are used and the kinetic energies of the incident electrons are kept constant. It is observed that the minimum wavelength λ_{min} is independent of target material.

TRAIN YOUR BRAIN

Example 30: The potential difference between the cathode and the target in a coolidge tube is 120 KV. What can be the minimum wavelength (in Å) of the x-rays emitted by this tube? What will be the momentum of this photon?

(*a*) 0.01 Å, 6.6×10^{-23} kg m/s

(*b*) 0.1 Å, 6×10^{-23} kg m/s

(*c*) 0.1 Å, 6.6×10^{-23} kg m/s

(*d*) 0.01 Å, 66×10^{-23} kg m/s

Sol. When electrons accelerated through a potential difference V strike a target, the maximum frequency of the emitted x-rays is given by-

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}} \Rightarrow \lambda_{min} = \frac{hc}{eV}$$

$$\Rightarrow \lambda_{min} = \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times120\times10^{8}}$$

$$\Rightarrow \lambda_{min} = 0.1 \times 10^{-10}\ m = 0.1\ Å$$

The momentum of the emitted x-ray photon is

$$p = \frac{h}{\lambda_{min}} = \frac{6.6\times10^{-31}}{0.1\times10^{-10}}$$

$p = 6.6 \times 10^{-23}$ kg m/s

Example 31: The maximum frequency of the x-ray emitted by an x-ray tube operating at 30 KV is-

(*a*) 2.47×10^{15} Hz (*b*) 4.94×10^{16} Hz

(*c*) 7.24×10^{18} Hz (*d*) 0.41×10^{10} Hz

Sol. $\nu = \frac{eV}{h} = \frac{1.6\times10^{-19}\times30\times10^{3}}{6.63\times10^{-34}}$

$\Rightarrow \nu = \frac{48\times10^{-16+34}}{6.63} = 7.24 \times 10^{18}$ Hz

Example 32: If the x-ray tube is working at 25 KV then the minimum wavelength of x-rays will be

(*a*) 0.49 Å (*b*) 0.29 Å

(*c*) 0.19 Å (*d*) 0.39 Å

Sol. $\lambda_{min} = \frac{12400}{V}$ Å $\Rightarrow \lambda_{min} = \frac{12400}{25\times10^{3}} = 0.49$ Å

Example 33: The ratio of energies of x-rays of the wavelength 0.01Å and 0.5 Å will be-

(*a*) 1 : 1 (*b*) 1 : 2

(*c*) 1 : 5 (*d*) 50 : 1

Sol. $\frac{E_1}{E_2} = \frac{hc}{\lambda_1}\cdot\frac{\lambda_2}{hc} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{E_1}{E_2} = \frac{50}{1}$

Example 34: When the minimum wavelength of x-rays is 2Å then the applied potential difference between cathode and anode will be-

(*a*) 6.2 KV (*b*) 2.48 KV

(*c*) 24.8 KV (*d*) 62 KV

Sol. $V = \frac{12400}{\lambda_{min}(Å)}$ Volt $\Rightarrow V = \frac{12400}{2} = 6.2$ KV

Example 35: An x-ray tube is operated at 15 KV. Calculate the upper limit of the speed of the electrons striking the target.

(*a*) 7.26×10^{7} m/s (*b*) 7.62×10^{7} m/s

(*c*) 7.62×10^{7} cm/s (*d*) 7.26×10^{9} m/s

Sol. The maximum kinetic energy of an electron accelerated through a potential difference of V volt is $\frac{1}{2}mv^2 = eV$

$\therefore$ maximum velocity $v = \sqrt{\frac{2eV}{m}}$

$v = \sqrt{\frac{2\times1.6\times10^{-19}\times15000}{9.1\times10^{-31}}}$

$v = 7.26 \times 10^{7}$ m/s

Example 36: An x-rays tube is being operated at 20 KV, the maximum speed of electrons striking the anticathode will be-

(*a*) 8.4 m/s (*b*) 8.4×10^{7} m/s

(*c*) 4.4×10^{7} m/s (*d*) zero

Sol. $v_{max} = \sqrt{\frac{2eV}{m}}$

$= \sqrt{\frac{2\times1.6\times10^{-19}\times20\times10^{3}}{9.1\times10^{-31}}}$

$v_{max} = 8.4 \times 10^{7}$ m/s

Example 37: The maximum kinetic energy of the electrons hitting a target so as to produce x-ray of wavelength 1 Å is

(*a*) 1.24 KeV (*b*) 12.4 KeV

(*c*) 124 KeV (*d*) None of these

Sol. $\lambda_{min} = 1$ Å (given)

$\because \lambda_{min} = \frac{1240}{E}$ (eV) (nm)

Thus, $E = \frac{1240(eV)(nm)}{.01(nm)} = 12400$ eV

E = 12.4 KeV

Example 38: When the x-ray tube is operated at 1KV, then x-rays of minimum wavelength 6.22 Å are produced. If the tube is operated at 10 KV, then the minimum wavelength of x-rays will be-

(*a*) 0.622 Å (*b*) 6.22 Å

(*c*) 3.11 Å (*d*) zero

Sol. $\frac{\lambda_{m_2}}{\lambda_{m_1}} = \frac{V_1}{V_2}$

$\Rightarrow \lambda_{m_2} = \frac{6.22\times10^{3}}{10^{4}} = 0.622$ Å

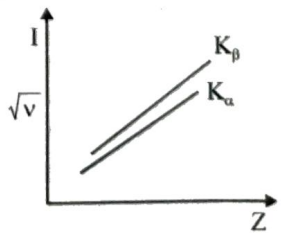
MOSELEY'S LAW

(*i*) In 1913, Moseley did experiments on known elements & calculated the K_α frequencies of the various elements.

(*ii*) This law exhibits relation between frequency of characteristic X-rays & atomic number Z of target element.

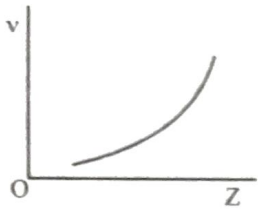
(*iii*) Moseley noted that the characteritic lines shifted systematically as the target material is changed.

(*iv*) According to this law square root of frequency of characteristic X-rays is linearly dependent on atomic number Z.



$\sqrt{\nu} = a(Z - b)$ where a, b are constants which are independent of material used.

The graph between ν & Z or energy of X-ray photon $E = h\nu$ versus Z for characteristic X-ray lines is parabolic as shown in fig.



(*v*) When an electron in n = 1 level is ejected, then for the remaining electron the electric field due to nucleus is screened by the remaining electron in the n = 1 level.

(*vi*) Moseley estimated that the effective nuclear charge for the K_α transition is (Z – 1) e.

(*vii*) According to Bohr's model, the energy released during the transition from
n = 2 to n = 1 is given by

$$\Delta E = h\upsilon_{k\alpha} = Rch\, Z^2_{eff}\left(1-\frac{1}{2^2}\right)$$

$$\Rightarrow \nu_{K_\alpha} = \frac{3Rc}{4}(Z-1)^2$$

$$\sqrt{\nu_{K_\alpha}} = \sqrt{\frac{3Rc}{4}}(Z-1)$$

$$\sqrt{\nu} = a(Z-b)$$

where a & b depend on transition & do not depend on target material.

$$a = \frac{3Rc}{4} = 2.47 \times 10^{15}\ Hz$$

b = screening const.

for K_α X-ray line b = 1

for L_α X-ray line b = 7.4

Uses of X-Rays

(*i*) **Medical uses:** The X-rays quite freely pass through the flesh but are stopped by the bones. So we can photograph the bones inside the body on a photographic film. This is used to detect and study bone fractures due to an accident.

Chest radiograph are used to study diseases in lungs.

Dentist also use X-rays to study teeth-decay.

X-ray are also used in cancer therapy. These damage the tumour cells.

(*ii*) **Art and Science:** X-rays are used to detect covered paintings. It is also used for determining crystal structures. Structure of DNA was also determined using X-ray diffraction

(*iii*) **Industry:** X-rays have been used to detect minute cracks and faults in concrete and metal structure which otherwise are not visible.

X-rays have been used as a quality control in rubber industry (e.g.) if there are bubble in rubber tyres, X-rays will indicate it.

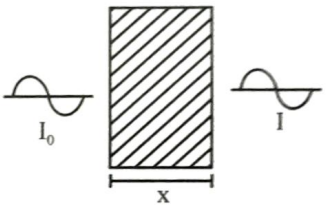
X-ray machines are used to inspect suitcases, wooden boxes etc. without opening them and can be typically found at the custom, security counters at airports etc.

Absorption of X-rays:

(1) When X-rays passes through medium, intensity of X-rays decreases.

(2) If I_0 is the initial intensity of X-rays then intensity I after traveling x distance in medium will be

$I = I_0 e^{-\mu x}$



where

I_0 = Intensity of incident X-Ray photon

I = Intensity of transmitted X-Ray photon

x = Thickness of given sheet

μ = Absorption or attenuation coefficient for given sheet material

Graph of I v/s x



(3) Absorption of X-rays is high in heavy material but low in light material.

(4) Absorber of X-rays are Pb, $BaSO_4$.

(5) Pb is best absorber for X -rays

(6) High λ of X -rays

$\Rightarrow$ more absorption ($\mu\uparrow$)

Low λ of X -rays

$\Rightarrow$ less absorption because low μ

Absorption coefficient (μ)

$I = I_0 e^{-\mu x}$ If $x = \frac{1}{\mu}$

then $I = I_0 e^{-1} = \frac{I_0}{e} = \frac{I_0}{2.718} = 0.3676\, I_0$

So when $\mu = \frac{1}{x} = x^{-1}$ then $I = \frac{I_0}{e} \approx 0.37\, I_0$

Absorption coefficient for a given material is equal to inverse of that thickness that reduces the intensity of transmitted x-ray to 37% of its original value.

Soft and Hard X-Ray:

Properties	Soft X-ray	Hard X-ray
Wavelength	10Å to 100Å	0.1Å – 10Å
Energy	$\frac{12400}{\lambda}$ eV – Å	$\frac{12400}{\lambda}$ eV – Å
Penetrating Power	Less	More
Use	Radio photography	Radio therapy

Diffraction of X-ray:

Diffraction of X-ray is possible by crystals because the interatomic spacing in a crystal lattice is order of wavelength of X-rays. It was first varified by Lauve.

Diffraction of X-ray take place according to Bragg's law

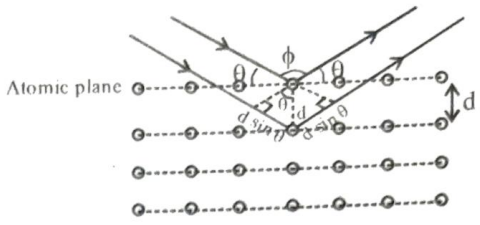
$2d \sin\theta = n\lambda$

d = spacing of crystal plane or lattice constant or distance between adjacent atomic plane

θ = Bragg's angle or glancing angle

ϕ = Diffracting angle

n = 1, 2, 3, ...



For Maximum Wavelength

$\sin\theta = 1$, $n = 1$

$\therefore\ \lambda_{max} = 2d$

so if $\lambda > 2d$, diffraction is not possible i.e. solution of Bragg's equation is not possible.

TRAIN YOUR BRAIN

Example 39: When x-rays of wavelength 0.5 Å are transmitted by an aluminium tube of thickness 7 mm, its intensity becomes one-fourth. The attenuation coefficient of aluminium for these X-rays-

(*a*) 0.188 mm^{-1} (*b*) 0.189 mm^{-1}

(*c*) 0.198 mm^{-1} (*d*) None of these

Sol. $\frac{I_0}{I} = 4$, x = 7mm (given)

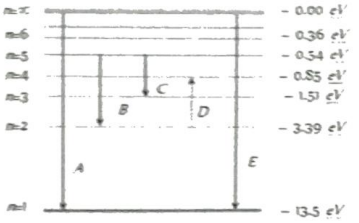
$$\Rightarrow \mu = \frac{2.303 \ \log_{10}\frac{I_0}{I}}{x}$$

$$\mu = \frac{2.303 \ \log_{10} 4}{7} = \frac{2.303 \times 0.6023}{7}$$

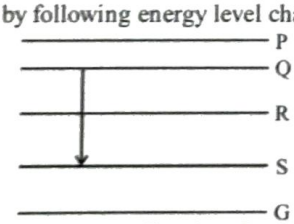
$\mu = 0.198\ mm^{-1}$

BOHR'S MODEL

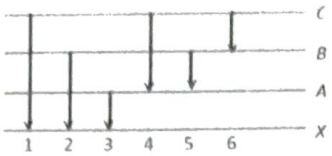
1. In the Bohr's hydrogen atom model, the radius of the stationary orbit is directly proportional to (n = principle quantum number)
   (a) n^{-1} (b) n
   (c) n^{-2} (d) n^{2}
2. The Lyman series of hydrogen spectrum lies in the region
   (a) Infrared (b) Visible
   (c) Ultraviolet (d) Of X – rays
3. The energy levels of the hydrogen spectrum is shown in figure. There are some transitions A, B, C, D and E. Transition A, B and C respectively represent



   (a) First member of Lyman series, third spectral line of Balmer series and the second spectral line of Paschen series
   (b) Ionization potential of hydrogen, second spectral line of Balmer series and third spectral line of Paschen series
   (c) Series limit of Lyman series, third spectral line of Balmer series and second spectral line of Paschen series
   (d) Series limit of Lyman series, second spectral line of Balmer series and third spectral line of Paschen series
4. Figure shows the energy levels P, Q, R, S and G of an hypothetical atom where G is the ground state. A red line in the emission spectrum of the atom can be obtained by an energy level change from Q to S. A blue line can be obtained by following energy level change



   (a) P to Q (b) Q to R
   (c) R to S (d) R to G
5. The figure indicates the energy level diagram of an atom and the origin of six spectral lines in emission (e.g. line no. 5 arises from the transition from level B to A). The following spectral lines will also occur in the absorption spectrum



   (a) 1, 4, 6 (b) 4, 5, 6
   (c) 1, 2, 3 (d) 1, 2, 3, 4, 5, 6
6. The absorption transitions between the first and the fourth energy states of hydrogen atom are 3. The emission transitions between these states will be
   (a) 3 (b) 4
   (c) 5 (d) 6
7. The third line of Balmer series of an ion equivalent to hydrogen atom has wavelength of 108.5 *nm*. The ground state energy of an electron of this ion will be
   (a) 3.4 eV (b) 13.6 eV
   (c) 54.4 eV (d) 122.4 eV

DE-BROGLIE WAVELENGTH

8. Wave is associated with matter
   (a) When it is stationary
   (b) When it is in motion with the velocity of light only
   (c) When it is in motion with any velocity
   (d) None of the above
9. A photon, an electron and a uranium nucleus all have the same wavelength. The one with the most energy
   (a) Is the photon
   (b) Is the electron
   (c) Is the uranium nucleus
   (d) Depends upon the wavelength and the properties of the particle.
10. When the kinetic energy of an electron is increased, the wavelength of the associated wave will
    (a) Increase
    (b) Decrease
    (c) Wavelength does not depend on the kinetic energy
    (d) None of the above
11. If the de-Broglie wavelengths for a proton and for a α-particle are equal, then the ratio of their velocities will be
    (a) 4 : 1 (b) 2 : 1
    (c) 1 : 2 (d) 1 : 4
12. The de-Broglie wavelength of an electron having 80 eV of energy is nearly (1 eV = 1.6×10^{-19} J, Mass of electron = 9×10^{-31} kg, Plank's constant = 6.6×10^{-34} J-sec)
    (a) 140 Å (b) 0.14 Å
    (c) 14 Å (d) 1.4 Å

13. The kinetic energy of electron and proton is 10^{-32} J. Then the relation between their de-Broglie wavelengths is
(a) $\lambda_p < \lambda_e$ (b) $\lambda_p > \lambda_e$
(c) $\lambda_p = \lambda_e$ (d) $\lambda_p = 2\lambda_e$

14. The de-Broglie wavelength of a particle accelerated with 150 volt potential is 10^{-10} m. If it is accelerated by 600 volts p.d., its wavelength will be
(a) 0.25 Å (b) 0.5 Å
(c) 1.5 Å (d) 2 Å

15. The de-Broglie wavelength of a neutron at 27°C is λ. What will be its wavelength at 927°C
(a) λ / 2 (b) λ / 3
(c) λ / 4 (d) λ / 9

Photoelectric Effect

16. The momentum of a photon is 3.3×10^{-29} kg-m/sec. Its frequency will be
(a) 3×10^3 Hz (b) 6×10^3 Hz
(c) 7.5×10^{12} Hz (d) 1.5×10^{13} Hz

17. If the momentum of a photon is p, then its frequency is- Where m is the rest mass of the photon
(a) $\frac{ph}{c}$ (b) $\frac{pc}{h}$
(c) $\frac{mh}{c}$ (d) $\frac{mc}{h}$

18. There are n_1 photons of frequency γ_1 in a beam of light. In an equally energetic beam, there are n_2 photons of frequency γ_2. Then the correct relation is
(a) $\frac{n_1}{n_2} = 1$ (b) $\frac{n_1}{n_2} = \frac{\gamma_1}{\gamma_2}$
(c) $\frac{n_1}{n_2} = \frac{\gamma_2}{\gamma_1}$ (d) $\frac{n_1}{n_2} = \frac{\gamma_1^2}{\gamma_2^2}$

19. Einstein's photoelectric equation states that $E_k = h\nu - \phi$. In this equation E_k refers to
(a) Kinetic energy of all the emitted electrons
(b) Mean kinetic energy of the emitted electrons
(c) Maximum kinetic energy of the emitted electrons
(d) Minimum kinetic energy of the emitted electrons

20. Threshold frequency for a metal is 10^{15}Hz. Light of λ = 4000Å falls on its surface. Which of the following statements is correct
(a) No photoelectric emission takes place
(b) Photo-electrons come out with zero speed
(c) Photo-electrons come out with 10^3 m/sec speed
(d) Photo-electrons come out with 10^5 m/sec speed

21. When light falls on a metal surface, the maximum kinetic energy of the emitted photo-electrons depends upon
(a) The time for which light falls on the metal
(b) Frequency of the incident light
(c) Intensity of the incident light
(d) Velocity of the incident light

22. The work function of a metal is 1.6×10^{-19} J. When the metal surface is illuminated by the light of wavelength 6400 Å, then the maximum kinetic energy of emitted photo-electrons will be (Planck's constant $h = 6.4 \times 10^{-34}$ Js)
(a) 14×10^{-19} J (b) 2.8×10^{-19} J
(c) 1.4×10^{-19} J (d) 1.4×10^{-19} eV

Experimental Study of Photoelectric Effect

23. A metal surface of work function 1.07 eV is irradiated with light of wavelength 332 nm. The retarding potential required to stop the escape of photo-electrons is
(a) 4.81 eV (b) 3.74 eV
(c) 2.66 eV (d) 1.07 eV

24. A photo cell is receiving light from a source placed at a distance of 1 m. If the same source is to be placed at a distance of 2 m, then the ejected electron
(a) Moves with one-fourth energy as that of the initial energy
(b) Moves with one-fourth of momentum as that of the initial momentum
(c) Will be half in number
(d) Will be one-fourth in number

25. In a photoelectric experiment for 4000 Å incident radiation, the potential difference to stop the ejection is 2 V. If the incident light is changed to 3000 Å, then the potential required to stop the ejection of electrons will be
(a) 2 V (b) Less than 2 V
(c) Zero (d) Greater than 2 V

26. Stopping potential for photoelectrons
(a) Does not depend on the frequency of the incident light
(b) Does not depend upon the nature of the cathode material
(c) Depends on both the frequency of the incident light and nature of the cathode material
(d) Depends upon the intensity of the incident light

27. If in a photoelectric experiment, the wavelength of incident radiation is reduced from 6000 Å to 4000 Å then
(a) Stopping potential will decrease
(b) Stopping potential will increase
(c) Kinetic energy of emitted electrons will decrease
(d) The value of work function will decrease

28. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential in volts is
(a) 2 (b) 4
(c) 6 (d) 10

Learning Plus

1. If a_0 is the Bohr radius, the radius of the n = 2 electronic orbit in triply ionized beryllium is -
   (*a*) $4a_0$ (*b*) a_0
   (*c*) $a_0/4$ (*d*) $a_0/16$
2. The angular momentum of an electron in the hydrogen atom is $\frac{3h}{2\pi}$. Here h is Planck's constant. The kinetic energy of this electron is:
   (*a*) 4.53 eV (*b*) 1.51 eV
   (*c*) 3.4 eV (*d*) 6.8 eV
3. If an orbital electron of the hydrogen atom jumps from the ground state to a higher energy state, its orbital speed reduces to half its initial value. If the radius of the electron orbit in the ground state is *r*, then the radius of the new orbit would be:
   (*a*) 2 *r* (*b*) 4 *r*
   (*c*) 8 *r* (*d*) 16 *r*
4. Three photons coming from emission spectra of hydrogen sample are picked up. Their energies are 12.1eV, 10.2eV and 1.9eV. These photons must come from
   (*a*) a single atom
   (*b*) two atoms
   (*c*) three atom
   (*d*) either two atoms or three atoms
5. In a hypothetical atom, if transition from n = 4 to n = 3 produces visible light then the possible transition to obtain infrared radiation is :
   (*a*) n = 5 to n = 3 (*b*) n = 4 to n = 2
   (*c*) n = 3 to n = 1 (*d*) none of these
6. If the electron in a hydrogen atom were in the energy level with n = 3, how much energy in joule would be required to ionise the atom? (Ionisation energy of H-atom is 2.18×10^{-18} J):
   (*a*) 6.54×10^{-19} (*b*) 1.43×10^{-19}
   (*c*) 2.42×10^{-19} (*d*) 3.14×10^{-20}
7. The wavelength of the first line in balmer series in the hydrogen spectrum is λ. What is the wavelength of the second line :
   (*a*) $\frac{20\lambda}{27}$ (*b*) $\frac{3\lambda}{16}$
   (*c*) $\frac{5\lambda}{36}$ (*d*) $\frac{3\lambda}{4}$
8. In a sample of hydrogen like atoms all of which are in ground state, a photon beam containing photons of various energies is passed. In absorption spectrum, five dark lines are observed. The number of bright lines in the emission spectrum will be (Assume that all transitions take place)
   (*a*) 5 (*b*) 10
   (*c*) 15 (*d*) None of these
9. Consider the spectral line resulting from the transition n = 2 → n = 1 in the atoms and ions given below. The shortest wavelength is produced by :
   (*a*) hydrogen atom
   (*b*) deuterium atom
   (*c*) singly ionized helium
   (*d*) doubly ionized lithium
10. Radius of the second Bohr orbit of singly ionised helium atom is
    (*a*) 0.53 Å (*b*) 1.06 Å
    (*c*) 0.265 Å (*d*) 0.132 Å
11. If radiation of allow wavelengths from ultraviolet to infrared is passed through hydrogen gas at room temperature, absorption lines will be observed in the:
    (*a*) Lyman series (*b*) Balmer series
    (*c*) Both (*a*) and (*b*) (*d*) Neither (*a*) nor (*b*)
12. The de Broglie wavelength of an electron moving with a velocity 1.5×10^8 ms^{-1} is equal to that of a photon. The ratio of the kinetic energy of the electron to that of the energy of photon is
    (*a*) 2 (*b*) 4
    (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$
13. The de Broglie wavelength of a neutron correspoding to root mean square speed at 927°C is λ. What will be the de Broglie wavelength of the neutron correspoding to root mean square speed at 27°C?
    (*a*) $\frac{\lambda}{2}$ (*b*) λ
    (*c*) 2 λ (*d*) 4 λ
14. The wavelength λ of de Broglie waves associated with an electron (mass *m*, charge *e*) accelerated through a potential difference of *V* is given by (*h* is Planck's constant) :
    (*a*) $\lambda = h/mV$ (*b*) $\lambda = h/2\ meV$
    (*c*) $\lambda = h/\sqrt{meV}$ (*d*) $\lambda = h/\sqrt{2meV}$
15. An electron with initial kinetic energy of 100eV is acceleration through a potential difference of 50V. Now the de-Broglie wavelength of electron becomes.
    (*a*) 1 Å (*b*) $\sqrt{1.5}$Å
    (*c*) $\sqrt{3}$ Å (*d*) 12.27 Å
16. Let n_r and n_b be respectively the number of photons emitted by a red bulb and a blue bulb of equal power in a given time.
    (*a*) $n_r = n_b$ (*b*) $n_r < n_b$
    (*c*) $n_r > n_b$ (*d*) data insufficient

17. The stopping potential for the photo electrons emitted from a metal surface of work function 1.7eV is 10.4 V. Identify the energy levels corresponding to the transitions in hydrogen atom which will result in emission of wavelength equal to that of incident radiation for the above photoelectric effect

(*a*) n = 3 to 1 (*b*) n = 3 to 2

(*c*) n = 2 to 1 (*d*) n = 4 to 1

18. A photon of light enters a block of glass after travelling through vacuum. The energy of the photon on entering the glass block

(*a*) Increases because its associated wavelength decreases

(*b*) Decreases because the speed of the radiation decreases

(*c*) Stays the same because the speed of the radiation and the associated wavelength do not change

(*d*) Stays the same because the frequency of the radiation does not change

19. Two separate monochromatic light beams A and B of the same intensity (energy per unit area per unit time) are falling normally on a unit area of a metallic surface. Their wavelength are λ_A and λ_B respectively. Assuming that all the incident light is used in ejecting the photoelectrons, the ratio of the number of photoelectrons from beam A to that from B is

(*a*) $\left(\frac{\lambda_A}{\lambda_B}\right)$ (*b*) $\left(\frac{\lambda_B}{\lambda_A}\right)$

(*c*) $\left(\frac{\lambda_A}{\lambda_B}\right)^2$ (*d*) $\left(\frac{\lambda_B}{\lambda_A}\right)^2$

20. The energy of a photon of frequency n is $E = hn$ and the momentum of a photon of wavelength l is $p = h/l$. From this statement one may conclude that the wave velocity of light is equal to :

(*a*) $3 \times 10^8\ ms^{-1}$ (*b*) $\frac{E}{p}$

(*c*) $E\,p$ (*d*) $\left(\frac{E}{p}\right)^2$

21. Let p and E denote the linear momentum and the energy of a photon. For another photon of smaller wavelength (in same medium)

(*a*) both p and E increase

(*b*) p increases and E decreases

(*c*) p decreases and E increases

(*d*) both p and E decreases

22. By increasing the intensity of incident light keeping frequency ($v > v_0$) fixed on the surface of metal

(*a*) kinetic energy of the photoelectrons increases

(*b*) number of emitted electrons increases

(*c*) kinetic energy and number of electrons increases

(*d*) no effect

23. In a photoelectric experiment, the frequency and intensity of a light source are both doubled. Then consider the following statements.

(i)The saturation photocurrent remains almost the same.

(ii) The maximum kinetic energy of the photoelectrons is doubled.

(*a*) Both (i) and (ii) are true

(*b*) (i) is true but (ii) is false

(*c*) (i) is false but (ii) is true

(*d*) both (i) and (ii) are false

24. The work function for aluminium surface is 4.2 eV and that for sodium surface is 2.0 ev. The two metals were illuminated with appropriate radiations so as to cause photo emission. Then :

(*a*) Both aluminium and sodium will have the same threshold frequency

(*b*) The threshold frequency of aluminium will be more than that of sodium

(*c*) The threshold frequency of aluminium will be less than that of sodium

(*d*) The threshold wavelength of aluminium will be more than that of sodium

25. If the frequency of light in a photoelectric experiment is doubled, the stopping potential will

(*a*) be doubled (*b*) halved

(*c*) become more than doubled

(*d*) become less than double

26. 10^{-3} W of 5000 Å light is directed on a photoelectric cell. If the saturationcurrent in the cell is 0.16 mA, the percentage of incident photons which produce photoelectrons, is

(*a*) 0.4% (*b*) 0.04%

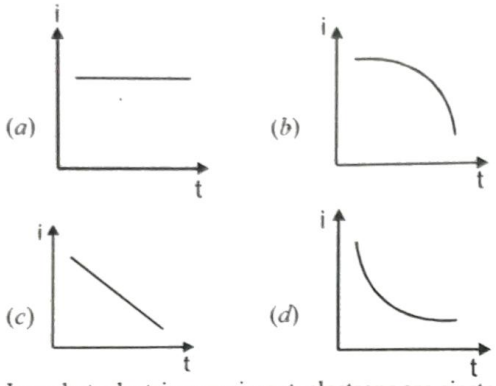
(*c*) 20% (*d*) 10%

27. When a centimetre thick surface is illuminated with light of wavelength λ, the stopping potential is V. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is $V/3$. The threshold wavelength for the surface is :

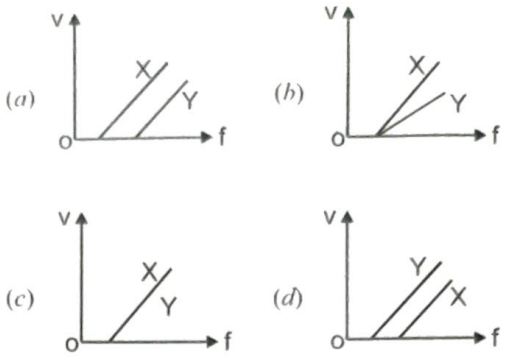
(*a*) $\frac{4\lambda}{3}$ (*b*) 4λ

(*c*) 6λ (*d*) $\frac{8\lambda}{3}$

28. A point source causes photoelectric effect from a small metal plate. Which of the following curves may represent the saturation photocurrent as a function of the distance between the source and the metal?



29. In a photoelectric experiment, electrons are ejected from metals X and Y by light of intensity I and frequency f. The potential difference V required to stop the electrons is measured for various frequencies. If Y has a greater work function than X ; which one of the following graphs best illustrates the expected results?



30. When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When X-rays are incident on the same cell, then :
(a) V_0 and K_{max} both increase
(b) V_0 and K_{max} both decrease
(c) V_0 increases but K_{max} remains the same
(d) K_{max} increases but V_0 remains the same

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/ NUMERICAL

1. In photoelectric effect, stopping potential depends on
(a) frequency of the incident light
(b) intensity of the incident light by varies source distance
(c) emitter's properties
(d) frequency and intensity of the incident light

2. Select the correct alternative(s):
When photons of energy 4.25 eV strike the surface of a metal A, the ejected photo electrons have maximum kinetic energy T_A eV and de Broglie wave length λ_A. The maximum kinetic energy of photo electrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de-Brogleie wave length of these photo electrons is $\lambda_B = 2\lambda_A$, then :
(a) the work function of A is 2.25 eV
(b) the work function of B is 4.20 eV
(c) $T_A = 2.00$ eV
(d) $T_B = 2.75$ eV

3. An electron in hydrogen atom first jumps from second excited state to first excited state and then, from first excited state to ground state. Let the ratio of wavelength, momentum and energy of photons in the two cases by x, y and z, then select the wrong answers:
(a) $z = 1/x$ (b) $x = 9/4$
(c) $y = 5/27$ (d) $z = 5/27$

4. A beam of ultraviolet light of all wavelengths passes through hydrogen gas at room temperature, in the x-direction. Assume that all photons emitted due to electron transition inside the gas emerge in the y-direction. Let A and B denote the lights emerging from the gas in the x and y directions respectively.
(a) Some of the incident wavelengths will be absent in A
(b) Only those wavelengths will be present in B which are absent in A
(c) B will contain some visible light.
(d) B will contain some infrared light.

5. Let A_n be the area enclosed by the nth orbit in a hydrogen atom. The graph of $\ln (A_n / A_1)$ against $\ln(n)$
(a) will pass through the origin
(b) will have certain points lying on a straight line with slope 4
(c) will be a monotonically increasing nonlinear curve
(d) will be a circle

6. A neutron collides head-on with a stationary hydrogen atom in ground state. Which of the following statements are correct (Assume that the hydrogen atom and neutron has same mass)
(a) If kinetic energy of the neutron is less than 20.4 eV collision must be elastic
(b) If kinetic energy of the neutron is less than 20.4 eV collision may be inelastic
(c) Inelastic collision may be take place only when initial kinetic energy of neutron is greater than 20.4 eV.
(d) Perfectly inelastic collision can not take place.

7. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,
   (*a*) the intensity increases
   (*b*) the minimum wavelength increases
   (*c*) the intensity remains unchanged
   (*d*) the minimum wavelength decreases

8. In an x–ray tube the voltage applied is 20KV. The energy required to remove an electron from L shell is 19.9 KeV. In the x–rays emitted by the tube (hc = 12420 evÅ)
   (*a*) minimum wavelength will be 62.1 pm
   (*b*) energy of the characterstic x–rays will be equal to or less than 19.9 KeV
   (*c*) L_α x–ray may be emitted
   (*d*) L_α x–ray will have energy 19.9 KeV

9. Two electrons are moving with the same speed v. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after sometime if the de-Broglie wavelengths of the two are λ_1 and λ_2 then :
   (*a*) $\lambda_1 = \lambda_2$
   (*b*) $\lambda_1 > \lambda_2$
   (*c*) $\lambda_1 < \lambda_2$
   (*d*) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

Comprehension–1 (Q. No. Q.10 to Q.12)

A physicist wishes to eject electrons by shining light on a metal surface. The light source emits light of wavelength of 450 nm. The table lists the only available metals and their work functions.

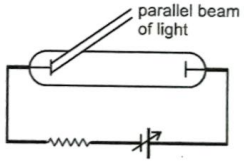
Metal	W_0(eV)
Barium	2.5
Lithium	2.3
Tantalum	4.2
Tungsten	4.5

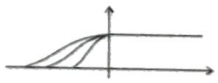
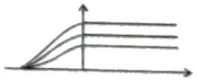
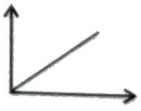
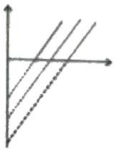
10. Which metal(s) can be used to produce electrons by the photoelectric effect from given source of light?
    (*a*) Barium only
    (*b*) Barium or lithium
    (*c*) Lithium,tantalum or tungsten
    (*d*) Tungsten or tantalum

11. Which option correctly identifies the metal that will produce the most energetic electrons and their energies?
    (*a*) Lithium, 0.45 eV (*b*) Tungston, 1.75 eV
    (*c*) Lithium, 2.30 eV (*d*) Tungston, 2.75 eV

12. Suppose photoelectric experiment is done separately with these metals with light of wavelength 450 nm. The maximum magnitude of stopping potential amongst all the metals is
    (*a*) 2.75 volt (*b*) 4.5 volt
    (*c*) 0.45 volt (*d*) 0.25 volt

13. In the shown experimental setup to study photoelectric effect, two conducting electrodes are enclosed in an evacuated glass-tube as shown. A parallel beam of monochromatic radiation, falls on photosensitive electrode. Assume that for each photons incident, a photoelectron is ejected if its energy is greater than work function of electrode. Match the statements in Column-I with corresponding graphs in Column-II.



Column-I	Column-II
(*a*) Saturation photocurrent (for same metal) versus intensity of radiation is represented by	(p) 
(*b*) Maximum kinetic energy of ejected photoelectrons versus frequency for electrodes of different work function is represented by	(q) 
(*c*) Photo current versus applied voltage for different intensity of radiation (for same metal) is represented by	(r) 
(*d*) Photo current versus applied voltage at constant intensity of radiation for electrodes of different work function.	(s) 

NUMERICAL VALUE BASED

14. The electron in the hydrogen atom is in the $n = 2$ state. What is its kinetic energy in eV.
15. What is the shortest wavelength in the Lyman series in Å?
16. Calculate the average binding energy per nucleon in MeV of the $^{40}_{20}Ca$ nuclei:

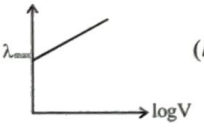
    Mass of $^{40}_{20}Ca$ nucleus = 39.9626 u

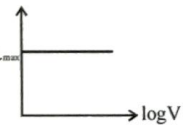
    Mass of neutron = 1.00886 u mass of proton = 1.007854
17. The radioactive isotope $^{60}_{27}Co$ is used in the treatment of tumors. It undergoes β^- decay with a half-life of 5.25 y. The initial decay rate of a 0.01 g sample is $x \times 10^{11}$ Bq then x is -
18. The threshold wavelength for cesium is 686 nm. If light of wavelength 470 nm illuminates the surface, the maximum speed of the photoelectrons in ms^{-1} is $x \times 10^5$ m then value of x is -
19. The work function for lithium is 2.3 eV. What is the maximum kinetic energy of photoelectrons in eV when the surface is illuminated with light of wavelength 400 nm?
20. When radiation of wavelength 350 nm is incident on a surface, the maximum kinetic energy of the photoelectrons is 1.2 eV. What is the stopping potential in volt for a wavelength of 230 nm?
21. What is the power output of $_{92}U^{235}$ reactor in mega watt if it takes 30 days to use up 2 kg of fuel, and if each fission gives 185 MeV of usable energy? Avogadro's number $N = 6.02 \times 10^{26}$ per kilomole.
22. A gas of identical hydrogen like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transition to higher energy level by absorbing monochromatic light of photon 2.7 eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have energy more and some have less than 2.7 eV. Find the principal quantum number of the initially excited level B.
23. When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area 1.0×10^{-4} m^2 andwork function 5.6 eV, 0.53% of the incident photons eject photoelectrons. If the number of photoelectrons emitted per second is $x \times 10^{11}$ then x is- (Take : 1 eV = 1.6×10^{-19} J)

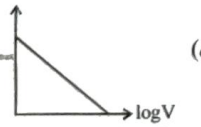
JEE Mains & Advanced Past Years Questions

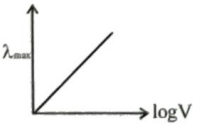
JEE-MAIN PREVIOUS YEAR'S

1. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log \lambda_{min}$ with log V is correctly represented in: *[JEE Main - 2017]*

(a) 

(b) 

(c) 

(d) 

2. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by : *[JEE Main - 2017]*

(a) $r = \frac{3}{4}$ (b) $r = \frac{1}{3}$

(c) $r = \frac{4}{3}$ (d) $r = \frac{2}{3}$

3. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the nth state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A, B are constants) *[JEE Main - 2018]*

(a) $\Lambda_n \approx A + B\lambda_n$ (b) $\Lambda_n^2 \approx A + B\lambda_n^2$

(c) $\Lambda_n^2 \approx \lambda$ (d) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

4. If the series limit frequency of the Lyman series is v_L, then the series limit frequency of the Pfund series is: *[JEE Main - 2018]*

(a) $16\ v_L$ (b) $v_L/16$
(c) $v_L/25$ (d) $25\ v_L$

5. In a hydrogen like atom, when an electron jumps from the M - shell to the L- shell the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell the wavelength of emitted radiation will be: *[JEE Main - 2019 (January)]*

(a) $\frac{27}{20}\lambda$ (b) $\frac{16}{25}\lambda$

(c) $\frac{25}{16}\lambda$ (d) $\frac{20}{27}\lambda$

6. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state, in terms of Bohr radius a_0, will (hc = 12500 eV – Å) [JEE Main-2019 (January)]
(a) $25\ a_0$ (b) $9\ a_0$
(c) $16\ a_0$ (d) $4\ a_0$

7. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbits and energy levels vary with quantum number n as:
[JEE Main - 2019 (January)]
(a) $r_n \propto \sqrt{n},\ E_n \propto n$ (b) $r_n \propto \sqrt{n},\ E_n \propto \frac{1}{n}$
(c) $r_n \propto n, E_n \propto n$ (d) $r_n \propto n^2, E_n \propto \frac{1}{n^2}$

8. In a Frank-hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to:
[JEE Main - 2019 (January)]
(a) 1700 nm (b) 2020 nm
(c) 220 nm (d) 250 nm

9. A He^+ ion is in its first excited state. Its ionization energy is : [JEE Main-2019 (April)]
(a) 6.04 eV (b) 13.60 eV
(c) 54.40 eV (d) 48.36 eV

10. Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms fall on He^+ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is: [JEE Main-2019 (April)]
(a) $n = 1 \rightarrow n = 4$ (b) $n = 2 \rightarrow n = 4$
(c) $n = 2 \rightarrow n = 5$ (d) $n = 2 \rightarrow n = 3$

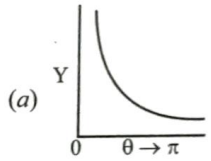
11. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets de-excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ? [JEE Main - 2019 (April)]
(**Given :** $h = 6.63 \times 10^{-34}$ Js ; $c = 3 \times 10^8\ ms^{-1}$)
(a) 9.4 nm (b) 12.3 nm
(c) 10.8 nm (d) 11.4 nm

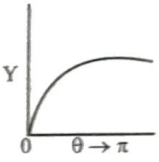
12. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is: [JEE Main-2019 (April)]
(a) 9/7 (b) 7/5
(c) 27/5 (d) 20/7

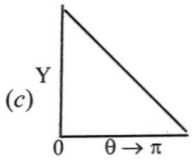
13. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65Å). The de-Broglie wavelength of this electron is:
[JEE Main-2019 (April)]
(a) 12.9 Å (b) 3.5 Å
(c) 9.7 Å (d) 6.6 Å

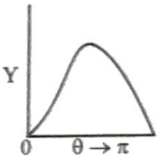
14. An excited He^+ ion emits two photons in succession, with wavelength 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy
$E = \frac{1240\text{eV}}{\lambda(\text{in nm})}$: [JEE Main-2019 (April)]
(a) n = 5 (b) n = 4
(c) n = 6 (d) n = 7

15. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron in its first excited state (in s^1) is: [JEE Main-2020 (January)]
(a) 6.2×10^{15} (b) 5.6×10^{12}
(c) 7.8×10^{14} (d) 1.6×10^{14}

16. The graph which depicts the results of Rutherford gold foil experiment with α-particles is : α-particles is :
Y = Number of scattred α-particles detected
(Plots are schematic and not to scale)
[JEE Main-2020 (January)]
(a) 

(b) 

(c) 

(d) 

17. The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is
[JEE Main-2020 (January)]

18. The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state :
[JEE Main-2020 (January)]
(a) 11.4 nm (b) 24.2 nm
(c) 35.8 nm (d) 8.6 nm

19. In a hydrogen atom the electron makes a transition from $(n + 1)^{th}$ level to the nth level. If $n >> 1$, the frequency of radiation emitted is proportional to
[JEE Main-2020 (September)]
(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$
(c) $\frac{1}{n^3}$ (d) $\frac{1}{n^4}$

20. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 304 Å. The corresponding difference for the Paschen series in Å is _____.

[*JEE Main-2020 (September)*]

21. A particle of mass 200 MeV/c^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is (Given the mass of the hydrogen atom to be 1 GeV/c^2) ___.

[*JEE Main-2020 (September)*]

22. Radiation of wavelength ë, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be:

[*JEE Main - 2016*]

(*a*) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (*b*) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(*c*) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (*d*) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

23. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de–Broglie wavelengths λ_A to λ_B after the collision is :

[*JEE Main-2017*]

(*a*) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (*b*) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

(*c*) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (*d*) $\frac{\lambda_A}{\lambda_B} = 2$

24. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to

$\left(\frac{hc}{e} = 1240\text{nm} - \text{V}\right)$

[*JEE Main - 2019 (January)*]

(*a*) 0.5V (*b*) 1.5V

(*c*) 1.0V (*d*) 2.0V

25. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50 V. Another particle B of mass '4 m' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelength $\frac{\lambda_A}{\lambda_B}$ is close to:

[*JEE Main - 2019 (January)*]

(*a*) 10.00 (*b*) 0.07

(*c*) 14.14 (*d*) 4.47

26. When a certain photosensitive surface is illuminated with monochromatic light of frequency v, the stopping potential for the photo current is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency v/2, the stopping potnetial is $-V_0$. The threshold frequency for photoelectric emission is :

[*JEE Main-2019 (January)*]

(*a*) $\frac{5v}{3}$ (*b*) $\frac{4}{3}$v

(*c*) 2v (*d*) $\frac{3v}{2}$

27. In an electron microscope, the resolution that can be achieved is of the order of the wavelength used. To resolve a width of 7.5×10^{-12}m, the minimum electron energy require is close to:

[*JEE Main-2019 (January)*]

(*a*) 500 keV (*b*) 100 keV

(*c*) 1 keV (*d*) 25 keV

28. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be :

[1 eV = 1.6×10^{-19} J]

[*JEE Main-2019 (January)*]

(*a*) 10^{14} and 10 eV (*b*) 10^{12} and 5 eV

(*c*) 10^{11} and 5 eV (*d*) 10^{10} and 5 eV

29. Surface of certain metal is first illuminated with light of wavelength λ_1=350 nm and then the light of wavelength λ_2=540 nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to :

(Energy of photon = $\frac{1240}{\lambda(\text{in nm})}$eV)

[*JEE Main-2019 (January)*]

(*a*) 1.8 (*b*) 2.5

(*c*) 5.6 (*d*) 1.4

30. If the deBroglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to : (Speed of light = 3×10^8 m/s ; Planck's constant = 6.63×10^{-34} J.s ; Mass of electron = 9.1×10^{-31} kg)

[*JEE Main-2019 (January)*]

(*a*) 1.1×10^6 m/s (*b*) 1.7×10^6 m/s

(*c*) 1.8×10^6 m/s (*d*) 1.45×10^6 m/s

31. The magnetic field associated with a light wave is given, at the origin,

by B = B_0 [sin (3.14×10^7)ct + sin (6.28×10^7) ct]

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons?

[*JEE Main-2019 (January)*]

(*a*) 6.82 eV (*b*) 12.5 eV

(*c*) 8.52 eV (*d*) 7.72 eV

32. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'λ_x' and 'λ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is: *[JEE Main-2019 (April)]*

(a) $\lambda_x + \lambda_y$ (b) $\frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y}$

(c) $\frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$ (d) $\lambda_x - \lambda_y$

33. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to (c = 3 × 10^8 m/s): *[JEE Main - 2019 (April)]*

(a) 15 × 10^{-8} N (b) 35 × 10^{-8} N

(c) 10 × 10^{-8} N (d) 20 × 10^{-8} N

34. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ, of the final particle, is given by: *[JEE Main - 2019 (April)]*

(a) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$ (b) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(c) $\lambda = \sqrt{\lambda_1 \lambda_2}$ (d) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

35. Light is incident normally on a completely absorbing surface with an energy flux of 25 Wcm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be: *[JEE Main - 2019 (April)]*

(a) 5.0 × 10^{-3} Ns (b) 3.5 × 10^{-6} Ns

(c) 1.4 × 10^{-6} Ns (d) 6.3 × 10^{-4} Ns

36. A 2 mW laser operates at a wavelength of 500nm. The number of photons that will be emitted per second is : [Given Planck's constant h = 6.6 ×10^{-34} Js,] *[JEE Main - 2019 (April)]*

(a) 2 ×10^{16} (b) 1.5 × 10^{16}

(c) 5× 10^{15} (d) 1 × 10^{16}

37. The electric field of light wave is given as $\vec{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{x}\frac{N}{C}$. This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is : *[JEE Main - 2019 (April)]*

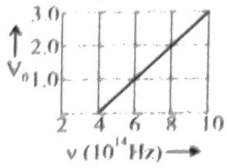
Given : $E\,(\text{in eV}) = \frac{12375}{\lambda(\text{in Å})}$

(a) 0.48 V (b) 2.0 V

(c) 2.48 V (d) 0.72 V

38. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be

(**Given** : Planck's constant (h) = 6.63 ×10^{-34} Js. electron charge e = 1.6×10^{-19} C) *[JEE Main - 2019 (April)]*



(a) 1.95 eV (b) 1.82 eV

(c) 1.66 eV (d) 2.12 eV

39. In a photoelectric effect experiment the threshold wavelength of the light is 380 nm. If the wavelength of incident light is 260nm, the maximum kinetic energy of emitted electrons will be: *[JEE Main - 2019 (April)]*

[**Given :** $E\,(\text{in eV}) = \frac{1237}{\lambda(\text{in nm})}$]

(a) 1.5 eV (b) 4.5 eV

(c) 15.1 eV (d) 3.0eV

40. A beam of electromagnetic radiation of intensity 6.4 × 10^{-5} W/cm^2 is comprised of wavelength, λ = 310 nm. It falls normallyon a metal (work function ϕ = 2eV)of surface area of 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x . (hc = 1240 eVnm, 1eV = 1.6 × 10^{-19} J), then x is ..... *[JEE Main-2020 (January)]*

41. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (C = speed of light in vacuum) *[JEE Main-2020 (January)]*

(a) $\frac{1}{c}\left(\frac{2E}{m}\right)^{\frac{1}{2}}$ (b) $\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

(c) $c(2mE)^{\frac{1}{2}}$ (d) $\frac{1}{c}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

42. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is: *[JEE Main-2020 (January)]*

(a) 2 eV (b) 3 eV

(c) 1.5 eV (d) 4 eV

43. Radiation, with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to: [JEE Main-2020 (January)]

(a) 1.8 eV (b) 0.8 eV
(c) 1.6 eV (d) 1.1 eV

44. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE is added to its energy, the wavelength become become λ/2. Value of ΔE, is: [JEE Main-2020 (January)]

(a) 4E (b) 3E
(c) 2E (d) E

45. An electron of mass m and magnitude of charge |e| initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t ignoring relativistic effects is : [JEE Main-2020 (January)]

(a) $-\frac{h}{|e|Et}$ (b) $-\frac{h}{|e|E\sqrt{t}}$

(c) $\frac{-h}{|e|Et^2}$ (d) $\frac{|e|Et}{h}$

46. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3λ, then stopping potential is $\frac{V}{4}$. If the threshold wavelength for the metallic surface is nλ then value of n will be _____. [JEE Main-2020 (September)]

47. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is Planck's constant $h = 6.63 \times 10^{-34}$ m/s Speed of light $c = 3 \times 10^8$ J.s. [JEE Main-2020 (September)]

(a) 1.1 eV (b) 2.0 eV
(c) 3.1 eV (d) 1.5 eV

48. Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light of the given wavelengths is [JEE Main-2020 (September)]

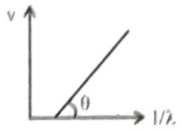
(a) $\frac{1}{250}$ (b) $\frac{1}{500}$

(c) 500 (d) 250

49. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger The work function of the metal is close to [JEE Main-2020 (September)]

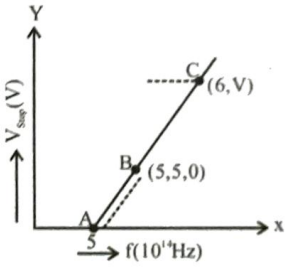
(a) 0.52 eV (b) 1.02 eV
(c) 0.61 eV (d) 0.81 eV

50. In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased [JEE Main-2020 (September)]



(a) Slope of the straight line get more steep
(b) Graph does not change
(c) Straight line shifts to left
(d) Straight line shifts to right

51. Given figure shows few data points in a photo electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is (Plancks constant $h = 6.62 \times 10^{-34}$ J.s) [JEE Main-2020 (September)]



(a) 2.59 eV (b) 2.27 eV
(c) 1.93 eV (d) 2.10 eV

52. Particle A of mass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. If both particles move along the x-axis after the collision, the change Δλ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is [JEE Main-2020 (September)]

(a) $\Delta\lambda = 2\lambda_0$ (b) $\Delta\lambda = 4\lambda_0$

(c) $\Delta\lambda = \frac{3}{2}\lambda_0$ (d) $\Delta\lambda = \frac{5}{2}\lambda_0$

53. The surface of a metal is illuminated alternately with photons of energies $E_1 = 4$ eV and $E_2 = 2.5$ eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in (eV) is _____.

[JEE Main-2020 (September)]

54. A beam of electrons of energy E scatters from a target having atomic spacing of 1 Å. The first maximum intensity occurs at $\theta = 60^\circ$. Then E (in eV) is _____.
(Planck constant $h = 6.64 \times 10^{-34}$ Js, 1 eV = 1.6×10^{-19} J, electron mass $m = 9.1 \times 10^{-31}$ kg)

[JEE Main-2020 (September)]

55. With increasing biasing voltage of a photodiode, the photocurrent magnitude [JEE Main-2020 (September)]
(a) Increases initially and after attaining certain value, it decreases
(b) Increases linearly
(c) Increases initially and saturates finally
(d) Remains constant

56. Assuming the nitrogen molecule is moving with r.m.s. velocity at 400 K, the de-Broglie wavelength of nitrogen molecule is close to (Given : nitrogen molecule weight : 4.64×10^{-26} kg, Boltzman constant : 1.38×10^{-23} J/K, Planck constant : 6.63×10^{-34} J.s)

[JEE Main-2020 (September)]

(a) 0.24 Å (b) 0.20 Å
(c) 0.34 Å (d) 0.44 Å

57. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths $\lambda_e, \lambda_{He^{++}}$ and λ_p is [JEE Main-2020 (September)]

(a) $\lambda_e > \lambda_{He^{++}} > \lambda_p$ (b) $\lambda_e < \lambda_p < \lambda_{He^{++}}$

(c) $\lambda_e > \lambda_p > \lambda_{He^{++}}$ (d) $\lambda_e < \lambda_{He^{++}} = \lambda_p$

58. Two identical photocathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are v_1 and v_2 respectively, then [JEE Main-2021 (March)]

(a) $v_1 - v_2 = \left[\frac{2k}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$

(b) $v_1^2 + v_2^2 = \frac{2k}{m}[f_1 + f_2]$

(c) $v_1 + v_2 = \left[\frac{2k}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(d) $v_1^2 - v_2^2 = \frac{2k}{m}[f_1 - f_2]$

JEE-ADVANCED PREVIOUS YEAR'S

1. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30mW and the speed of light is 3×10 ms^{-1}. The final momentum of the object is: [JEE Advanced - 2013]
(a) 0.3×10^{-17} kg ms^{-1} (b) 1.0×10^{-17} kg ms^{-1}
(c) 3.0×10^{-17} kg ms^{-1} (d) 9.0×10^{-17} kg ms^{-1}

2. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is: [JEE Advanced-2013]

3. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are) :

[JEE Advanced - 2013]

(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$

(c) $\frac{9}{5R}$ (d) $\frac{4}{3R}$

4. If λ_{Cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_α X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to [JEE Advanced - 2014]
(a) 1.99 (b) 2.14
(c) 0.50 (d) 0.48

5. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and hc = 1240 eV nm, the work function of the metal is nearly [JEE Advanced-2014]
(a) 3.7 eV (b) 3.2 eV
(c) 2.8 eV (d) 2.5 eV

6. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below: [JEE Advanced-2016]

λ (μm)	V_0(Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 136 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is

(*a*) 6.0×10^{-34} (*b*) 6.4×10^{-34}

(*c*) 6.6×10^{-34} (*d*) 6.8×10^{-34}

7. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where $n >> 1$. Which of the following statement(s) is (are) true?

[*JEE Advanced - 2016*]

(*a*) Relative change in the radii of two consecutive orbitals does not depend on Z

(*b*) Relative change in the radii of two consecutive orbitals varies as $1/n$

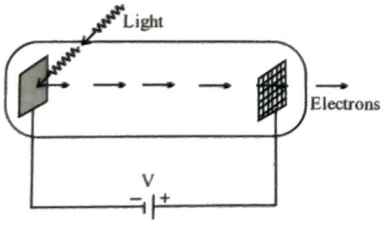
(*c*) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(*d*) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

8. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as -13.6 eV, the number of lines present in the emission spectrum is **[*JEE Advanced - 2016*]**

9. *Light of wavelength* λph falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between *the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is* λ_e, which of the following statement(s)is (are) true?

[*JEE Advanced - 2016*]



(*a*) For large potential difference ($V >> \phi/e$), λ_e is approximately halved if V is made four times

(*b*) λ_e decreases with increase in ϕ *and* λ_{ph}

(*c*) λ_e *increases at the same rate as* λ_{ph} *for* $\lambda_{ph} < hc/\phi$

(*d*) λ_e is a approximately halved, if d is doubled

10. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible n_f is: **[*JEE Advanced - 2017*]**

11. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\frac{\Delta\lambda_d}{\Delta\lambda}$ is proportional to: **[*JEE Advanced - 2017*]**

(*a*) $\frac{\lambda_d^3}{\lambda^2}$ (*b*) $\frac{\lambda_d^3}{\lambda}$

(*c*) $\frac{\lambda_d^2}{\lambda^2}$ (*d*) $\frac{\lambda_d}{\lambda}$

12. In a photoelectric experiment a parallel beam of monochromatic light with power of 2300W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Asume that the photoelectron emission efficinecy is 100% A potential difference of 500V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is...Mass of the electron $m_e = 9 \times 10^{-31}$ kg and 1.0 eV $= 1.6 \times 10^{-19}$ J.? **[*JEE Advanced - 2018*]**

13. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is ..... **[*JEE Advanced - 2018*]**

14. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1to the state n = 4. Immediately after that the electron jumps to the n = m state by emitting a photon of wavelength λ_e. Let and the emission are Δp_a and Δp_e, respectively. If $\lambda_a/\lambda_e = \frac{1}{5}$. Which of the option(s) is/are correct?

[JEE Advanced-2019]

[Use hc = 1242 eV nm; 1 nm = 10^{-9} m, h and c are Planck's constant and speed of light, respectively]

(*a*) $\lambda_e = 418$ nm

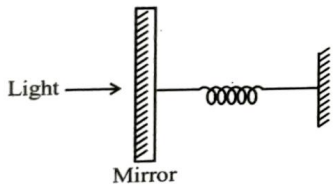
(*b*) The ratio of kinetic energy of the electron in the state n = m to the state n = 1 is $\frac{1}{4}$

(*c*) m = 2

(*d*) $\Delta p_a/\Delta p_e = \frac{1}{2}$

15. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24}\,m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1μm. If the value of N is $x \times 10^{12}$, then the value of x is_____. [Consider the spring as massless]

[JEE Advanced - 2019]



16. A particle of mass m moves in circular orbits with potential energy V(r)=*Fr*, where F is a positive constant and r is its distance from the origin. Its energies are calculated using the Bohr model. If the radius of the particle's orbit is denoted by R and its speed and energy are denoted by v and E, respectively, then for the nth orbit (here h is the Planck's constant)-

[JEE(Advanced) - 2020]

(*a*) $R \propto n^{1/3}$ and $v \propto n^{2/3}$ (*b*) $R \propto n^{2/3}$ and $v \propto n^{1/3}$

(*c*) $E = \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2m}\right)^{1/3}$ (*d*) $E = 2\left(\frac{n^2h^2F^2}{4\pi^2m}\right)^{1/3}$

17. In an X-ray tube, electrons emitted from a filament (cathode) carrying current I hit a target (anode) at a distance d from the cathode. The target is kept at a potential V higher than the cathode resulting in emission of continuous and characteristic X-rays. If the filament current I is decreased to $\frac{I}{2}$, the potential difference V is increased to 2V, and the separation distance d is reduced to $\frac{d}{2}$, then

[JEE(Advanced) - 2020]

(*a*) the cut-off wavelength will reduce to half, and the wavelengths of the characteristic X-rays will remain the same

(*b*) the cut-off wavelength as well as the wavelengths of the characteristic X-rays will remain the same

(*c*) the cut-off wavelength will reduce to half, and the intensities of all the X-rays will decrease

(*d*) the cut-off wavelength will become two times larger, and the intensity of all the X-rays will decrease

ANSWER KEY

Topicwise Questions

1. (*d*)	**2.** (*c*)	**3.** (*c*)	**4.** (*d*)	**5.** (*c*)	**6.** (*d*)	**7.** (*c*)	**8.** (*c*)	**9.** (*a*)	**10.** (*b*)
11. (*a*)	**12.** (*d*)	**13.** (*a*)	**14.** (*b*)	**15.** (*a*)	**16.** (*d*)	**17.** (*b*)	**18.** (*c*)	**19.** (*c*)	**20.** (*a*)
21. (*b*)	**22.** (*c*)	**23.** (*c*)	**24.** (*d*)	**25.** (*d*)	**26.** (*c*)	**27.** (*b*)	**28.** (*b*)		

Learning Plus

1. (*b*)	**2.** (*b*)	**3.** (*b*)	**4.** (*d*)	**5.** (*d*)	**6.** (*c*)	**7.** (*a*)	**8.** (*c*)	**9.** (*d*)	**10.** (*b*)
11. (*a*)	**12.** (*d*)	**13.** (*c*)	**14.** (*d*)	**15.** (*a*)	**16.** (*c*)	**17.** (*a*)	**18.** (*d*)	**19.** (*a*)	**20.** (*b*)
21. (*a*)	**22.** (*b*)	**23.** (*b*)	**24.** (*b*)	**25.** (*c*)	**26.** (*b*)	**27.** (*b*)	**28.** (*d*)	**29.** (*a*)	**30.** (*a*)

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. (*a,c*)	**2.** (*a,b,c*)	**3.** (*b*)	**4.** (*a,c,d*)	**5.** (*a, b*)	**6.** (*a,c*)	**7.** (*a,d*)	**8.** (*a,b,c,d*)	**9.** (*d*)	**10.** (*b*)
11. (*a*)	**12.** (*c*)	**13.** (*a*)-r, (*b*)-s, (*c*)-p (*d*)-q			**14.** [3.4 eV]	**15.** [914Å]	**16.** [8.65 MeV]		**17.** [4.2]
18. [5.4]	**19.** [0.8 eV]	**20.** [3.05 V]	**21.** [58.46 MW]		**22.** [Z = 1]	**23.** [6.25]			

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREIVOUS YEAR'S

1. (*c*)	**2.** (*b*)	**3.** (*d*)	**4.** (*c*)	**5.** (*d*)	**6.** (*c*)	**7.** (*a*)	**8.** (*d*)	**9.** (*b*)	**10.** (*b*)
11. (*c*)	**12.** (*d*)	**13.** (*c*)	**14.** (*a*)	**15.** (*c*)	**16.** (*a*)	**17.** (486)	**18.** (*a*)	**19.** (*c*)	
20. [10553.14]		**21.** (51.00)	**22.** (*a*)	**23.** (*d*)	**24.** (*c*)	**25.** (*c*)	**26.** (*d*)	**27.** (*d*)	**28.** (*c*)
29. (*a*)	**30.** (*d*)	**31.** (*d*)	**32.** (*c*)	**33.** (*d*)	**34.** (*d*)	**35.** (*a*)	**36.** (*c*)	**37.** (*a*)	**38.** (*c*)
39. (*a*)	**40.** [11]	**41.** (*d*)	**42.** (*d*)	**43.** (*d*)	**44.** (*b*)	**45.** (*c*)	**46.** (9)	**47.** (*c*)	**48.** (*b*)
49. (*c*)	**50.** (*b*)	**51.** (*b*)	**52.** (*b*)	**53.** [02.00]	**54.** [50.47]	**55.** (*c*)	**56.** (*a*)	**57.** (*c*)	**58.** (*d*)

JEE-ADVANCED PREVIOUS YEAR'S

1. (*b*)	**2.** [1]	**3.** (*a,c*)	**4.** (*b*)	**5.** (*a*)	**6.** (*b*)	**7.** (*a,b,d*)	**8.** [N = 1, 2 or 3]	**9.** (*a*)
10. [5]	**11.** (*a*)	**12.** [24]	**13.** [3]	**14.** (*b,c*)	**15.** [1.00]	**16.** (*b,c*)	**17.** (*a, c*)	

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26 NUCLEI

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INTRODUCTION

So far the only knowledge of nucleus we have as a tiny positively charged object whose primary contributions are to provide the atom with most of its mass and to hold its electrons in captivity. The chief properties of atoms, molecules, solids and liquids can all be traced to the behaviour of atomic electrons and not to the behaviour of nuclei. However, the nucleus turns out to be of paramount importance in the grand scheme of things. To start with the very existence of the various elements is due to the ability of nuclei to possess multiple elecric charge. Furthermore, the energy involved in almost all natural processes can be traced to nuclear reactions and transformations. In the following sections we will study about the nucleus and phenomenon associated with nucleus.

Nuclear characteristics

(i) Rutherford's α-scattering experiment established that the mass of atom is concentrated within small positively charged region at the centre which is called 'nucleus'.

(ii) Radius of nucleus (R)

Volume of nucleus is found to be proportional to number of nucleons in it.

$\therefore$ Volume $\propto$ A

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3$$

$$R \propto A^{1/3}$$

$$R = R_0 A^{1/3}$$

where A = mass number

where $R_0 = 1.1 \times 10^{-15}$ m

(iii) Any element X with mass number A and charge number Z can be represented by ${}_ZX^A$ or ${}^A_Z X$.

Number of neutron = A – Z

Mass number = A = P + N

(iv) 1 amu = $\frac{1}{12^{th}}$ of mass of one carbon-12 Atom

Also written as 1u. e.g. mass of C-12 Atom = 12.000 u.

(v) Density of nucleus (ρ)

(a) $\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$

(b) ρ is of the order of $\simeq 10^{17}$ kg/m^3 for almost all nuclei's.

(c) Nuclear density is independent of its mass number.

DIFFERENT TYPES OF NUCLEI

(i) Isotopes → Those set of nuclei's containing same number of protons but different numbers of neutrons.

(ii) Isobars → Those set of nuclei's containing same mass number but different atomic number are known as isobars.

Example : ${}_6C^{14}$ and ${}_7N^{14}$

(iii) Isotones → Those set of nuclei's containing same numbers of neutrons are known as isotones.

Example : ${}_6C^{14}$ and ${}_8O^{16}$

(iv) Mirror nuclei → Nuclei with same mass number A but with proton and neutron numbers interchanged i.e. the number of protons in one is equal to the number of neutrons in the other are called mirror nuclei.

Example : ${}_4Be^7$ (Z = 4, N = 3) and ${}_3Li^7$ (Z = 3, N = 4)

TRAIN YOUR BRAIN

Example 1: The ratio of the radii of the nuclei ${}^{27}_{13}Al$ and ${}_{52}Te^{125}$ is approximately:

(*a*) 6 : 10 (*b*) 13 : 52

(*c*) 40 : 177 (*d*) 14 : 73

Sol. $R_{A_l} / R_{T_e} = \frac{(27)^{1/3}}{(125)^{1/3}} = \frac{3}{5} = \frac{6}{10}$

Example 2: How many electrons, protons, and neutrons are there in a nucleus of atomic number 11 and mass number 24?

(*a*) 11, 12, 13 (*b*) 11, 11, 13

(*c*) 12, 11, 13 (*d*) 11, 13, 12

Sol. Number of protrons in nucleus

= atomic number = 11

Number of electrons = number of protons = 11.

Number of neutrons

= mass number A – atomic number Z

N = 24 – 11 = 13

Nuclear forces →

Forces acting between n–n, n–p and p–p inside the nucleus is called nuclear forces.

(a) They are short ranging forces (10^{-15} m – 10^{-14} m)

(b) They are the strongest in nuclear ranges and their strength becomes zero when distance between nucleons becomes of the order of 10^{-12} m or more.

(c) Order of strength of various forces.

$F_{nuclear} : F_{electromegnetic} : F_{weak} : F_{gravitation}$

$:: 10^{19} : 10^{16} : 10^{33} : 1$

(d) Nuclear forces are charge independent or they act alike between various nucleons.

$F_{n-n} = F_{n-p} = F_{pp}$

(e) Net force between the nucleon

$F_{n-n} = F_{n-p} > F_{pp}$

Mass Defect

When proton and neutron combines together in order to form nucleus, then in this process some mass gets lost and this lost mass is known as mass defect (Δm).

Mathematically:

Δm = Mass of nucleons - mass of nucleus

$= A - M$

$= [Zm_p + Nm_n] - M$

$= [Zm_p + (A - Z)\, m_n] - [\text{Mass of atom}]$

Packing fraction (f) = mass defect per nucleons

$$= \frac{\Delta m}{A} = \frac{[Zm_p + (A-Z)m_n] - M}{A}$$

$$\left\{\begin{array}{l}\text{Mass of nucleus} \simeq \text{Mass of Atom} \\ \text{as } m_e \text{ is negligible}\end{array}\right\}$$

Note : "More the value of packing fraction, more is the stability of product nuclei".

TRAIN YOUR BRAIN

Example 3: Consider the decay of radium-226 atom into an alpha particle and radon-222. Then, what is the mass defect of the reaction-

Mass of radium -226 atom = 226.0256 u

Mass of radon - 222 atom = 222.0715 u

Mass of helium - 4 atom = 4.0026 u

(*a*) 0.0053 u (*b*) 0.0083 u

(*c*) 0.083 u (*d*) None

Sol. Mass defect =

$\Delta m = M(Ra\ 226) - M(Rn\ 222) - M(\alpha)$

$= 226.0256 - 222.0175 - 4.00026$

$= 0.0053$ u.

NUCLEAR BINDING ENERGY

In nucleus, proton and neutron are contained in very small sphere of radius of the order 10^{-15} m. At such small distance, two proton exerts a very large force of repulsion on each other. Therefore certain amount of energy is needed to bind the nucleus. This energy is known as binding energy.

(i) When nucleus is formed, the mass of nucleons forming it decreases. The mass defect supplies the required binding energy.

(ii) The energy equivalent to mass defect is called binding energy.

By Einstein mass energy relation Binding energy = (Mass defect) x (velocity of light)^2 $\Rightarrow \Delta E = (\Delta m)c^2$

Here Δm : mass defect

1 amu mass = 931.49 MeV of energy

(iii) Binding energy of a nucleus may also be defined as the amount of work required to separate the nucleons to infinite distance.

Binding Energy/Nucleon

$$\text{Binding energy per nucleon} = \frac{\Delta E}{A} = \frac{\Delta mc^2}{A}$$

It is the measure of stability of nucleus.

Greater the binding energy per nucleon greater will be nucleus stability.

TRAIN YOUR BRAIN

Example 4: If mass equivalent to one mass of proton is completely converted into energy then determine the energy produced?

Sol. $E = mc^2$

$= (1.66 \times 10^{-27})(3 \times 10^8)^2$ J

$= 1.49 \times 10^{-10}$ J

$$= \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-13}} \text{ MeV} = 931.49 \text{ MeV}$$

Example 5: If mass equivalent to one mass of electron is completely converted into energy then determine the energy liberated.

Sol. $E = mc^2$

$= (9.1 \times 10^{-31})(3 \times 10^8)^2$ J

$= 0.51$ MeV

Example 6: If the mass defect in the formation of helium from hydrogen is 0.5 %, then the energy obtained in kWh, in forming helium from 1 kg of hydrogen will be-

(*a*) 1.25 (*b*) 125×10^4

(*c*) 1.25×10^8 (*d*) 1.25×10^6

Sol. $\Delta E = \Delta mc^2$

$$\Delta m = \frac{0.5}{100} \text{kg} = 0.005 \text{ kg}$$

$c = 3 \times 10^8$ m/s

$\Delta E = 0.005 \times (3 \times 10^8)^2$

$\Delta E = 4.5 \times 10^{14}$ J or watt-sec

$$\Delta E = \frac{4.5 \times 10^{14}}{60 \times 60} = 1.25 \times 10^{11} \text{ watt hour}$$

$\Delta E = 1.25 \times 10^8$ kWh

Example 7: Binding energy per nucleon of an α-particle from the following data:

Mass of the helium nucleus = 4.001265 amu

Mass of proton = 1.007277 amu

Mass of neutron = 1.00866 amu

(1 amu = 931.4812 MeV)

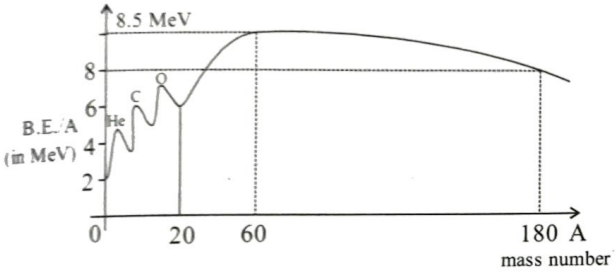
(*a*) 28.5 MeV (*b*) 7.1 MeV

(*c*) 14.25 MeV (*d*) 32.4 MeV

Sol. Mass of two protons
$= 2 \times 1.007277 = 2.014554$ amu
Mass of two neutron
$= 2 \times 1.008666 = 2.017332$ amu
Total initial mass of two protons and neutrons $= 2.014554 + 2.017332 = 4.031886$ amu
Mass defect $\Delta m = 4.031816 - 4.001265$,
$\Delta m = 0.030621$ amu
$\therefore$ Binding energy of α particle
$= 0.030621 \times 931.4812 = 28.5221$ MeV
Binding energy per nucleon
$= 28.5221/4 = 7.10525$ MeV

BINDING ENERGY CURVE

(i) Nature of the curve:



(ii) The curve plotted between binding energy per nucleon and mass number is known as binding energy curve.

(iii) The curve shows that BE per nucleon first increases, attains a maximum value and then decreases

(iv) The average binding energy per nucleon in a heavy nucleus is approximately 8 MeV.

(v) The heaviest stable nuclide is $^{209}_{83}Bi$ (Half life $> 2 \times 10^{16}$ years greater than the age of the universe). All nuclide with $Z > 83$ are unstable.

NUCLEAR FISSION

(i) It was discovered by **Scientist Otto Hann and F. Strassman.**

(ii) Process of splitting up of heavier nuclei by the bombardment of neutron into two almost equal weight nuclei is known as nuclear fission and in this process on account of mass defect huge energy is liberated.

(iii) Natural uranium is obtained in the following proportions
$_{92}U^{234} : {}_{92}U^{275} : {}_{92}U^{238} :: 0.006 : 0.7 : 99.3$ (approx)

(iv) U^{238} is not used in fission as its fission is possible only by neutron of energy approximately 1 MeV.

(v) Fission of U^{235} is possible by slow neutrons of energy (~ 1eV) or even by thermal neutrons of energy (0.025 eV)

(vi) Fission reaction $\rightarrow$
$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236}$ (unstable nucleus)

(vii) (a) Because of unstability, $_{92}U^{236}$ again fissions into two stable nucleus Ba and Kr with the three other neutrons.

$$_{92}U^{236} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + (200 \text{ MeV energy})$$

(b) During $_{92}U^{235}$ fission we don't get only isotopes of Ba and Kr but we can get other type of pairs also like

$$_{92}U^{235} + {}_0n^1 \longrightarrow {}_{92}U^{236} \text{ (Unstable)}$$

$$\longrightarrow {}_{38}Sr^{95} + {}_{54}Xe^{139} + 2{}_0n^1 + 200 \text{ MeV (energy)}$$

(c) By fission of one $_{92}U^{235}$ we get approximately 200 MeV energy.

(viii) By fission of $_{92}U^{235}$, on an average 2.5 neutrons are liberated.

(ix) Most part of liberated energy is in the form of kinetic energy of fragmented nuclei.

(x) Neutrons emitted from nuclear fission have energy equal to 2.0 MeV. These fast neutrons can escape from the reaction so as to proceed the chain reaction they are needed to slow down.

Nuclear Chain Reaction:

The equation of fission of U^{235} is

$$U^{235} + {}_0n^1 \longrightarrow Ba + Kr + 3{}_0n^1 + Q$$

These three secondary neutrons produced in the reaction may cause of fission of three more U^{235} and given 9 neutrons, which in turn, may cause of nine more fission of U^{235} and so on. Thus a continuous **'Nuclear Chain reaction'** would start.

- If there is no control on chain reaction then in a short time ($\simeq 10^{-6}$ sec.) a huge amount of energy will be released. (This is the principle of 'Atom bomb')
- If chain is controlled the produced energy can be used for peaceful purposes. For example nuclear reactor (Based on fission) are generating electricity.

Critical mass

In order to sustain chain reaction in a sample of enriched uranium, it is required that the number of lost neutrons should be much smaller than the number of neutrons produced in a fission process. For it the size of uranium block should be equal or greater than a certain size called critical size.

$$\textbf{Reproduction factor (K)} = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

(i) If size of Uranium used is 'Critical' then $K = 1$ and the chain reaction will be steady or sustained (As in nuclear reaction).

(ii) If size of Uranium used is 'Super critical' then K > 1 and chain reaction will accelerate resulting in a explosion (As in atom bomb)

(iii) If size of Uranium used is 'Sub Critical' then K < 1 and chain reaction will retard and will stop.

Nuclear Reactor (K = 1)

Construction:

- **Nuclear fuel:** Commonly used are U^{235}, Pu^{239}. Pu^{239} is the best. Its critical size is less than critical size of U^{235}.
  But P^{239} is not naturally available and U^{235} is used in most of the reactors.
- **Moderator:** Its function is to slow down the fast secondary neutrons. Because only slow neutrons can bring the fission of U^{235}. The moderator should be light and the it should not absorb the neutrons. Commonly, Heavy water (D_2O, molecular wight 20gm.) Graphite etc. are used. These are rich of proton. Neutrons collide with the proton and interchange their energy. The neutrons get slow down.
- **Control rods:** They have the ability to capture the slow neutrons and can control the chain reaction at any stage. Boron and Cadmium are best absorber fo neutrons.
- **Coolant:** A substance which absorb the produced heat and transfer it to water for further use. Generally coolant is water at high pressure.

NUCLEAR FUSION

(i) Process of combining two light weight nucleus into one heavy weight nucleus at high temperature and pressure is called nuclear fussion .

(ii) This reaction is possible only at high pr. ($\cong 10^6$ atm) and high temperature ($\cong 10^7 - 10^8$ K)

(iii) The vast amount of energy emitted in this reaction is also called thermonuclear energy and this reaction is called thermonuclear reaction.

(iv) Sun and stars get energy produced from nuclear fusion.

(v) Four hydrogen atom combined into one $_2He^4$ nucleus and 24.7 MeV energy emits in the whole reaction.

$$4\ {}_1H^1 \longrightarrow {}_2He^4 + 2\ {}_{-1}e^0 + 24.7\ MeV$$

(vi) Hydrogen bomb is also based on above principle and it is much more disastrous than the atom bomb.

(vii) Hydrogen bomb explosion needs atom bomb explosion so that $10^7 - 10^8$K temperature could be reached for fusion reaction.

(viii) Fusion reactor is still not available to get energy from nuclear fusion.

(ix) Energy per nucleon in fission $\left(=\frac{200}{235}=0.85\ MeV\right)$

<< energy per nucleon in fusion $\left(=\frac{26}{4}=6.5\ MeV\right)$

TRAIN YOUR BRAIN

Example 8: In a nuclear fusion process 4 hydrogen atom combine to form a helium atom and two positrons. Calculate the energy released in the process.

(a) 13.92 MeV (b) 25.69 MeV

(c) 20.69 MeV (d) None

Sol. Initial mass = 4 × 1.007825
= 4.091300 amu

Total final mass = 4.003702 amu
= 4.003702 amu

Decreases in mass = 4.031300 – 4.003702
= 0.026598 amu

1amu = 931 MeV

Energy released = 0.027598 x 931
= 25.69 MeV

Example 9: In the nuclear reaction,

$_{92}U^{238} \rightarrow {}_ZTh^A + {}_2He^4$,

the values of A and Z are-

(a) A = 234, Z = 94 (b) A = 234, Z = 90

(c) A = 238, Z = 94 (d) A = 238, Z = 90

Sol. A = 238 – 4 = 234, Z = 92 - 2 = 90

Example 10: The mass of helium nucleus is less than that of its constituent particles by 0.03 a.m.u. The binding energy per nucleon of $_2He^4$ nucleus will be-

(a) 7 MeV (b) 14 MeV

(c) 3.5 MeV (d) 21 MeV

Sol. $\Delta m = 0.03$ a.m.u., A = 4

$$\Rightarrow \frac{\Delta E}{A} = \frac{\Delta m \times 931}{A}$$

$$\Rightarrow \frac{\Delta E}{A} = \frac{0.03 \times 931}{4} = 7\ MeV$$

Example 11: The binding energies of deutron ($_1H^2$) and α-particle ($_2He^4$) are 1.125 and 7.2 MeV/nucleon respectively. In the process $_1H^2 + {}_1H^2 \rightarrow {}_2He^4$, Amount of energy transferred is & which nucleus is more stable ?

(a) 2.43 MeV liberated, $_1H^2$ (b) 24.3 MeV liberated, $_2He^4$

(c) 2.43 MeV supplied, $_1H^2$ (d) 24.3 MeV supplied, $_2He^4$

Sol. Binding energy of deutron
= 2 × 1.125 = 2.25 MeV

∴ Total Binding energy ($_1H^2 + {}_1H^2$)
= 2.25 + 2.25 = 4.5 MeV

Binding energy of $_2He^4$ is
= 4 × 7.2 = 28.8 MeV

Thus $_2He^4$, in comparison to ($_1H^2 + {}_1H^2$), has (28.8 – 4.5) = 24.3 MeV more energy. So, 24.3 MeV energy will be libereated.

Also higher the binding energy per nucleon for a nucleus, more stable is the nucleus so $_2He^4$ is more stable.

Example 12: Deuterium is an isotope of hydrogen having a mass of 2.01470 amu. Find binding energy in MeV of this isotope -

(*a*) 2.741 MeV (*b*) 2.174 MeV
(*c*) 1.741 MeV (*d*) 0.741 MeV

Sol. Deuterium, the isotope of hydrogen consist of one proton and neutron. Therefore mass of nuclear constituents of deuterium = mass of proton + mass of neutron

= 1.00759 + 1.00898 = 2.01657 amu.

mass of nucleus of deuterium = 2.01470 amu.

Mass defect = 2.01657 – 2.01470 = 0.00187 amu.

Binding energy = ΔE = 0.00187 × 931 MeV

= 1.741 MeV.

Example 13: The binding energy per nucleon for $_3Li^7$ will be, if the mass of $_3Li^7$ is 7.0163 amu

(*a*) 5.6 MeV (*b*) 39.25 MeV
(*c*) 1 MeV (*d*) zero

Sol. $E = \frac{\Delta E}{A} = \frac{\Delta m \times 931}{A}$ MeV

$\Delta m = (3m_p + 4m_n)$ – mass of Li^7

$\Delta m = (3 \times 1.00759 + 4 \times 1.00898) - 7.01653$

$\Delta m = 0.04216$ a.m.u.

$E = \frac{0.04216 \times 931}{7} = \frac{39.25}{7} = 5.6$ MeV

Example 14: The energy released per fission of uranium 235 is about 200 MeV. A reactor using U-235 as fuel is producing 1000 kilowatts power. The number of U-235 nuclei undergoing fission per sec is, approximately-

(*a*) 10^6 (*b*) 2×10^8
(*c*) 3×10^{16} (*d*) 931

Sol. The energy produced per second is

$= 1000 \times 10^3$ J $= \frac{10^6}{1.6 \times 10^{-19}}$ eV $= 6.25 \times 10^{24}$ eV

The number of fissions should be, thus:

number $= \frac{6.25 \times 10^{24}}{200 \times 10^6} = 3.125 \times 10^{16}$

Example 15: Power output of $_{92}U^{235}$ reactor if it takes 30 days to use up 2kg of fuel, and if each fission gives 185MeV of usable energy is:

(*a*) 5.846 KW (*b*) 58.46 MW
(*c*) .5846 KW (*d*) None of these

Sol. No. of atoms in 2kg $_{92}U^{235}$ is $= \frac{2}{235} \times N_A$

$= \frac{2}{235} \times (6.02 \times 10^{26}) = 5.12 \times 10^{24}$

Fission rate $= \frac{5.12 \times 10^{24}}{30 \times 24 \times 60 \times 60}$

$= 1.975 \times 10^{18}$ per sec

Usable energy per fission = 185 MeV

∴ Power output

$= (185 \times 10^6)(1.975 \times 10^{18})(1.6 \times 10^{-19})$ watt $= 58.4 \times 10^6$ watt = 58.46 MW

RADIOACTIVITY

Definition:

The self, spontaneous, disintegration (i e. decay) of unstable radioactive nuclei is known as radioactivity & the nuclei exhibiting this phenomenon are known as radioactive nuclei.

(1) The phenomenon of radioactivity was discovered by Henry Becquerel in uranium salts. (1896)

(2) Some examples of radio active elements are :
Uranium, Radium, Thorium, Polonium, Neptumium etc.

(3) All elements having atomic number greater than 83 are radio active elements.

(4) Decay processes are random. Here one simply knows that in a radioactive element, radioactivity is taking place or it is definite that a certain no. of atoms will decay in a given time interval but one never knows that which particular radioactive nuclei will decay. It is just a matter of chance i.e. probability which is explained by quantum mechanics. It is statistical in nature.

(5) This process is spontaneous i.e. it can neither be started, stopped, accelerated nor retarded by any physical (i.e. by changing temperature, pressure & force) or chemical change (catalyst).

(6) In this phenomenon parent-daughter chain continues. Original radio active element are called parent element or nucleus but new element is called Daughter element or disintegrated nucleus.

$_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_2\alpha^4$
(Parent) (Daughter)

$_{90}Th^{234} \rightarrow {}_{91}Pa^{234} + {}_{-1}\beta^0 + \bar{\nu}$
(Parent) (Daughter) (Antineutrino)

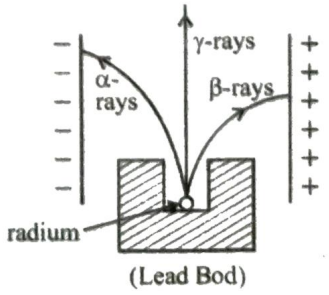
(7) In this phenomenon, α, β particles and γ-rays are emitted. For a given nuclei at a particular time emission of either α or β takes place, never both at time.

(8) Due to emission of α, β, γ during radioactive decay, mass of parent nuclei goes on decreasing.

RUTHERFORD'S EXPERIMENT

He put a sample of radioactive substance in a lead box and allowed the emission of radioactive rays through a small hole only. When the radiation enter into the external electric field, they split into three parts.

(i) Radiations which deflect towards negative plate are called α-rays.



(ii) Radiations which deflect towards positive plate are called β-rays.

(iii) Radioations which are undeflected, called γ-rays.

CHARACTERISTICS OF RADIOACTIVE RADIATIONS

Characteristics of α - decay:

(1) α- particle are two times ionised Helium atoms.

(2) α- particle has 2- protons and 2 neutrons.

(3) $_2He^4$ (Nucleus) = α-particle

(4) In general α - decay is given by

$$_ZX^A \xrightarrow{\alpha-\text{decay}} {}_{Z-2}Y^{A-4} + {}_2He^4 + \gamma \text{ (Energy)},$$

(α-particle)

atomic number decreases by 2 & mass no. decreases by 4.

(5) Energy in α- particle emission

$$= (M_x - M_y - M_{_2He^4})\, c^2$$

Energy in α- particle emission varies from 4.5 MeV to 11MeV. Through calculation α- Particle has to cross potential barrier of 21 MeV for their emission. Hence α-particle emission can not be explained on the basis of classical theory.

(6) Mass of α- particle = (2p + 2n)

$= 6.68 \times 10^{-27}$ kg

Charge of α- particle = + 2e

$= 2 \times 1.6 \times 10^{-19}$ coulomb, (+) Ve.

(7) In α- particle emission an element goes two column backward in periodic table.

(8) Emission of α-particle can be explained on the basis of quantum mechanics (Tunnel effect).

(9) Energy spectrum of α- particle is line spectrum.

(10) When α-particle emission takes place, γ-rays are also emitted.

Characteristics of β- decay:

(1) β-particle are high energy electron or positron.

(a) β^- or $_{-1}e^0$ (electron)

(b) β^+ or $_{+1}e^0$ (Positron)

(2) Resultant charge on β-particle

$= \pm 1.6 \times 10^{-19}$ coulomb

(3) Rest mass of β-particle is equal to mass of electron.

(4) β-particle emission can be represented by following reactions:

$$_ZX^A \xrightarrow{\beta^- -\text{decay}} {}_{Z+1}Y^A + {}_{-1}e^0 + Q$$

$$_ZX^A \xrightarrow{\beta^+ -\text{decay}} {}_{Z-1}Y^A + {}_{+1}e^0 + Q$$

where Q = Energy

(5) In β^- decay, atomic no. increases by one and in β^+ emission, atomic no. decreases by one.

(6) Mass number does not change in β-emission.

(7) Emission of β particle can be explained by conversion of neutron in proton & vice versea in the nucleus.

$_1p^1 \rightarrow {}_0n^1 + {}_1e^0$ (β^+)

or $_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0$ (β^-)

(8) β neutrino hypothesis , emission of β particle reaction

$_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{\nu}$ (Anti-Neutrino)

$_1p^1 \rightarrow {}_0n^1 + {}_1e^0 + \nu$ (Neutrino)

Hence reaction $_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu + Q$

$_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu} + Q$

(9) Existence of nutrino is practically explained by Rein's Collin.

(10) Energy range of emitted β- particle has all possible energies.

(11) Maximum value of energy of β-particle is called end point energy.

Characteristics of γ- rays:

(1) γ- rays are electromagnetic waves of short wavelength ($\cong 10^{-12}$m)

(2) They emit from nucleus.

(3) They travel with speed of light = (3×10^8 m/sec)

(4) These are high energy rays (of photons)

(5) When α or β particle emission takes place, nucleus achieves an excited state and during coming back to normal state γ emission takes place

$_{27}Co^{60} \rightarrow {}_{28}Ni^{60}$(exicited state) + $_{-1}e^0$

$_{28}Ni^{60}$(exicited state) $\rightarrow {}_{28}N^{60}$ (ground state) + Q (gama rays)

(6) In γ-decay atomic number and mass number does not changes.

(7) Energy spectrum of γ-rays is line spectrum.

(8) It affects photographic plate.

(9) It has ionising power (least among α, β and γ).

(10) It also has penetration power (highest among α, β and γ).

(11) It is not affected by electric & magnetic field.

(12) γ-rays after entering into substance are absorbed in three processes depending upon energy

(A) Photoelectric effect

(B) Compton effect

(C) Pair production.

LAWS OF RADIOACTIVE DISINTEGRATION

(1) If N = No. of active nuclei at time t

N – dN = No. of active nuclei after time interval of dt.

$\therefore$ Rate of decay = $-\frac{dN}{dt}$

(2) According to the Rutherford and Soddy law for radio active decay, "At any instant the rate of decay of

radioactive atoms is proportional to the number of atoms present at that instant " Rate of decay $\propto$ N

$$-\frac{dN}{dt} \propto N$$

or $$-\frac{dN}{dt} = \lambda N$$

λ is called decay constant.

(3) If N_i = No of active nuclei at time t_i.

N_f = No of active nuclei left at time t_f

$$\int_{N_i}^{N_f} \frac{dN}{N} = -\int_{t_i}^{t_f} \lambda dt$$

$$[\ell n\ n]_{N_i}^{N_f} = -\lambda\ [t]_{t_i}^{t_f}$$

$$\ell n\ \frac{N_f}{N_i} = -\lambda\ [t_f - t_i]$$

$$\frac{N_f}{N_i} = e^{-\lambda[t_f - t_i]}$$

If $t_f = t$ & $N_f = N$

& $t_i = 0$ $N_i = N_0$

then $$\frac{N}{N_0} = e^{-\lambda t} \qquad N = N_0 e^{-\lambda t} \qquad \text{..... (1)}$$

(4) eq. (1) is known as Rutherford & Soddy's exponential equation of radioactivity.

(5) If $t = \infty$ then $N = 0$

Therefore, for a complete decay of a radioactive nuclei, it takes infinite time.

(6) **Half life ($T_{1/2}$) →**

Half life of a radioactive element is the time or time interval is which 50% of radioactive nuclei will disintegrate.

or when $t = T_{1/2}$ then $N = \frac{N_0}{2}$

From $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0\ e^{-\lambda T_{1/2}}$$

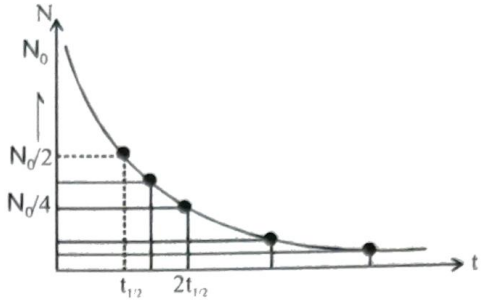
$$T_{1/2} = \frac{\ell n\, 2}{\lambda} = \frac{0.693}{\lambda}$$

(7) **Physical significance of Half life.**

If half life of a radio active nuclei is more then in a given time interval probability of its availability is also more. Half life of a element does not depend on physical & chemical circumstances.

$$\frac{N}{N_0} = \frac{1}{(2)^{t/T}}.$$

(8) Graph between no. of active nuclei left & time of decay



(9) **Average life or mean life (τ) :**

The average of times for which a radioactive material remains active, is defined as mean life of that material.

$$\tau = \frac{\text{Sum of lives of all nuclei's}}{\text{Total no. of nuclei present}}$$

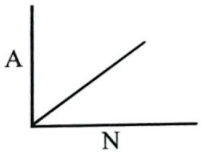
$$\tau = \frac{1}{\lambda}$$

(10) **Activity (A or R):**

(i) Activity of any element shows instability of the element at any time (t).

(ii) Activity is defined as rate of decay at given instant.

$$A = -\frac{dN}{dt} = N\lambda$$

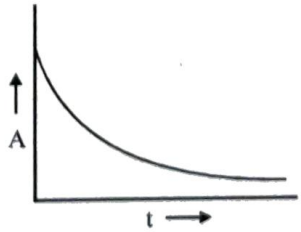


$\therefore$ $A \propto N$

(iii) Original activity

$A_0 = \lambda N_0$

$$\therefore \frac{A}{A_0} = \frac{N}{N_0} = e^{-\lambda t} = \frac{1}{(2)^{t/T}}$$



(iv) Units of Activity

(a) 1 disintegration / sec = 1 becquerel (Bq)

(b) 1 curie = 3.7×10^{10} dps

(c) 1 Ruther ford = 10^6 dps

(d) 1 mille curie = 37 Rutherford

(e) Specific Activity: Activity per gram of a substance is known as specific activity. The specific activity for 1 gm of radium 226 is 1 curie.

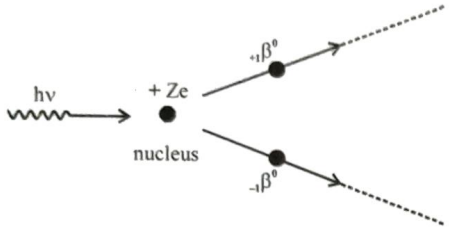
(v) Fraction of active nuclei left $= \frac{N}{N_0}$ (probability of survival)

(vi) Fraction of decayed nuclei $= 1 - \frac{N}{N_0} = \frac{N_0 - N}{N_0}$

(Probability of decay)

PAIR PRODUCTION & PAIR-ANNIHILATION

Collision of γ-ray photon by a nucleus & production of electron positron pair is known as pair production.



The rest mass of each of the electron & the positron is 9.1×10^{-31} kg. So, the rest mass energy of each of them is

$E_0 = m_0c^2 = (9.1 \times 10^{-31})(3 \times 10^8)^2$

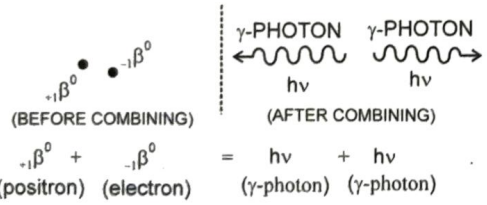
$= 8.2 \times 10^{-14}$ Joule

$= 0.51$ MeV

Hence for pair-production, it is essential that the energy of γ-photon must be at least

$2 \times 0.51 = 1.02$ MeV.

when in positronium ion an electron and positron combines together two γ-photons are produced. This is called pair annhilation.



TRAIN YOUR BRAIN

Example 16: The half life of polonium is 140 days. In what time will 15 gm of polonium be disintegrated out of its initial mass of 16 gm?

(*a*) 500 days (*b*) 480 days

(*c*) 560 days (*d*) 280 days

Sol. Suppose the initial mass of a radio active element is N_0. The mass of the element left after n half-lives is given by $N = N_0\left(\frac{1}{2}\right)^n$

$\therefore \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$\because N_0 = 16$ gm. The mass of the disintegrated element is 15 gm So, the mass of the element left is $N = 16 - 15 = 1$ gm. Thus

$\Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$

The half life of polonium is 140 days. Hence, the time taken in the disintegration of 150 gm polonium = half life × no. of half lives $= 140 \times 4 = 560$ days

Example 17: The half-life of radium is 1600 years. After how many years 25% of radium block remains undecayed?

(*a*) 3200 years (*b*) 4800 years

(*c*) 7200 years (*d*) 9600 years

Sol. Suppose the initial quantity of radium is N_0. Then the quantity left after n half-lives will be $N = N_0\left(\frac{1}{2}\right)^n$

Here, $N = 25\%$ of $N_0 = \frac{N_0}{4}$

$\therefore \frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^n, \frac{1}{4} = \left(\frac{1}{2}\right)^n$

$\Rightarrow \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2 \therefore n = 2$

time of disintegration = half life × number of half lives $= 1600 \times 2 = 32\,00$ years

Example 18: If a radio active material contains 0.1 mg of Th^{234}, how much of it will remain unchanged after 120 days? Its half life is 24 days.

(*a*) 0.0312 gm (*b*) 0.0312 mg

(*c*) .00312 mg (*d*) .00312 gm

Sol. T = 24 days, t = 120 days

$\frac{M_0}{m} = 2^{120/24} = 2^5 = 32, N = \frac{N_0}{32}$,

Mass left $= \frac{\text{Original mass}}{32} = \frac{0.1}{32}$

$= .003125$ mg

Example 19: The half life of a radioactive material is 12.7 hr. What fraction of the original active material would become inactive in 63.5 hr?

(*a*) 1/32 (*b*) 1/23

(*c*) 31/32 (*d*) 23/32

Sol. t = 63.5 Hr, T = 12.7 Hr.

$\frac{N}{N_0} = \frac{1}{2^{t/T}} = \frac{1}{2^{(63.5/12.7)}} = \frac{1}{2^5} \Rightarrow \frac{N}{N_0} = \frac{1}{32}$

$\therefore$ Inactive fraction $= 1 - \frac{N}{N_0}$

$= 1 - \frac{1}{32} = \frac{31}{32}$

Example 20: A radio active sample contains 10^6 radioactive nucleus. It's half life is 20 sec. Number of remaining nucleus after 10 seconds.

(*a*) 7.09 (*b*) 7.09×10^5

(*c*) 79 (*d*) 709

Sol. $N = 10^6 \left(\frac{1}{2}\right)^{t/T}$

$$N = 10^6 \left(\frac{1}{2}\right)^{10/20}$$

$\Rightarrow N = 10^6/\sqrt{2} = 10^6/1.41 \Rightarrow N = 7.9 \times 10^5$

Example 21: The count rate of a radio active source at $t = 0$ was 1600 count/s and at $t = 8$ sec, it was 100 counts/s. The count rate (in counts) at $t = 6$ sec will be-

(*a*) 150 (*b*) 200

(*c*) 300 (*d*) 400

Sol. $t = \frac{T \log \frac{N_0}{N}}{\log 2} \Rightarrow 8 = \frac{T \log \frac{1600}{100}}{\log 2} = \frac{T \log 16}{\log 2}$

$\Rightarrow 8 = \frac{T \log_2 {}^4}{\log 2} \Rightarrow 8 = 4T \frac{\log 2}{\log 2}$

$\therefore T = 2\text{sec}$

$N = \frac{N_0}{2^{t/T}} = \frac{1600}{2^{6/2}} \Rightarrow N = 200$ counts/s

Example 22: 1 milligram radium has 2.68×10^{18} atoms. Its half-life is 1620 years. How many radium atoms will disintegrate from 1 milligram of pure radium in 3240 years ?

(*a*) 2.01×10^{18} (*b*) 0.75×10^{18}

(*c*) cannot be predicted (*d*) None

Sol. If the initial quantity of a radio-active element be N_0, then the quantity left after n half-lives is given by $N = N_0 \left(\frac{1}{2}\right)^n$

The half life of radium is 1620 years. The number of half-lives in 3240 years is

$n = \frac{3240}{1620} = 2$

$N = 1 \times \left(\frac{1}{2}\right)^2$

$(\therefore N_0 = 1\text{mg}),\ N = 1/4 = 0.25$ m

$\therefore$ mass of disintegrated radium

$= 1 - 0.25 = 0.75$ m

Number of atoms in it $= 0.75 \times (2.86 \times 10^{18})$

$= 2.01 \times 10^{18}$

Example 23: When ${}_{90}Th^{228}$ gets converted into ${}_{83}Bi^{212}$, then the number of α-and β-particles emitted will respectively be-

(*a*) 4 α, 7 β (*b*) 4 α, 1 β

(*c*) 8 α, 7 β (*d*) 4 β, 4 α

Sol. ${}_{90}Th^{228} = {}_{83}Bi^{212} + x\,({}_2He^4) + y(-1e^0)$

According to law of conservation of charge

$90 = 83 + 2x - y$

$2x - y = 7$

According to law of conservation of mass number

$228 = 212 + 4x$

$4x = 16,\ x = 4,\ 2 \times 4 - y = 7,\ y = 1$

Hence 4α and 1β will be emitted.

Example 24: A radioactive Nucleus decays as follows:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and charge number of A are 180 and 72 respectively, then for A_4 these values will respectively be:

(*a*) 172, 69 (*b*) 108, 252

(*c*) 108, 72 (*d*) None

Sol. ${}_{72}A^{180} = {}_2He^4 + {}_{70}A_1{}^{176}$

${}_{70}A_1{}^{176} = {}_{-1}e^0 + {}_{71}A_2{}^{176}$

${}_{71}A_2{}^{176} = {}_2He^4 + {}_{69}A_3{}^{172}$, ${}_{69}A_3^{172}$

$= \gamma + {}_{69}A_4^{172}$

Example 25: If the activity of radioactive sample drops to 1/32 of its initial value in 7.5 Hr. Half life will be-

(*a*) 3 Hr (*b*) 4.5 Hr

(*c*) 7.5 Hr (*d*) 1.5 Hr.

Sol. $A = A_0/32$, $t = 7.5$ Hr, $T = ?$

$\because \frac{A_0}{A} = 2^{t/T}$

$\Rightarrow 32 = 2^{t/T} \Rightarrow 2^5 = 2^{t/T} \Rightarrow 5 = t/T$

$\Rightarrow T = t/5 \Rightarrow T = 7.5/5 = 1.5$ Hr.

Example 26: In 420 days, the activity of a sample of polonium, Po fell to one lights of its initial value. Nuclear reaction is following ${}^a_b Po \rightarrow {}^d_c\alpha + {}^{206}_{82}Pb + {}^e_f\gamma$ Half life of polonium & value of a,b, c, d, e, f will be-

(*a*) 140, 210, 84, 4, 2, 0, 0

(*b*) 420, 208, 83, 2, 1, 0,0

(*c*) 420, 210, 84, 2, 4, 0, 0

(*d*) 140, 210, 84, 2, 4, 0, 0

Sol. $\frac{N_0}{N} = 2^{t/T},\ 8 = 2^{t/T} \Rightarrow 2^3 = 2^{t/T}$

$\therefore 3 = t/T \Rightarrow T = t/3 = 420/3 = 140$ days

Reaction will be, ${}^{210}_{84}Po \rightarrow {}^4_2\alpha + {}^{206}_{82}Pb + {}^0_0\gamma$

Topicwise Questions

Propertices of Nucleus

1. The stable nucleus that has a radius 1/3 that of O_5^{189} is
   (a) $_3Li^7$ (b) $_2He^4$
   (c) $_5B^{10}$ (d) $_6C^{12}$
2. Order of magnitude of density of uranium nucleus is ($m_p = 1.67 \times 10^{-27}$ kg)
   (a) 10^{20} kg / m^3 (b) 10^{17} kg / m^3
   (c) 10^{14} kg / m^3 (d) 10^{11} kg / m^3
3. Nucleus of an atom whose atomic mass is 24 can consists of
   (a) 11 electrons, 11 protons and 13 neutrons
   (b) 11 electrons, 13 protons and 11 neutrons
   (c) 11 protons and 13 neutrons
   (d) 11 protons and 13 electrons

Mass defect and Binding energy

4. Masses of nucleus, neutron and protons are M, m_n and m_p respectively. If nucleus has been divided into neutrons and protons, then
   (a) $M = (A - Z)\, m_n + Zm_p$
   (b) $M = Zm_n + (A - Z)\, m_p$
   (c) $M < (A - Z)\, m_n + Zm_p$
   (d) $M > (A - Z)m_n + Zm_p$
5. Equivalent energy of mass equal to 1 a.m.u. is
   (a) 931 KeV (b) 931 eV
   (c) 931 MeV (d) 9.31 MeV
6. Outside a nucleus
   (a) Neutron is stable
   (b) Proton and neutron both are stable
   (c) Neutron is unstable
   (d) Neither neutron nor proton is stable

Radio activity decay and displacement

7. A reaction between a proton and $_8O^{18}$ that produces $_9Fe^{18}$ must also liberate
   (a) $_0n^1$ (b) $_1e^0$
   (c) $_1n^0$ (d) $_0e^1$
8. In the nuclear reaction $_{85}X^{297} \rightarrow Y + 4\alpha$, Y is -
   (a) $_{76}Y^{287}$ (b) $_{77}Y^{285}$
   (c) $_{77}Y^{281}$ (d) $_{77}Y^{289}$
9. In the nuclear reaction $_6C^{11} \rightarrow _5B^{11} + \beta^+ + X$, what does X stand for
   (a) An electron (b) A proton
   (c) A neutron (d) A neutrino

Statistical law of radioactive decay

10. The half life of Thorium (Th^{232}) is 1.4×10^{10} years. Then the fraction of Thorium atoms decaying per year is very nearly-
    (a) 1×10^{-11} (b) 4.95×10^{-11}
    (c) 0.69×10^{-11} (d) 7.14×10^{-11}
11. The specific activity (per gm) of Radium is nearly:
    (a) 1 Bq (b) 1 Ci
    (c) 3.7×10^{10} Ci (d) 1 mCi
12. Beta rays emitted by a radioactive material are
    (a) Electromagnetic radiation
    (b) The electrons orbiting around the nucleus
    (c) Charged particles emitted by nucleus
    (d) Neutral particles
13. If T is the half life of a radioactive material, then the fraction that would remain after a time $\frac{T}{2}$ is
    (a) $\frac{1}{2}$ (b) $\frac{3}{4}$
    (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{2}-1}{\sqrt{2}}$
14. If the decay or disintegration constant of a radioactive substance is λ, then its half life and mean life are respectively
    (a) $\frac{1}{\lambda}$ and $\frac{\log_e 2}{\lambda}$
    (b) $\frac{\log_e 2}{\lambda}$ and $\frac{1}{\lambda}$
    (c) $\lambda \log_e 2$ and $\frac{1}{\lambda}$
    (d) $\frac{\lambda}{\log_e 2}$ and $\frac{1}{\lambda}$ ($\log_e 2$ can be written as ln 2)
15. A radioactive nucleus undergoes a series of decay according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

    If the mass number and atomic number of A are 180 and 72 respectively, then what are these number for A_4
    (a) 172 and 69 (b) 174 and 70
    (c) 176 and 69 (d) 176 and 70
16. If half life of a radioactive element is 3 hours, after 9 hours its activity becomes
    (a) 1/9 (b) 1/27
    (c) 1/6 (d) 1/8

Nuclear Fission and Fusion

17. The binding energy of deuteron $^{2}_{1}H$ is 1.112 MeV per nucleon and an α -particle $^{4}_{2}He$ has a binding energy of 7.047 MeV per nucleon. Then in the fusion reaction $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{4}_{2}He + Q$, the energy Q released is
(*a*) 1 MeV
(*b*) 11.9 MeV
(*c*) 23.8 MeV
(*d*) 931 MeV

18. Atomic reactor is based on
(*a*) controlled chain reaction
(*b*) uncontrolled chain reaction
(*c*) nuclear fission
(*d*) nuclear fusion

19. The nuclear reaction $^{2}H + ^{2}H \rightarrow ^{4}He$ (mass of deuteron = 2.0141 a.m.u. and mass of He = 4.0024 a.m.u.) is
(*a*) Fusion reaction releasing 24 MeV energy
(*b*) Fusion reaction absorbing 24 MeV energy
(*c*) Fission reaction releasing 0.0258 MeV energy
(*d*) Fission reaction absorbing 0.0258 MeV energy

Learning Plus

1. The mass number of a nucleus is
(*a*) always less than its atomic number
(*b*) always more than its atomic number
(*c*) equal to its atomic number
(*d*) sometimes more than and sometimes equal to its atomic number

2. As the mass number A increases, the binding energy per nucleon in a nucleus
(*a*) increases
(*b*) decreases
(*c*) remains the same
(*d*) varies in a way that depends on the actual value of A.

3. The atomic weight of boron is 10.81 g/mole and it has two isotopes $^{10}_{5}B$ and $^{11}_{5}B$. The ratio (by number) of $^{10}_{5}B$: $^{11}_{5}B$ in nature would be :
(*a*) 19 : 81 (*b*) 10 : 11
(*c*) 15 : 16 (*d*) 81 : 19

4. Let u be denote one atomic mass unit. One atom of an element of mass number A has mass exactly equal to Au.
(*a*) for any value of A
(*b*) only for A = 1
(*c*) only for A = 12
(*d*) for any value of A provided the atom is stable

5. Consider the nuclear reaction $X^{200} \rightarrow A^{110} + B^{90}$ If the binding energy per nucleon for X, A and B is 7.4 MeV, 8.2 MeV and 8.2 MeV respectively, what is the energy released?
(*a*) 200 MeV (*b*) 160 MeV
(*c*) 110 MeV (*d*) 90 MeV

6. The binding energies of the nuclei of $^{4}_{2}He$, $^{7}_{3}Li$, $^{12}_{6}C$ & $^{14}_{7}N$ are 28, 52, 90, 98 MeV respectively. Which of these is most stable.
(*a*) $^{4}_{2}He$, (*b*) $^{7}_{3}Li$,
(*c*) $^{12}_{6}C$ (*d*) $^{14}_{7}N$

7. The rest mass of the deuteron, $^{2}_{1}H$, is equivalent to an energy of 1876 MeV, the rest mass of a proton is equivalent to 939 MeV and that of a neutron to 940 MeV. A deuteron may disintegrate to a proton and a neutron if it:
(*a*) emits a g-ray photon of energy 2 MeV
(*b*) captures a g-ray photon of energy 2 MeV
(*c*) emits a g-ray photon of energy 3 Me V
(*d*) captures a g-ray photon of energy 3 MeV

8. In an a-decay the Kinetic energy of a particle is 48 MeV and Q-value of the reaction is 50 MeV. The mass number of the parent nucleus is : (Assume that daughter nucleus is in ground state)
(*a*) 96 (*b*) 100
(*c*) 104 (*d*) none of these

9. A free neutron decays into a proton, an electron and:
(*a*) A neutrino (*b*) An antineutrino
(*c*) An a-particle (*d*) A b-particle

10. Nuclei X decay into nuclei Y by emitting α particles. Energies of a particle are found to be only 1 MeV & 1.4 MeV. Disregarding the recoil of nuclei Y. The energy of γ photon emitted will be
(*a*) 0.8 MeV (*b*) 1.4 MeV
(*c*) 1 MeV (*d*) 0.4 MeV

11. 10 grams of ^{57}Co kept in an open container beta-decays with a half-life of 270 days. The weight of the material inside the container after 540 days will be very nearly -
(*a*) 10 g (*b*) 7.5 g
(*c*) 5 g (*d*) 2.5 g

12. The radioactive sources A and B of half lives of 2 hr and 4 hr respectively, initially contain the same number of radioactive atoms. At the end of 2 hours, their rates of disintegration are in the ratio :
(*a*) 4 : 1 (*b*) 2 : 1
(*c*) $\sqrt{2}$:1 (*d*) 1 : 1

13. Activity of a radioactive substance is R_1 at time t_1 and R_2 at time $t_2 (t_2 > t_1)$. Then the ratio $\frac{R_2}{R_1}$ is :

(a) $\frac{t_2}{t_1}$ (b) $e^{-\lambda(t_1+t_2)}$

(c) $e\left(\frac{t_1 - t_2}{\lambda}\right)$ (d) $e^{\lambda(t_1-t_2)}$

14. The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 $(t_2 > t_1)$. Its mean life is T.

(a) $A_1t_1 = A_2t_2$ (b) $\frac{A_1 - A_2}{t_2 - t_1}$ = constant

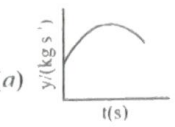
(c) $A_2 = A_1e^{(t_1-t_2)/T}$ (d) $A_2 = A_1e^{(t_1/Tt_2)}$

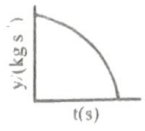
15. A fraction f_1 of a radioactive sample decays in one mean life, and a fraction f_2 decays in one half-life.
(a) $f_1 > f_2$
(b) $f_1 < f_2$
(c) $f_1 = f_2$
(d) May be (1), (2) or (3) depending on the values of the mean life and half life.

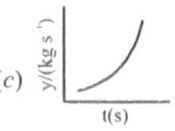
16. A radioactive nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2. The effective decay constant of the nuclide is λ, then:
(a) $\lambda = \lambda_1 + \lambda_2$ (b) $\lambda = 1/2(\lambda_1 + \lambda_1)$

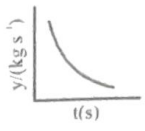
(c) $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ (d) $\lambda = \sqrt{\lambda_1\lambda_2}$

17. The radioactive nucleus of an element X decays to a stable nucleus of element Y. a graph of the rate of formation of Y against time would look like

(a) 

(b) 

(c) 

(d) 

18. Assuming that about 20 MeV of energy is released per fusion reaction, ${}_1H^2 + {}_1H^3 \rightarrow {}_0n^1 + {}_2He^4$, the mass of ${}_1H^2$ consumed per day in a future fusion reactor of power 1 MW would be approximately
(a) 0.1 gm (b) 0.01 gm
(c) 1 gm (d) 10 gm

19. Fusion reaction is possible at high temperature because
(a) atoms are ionised at high temperature
(b) molecules break-up at high temperature
(c) nuclei break-up at high temperature
(d) kinetic energy is high enough to overcome repulsion between nuclei.

20. In a uranium reactor whose thermal power is P = 100 MW, if the average number of neutrons liberated in each nuclear splitting is 2.5. Each splitting is assumed to release an energy E = 200 MeV. The number of neutrons generated per unit time is -
(a) 4×10^{18} s^{-1} (b) 8×10^{23} s^{-1}

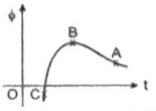
(c) 8×10^{19} s^{-1} (d) $\frac{125}{16} \times 10^{18}$ s^{-1}

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/STATEMENT/MATCHING

1. The heavier stable nuclei tend to have larger N/Z ratio because -
(a) a neutron is heavier than a proton
(b) a neutron is an unstable particle
(c) a neutron does not exert electric repulsion
(d) Coulomb forces have longer range compared to nuclear forces

2. The graph shown by the side shows the variation of potential energy ϕ of proton with its distance 'r' from a fixed sodium nucleus, as it approaches the nucleus, placed at origin O. Then the portion.



(a) AB indicates nuclear repulsion
(b) AB indicates electrostatic repulsion
(c) BC indicates nuclear attraction
(d) BC represents electrostatic interation

3. Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}^{20}_{10}Ne$ nucleus and M_2 the mass of a ${}^{40}_{20}Ca$ nucleus. Then
(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$
(c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

4. When a nucleus with atomic number Z and mass number A undergoes a radioactive decay process:
   (*a*) both Z and A will decrease, if the process is α decay
   (*b*) Z will decrease but A will not change, if the process is β^- decay
   (*c*) Z will decrease but A will not change, if the process is β^+ decay
   (*d*) Z and A will remain unchanged, if the process is γ decay.

5. A nitrogen nucleus ${}_7N^{14}$ absorbs a neutron and can transform into lithium nucleus ${}_3Li^7$ under suitable conditions, after emitting
   (*a*) 4 protons and 4 neutrons
   (*b*) 5 protons and 1 negative beta particle
   (*c*) 2 alpha particles and 2 gamma particles
   (*d*) 1 alpha particle, 4 protons and 2 negative beta particles.

6. In β-decay, the Q-value of the process is E. Then
   (*a*) K.E. of a β-particle cannot exceed E.
   (*b*) K.E. of anti neutrino emitted lies between Zero and E.
   (*c*) N/Z ratio of the nucleus is altered.
   (*d*) Mass number (A) of the nucleus is altered.

7. A U^{238} sample of mass 1.0 g emits alpha particles at the rate 1.24×10^4 particles per second. ($N_A = 6.023 \times 10^{23}$)
   (*a*) The half life of this nuclide is 4.5×10^9 years
   (*b*) The half life of this nuclide is 9×10^9 years
   (*c*) The activity of the prepared sample is 2.48×10^4 particles/sec
   (*d*) The activity of the prepared sample is 1.24×10^4 particles/sec.

Comprehension – 1 (8 to 10)

Consider the following nuclear decay : (initially ${}^{236}U_{92}$ is at rest)

$$^{236}_{92}U \longrightarrow {}^{232}_{90}Th + X$$

8. Regarding this nuclear decay select the correct statement:
   (*a*) The nucleus X may be at rest.
   (*b*) The ${}^{232}_{90}Th$ nucleus may be in excited state.
   (*c*) The X may have kinetic energy but ${}^{232}_{90}Th$ will be at rest
   (*d*) The Q value is Δmc^2 where Δm is mass difference of ($^{236}_{92}U$ and $^{232}_{90}Th$) and c is speed of light.

9. If the uranium nucleus is at rest before its decay, which one of the following statement is true concerning the final nuclei?
   (*a*) They have equal kinetic energies, but the thorium nucleus has much more momentum.
   (*b*) They have equal kinetic energies and momenta of equal magnitudes.
   (*c*) The have momenta of equal magnitudes, but the thorium nucleus has much more kinetic energy.
   (*d*) They have momentum of equal magnitudes, but X has much more kinetic energy.

10. Following atomic masses and conversion factor are provided
    $^{236}_{92}U = 236.045\ 562$ u ; $^{232}_{90}Th = 232.038054$ u ;
    $^{1}_{0}n = 1.008665$ u ; $^{1}_{1}p = 1.007277$ u ; $^{4}_{2}He = 4.002603$ u and 1 u = 1.5×10^{-10} J. The amount of energy released in this decay is equal to:
    (*a*) 3.5×10^{-8} J (*b*) 4.6×10^{-12} J
    (*c*) 6.0×10^{-10} J (*d*) 7.4×10^{-13} J

11. Match the Column-I of properties with Column-II of reactions:

Column-I	Column-II
(*a*) Mass of product formed is less than the original mass of the system in	(p) α-decay
(*b*) Binding energy per nucleon increase in	(q) β-decay
(*c*) Mass number is conserved in	(r) Nuclear fission
(*d*) Charge number is conserved in	(s) Nuclear fusion

12. In Column-I, consider each process just before and just after it occurs. Initial system is isolated from all other bodies. Consider all product particles (even those having rest mass zero) in the system. Match the system in Column-I with the result they produce in Column-II.

Column-I	Column-II
(*a*) Spontaneous radioactive decay of an uranium nucleus initially at rest as given by reaction $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + ...$	(p) Number of protons is increased
(*b*) Fusion reaction of two hydrogen nuclei as given by reaction $^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + ...$	(q) Momentum is conserved
(*c*) Fission of U^{235} nucleus initiated by a thermal neutron as given by reaction $^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n + ...$	(r) Mass is converted to energy or vice versa
(*d*) β^- decay (negative beta decay)	(s) Charge is conserved

NUMERICAL VALUE BASED

13. When the voltage applied to an X-ray tube is increased from V_1 = 15.5 KV to V_2 = 31 KV, the wavelength interval between the K_α line and the short wavelength cut-off of the continuous X-ray spectrum increases by a factor of 1.3. Find the atomic number of the element of the target. (**Take:** hc = 1240 eV nm and R = 1×10^7 /m)

14. In a sample initially there are equal number of atoms of two radioactive isotopes A and B. 3 days later the number of atoms of A is twice that of B. Half life of B is 1.5 days. What is half life of isotope A ? (in days)

15. Find the reaction energy (in KeV) of the following reaction. Assume that all the products were produced in ground state. Neglect the mass of neutrino.

Take energy of 1 amu = 931.5 MeV.

$^{64}_{29}Cu \rightarrow ^{64}_{28}Ni + ^{0}_{+1}e + \nu$

M_{Cu} = 63.9297642 amu

M_{Ni} = 63.9279660 amu

M_e = 0.0005486 amu

16. The nuclear radius of a certain nucleus is 7.2 fm and it has charge of 1.28×10^{-17} C. Find the number of neutrons inside the nucleus.

17. Water kept in a porus pot evaporates through the walls of pot. Rate of evaporation is proportional to volume of water. When water is kept in the pot 75% water get evaporated in 16 hrs. A suction mechanism attached in the pot sucks water at the rate which is also proportional to volume of water in the pot. Without evaporations, half of the water kept in pot is sucked in 24 hr. The pot is filled with 16 kg of water, with evaporation and suction acting simultaneously, what amount of water (in kg) will be left is pot after one day.

18. The reaction

$^{7}_{3}Li + ^{1}_{1}H \rightarrow ^{7}_{4}Be + ^{1}_{0}n$

is endothermic. Assuming that Li nuclei is free and at rest. What is the minimum kinetic energy (in keV) of incident proton so that this reaction occurs? Take Q value of this reaction as –1645 keV.

19. Proton separation energy is the amount of energy we must supply to remove a proton from a nucleus. Find the proton separation energy of $^{4}_{2}He$ (in MeV). The mass of

$m\left(^{1}_{1}H\right)$ = 1.007825 amu

$m\left(^{3}_{1}H\right)$ = 3.016049 amu

$m\left(^{4}_{2}He\right)$ = 4.002603 amu

Round off to nearest integer.

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be:

[JEE Main-2016]

(*a*) 1 : 16 (*b*) 4 : 1

(*c*) 1 : 4 (*d*) 5 : 4

2. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by:

[JEE Main-2017]

(*a*) $t = T \log(1.3)$ (*b*) $t = \frac{T}{\log(1.3)}$

(*c*) $t = T\frac{\log 2}{\log 1.3}$ (*d*) $t = T\frac{\log 1.3}{\log 2}$

3. At a given instant, say t = 0, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half-life of A is $\ell n2$, the half-life of B is:

[JEE Main-2019 (January)]

(*a*) $4\ell n2$ (*b*) $\frac{\ell n2}{2}$

(*c*) $\frac{\ell n2}{4}$ (*d*) $2\ell n2$

4. An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is:

[JEE Main-2019 (January)]

(*a*) 2 m (*b*) 3.5 m

(*c*) 1.5 m (*d*) 4 m

5. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci 3.7×10^{10} decays/s), has twice the number of nuclei as another sample of different radioactive material B which has an activity of 20 mCi. The correct choices for half lives of A and B would then be respectively: [JEE Main-2019 (January)]
(a) 5 days and 10 days (b) 10 days and 40 days
(c) 20 days and 5 days (d) 20 days and 10 days

6. In a radioactive decay chain, the initital nucleus is $^{232}_{90}Th$. At the end there are 6 α-particles and 4 β-particles with are emitted. If the end nucleus is $^{A}_{Z}X$, A and Z are given by : [JEE Main-2019 (January)]
(a) A = 208 ; Z = 80 (b) A = 202 ; Z = 80
(c) A = 208 ; Z = 82 (d) A = 200 ; Z = 81

7. Consider the nuclear fission
$Ne^{20} \rightarrow 2He^4 + C^{12}$.
Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV, and 7.86 MeV, identify the correct statement:
[JEE Main-2019 (January)]
(a) energy of 12.4 MeV will be supplied
(b) 8.3 MeV energy will be released
(c) energy of 3.6 MeV will be released
(d) energy of 11.9 MeV has to be supplied

8. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 count rate observed, as counts per second, and at t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to: [JEE Main-2019 (January)]
(a) 200 (b) 150
(c) 400 (d) 360

9. Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be:
[JEE Main-2019 (April)]
(a) $1 / 4\lambda$ (b) $1 / \lambda$
(c) $1 / 2\lambda$ (d) $2 / \lambda$

10. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to:
(a) 1 (b) 2
(c) 0.1 (d) 5

11. A nucleus A, with a finite de-broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C are respectively:
[JEE Main-2019 (April)]
(a) $2\lambda_A, \lambda_A$ (b) $\lambda_A, 2\lambda_A$
(c) $\lambda_A, \frac{\lambda_A}{2}$ (d) $\frac{\lambda_A}{2}, \lambda_A$

12. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be:
[JEE Main-2019 (April)]
(a) 9 : 8 (b) 1 : 8
(c) 8 : 1 (d) 3 : 8

13. Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time:
[JEE Main-2019 (April)]
(a) $\frac{11}{10\lambda}$ (b) $\frac{1}{9\lambda}$
(c) $\frac{1}{10\lambda}$ (d) $\frac{1}{11\lambda}$

14. The activity of a radioactive sample falls from $700 s^{-1}$ to $500 s^{-1}$ in 30 minutes. Its half life is close to:
[JEE Main-2020 (January)]
(a) 62 min (b) 66 min
(c) 72 min (d) 52 min

15. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, $N = 6.023 \times 10^{26}$ per kilo mole and 1 eV $= 1.6 \times 10^{-19}$ J. The power output of the reactor is close to [JEE Main-2020 (September)]
(a) 125 MW (b) 35 MW
(c) 60 MW (d) 54 MW

16. The radius R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15})A^{1/3}$ m. It follows that the mass density of a nucleus is of the order of ($M_{prot.} \cong M_{neut.} = 1.67 \times 10^{-27}$ kg)
[JEE Main-2020 (September)]
(a) 10^{24} kg m^{-3} (b) 10^{10} kg m^{-3}
(c) 10^{17} kg m^{-3} (d) 10^{3} kg m^{-3}

17. In a radioactive material, fraction of active material remaining after time t is 9/16. The fraction that was remaining after t/2 is: [JEE Main-2020 (September)]
(a) $\frac{3}{4}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) $\frac{7}{8}$

18. Find the binding energy per nucleon for $^{120}_{50}Sn$. Mass of proton $m_p = 1.00783$ U, mass of neutron $m_n = 1.00867$ U and mass of tin nucleus $m_{Sn} = 119.902199$ U.
(Take 1U = 931 MeV) [JEE Main-2020 (September)]
(a) 9.0 MeV (b) 8.5 MeV
(c) 8.0 MeV (d) 7.5 MeV

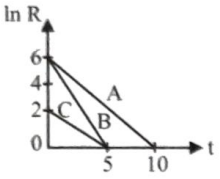
19. A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100 s. The effective half life of the nucleus is close to [JEE Main-2020 (September)]

(a) 12 sec (b) 9 sec

(c) 55 sec (d) 6 sec

20. Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives $T_{\frac{1}{2}}(A) : T_{\frac{1}{2}}(B) : T_{\frac{1}{2}}(C)$ are in the ratio

[JEE Main-2020 (September)]



(a) 2 : 1 : 3 (b) 4 : 3 : 1

(c) 2 : 1 : 1 (d) 3 : 2 : 1

21. Given the masses of various atomic particles $m_p = 1.0072$ u, $m_n = 1.0087$ u, $m_e = 0.000548$ u, $m_{\bar{\nu}} = 0$, $m_d = 2.0141$u, where p = proton, n = neutron, e = electron, $\bar{\nu}$ = antineutrino and d = deuteron. Which of the following process is allowed by momentum and energy conservation? [JEE Main-2020 (September)]

(a) $n + n \rightarrow$ deuterium atom (electron bound to the nucleus)

(b) $n + p \rightarrow + \gamma$

(c) $p \rightarrow n + e^+ + \bar{\nu}$

(d) $e^+ + e^- \rightarrow \gamma$

22. You are given that Mass of $^7_3Li = 7.0160$u,

Mass of $^4_2He = 4.0026$u

ans Mass of $^1_1H = 1.0079$u.

When 20 g of ^7_3Li is coverted into ^4_2He by proton capture, the energy liberated. (in kWh), is

[Mass of nucleon = 1 GeV/C^2]

[JEE Main-2020 (September)]

(a) 1.33×10^6 (b) 4.5×10^5

(c) 8×10^6 (d) 6.82×10^5

23. A proton and an α -particle, having kinetic energies K_p and K_α respectively, enter into a magnetic field at right angles.

The ratio of the radii of trajectory of proton to that of α -particle is \$2: 1\$. The ratio of $K_p : K_\alpha$ is:

[JEE Main-2021 (March)]

(a) 8 : 1 (b) 4 : 1

(c) 1 : 8 (d) 1 : 4

24. The decay of a proton to neutron is:

[JEE Main-2021 (March)]

(a) Always possible as it is associated only with β^+ decay

(b) Possible only inside the nucleus

(c) Not possible as proton mass is less than the neutron mass

(d) Not possible but neutron to proton conversation is possible

25. The speed of electrons in a scanning electron microscope is 1×10^7 ms^{-1}. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of: [JEE Main-2021 (March)]

(a) $\frac{1}{1837}$ (b) 1837

(c) $\sqrt{1837}$ (d) $\frac{1}{\sqrt{1837}}$

JEE-ADVANCED PREVIOUS YEAR'S

1. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Brogle wavelength (in units of fm) of the proton at its start is:

(take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg, $h/e = 4.2 \times 10^{-15}$ J.s/C ; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm = 10^{-15}m) [IIT JEE-2012]

Comprehension – 1 (2 and 3)

The β^- decay process, discovered around 1900, is basically the decay of a neutron (n), In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. therefore, considering the decay of a neutron as a tro-body dcay process, it was predicted theoretically that thekinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e. $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the lectron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

2. What is the maximum energy of the anti-neutrino?

[IIT JEE-2012]

(a) Zero

(b) Much less than 0.8×10^6 eV

(c) Nearly 0.8×10^6 eV

(d) Much larger than 0.8×10^6 eV

3. If the anti-neutrino had a mass of $3eV/c^2$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(a) $0 \le K \le 0.8 \times 10^6$ eV

(b) $3.0 \text{ eV} \le K \le 0.8 \times 10^6$ eV

(c) $3.0 \text{ eV} \le K < 0.8 \times 10^6$ eV

(d) $0 \le K < 0.8 \times 10^6$ eV

4. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that ln 2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80s after preparation of the sample is:

[JEE Advanced-2013]

5. Match List-I of the nuclear processes with List-II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists : [JEE Advanced-2013]

List-I	List-II
P. Alpha decay	1. $^{15}_{8}O \rightarrow ^{15}_{7}N + ...$
Q. β^- decay	2. $^{238}_{92}U \rightarrow ^{234}_{90}Th + ...$
R. Fission	3. $^{185}_{83}Bi \rightarrow ^{184}_{82}Pb + ...$
S. Proton emission	4. $^{239}_{94}Pu \rightarrow ^{140}_{57}La + ...$

Codes:

	P	Q	R	S
(a)	4	2	1	3
(b)	1	3	2	4
(c)	2	1	4	3
(d)	4	3	2	1

Comprehension – 2 (6 and 7)

The mass of a nucleus $^{A}_{Z}X$ is less that the sum of the masses of (A – Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below:

[JEE Advanced-2013]

$^{1}_{1}H$	1.007825u	$^{2}_{1}H$	2.014102u	$^{3}_{1}H$	3.016050u	$^{4}_{2}He$	4.002603u
$^{6}_{3}Li$	6.015123u	$^{7}_{3}Li$	7.016004u	$^{70}_{30}Zn$	69.925325u	$^{82}_{34}Se$	81.916709u
$^{150}_{64}Gd$	151.919803u	$^{206}_{82}Pb$	205.974455u	$^{209}_{83}Bi$	208.980388u	$^{210}_{84}Po$	209.982876u

6. The correct statement is :

(a) The nucleus $^{8}_{3}Li$ can emit an alpha particle

(b) The nucleus $^{210}_{84}P_0$ can emit a proton

(c) Deuteron and alpha particle can undergo complete fusion.

(d) The nuclei $^{70}_{30}Zn$ and $^{82}_{34}Se$ can undergo complete fusion.

7. The kinetic energy (in keV) of the alpha particle, when the nucleus $^{210}_{84}P_0$ at rest undergoes alpha decay, is:

(a) 5319 (b) 5422

(c) 5707 (d) 5818

8. The isotope $^{12}_{5}B$ having a mass 12.014 u undergoes β-decay to $^{12}_{6}C$. $^{12}_{6}C$ has an excited state of the nucleus ($^{12}_{6}C^0$) at 4.041 MeV above its ground state. If $^{12}_{5}B$ decays to $^{12}_{6}C$, the (1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum). [JEE Advanced-2016]

9. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by $E=\frac{3}{5}=\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$. The measured masses of the neutron, $^{1}_{1}H$, $^{15}_{7}N$ and $^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the $^{15}_{7}N$, and $^{15}_{8}O$ nuclei are same, 1 u = 931.5 MeV/c^2 (c is the speed of light) and $e^2 / (4\pi\varepsilon_0)$ = 1.44 MeV fm. Assuming that the difference between the binding energies of $^{15}_{7}N$ and $^{15}_{8}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1 fm = 10^{-15} m)

[JEE Advanced-2016]

(a) 2.85 fm (b) 3.03 fm

(c) 3.42 fm (d) 3.80 fm

10. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

[JEE Advanced-2016]

(a) 64 (b) 90

(c) 108 (d) 120

11. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours., 2.5 ml of blood is drawn from the person's body and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use $e^x \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$).

[*JEE Advanced-2017*]

12. In a radioactive decay chain, $^{232}_{90}Th$ nucleus decays to $^{212}_{82}Pb$ nucleus. Let N_α and N_β be the number of a and b particles, respectively, emitted in this decay process. Which of the following statements is (are) true?

[*JEE Advanced-2018*]

(*a*) $N_\alpha = 5$ (*b*) $N_\alpha = 6$
(*c*) $N_\beta = 2$ (*d*) $N_\beta = 4$

13. In a radioactive sample, $^{40}_{19}K$ nuclei either decay into stable $^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable $^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei are produced by the $^{40}_{19}K$ nuclei only. In time $t \times 10^9$ years. if the ratio of the sum of stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei to the radioactive $^{40}_{19}K$ nuclei is 99, the value of t will be : [Given $\ln 10 = 2.3$]

[*JEE Advanced-2019*]

(*a*) 9.2 (*b*) 1.15
(*c*) 4.6 (*d*) 2.3

14. Suppose a $^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α-decay to a $^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. $^{222}_{86}Rn$ nucleus then goes to its ground state by γ-decay. The energy of the emitted γ-photon is_____ keV,

[Given : atomic mass of $^{226}_{88}Ra = 226.005u$, atomic mass of $^{222}_{86}Rn = 222.00u$, atomic mass of a particle = 4.000u, $1u = 931\ MeV/c^2$, c is speed of the light]

[*JEE Advanced-2019*]

15. A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with probability of 60% and beta-decay with probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha decays of Q in the first one hour is [*JEE Advanced-2021*]

(*a*) 50 (*b*) 75
(*c*) 350 (*d*) 525

16. A heavy nucleus N, at rest, undergoes fission $N \to P + Q$, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P, M_Q and M_N are the masses of P, Q and N, respectively. E_P And E_Q are the kinetic energies of P and Q, respectively. The speed of P and Q are v_P and v_Q, respectively. If c is the speed of light, which of the following statement(s) is (are) correct?

[*JEE Advanced-2021*]

(*a*) $E_P + E_Q = c^2\delta$

(*b*) $E_P = \left(\frac{M_P}{M_P + M_Q}\right)c^2\delta$

(*c*) $\frac{v_P}{v_Q} = \frac{M_Q}{M_P}$

(*d*) The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $\mu = \frac{M_P M_Q}{(M_P + M_Q)}$

17. Which of the following statement(s) is(are) correct about the spectrum of hydrogen atom? [*JEE Advanced-2021*]

(*a*) The ratio of the longest wavelength to the shortest wavelength in Balmer series is 9 / 5

(*b*) There is an overlap between the wavelength ranges of Balmer and Paschen series.

(*c*) The wavelengths of Lyman series are given by $\left(1 + \frac{1}{m^2}\right)\lambda_0$, where λ_0 is the shortest wavelength of Lyman series and m is an integer

(*d*) The wavelength ranges of Lyman and Balmer series do not overlap

ANSWER KEY

Topicwise Questions

1. (*a*) **2.** (*b*) **3.** (*c*) **4.** (*c*) **5.** (*c*) **6.** (*c*) **7.** (*a*) **8.** (*c*) **9.** (*d*) **10.** (*b*)
11. (*b*) **12.** (*c*) **13.** (*c*) **14.** (*b*) **15.** (*a*) **16.** (*d*) **17.** (*c*) **18.** (*a*) **19.** (*a*)

Learning Plus

1. (*d*) **2.** (*d*) **3.** (*a*) **4.** (*c*) **5.** (*b*) **6.** (*c*) **7.** (*d*) **8.** (*b*) **9.** (*b*) **10.** (*d*)
11. (*a*) **12.** (*c*) **13.** (*d*) **14.** (*c*) **15.** (*a*) **16.** (*a*) **17.** (*d*) **18.** (*a*) **19.** (*d*) **20.** (*d*)

Advanced Level Multiconcept Questions

1. (*c, d*) **2.** (*b, c*) **3.** (*c*) **4.** (*a, b, d*) **5.** (*a, c, d*) **6.** (*a, b, c*) **7.** (*a, d*) **8.** (*b*) **9.** (*d*) **10.** (*d*)
11. (*a*) p, q, r, s (*b*) p, q, r, s (*c*) p, q, r, s (*d*) p, q, r, s **12.** (*a*) q, r, s, (*b*) q, r, s (*c*) q, r, s (*d*) p, q, r, s **13.** [0026]
14. [0003] **15.** [0653] **16.** [0136] **17.** [0001] **18.** [1880] **19.** [0020]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*d*) **2.** (*d*) **3.** (*c*) **4.** (*d*) **5.** (*c*) **6.** (*c*) **7.** (*d*) **8.** (*a*) **9.** (*c*) **10.** (*a*)
11. (*d*) **12.** (*a*) **13.** (*b*) **14.** (*a*) **15.** (*c*) **16.** (*c*) **17.** (*a*) **18.** (*b*) **19.** (*b*) **20.** (*a*)
21. (*b*) **22.** (*a*) **23.** (*b*) **24.** (*b*) **25.** (*b*)

JEE-ADVANCED PREVIOUS YEAR'S

t.me/realprayasians2

1. [7] **2.** (*c*) **3.** (*d*) **4.** (*d*) **5.** (*c*) **6.** (*c*) **7.** (*a*) **8.** [9] **9.** (*c*) **10.** (*c*)
11. [5] **12.** (*a, c*) **13.** (*a*) **14.** (135.00) **15.** (*d*) **16.** (*a, c, d*) **17.** (*a, d*)

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SEMICONDUCTOR AND DEVICES

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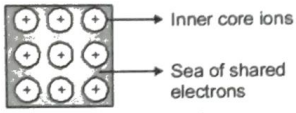
INTRODUCTION

The energy of an electron in an isolated atom is decided by the orbit in which it is revolving. In a solid, these electron energies would be different because of the presence of many atoms placed close to each other. This would make the nature of electron motion in a solid different from that in an isolated atom.

In a solid, the atoms are held together closely in a well defined 3-dimensional array or lattice by strong forces as the separation between them is quite small, nearly twice the radii of the outermost electronic orbits. In conducting metals like Al, Cu etc, the outer orbit electrons are shared by all the atoms and the metallic crystal can be visualised as positive inner cores embedded in a regular fashion in a sea of shared electrons.



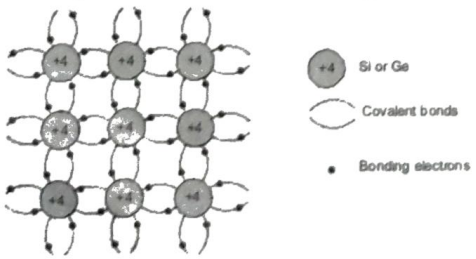
Such electrons are obviously free to travel.

For insulators the outermost electron remains bound to their parent atoms almost at all temperatures and very high energies are required for these electrons to breakaway. Therefore conductivity is low or resistivity is high.

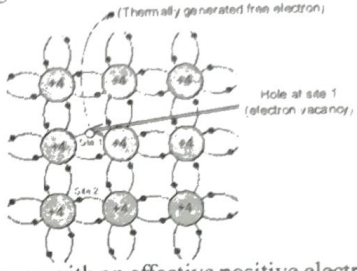
The bonding in semiconductors is such that at low temperatures the conductivity of most of the semiconductor is low and comparable to those of insulators but under thermal excitation at moderate temperatures bonded electrons can freely give moderate values of conductivity.

VALENCE-BOND DESCRIPTION OF INTRINSIC SEMICONDUCTOR

The term intrinsic semiconductor refers to pure materials. Si and Ge have four valence electrons. In its crystalline structure, every Si or Ge atom tends to share one of its four valence electrons with each of its four nearest neighbours atoms. These shared electron pairs are referred to as forming a covalent bond.



As the temperature increases, more thermal energy becomes available to these electrons and some of these electrons may break away, becoming free electrons contributing to conduction.



This vacancy with an effective positive electronic charge is called hole. Creation or liberation one free electron by the thermal energy has created one hole. Therefore intrinsic charge carriers such that

$$n_e = n_h = n_i$$

Semiconductors posses the unique property in which, a part from electrons, the holes also move. These holes move towards negative potential, giving the hole current I_h and the free electron moves completely independently and gives rise to an electron current (I_e) under an applied electric field. The total current is the sum of the electron current I_e due to the thermally generated conduction electrons and the hole current I_h.

VALENCE BAND DESCRIPTION OF EXTRINSIC SEMICONDUCTOR

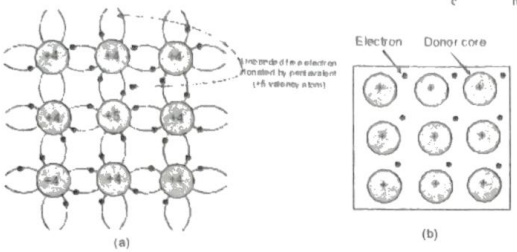
The intrinsic semiconductor materials, charge carries are always thermally generated can never have either predominant hole or electron for conduction, this limits the usefulness of such materials.

To overcome this problem, a small amount say ~ 1 part per million (PPM) of suitable impurity is added to the pure semiconductor. Such semiconductor are known as extrinsic semiconductors, the impurity atoms are called dopants.

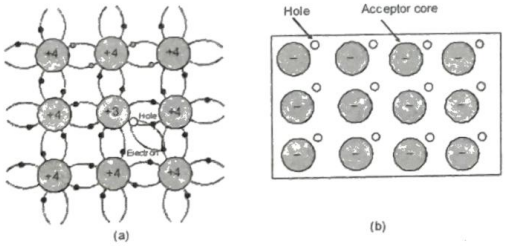
We choose the dopant element from nearby fifteen or thirteenth group, so that the size of the dopant atoms is nearly the same as that of Si and Ge. Interestingly, the pentavalent and trivalent dopants in Si or Ge give two entirely different types of semiconductors.

(a) n-type semiconductor: If we dope Si or Ge (valency 4) with a pentavalent (valency 5) four of its electrons bond with the four silicon neighbours while the fifth electron is free to move. Thus, the pentavalent dopant is donating

one extra electron for conduction and hence is known as n-type semiconductors. For such materials $n_e >> n_h$. The total number of conduction electrons n_e is due to combined contribution of the donors as well as the thermally generated carries while the holes n_h continue to be only due to the thermal process. These materials are known as n-type semiconductors. For such materials $n_e >> n_h$.



(b) p-type semiconductor: When Si or Ge (valency 4) is doped with group thirteen impurity, trivalent has one outer electron less than Si or Ge. The dopant has one outer electron less than Si or Ge and therefore, this atom can from bonds from three sides with Si and fails to form bond on one side. Some of the outer bound electrons in the neighbourhood have a tendency to slide into this vacant bond as shown in figure, leaving a vacancy or hole at its own site. This hole is available for conduction. These holes are in addition to the thermally generated holes. Thus, for such a material, the holes are the majority carriers. For *p*-type material $n_h >> n_e$ or $p >> n$.



The neighbourhood, from which the free electron (with charge–q) has come out leaves a vacancy with an effective charge ($+q$) to slide into this vacant bond as shown in figure, leaving a vacancy or hole at its own site. It is obvious that these holes are in addition to the thermally generated holes whiles the source of conduction electrons is only thermal generations. The holes are the majority charge carriers and electrons are minority carriers for such a material.

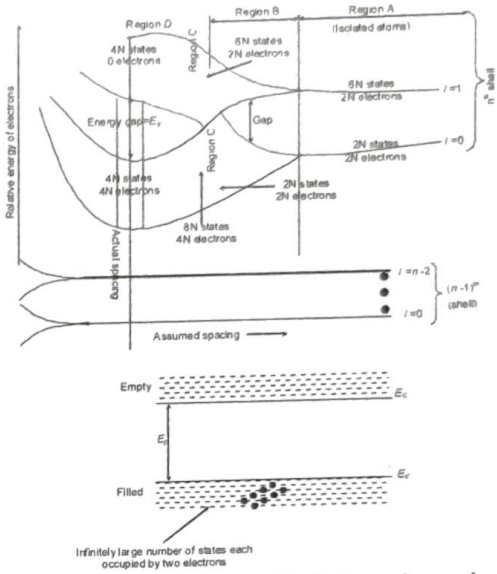
$$n_h >> n_e$$

ENERGY BAND DESCRIPTION OF SOLIDS-METALS, INSULATORS AND SEMICONDUCTORS

In the bond description of solids the bonding electrons and holes have been considered as highly localised, which is not exact due to strong overlapping of the orbitals, alternative approach is termed as energy-band description of solids. The electron energies in solids, in view of strong overlap of different atomic orbitals, will be more like an energy band instead of discrete energy levels of single isolated atoms.

In crystal, the atoms are close to each other and therefore the electrons interact with each other and also with the neighbouring atomic cores. The overlap or interaction will be more felt by the electrons in the outermost orbit while the inner orbit or core electron energies may remain unaffected.

When these atoms start coming nearer to each other to form a solid. The total number of available energy states 8N has been re-apportioned between the two bonds (4N states each in the lower and upper energy bands). Here the significant point is that there are exactly as many states in the lower band (4N) as there are available valence electrons from the atom (4N)



Therefore, this band (called the valence band) is completely filled while the upper band is completely empty. The upper band is called the conduction band.

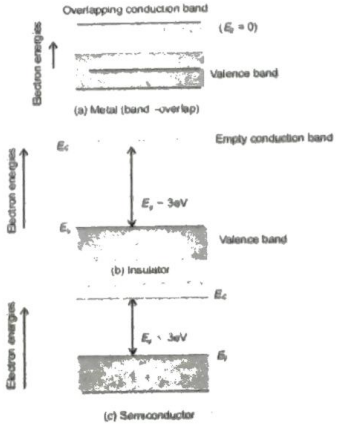
The gap between the top of the valence band and bottom of the conduction band is called the energy band gap (energy gap)

In case of metal, bands are overlapping $E_g = 0$

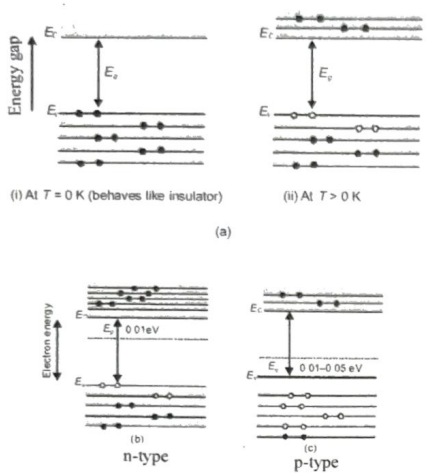
For insulator large band gap E_g exists ($E_g > 3eV$).

For semiconductor finite but small band gas ($E_g < 3$eV)

An intrinsic semiconductor will behave like an insulator at $T = 0$ K. It is the thermal energy at high temperature ($T > 0$K) which excites some electrons from the valence band (thus creating an equal number of holes in the valence band) to the conduction band.



In case of extrinsic semiconductors, additional energy states, apart from E_c and E_i due to donor impurities (E_D) and acceptor impurities (E_A) also exist. We known that very small energy (~0.1eV) is required for the electrons to be released from the donor impurity in the n-type semiconductor. Hence the donor energy E_D lies very near the bottom of the conduction band.

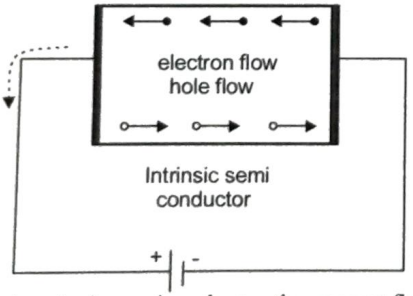


Energy–Band diagrams description is grossly approximate and hypothetical. It helps in understanding the difference between metals insulators and semiconductors in a simple manner.

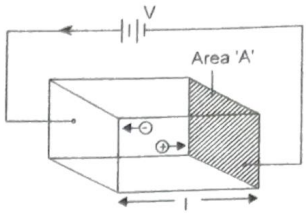
CONDUCTION IN INTRINSIC SEMICONDUCTORS :

In intrinsic semiconductor , the number of free electrons and holes are equal. Both electrons and holes contribute in current conduction. For the purpose of flow of current , a hole , behaves like a positively charged particle having some effective mass. Therefore while the electron moves from negative electrode of the battery to the positive electrode though the semiconductor, the hole moves in opposite side.

The holes exist only inside a semiconductor. There are no holes in a metal. There , electric conduction through holes takes place inside the semiconductor only. Outside, in the metal wires, the electric current flow is due to electrons only. (In cell current flow is due to the motion of positive and negative ions).



In an instrinsic semiconductor the current flow occurs due to the motion of both, the electrons and the holes,. Let e = magnitude of charge on the electrons, n_h= number density of holes, μ_e = mobility of electrons an μ_b = mobility of holes, then the conductivity of intrinsic semiconductor is $\sigma = e (n_e\mu_e + n_h \mu_h)$. A block of semiconductor of length ℓ, area of cross section A and having density of electron and holes as n_e and n_h respecitively when a potential difference say V is applied across it, current I flows through it as shown in fig. The current I is made of electron current I_e and hole current I_h.



Thus, $I = I_e + I_h$ ....(i)

If v_e is drift velocity of electrons,

then $I_e = en_e Av_e$ ....(ii)

Similarly , the hole current is given by

$I_h = en_eAv_h$ ....(iii)

Using equations (ii) and (iii), the equation (i) becomes

$I = eA (n_e v_e + n_h v_h)$ ....(iv)

if R is the resistance offered by the simiconductor to the flow of current, then

$$I = \frac{V}{R} \text{ or } \frac{V}{R} = eA (n_e v_e + n_h v_h) \quad .....(v)$$

The electric field set up across the semiconductor is given by

$$E = \frac{V}{\ell} \qquad \text{or } V = E\ell$$

Therefore, equation (v) becomes

$$\frac{E\ell}{R} = eA (n_e v_e + n_h v_h)$$

$$\text{or } \frac{E}{R\frac{A}{\ell}} = e (n_e v_e + n_h v_h)$$

But $R\frac{A}{\ell} = \rho$ = resistivity of the material of semiconductor

$$\text{Therefore, } \frac{E}{\rho} = e (n_e v_e + n_h v_h) \quad .....(vi)$$

Mobility of electrons or holes is defined as the drift velocity acquired per unit electric field. Therefore, mobility of electrons and holes is given by

$$\mu_e = \frac{v_e}{E} \qquad \text{and } \mu_h = \frac{v_h}{E}$$

From equation (iv) ,we have

$$\frac{1}{\rho} = e \left\{ n_e \cdot \frac{ve}{E} + n_h \cdot \frac{v_h}{E} \right\}$$

$$\text{or } \sigma = e (n_e \mu_e + n_h \mu_h) \quad ....(vii)$$

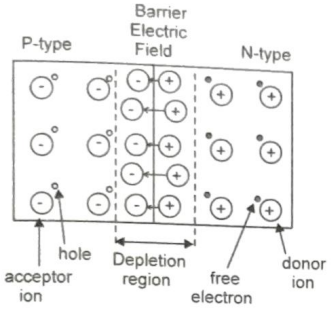
where $\sigma = \frac{1}{\rho}$ is called conductivity of the material of semiconductor and μ_e and μ_h are electron and hole mobilities respectively.

PN JUNCTION :

When a piece of p-type material and piece of n-type material are joined in such a manner that cyrstal structure remains continuous at the boundary, then a pn junction is formed.It is also called a pn junction (PN junction) diode.

A PN junction can not be made by simply pushing the two pieces together as it would not lead to a single crystal structure .There are special fabricaton techniques to form a PN junction.

Immediately after a PN junction is formed, the following process are initiated :



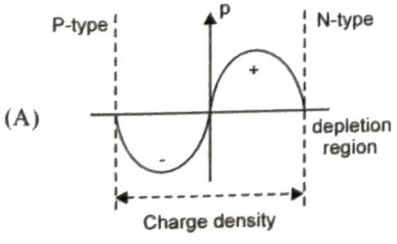
(i) The negative ions on P-sides and positive ions on N-sides are immobile.The majority holes from P region diffuse into N region, and the majority electrons from N region diffuse into P region.

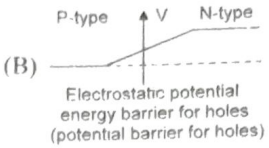
(ii) Due to the above, the electrons and holes at the juction region recombine and disappear (i.e., covalent bonds are completed).

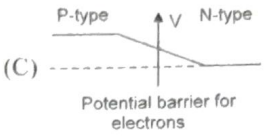
(iii) As a result, a layer of negative ions on P-side and a layer of positive ions on N-side is formed at the junction. In this region, due to recombination of electrons and holes, depletion of free charge carriers occurs. So this region is called depletion region. The charge density on the two sides of the junction (due to ion layers) is shown in Fig.

(iv) The uncompensated ion layers in the depletion region generates an electric field in this region. The electric field points from N side to P side. This electric field prevents further diffusion of holes from P-sides. It also prevents further diffusion of electrons from the N side to P side . The electric field is called barrier electric field.

(v) The barrier electric field gives rise to a difference of potential from one side to the other side. This is called barrier potential (or potential barrier). For silicon PN junction the barrier potential is about 0.7 V while for germanium PN junction, it is about 0.3V.







(vi) For holes the potential on the N-sides is higher . Holes can not cross the deplection region because of this barrier potential. Fig (B). For electrons the potential barrier is shown in Fig. (C)

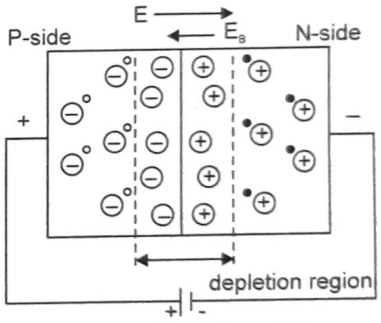
(vii) On the average the potential barrier height in PN junction is ~ 0.5 V and the width of the depletion region ~1 μm or 10^{-6} m. The barrier electric field is thus

$$E = \frac{V}{d} = \frac{0.5}{10^{-6}}$$

$$= 5 \times 10^5 \text{ volt /m}$$

PN JUNCTION WITH FORWARD BIAS :

(i) When the positive terminal of a battery is connected to the P-side and the negative terminal to the N-side of a PN-junction, then it is said to be forward biased (Fig).



(ii) The holes are repelled from the positive terminal and compelled to move towards the junction. The electrons are also repelled from the negative terminal and move towards the junction . This reduces the depletion region for a forward biased PN-junction

(iii) The potential barrier is reduced . More charge carriers diffuse across the junction.

(iv) In the P-type material, near the positive terminal , an electron breaks the covalent bond and goes to battery. As a result a hole is created in P-sides . At the same time an electron enters the N-sides from the negative terminal. The currrent in the P - region arises due to flow of the holes while the currrent in the N-region is due to electrons.

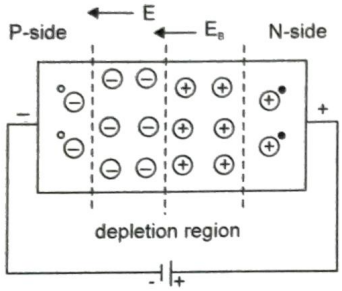
(v) The electric field at the barrier, due to the battery is from P side to N side (forward bias). This is in opposition to the barrier electric field.

(vi) If battery potential is increased, the potential barrier is further reduced. More majority carriers diffuse across the junction. The current increases.

PN JUNCTION WITH REVERSE BIAS :

(i) When the positive terminal of a battery is connected to the N-side and the negative terminal is connected to the P-side of the PN junction, then it is said to be reverse biased.

(ii) The holes in the P-region are attracted towards the negative terminal and the electrons in the N-region are attracted towards the positive terminal . Thus the majority carriers move away from the junction. The depletion region increases for a reverse biased PN-junction,



(iii) The barrier potential increases, this makes it more difficult for the majority carriers to diffuse across the junction .

(iv) A very little current called reverse saturation current flows due to minority carrier flow.It is of the order of nanoamperes (10^{-9}A)for silicon and microamperes (10^{-6} A) for germanium PN -diodes.

(v) In reverse bias situation , the junction behaves like a high resistivity material sandwiched in between two regions.

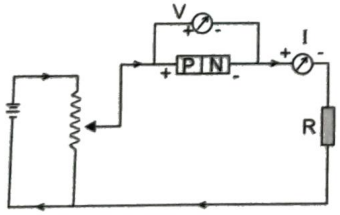
(vi) The effective capacitance of PN junction in the reverse bias condition is of the order of few pico farads.

PN JUNCTION DIODE CHARACTERISTICS :

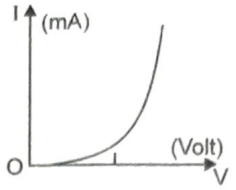
Forward bias characteristics :

The circuit diagram for studying the V-I characteristics of a PN junction diode in forward bias is shown in Fig.

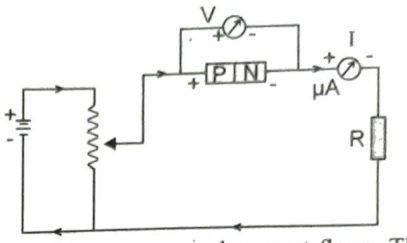
In forward bias the depletion region decreases, the barrier potential decreases, and current flows due to diffusion of charge carriers across the junction . Majority holes from P side cross over to N side, and majority electrons from N side cross over to P side, The current voltage chracteristics is shown in Fig.



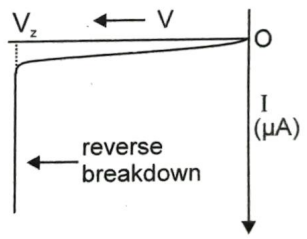
The diode current is negligibly small for first few tenths of a volt. The reason is that the diode does not conduct till the external voltageV, overcomes the barrier potential . The voltage at which the current starts to increase rapidly is called cut-in or Knee voltage (V_0) of the diode. For a silicon diode V_0~0.7 volt and for germanium V_0~ 0.3 volt. Junction diode in forward bias is shown in Fig.



In reverse bias state, the depletion region increases and potential barrier also increases. The majority holes in P region and majority electrons in N region , now do not cross the junction . They do not give rise to any current.

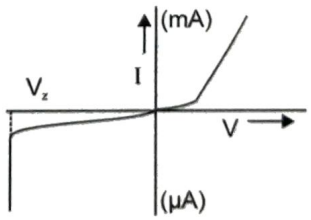


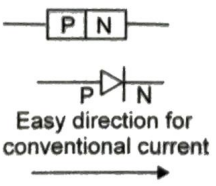
In reverse bias a very smal current flows. This arises due to the flow of minority charge carriers across the junction . The reverse current is only few μ A for germanium diodes and only a few nA for silicon diodes. It remains small and almost constant for all reverse bias voltages less than the break down voltage V_Z. At breakdown, the current increses rapidly for small increase in voltage



The full characteristics, forward and reverse bias are shown in Fig. The PN junction diode thus is a unidirectional device. Large current (mA) flows in one direction, but neglible current flows in the reverse direction.

The symbol used for PN junction diode is shown in Fig.





REVERSE BREAKDOWN

If the reverse bias voltage is made too high, the current through the PN junction increases rapidly at V_Z(see fig.). The voltage at which this happens is called breakdown voltage or Zener voltage.

There two mechanism which causes this breakdown.

One is called Zener breakdown and the other is called avalanche breakdown

Zener breakdown: When reverse bias is increased the electric field at then junction also increases. At some stage the electric field becomes so high that it breaks the covalent bonds creating electron,hole pairs. thus a large number of carriers are generated. This causes a large current to flow. This machanism is know as Zener breakdown.This phenomenon is used in Zener diode and used in voltage regulator.

Avalanche breakdown: At high reverse voltage, due to high electric field, the miniority charge carriers, while crossing the junction acquires very high velocities . These by collision breaks down the covalent bonds , generating more carriers. A chain reaction is established , giving rise to high current. This mechanism is called **avalanche breakdown .**

TRAIN YOUR BRAIN

Example 1: In a pure germanium sample, the electron and hole mobilities at room temperature are 3600 cm^2/volt-sec, 1700 cm^2/volt-sec respectively. If the electron-hole concentration is 2.5×10^{19} m^{-3}, then the conductivity of Ge is-

(*a*) 15.7 mho-m^{-1} (*b*) 2.12 mho-m^{-1}

(*c*) 1.21 mho-m^{-1} (*d*) 0.5 mho-m^{-1}

Sol. $\sigma = n_i e (\mu_e + \mu_n)$

$\because n_i = p_i$

$\Rightarrow \sigma = 2.5 \times 10^{19} \times 1.6 \times 10^{-19} (.36 + .17)$

$= 2.12$ mho-m^{-1}

Example 2: In a pure silicon sample, 10^{13} atoms of phosphorous are doped per cm^3. If all the doner atoms produce carries and $\mu_e = 1200$ cm^2/volt-sec, then the resistivity of the sample is-

(*a*) 52.08 ohm-m (*b*) 520.8 ohm-m

(*c*) 5.208 ohm-m (*d*) 5208 ohm-m

Sol. $\rho = 5.208$ ohm-m

TRANSISTOR :

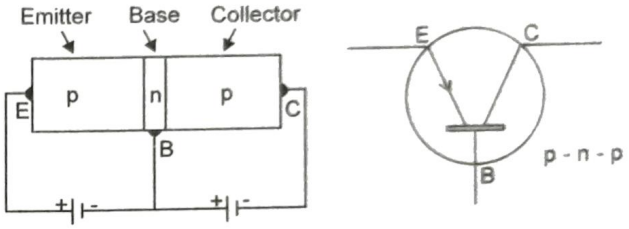
A transistor is an electronic device formed by p and n type of semiconductor which is used in place of a triode valve. It was discovered in 1948 by American scientists Bardeen, Shockley and Barattain .

Transistors are of two types :

p-n-p transistor and n-p-n transistor. The transistor has three components, emitter, base and collector. The base is sandwiched between the emitter and collector. The emitter is heavily doped, base is very lightly doped and collector is moderately doped. The Collector has largest size while base is very thin (only a few microns). This difference in doping levels and size of base gives interesting properties to a transistor.

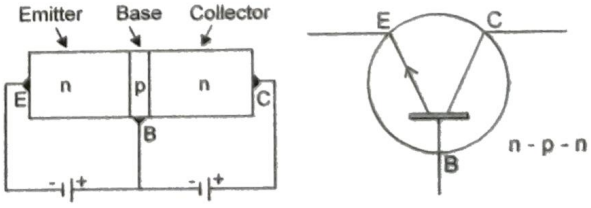
p-n-p Transistor :

It consists of a very thin slice of n-type semiconductor sandwiched between two small crystals of p-type semiconductor (fig). The central slice is called the 'base' while the left and right crystals are called the 'emitter' and the 'collector' respectively. The emitter is given a positive potential and collector is negative potential with respect to the base. Thus , the emitter -base (p-n) junction on the left is under forward-bias (high resistance). The symbol for this transistor is shown in Fig. 12(b) in which the direction of the arrow indicates the direction of current (direction of flow of holes).



n-p-n Transistor :

It consists of a very thin slice of p-type semiconductor (fig). In this transistor the emmitter is given a negative potential while the collector is given a positive potential with respect to the base. Again, the emitter-base (n-p) junction on the left is under forward-bias, while the base collector (p-n) junction on the right is denser reverse-bias.* The symbol for the n-p-n transistor is shown in Fig. (b) in which the direction of the arrow indicates the direction of current (opposite of the direction of flow of electrons).



(a) (b)

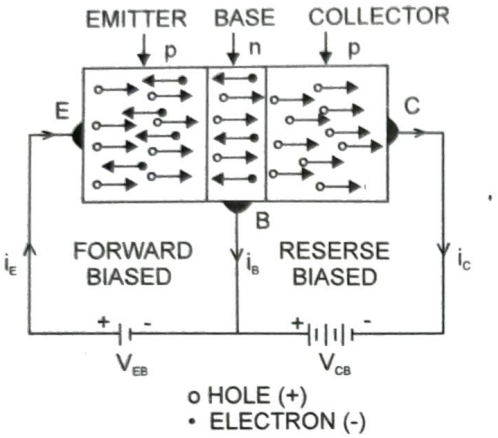
A transistor can be connected in a circuit in three difference ways. They are :

(i) common-base configuration, (ii) common-emitter configuration and (iii) common-collector configuration. The work 'common' is related with that electrode which is common in input and output circuits. This common electrode is generally grounded. Hence the above three

configurations of connection are also called respectively as grounded -base configuration, grounded-emitter configuration and ground collector configuration. Each configuration has its own characteristics.

Working of p-n-p Transistor

A 'common-base ' circuit of a p-n-p transistor is shown in Fig. The emitter- base (p-n) junction on the left is given a small forward bias (fraction of a volt) while the base-collector (n-p) junction is given a large reverse-bias (a few volts).



Holes are the charge-carriers with in the p-n-p transistor, while electrons are the charge-carriers in the external circuit.

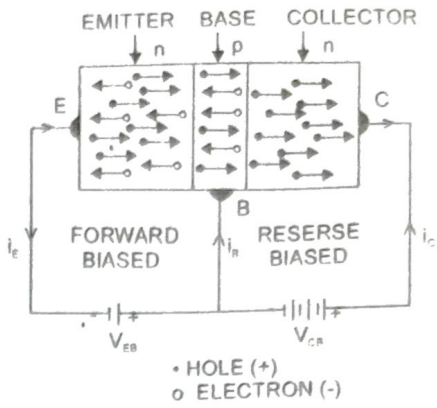
The small current which leaves the base terminal B is called the 'i_B 'base-current' the larger current which leaves the collector terminal is called the 'collector -current ' i_C.Both these currents combine to enter the emitter terminal E and constitute the emitter-current i_E. Clearly,

$$i_E = i_B + i_C$$

The base being very thin, the number of hole-electron combinations in it is very small, and almost all the holes entering the base from the emitter reach the collector. . Hence the collector-current i_C is only very slightly less than the emitter current i_E.

Working of n-p-n Transistor

A circuit known as 'common-base' circuit of n-p-n transistor is shown in Fig. The two n-regions contain the mobile electrons while the central thin p-region contains the mobile (positive) holes. The emitter-base by means of a battery V_{EB}, while the base-collector (p-n)junction on the right has been given a large reverse-bias by means of battery V_{CB}.



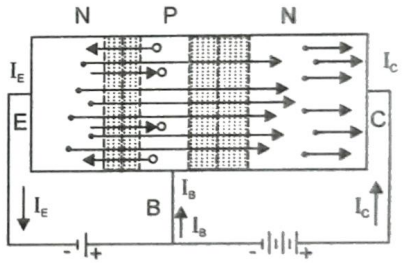
The electrons are the majority charge-carriers with in the n-p-n transistor as well as in the external circuit (whereas holes are the charge-carriers with in p-n-p transistor).

The small current entering the base terminal B is the base current i_B, while the larger current entering that collector terminal C is the collector-current i_C. Both currents combine to leave the emitter terminal E and constitute the emitter current i_E. Thus

$$i_E = i_B + i_C$$

TRANSISTOR ACTION

There are four possible ways of biasing the two PN junctions (emitter junction and collector junction) of a transistor .These are tabulated below.



$I_E = I_C + I_B$

$I_B << I_C$, and

$I_B << I_E$

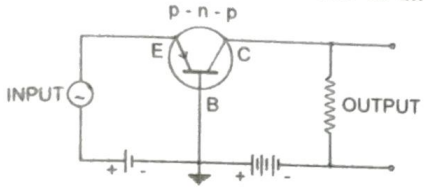
Thus I_C is always less than I_E, but the difference is small.

Since the emitter junction is forward biased its resistance is small, while the collector jucntion is reverse biased, therefore its resistance is large. Thus , a transistor is a device which transfers I_E current from low resistance circuit to a high resistance circuit ($I_C \sim I_E$). Thus it is, transfer + resistor = transistor

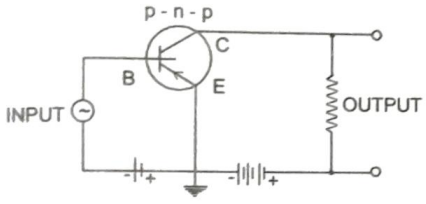
(The name transistor originated from the above action of the transistor).

TRANSISTOR CONFIGURATION AND ITS USE AS AN AMPLIFIER

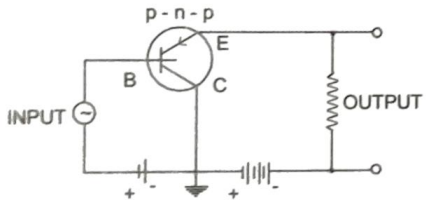
A junction diode cannot amplify a signal . A transistor consisting of two p-n junctions, one forward-biased and the other reverse-biased can however , be used for amplifying a weak signal The forward-biased junction has a low-resistance path whereas the reverse-biased junction has a high-resistances path. The weak input signal is applied across the forward-biased (low resistance) junction and the output currents are signal is taken appears with a much higher voltage. The transistor thus acts as an amplifier .



(a) COMMON-BASE



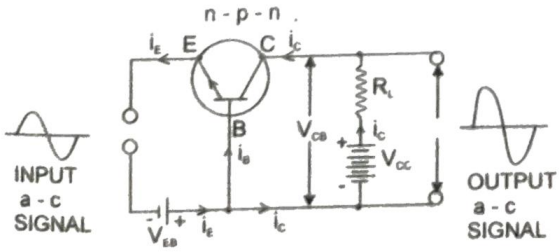
(b) COMMON-EMITTER



(b) COMMON-COLLECTOR

When a transistor is to be operated as amplifier, three different basic circuit connection are possible, as illustrated in fig. These are (a) common-base (b) common emitter and (c) common-collector circuits.

Transistor as Common-Base Amplifier :



Common-Base Amplifier using a n-p-n Transistor:

Fig. shows the common-base amplifier circuit using an n-p-n transistor. The base is made common to the input and the output circuits.

The emitter-base input circuit is forward-biased by a low-voltage battery V_{EB} so that the resistance of the input circuit is small The collector -base output circuit is reverse-biased by means of a high-voltage battery V_{CC} so the at the resistance of the output circuit is quite large. R_L is a load resistance connected in the collector-base circuit. The weak input a-c voltage signal is applied across the emitter-base circuit and the amplified output signal is obtained across the collector-base circuit.

Let i_E, i_B and i_C be the emitter -current and collector-current irrespectively when no a-c voltage signal is applied to the input circuit. (The arrows represent the direction of hole current, that is conventional current which is opposite to the direction of electron current) . By Kirchhoff's first law, we have

$$i_E = i_B + i_C \qquad \text{.....(1)}$$

Due to the collector- current i_C, the voltage drop across R_L is $i_C R_L$. Therefore, the collector-to- base voltage (potential difference between collector and base) V_{CB} would be given by

$$V_{CB} = V_{CC} - i_C R_L \qquad \text{.....(2)}$$

When the input a-c voltage signal is applied across the emitter-base circuit, it changes the emitter-base voltage and hence the emitter-current i_E which, in turn , changes the collector current i_C. Consequently, the collector to base voltage V_{CB} varies in accordance with equation (2). This variation in V_{CB}, when the input signal is applied, appears as an amplified output.

Phase Relationship between Input and Output voltage Signals in CB circuit:

The output voltage signal is in phase with the input voltage signal in the common-base amplifier.

Gains in Common-Base Amplifier:

The various gains in a common-base amplifier are as follow:

(i) A - c Current Gain: It is defined as the ratio of the change in the colllector-current to the change in the emitter-current at a constant collector-to-base voltage, and is denoted by α. Thus

$$\alpha = \left(\frac{\Delta i_C}{\Delta i_E}\right)_{V_{CB}}$$

The value of α is slightly less than 1 (actually, there is a little current loss).

(ii) A - c Voltage Gain: It is defined as the ratio of the changes in the output voltage to the change in the input voltage, and is denoted by A_V.

Suppose, on applying an a-c input voltage signal, the emitter current changes by Δi_E and correspondingly the collector-current changes by Δi_C. If R_{in} and R_{out} be the resistances of the input and the output circuits respectively, then

$$A_V = \frac{\Delta i_C \times R_{out}}{\Delta i_E \times R_{in}} = \frac{\Delta i_C}{\Delta i_E} \times \frac{R_{out}}{R_{in}}$$

Now , $\Delta i_C / \Delta i_E$ is the a-c current-gain and R_{out}/R_{in} is called the 'resistance gain'.

$\because$ $A_V = \alpha \times$ resistance gain

Since the resistance gain is quite high, A_V is also quite high although α is slightly less than 1.

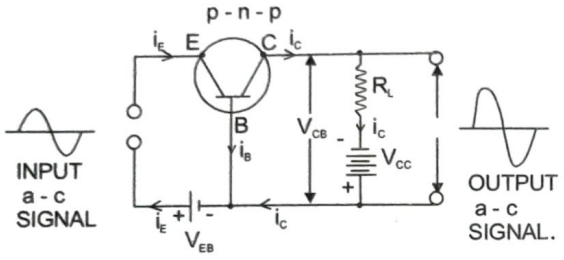
(iii) a-c Power Gain: it is defined as the ratio of the change in the output power to the change in the input power.

Since power = current x voltage, we have

a-c power gain = a-c current gain × a-c voltage-gain

$= \alpha^2 \times$ resistance gain

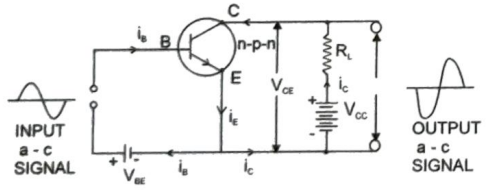
Common-Base Amplifier using a p-n-p Transistor :



Transistor as Common-Emitter Amplifier :

Common-Emitter Amplifier using an n-p-n Transistor: Fig.shows the common-emitter amplifier circuit using an n-p-n transistor. The emitter is made common to the input and the output circuits.

The input (base-emitter) circuit is forward-biased by a low-voltage battery V_{BE} so that the resistance of the input circuit is small. The output (collector-emitter) circuit is reverse-biased by means of a high voltage battery V_{CC} so that the resistance of the output circuit is high. R_L is a load resistance connected in the collector-emitter output circuit. The weak input a-c signal is applied across the base-emitter circuit and the amplified output signal is obtained across the collector -emitter circuit.



Let i_E, i_B and i_C be the emitter-current, base-current and collector- current respectively when no a-c voltage signal is applied to the input circuit. (The arrows represent the direction of the hole current, that is conventional current which is opposite to the direction of electron current.) By Kirchhoff's first law, we have

$i_E = i_B + i_C$

Due to the collector current i_C (Which is only slightly smaller that i_E), the voltage drop across R_L is $i_C R_L$. Therefore, the collector -to -emitter voltage (potential between collector and emitter) V_{CE} would be given by

$V_{CE} = V_{CC} - i_C R_L$

When the input a-c voltage signal is applied across the base-emitter circuit, it changes the base-emitter voltage and hence the emitter -current i_E which, in turn , changes the collector current i_C. Consequently, the collector-to-emitter voltage V_{CE} varies in accordance with equation. (ii). This variation in V_{CE}, when the input signal is applied, appears as an amplified output.

Phase Relationship between Input an Output Voltage Signals

In a common-emitter amplifier the input voltage signal and the output voltage signal obtained across the collector and the emitter are out of phase with each other.

The output voltage signal is 180° out of phase with the input voltage signal in the common-emitter amplifier.

GAINS IN COMMON-EMITTER AMPLIFIER

The various gains in a common-emitter amplifier are as follows:

(i) D-c Current Gains: it is defined as the ratio of the collector current to the base current, and is denoted by β (d-c) . Thus

$$\beta \text{ (d-c)} = \frac{i_C}{i_B}$$

In a typical transistor , a small base- current ($\approx 10\mu A$) produces a large collector -current ($\approx 500\mu A$). Thus

$$\beta \text{ (d-c)} = \frac{500}{10} = 50$$

(ii) A-c Current Gain: It is defined as the ratio of the change in the collector -current to the change in the in the base -current at a constant collector to emitter voltage , and is denoted by β (a-c) . Thus

$$\beta \text{ (a -c)} = \left(\frac{\Delta i_C}{\Delta i_B}\right)_{V_{CE}}$$

(iii) Voltage gain : Suppose , on applying an a-c input voltage signal, the input base-current changes by Δi_B and correspondingly the output collector-current changes by Δi_C. If R_{in} and R_{out} be the resistance of the input an the output circuits respectively , then

$$A_V = \frac{\Delta i_C \times R_{out}}{\Delta i_B \times R_{in}} = \frac{\Delta i_C}{\Delta i_B} \times \frac{R_{out}}{R_{in}}$$

Now, $\Delta i_C / \Delta i_B$ is the a-c current gain (a-c) and R_{out} / R_{in} is the resistance gain

$\therefore$ $A_V = \beta$ (a-c) × resistance gain.

Since β (a-c) >> α (a-c), the a-c voltage gain in common-emitter amplifier is larger compared to that is common -base amplifier., although the resistance gain is smaller.

(iv) **a-c Power Gain** : It is defined as the ratio of the change in the output power to the change in the input power.

Since power = current × voltage , we have

a-c power gain = a-c current gain × a-c voltage gain

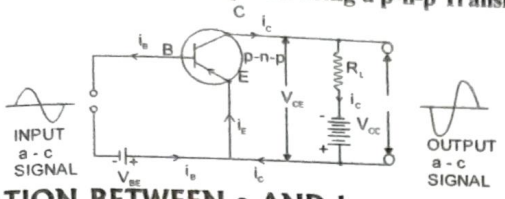
$= \beta$ (a-c) × A_V

$= \beta$ (a -c) × { β (a-c) x resistance gain}

$= \beta^2$ (a-c) × resistance gain.

Since β (a-c) >> α (a-c) , the a-c power gain in common-emitter amplifier is extremely large compared to that in common-base amplifier.

(v) **Common-Emitter Amplifier using a p-n-p Transistor:**



RELATION BETWEEN a AND b :

CB current gain (α)

CB current gain (α) is the ratio of output current to the input current in common base configuration of a transistor.

$$\alpha_{dc} = \frac{I_C}{I_E} \; ; \qquad \alpha_{ac} = \frac{\Delta I_C}{\Delta I_E}$$

CE current gain (β)

CE current gain (β) is the ratio of the output current to the input current in emitter cofiguration of the transistor.

$$\beta_{dc} = \frac{I_C}{I_B} \; ; \qquad \beta_{ac} = \frac{\Delta I_C}{\Delta I_B}$$

The CB current gain α and CE current gain β are related by the rollowing relations.

$$\frac{5}{1.6 \times 39 \times 10^{-17}} = 1 + \frac{1}{\beta}$$

$$\alpha = \frac{\beta}{\beta+1} \; ; \qquad \beta = \frac{\alpha}{1-\alpha}$$

The above relations are applicable for both dc and ac current gains.

Comment. The value or α is always less than 1. $\alpha \sim 0.9$ to 0.99 or more. The value of β is always much greater than 1. $\beta \sim 95$ to 999 or more.

TRAIN YOUR BRAIN

Example 3: The dc current gain of a transistor in CB configuration is 0.99. Find its dc current gain in CE configuration.

(*a*) 99 (*b*) 400

(*c*) 300 (*d*) 50

Sol. Given $\alpha_{dc} = 0.99$

$$\text{Therefore, } \beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}} = \frac{0.99}{1-0.99}$$

$$= \frac{0.99}{.01} \text{ or } \qquad \beta_{dc} = 99$$

Example 4: When the emitter current of a transistor is changed by 1 mA, its collector current changes by 0.995 mA . Calculate (i) its common base current gain and (ii) its common emitter current gain

Sol. (i) $\alpha = \dfrac{\Delta I_C}{\Delta I_E} = \dfrac{0.995 \text{ (mA)}}{1 \text{ (mA)}} = 0.995$

(ii) $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.995}{1-0.995} = \dfrac{0.995}{.005} = 199$

Example 5: In certain transistor $\alpha_{dc} = 0.98$ and $I_E = 1$ mA . Find the corresponding values of base current and collector current.

Sol. (i) $\alpha_{dc} = \dfrac{I_C}{I_E}$ or $0.98 = \dfrac{I_C}{I(mA)}$

thus $I_C = 0.98$ mA

(ii) Using $I_B = I_E - I_C$

we get $I_B = (1 - 0.98)$ mA $= 0.02$ mA

Example 6: For a common base transistor if the values of I_E and I_c are 10^3 μA and 0.96 mA respectively then the value of I_B will be-

(*a*) 0.04 mA (*b*) 4 mA

(*c*) 0.4 mA (*d*) 0.004 mA

Sol. ∵ $I_E = I_B + I_C$

$I_B = I_E - I_C$

$= 1000 \times 10^{-6} - 0.96 \times 10^{-3}$

$= 1$ mA $- 0.96$ mA $= 0.04$ mA

Example 7: The value of β for a transistor for which α=0.95, will be-

(*a*) 19 (*b*) 91

(*c*) 1.9 (*d*) 0.19

Sol. $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.95}{1-0.95} = \dfrac{0.95}{0.05} = 19$

Example 8: Mobility of electrons in Germanium of N types & their conductivity are 3900 cm^2/volt-sec & 5 mho/cm respectively. If effect of holes are negligible then concentration of impurity will be-

(a) 8×10^{15} per cm^3 (*b*) 9.25×10^{14} per cm^3

(*c*) 6×10^{13} per cm^3 (*d*) 9×10^{13} per cm^3

Sol. $\mu_n = 3900$ cm^2/volt sec ; $\sigma_n =$ 5mho/cm

$\sigma_n = n_d e \mu_n$

$5 = n_d \times 1.6 \times 10^{-19} \times 3900$

$\therefore \; n_d = \dfrac{5}{1.6 \times 39 \times 10^{-17}} = 8 \times 10^{15}$

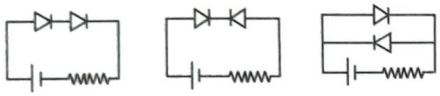
Topicwise Questions

1. Germanium is doped with arsenic, what will be the result?
   (*a*) *p*-type semi-conductor
   (*b*) *n*-type semi-conductor
   (*c*) Intrinsic semi-conductor
   (*d*) none of these

2. A transistor in common base configuration has current gain $\alpha = 0.95$. It has a change in emitter current of 100 milliampere. Then the change in collector current is
   (*a*) 95 mA (*b*) 100 mA
   (*c*) 99.05 mA (*d*) 100.95 mA

3. The current gain for a transistor working as common-base amplifier is 0.96. If the emitter current is 7.2 mA, then the base current is
   (*a*) 0.29 mA (*b*) 0.35 mA
   (*c*) 0.39 mA (*d*) 0.43 mA

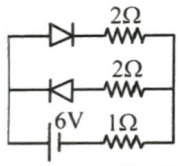
4. Assuming the diodes are ideal, current through the battery is zero



(i) (ii) (iii)

   (*a*) in (i) and (iii) (*b*) in (ii) and (iii)
   (*c*) in only (ii) (*d*) in only (iii)

5. Assuming ideal diode the current through the 1Ω resistance in the circuit as shown in the figure is



   (*a*) 2A (*b*) 1A
   (*c*) 3A (*d*) none of the above

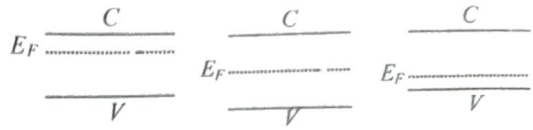
6. The majority of current carrier in an *n*-type semiconductor are
   (*a*) holes (*b*) electrons
   (*c*) negative ions (*d*) positive ions

7. In a forward biased *p-n* junction, the potential barrier
   (*a*) becomes zero (*b*) decreases
   (*c*) increases (*d*) remain constant

8. A transistor can be used as an amplifier when
   (*a*) base-emitter is forward biased and base-collector is reverse biased
   (*b*) both base-emitter and base-collector are reverse biased
   (*c*) both base-emitter and base-collector forward biased
   (*d*) none of these

9. The current gain in the common emitter mode of a transistor is 10. The input impedance is 20 kΩ and load of resistance is 100 kΩ. The power gain is
   (*a*) 300 (*b*) 500
   (*c*) 200 (*d*) 100

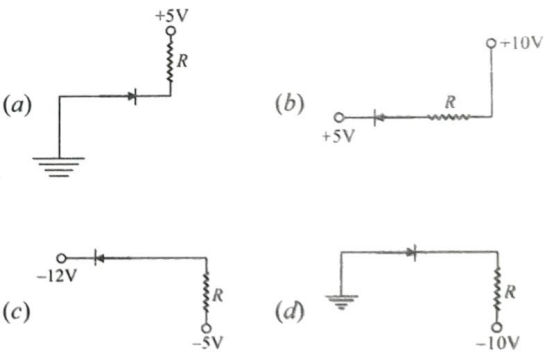
10. The band diagrams of the three semiconductors are given in the figure. From left to right, they are respectively



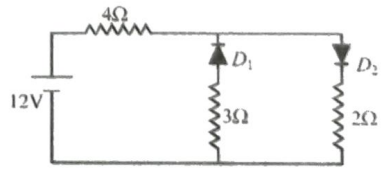
    (*a*) *n*-intrinsic-*p* (*b*) *p*-intrinsic-*n*
    (*c*) intrinsic-*p*-*n* (*d*) *p*-*n*-intrinsic

11. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by
    (*a*) Metallic bonding
    (*b*) Ionic bonding
    (*c*) Covalent bonding
    (*d*) Vander Waals bonding

12. In the following, which one of the diodes is reverse biased?



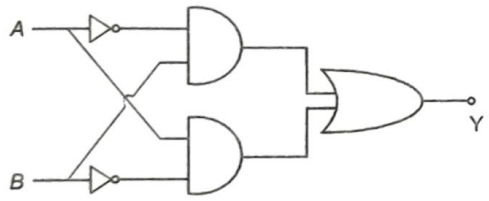
13. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit?



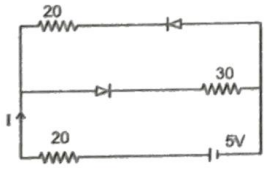
    (*a*) 01.33 A (*b*) 1.71 A
    (*c*) 2.00 A (*d*) 2.31 A

Learning Plus

1. A piece of copper and other of germanium are cooled from room temperature to 80K, then :
   (*a*) resistance of each will increase
   (*b*) resistance of germanium will decrease
   (*c*) resistance of copper will increase while that of germanium will decrease
   (*d*) resistance of copper will decrease while that of germanium will increase
2. Which of the following is added as an impurity, into the silicon, produces n-type semiconductor?
   (*a*) Phosphorous (*b*) Aluminium
   (*c*) Magnesium (*d*) Both (*a*) and (*c*)
3. The cause of the potential barrier in a p-n diode is :
   (*a*) depletion of positive charges near the junction
   (*b*) concentration of positive charges near the junction
   (*c*) depletion of negative charges near the junction
   (*d*) concentration of positive and negative charges near the junction
4. Which of the following when added as an impurity into silicon produces n-type semiconductor ?
   (*a*) P (*b*) As
   (*c*) B (*d*) Mg
5. The following circuit represents :

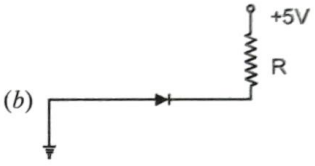


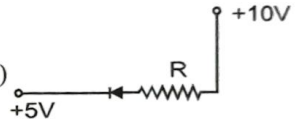
   (*a*) OR gate (*b*) XOR gate
   (*c*) AND gate (*d*) NAND gate
6. The current (I)in the circuit will be :-

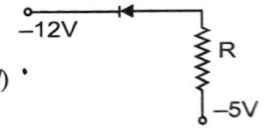


   (*a*) $\frac{5}{40}$ A (*b*) $\frac{5}{50}$ A
   (*c*) $\frac{5}{10}$ A (*d*) $\frac{5}{20}$ A
7. Copper and silicon is cooled from 300 K to 60K, the specific resistance :-
   (*a*) Decrease in copper but increase in silicon
   (*b*) Increase in copper but increase in silicon
   (*c*) Increase in both
   (*d*) Decrease in both
8. Of the diodes shown in the following diagrams, which one of the diode is reverse biased ?

   (*a*)


   (*b*)


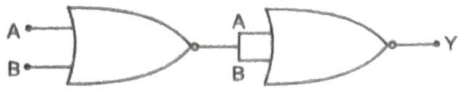
   (*c*)


   (*d*)


9. Zener diode is used for :-
   (*a*) Rectification
   (*b*) Stabilisation
   (*c*) Amplification
   (*d*) producing oscillations in an oscillator
10. Application of a forward bias to a p-n junction –
    (*a*) Widens the depletion zone
    (*b*) Increases the number of donors on the n side
    (*c*) Increases the potential difference across the depletion zone
    (*d*) Increases the electric field in the depletion zone
11. A forward biased diode is:-
    (*a*) – 4V – 3V
    (*b*) 3V 5V
    (*c*) –2V +2V
    (*d*) 0V –2V

12. Semiconductor Ge has forbidden gap of 1.43 eV. Calculate maximum wavelength in Å which result from electron hole combination.

13. In the following circuit, the output Y for all possible inputs A and B is expressed by the truth table:



(a)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(b)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

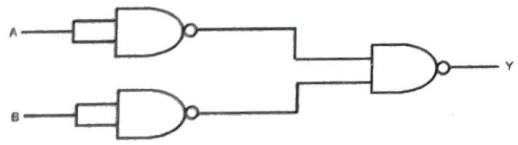
(c)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

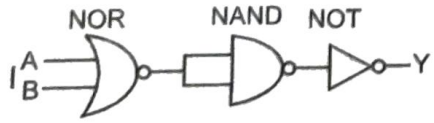
(d)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

14. Draw the truth table for the logic gate arrangement shown in the figure.

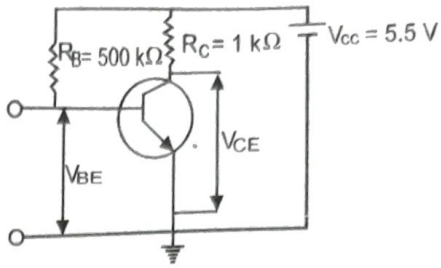


15. The circuit is equivalent to



(a) AND gate
(b) NAND gate
(c) NOR gate
(d) OR gate

16. For given transistor circuit, the base current is 10 μA and the collector current is 5.2 mA. Can this transistor circuit be used as an amplifier. Your answer must be supported with proper explanation.



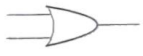
For a common emitter amplifier,current gain is 69. If the emitter current is 7 mA then calculate the base current and collector current.

17. A p-n photodiode is fabricated from a semiconductor with a band gap of 2.5 eV. It can detect a signal of wavelength

(a) 6000 Å (b) 4000 nm
(c) 6000 nm (d) 4000 Å

18. The symbolic representation of four logic gates

(i)  (ii) 

(iii)  (iv) 

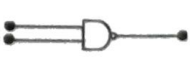
The logic symbol for OR, NOT and NAND gates are respectively

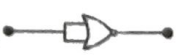
(a) (iii), (iv), (ii) (b) (iv), (i), (iii)
(c) (iv), (ii), (i) (d) (i), (iii), (iv)

19. When the light of frequency $2v_0$ (where v_0 is threshold frequency), is incident on a metal plate, the maximum velocity of electrons emitted is v_1. When the frequency of the incident radiation is incrased to $5v_0$, the maximum velocity of electrons emitted from the same plate is v_2. The ratio of v_1 to v_2 is

(a) 4 : 1 (b) 1 : 4
(c) 1 : 2 (d) 2 : 1

20. Which of the following gives a reversible operation ?

(a)  (b) 

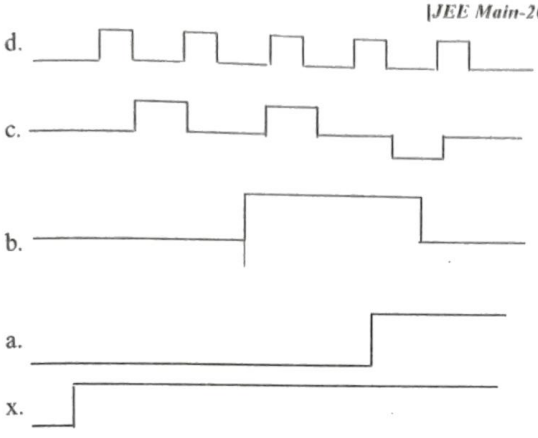
(c)  (d) 

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is:

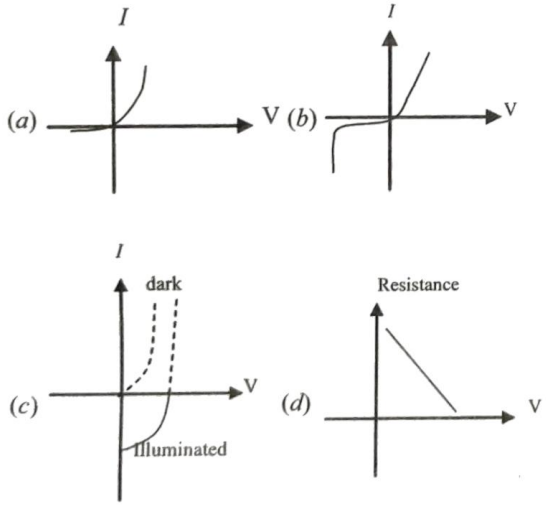
[*JEE Main-2016*]



(*a*) NOT (*b*) AND
(*c*) OR (*d*) NAND

2. Identify the semiconductor devices whose characteristics are given below, in the order (*a*), (*b*), (*c*), (*d*):

[*JEE Mains-2016*]



(*a*) Simple diode, Zener diode, Solar cell, Light dependent resistance
(*b*) Zener diode, Simple diode, Light dependent resistance, Solar cell
(*c*) Solar cell Light dependent resistance, Zener diode, Simple diode
(*d*) Zener diode, Solar cell, Simple diode, Light dependent resistance

3. *For a common emitter configuration, if* α and β *have their usual meanings, the* **incorrect** *relationship between* α *and* β *is :* [*JEE Mains - 2016*]

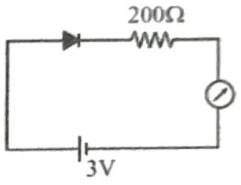
(*a*) $\frac{1}{\alpha}=\frac{1}{\beta}+1$ (*b*) $\alpha=\frac{\beta}{1-\beta}$

(*c*) $\alpha=\frac{\beta}{1+\beta}$ (*d*) $\alpha=\frac{\beta^2}{1+\beta^2}$

4. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be : [*JEE Main-2017*]

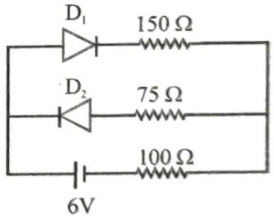
(*a*) 135° (*b*) 180°
(*c*) 45° (*d*) 90°

5. The reading of the ammeter for a silicon diode in the given circuit is : [*JEE Main-2018*]



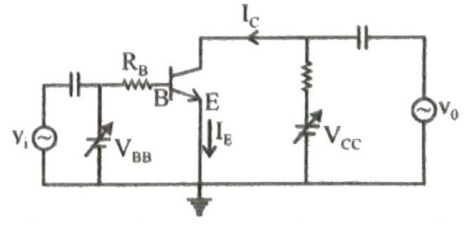
(*a*) 15 mA (*b*) 11.5 mA
(*c*) 13.5 mA (*d*) 0

6. The circuit shown below cotains two ideal diodes, each with a forward resistance of 50Ω. If the battery voltage is 6V, the current through the 100Ω resistance (in Amperes) is: [*JEE Main - 2019 (January)*]



(*a*) 0.036 (*b*) 0.020
(*c*) 0.027 (*d*) 0.030

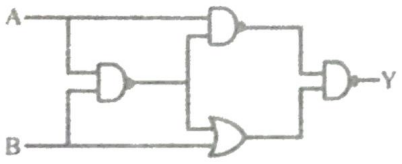
7. In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, V_{CC} = 5V, β_{dc} = 200, R_B = 100, R_C = 1 kΩ and V_{BE} = 1.0 V. The minimum base current and the input voltage at which the transistor will go to saturation, will be respectively : [*JEE Main - 2019 (January)*]



(*a*) 25 μA and 3.5 V (*b*) 20 μA and 3.5 V
(*c*) 25 μA and 2.8 V (*d*) 20 μA and 2.8 V

8. The output of the given logic circuit is.

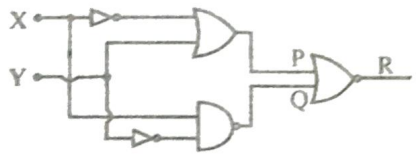
[JEE Main - 2019 (January)]



(a) $A\bar{B} + \bar{A}B$ (b) $AB + \bar{A}\bar{B}$

(c) $A\bar{B}$ (d) $\bar{A}B$

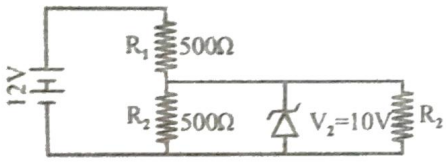
9. To get output '1' at R, for the given logic gate circuit the input values must be: [JEE Main - 2019 (January)]



(a) X = 0, Y = 1 (b) X = 1, Y = 1

(c) X = 1, Y = 0 (d) X = 0, Y = 0

10. In the given circuit the current through Zener Diode is close to : [JEE Main-2019 (January)]

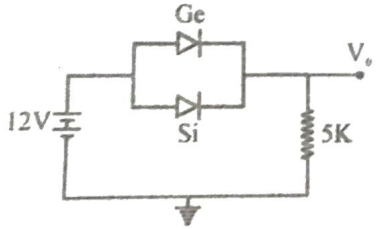


(a) 0.0 mA (b) 6.7 mA

(c) 4.0 mA (d) 6.0 mA

11. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_0 changes by: (assume that the Ge diode has large breakdown voltage)

[JEE Main - 2019 (January)]



(a) 0.8 V (b) 0.6 V

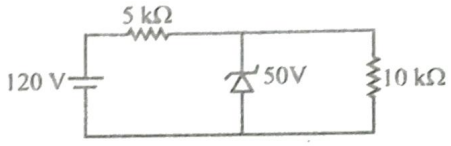
(c) 0.2 V (d) 0.4 V

12. Mobility of electron in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is 1.6 m^2(V.s) then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to : [JEE Main - 2019 (January)]

(a) 2 Ωm (b) 4 Ωm

(c) 0.4 Ωm (d) 0.2 Ωm

13. For the circuit shown below, the current through the Zener diode is ; [JEE Main - 2019 (January)]

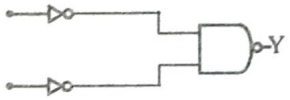


(a) 9 mA (b) 5 mA

(c) Zero (d) 14 mA

14. The logic gate equivalent to the given logic circuit is:-

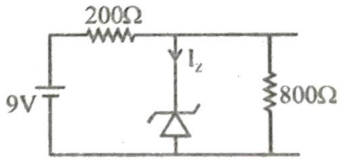
[JEE Main - 2019 (April)]



(a) OR (b) AND

(c) NOR (d) NAND

15. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit. The current I_z through the Zener is : [JEE Main - 2019 (April)]



(a) 7 mA (b) 17 mA

(c) 10 mA (d) 15 mA

16. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6V and the load resistance is R_L = 4 kΩ. The series resistance of the circuit is R_s = 1kΩ. If the battery voltage V_B varies from 8V to 16V, what are the minimum and maximum values of the current through Zener diode

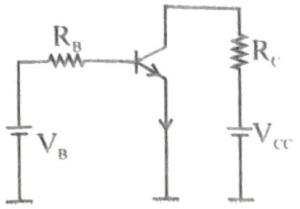
[JEE Main - 2019 (April)]

(a) 0.5 mA ; 6 mA (b) 0.5 mA ; 8.5 mA

(c) 1.5 mA ; 8.5 mA (d) 1 mA : 8.5 mA

17. A common emitter amplifier circuit, built using an n-p-n transistor, is shown in the figure. Its dc current gain is 250, R_C = 1kW and V_{CC} = 10 V. What is the minimum base current for VC_{CE} to reach saturation?

[*JEE Main - 2019 (April)*]



(*a*) 100 μA (*b*) 7μA
(*c*) 40μA (*d*) 10μA

18. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to: [*JEE Main - 2019 (April)*]

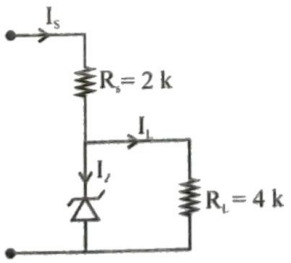
(*a*) 0.7% (*b*) 0.2%
(*c*) 3.5% (*d*) 6.8%

19. An n-p-n transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15μA change in the base current of the amplifier. The input resistance and voltage gain are :

[*JEE Main - 2019 (April)*]

(*a*) 0.33 kΩ, 1.5 (*b*) 0.67 kΩ, 200
(*c*) 0.33 kΩ, 300 (*d*) 0.67 kΩ, 300

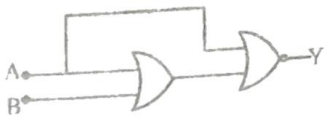
20. Figure shown a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6V. If the unregulated input voltage varies between 10 V to 16V, then what is the maximum Zener current? [*JEE Main - 2019 (April)*]



(*a*) 2.5 mA (*b*) 3.5 mA
(*c*) 7.5 mA (*d*) 1.5 mA

21. The truth table for the circuit given in the figure is :

[*JEE Main - 2019 (April)*]



(*a*)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(*b*)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

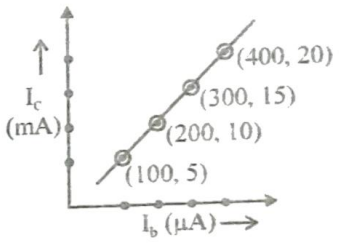
(*c*)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(*d*)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

22. The transfer characteristic curve of a transistor, having input and output resistance 100Ω and 100kΩ respectively, is shown in the figure. The Voltage and Power gain, are respectively : [*JEE Main - 2019 (April)*]



(*a*) 5×10^4, 5×10^5 (*b*) 5×10^4, 5×10^6
(*c*) 5×10^4, 2.5×10^6 (*d*) 2.5×10^4, 2.5×10^6

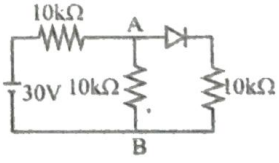
23. An n–p–n transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100W and the output load resistance is 10kΩ. The common emitter current gain β is :

[*JEE Main - 2019 (April)*]

(*a*) 60 (*b*) 10^4
(*c*) 6×10^2 (*d*) 10^2

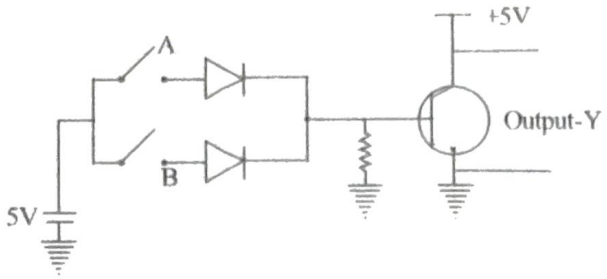
24. In the figure, potential difference between A and B is:

[*JEE Main-2020 (January)*]



(*a*) 15V (*b*) Zero
(*c*) 10 V (*d*) 5 V

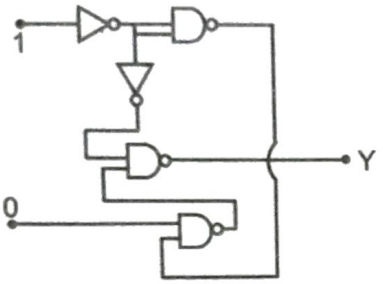
25. Boolean relation at the output stage-Y for the following circuit is: [JEE Main-2020 (January)]



(a) $\bar{A}+\bar{B}$ (b) $\bar{A}.\bar{B}$

(c) A.B (d) A + B

26. In the given circuit, value of Y is :

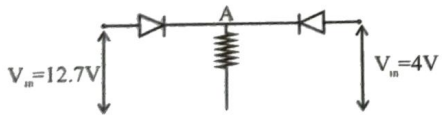


[JEE Main-2020 (January)]

(a) toggles between 0 and 1
(b) 0
(c) will not execute
(d) 1

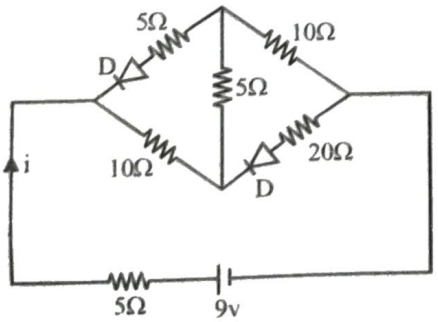
27. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased, built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in volts)at point A is ..... .

[JEE Main-2020 (January)]



28. The current i in the network is :

[JEE Main-2020 (January)]

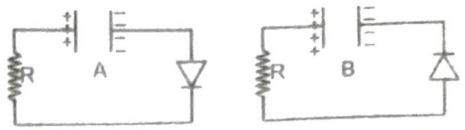


(a) 0.2A (b) 0.3A

(c) 0A (d) 0.6A

29. Two identical capacitor A and B, charged to the same potential 5V are connected in two different circuits as shown below at time t = 0. If the charge on capacitors A and B at time t = CR is Q_A and Q_B respectively, then (Here e is the base of natural logarithm)

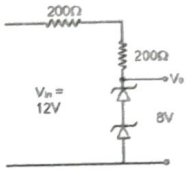
[JEE Main-2020 (January)]



(a) $Q_A = VC, Q_B = CV$ (b) $Q_A = VC, Q_B = VC/e$

(c) $Q_A = VC/e, Q_B = CV/2$ (d) $Q_A = CV/2, Q_B = VC/e$

30. The circuit shown is working as a 8V dc regulated voltage source. When 12V is used as input, the power dissipated (in mW)in each diode is: (considering both diodes are identical) [JEE Main-2020 (January)]



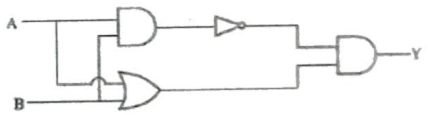
31. The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is:

[JEE Main-2021 (March)]

(a) $\beta=\frac{\alpha}{1+\alpha}$ (b) $\alpha=\frac{\beta}{1-\alpha}$

(c) $\alpha=\frac{\beta}{1+\beta}$ (d) $\beta=\frac{1}{1-\alpha}$

32. Which one of the following will be the output of the given circuit? [JEE Main-2021 (March)]

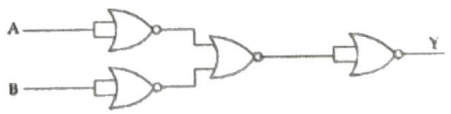


(a) XOR Gate (b) NOR Gate

(c) NAND Gate (d) AND Gate

33. The output of the given combination gates represents:

[JEE Main-2021 (March)]

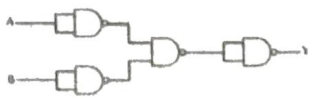


(a) AND Gate (b) NOR Gate

(c) NAND Gate (d) XOR Gate

34. The following logic gate is equivalent to:

[JEE Main-2021 (March)]



(a) NOR Gate (b) NAND Gate

(c) OR Gate (d) AND Gate

ANSWER KEY

Topicwise Questions

1. (*b*) 2. (*a*) 3. (*a*) 4. (*c*) 5. (*a*) 6. (*b*) 7. (*b*) 8. (*a*) 9. (*b*) 10. (*a*)
11. (*c*) 12. (*a*) 13. (*b*)

Learning Plus

1. (*d*) 2. (*a*) 3. (*d*) 4. (*b*) 5. (*b*) 6. (*b*) 7. (*a*) 8. (*b*) 9. (*b*) 10. (*b*)

11. (*d*) 12. [8654] 13. (*d*) 14.

Input		output
A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1

15. (*c*) 16. (I_C = 6.9 mA, I_B = 0.1 mA)

17. (*d*) 18. (*c*) 19. (*c*) 20. (*d*)

JEE Mains & Advanced Past Years Questions

JEE-MAINS PREVIOUS YEAR'S

1. (*c*) 2. (*a*) 3. (*b,d*) 4. (*b*) 5. (*b*) 6. (*b*) 7. (*a*) 8. (*c*) 9. (*c*) 10. (*a*)
11. (*d*) 12. (*b*) 13. (*a*) 14. (*a*) 15. (*c*) 16. (*b*) 17. (*c*) 18. (*d*) 19. (*d*) 20. (*b*)
21. (*a*) 22. (*c*) 23. (*d*) 24. (*c*) 25. (*b*) 26. (*b*) 27. [12] 28. (*b*) 29. (*b*) 30. [40]
31. (*c*) 32. (*a*) 33. (*c*) 34. (*a*)

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28 COMMUNICATION SYSTEMS

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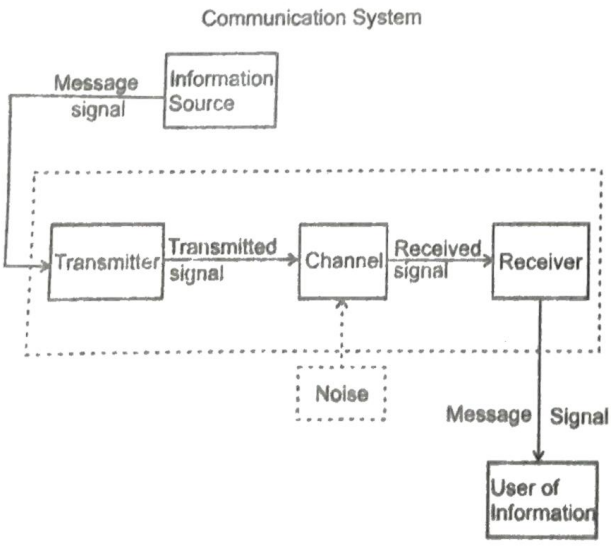
INTRODUCTION

Communication is the act of transmission of information. Every living creature in the world experiences the need to impart or receive information almost continuously with others in the surrounding world. For communication to be successful, it is essential that the sender and the receiver understand a common language. Man has constantly made endeavors to improve the quality of communication with other human beings. Languages and methods used in cummunication have kept evolving from prehistoric to modern times, to meet the growing demands in terms of speed and complexity of information.

ELEMENTS OF A COMMUNICATION SYSTEM

Communication pervades all stages of life of all living creatures. Irrespective of its nature, every communication system has three essential elements transmitter, medium/ channel and receiver. The block diagram shown in figure depicts the general form of a communication system.



In a communication system, the transmitter is located at one place, the receiver is located at some other place (far or near) separate from the transmitter and the channel is the physical medium that connects them. Depending upon the type of communication system, a channel may be in the form of wires or cables connecting the transmitter and the receiver or it may be wireless. The purpose of the transmitter is to convert the message signal produced by the source of information into a form suitable for transmission through the channel. If the output of the information source is a non-electrical signal like a voice signal, a tranducer converts it to electrical form before giving it as an input to the transmitter. When a transmitted signal propagates along the channel it may get distorted due to channel imperfection. Moreover, noise adds to the transmitted signal and the receiver receives a corrupted version of the transmitted signal. The receiver has the task of operating on the received signal. It reconstructs a recognisable form of the original message signal for delivering it to the user of information.

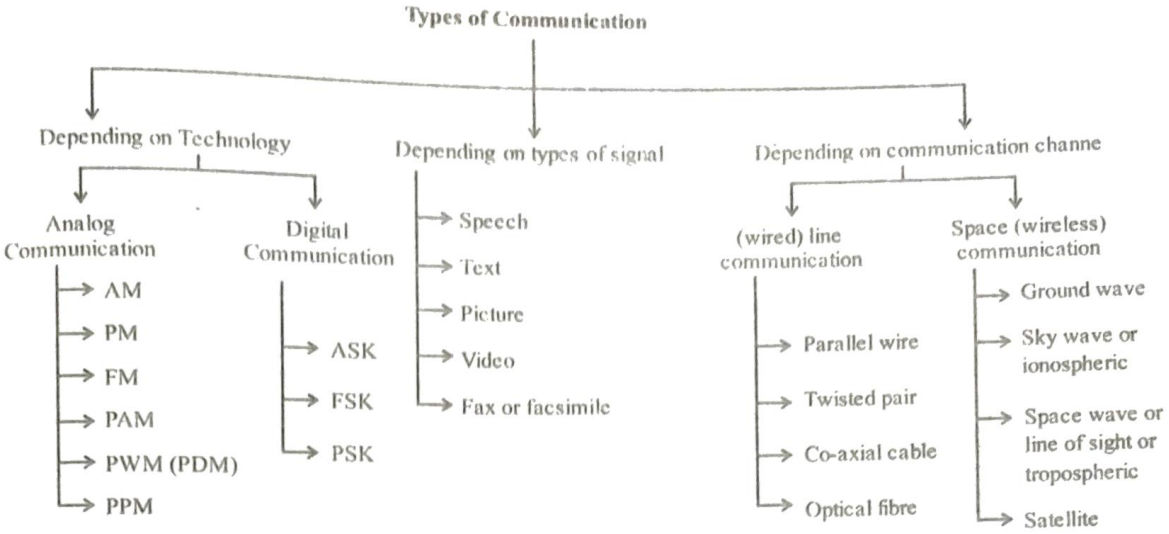
There are two basic modes of communication : Point -to-point and broadcast.

In point-to-point communication mode, communication takes place over a link between a single transmitter and a receiver. Telephony is an example of such a mode of communication. In contrast, in the broadcast mode, there are a large number of receivers corresponding to a single transmitter. Radio and television are example of broadcast mode of communication.

(1) Digital and analog signals to be transmitted are usually of low frequency and hence cannot be transmitted as such.

(2) These signals require some carrier to be transported. These carriers are known as carrier waves or high frequency signals.

(3) The process of placement of a low frequency (LF) signal over the high frequency (HF) signal is known as **modulation.**



BASIC TERMINOLOGY USED IN ELECTRONIC COMMUNICATION SYSTEMS

(i) **Transducer:** It is a device that converts one form of energy into another. In electronic communication systems, we usually come across devices that have either their inputs or outputs in the electrical form. An electrical transducer may be defined as a device that converts some physical variable (pressure, displacement, force, temperature, etc.) into corresponding variations in the electrical signal at its output.

(ii) **Signal :** It is the Information converted in electrical form and suitable for transmission. Signals can be either *analog* or *digital*. Analog signals are continuous variations of voltage or current. *They are essentially single-valued functions of time.* Sine wave is a fundamental analog signal. Digital signals are those which can take only discrete stepwise values. Binary system that is extensively used in digital electronics employs just two levels of a signal. '0' corresponds to a low level and '1' corresponds to a high level of voltage/ current.

(iii) **Noise:** The unwanted signals that tend to disturb the transmission and processing of message signals in a communication system is known as noise. The source generating the noise may be located inside or outside the system.

(iv) **Transmitter:** It processes the incoming message signal so as to make it suitable for transmission through a channel and subsequent reception.

(v) **Receiver:** It extracts the desired message signals from the received signals at the channel output.

(vi) **Attenuation :** It is the loss of strength of a signal while propagating through a medium.

(vii) **Amplification:** Amplification refers to the process of increasing the amplitude (and consequently the strength) of a signal using an electronic circuit called the amplifier. Amplification is necessary to compensate for the attenuation of the signal in communication systems.

(viii) **Range :** It is the largest distance between a source and a destination up to which the signal is received with sufficient strength.

(ix) **Bandwidth :** It is the frequency range over which an equipment operates or the portion of the spectrum occupied by the signal.

(x) **Modulation :** The original low frequency message/ information signal cannot be transmitted to long distances because of obvious reasons. Therefore, at the transmitter, information contained in the low frequency message signal is superimposed on a high frequency wave, which acts as a carrier of the information. This process is known as modulation. There are several types of modulation, abbreviated as AM, FM and PM.

(xi) **Demodulation :** It is the process of extraction of information from the carrier wave at the receiver. It is the reverse process of modulation.

(xii) **Repeater:** It is a combination of a receiver and a transmitter. A repeater, picks up the signal from the transmitter, amplifies and retransmits it to the receiver sometimes with a change in carrier frequency. Repeaters are used to extend the range of a communication system as shown in figure. A communication satellite is essentially a repeater station in space.

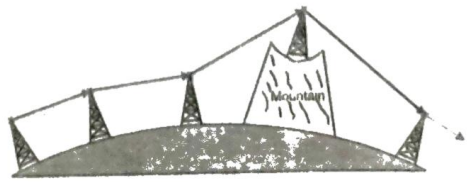


Fig. Use of repeater station to increase the range of communication

Note: The undesirable effects in the course of signal transmission are

(i) **Attenuation:** decrease in signal strength due to energy loss.

(ii) **Distortion:** waveform perturbation.

(iii) **Interference:** contamination by extraneous signals.

(iv) **Noise:** due to random electrical instrumeuted signals.

BANDWIDTH OF SIGNALS

The message signal can be voice, music, picture or computer data. In a communication system, each of these signal has different ranges of frequencies.

Table : Some important signal's frequency range

S. No.	Signals	Frequency band width
1	Speech	(3100 Hz-300 Hz) 2800 Hz
2	Music	20 kHz
3	Audible sound	(20 Hz to 20 kHz) = 20 kHz
4	Video	4.2 MHz
5	Voice + Picture	6 MHz

BANDWIDTH OF TRANSMISSION MEDIUM

Similar to massage signals, different types of transmission media offer different bandwidths. The commonly used transmission media are wire, free space and fiber optic cable. Coaxial cable is a widely used wire medium, which offers a bandwidth of approximately 750 MHz. Such cables are normally operated below 18 GHz. Communication through free space using radio waves takes place over a very wide range of frequencies: from a few hundreds of kHz to a few GHz. This range of frequencies is further subdivided and allocated for various services as indicated in Table 2. An optical fiber can offer a transmission bandwidth in excess of 100 GHz.

Table : Some important wireless communication frequency bands

Service	Frequency bands	Comments
Standard AM broadcast	540-1600 kHz	
FM broadcast	88-108 MHz	
Television	54-72 MHz	VHF (very high frequencies)
	76-88 MHz	TV
	174-216 MHz	UHF (ultra high frequencies)
	420-890 MHz	TV
Cellular Mobile Radio	896-901 MHz	Mobile to base station
	840-935 MHz	Base station to mobile
Satellite Communication	5.925-6.425 GHz	Uplink
	3.7-4.2 GHz	Downlink

PROPAGATION OF ELECTROMAGNETIC WAVES

An antenna at the transmitter in communication using radio waves, radiates the electromagnetic waves which travel through space and reach the receiving antenna at the other end. Several factors affect the propagation of em waves and the path, they follow.

Ground Wave

The antennas should have a size comparable to the wavelength λ of the signal (at least ~ $\lambda/4$) to radiate signals with high efficiency. At longer wavelengths (*i.e.*, at lower frequencies), the antennas have large physical size and they are located on or very near to the ground. In standard AM broadcast, ground based vertical, towers are generally used as transmitting antennas. The maximum range of coverage depends on the transmitted power and frequency (less than a few MHz).

Sky waves

Long distance communication can be achieved by ionospheric reflection of radio waves back towards the earth. Sky wave propagation is used by short wave broadcast services. The ionosphere is so called because of the presence of a large number of ions or charged particles. It extends from a height of ~ 65 km to about 400 km above the earth's surface. The ionosphere is further subdivided into several layers, the details of which are given in Table 4. The degree of ionisation varies with the height. The density of atmosphere decreases with height. The ionospheric layer acts as a reflector for a certain range of frequencies (3 to 30 MHz). Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape. These phenomena are shown in the figure. The phenomenon of bending of em waves so that they are diverted towards the earth is similar to total internal reflection in optics.

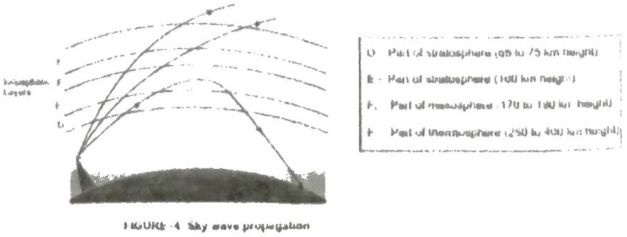


Figure : Sky wave propagation. The layer nomenclature is given in table

Table : Different layers of atmosphere and their interaction with the propagating electromagnetic waves

Name of the stratum (layer)	Approximate height over earth's surface	Exists during	Frequencies most affected
Troposphere	10 km	Day and night	VHF (up to several GHz)
D (part of stratosphere)	65-75 km	Day only	Reflects LF, absorbs MF and HF to some degree
E (part of Stratosphere)	100 km	Day only	Helps surface waves, reflects HF
F_1 (Part of Mesosphere)	170-190 km	Daytime, merges with F_2 at night	Partially absorbs HF waves yet allowing them to reach F_2
F_2 (Thermosphere)	300 km at night, 250-400 km during daytime	Day and night	Efficiently reflects HF waves, particularly at night

Space wave

Another mode of radio wave propagation is by space waves. A space wave travels in a straight line from transmitting antenna to the receiving antenna. Space waves are used for line-of-sight (LOS) communication as well as satellite communication. At frequencies above 40 MHz, communication is essentially limited to line-of-sight paths. At these frequencies, the antennas are relatively smaller and can be placed at heights of many wavelengths above the ground. Because of line-of-sight nature of propagation, direct waves get blocked at some point by the curvature of the earth as illustrated in figure.

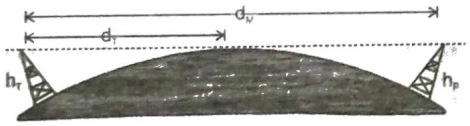


FIGURE-5: Line of sight communication by space waves.

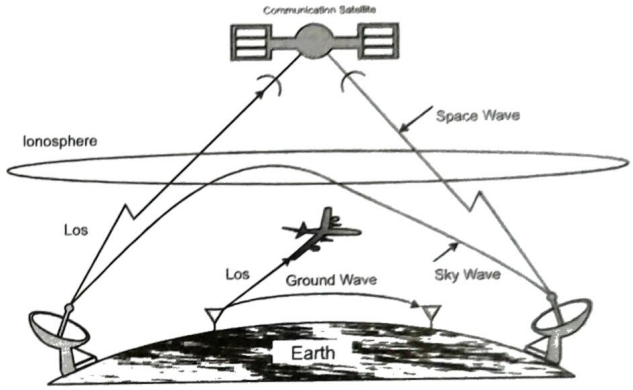
Fig. Line of sight communication by space waves

If the transmitting antenna is at a heigth h_T, then you can show that the distance to the horizon d_T is given as $d_T = \sqrt{2Rh_T}$, where R is the radius of the earth (approximately 6400 km). d_T is also called the radio horizon of the transmitting antenna. With reference to figure above the maximum line-of–sight distance d_M between the two antennas having height h_T and h_R above the earth is given by

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where h_R is the height of receiving antenna.

Television broadcast, microwave links and satellite communication are some examples of communication systems that use space wave mode of propagation. Figure summarises the various modes of wave propagation discussed so far.



FIGURE–6: Various propagation modes for em waves

TRAIN YOUR BRAIN

Example 1: In which frequency range, space waves are normally propagated

(*a*) HF (*b*) VHF

(*c*) UHF (*d*) SHF

Ans. (*c*)

Example 2: An antenna behaves as resonant circuit only when its length is

(*a*) $\frac{\lambda}{2}$

(*c*) $\frac{\lambda}{4}$

(*c*) λ

(*d*) $\frac{\lambda}{2}$ or integral multiple of $\frac{\lambda}{2}$

Ans. (*d*)

Example 3: The process of superimposing signal frequency (i.e. audio wave) on the carrier wave is known as

(*a*) Transmission (*b*) Reception
(*c*) Modulation (*d*) Detection

Ans. (*c*) Carrier + signal → modulation.

Example 4: Long distance short-wave radio broadcasting uses

(*a*) Ground wave (*b*) Ionospheric wave
(*c*) Direct wave (*d*) Sky wave

Ans. (*c*)

Example 5: The maximum distance up to which TV transmission from a TV tower of height h can be received is proportional to

(*a*) $h^{1/2}$ (*b*) h
(*c*) $h^{3/2}$ (*d*) h^2

Sol. (*a*) $d = \sqrt{2hR} \Rightarrow d \propto h^{1/2}$

Example 6: A transmitting antenna at the top of a tower has a height 32 m and the height of the receiving antenna is 50 m. What is the maximum distance between them for satisfactory communication in LOS mode? Given radius of earth 6.4×10^6 m.

Sol. $d_m = \sqrt{2\times64\times10^5\times32} + \sqrt{2\times64\times10^5\times50}$ m

$= 64 \times 10^2 \times \sqrt{10} + 8 \times 10^3 \times \sqrt{10}$ m

$= 144 \times 10^2 \times \sqrt{10}$ m $= 45.5$ km

Example 7: In a communication system, noise is most likely to affect the signal

(*a*) At the transmitter
(*b*) In the channel or in the transmission line
(*c*) In the information source
(*d*) At the receiver

Ans. (*b*)

Example 8: The waves used in telecommunication are

(*a*) IR (*b*) UV
(*c*) Microwave (*d*) Cosmic rays

Sol. (*c*) In telecommunication microwaves are used.

Example 9: Television signals on earth cannot be received at distances greater than 100 km from the transmission station. The reason behind this is that

(*a*) The receiver antenna is unable to detect the signal at a distance greater then 100 km
(*b*) The TV programme consists of both audio and video signals
(*c*) The TV signals are less powerful than radio signals
(*d*) The surface of earth is curved like a sphere

Ans. (*d*)

Modulation and its necessity

As already mentioned, the purpose of a communication system is to transmit information or message signals. Message signals are also called baseband signals, which essentially designate the band of frequencies representing the original signal, as delivered by the source of information. No signal, in general, is a single frequency sinusoid, but it spreads over a range of frequencies called the signal bandwidth. Suppose we wish to transmit an electronic signal in the audio frequency (AF) range (baseband signal frequency less than 20 kHz) over a long distance directly. Let us find what factors prevent us from doing so and how we overcome these factors.

Size of the antenna or aerial

For transmitting a signal, we need an antenna or an aerial. This antenna should have a size comparable to the wavelength of the signal (at least $\lambda/4$ in dimension) so that the antenna properly senses the time variation of the signal. For an electromagnetic wave of frequency 20 kHz. the wavelength λ is 15 km. Obviously, such a long antenna is not possible to construct and operate. Hence direct transmission of such baseband signals is not practical. We can obtain transmission with reasonable antenna lengths if transmission frequency is high (for example, if υ is 1MHz, then λ is 300 m). Therefore, there is a need of translating the information contained in our original low frequency baseband signal into high or radio frequencies before transmission.

Effective power radiated by an antenna

A theoretical study of radiation from a linear antenna (length ℓ) shows that the power radiated is proportional to $(\ell/\lambda)^2$. This implies that for the same antenna length, the power radiated increases with decreasing λ, i.e., increasing frequency. Hence, the effective power radiated by a long wavelength baseband signal would be small. For a good transmission, we need high powers and hence this also points out to the need of using high frequency transmission.

Mixing up of signals from different transmitters

Another important argument against transmitting basband signals directly is more practical in nature. Suppose many people are talking at the same time or many transmitters are transmitting baseband information signals simultaneously. As these signals will get mixed up and there is no simple way to distinguish between them. This points out towards a possible solution by using communication at high frequencies and allotting a band of frequencies to each message signal for its transmission.

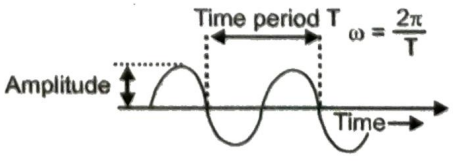


Figure : Sinusoidal

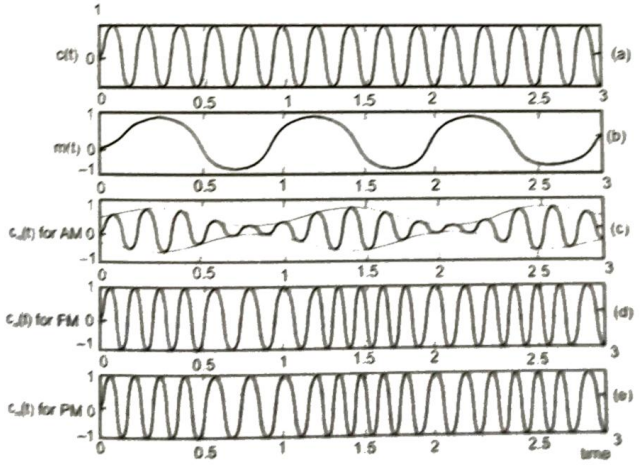
The above arguments suggest that there is a need for translating the original low frequency baseband message or information signal into high frequency wave before transmission such that the translated signal continues to possess the information contained in the original signal. In doing so, we take the help of a high frequency signal, known as the carrier wave, and a process known as modulation which attaches information to it. The carrier wave may be continuous (sinusoidal) or in the form of pulses as shown in figure below.



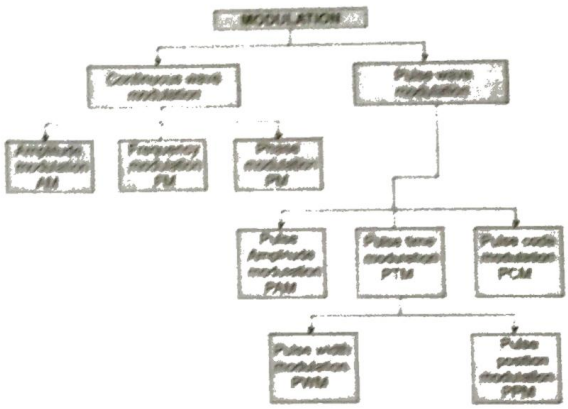
Figure: Pulse shaped signals

A sinusoidal carrier wave can be represented as $c(t) = A_C \sin(\omega_c t + \phi)$

where c(t) is the signal strength (voltage or current), A_C is the amplitude, ω_c ($= 2\pi\upsilon_c$) is the angular frequency and ϕ is the initial phase of the carrier wave. During the process of modulation, any of the three parameters, i.e. A_c, ω_c and ϕ, of the carrier wave can be controlled by the message or information signal. This results in three types of modulation. (i) Amplitude modulation (AM), (ii) frequency modulation (FM) and (iii) pulse modulation (PM) as shown below.



Similarly, the significant characteristics of a pulse are : pulse amplitude, pulse duration or pulse Width, and pulse position (denoting the time of rise or fall of the pulse amplitude) as shown in Fig. 15.7(b). Hence, different types of pulse modulation are : (a) pulse amplitude modulation (PAM), (b) pulse duration modulation (PDM) or pulse width modulation (PWM), and (c) pulse position modulation (PPM). In this chapter, we shall confine to amplitude modulation only.



Amplitude Modulation

In amplitude modulation the amplitude of the carrier is varied in accordance with the information signal. Here we explain amplitude modulation process using a sinusoidal signal as the modulating signal.

Let $c(t) = A_c \sin \omega_c t$ represent carrier wave and $m(t) = A_m \sin \omega_m t$ represent the message or the modulating signal where $\omega_m = 2\pi f_m$ is the angular frequency of the message signal. The modulated signal $c_m(t)$ can be written as

$$c_m(t) = (A_C + A_m \sin \omega_m t) \sin \omega_c t$$

$$= A_C \left(1 + \frac{A_m}{A_c} \sin \omega_m t\right) \sin \omega_c t \qquad ...(i)$$

Note that the modulated signal now contains the message signal. This can also be seen from figure. From Eq. (i), we can write,

$$c_m(t) = A_c \sin \omega_c t + \mu A_c \sin \omega_m t \sin \omega_c t \qquad ...(ii)$$

Here $\mu = A_m/A_c$ is the modulation index; in practice, μ is kept ≤ 1 to avoid distortion.

Using the trignomatric relation $\sin A \sin B = 1/2 (\cos (A - B) - \cos (A + B))$,

we can write $c_m(t)$ of Eq. (ii) as

$$c_m(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2} \cos(\omega_C - \omega_m) t$$

$$- \frac{\mu A_c}{2} \cos(\omega_C + \omega_m) t \qquad ...(iii)$$

Here $\omega_C - \omega_m$ and $\omega_C + \omega_m$ are respectively called the lower side and upper side frequencies. The modulated signal now consists of the carrier wave of frequency ω_c plus two sinusoidal waves each with a frequency slightly different from, known as side bands. The frequency spectrum of the amplitude modulated signal is shown in figure below.

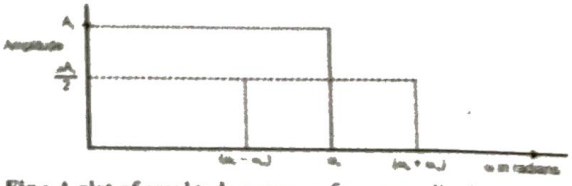


Fig.: A plot of amplitude versus ω for an amplitude modulated signal

As long as the broadcast frequencies (carrier waves) are sufficiently spaced out so that side bands do not overlap, different stations can operate without interferring with each other.

Modulation index : The ratio of change of amplitude of carrier wave to the amplitude of original carrier wave is called the modulation factor or degree of modulation or modulation index (m_a).

$$m_a = \frac{\text{Change in amplitude of carrier wave}}{\text{Amplitude of original carrier wave}} = \frac{kE_m}{E_c}$$

where k = A factor which determines the maximum change in the amplitude for a given amplitude E_m of the modulating. If k = 1 then

$$m_a = \frac{E_m}{E_c} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

If a carrier wave is modulated by several sine waves the total modulated index m_t is given by

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + ...}$$

Side band frequencies and band width in AM wave

(i) Side band frequencies : The AM wave contains three frequency f_c, $(f_c + f_m)$ and $(f_c - f_m)$, f_c is called carrier frequency,

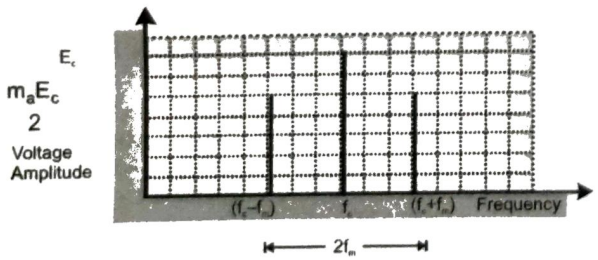
$(f_c + f_m)$: Upper side band (USB) frequency

$(f_c - f_m)$: Lower side band (LBS) frequency

Side band frequency are generally close to the carrier frequency.

(ii) Band width : The two side bands lie on either side of the carrier frequency at equal frequency interval f_m.

So, band width = $(f_c + f_m) - (f_c - f_m) = 2f_m$



Power in AM waves : Power dissipated in any circuit

$$P = \frac{V_{rms}^2}{R}.$$

Hence (i) carrier power $P_c = \frac{\left(\frac{E_c}{\sqrt{2}}\right)^2}{R} = \frac{E_c^2}{2R}$

(ii) Total power of side bands

$$P_{sb} = \frac{\left(\frac{m_a E_c}{2\sqrt{2}}\right)^2}{R} = \frac{m_a^2 E_c^2}{8R}$$

(iii) Total power of AM wave $P_{Total} = P_c + P_{ab}$

$$= \frac{E_c^2}{2R}\left(1 + \frac{m_a^2}{2}\right)$$

(iv) $\frac{P_t}{P_c} = \left(1 + \frac{m_a^2}{2}\right)$ and $\frac{P_{sb}}{P_t} = \frac{m_a^2/2}{\left(1 + \frac{m_a^2}{2}\right)}$

(v) Maximum power in the AM (without distortion) will occur when $m_a = 1$ i.e., $P_t = 1.5\ P = 3P_{ab}$

(vi) If I_c = Unmodulated current and I_t = total or modulated current

$$\Rightarrow \frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} \quad \Rightarrow \frac{I_t}{I_c} = \sqrt{\left(1 + \frac{m_a^2}{2}\right)}$$

Limitation of amplitude modulation

(i) Noisy reception

(ii) Low efficiency

(iii) Small operating range

(iv) Poor audio quality

PRODUCTION OF AMPLITUDE MODULATED WAVE

Amplitude modulation can be produced by a variety of methods. A conceptually simple method is shown in the block diagram of figure.

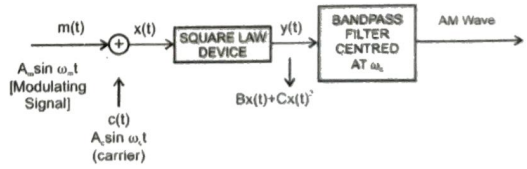


FIGURE-10: Block diagram of a simple modulator for obtaining an AM signal

Figure : Block diagram of simple modulator for obtaining and AM signal

Here the modulating signal $A_m \sin \omega_m t$ is added to the carrier signal $A_c \sin \omega_c t$ to produce the signal x(t). This signal $x(t) = A_m \sin \omega_m t + A_c \sin \omega_c t$ is passed through a square law device which is a non–linear device which produces an output

$y(t) = B\,x(t) + Cx^2(t)$

where B and C are constants. Thus,

$y(t) = BA_m \sin \omega_m t + BA_c \sin \omega_c t$

$$+ C[A_m^2 \sin^2 \omega_m t + A_c^2 \sin^2 \omega_c t + 2A_m A_c \sin\omega_m t \sin \omega_c t]$$
$$= BA_m \sin \omega_m t + BA_c \sin \omega_c t$$
$$+ \frac{CA_m^2}{2} + A_c^2 - \frac{CA_m^2}{2} \cos 2\omega_m t - \frac{CA_c^2}{2} \cos 2\omega_c t$$
$$+ CA_m A_c \cos(\omega_c - \omega_m)t - CA_m A_c \cos(\omega_c + \omega_m)\, t$$
$$\ldots.(v)$$

where the trigonometric relations

$\sin^2 A = (1 - \cos 2A)/2$ and the relation for sin A sin B mentioned earlier are used.

In equation (v), there is a dc term C/2 $(A_m^2 + A_c^2)$ and sinusoids of frequencies $\omega_m, 2\omega_m, \omega_c, 2\omega_c, \omega_c - \omega_m$ and $\omega_c + \omega_m$. As shown in figure this signal is passed through a band pass filter which rejects dc and the sinusoids of frequencies ω_m, $2\omega_m$ and $2\omega_c$ and retains the frequencies ω_c, $\omega_c - \omega_m$ and $\omega_c + \omega_m$. The output of the band pass filter therefore is of the same form as equation (iii) and is therefore an AM wave.

It is to be mentioned that the modulated signal cannot be transmitted as such. The modulator is to be followed by a power amplifier which provides the necessary power and then the modulated signal is fed to an antenna of appropriate size for radiation as shown in figure.

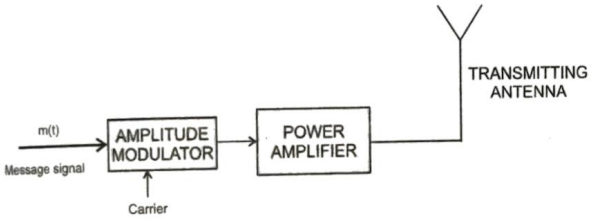


FIGURE-11: Block diagram of a transmitter

Figure : Block diagram of a transmitter

DETECTION OF AMPLITUDE MODULATED WAVE

The transmitted message gets attenuated in propagating through the channel. The receiving antenna is therefore to be followed by an amplifier and a detector. In addition, to facilitate further processing, the carrier frequency is usually changed to a lower frequency by what is called an intermediate frequency (IF) stage preceding the detection. The detected signal may not be strong enough to be made use of and hence is required to be amplified. A block diagram of a typical receiver is shown in figure.

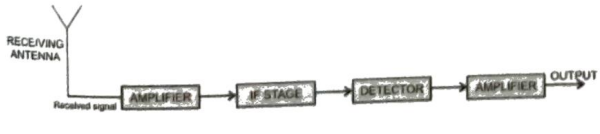


FIGURE-12: Block diagram of a receiver

Figure : Block diagram of a receiver

Detection is the process of recovering the modulating signal from the modulated carrier wave. We just saw that the modulated carrier wave contains the frequencies ω_c and $\omega_c \pm \omega_m$. In order to obtain the original message signal m(t) of angular frequency ω_m a simple method is shown in the form of a block diagram in figure below.

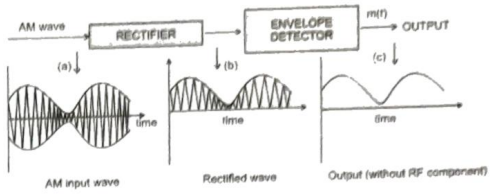


FIGURE-13: Block diagram of a detector for AM signal. The quantity on y-axis can be current or voltage

Figure: Block diagram of a detector for AM signal. The quantity of y-axis can be current or voltage

The modulated signal of the form given in (a) of figure-13 is passed through a rectifier to produce the output shown in (b). This envelope of signal (b) is the message signal. In order to retrieve m(t), the signal is passed through an envelope detector (which may consist of a simple RC circuit).

TRAIN YOUR BRAIN

Example 10: AM is used for broadcasting because

(*a*) It is more noise immune than other modulation systems

(*b*) It requires less transmitting power compared with other systems

(*c*) Its use avoids receiver comlexity

(*d*) No other modulation system can provide the necessary bandwidth faithful transmission

Ans. (*c*)

Example 11: Range of frequencies allotted for commercial FM radio broadcast is

(*a*) 88 to 108 MHz (*b*) 88 to 108 kHz

(*c*) 8 to 88 MHz (*d*) 88 to 108 GHz

Ans. (*a*)

Example 12: The velocity factor of a transmission line x. If dielectric constant of the medium is 2.6, the value of x is

(*a*) 0.26 (*b*) 0.62

(*c*) 2.6 (*d*) 6.2

Sol. velocity factor $= \frac{1}{\sqrt{k}} = \frac{1}{\sqrt{2.6}} = 0.62$

Example 13: If μ_1 and μ_2 are the refractive indices of the materials of core and cladding of an optical fibre, then refraction of light due to its leakage can be minimised by having

(*a*) $\mu_1 > \mu_2$ (*b*) $\mu_1 < \mu_2$

(*c*) $\mu_1 = \mu_2$ (*d*) None of these

Ans. (*a*)

Example 14: The radio waves of frequency 300 MHz to 3000 MHz belong to

(*a*) High frequency band

(*b*) Very high frequency band

(*c*) Ultra high frequency band

(*d*) Super high frequency band

Ans. (*c*)

Example 15: The maximum peak to peak voltage of an AM wire is 24 mV and the minimum peak to peak voltage is 8 mV. The modulation factor is

(*a*) 10% (*b*) 20%

(*c*) 25% (*d*) 50%

Sol. (*d*) Here, $V_{max} = \frac{24}{2} = 12$ mV

and $V_{min} = \frac{8}{2} = 4$ mV

Now, $m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{12-4}{12+4} = \frac{8}{16} = \frac{1}{2} = 0.5$

$= 50\%$

Example 16: If a number of sine waves with modulation indices n_1, n_2, n_3 .....modulate a carrier wave, then total modulation index (n) of the wave is

(*a*) $n_1 + n_2$ .....+ $2(n_1 + n_2$ .....)

(*b*) $\sqrt{n_1 - n_2 + n_3 .....}$

(*c*) $\sqrt{n_1^2 + n_2^2 + n_3^2 .....}$

(*d*) None of these

Ans. (*c*)

Example 17: An AM wave has 1800 watt of total power content, for 100% modulation the carrier should have power content equal to

(*a*) 1000 watt (*b*) 1200 watt

(*c*) 1500 watt (*d*) 1600 watt

Sol. (*b*) $P_t = P_c\left(1+\frac{m_a^2}{2}\right)$; Here $m_a = 1$

$\Rightarrow 1800 = P_c\left(1+\frac{(1)^2}{2}\right) \Rightarrow P_c = 1200$ W

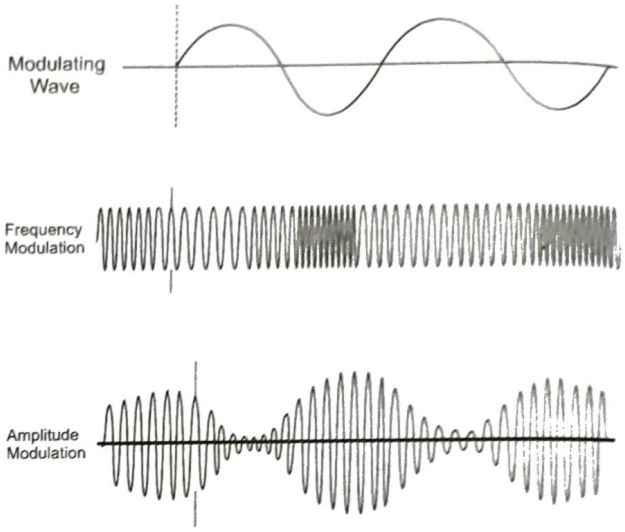
FREQUENCY MODULATION

It is the second type of continuous wave or sinusoidal wave modulation.

In this mode of modulation, the frequency of the carrier signal varies in accordance with the modulating signal. The amplitude of the carrier wave is fixed while its frequency is changing.

In frequency modulation, the frequency of the carrier wave is modified in accordance with the amplitude of the modulating wave.

The amplitidue of the carrier remains unchanged at all times. In other words, the amplitude of the modulated wave remains the same as the amplitude of the carrier wave. The frequency of the carrier is made to fluctuate symmetrically above and below its unmodulated frequency. As an example, a carrier frequency, of 1000 kHz may be caused to swing between 925 kHz and 1075 kHz



or any other amount chosen in accordance with the signal voltage. In frequency modulation, the deviation of the carrier frequency from its average value is proportional to the instantaneous amplitude of the modulating signal. When the signal voltage is zero, the carrier frequency is unchanged. When the signal approaches its positive peaks, the carrier frequency is increased to maximum as indicated by the closely spaced cycles. However, during the negative peaks of signal, the carrier frequency is reduced to minimum as shown by widely spaced cycles.

NECESSITY OF FREQUENCY MODULATION

1. Various electrical machines and noises cause amplitude disturbance in the transmission of amplitude-modulated wave. This makes the reception noisy. So, there is a need for different type of modulation which can reduce the noise factor. Frequency modulation (FM) was proposed as a means of improving the signal-to-noise ratio of a radio system. The first practical system was put forward in 1936 as an alternative to AM in an effort to make radio transmissions more resistant to noise.
2. Fidelity or audio quality of amplitude modulated transmission is poor. This type of transmission is also not good for musical programmes. There is a need to eliminate amplitude-sensitive noise. This is possible if we eliminate amplitude variation. In other words, there is a need to keep the amplitude of the carrier constant. This is precisely what we do in frequency modulation.

FM is preferred for transmission of music

Frequency modulation (FM) gives better quality transmission and has a larger bandwidth. In FM signals, the intelligence (information or message signal) is in the form of frequency variations and, therefore, the atmospheric or man-made noises (which are generally amplitude changes) do little harm. It is preferred for transmission of music. The range of frequencies allotted for commercial FM radio and TV -broadcast are given in table below

Range of Frequencies allotted for FM Radio and TV Broadcast:

Nature of broadcast	Frequency band
FM radio	88 to 108 MHz
VHF TV	47 to 230 MHz
UHF TV	470 to 960 MHz

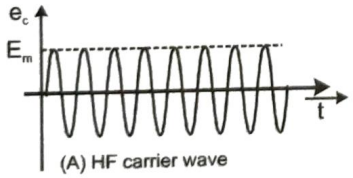
Frequency Modulation (FM)

The process of changing the frequency of a carrier wave in accordance with the audio frequency signal is known as frequency modulation

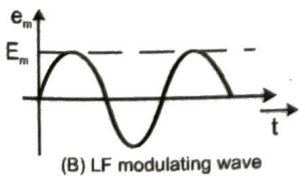
(1) Audio quality of AM transmission is poor. There are need to eliminate amplitude sensitive noise. This is possible if we eliminate amplitude variation. (i.e., a need to keep the amplitude of the carrier constant). This is precisely what we do in FM.

(2) In FM the overall amplitude of FM wave remains constant at all times.

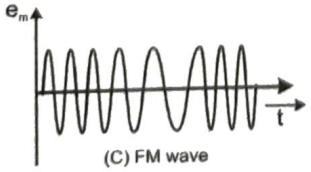
(3) In FM the total transmitted power remains constant.



(A) HF carrier wave



(B) LF modulating wave



(C) FM wave

(4) **Frequency deviation:** The maximum change in frequency from mean value (v_c) is known as frequency deviation. This is also the change or shift either above or below the frequency v_c and is called as frequency deviation.

$$\therefore \delta = (f_{max} - f_c) = f_c - f_{min} = k_f \cdot \frac{E_m}{2\pi}$$

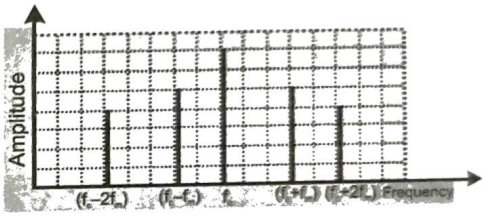
k_f = Constant of proportionality. It determines the maximum variation in frequency of the modulated wave for a given modulating signal.

(5) **Carrier swing (CS):** The total variation in frequency from the lowest to the highest is called the carrier swing i.e., $CS = 2 \times Df$

(6) **Frequency modulation index (m_f) :** The ratio of maximum frequency deviation to the modulating frequency is called modulation index.

$$m_f = \frac{\delta}{f_m} = \frac{f_{max} - f_c}{f_m} = \frac{f_c - f_{min}}{f_m} = \frac{k_f E_m}{f_m}$$

(7) **Frequency spectrum:** FM side band modulated signal consist of infinite number of side bands whose frequencies are $(f_c \pm f_m)$, $(f_c \pm 2f_m)$, $(f_c \pm 3f_m)$..... The number of side bands depends on the modulation index m_f.



In FM signal, the information (audio signal) is contained in the side bands. Since the side bands are separated from each other by the frequency of modulating signal f_m so

Band width $= 2n \times f_m$;

where n = number of significant side band pairs

(8) **Deviation ratio:** The ratio of maximum permitted frequency deviation to the maximum permitted audio frequency is known as deviation ratio. Thus, deviation

$$\text{ratio} = \frac{(\Delta f)_{max}}{(f_m)_{max}}$$

(9) **Percent modulation :** The ratio of actual frequency deviation to the maximum allowed frequency deviation is defined as percent modulation. Thus,

$$\text{percent modulation, } m = \frac{(\Delta f)_{actual}}{(\Delta f)_{max}}$$

FREQUENCY-MODULATED COMMUNICATION (Height of Transmitting Antenna)

The TV signals are frequency-modulated. Their transmission cannot be obtained by ground wave propagation. This is because the signals get absorbed by ground due to their high frequency. The transmission via sky wave propagation is also not desirable. This is becasuse the ionosphere is unable to reflect radio waves of frequencies greater than 40 MHz.

The only way for the transmission of TV signals is that the receiving antenna should directly intercept the signal from the transmitting antenna.

The transmitted waves, travelling in a straight line, directly reach the receiver end and are then picked up by the receiving antenna as shown in figure. It can be seen that due to the finite curvature of the earth, such waves cannot be seen beyond the tangent points R_1 and R_2. The effective reception range of the broadcast is essentially the region from R_1 to R_2 which is covered by the line of sight in a conventional sense. Hence, sometimes this mode of communication is termed as **line of sight communication.**

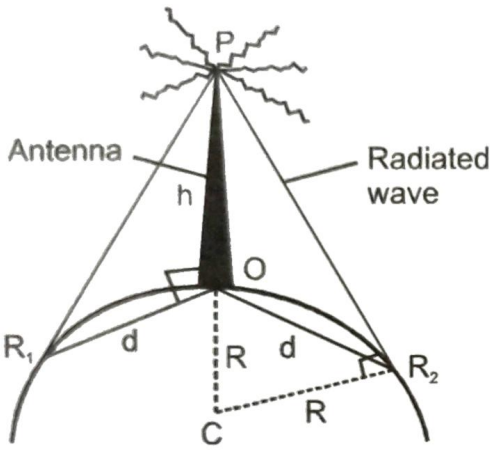


Figure: Ray-path of transmitted waves following space-wave (or line of sight) mode of propagation. The transmitter is located at the ground on a tall tower.

For large TV coverage, the transmission of TV signal is done from a tall antenna.

Consider a TV antenna OP of height h. The transmitted signal cannot be received beyond points R_1 and R_2. This is due to curvature of earth.

Clearly, OR_1 and OR_2 are the maximum distances, from the antenna, upto which the transmission signal can be received.

Let $OR_1 = OR_2 = d$[d is half the total range]

The TV signal will be received in a circle of radius d.

The relation between height h of the TV antenna and the maximum distance d upto which the TV signal can be received can be derived on the basis of geometrical considerations.

From right-angled triangle CR_2P, $CP^2 = CR_2^2 + PR_2^2$

In right-angled trianglePOR_2, $PR_2^2 = h^2 + d^2$

[**Note** that $\angle POR_2$ can be taken as right angle.]

Also, $CR_2 = CO = R$ and $CP = R + h$

From Equation (1),

$$(R + h)^2 = R^2 + (h^2 + d^2)$$

or $R^2 + h^2 + 2Rh = R^2 + h^2 + d^2$ or

$$d = \sqrt{2Rh}$$

It is clear from this equation that if h is large, d will be large. This explains as to why the television broadcasts are made from tall antennas.

If one wishes to send signals at far away stations, then either Repeater transmitting stations are necessary or h is increased (by locating the transmitter on a satellite). However, much before the advent of satellites, radio broadcast covered distances much longer than the line of sight propagation. This was found possible due to the presence of an ionised layer in the upper portion of earth's atmosphere called ionosphere. This mode of propagation known as ionospheric propagation or sky wave propagation.

TRAIN YOUR BRAIN

Example 18: A TV tower has a height of 75m. What is the maximum distance and area up to which this TV transmission can be received? Take radius of the earth as 6.4×10^6 m.

Sol. $d = \sqrt{2Rh}$

$$= \sqrt{2 \times 6.4 \times 10^6 \times 75} = 3.1 \times 10^4 \text{ m} = 31 \text{ km}$$

Area covered $= \pi r^2 = 3018 \text{ km}^2 = \pi d^2$

Example 19: A TV tower has a height of 100 m. How much population is covered by the TV broadcast if the average population density around the tower is 1000 km^{-2} ?

Given : radius of earth = 6.37×10^6 m.

Sol. h = 100 m, R = 6.37×10^6 m, Average population density = 1000 km^{-2} = 1000 $(10^3)^{-2}$ m^{-2} = 10^{-3} m^{-2}

Distance up to which the transmission could be viewed,

$d = \sqrt{2hR}$

Total area over which transmission could be viewed

$= \pi d^2 = 2\pi hR$

Population covered = $10^{-3} \times 2\pi hR$

= $10^{-3} \times 2 \times 3.14 \times 100 \times 6.37 \times 10^6$ = 40 lakh

Pulse Modulation

Here the carrier wave is in the form of pulses.

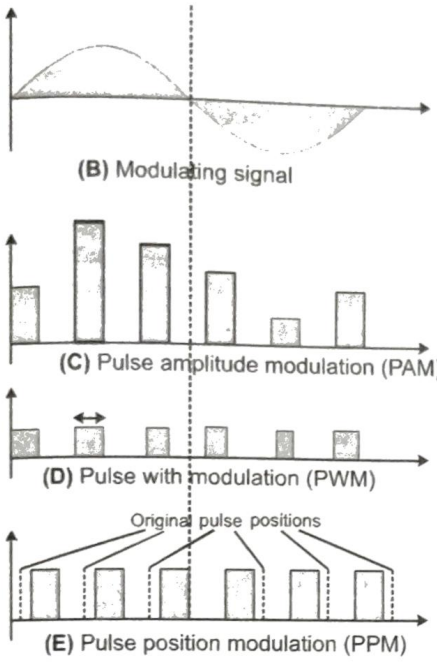
(1) **Pulse amplitude modulation (PAM) :** The amplitude of the pulse varies in accordance with the modulating signal.

(2) **Pulse width modulation (PWM) :** The pulse duration varies in accordance with the modulating signal.

(3) **Pulse position modulation (PPM) :** In PPM, the position of the pulses of the carrier wave train is varied in accordance with instantaneous value of the modulating signal.



(A) Unmodulated pulse



Pulse code Modulation

The pulse amplitude, pulse width and pulse position modulations not completely digital.

A completely digital modulation is obtained by pulse code modulation (PCM).

An analog signal is pulse code modulated by following three operation.

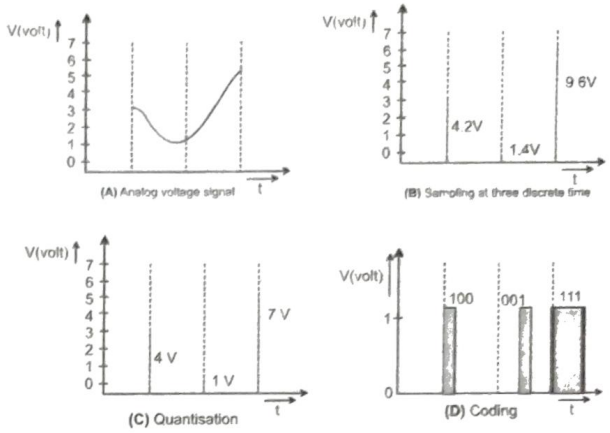
(1) **Sampling:** It is the process of generating pulses of zero width and of amplitude equal to the instantaneous amplitude of the analog signal.
The number of samples taken per second is called sampling rate.

(2) **Quantisation:** The process of dividing the maximum amplitude of the analog voltage signal into a fixed number of levels is called quantisation.
i.e. amplitude 5 V of the analog voltage signal divides into six. Quantisation level viz 0,1, 2, 3, 4, 5.
Pulses having amplitude between – 0.5 V to 0.5 V are approximated (quantised) to a value 0 V, amplitude between 0.5 V to 1.5 V are approximated to a value of 1 V and so on.

(3) **Coding:** The process of digitising the quantised pulses according to some code is called coding.

Quantized Level	0	1	2	3	4	5	6	7
Binary Code	000	001	010	011	100	101	110	111

For example consider that voltage amplitude of analog signal varies between 0 and 7 V.



Demodulation

The process of extracting the audio signal from the modulated wave is known as demodulation or detection.

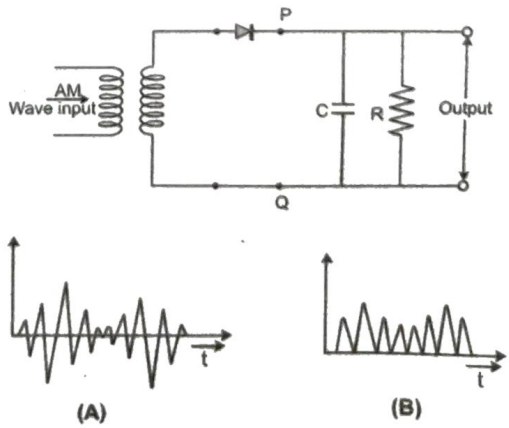
The wireless signals consist of radio frequency (high frequency) carrier wave modulated by frequency (low frequency). The diaphragm of a telephone receiver of a loud speaker cannot vibrate with high frequency. So it is necessary to separate the audio frequenies from the radio frequency carrier wave.

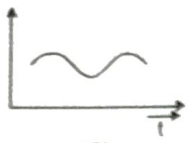
Simple demodulator circuit

A diode can be used to detect or demodulate an amplitude modulated (AM) wave. A diode basically acts as a rectifier i.e., it reduces the modulated carrier wave into positive envelope only.

The AM wave input is shown in figure. It appears at the output of the diode across PQ as a rectified wave (since a diode conducts only in the positive half cycle). This rectified wave after passing through the RC network does not contain the radio frequency carrier componet. Instead, it has only the envelope of the modulated wave.

In the actual circuit the value of RC is chosen such that $\frac{1}{f_c} << RC$; where f_c = frequency of carrier signal.





(C)

Data Transmission and Retrieval

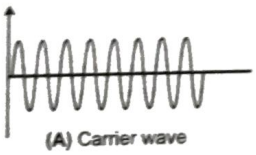
The term data is applied to a representation of facts, concepts or instructions suitable for communication interpretation or processing by human beings or by automatic means. Data in most cases consists of pulse type of signals.

The pulse code modulated (PCM) signal is a series of 1's and 0's. The following three modulation techinques are used to transmit a PCM signal.

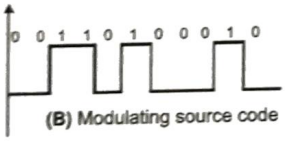
(1) **Amplitude shift keying (ASK) :** Two different amplitudes of the carrier represent the two binary values of the PCM signal. This method is also known as on-off keying (OOK)
(1) : Presence of carrier of same constant amplitude.
(0) : Carrier of zero amplitude.

(2) **Frequency shift keying (FSK) :** The binary values of the PCM signal are represented by two frequencies.
(1) : Increase in frequency
(0) : Frequency unaffected

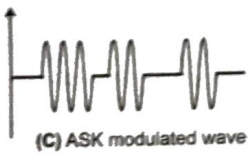
(3) **Phase shift keying (PSK) :** The phase of the carrier wave is changed in accordance with modulating data signal.
(1) : Phase changed by p
(0) : Phase remains unchanged.



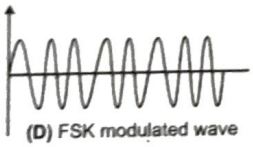
(A) Carrier wave



(B) Modulating source code



(C) ASK modulated wave



(D) FSK modulated wave



(E) PSK modulated wave

TRAIN YOUR BRAIN

Example 20: What is the modulation index of an over modulated wave

(*a*) 1 (*b*) zero
(*c*) < 1 (*d*) > 1

Sol. (*d*) When $m_a > 1$ then carrier is said to be over modulated.

Example 21: Which of the following is the disadvantage of FM over AM

(*a*) Larger band width requirement
(*b*) Larger noise
(*c*) Higher modulation power
(*d*) Low efficiency

Sol. (*a*) Frequency modulation requires much wider channel (7 to 15 times) as compared to AM.

Example 22: When the modulating frequency is doubled, the modulation index is halved and the modulating voltage constant the modulation system is

(*a*) Amplitude modulation
(*b*) Phase modulation
(*c*) Frequency modulation
(*d*) All of the above

Ans. (*c*)

Example 23: Indicate which one of the following system is digital

(*a*) Pulse position modulation
(*b*) Pulse code modulation
(*c*) Pulse width modulation
(*d*) Pulse amplitude modulation

Ans. (*b*)

Example 24: In an FM system a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is

(*a*) 7.143 (*b*) 8
(*c*) 0.71 (*d*) 350

Ans. (*a*)

Example 25: Sinusoidal carrier voltage of frequency 1.5 MHz and amplitude 50 V is amplitude modulated by sinusiodal voltage of frequency 10 kHz producing 50% modulation. The lower and upper side-band frequencies in kHz are

(*a*) 1490, 1510 (*b*) 1510, 1490
(*c*) $\frac{1}{1490}, \frac{1}{1510}$ (*d*) $\frac{1}{1510}, \frac{1}{1490}$

Sol. (*a*) Here, $f_c = 1.5$ MHz = 1500 kHz, $f_m = 10$ kHz

$\therefore$ Lower side band frequency

$= f_c - f_m = 1500 \text{ kHz} - 10 \text{ kHz} = 1490 \text{ kHz}$

Upper side band frequency

$= f_c + f_m = 1500 \text{ kHz} + 10 \text{ kHz} = 1510 \text{ kHz}$

Example 26: If f_0 and f_f represent the carrier wave frequencies for amplitude and frequency modulations respectively, then

(*a*) $f_0 > f_f$ (*b*) $f_0 < f_f$
(*c*) $f_0 = f_f$ (*d*) $f_0 \geq f_f$

Ans. (*b*)

Example 27: The frequency of a FM transmitter without signal input is called

(*a*) Lower side band frequency
(*b*) Upper side band frequency
(*c*) Resting frequency
(*d*) None of these

Ans. (*c*)

Example 28: What type of modulation is employed in India for radio transmission

(*a*) Amplitude modulation (*b*) Frequency modulation
(*c*) Pulse modulation (*d*) None of these

Ans. (*a*)

Example 29: While tuning in a certain broadcast station with a receiver, we are actually

(*a*) Varying the local oscillator frequency
(*b*) Varying the frequency of the radio signal to be picked up
(*c*) Tuning the antenna
(*d*) None of these

Ans. (*a*)

Example 30: Consider telecommunication through optical fibres. Which of the following statements is not true

(*a*) Optical fibres may have homogeneous core with a suitable cladding
(*b*) Optical fibres can be of graded refractive index
(*c*) Optical fibres are subject to electromagnetic interference from outside
(*d*) Optical fibres have extremely low transmission loss

Ans. (*c*)

Example 31: The phenomenon by which light travels in an optical fibres is

(*a*) Reflection
(*b*) Refraction
(*c*) Total internal reflection
(*d*) Transmission

Ans. (*c*)

Topicwise Questions

MODULATION, ANTENNA, OPTICAL FIBRE

1. The process of superimposing signal frequency (i.e. audio wave) on the carrier wave is known as
   (*a*) Transmission (*b*) Reception
   (*c*) Modulation (*d*) Detection
2. A step index fibre has a relative refractive index of 0.88%. What is the critical angle at the corecladding interface
   (*a*) 60° (*b*) 75°
   (*c*) 45° (*d*) None of these
3. In short wave communication waves of which of the following frequencies will be reflected back by the ionospheric layer, having electron density 10^{11} per m^3
   (*a*) 2 MHz (*b*) 10 MHz
   (*c*) 12 MHz (*d*) 18 MHz
4. An antenna behaves as resonant circuit only when its length is
   (*a*) $\frac{\lambda}{2}$
   (*b*) $\frac{\lambda}{4}$
   (*c*) λ
   (*d*) $\frac{\lambda}{2}$ or integral multiple of $\frac{\lambda}{2}$
5. An oscillator is producing FM waves of frequency 2 kHz with a variation of 10 kHz. What is the modulating index
   (*a*) 0.20 (*b*) 5.0
   (*c*) 0.67 (*d*) 1.5
6. Which of the following is the disadvantage of FM over AM
   (*a*) Larger band width requirement
   (*b*) Larger noise
   (*c*) Higher modulation power
   (*d*) Low efficiency
7. When the modulating frequency is doubled, the modulation index is halved and the modulating voltage remains constant, the modulation system is
   (*a*) Amplitude modulation
   (*b*) Phase modulation
   (*c*) Frequency modulation
   (*d*) All of the above
8. What type of modulation is employed in India for radio transmission?
   (*a*) Amplitude modulation
   (*b*) Frequency modulation
   (*c*) Pulse modulation
   (*d*) None of these

Learning Plus

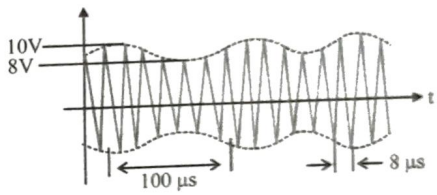
1. The phenomenon by which light travels in an optical fibres is
   (*a*) Reflection (*b*) Refraction
   (*c*) Total internal reflection (*d*) Transmission
2. Consider telecommunication through optical fibres. Which of the following statements is not true
   (*a*) Optical fibres may have homogeneous core with a suitable cladding
   (*b*) Optical fibres can be of graded refractive index
   (*c*) Optical fibres are subject to electromagnetic interference from outside
   (*d*) Optical fibres have extremely low transmission loss
3. In an amplitude modulated wave for audio frequency of 500 cycle/second, the appropriate carrier frequency will be
   (*a*) 50 cycle/second (*b*) 100 cycle/second
   (*c*) 500 cycle/second (*d*) 50,000 cycle/second
4. In which frequency range, space waves are normally propagated
   (*a*) HF (*b*) VHF
   (*c*) UHF (*d*) SHF
5. Sinusoidal carrier voltage of frequency 1.5 MHz and amplitude 50 V is amplitude modulated by sinusoidal voltage of frequency 10 kHz producing 50% modulation. The lower and upper side-band frequencies in kHz are
   (*a*) 1490, 1510 (*b*) 1510, 1490
   (*c*) $\frac{1}{1490}, \frac{1}{1510}$ (*d*) $\frac{1}{1510}, \frac{1}{1490}$
6. Basically, the product modulator is
   (*a*) An amplifier
   (*b*) A mixer
   (*c*) A frequency separator
   (*d*) A phase separator

7. An antenna is a device
   (*a*) That converts electromagnetic energy into radio frequency signal
   (*b*) That converts radio frequency signal into electromagnetic energy
   (*c*) That converts guided electromagnetic waves into free space electromagnetic waves and vice-versa
   (*d*) None of these
8. While tuning in a certain broadcast station with a receiver, we are actually
   (*a*) Varying the local oscillator frequency
   (*b*) Varying the frequency of the radio signal to be picked up
   (*c*) Tuning the antenna
   (*d*) None of these
9. In an FM system a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is
   (*a*) 7.143
   (*b*) 8
   (*c*) 0.71
   (*d*) 350
10. Advantage of optical fibre
   (*a*) High bandwidth and EM interference
   (*b*) Low bandwidth and EM interference
   (*c*) High bandwidth, low transmission capacity and no EM interference
   (*d*) High bandwidth, high data transmission capacity and no EM interference

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. Choose the correct statement:
   (*a*) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
   (*b*) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
   (*c*) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
   (*d*) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.
2. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?
   (*a*) $\omega_m + \omega_c$ (*b*) $\omega_c - \omega_m$
   (*c*) ω_m (*d*) ω_c
3. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?
   (*a*) 2×10^4 (*b*) 2×10^5
   (*c*) 2×10^6 (*d*) 2×10^3
4. An amplitude modulated signal is plotted below:



   Which one of the following best describes the above signal?
   (*a*) $(9 + \sin(2.5\pi \times 10^5 t))\sin(2\pi \times 10^4 t)$ V
   (*b*) $(1 + 9\sin(2\pi \times 10^4 t))\sin(5\pi \times 10^5 t)$ V
   (*c*) $(9 + \sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)$ V
   (*d*) $(9 + \sin(4\pi \times 10^4 t))\sin(5\pi \times 10^5 t)$ V
5. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier waves. If another AM station approaches you for license what broadcast frequency will you allot?
   (*a*) 2750 kHz (*b*) 2900 kHz
   (*c*) 2250 kHz (*d*) 2000 kHz
6. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accommodated for transmitting TV signals of band width 6 MHz are
   (Take velocity of light $c = 3 \times 10^8$ m/s, $h = 6.6 \times 10^{-34}$ J-s)
   (*a*) 3.75×10^6 (*b*) 3.86×10^6
   (*c*) 6.25×10^5 (*d*) 8.87×10^5

7. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index?
(a) 0.3 (b) 0.5
(c) 0.6 (d) 0.4

8. To double the covering range of a TV transmission tower, its height should be multiplied by :
(a) $\frac{1}{\sqrt{2}}$ (b) 2
(c) 4 (d) $\sqrt{2}$

9. An amplitude modulated signal is given by $V(t) = 10\,[1 + 0.3\cos(2.2 \times 10^4\, t)\sin(5.5 \times 10^5)t]$. Here t is in seconds.
The sideband frequencies (in kHz) are, [Given $\pi = 22/7$]
(a) 1785 and 1715 (b) 178.5 and 171.5
(c) 89.25 and 85.75 (d) 892.5 and 857.5

10. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode?
(**Given** : radius of earth = 6.4×10^6 m).
(a) 65 km (b) 48 km
(c) 80 km (d) 40 km

11. The physical sizes of the transmitter and receiver antenna in a communication system are:
(a) proportional to carrier frequency
(b) inversely proportional to modulation frequency
(c) inversely proportional to carrier frequency
(d) independent of both carrier and modulation frequency

12. The wavelength of the carrier waves in a modern optical fiber communication network is close to:
(a) 600 nm (b) 900 nm
(c) 2400 nm (d) 1500 nm

13. In a line of sight radio communication, a distance of about 50km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be :
(Radius of the Earth = 6.4×10^6 m).
(a) 40 m (b) 51 m
(c) 32 m (d) 20m

14. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is:
(a) $v_0 \sin\omega_0 t + A\cos\omega t$
(b) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$
(c) $(v_0 + A)\cos\omega t \sin\omega_0 t$
(d) $v_0 \sin[\omega_0(1 + 0.01A\sin\omega t)]$

15. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are :
(a) 4 ; 1×10^8 Hz (b) 0.25 ; 1×10^8 Hz
(c) 4 ; 2×10^8 Hz (d) 0.25 ; 2×10^8 Hz

16. In an amplitude modulator circuit, the carrier wave is given by, $C(t) = 4\sin(20000\,\pi t)$ while modulating signal is given by, $m(t) = 2\sin(2000\,\pi t)$. The values of modulation index and lower side band frequency are:
(a) 0.5 and 9 kHz (b) 0.5 and 10 kHz
(c) 0.3 and 9 kHz (d) 0.4 and 10 kHz

17. An amplitude modulated wave is represented by the expression $v_m = 5(1 + 0.6\cos 6280t)\sin(211 \times 10^4 t)$ volts The minimum and maximum amplitudes of the amplitude modulated wave are, respectively.
(a) 5 V, 8 V (b) $\frac{3}{2}$ V, 5V
(c) 3 V, 5 V (d) $\frac{5}{2}$ V, 8V

18. The correct match between the entries in column I and column II are

Column-I	**Column-II**
Radiation	**Wavelength**
(a) Microwave	(i) 100 m
(b) Gamma rays	(ii) 10^{-15} m
(c) A.M. radio waves	(iii) 10^{-10} m
(d) X = rays	(iv) 10^{-3} m

(a) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(b) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)
(c) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)
(d) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

ANSWER KEY

Topicwise Questions

1. (*c*) **2.** (*d*) **3.** (*a*) **4.** (*d*) **5.** (*b*) **6.** (*a*) **7.** (*c*) **8.** (*a*)

Learning Plus

1. (*c*) **2.** (*b*) **3.** (*d*) **4.** (*c*) **5.** (*a*) **6.** (*b*) **7.** (*c*) **8.** (*a*) **9.** (*a*) **10.** (*d*)

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*a*) **2.** (*c*) **3.** (*b*) **4.** (*c*) **5.** (*d*) **6.** (*c*) **7.** (*c*) **8.** (*c*) **9.** (*c*) **10.** (*a*)
11. (*c*) **12.** (*d*) **13.** (*c*) **14.** (*b*) **15.** (*d*) **16.** (*a*) **17.** (None of the option) **18.** (*b*)

t.me/realprayasians2