## HIGHER SECONDARY FIRST YEAR REVISION EXAMINATION - JANUARY 2024 PHYSICS KEY ANSWER

## Note:

1. Answers written with Blue or Black ink only to be evaluated.
2. Choose the most suitable answer in Part A, from the given alternatives and write the option code and the corresponding answer.
3. For answers in Part-II, Part-III and Part-IV like reasoning, explanation, narration, description and listing of points, students may write in their own words but without changing the concepts and without skipping any point.
4. In numerical problems, if formula is not written, marks should be given for the remaining correct steps.
5. In graphical representation, physical variables for X -axis and Y -axis should be marked.

## PART - I

Answer all the questions.
$15 \times 1=15$

| Q. <br> No. | OPTION | ANSWER | Q. <br> No. | OPTION | ANSWER |  |
| :---: | :---: | :--- | :---: | :---: | :--- | :---: |
| 1 | (a) | Kg²$^{2}$ | 9 | (a) | PV / KT |  |
| 2 | (b) | Inertia of direction | 10 | (a) | Number of moles and T |  |
| 3 | (c) | $6 \mathrm{~ms}^{-2}$ | 11 | (d) | A straight line |  |
| 4 | (b) | Zero | 12 | (d) | Sin(x+vt) |  |
| 5 | (a) | Only in rotating frames | 13 | (d) | $\left[\mathrm{ML}^{-1} \mathrm{~T}^{\circ}\right]$ |  |
| 6 | (a) | Perihelion and aphelion | 14 | (b) | work |  |
| 7 | (a) | 1 | 15 | (b) | g/2 |  |
| 8 | (b) | Adiabatic |  |  |  |  |

## PART - II

Answer any six questions. Question number 24 is compulsory.

$$
6 \times 2=12
$$

## Principle of homogeneity of dimensions.

The principle of homogeneity of dimensions' states that the dimensions of all the terms in a physical expression should be the same. For example, in the physical expression $\mathbf{v}^{\mathbf{2}}=\mathbf{u}^{\mathbf{2}} \mathbf{+} \mathbf{2 a s}$, the dimensions of $\mathrm{v}^{2}, \mathbf{u}^{2}$
and 2 as are the same and equal to $\left[\mathbf{L}^{\mathbf{2}} \mathbf{T}^{\mathbf{- 2}}\right]$.

|  | Projectile and Examples <br> When an object is thrown in the air with some initial velocity and then <br> allowed to move under the action of gravity alone, the object is known as <br> a projectile. <br> Examples: <br> 1. An object dropped from window of a moving train <br> 2. A bullet fired from a rifle. <br> 3. A ball thrown in any direction. <br> 4. A javelin or shot put thrown by an athlete. <br> 5. A jet of water issuing from a hole near the bottom of a water tank. | 1 |  |
| :--- | :--- | :--- | :--- |
| 18 | Impulse or Impulse Force: <br> If a very large force acts on an object for a very short duration, then the <br> force is called impulsive force or impulse. | 2 | 2 |
|  | No lunar eclipse and solar eclipse every month: <br> Moon's orbit is tilted 50 with respect to Earth's orbit, only during certain <br> periods of the year; the Sun, Earth and Moon align in straight line leading <br> to either lunar eclipse or solar eclipse depending on the alignment | 2 | 2 |
|  | Cohesive and adhesive force: <br> The force between the like molecules which holds the liquid together is <br> called 'cohesive force'. <br> When the liquid is in contact with a solid, the molecules of the these <br> solid and liquid will experience an attractive force which is called <br> 'adhesive force'. | 1 | 1 |


| 23 | Resonance: <br> The frequency of external periodic force (or driving force) matches with the natural frequency of the vibrating body (driven). As a result, the oscillating body begins to vibrate such that its amplitude increases at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations. | 2 | 2 |
| :---: | :---: | :---: | :---: |
| 24 | $\begin{aligned} & \text { Centrifugal force is given by, } \mathrm{F}_{\mathrm{cf}}=\frac{m v^{2}}{r} \\ & =\frac{60 \times 50 \times 50}{10} ;=6 \times 2500 \\ & \mathrm{~F}_{\mathrm{cf}}=15000 \mathrm{~N} \end{aligned}$ | $\begin{gathered} 1 \\ 1 / 2 \\ 1 / 2 \end{gathered}$ | 2 |
| PART - II $6 \times 3=18$ |  |  |  |
| 25 | Applications (uses) of dimensional analysis <br> 1. Convert a physical quantity from one system of units to another. <br> 2. Check the dimensional correctness of a given physical equation. <br> 3. Establish relations among various physical quantities. | $3 \times 1=3$ | 3 |
| 26 | Properties of Dot product or Scalar Product: <br> 1) The product quantity $\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}$ is always a scalar. It is positive if the angle between the vectors is acute (i.e., $<90^{\circ}$ ) and negative if the angle between them is obtuse (i.e. $90^{\circ}<\theta<180^{\circ}$ ). <br> 2) The scalar product is commutative, i.e. $\vec{A} \cdot \vec{B}=\vec{B} \cdot \vec{A}$ <br> 3) The vectors obey distributive law i.e. $\vec{A} \cdot(\vec{B}+\vec{C})=\vec{A} \cdot \vec{B}+\vec{A} \cdot \vec{C}$ <br> 4) The angle between the vectors $\theta=\cos -1\left[\frac{\vec{A} \cdot \vec{B}}{A B}\right]$ <br> 5) The scalar product of two vectors will be maximum when $\operatorname{Cos} \theta=1$, i.e. $\theta=0^{\circ}$, i.e., when the vectors are parallel; $(\vec{A} \cdot \vec{B})_{\max }=A B$ <br> 6)The scalar product of two vectors will be minimum, when $\operatorname{Cos} \theta=-1$, i.e. $\theta=180^{\circ}(\vec{A} \cdot \vec{B})_{\text {min }}=-A B$ when the vectors are anti-parallel. <br> 7) If two vectors $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$, are perpendicular to each other than their scalar Product $\overrightarrow{\mathrm{A}} \cdot \overrightarrow{\mathrm{B}}=0$, because $\operatorname{Cos} 90^{\circ}=0$. Then the vectors $\overrightarrow{\mathrm{A}}$ and $\overrightarrow{\mathrm{B}}$. are said to be mutually orthogonal. <br> 8) The scalar product of a vector with itself is termed as self-dot product and is given by $(\overrightarrow{\mathrm{A}})^{2}=\overrightarrow{\mathrm{A}} \cdot \overrightarrow{\mathrm{A}}=\mathrm{AA} \operatorname{Cos} \theta=\mathrm{A}^{2}$. Here angle $\theta=0^{\circ}$ <br> The magnitude or norm of the vector $\overrightarrow{\mathbf{A}}$ is $\|\overrightarrow{\mathbf{A}}\|=\mathbf{A}=\sqrt{\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{A}}}$ <br> 9) In case of a unit vector $\hat{n}, \hat{n} \cdot \hat{n}=1 \times 1 \times \operatorname{Cos} 0=1$. <br> For example, $\hat{\imath} . \hat{\imath}=\hat{\jmath} . \hat{\jmath}=\hat{k} . \hat{k}=1$ <br> 10) In the case of orthogonal unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}, \hat{\imath} \cdot \hat{\jmath}=\hat{\jmath} . \hat{k}=\hat{k} . \hat{\imath}=1.1$ $\cos 90^{\circ}=0$ <br> 11) In terms of components the scalar product of $\vec{A}$ and $\vec{B}$ can be written As $\vec{A} \cdot \vec{B}=\left(A_{x} \hat{l}+A_{y} \hat{\jmath}+A_{z} \hat{k}\right) \cdot\left(B_{x} \hat{l}+B_{y} \hat{\jmath}+B_{z} \hat{k}\right)=A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z}$ with all other terms zero. The magnitude of vector $\|\vec{A}\|$ is given by $\|\vec{A}\|=A=\sqrt{A_{x}^{2}+A_{y}^{2}+A_{z}^{2}}$ | $\begin{gathered} \text { Any } 6 \\ 6 \times 1 / 2 \\ =3 \end{gathered}$ | 3 |


| 27 | Newton's First Law: <br> i) Every object continues to be in the state of rest or of uniform motion (constant velocity) unless there is external force acting on it. <br> Newton's Second Law: <br> i) The force acting on an object is equal to the rate of change of its momentum $\overrightarrow{\mathrm{F}}=\frac{\mathrm{d} \overrightarrow{\mathrm{p}}}{\mathrm{dt}}$ <br> Newton's Third law: <br> i) Newton's third law assures that the forces occur as equal and opposite pairs. An isolated force or a single force cannot exist in nature. <br> ii) Newton's third law states that for every action there is an equal and opposite reaction. |  | 3 |
| :---: | :---: | :---: | :---: |
| 28 | Elastic Collision ${ }^{\text {a }}$ Inelastic Collision | $\begin{aligned} & \text { Any } 3 \\ & 3 \times 1=3 \end{aligned}$ | 3 |
|  | Total momentum is conserved Total momentum is conserved |  |  |
|  | Total kinetic energy is conserved $\begin{array}{l}\text { Total kinetic energy is not } \\ \text { conserved }\end{array}$ <br> 俍  |  |  |
|  | Forces involved are conservative <br> forces Forces involved are non- <br> conservative Forces |  |  |
|  | Mechanical energy is not <br> dissipated Mechanical energy is dissipated <br> into heat, light, sound etc. |  |  |
| 29 | (1) The presence of any contamination or impurities considerably affects the force of surface tension depending upon the degree of contamination. <br> (2) The presence of dissolved substances can also affect the value of surface tension. For example, a highly soluble substance like sodium chloride ( NaCl ) when dissolved in water $\left(\mathrm{H}_{2} \mathrm{O}\right)$ increases the surface tension of water. But the sparingly soluble substance like phenol or soap solution when mixed in water decreases the surface tension of water. <br> (3) Electrification affects the surface tension. When a liquid is electrified, surface tension decreases. Since external force acts on the liquid surface due to electrification, area of the liquid surface increases which acts against the contraction phenomenon of the surface tension. Hence, it decreases. <br> (4) Temperature plays a very crucial role in altering the surface tension of a liquid. Obviously, the surface tension decreases linearly with the rise of temperature. | $\begin{gathered} \text { Any } 3 \\ 3 \times 1 \\ =3 \end{gathered}$ | 3 |


| Linear expansion of solid: |  |  |
| :--- | :--- | :--- | :--- |
| 30 |  | 1 |


| 33 | $A=\pi r^{2} ;$ | 1 |  |
| :--- | :--- | :--- | :--- |
|  | $=3.14 \times 3.12 \times 3.12 ;=30.57 \mathrm{~m}^{2}$ | 1 | 3 |
|  | A $=30.6 \mathrm{~m}^{2}$ (rounding off with significant figure3 | 1 |  |

## PART - IV

Answer all the questions.

## 34 Measurement of large distances:

(a) For measuring larger distances such as the height of a tree, distance of the Moon or a planet from the Earth, some special methods are adopted. Triangulation method, parallax method and radar method are used to determine very large distances.
Triangulation method for the height of an accessible object:
Let $\mathbf{A B}=\mathbf{h}$ be the height of the tree or tower to be measured.
Let $C$ be the point of observation at distance $x$ from $B$. Place a range finder at $C$ and measure the angle of elevation, $\angle A C B=\theta$ as shown in Figure. From right angled triangle ABC, $\tan \theta=\frac{\mathrm{AB}}{\mathrm{BC}}=\frac{\mathrm{h}}{\mathrm{x}}$ (or) height $\mathrm{h}=x \tan \theta$
Knowing the distance $x$, the height $h$ can be
 determined.

## RADAR method:

The word RADAR stands for Radio Detection and Ranging. Radar can be used to measure accurately the distance of a nearby planet such as Mars. In this method, radio waves are sent from transmitters which, after reflection from the planet, are detected by the receiver.
By measuring, the time interval ( t ) between the instants the radio waves are sent and received, the distance of the planet can be determined as $d=\frac{v \times t}{2}$. where $\mathbf{v}$ is the speed of the radio wave.
As the time taken ( t ) is for the distance covered during the forward and backward path of the radio waves, it is divided by 2 to get the actual distance of the object. This method can also be used to determine the height, at which an aero-

(b) The triangle law of addition.

1) Represent the vectors $\vec{A}$ and $\vec{B}$ by the two adjacent sides of a triangle taken in the same order. Then the resultant is given by the third side of the triangle taken in the opposite order.
2) The head of the first vector $\vec{A}$ is connected to the tail of the second vector $\vec{B}$. Let $\theta$ be the angle between $\vec{A}$ and $\vec{B}$. Then $\vec{R}$ is the resultant vector connecting the tail of the first vector $\overrightarrow{\mathrm{A}}$ to the head of the second vector $\overrightarrow{\mathrm{B}}$.
3) The magnitude of $\vec{R}$ (resultant) is given
 geometrically by the length of $\vec{R}(\mathrm{OQ})$ and the direction of the resultant vector is the angle between
$\overrightarrow{\mathrm{R}}$ and $\overrightarrow{\mathrm{A}}$ Thus we write $\overrightarrow{\mathbf{R}}=\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}} . \because \overrightarrow{\mathbf{0 Q}}=\overrightarrow{\mathbf{O P}}+\overrightarrow{\mathbf{P Q}}$
Magnitude of resultant vector:
4) Consider the triangle $A B N$, which is obtained by extending the side

OA to $O N$. ABN is a right angled triangle.
$\operatorname{Cos} \theta=\frac{\mathrm{AN}}{\mathrm{B}} \therefore \mathrm{AN}=\mathrm{B} \operatorname{Cos} \theta$ and
$\operatorname{Sin} \theta=\frac{B N}{B} \therefore B N=B \operatorname{Sin} \theta$
For $\triangle \mathrm{OBN}$,
we have $\mathrm{OB}^{2}=\mathrm{ON}^{2}+\mathrm{BN}^{2}$

$\Rightarrow R^{2}=(A+B \operatorname{Cos} \theta)^{2}+(B \operatorname{Sin} \theta)^{2}$
$\Rightarrow R^{2}=A^{2}+B^{2} \cos ^{2} \theta+2 A B \cos \theta+B^{2} \sin ^{2} \theta$
$\Rightarrow R^{2}=A^{2}+B^{2}\left(\cos ^{2} \theta+\sin ^{2} \theta\right)+2 A B \cos \theta$
$\Rightarrow R=\sqrt{A^{2}+B^{2}+2 A B \operatorname{Cos} \theta}$
which is the magnitude of the resultant of $A$ and $B$

## Direction of resultant vectors:

5) If $\theta$ is the angle between $\overrightarrow{\mathrm{A}}$ and $\overrightarrow{\mathrm{B}}$, then
$|\vec{A}+\vec{B}|=\sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}+2 \mathrm{AB} \operatorname{Cos} \theta}$
If $\overrightarrow{\mathrm{R}}$ makes an angle $\alpha$ with $\overrightarrow{\mathrm{A}}$, then in $\triangle \mathrm{OBN}, \tan \alpha=\frac{\mathrm{BN}}{\mathrm{ON}}=\frac{\mathrm{BN}}{\mathrm{OA}+\mathrm{AN}}$
$\boldsymbol{\operatorname { t a n }} \alpha=\left(\frac{\mathrm{B} \operatorname{Sin} \theta}{\mathrm{A}+\mathrm{B} \operatorname{Cos} \theta}\right) ; \alpha=\tan ^{-1}\left(\frac{\mathrm{~B} \operatorname{Sin} \theta}{\mathrm{~A}+\mathrm{B} \operatorname{Cos} \theta}\right)$

\begin{tabular}{|c|c|c|}
\hline \[
\begin{array}{|l|}
\hline 35 \\
\text { (a) }
\end{array}
\] \& \begin{tabular}{l}
A particle moving in an Inclined Plane: \\
i) To draw the free body diagram, the block is assumed to be a point mass Since the motion is on the inclined surface, we have to choose the coordinate system parallel to the inclined surface as shown in Figure. \\
ii) The gravitational force mg is resolved in to parallel component \(\mathrm{mg} \sin \theta\) along the inclined plane and perpendicular component \(\mathrm{mg} \cos \theta\) perpendicular to the inclined surface Figure. \\
iii) Note that the angle made by the gravitational force (mg) with the perpendicular to the surface is equal to the angle of inclination \(\theta\) \\
iv) There is no motion (acceleration) along \\
the \(\mathbf{y}\) axis. Applying Newton's second law in the direction \\
\(-m g \cos \theta \hat{\jmath}+N \hat{j}=0\) (No acceleration) \\
By comparing the components on both sides, \(N-m g \cos \theta=0 N=m g \cos \theta\) \\
v) The magnitude of normal force ( N ) \\
exerted by the surface is equivalent to mg \(\cos \theta\). The object slides (with an acceleration) along the x direction. Applying Newton's second law in the \(x\) direction \(m g \sin \theta \hat{\imath}=m a \hat{\imath}\) \\
By comparing the components on both sides, we can equate \(\mathrm{mg} \sin \theta=\mathrm{ma}\). The acceleration of the sliding object is \(\mathrm{a}=\mathrm{g} \sin \theta\) \\
vi) Note that the acceleration depends on the angle of inclination \(\theta\). If the angle \(\theta\) is 90 degrees, the block will move vertically with acceleration a \\
\(=g\). Newton's kinematic equation is used to find the speed of the object when it reaches the bottom. The acceleration is constant throughout the motion. \(v^{2}=u^{2}+2\) as along the \(x\) direction \\
vii) The acceleration a is equal to \(g \sin \theta\). The initial speed \((u)\) is equal to zero as it starts from rest. Here, \(s\) is the length of the inclined surface. \\
The speed ( \(v\) ) when it reaches the bottom is \(v=\sqrt{2 s g \sin \theta}\)
\end{tabular} \& 1
1
1

1
$11 / 2$
$1 / 2$
1 <br>

\hline \[
$$
\begin{aligned}
& \hline 35 \\
& \text { (b) }
\end{aligned}
$$

\] \& | Work - Kinetic Energy Theorem: |
| :--- |
| 1) It states that work done by the force acting on a body is equal to the change produced in the kinetic energy of the body. |
| 2) Consider a body of mass $m$ at rest on a frictionless horizontal surface. |
| 3) The work (W) done by the constant force (F) for a displacement (s) in the same direction is, $\mathrm{W}=\mathrm{Fs}$ $\qquad$ (1) |
| The constant force is given by the equation, $\mathrm{F}=\mathrm{ma}-------$ - (2) |
| The third equation of motion can be written as, $v^{2}=u^{2}+2$ as $\begin{equation*} a=\frac{v^{2}-u^{2}}{2 s}-------\quad \text { (3) } \tag{4} \end{equation*}$ |
| Substituting for a in equation (2), $F=m\left(\frac{v^{2}-u^{2}}{2 s}\right)$ |
| Substituting equation |
| (4) in (1), W $=m\left(\frac{v^{2}}{2 s} s\right)-m\left(\frac{\mathbf{u}^{2}}{2 s} s\right)$ | \& 1

1 <br>
\hline
\end{tabular}

|  | $\begin{equation*} W=1 / 2 m v^{2}-1 / 2 m u^{2} . \tag{5} \end{equation*}$ $\qquad$ <br> The expression for kinetic energy: <br> i) The term $1 / 2\left(\mathrm{mv}^{2}\right)$ in the above equation is the kinetic energy of the body of mass (m) moving with velocity (v). KE $=1 / 2 \mathrm{mv}^{2}$ $\qquad$ (6) <br> ii) Kinetic energy of the body is always positive. <br> From equations (5) and (6) $\Delta K E=1 / 2 m v^{2}-1 / 2 m u^{2} .$ $\qquad$ (7) thus, $\mathrm{W}=\Delta \mathrm{KE}$ <br> iii) The expression on the right hand side (RHS) of equation (7) is the change in kinetic energy ( $\triangle \mathrm{KE}$ ) of the body. <br> iv) This implies that the work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem. | 1 $1 / 2$ $1 / 2$ 1 $1 / 2$ |  |
| :---: | :---: | :---: | :---: |
| $36$ <br> (a) | Moment of Inertia of a Rod: <br> 1) Let us consider a uniform rod of mass (M) and length (I) as shown in Figure. Let us find an expression for moment of inertia of this rod about an axis that passes through the center of mass and perpendicular to the rod. <br> 2) First an origin is to be fixed for the coordinate system so that it coincides with the center of mass, which is also the geometric center of the rod. The rod is now along the $x$ axis. <br> 3) We take an infinitesimally small mass (dm) at a distance ( $x$ ) from the origin. The moment of inertia (dl) of this mass (dm) about the axis is, $\mathrm{dl}=(\mathrm{dm}) \mathrm{x}^{2}$ <br> As the mass is uniformly distributed, the mass per unit length $(\lambda)$ of the rod is, $\lambda=\frac{M}{l}$ <br> The (dm) mass of the infinitesimally small length as, $\mathrm{dm}=\lambda, \mathrm{dx}=\frac{M}{l} \mathrm{dx}$. <br> The moment of inertia (I) of the entire rod can be found by integrating dl , $\begin{aligned} & \mathrm{I}=\int d I=\int(d m) x^{2} \\ & \int\left(\frac{\boldsymbol{M}}{l} \boldsymbol{d} \boldsymbol{x}\right) \boldsymbol{x}^{2} \\ & \mathrm{I}=\frac{M}{l} \int x^{2} d x \end{aligned}$ <br> 4) As the mass is distributed on either side of the origin, the limits for integration are taken from $-\frac{l}{2}$ to $\frac{l}{2}$ $\mathrm{I}=\frac{M}{l} \int_{\frac{-l}{2}}^{\frac{l}{2}} x^{2} d x$ | 1 | 5 |


|  | $\begin{aligned} & =\frac{M}{l}\left[\frac{x^{3}}{3}\right]_{\frac{-l}{2}}^{\frac{l}{2}} \\ \mathrm{I} & =\frac{M}{l}\left[\frac{l^{3}}{24}-\left(-\frac{l^{3}}{24}\right)\right] \quad=\frac{M}{l}\left[\frac{l^{3}}{24}+\frac{l^{3}}{24}\right] \\ \mathrm{I} & =\frac{M}{l}\left[2\left(\frac{l^{3}}{24}\right)\right] \\ \mathrm{I} & =\frac{1}{12} \mathrm{~m} \mathrm{l}^{2} \end{aligned}$ | 1 |  |
| :---: | :---: | :---: | :---: |
| $36$ <br> (b) | Escape speed. <br> 1) Consider an object of mass $M$ on the surface of the Earth. When it is thrown up with an initial speed $\mathbf{v}_{\mathbf{i}}$, the initial total energy of the object is $E_{i}=1 / 2 M V_{i}^{2}-\frac{G M M_{E}}{R_{E}} \ldots-\ldots-1$ <br> Where $\mathrm{M}_{\mathrm{E}}$, is the mass of the Earth and $\mathrm{R}_{\mathrm{E}}$ - the radius of the Earth. <br> The term $-\frac{G M M_{E}}{R_{E}}$ is the potential energy of the mass $M$. <br> 2) When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero [ $\mathrm{U}(\infty)=0$ ] and the kinetic energy becomes zero as well. Therefore, the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape. Otherwise Kinetic energy can be non-zero. <br> $\mathrm{E}_{\mathrm{f}}=0$, According to the law of energy conservation, $\mathrm{E}_{\mathrm{i}}=\mathrm{E}_{\mathrm{f}}-----2$ <br> Substituting (1) in (2) we get, $\begin{aligned} & 1 / 2 M V i^{2}-\frac{G M M_{E}}{R_{E}}=0 \\ & 1 / 2 M V_{i}{ }^{2}=\frac{\text { GMM }_{E}}{R_{E}}- \end{aligned}$ <br> 3) The escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace, $\mathrm{V}_{\mathrm{i}}$ with $\mathrm{V}_{\mathrm{e}}$. i.e, $\begin{aligned} & 1 / 2 \mathrm{MV}_{\mathrm{e}}{ }^{2}=\frac{\mathrm{GMM}_{\mathrm{E}}}{\mathrm{R}_{\mathrm{E}}} \\ & \mathrm{~V}_{\mathrm{e}}^{2}=\frac{\mathrm{GMM}_{\mathrm{E}}}{\mathrm{R}_{\mathrm{E}}} \cdot \frac{2}{M} ; \mathrm{V}_{\mathrm{e}}^{2}=\frac{2 \mathrm{GM}_{\mathrm{E}}}{\mathrm{R}_{\mathrm{E}}}-\ldots \end{aligned}$ <br> Using $g=\frac{\mathrm{GM}_{\mathrm{E}}}{\mathrm{R}_{\mathrm{e}}}---------------5$ $\mathrm{V}_{\mathrm{e}}{ }^{2}=2 \mathrm{gR}_{\mathrm{E}} ; \mathrm{V}_{\mathrm{e}}=\sqrt{2 \mathrm{gRE}} \ldots-\cdots$ <br> From equation (6) the escape speed depends on two factors: acceleration due to gravity and radius of the Earth. It is completely independent of the mass of the object. | 1 | 5 |

## 37(a)

Bernoulli's theorem:
According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, nonviscous fluid in a streamlined flow remains a constant.
$\frac{\mathbf{P}}{\boldsymbol{\rho}}+\frac{\mathbf{1}}{\mathbf{2}} \mathbf{v}^{2}+\mathbf{g h}=$ Constant, this is known as Bernoulli's equation.

## Proof:

Let us consider a flow of liquid through a pipe $A B$ as shown in Figure. Let V be the volume of the liquid when it enters $A$ in a time $t$ which is equal to the volume of the liquid leaving $B$ in the same time. Let $a_{A}, v_{A}$ and PA be the area of cross section of the tube, velocity of the liquid and pressure

exerted by the liquid at $A$ respectively.
Let the force exerted by the liquid at $A$ is $F_{A}=P_{A} a_{A}$
Distance travelled by the liquid in time $t$ is $d=v_{A} t$
Therefore, the work done is $W=F_{A} d=P_{A} a_{A} V_{A} t$
But $\mathbf{a}_{\mathbf{A}} \mathbf{V}_{\mathbf{A}} \mathbf{t}=\mathbf{a}_{\mathbf{A}} \mathbf{d} \mathbf{=} \mathbf{V}$, volume of the liquid entering at A .
Thus, the work done is the pressure energy (at $A$ ), $\mathbf{W}=F_{A} d=P_{A} V$
Pressure energy per unit volume at $A=\frac{\text { Pressure energy }}{\text { Volume }}=\frac{P_{A} V}{V}=P_{A}$
Pressure energy per unit mass at $A=\frac{\text { Pressure energy }}{\text { Mass }}=\frac{P_{A} V}{m}=\frac{P_{A}}{\frac{m}{\mathrm{~V}}}=\frac{P_{A}}{\rho}$
Since $m$ is the mass of the liquid entering at $A$ in a given time, therefore, pressure energy of the liquid at A is $\mathrm{E}_{\mathrm{PA}}=\mathrm{P}_{\mathrm{A}} \mathrm{V}=\mathrm{P}_{\mathrm{A}} \mathrm{V} \times\left(\frac{m}{m}\right)=\mathrm{m} \frac{P_{A}}{\rho}$
Potential energy of the liquid at $\mathrm{A}, \mathrm{P}_{\mathrm{EA}}=\mathrm{mg} \mathrm{h}_{\mathrm{A}}$,
Due to the flow of liquid, the kinetic energy of the liquid at $\mathbf{A}$,
$K E_{A}=1 / 2 m V_{A}{ }^{2}$
Therefore, the total energy due to the flow of liquid at $A$,
$E_{A}=E P_{A}+K E_{A}+P E_{A}$

$$
\mathrm{E}_{\mathrm{A}}=\mathrm{m} \frac{P_{A}}{\rho}+1 / 2 \mathrm{mV}_{\mathrm{A}^{2}}+\mathrm{mgh}_{\mathrm{A}}
$$

Similarly, let $a_{B}, v_{B}$, and $P_{B}$ be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at B . Calculating the total energy at $E_{B}$, we get $\mathbf{E}_{B}=\mathbf{m} \frac{\boldsymbol{P}_{B}}{\boldsymbol{\rho}}+\mathbf{1} / \mathbf{2} \mathbf{m V} \mathbf{V}^{\mathbf{2}}+\mathbf{m g h} \mathbf{m}_{\mathbf{B}}$
From the law of conservation of energy, $E_{A}=E_{B}$
$E_{A}=m \frac{P_{A}}{\rho}+1 / 2 m V_{A} 2+m g h_{A}=E_{B}=m \frac{P_{B}}{\rho}+1 / 2 m V_{B}^{2}+m g h_{B}$
$\frac{P_{A}}{\rho}+1 / 2 V_{A}{ }^{2}+g h_{A}=\frac{P_{B}}{\rho}+1 / 2 V_{B}{ }^{2}+g h_{B}=$ constant
Thus, the above equation can be written as $\frac{\mathbf{P}}{\mathbf{\rho g}}+\frac{\mathbf{1}}{\mathbf{2}} \frac{\mathbf{v}^{\mathbf{2}}}{\mathbf{g}}+\mathbf{h}=\mathbf{c o n s t a n t}$

\begin{tabular}{|c|c|c|}
\hline 37
(b) \& \begin{tabular}{l}
Meyer's relation \\
1) Consider \(\mu\) mole of an ideal gas in a container with volume \(V\), pressure \(P\) and temperature \(T\). \\
2) When the gas is heated at constant volume the temperature increases by dT. As no work is done by the gas, the heat that flows into the system will increase only the internal energy. Let the change in internal energy be dU. \\
If Cv is the molar specific heat capacity at constant volume,
\[
\mathrm{dU}=\boldsymbol{\mu} \mathrm{C}_{\mathbf{v}} \mathrm{dT}
\] \\
3) Suppose the gas is heated at constant pressure so that the temperature increases by \(d T\). If ' \(Q\) ' is the heat supplied in this process and ' \(d V\) ' the change in volume of the gas. \(\mathbf{Q}=\boldsymbol{\mu} \mathbf{C}_{\mathbf{p}} \mathbf{d T}-\ldots-----2\) \\
4) If \(W\) is the work done by the gas in this process, then
\[
\text { W = PdV }--------3
\] \\
But from the first law of thermodynamics, \(\mathbf{Q}=\mathbf{d U}+\mathbf{W}-------4\) \\
Substituting equations (1), (2) and (3) in (4), we get,
\[
\mu C_{p} d T=\mu C_{v} d T+P d V------1
\] \\
5) For mole of ideal gas, the equation of state is given by
\[
P V=\mu R T \Rightarrow P d V+V d P=\mu R d T--------6
\] \\
Since the pressure is constant, \(\mathrm{dP}=0\)
\[
\begin{aligned}
\& \therefore \mathrm{C}_{\mathrm{p}} \mathrm{dT}=\mathrm{C}_{\mathrm{v}} \mathrm{dT}+\mathrm{RdT} \\
\& \therefore \mathrm{C}_{\mathrm{P}}=\mathbf{C}_{\mathrm{v}}+\mathbf{R} \text { (or) } \mathrm{C}_{\mathrm{p}}-\mathbf{C}_{\mathrm{v}}=\mathbf{R}
\end{aligned}
\] \\
This relation is called Meyer's relation
\end{tabular} \& 1
1
1
1
1
1
1 \\
\hline 38
(a) \& \begin{tabular}{l}
Energy in Simple Harmonic Motion: \\
a. Expression for Potential Energy \\
1) For the simple harmonic motion, the force and the displacement are related by Hooke's law \(\overrightarrow{\mathrm{F}}=-\mathrm{k} \overrightarrow{\mathrm{r}}\) \\
2) Since force is a vector quantity, in three dimensions it has three components. Further, the force in the above equation is a conservative force field; such a force can be derived from a scalar function which has only one component. In one dimensional case \\
\(\mathrm{F}=-\mathrm{kx}\) \(\qquad\) (1) \\
The work done by the conservative force field is independent of path. The potential energy \(U\) can be calculated from the following expression.
\[
\mathrm{F}=\frac{d U}{d x}-\ldots--2
\] \\
Comparing (1) and (2), we get \(-\frac{d U}{d x}=-k x ; d U=k x d x\) \\
3) This work done by the force F during a small displacement dx stores as potential energy \(U(\mathrm{x})=\int_{0}^{x} k x^{\prime} d x=\left.\frac{1}{2}\left(x^{\prime}\right)^{2}\right|_{0} ^{x}=1 / 2 k x^{2}-\ldots-3\) \\
From equation \(\omega=\sqrt{\frac{k}{m}}\), we can substitute the value of force constant
\end{tabular} \& \(1 / 2\)

$1 / 2$ <br>
\hline
\end{tabular}

$k=m \omega^{2}$ in equation (3), $U(x)=m \omega^{2} x^{2}$
4) where $\omega$ is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation $x=A \sin \omega t$ $\mathrm{U}(\mathrm{t})=1 / 2 m \omega^{2} \mathrm{~A}^{2} \operatorname{Sin}^{2} \omega \mathrm{t}$ -4
This variation of $U$ is shown below.


## b. Expression for Kinetic Energy

Kinetic energy KE $=1 / 2 \mathrm{mv}_{\mathrm{x}}{ }^{2}=1 / 2 \mathrm{~m}\left(\frac{d x}{d t}\right)^{2}$
Since the particle is executing simple harmonic motion, from equation
$\mathrm{y}=\mathrm{A} \sin \omega t ; x=\mathrm{A} \sin \omega t$ Therefore, velocity is $\mathrm{v}_{\mathrm{x}}=\frac{d x}{d t} \mathrm{~A} \omega \cos \omega \mathrm{t}$

$$
=a \omega \sqrt{1-\left(\frac{x}{A}\right)^{2}} ; v_{x}=\omega \sqrt{A^{2}-x^{2}}
$$

Hence, $K E=1 / 2 m v_{x}{ }^{2}=1 / 2 m \omega^{2}\left(A^{2}-x^{2}\right) \cdots-\cdots$

$$
K E=1 / 2 m \omega^{2} A^{2} \cos ^{2} \omega t-\cdots--------7
$$

This variation with time is shown below.


## c. Expression for Total Energy

Total energy is the sum of kinetic energy and potential energy
$E=K E+U-------\quad ; \quad E=1 / 2 m \omega^{2}\left(A^{2}-x^{2}\right)+1 / 2 m \omega^{2} x^{2}$
Hence, cancelling $x^{2}$ term, $E=1 / 2 m \omega^{2} A^{2}=$ Constant -------9
Alternatively, from equation (4) and equation (7),
we get the total energy as $E=1 / 2 m \omega^{2} A^{2} \operatorname{Sin}^{2} \omega t+1 / 2 m \omega^{2} A^{2} \cos ^{2} \omega t$
$E=1 / 2 m \omega^{2} A^{2}\left(\operatorname{Sin} 2 \omega t+\cos ^{2} \omega t\right)$
From trigonometry identity,
$\left(\operatorname{Sin}^{2} \omega t+\operatorname{Cos}^{2} \omega t\right)=1$
$E=1 / 2 m \omega^{2} A^{2}=$ Constant.
which gives the law of conservation of total energy
This is depicted in Figure. Thus the
 amplitude of simple harmonic oscillator, can be expressed in terms of total energy. $\mathrm{A}=\sqrt{\frac{2 E}{m \omega^{2}}}=\sqrt{\frac{2 E}{k}}$


