Assignment no. 3

due: Monday, May 13th, 2011

Exercise 3.1 The clearance of a robot R moving among obstacles O, when the robot is at configuration q is the minimal distance between a point r in $R(q)$ and a point o in O over all such pairs (r, o). If $R(q)$ and O intersect, the clearance is 0.

(a, warm-up**)** Let R be a robot translating in the plane, and a configuration of R be given by the coordinates of a fixed reference point attached to R; here $R(q)$ is the robot with its reference point placed at q. Prove that if q is a free configuration then the clearance of R at q is equal to the minimal distance from q to the Minkowski sum $O \oplus -R(\textbf{origin})$.

Given a path γ in the configuration space of R, the clearance of R along γ is the minimal clearance of $R(q)$ over all configurations $q \in \gamma$.

(b) Give an algorithm to solve the following problem. We are given a simple polygonal robot R with m vertices translating among polygonal obstacles with a total of n vertices, and a path γ which is a single line segment connecting the start and goal position for R. Find the clearance of R along γ. Analyze the complexity of the algorithm.

Exercise 3.2 We are given a line segment (rod, ladder) translating and rotating among (not necessarily convex) polygonal obstacles in the plane. **(a)** Show that the maximum combinatorial complexity of the free configuration space in this case is $O(n^2)$. To show this bound you have to bound the number of semi-free triple contacts (namely placements of the rod where it touches the obstacles boundaries in three points without penetrating into the obstacles). Hint: Use the result for the single-segment "robot arm" in Exercise 2.6. **(b)** Show that the above bound is tight in the worst case. That is, describe a scene where the complexity of the free space is $\Omega(n^2)$.

Exercise 3.3 (p2) Solve the following motion-planning problem with PRM. The moving system is composed of two simple polygons and the obstacles are a collection of simple polygons in the plane. You can rely on and extend your solution to Exercise 2.2. Notice that in Exercise 2.2 the (auxiliary) moving body is a convex polygon and here each of the moving bodies is a *simple* polygon. (Eventually, in the next assignment, you will be expected to develop an effective collision detector as well as nearest-neighbor search, so you may prefer to no longer use these as black-boxes.)

Remark. In this exercise you are expected to treat the pair of moving polygons as one *composite* robot with four degrees of freedom. A free configuration is one where none of the robots intersects the obstacles nor its fellow robot.

(a) Write a brief description of your planner and its major ingredients.
(b) Present experimental results obtained with your program, depicting instances of varying difficulty for your planner, from easy to hard.
(c) Try two different distance measures (of your choice/design) between a pair of configurations. Design, carry out, and report on experiments that show the effect of the choice of distance measure on the performance of the planner.

Exercise 3.4 (p) (optional, bonus) Implement the algorithm of Exercise 3.1(b).